Chapter 12
A Beidou Based Multiple-GNSS Positioning Algorithm for Mission Critical Applications

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Abstract With the development of the Global Navigation Satellite Systems (GNSS), countries that own a GNSS have realised that critically national infrastructures using Position Navigation and Timing (PNT) services and a portion of the national economy associated with GNSS applications should not be over reliant on other countries. Recently, both China and Russia have made their systems mandatory for some applications. This paper addresses this issue and proposes a Beidou based multiple-GNSS positioning algorithm. It involves three stages: (1) Understanding of the quality of Beidou solutions. This was achieved by Receiver Autonomous Integrity Monitoring (RAIM) embedded in the Beidou positioning algorithm. (2) A real time validation and modelling algorithm for the measurements from the other constellations if Beidou solution is proved good in stage 1. The measurement residual errors relative to the Beidou position solution are assessed. (3) Introduction of measurements from the other constellations if there is not enough Beidou measurements. At this stage, the models derived in stage 2 are applied to the non-Beidou measurements. The tests were carried out using the Beidou and GPS data from a reference station. The signal blockage of Beidou and GPS constellation is simulated. The test results show that the proposed methods can benefit from the validated measurements from the GPS constellation. The performance can be significantly improved in terms of accuracy, continuity, integrity and availability in difficult environments. It can be extended for critical applications where any constellation is mandated.

Keywords Beidou · Multiple GNSS · Integrity · Continuity · Mission critical applications

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12.1 Introduction

With the modernization of the US Global Positioning System, the revitalization and modernization of the Russian GLONASS and the deployment of European Galileo and China's Beidou systems, it is expected that the full operation of multiple constellations is on the horizon. This will bring opportunities for user level positioning and integrity monitoring including more satellites, frequencies, message types, and better signal design and geometry. These will enable a multi-constellation receiver having the potential to support many more applications (e.g. urban rail operations, stand-alone precision aircraft operations and personal navigation services) than a single constellation receiver. Considerable research has been undertaken on positioning and integrity monitoring using multiple Global Navigation Satellite Systems (GNSS). However, due to the differences among the systems such as system time, spatial reference frame, signal quality and the performance of ephemeris, the best way to employ multiple constellations is still a challenge. For example the existing Solutions Separation method is a trade-off between two positioning solutions. It removes the negative impact of system time difference. However, it reaches a solution that is not as good as the better one of the two solutions from each individual constellation.

This paper addresses the technical issues of the existing methods and proposes a Beidou constellation based multiple-GNSS positioning and integrity monitoring algorithm. A Receiver Autonomous Integrity Monitoring (RAIM) is embedded in the positioning algorithm with the Beidou constellation. In order to benefit from the other constellations without compromising the trust on the Beidou constellation, a real time validation algorithm for the measurements from GPS constellations is developed using the solution from the Beidou constellation. The measurements from GPS constellation are pre-processed with conventional methods including correcting of satellite clock, the ionosphere effect and troposphere delay. The measurement residual errors relative to the position solution from the Beidou constellation are assessed. Upon pass the validation process, the measurements from other constellations are integrated to benefit from the feature of multiple constellations and generate a better solution.

The tests were carried out using the Beidou and GPS data from a reference station in China. Beidou was used as core constellation; while the errors in the GPS measurements are monitored and modelled. The test results show that the proposed methods can effectively identify potential issues and can benefit from the validated measurements from GPS. It can be used in critical applications where there are not enough Beidou measurements available.

12.2 Multi-Constellation Positioning and RAIM Algorithm

In single constellation, the positioning algorithm is usually based on least squares or filtering method. In any case, the measurements are pre-processed to mitigation various errors as much as possible by applying error models. It is generally assumed that the remaining un-modelled errors are acceptable. Therefore, the linearized form is:

$$\Delta\rho = H \cdot \Delta X + \epsilon \tag{12.1}$$

where $\Delta\rho$ is the difference between measured range (pre-processed) and expected range. $H = [\,G \quad (\mathbf{1})\,]$ is the design matrix. G is the matrix consists of the direction vectors. $(\mathbf{1})$ is a vector with the same number of row as G with all its elements being one. $\Delta X = [\,\Delta x \quad \Delta y \quad \Delta z \quad \Delta t_r\,]$ is the positioning and receiver clock error. ϵ is the measurement error vector.

