Chapter 3
Application Study of a Phase-Optimized Constant-Envelope Transmission (POCET) Optimization Algorithm for BDS B1 Signal

Minggui Cai, Jun Xie and Gang Wang

Abstract Firstly we describe an improved POCET algorithm and design a multiplexing plan for B1 signals using it. At B1 frequency, binary complex subcarrier is chosen to implement the center frequency difference between regional and global navigation systems. Regional BPSK(2) OS signal and global TMBOC signal are combined and the best multiplex efficiency is −1.010 dB. Regional BPSK(2) AS signal and global BOC signal are combined and the best multiplex efficiency is −1.593 dB. And then, we analyze the signal in band-limited systems from three aspects: multiplex efficiency, constellation diagram and signal's peak to average power ration (PAPR). And then, we analyze the multiplex signal in band-limited system from three aspects: multiplex efficiency, constellation diagram and signal's peak to average power ration (PAPR). The ideal constant-envelope signal has become nonconstant-envelope of which constellation has changed and PAPR has increased. Also the multiplex efficiency has decreased, and it has decreased more after 30 MHz filter by almost 0.55 dB. The results of this paper provide the constant-envelope multiplex scheme at B1 frequency and the multiplex efficiency, and the analysis in band-limited system which is of significance in engineering practice.

Keywords Constant-envelope multiplex · BDS · POCET · Multiplex efficiency · Band limit

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J. Sun et al. (eds.), *China Satellite Navigation Conference (CSNC) 2015 Proceedings: Volume II*, Lecture Notes in Electrical Engineering 341,
DOI 10.1007/978-3-662-46635-3_3

3.1 Introduction

With the continuous development of GNSS (Global Navigation Satellite Systems), the number of service signals is increasing fast [1]. But the navigation frequency resource is limited, so the multichannel ranging code should be modulated compositely on the limited carriers in order to deal with GNSS signal spectrum congestion. POCET (Phase-Optimized Constant-Envelope Transmission) is a kind of phase modulation technique. It computes the phase and amplitude of the modulation signal to maximize the multiplex efficiency by optimization algorithm on the premise of the power and phase relationship between the signals [2]. The multiplex efficiency of POCET is high and the implementation is quite simple. The GPS signals using POCET has been verified in semi-physical simulation [3], and the Aerospace has been studying the POCET modulation for multi-frequency signals [4].

3.2 POCET Algorithm

3.2.1 Theory of POCET

The multiplex efficiency of N-signals multiplex can be defined as the ration between total power of all sub signals to power of compound signal:

$$\eta = \frac{\sum_{n=1}^{N} |\mathrm{corr}_n|^2}{A^2} \tag{3.1}$$

where $P_{dn} = |\mathrm{corr}_n|^2$ is the power of nth sub signal from correlation receiver, and A^2 is the power of compound signal with A is the amplitude, and corr_n is the correlation of the nth sub signal:

$$\mathrm{corr}_n = \frac{A}{2^N} \sum_{k=0}^{2^N-1} [1 - 2b_n(k)]\exp(j\theta_k) \tag{3.2}$$

where $b_n(k)$ is the binary code of nth signal, and θ_k is the phase corresponding to the kth codes group.

POCET is an optimization problem with the constraints of the phase and power relationship between sub signals, and usually penalty function is used to change this constrained optimization into an unconstrained one.

3.2.2 Constrained Optimization Algorithm

An improved algorithm without amplitude A is given in 5, but it still uses the penalty function for optimization. In fact, the penalty factors will affect the solutions when using penalty function. But there is no simple liner relationship between them and the factors will affect each other. So to find the optimal solution, the factors should be continuously readjusted according to the last solutions which increases the optimization time.

Therefore we provide a constrained optimization algorithm. Firstly normalized correlation $corr_n$ and expected power p_n is defined as:

$$corr_n = \frac{1}{2^N}\sum_{k=0}^{2^N-1}[1-2b_n(k)]e^{j\theta_k} \tag{3.3}$$

$$p_n = \eta\frac{(corrd_n)^2}{\sum_{i=1}^{N}(corrd_n)^2} \tag{3.4}$$

where $(corrd_n)^2$ is the expected power of the nth sub signal, and $\eta = \sum_{n=1}^{N}|corr_n|^2$, that is the multiplex efficiency.

The power error and phase error can be written as:

$$\begin{aligned}\Delta P &= \sum_{n}^{N}\left||corr_n|^2 - p_n\right| \\ \Delta\varphi &= \sum_{m=1}^{N}\sum_{l=m+1}^{N}\left|imag(e^{j\varphi_{ml}}\frac{corr_m corr_l^*}{|corr_m||corr_l|})\right|\end{aligned} \tag{3.5}$$

Then the optimization problem can be written as:

$$\min f(\vec{\theta}) = 1-\eta, \quad s.t.\begin{cases}\Delta P \le \delta_1 \\ \Delta\varphi \le \delta_2\end{cases} \tag{3.6}$$

where δ_1 and δ_2 is the maximum power and phase error which can be set according to the practical requirements. The complex method is used to search the optimal solution.

