Chapter 17
Zenith Tropospheric Delay Modeling Method for Sparse Reference Station Network Considering Height Difference

Yang Yang, Guorong Yu, Shuguo Pan, Wang Gao and Weirong Chen

Abstract A new method of zenith tropospheric delay modeling (HTIM) for sparse reference station network is put forward in this paper. The zenith tropospheric hydrostatic delay (ZHD) is calculated by the UNB3m model. In addition, the zenith tropospheric wet delay (ZWD) with high precision is estimated by conventional precise point positioning (PPP) using ionosphere-free combination. Considering the relationship between height factor and zenith tropospheric delay (ZTD) in UNB3m model, the zenith tropospheric delay is divided into two parts, the height weakly correlated and the height strongly correlated. The new method of zenith tropospheric delay modeling is established using the two parts of the zenith tropospheric delay. Based on the results of regional modeling, the zenith tropospheric delay of rover station is augmented. The new method is compared with conventional modeling methods. The results show that the new zenith tropospheric delay modeling method is superior to other methods in the undulated region only using four reference stations, RMS is 0.020 m and the absolute deviation is nearly within 0.03 m. It also shows that the RMS is 0.008 m and the rate of absolute deviation within 0.01 m reaches 75.95 % in the sparse reference station network. In the new method, the interpolation precision is not limited by the number of reference stations and three reference stations also can be used to get the interpolation results with high reliability. The results show that the new zenith tropospheric delay modeling method is superior to conventional methods, especially for the undulated region in a sparse reference station network.

Keywords Sparse reference station · Zenith tropospheric delay · Height difference · PPP

Y. Yang (✉) · G. Yu · W. Gao · W. Chen
School of Transportation, Southeast University, Sipailou 2#,
Nanjing 210096, Jiangsu, China
e-mail: johnny2014@sina.com

S. Pan
School of Instrument Science and Engineering, Southeast University, Nanjing, China

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17.1 Introduction

Using the GNSS (Global Navigation Satellite System) observations, post-processed satellite ephemeris and satellite clock offset with high precision, precise point position (PPP) technology can get the positioning solution [1, 2]. Although the precision of satellite orbits and the satellite clock offset are high, it still costs PPP users at least 30 min to converge the positioning results within 10 cm [3, 4]. Because of the restriction of environmental factors, such as atmospheric delay, the promotion and application of PPP technology are greatly limited.

Through researches, domestic and foreign researchers indicate that the convergence speed of PPP can be significantly improved with the prior information of atmospheric delay [3, 5]. The real-time atmospheric delay with high precision of reference stations can be calculated with the known exact coordinates and the successive observations. PPP users in the region can get the prior information of atmospheric delay to greatly shorten the initialization time and improve the precision of positioning by interpolation modeling of the prior information [6].

More researches of conventional interpolation models are studied in network real time kinematic (NRTK), such as Linear Interpolation Model (LIM) [7], Linear Combination Model (LCM) [8] and considering Height Linear Interpolation Model (HLIM), etc. These models are put forward in connection with the double difference error correction in NRTK on the whole.

Chen compared the differences between conventional PPP methods and improved PPP method which is based the enhancement of continuously operating reference stations (CORS) network. The results showed that the improved PPP method is equivalent to conventional methods in precision, while the convergence time is shortened by 30 % [9]. But the precision of interpolation is limited by the number of reference stations and the distances between reference stations and the rover station. Li puts forward a method to enhance the PPP with the contribution of regional CORS network [5, 10], which is gradually developed into the popular PPP-RTK technology [11] nowadays. He also proposed the Modified Linear Combination Model (MLCM) [5] to interpolate the tropospheric delay. Based on the analysis of the relationship between the height factor and the zenith troposphere delay of different stations, Zhang presented six spatial regression models. The results showed that the H1QX1 model and H1QM3 model are superior to other models. Besides, interpolation precision is 10 mm in the flat region and 20 mm in the undulated region [12]. But his methods are still based on regional CORS and had certain requirements on the number of reference stations and spatial distances. The methods mentioned above are might not suitable for the remote areas which are augmented by a sparse reference station network.