With two constellations, the number of navigation satellites is roughly doubled. This will provide better geometry, more redundancy and further enhance the positioning performance if two constellations could be used in combination. If more constellations are used in combination, even better positioning performances may be expected. The major issues in combining multiple constellations for positioning are differences in the system reference and signal quality. There are two types of approach in combining two or more constellations for positioning. One is combining in the position domain; the other one is in measurement (range) domain.

The approach of combining two constellations in position domain is to perform positioning independently in each constellation, with the ultimate possibility of determining the final solution as a weighted average of independent solutions [1]. In theory, the position domain method provides a capability to support RAIM. The algorithm called Optimally Weighted Average Solution (OWAS) method [2] provides a capability to provide integrity in the presence of multiple faults in a single constellation by comparing one constellation to another. The use of this method is subject to the RAIM availability within each constellation. Obviously, if there is not enough visible satellites (e.g. less than 4) in either constellation. The weighted average method does not work at all. Even with enough number of visible satellites (e.g. more than 4) in each constellation, the weighted average method is a trade-off between positioning solutions from two constellations.

In the measurement domain, there are three methods. The major difference is the way of handling the differences between constellations. The first compensates for differences between constellations by using correction to GPS time relative to other constellation (e.g. Beidou) time broadcast by the system [3]. The second method in the measurement domain is to take the differences (e.g. reference time) as unknowns to be estimated as part of the positioning solution [4]. The third method in the measurement domain is to employ single differences technique between pairs of measurements from two or more constellations. Each pair of measurement is selected from different constellation for example one from GPS and the other one from Galileo [5]. In this case, the receiver clock bias and drift error are removed by the single difference. The inter-system time difference between any two constellations appears as a bias. In addition, a carefully selected pair can also mitigate correlated un-modelled errors such as tropospheric residual error.

In terms of RAIM, the measurements domain approach can fully exploit the redundancy provided by multiple constellations. Conventional Receiver Autonomous Integrity Monitoring (RAIM) assumes that only one failure occurs at a time. For the standard weighted least squares method [6, 7], the Sum of the Squared Errors (SSE) is used to construct a test statistic:

$$test\,statistic = \sqrt{SSE} \tag{12.2}$$

where $SSE = w^T w = \varepsilon^T S^2 \varepsilon = \varepsilon^T S \varepsilon$, $S = I - H \cdot (H^T \cdot W \cdot H)^{-1} \cdot H^T \cdot W$. The SSE has a chi-square distribution with $n - 4$ (n is the number of visible satellites) degrees of freedom (assuming that the measurement errors are independent and normally distribution with zero mean). Based on this assumption, a threshold can be derived for a given probability of false alert. This threshold is used to judge if there is any failure in the measurements

A protection level is an estimate of the up bound of positioning error. The Horizontal Protection Level (HPL) and Vertical Protection Level (VPL) are addressed as:

$$HPL = \max(Hslope(i)) * Pbias \tag{12.3}$$

$$VPL = \max(Vslope(i)) * Pbias \tag{12.4}$$

where $Hslope(i) = \sqrt{(A_{1i}^2 + A_{2i}^2)/S_{ii}}$, $Vslope(i) = A_{3i}/\sqrt{S_{ii}}$ is the sensitivity of the horizontal and vertical error to test statistics respectively. The *Pbias* is Minimum Detectable Bias (MDB) taking into account the threshold and probability of missed detection.

