Investigating the Convergence Characteristics of Harmony Search

Joong Hoon Kim, Ho Min Lee and Do Guen Yoo

Abstract Harmony Search optimization algorithm has become popular in many fields of engineering research and practice during the last decade. This paper introduces three major rules of the algorithm: harmony memory considering (HMC) rule, random selecting (RS) rule, and pitch adjusting (PA) rule, and shows the effect of each rule on the algorithm performance. Application of example benchmark function proves that each rule has its own role in the exploration and exploitation processes of the search. Good balance between the two processes is very important, and the PA rule can be a key factor for the balance if used intelligently.

Keywords Harmony search · Convergence · Exploration and exploitation · Harmony memory considering · Pitch adjustment

1 Introduction

Optimization is the process of selecting the best element from some sets of available alternatives under certain constraints. In each iteration of the optimization process, choosing the values from within an allowed set is done systematically until the minimum or maximum result is reached or when the stopping criterion is met [1]. Meta-heuristic algorithms are well known approximate algorithms which can solve optimization problems with satisfying results [2, 3]. The Harmony Search (HS) algorithm [4, 5] is one of the most recently developed optimization algorithm and at a same time, it is one the most efficient algorithm in the field of combinatorial

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optimization [6]. The HS algorithm can be conceptualized from a musical performance process involving searching for a best harmony. In the HS algorithm, random selecting (RS) rule, harmony memory considering (HMC) rule, and pitch adjusting (PA) rule are used for generation of new solution, and then adopts two parameters of harmony memory considering rate (HMCR) and pitch adjustment rate (PAR), which mean a selection probability of one of the processes. In addition, harmony memory size (HMS) representing the size of memory space (harmony memory, HM) and band width (BW) meaning the adjustment width during the pitch adjustment are used as the parameters. Recently, the HS algorithm's three rules were analyzed by using various parameters for applications of continuous benchmark functions by Ahangaran and Ramesani [7]. In this study, six benchmark functions have varied characteristics (e.g., continuous, discrete and mixed discrete functions) were used for analysis of the effect of each rule on the algorithm performance.

2 Three Rules of Harmony Search Algoritm

2.1 Random Selecting (RS) Rule

In the RS operation, the values of decision variables are generated randomly in the boundary condition with probability of (1-HMCR). The RS rule is one of the exploration (global search) parts in the optimization process. The role of the RS rule is inducement to escaping from local optima for new solution by using sketchy search with whole solution domain for each dicision variable.

2.2 Harmony Memory Considering (HMC) Rule

The HMC rule selects the solution value for each decision variable from the memory space (HM) of the HS algorithm. The probability of selecting HMC rule is HMCR and it can have a value between 0 and 1. In general, in the cases with between 0.70 and 0.95 of HMCR produce good results. The HMC rule is exploitation (local search) part in the optimization process of the HS algorithm.

2.3 Pitch Adjusting (PA) Rule

After finish the HMC operation, the PA operation can be selected with probability of PAR. In the PA operation, a selected solution value of decision variable from HMC operation is adjusted with upper or lower value. The parameter PAR can have a value between 0 and 1 and it is usually set between 0.01 and 0.30. The PA rule has composite role in the HS algorithm. It is an exploration part for escaping from local optima, and it is also an exploitation part in the optimization process for finding exact optimal point by using fine tuning of decision variables.

3 Methodology

In this study, the HS algorithm with various parameter combinations was applied for solving six unconstrained benchmark functions widely examined in the literature (two continuous benchmark functions, two descrete bechmark functions, and two mixed descrete benchmark functions). The optimization task was carried out using 30 individual runs for problems.