In this paper, a new method of zenith tropospheric delay modeling for sparse reference station network is put forward based on the research of the relation between height difference and zenith tropospheric delay. According to the relation between height factor and zenith tropospheric hydrostatic delay, zenith tropospheric wet delay in the UNB3m model [13], the zenith tropospheric delay of reference station is divided into two parts, the height weakly correlated and the height strongly correlated. The new method of zenith tropospheric delay modeling is established using the two parts of the zenith tropospheric delay. Based on the results of regional modeling, the zenith tropospheric delay of rover station is augmented. In sparse reference station region, the applicability and feasibility of the new method are analyzed by experiments.

17.2 Relation Between Height and ZTD

For a certain region, the troposphere in height direction is vertically mixed and the troposphere of horizontal direction is unevenly distributed, which is due to the meteorological characteristics of the troposphere. The 24 h observations of 513 stations of the US CORS on August 8th of 2013 are used to calculate the daily average ZTD by PPP using ionosphere-free combination. The ZTD of different stations are all based on the results after the convergence of ambiguity. The distribution of stations is shown in Fig. 17.1. The relation between height and ZTD of different stations is calculated in Fig. 17.2. It is obvious that ZTD has strong correlation with height. So, the impact of height must be considered when the regional tropospheric delay is modeled by interpolation.

When the ZTD is calculated by PPP using ionosphere-free combination, the ZHD is calculated by UNB3m model and the ZWD is obtained by parameter estimation. The UNB3m model is defined as follows:

$$ZHD(H) = \frac{10^{-6}K_1R}{g_m}P_0(1 - \frac{\beta H}{T_0})^{\frac{g}{R\beta}} \tag{17.1}$$

$$ZWD(H) = \frac{10^{-6}(T_mK_2 + K_3)R}{g_m - \beta R}\frac{e_0}{T_0}(1 - \frac{\beta H}{T_0})^{\frac{(\lambda+1)g}{R\beta}-1} \tag{17.2}$$

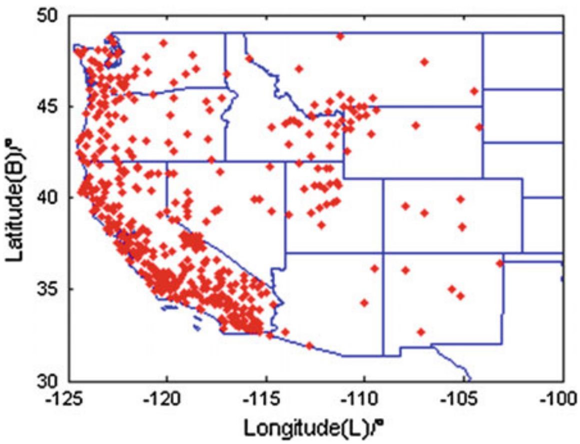
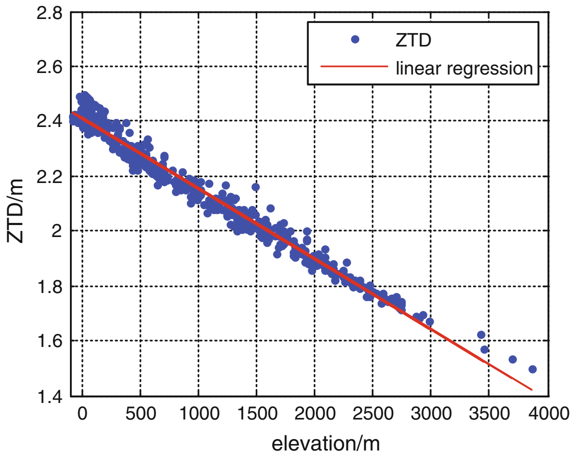