3.3 Multiplex Design for B1 Signals

For BDS B1 frequency band, there is OS (Open Service) signal using MBOC (1,6,1/11) modulation and AS (Authorized Service) signal using BOC(14,2) modulation [5]. We choose TMBOC modulation for OS signals, and set the power ratio of pilot signal to data signal as 3:1.

3.3.1 Smooth Transition for B1 Signals

The complex subcarrier shown as follows is taken to modulate the QPSK(2) at L1 frequency band (1575.42 MHz), to move down the center frequency:

$$\begin{aligned} SC_{\cos}(t) &= \text{sign}(\cos 2\pi f_{sc} t) \\ SC_{\sin}(t) &= \text{sign}(\sin 2\pi f_{sc} t) \\ SC(t) &= SC_{\sin}(t) + jSC_{\cos}(t) \end{aligned} \tag{3.7}$$

where $f_{sc} = 14.322\,\text{MHz}$.

3.3.2 POCET Phase Solution

3.3.2.1 Multiplex of Four B1 OS Signals

The power ratio between $S_{B1I\_I}(t)$, $S_{B1I\_Q}(t)$, $S_{B1C\_p}(t)$ and $S_{B1C\_d}(t)$ is set as 1:1:1.717:0.572. And there is 90° phase difference between $S_{B1I\_I}(t)$ and $S_{B1I\_Q}(t)$, and 0° phase difference between $S_{B1C\_p}(t)$ and $S_{B1C\_d}(t)$. The phase is worked out based on Eq. (3.6) with $\delta_1 = \delta_2 = 10^{-3}$. The optimized phase is shown in Table 3.1, and because of the symmetry of the codes value, the rest phases have 180° phase difference between the ones shown.

The multiplex efficiency is −1.010 dB, and PAPR is 1 which means that the compound signal is ideally constant-envelope. The constellation of the compound signal is shown in Fig. 3.1 from which we can visually denote all the phase points locate on the unit circle. The power error and phase error of the optimal phase solution is 10^{-3} and 10^{-7}.

3.3.2.2 Multiplex of Three B1 AS Signals

The power ratio between $S_{B1Q\_I}(t)$, $S_{B1Q\_Q}(t)$ and $S_{B1A}(t)$ is set as 1:1:2. And there is 90° phase difference between $S_{B1Q\_I}(t)$ and $S_{B1Q\_Q}(t)$.

Table 3.1 Phase of 4-signals multiplex

SB1C_d	SB1I_I	SB1I_Q	SB1C_p	θ_k(°)
0	0	0	0	0
0	0	0	1	−55.4
0	0	1	0	112.2
0	0	1	1	−23.1
0	1	0	0	242.1
0	1	0	1	291.7
0	1	1	0	127.2
0	1	1	1	−100.1

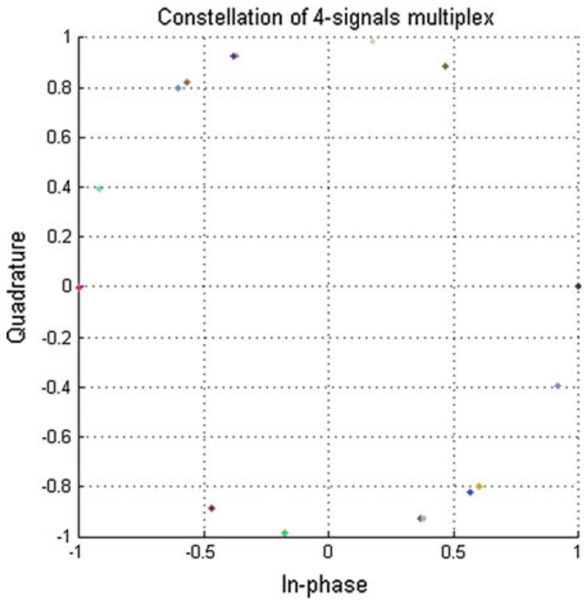

Fig. 3.1 Constellation of 4-signals multiplex

The phase is worked out based on Eq. (3.6) with $\delta_1 = \delta_2 = 10^{-3}$. The optimized phase is shown in Table 3.2.

The multiplex efficiency is −1.593 dB, and PAPR is 1. The constellation is shown in Fig. 3.2.