Table 1 Benchmark Functions (BFs)

BF 1 (continuous) : Six-hump camel back function
Minimize $f(x) = \left(4 - 2.1x_1^2 + x_1^4 / 3\right) x_1^2 + x_1 x_2 + (-4 + 4x_2^2) x_2^2$ $-3 \le x_1 \le 3, -2 \le x_2 \le 2$, min $f(x) = -1.0316$
BF 2 (continuous) : Goldstein price's function
Minimize $f(x) = \left[1 + (x_1 + x_2 + 1)^2 \left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6 + x_1 x_2 + 3x_2^2\right)\right] \times \left[30 + (2x_1 - 3x_2)^2 \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1 x_2 + 27x_2^2\right)\right]$ $-2 \le x_i \le 2, i \in \{1, 2\}$, min $f(x) = 3$
BF 3 (discrete) : Gear function
Minimize $f(x) = \left(\frac{1}{6.931} - \frac{x_1 x_2}{x_3 x_4}\right)$ $12 \le x_i \le 60$ (integer variables), $i \in \{1, 2, 3, 4\}$, min $f(x) = 0$
BF 4 (discrete) : Simpleton-25 function
Minimize $f(x) = -\sum_{i=1}^{n} x_i$ $0 \le x_i \le 10$ (integer variables), $i \in \{1, 2, \cdots, n\}$, $n = 25$, min $f(x) = -250$
BF 5 (mixed discrete) : Mixed Griewank function
Minimize $f(x) = (1/4000)\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos(x_i / \sqrt{i}) + 1$ $-600 \le x_i \le 600$ (continuous variables), $i \in \{1, 2, 3, 4\}$, $-600 \le x_i \le 600$ (integer variables), $i \in \{5, 6, 7, 8\}$, $n = 8$, min $f(x) = 0$
BF 6 (mixed discrete) : Mixed Ackley function
Minimize $f(x) = -a \exp\left(-b\sqrt{(1/n)\sum_{i=1}^{n} x_i^2}\right) - \exp\left((1/n)\sum_{i=1}^{n} \cos(cx_i)\right) + a + \exp(1)$ $-32 \le x_i \le 32$ (continuous variables), $i \in \{1, 2, 3, 4\}$, $-32 \le x_i \le 32$ (integer variables), $i \in \{5, 6, 7, 8\}$, $a = 20, b = 0.2, c = 2\pi, n = 8$, min $f(x) = 0$

Table 2 Applied Parameters

<table>
<tr><th>Cases</th><th>HMS</th><th>HMCR</th><th>PAR</th><th>BW</th><th>NFEs</th></tr>
<tr><td>Case 1</td><td rowspan="10">10
(for BFs 1-3),

30
(for BFs 4-6)</td><td>1.0</td><td rowspan="5">0.2</td><td rowspan="10">0.01</td><td rowspan="10">20,000</td></tr>
<tr><td>Case 2</td><td>0.8</td></tr>
<tr><td>Case 3</td><td>0.5</td></tr>
<tr><td>Case 4</td><td>0.2</td></tr>
<tr><td>Case 5</td><td>0.0</td></tr>
<tr><td>Case 6</td><td rowspan="5">0.8</td><td>1.0</td></tr>
<tr><td>Case 7</td><td>0.8</td></tr>
<tr><td>Case 8</td><td>0.5</td></tr>
<tr><td>Case 9</td><td>0.2</td></tr>
<tr><td>Case 10</td><td>0.0</td></tr>
</table>

Tables 1 and 2 show the definitions and specifications of benchmark functions and applied parameter combinations of HS algorithm in this study respectively. In this case study, HMS of 10 and 30 were applied to bechmark functions respectively in consideration of the number of decision variables in each function. HMCR and PAR were applied differently in each case as shown in Table 3. The total number of function evaluations (NFEs) was fixed value 20,000 and also BW is fixed value 0.01.

Case 1 has HMC and PA rules, Cases 2-4, 7-9 have RS, HMC and PA rulse in accordance with HMCR and PAR, Case 5 only has RS rule, Case 6 has RS and PA rules, and Case 9 has RS and HMC rules respectively. Cases 2 and 9 are same case as a default parameter combination for the comparison criterion of Cases 1-5 and Cases 6-9 respectively.

4 Results and Discussions

Table 3 and Figures 1-3 show the analysis results from appications of HS algorithm with various combinations of parameters for becnmark functions (Figure 1 for BFs 1, 2, Figure 2 for BFs 3, 4, and Figure 3 for BFs 5, 6).