12.3 Beidou Based Multi Constellation Algorithms

Core constellation is the constellation mandated or its integrity is monitored by independent systems and known by users. Figure 12.1 shows the function diagram of the Beidou based method. Even though the integrity of signals from Beidou constellation may be monitored independently, the user level integrity monitoring is still necessary because it is difficult for an independent system to monitor the errors coming from the user local environments such as multipath, local interference and receiver related errors. Therefore, RAIM must be used in the core constellation positioning algorithm in order to provide a trust or quality indicator of the corresponding solutions. In general, the core constellation only can provide positioning solutions if there are enough satellites in view. However, it is not always the case

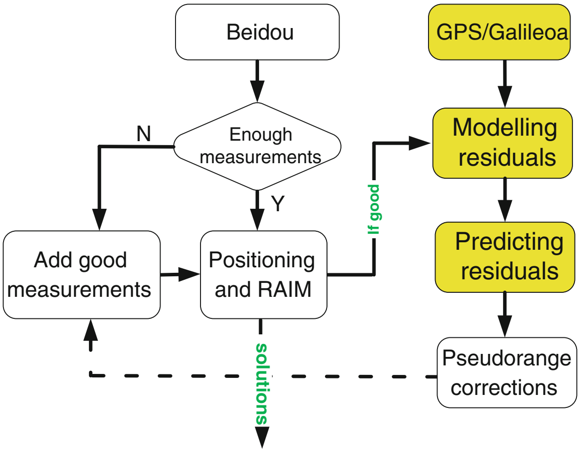


Fig. 12.1 The core constellation based method

where there are more than 4 satellites in view for example signal blockage. In order to provide continuous positioning solutions, measurements from other (non-core) constellations will be needed. In order to reach a good solution, only those measurements being validated can be used together with measurements from the core constellation. The positioning with the Beidou constellation can adopt the conventional least squares method or its variations. The corresponding RAIM is based on failure detection and the calculation of protection levels described in the previous section. Upon passing the RAIM tests, the positioning solution based on Beidou constellation is used to calculate the range residual errors for non-core constellations.

The error of range between a receiver and a satellite can be expressed as:

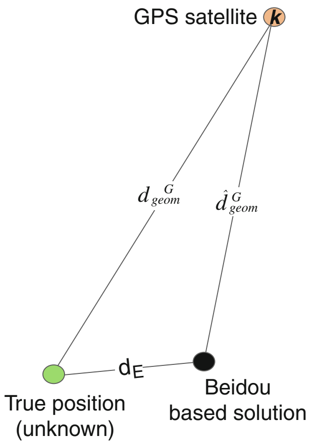
$$\rho^B - d^B_{geom} = ct - cT^B + d^B_{orb} + d^B_{ion} + d^B_{mp} + \varepsilon^B \tag{12.5}$$

$$\rho^G - d^G_{geom} = ct - cT^G + d^G_{orb} + d^G_{ion} + d^G_{trop} + d^G_{mp} + \varepsilon^G + ct_{sys} \tag{12.6}$$

where, the superscript B denotes Beidou constellation; G denotes GPS constellation, c is the speed of light, d is the range, t is the receiver clock offset, T is the satellite clock offset, ε is the measurement noise, t_{sys} is the system time difference between constellations, the subscript *orb* denotes orbit, *ion* denotes ionosphere, *trop* denotes troposphere, *mp* denotes multipath.

Figure 12.2 shows the geometry distance between a GPS satellite (k) and position of the user receiver. The true position where the pseudoranges are taken by a receiver is unknown. However, one of the estimates is the positioning solution from the Beidou constellation. Since the Beidou constellation may be monitored by an independent system and a RAIM algorithm is embedded, the estimate should be close to the true position. The position error d_E in Fig. 12.2 should be relatively small. As a result, the estimate of geometry distance $\hat{d}^G_{geom}$ should be close to the

Fig. 12.2 Geometry distance



true geometry distance d^G_{geom}. Therefore, the estimated pseudorange residual from a GPS satellite can be expressed as:

$$r = \rho^G - \hat{d}^G_{geom} = ct - cT^G + d^G_{orb} + d^G_{ion} + d^G_{trop} + d^G_{mult} + \varepsilon^G + ct_{sys} \tag{12.7}$$