Fig. 17.1 The distribution of US CORS stations (Date: August 8th of 2013)

Fig. 17.2 Linear regression between height and ZTD



where H (m) is the height of the station; g_m (m/s^2) is the acceleration of gravity of station; T_m (K) is the temperature around station; P_0 (mbar), T_0 (K), e_0 (mbar), β (K/m), λ (mbar/m) are respectively the atmospheric pressure, temperature, vapor pressure, temperature change rate, vapor pressure change rate at sea level, which are interpolated by meteorological parameters grid using the latitude (ϕ) of the station; $T_m = T_0[1 - \beta R/(g_m(\lambda + 1))]$; $K_1 = 77.60\ \text{k mbar}^{-1}$; $K_2 = 16.6\ \text{k mbar}^{-1}$; $K_3 = 377{,}600\ \text{k mbar}^{-1}$; $g = 9.80665\ \text{m/s}^2$; $R = 287.054\ \text{J kg}^{-1}\ \text{K}^{-1}$.

Besides, $g_m = 9.784(1 - 2.66 \times 10^{-3}\cos(2\phi) - 2.8 \times 10^{-7}\ H)$, the height and latitude of station have a little effect on the value of the acceleration of gravity. So, the acceleration of gravity of station is approximately equal to 9.784. It is also obvious that the ZWD and ZHD have strong correlation with height in the UNB3m model.

17.3 The ZTD Modeling Method Considering Height Difference

17.3.1 Tropospheric Delay Separation of Reference Stations

Based on the dual-frequency observations and the precise coordinates of regional reference stations, the zenith tropospheric wet delay of reference station (ZWD_r) is calculated by parameter estimation with the PPP filter using ionosphere-free combination. The zenith tropospheric hydrostatic delay (ZHD_r) is calculated by the UNB3m model before the parameter estimation. r is the number of reference stations.

Considering the relationship between height factor and zenith tropospheric delay (*ZHD* and *ZWD*) in UNB3m model, the zenith tropospheric delay is divided into two parts, the height weakly correlated ($ZHDD_r$ and $ZWDD_r$) and the height strongly correlated ($ZHDH_r$ and $ZWDH_r$). For zenith tropospheric hydrostatic delay, the two parts are calculated as follows:

$$\begin{aligned} ZHDD_r &= ZHD(H = 0) \\ ZHDH_r &= ZHD(H = h) - ZHDD_r = ZHD_r - ZHDD_r \end{aligned} \tag{17.3}$$

In a similar way, the zenith tropospheric wet delay is separated as follows:

$$\begin{aligned} ZWDD_r &= ZWD_r \cdot ZWD(H = 0)/ZWD(H = h) \\ ZWDH_r &= ZWD_r - ZWDD_r \end{aligned} \tag{17.4}$$

The height weakly correlated zenith tropospheric delay $ZTDD_r$ and the height strongly correlated zenith tropospheric delay $ZTDH_r$ are calculated as follows:

$$\begin{aligned} ZTDD_r &= ZHDD_r + ZWDD_r \\ ZTDH_r &= ZTDH_r + ZWDH_r \end{aligned} \tag{17.5}$$

17.3.2 The ZTD Modeling Method Considering Height Difference

According to the positional relationship between rover station and reference stations, height weakly correlated zenith tropospheric delay and the height strongly correlated zenith tropospheric delay are modeled separately.

17.3.2.1 The Height-Strongly Correlated ZTD Modeling Method

Suppose that $ZTDD_r = a_1 + a_2 x_r + a_3 y_r$, where $a_i(i = 1,\ 2,\ 3)$ are interpolation coefficients; x_r, y_r are the plane coordinates of stations; n is the number of reference stations. When interpolating, a local coordinate system is set up which is centered by the rover station. So, x_r and y_r are the values of the local coordinate system. Besides, x_r and y_r of rover station are both equal to 0. Thus:

$$ZTDH_u = a_1 \tag{17.6}$$

The interpolation coefficients satisfy the following formula:

$$[\,a_1 \quad a_2 \quad a_3\,]^T = (A^T A)^{-1} A^T V \tag{17.7}$$

where,

$$A = \begin{bmatrix} 1 & x_1 & y_1 \\ \vdots & \vdots & \vdots \\ 1 & x_n & y_n \end{bmatrix}, \ V = \begin{bmatrix} ZTDD_1 \\ \vdots \\ ZTDD_n \end{bmatrix} \tag{17.8}$$