Table 3 Analysis Results Comparison

Cases	BF 1	BF 2	BF 3	BF 4	BF 5	BF 6
	Avg. error	Avg. error	Avg. error	Avg. error	Avg. error	Avg. error
Case 1	1.09E-01	1.67E+01	9.19E-05	1.67E+00	1.10E+01	1.03E+01
Case 2	0.00E+00	7.88E-06	2.36E-05	0.00E+00	2.77E-01	2.18E-01
Case 3	0.00E+00	4.33E-05	3.03E-05	2.52E+01	2.34E+00	5.31E+00
Case 4	4.53E-06	6.73E-04	4.19E-05	5.55E+01	1.60E+01	1.34E+01
Case 5	3.23E-01	1.76E+01	6.71E-05	7.47E+01	9.38E+01	1.84E+01
Case 6	6.33E-07	2.76E-04	5.43E-05	1.29E+01	7.77E-01	2.80E+00
Case 7	0.00E+00	9.43E-06	1.85E-05	7.57E+00	7.47E-01	2.82E+00
Case 8	0.00E+00	4.76E-06	1.74E-05	9.60E+00	4.97E-01	1.64E+00
Case 9	0.00E+00	7.88E-06	2.36E-05	0.00E+00	2.77E-01	2.18E-01
Case 10	1.95E-04	7.71E-03	1.31E-04	8.67E-01	4.04E-01	1.22E+00

In most benchmark functions, the combined cases with three rules of the HS algorithm (Cases 2-4, 7-9) showed better efficiency than combined cases with two rules (Cases 1, 6 and 10) and cases with one rule (Cass 5). This results mean the importance of each rule in the HS algorithm and each rule has own role in the optimization process of HS algorithm. Meanwhile, the cases with the value of HMCR above 0.5 and the cases with the value of PAR below 0.5 showed better results of average error stably than other cases.

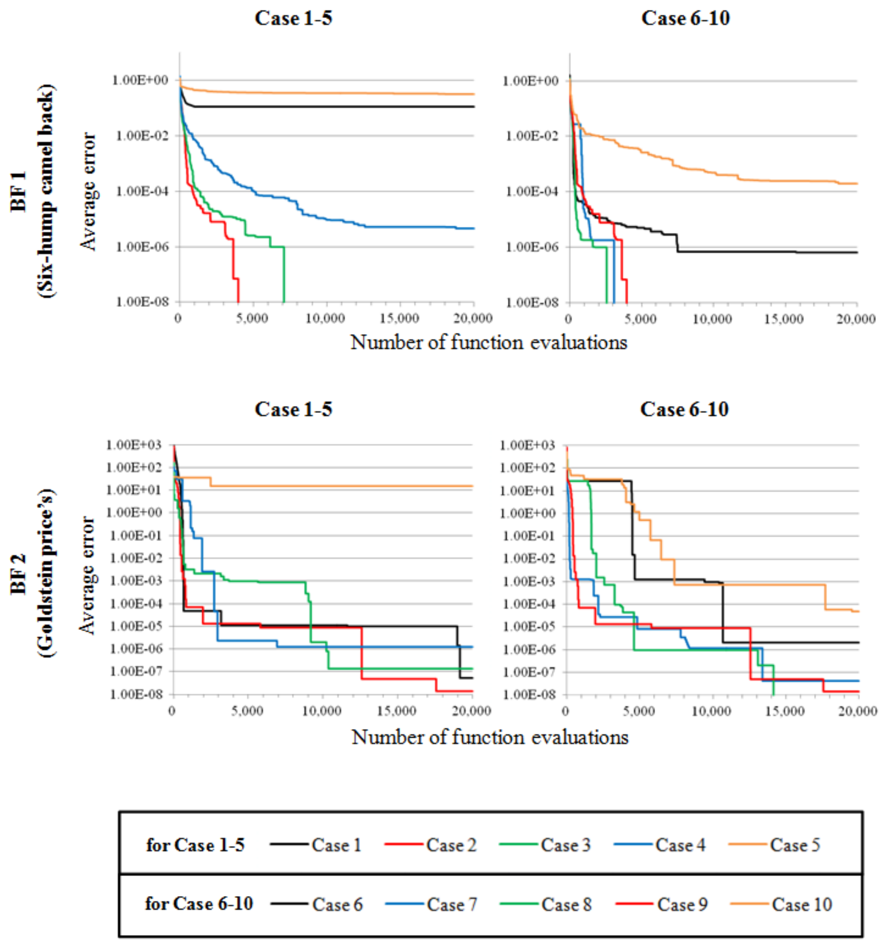

Fig. 1 Average Error Results for BFs 1, 2 (continuous functions)