The residual is considered as the correction for GPS measurements. Given the fact that the positioning error (d_E in Fig. 12.2) is relatively small, most of the errors are highly correlated in spatial. These errors are also temporally correlated. The correction for measurement from a GPS satellite can be modelled as:

$$r(t) = \alpha_0 + a_1(t - t_0) + a_2(t - t_0)^2 \tag{12.8}$$

The coefficients a_0, a_1 and a_2 are determined when the Beidou solutions continuously passes integrity monitoring test since t_0. This model is then used in perdition mode when measurements from non-core constellation are needed. If the predicted correction (12.8) is applied to (12.6), the receiver clock and system clock difference error are included in the correction. The remaining errors are the multipath and noise. Therefore, the design matrix is written as

$$H = \begin{bmatrix} G^B & 1 \\ G^G & 0 \end{bmatrix} \tag{12.9}$$

where, matrix G consists of the direction vectors. (**1**) is a vector with the same number of row as G with all its element being one. (**0**) is a vector with the same number of row as G with all its element being zero.

The corresponding integrity monitoring needs to detect potential problems associated with the Beidou and GPS measurements. In addition, if the residual error models are not accurate, the un-modelled errors will appear in the test statistic. Therefore, the un-modelled error will not escape the detection.

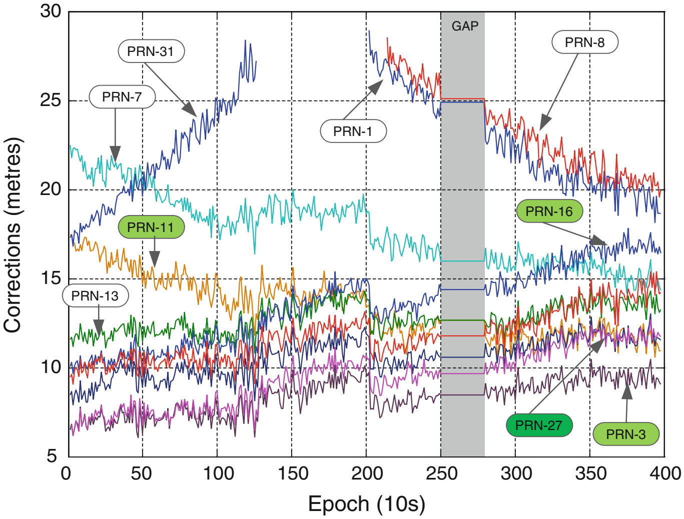


Fig. 12.3 Residuals of GPS measurements

12.4 Tests and Results

The data from a reference station in China which contain Beidou, GPS and GLONASS measurements were selected to verify the method proposed. Beidou was used as a core constellation, while GPS was used as a non-core constellation. Signal blockages were simulated for a period. Figure 12.3 shows the residuals of GPS measurements. During period when Beidou was not able to monitor its integrity, the GPS residuals were not monitored and modelled.

Two scenarios were tested. The first one was designed with four satellites in each constellation; while the second scenario was designed with three satellites in each constellation. For comparison purpose, six types of results are shown to demonstrate the performance of method proposed.

12.4.1 Results for the First Scenario

This scenario was designed to show the performance of the proposed method when each constellation has the capability to derive positioning solutions even during partial blockage period. Figure 12.4 shows the number of satellites during the test period. Figure 12.5 shows the sky plot during the signal partial blockage period. It can be seen that the geometry of both Beidou and GPS are poor. This also demonstrates that the positioning errors are relatively big during the same period as shown in Figs. 12.6 and 12.7.