17.3.2.2 The Height Weakly Correlated ZTD Modeling Method

Suppose that $ZTDH_u = b_1 ZTDH_1 + b_2 ZTDH_2 \cdots + b_n ZTDH_n$, where $b_i(i = 1, 2, \ldots, n)$ is interpolation coefficients, which satisfy the formulas that $\sum_{i=1}^{n} b_i = 1$, $\sum_{i=1}^{n} b_i h_i = 0$, $\sum_{i=1}^{n} b_i^2 = \min$. h_i are the height of reference stations. When interpolating, a local coordinate system is set up which is centered by the rover station. So, h_i are the values in the local coordinate system. Besides, the interpolation coefficients satisfy the following formula:

$$\begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix}^T = N(NN)^{-1} B^T W, \ N = B^T B \tag{17.9}$$

$$B = \begin{bmatrix} h_1 & h_2 & \cdots & h_n \\ 1 & 1 & \cdots & 1 \end{bmatrix}, \ W = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{17.10}$$

Through the interpolation model of height strongly correlated and height weakly correlated ZTD which are extracted from reference stations, the height strongly correlated ZTD ($ZTDH_u$) and height weakly correlated ZTD ($ZTDD_u$) of rover station can be obtained. The zenith tropospheric delay of rover station (ZTD_u) can be gained via combining the two parts together.

$$ZTD_u = ZTDH_u + ZTDD_u \tag{17.11}$$

17.4 Experiment Results and Analyses

17.4.1 Interpolation Modeling in Undulated Region

The area of 34°–36° north latitude and 119°–121° west longitude of the US CORS is selected as the experimental area aiming at the area where there is a bigger height difference. As is shown in the Fig. 17.3, five stations are chosen, among which, the middle one is used as a rover station (triangle), and the others as reference stations (circle). The maximum value of the height of the reference stations is 1526.119 m, minimum 284.566 m. And the height of the rover station is 1709.067 m, mean distance between the rover and reference stations 66.06 km. Data used in the experiment are the observations of 24 h on August 8th of 2013, with a sampling rate of 15 s.

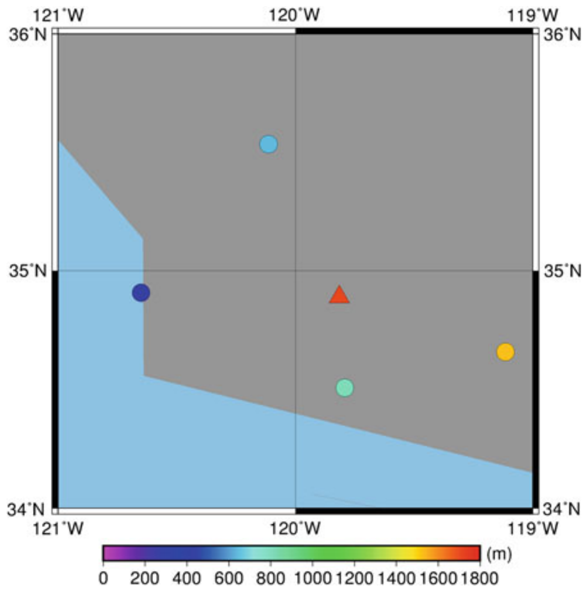


Fig. 17.3 The distribution of stations

2000 epochs of ZTD values are real-time regional modeled, which are based on the convergence of ambiguity in the PPP solutions of stations. The ZTD value of the rover station via PPP is as the true value. At the same time, the HTIM method proposed in this paper is compared with HLIM and MLCM methods. The result is shown in Fig. 17.4. The absolute deviation and root mean square error (RMS) of 2000 epochs are counted in Table 17.1.