Average error results of benchmark functions with parameter combination Case 1, only includes RS rule, showed the effect of the RS rule in early stages is far more than the final iterations. Therfore, we can conclude that in early stages of optimization process RS and PA rules work together as an exploration part, and during optimization progresses the influences of HMC and PA rules are increased gradually for exploitation.

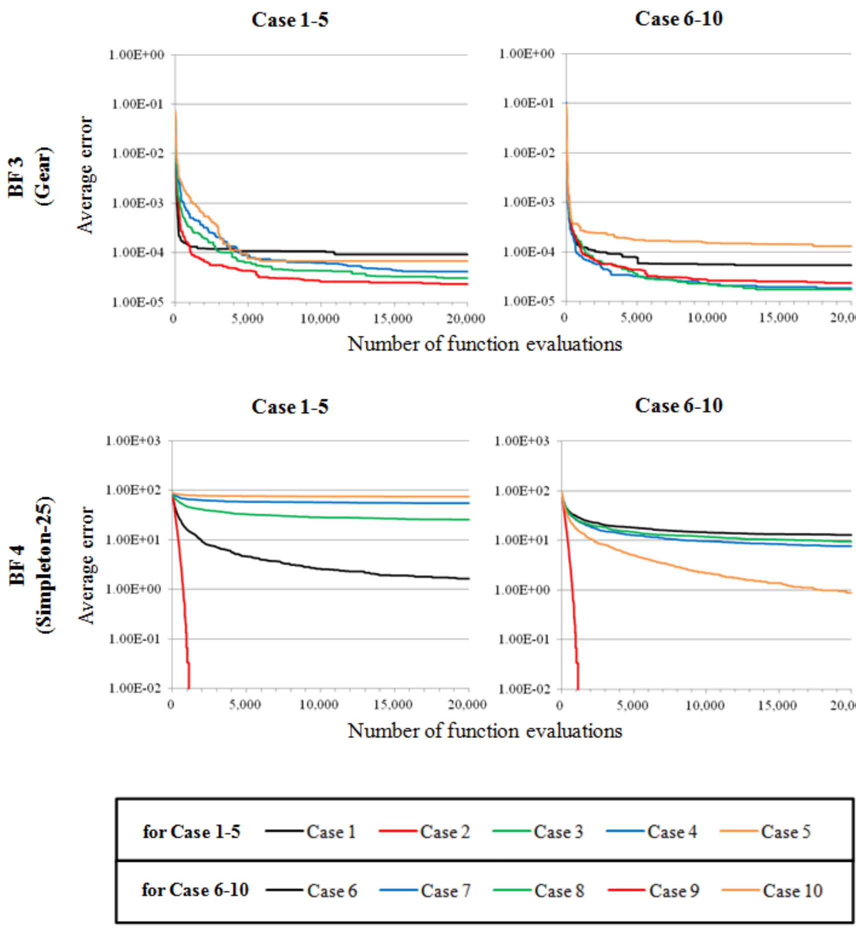

Fig. 2 Average Error Results for BFs 3, 4 (discrete functions)

Meanwhile, for the optimization results of BF 4 (Figure 2), Case 1 without RS rule and Case 10 without HMC rule produced second ranking results among Cases 1-5 and Cases 6-10 respectivley. The reason is characteristics of BF 4. This function has 25 dicision variables, more dicision variables than other benchmark functions, however BF 4 does not include local optima. Moreover BF 4 is discrete problem with 10 possible solutions for each dicision variable. Therefor we should consider the problem characteristics in the optimization when we apply optimization algorithms for particular problem.