3.4 Simulation and Analysis in Band-Limited System

There has been much research about constant-envelope multiplex techniques for navigation signals. For example, the BDS B1 4-codes multiplex plan proposed by Zhang Kai using optimized POCET [6] and the BDS B1 4-codes multiplex plan proposed by Zhu Liang et al. using asymmetric AltBOC [7], both of which multiplex efficiency reaches −1 dB. But the bandwidth limit in real satellite navigation

Table 3.2 Phase of 3-signals multiplex

SB1Q_I	SB1Q_Q	SB1A	$\theta_k(°)$
0	0	0	0
0	0	1	−226.1
0	1	0	113.4
0	1	1	−193.7

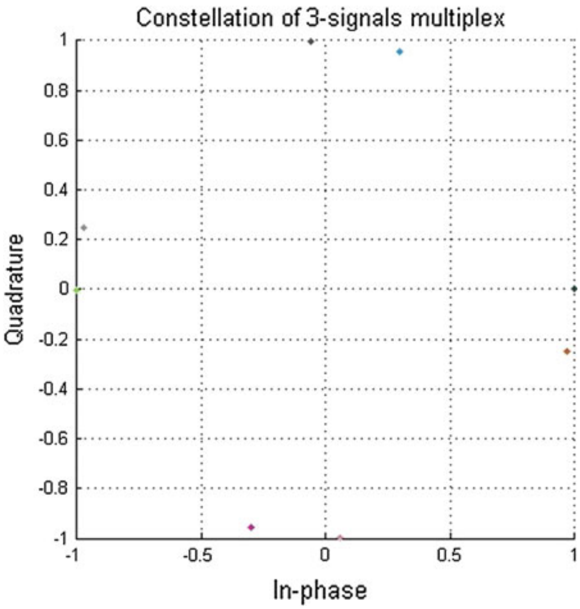

Fig. 3.2 Constellation of 3-signals multiplex

systems was not considered in all these multiplex plans. So in this chapter the multiplex performance in band-limited system is further simulated and analyzed.

According to Eq. (3.1), the multiplex efficiency in band-limited system can be written as:

$$\eta = \frac{\sum_{n=1}^{N} |\mathrm{corr}_{\mathrm{real\_n}}|^2}{\bar{A}^2} \tag{3.8}$$

where $\bar{A}^2$ is the compound signal's power, and $\mathrm{corr}_{\mathrm{real\_n}}$ is the correlation of the compound signal and the nth sub signal:

$$\mathrm{corr}_{\mathrm{real\_n}} = \frac{\sum_{k=0}^{N_s-1} s(k)b_n(k)}{N_s} \tag{3.9}$$

where s(k) is the compound signal, N_s is the length of the signal, and $b_n(k)$ is the nth ideal sub signal.

3.4.1 Baseband Equivalent Model

The baseband equivalent model for simulation is shown in Fig. 3.3 [8]. The filter is 8th order low-pass elliptic filter, and the HPA (high-power amplifier) adopts normalized Saleh model [9]. Two groups of band-limited signals with 30 MHz bandwidth and 60 MHz bandwidth are simulated separately for comparison.

3.4.2 Analysis of Multiplex Performance

We focus on the multiplex of four B1 OS signals with band limit. The constellation of the compound signal after pre-filter is shown as Figs. 3.4 and 3.5.