As the table shows, aiming at the area which is undulated, when there are fewer reference stations, the method of HTIM is better than the other two methods in that the interpolation accuracy is higher with the absolute deviation within 0.03 m

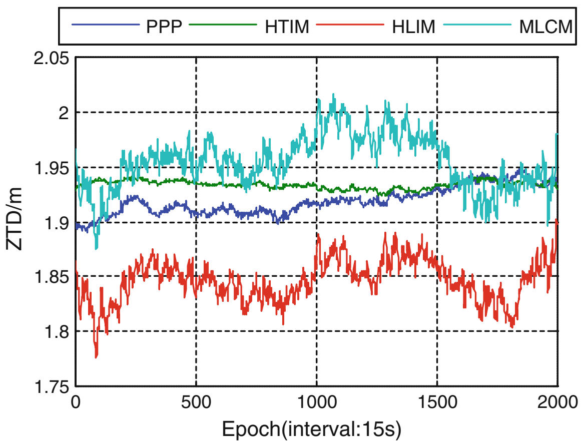


Fig. 17.4 The ZTD interpolated by different models and the true value

Table 17.1 Absolute deviation and RMS of different models

Absolute deviation	HTIM	HLIM	MLCM
<0.03 m	91.40 %	0.40 %	31.70 %
<0.02 m	56.75 %	0.00 %	21.75 %
<0.01 m	39.45 %	0.00 %	12.45 %
RMS/m	0.020	0.076	0.046

basically above 90 %, and within 0.01 m reaching 40 % nearly, and in that the mean square error is obviously smaller. Also, while interpolating using the HTIM, there is less fluctuation. Thus, evidently, under the circumstance where the height difference is bigger, only a few reference stations are needed to conduct regional interpolation that could generate higher precision compared with conventional methods.

17.4.2 Interpolation Modeling with Sparse Reference Stations

The area of 40°–45° north latitude and 118°–125° west longitude of the US CORS is selected as the experimental area aiming at the large area with sparse reference stations. As is shown in the Fig. 17.5, five stations are chosen, among which, the middle one is used as a rover station (triangle), and the others as reference stations (circle). The maximum value of the height of the reference stations is 1584.542 m, minimum 27.092 m. And the height of the rover station is 1390.816 m, mean distance between the rover and reference stations 233.79 km. Data used in the experiment are the observations of 24 h on August 8th of 2013, with a sampling rate of 15 s.