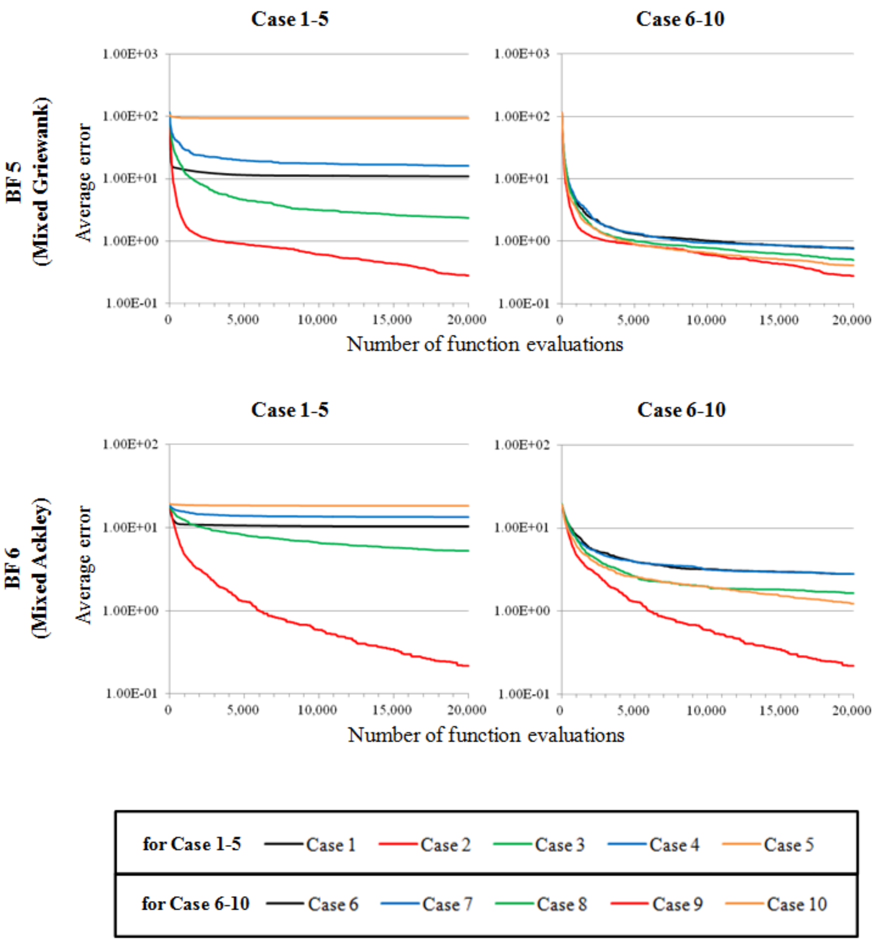

Fig. 3 Average Error Results for BFs 5, 6 (mixed discrete functions)

In this study the varied combinations of parameters HMCR and PAR were applied to benchmark functions to compare the convergence characteristics. However, the HS algorithm includes two more parameters HMS and BW, also important parameters in optimization process. Therfore the anlysis of results by considering various combinations of HMS and BW should be studied on our future research.

5 Conclusion

Optimization is the process of selecting the best solution among available alternatives under certain constraints. Meta-heuristic algorithms are well known approximate algorithms which can solve optimization problems with satisfying results and have own rules for finding best solution. The Harmony Search (HS) algorithm is one of the most recently developed optimization algorithm, and it has three rules in the optimization. The optimization process of HS algorithm includes three operation rules, harmony memory considering (HMC) rule, random selecting (RS) rule, and pitch adjusting (PA) rule. In this study, six benchmark functions have varied characteristics were selected for analysis of the effect of each rule on the algorithm performance.

Applications of benchmark functions prove that each rule has its own role in the exploration and exploitation processes of the optimization. In addition, the selection of suitable parameter combination with considering characteristics of object problem is essential for using optimization algorithms. In early stages of optimization process RS and PA rules have a leading role for exploration, and as optimization progresses the roles of HMC and PA rules are important for exploitation. Good balance between exploration and exploitation is very important for every optimization algorithm, and the intelligent use of PA rule can be a key factor for the balance in the HS algorithm.

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