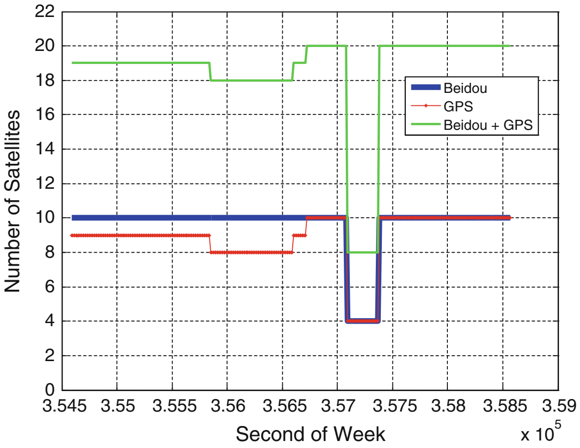


Fig. 12.4 Number of visible satellite

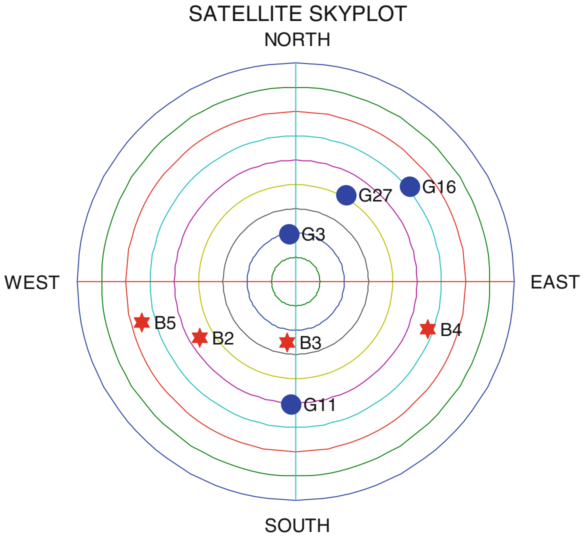


Fig. 12.5 Skyplot during blockage

In terms of integrity monitoring, Fig. 12.8 shows that even during the partial blockage period, the proposed algorithm still has the failure (or inconsistence of modelled GPS with Beidou) detection capability. Figure 12.9 shows the horizontal protection levels (HPL). The HPLs overbound the horizontal position error (in Fig. 12.8) all the time.

During the partial blockage period, the positioning accuracy was very poor. Neither Beidou nor GPS was able to performance integrity monitoring. The proposed method can improve the accuracy and be able to carry out integrity monitoring.

Fig. 12.6 3D positioning errors

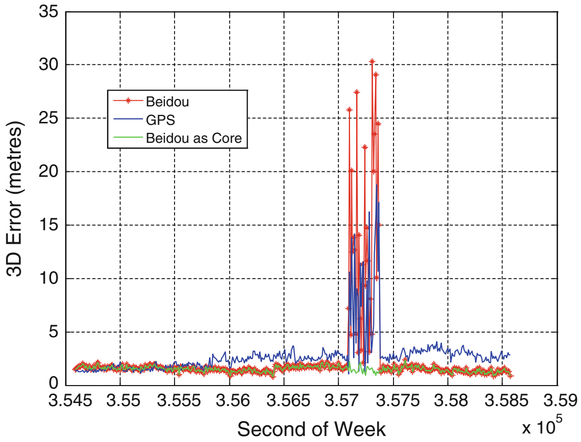
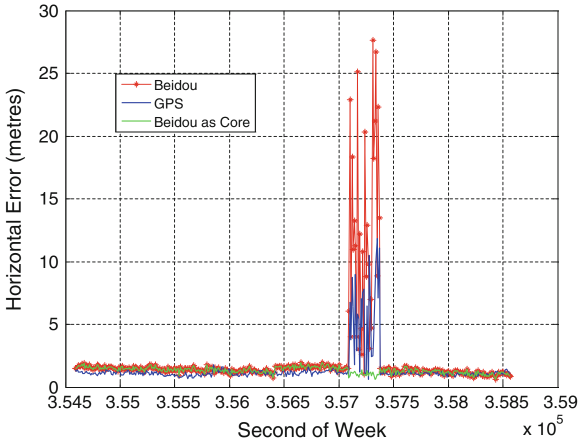


Fig. 12.7 Horizontal positioning errors



12.4.2 Results for the Second Scenario

This scenario was designed to show the performance of the proposed method when each constellation doesn't have the capability to derive positioning solutions during partial blockage period. Figure 12.10 shows the number of satellites during the test period. Figure 12.11 shows the sky plot during the signal partial blockage period. Figures 12.12 and 12.13 show the positioning error. During partial blockage period, the proposed method can still produce good solutions. In terms of integrity monitoring, Fig. 12.14 shows that the proposed algorithm still has the failure detection capability even during the partial blockage period. Figure 12.15 shows the horizontal protection levels (HPL). The HPLs overbound the horizontal position error (in Fig. 12.13) all the time.