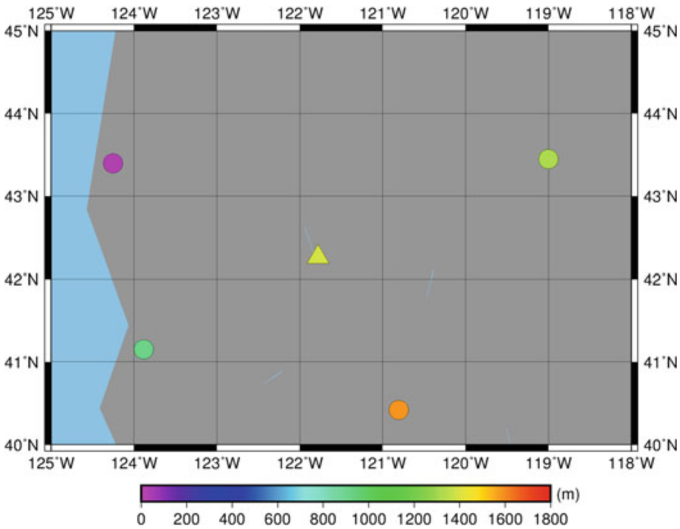


Fig. 17.5 The distribution of stations

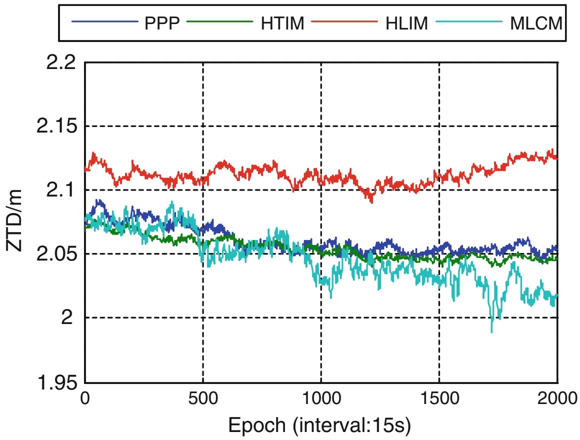


Fig. 17.6 The ZTD interpolated by different models and the true value

Table 17.2 Absolute deviation and RMS of different models

Absolute deviation	HTIM	HLIM	MLCM
<0.03 m	100.00 %	1.25 %	87.95 %
<0.02 m	99.95 %	0.00 %	68.30 %
<0.01 m	75.95 %	0.00 %	35.30 %
RMS/m	0.008	0.053	0.020

The analytical methods are same as which used in Sect. 17.4.1. The result is shown in Fig. 17.6. The absolute deviation and root mean square error (RMS) of 2000 epochs are counted in Table 17.2.

As the table shows, aiming at the large area with sparse reference stations, the method of HTIM is better than the other two methods in that the interpolation accuracy is higher with the absolute deviation is totally within 0.03 m, and within 0.01 m reaching 75.95 %, and in that the mean square error is obviously smaller. Thus, evidently, under the circumstance where reference stations are spare, only a few reference stations are needed to conduct regional interpolation that could generate higher precision compared with conventional methods.

17.4.3 Interpolation Modeling with Different Number of Reference Stations

The number of reference stations has an impact on the interpolation accuracy. The area of 34°–36° north latitude and 118°–121° west longitude of the US CORS is selected as the experimental area to analysis the impact of the number of reference stations on the interpolation precision of HTIM model. As is shown in the Fig. 17.7, eight stations are chosen, among which, the middle one is used as a rover station

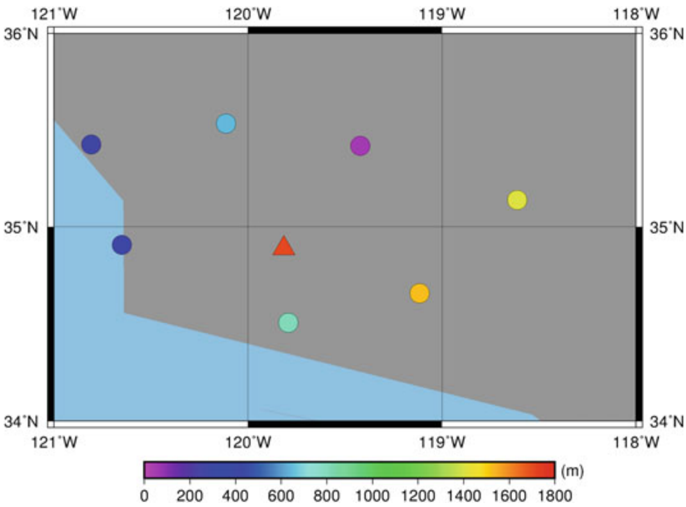


Fig. 17.7 The distribution of stations

(triangle), and the others as reference stations (circle). The maximum value of the height of the reference stations is 1526.119 m, minimum 56.588 m. And the height of the rover station is 1709.067 m, mean distance between the rover and reference stations 79.25 km. Data used in the experiment are the observations of 24 h on August 8th of 2013, with a sampling rate of 15 s.

The analytical methods are same as which used in Sect. 17.4.1. The result is shown in Fig. 17.8. The absolute deviation and root mean square error (RMS) of 2000 epochs are counted in Table 17.3.