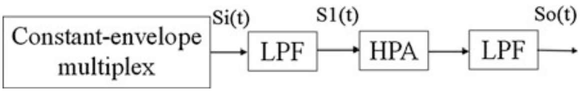

Fig. 3.3 Baseband equivalent model

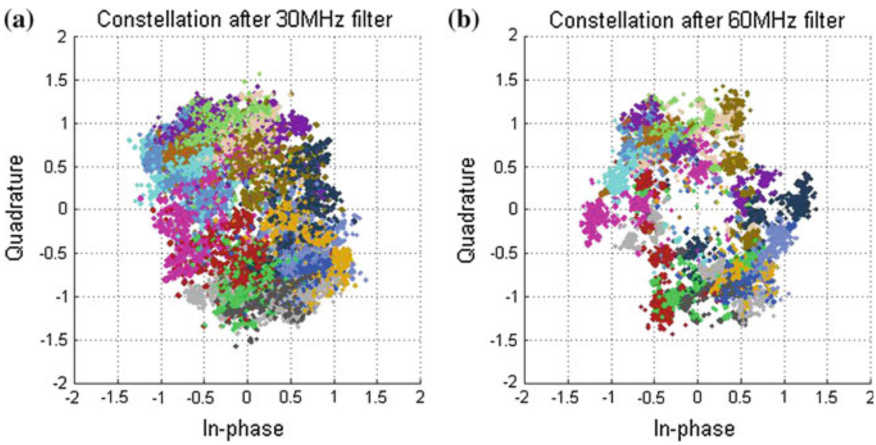

Fig. 3.4 Contrast of constellation after pre-filter

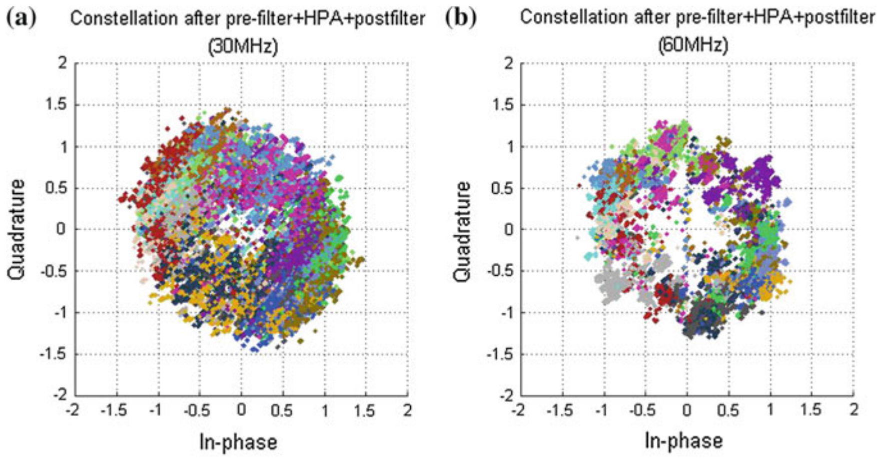

Fig. 3.5 Contrast of constellation after pre-filter, HPA and post-filter

We can see that band limit will break the constant-envelope feature: the amplitude is not constant anymore and the phase points spread. Comparing Fig. 3.4a with b, we can see that the deformation of the signal with 30 MHz bandwidth limit is severer, and overlapping region of phase points is lager. Also Quadrature branch varies more than In-phase branch for 30 MHz band-limited signal, while the fluctuations of both branches are basically the same for the 60 MHz one. Multiplex efficiency is −1.430 dB after 30 MHz bandwidth filter, 0.4 dB less than in ideal conditions, which is an obvious loss. And PAPR of it is 2.8386, which shows the signal is no longer constant-envelope and the amplitude range is relatively wide. For 60 MHz band-limited signal, multiplex efficiency is −1.234 dB, 0.2 dB less than in ideal conditions. PAPR of it is 2.3947.

The constellation of the compound signal after pre-filter is shown as Fig. 3.4.

Comparing with the constellation only after pre-filter, we can see that deformation of signal becomes severer, and overlapping region of phase points becomes lager. Especially with the phase characteristic of HPA, the constellation has rotated. With 30 MHz bandwidth limit, the multiplex efficiency turns into −1.542, 0.55 dB less than in ideal conditions and 0.1 dB less than only after pre-filter. And PAPR is 2.4007. For 60 MHz band-limited signal, the multiplex efficiency turns into −1.334, 0.3 dB less than in ideal conditions and 0.1 dB less than only after pre-filter. And PAPR is 1.9544. Although the PAPR of both signals has decreased a little, we can see from the constellation obviously that phase points spread much more seriously and overlapping region is lager, especially for 30 MHz band-limited signal, the phase points in close proximity mostly overlap.

3.5 Conclusions

POCET computes the phase of the modulation signal on the premise of the power and phase relationship between the signals to maximize the multiplex efficiency, so it can reach the highest efficiency in theory. In traditional POCET, penalty function is used to change constrained optimization into unconstrained optimization. But finding appropriate penalty factors will cause an extra increase in solving iterations which takes much time. While the constrained optimization algorithm presented in this article can simplify resolution process and reduce iterations. Besides, the form of setting power and phase errors is more suitable for practical use.

We provide a constant-envelope multiplex plan for B1 signals using the improved POCET algorithm: multiplex efficiency of four OS signals is −1.010 dB and of three AS signals is −1.593 dB. And according to the simulation results of 4-signals multiplex, the given power and phase constrains can be satisfied perfectly using the improved POCET. The improved algorithm greatly simplifies the process of searching for optimal solution.

At present, there have been lots of studies of constant-envelope multiplex techniques of satellite navigation signal, but the bandwidth limit in navigation systems is not considered in all of them. Thus we simulate the multiplex of four B1 OS signals in band-limited system. With bandwidth limit, the constant-envelope signal becomes nonconstant and multiplex efficiency decreases: multiplex efficiency decreases to −1.542 dB in 30 MHz band-limited system and −1.334 dB in 60 MHz system. The influence of band limit is quite obvious, thus analysis of the multiplex performance in band-limited system is of more significance in engineering practice.

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