Fig. 12.8 Test statistics and thresholds

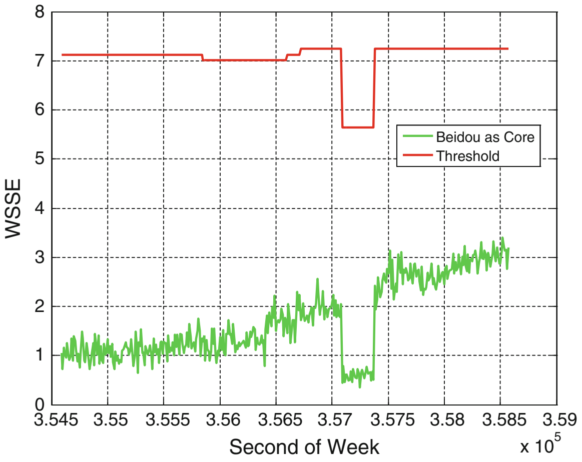


Fig. 12.9 Horizontal protection levels

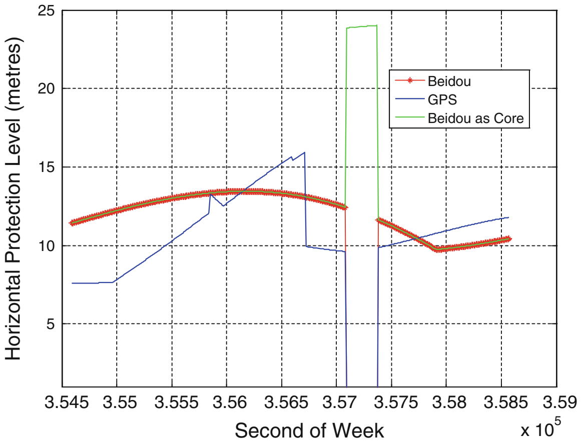


Fig. 12.10 Number of visible satellite

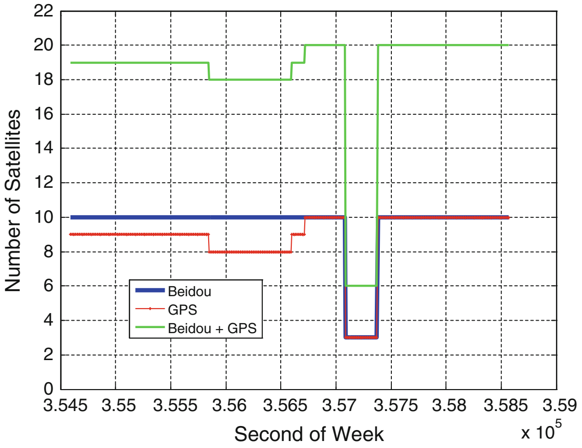


Fig. 12.11 Skyplot during blockage

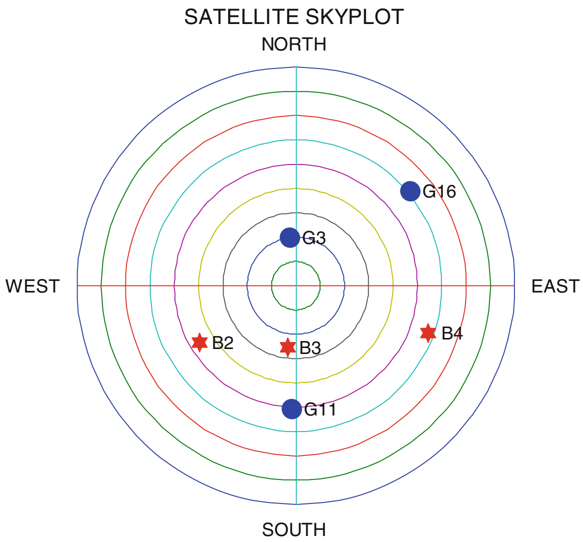
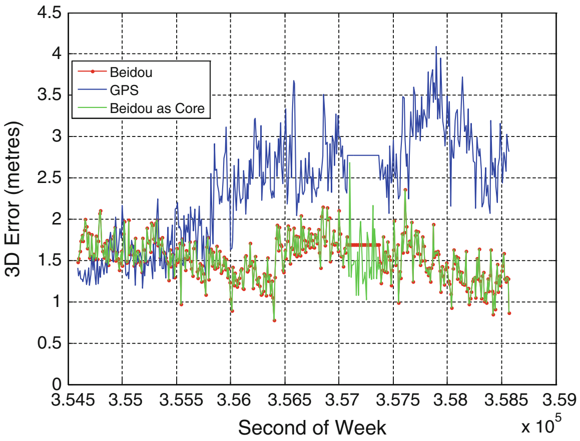