As the table shows, with the increase of the number of reference stations, the interpolation accuracy of the HTIM model is improved, especially in absolute

Fig. 17.8 The ZTD interpolated by different models and the true value

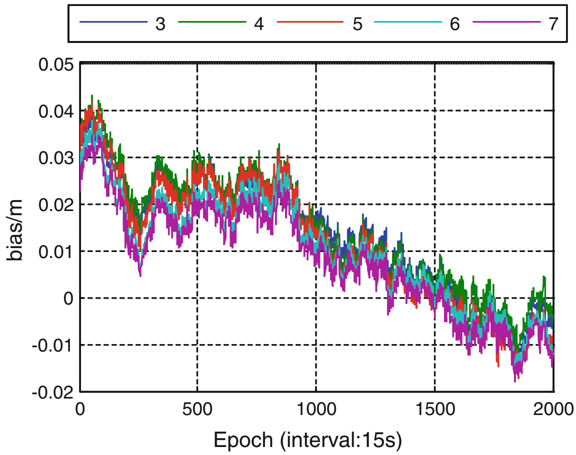


Table 17.3 Absolute deviation and RMS of different models

Absolute deviation	Number of reference stations				
	3	4	5	6	7
<0.03 m	93.85 %	91.40 %	93.25 %	95.40 %	97.00 %
<0.02 m	58.80 %	56.75 %	62.25 %	75.35 %	85.95 %
<0.01 m	34.60 %	39.45 %	38.65 %	43.60 %	45.50 %
RMS/m	0.019	0.020	0.018	0.016	0.014

deviation. It is three reference stations that the HTIM model can perform the interpolation model by. And the interpolation accuracy of HTIM model using three reference stations has reached the need of zenith tropospheric delay modeling, with absolute deviation within 0.03 m basically above 93.85 % and the RMS value within 0.020 m. It is should pointed out that the accuracy of true value is within several millimeters. So, it is accepted that the RMS of three reference stations is superior to the result of four reference stations. Besides, the absolute deviation within 0.01 m of four reference stations is far more than the percentage of three stations, although the other two indicators of absolute deviation of three reference stations are slightly better than four stations.

17.5 Concluding Remarks

Based on the analysis of the relationship between zenith tropospheric delay and the height of station, the new method of zenith tropospheric delay modeling (HTIM) for sparse reference station network is put forward. According to the relation between height factor and zenith tropospheric hydrostatic delay, zenith tropospheric wet delay in the UNB3m model, the zenith tropospheric delay of reference station is divided into two parts, the height weakly correlated and the height strongly correlated. The new method of zenith tropospheric delay modeling is established using the two parts of the zenith tropospheric delay. Based on the results of regional modeling, the zenith tropospheric delay of rover station is obtained. Through experiment and analysis, the method shows fair results and the conclusions are as follows:

1. In the certain region, there is a strong correlation between the height and the zenith tropospheric delay that the higher height is, and the smaller the zenith tropospheric delay is.
2. In the undulated area, only using four reference stations, the method of HTIM is better than the other two methods (HLIM and MLCM) in that the interpolation accuracy is higher with the absolute deviation within 0.03 m basically above 90 %, and within 0.01 m reaching 40 % nearly, and in that the RMS reaches 0.020 m.

3. In the large area with four sparse reference stations, the method of HTIM is better than the other two methods (HLIM and MLCM) in that the interpolation accuracy is higher with the absolute deviation is totally within 0.03 m, and within 0.01 m reaching 75.95 %, and in that the RMS is only 0.008 m.
4. The HTIM modeling method is basically unlimited by the number of reference stations. Using three reference stations, the RMS of interpolation is guaranteed to be less than 0.02 m and the absolute deviation is basically within 0.03 m.

In the paper, the zenith tropospheric delay of reference stations is not completely separated. The region for the experiment is in the US CORS, and the result in the other area has not been studied. So, there are further analysis and researches.

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