Fig. 12.12 3D positioning errors



During the partial blockage period, neither Beidou nor GPS was able to perform positioning and integrity monitoring. The proposed method can perform both positioning and integrity monitoring. What should be highlighted here is that the inconsistence of modelled GPS error with Beidou measurements is also taken into account. This prevents the unmonitored GPS measurements contaminating the overall solutions.

Fig. 12.13 Horizontal positioning errors

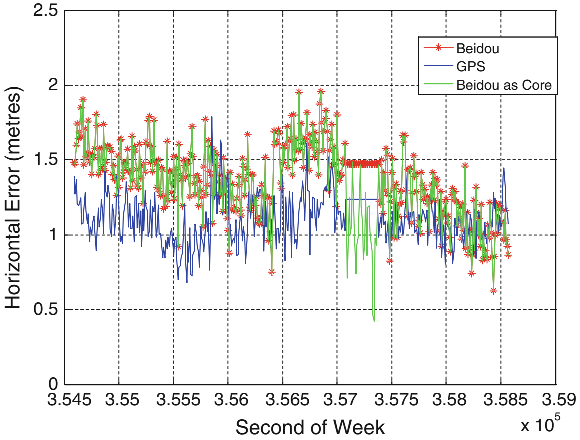


Fig. 12.14 Test statistics and thresholds

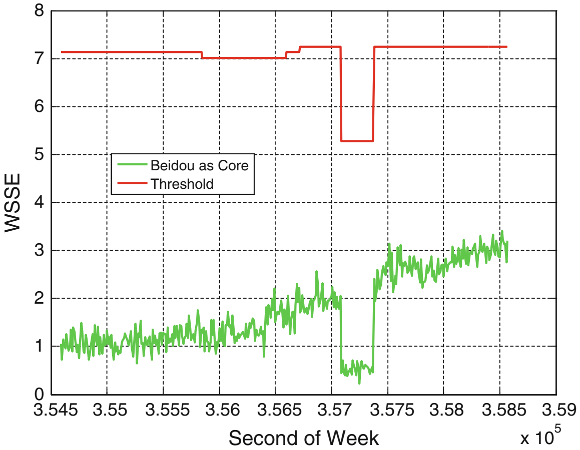
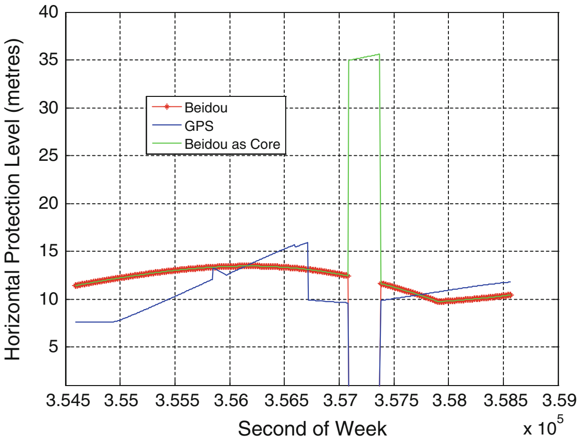


Fig. 12.15 Horizontal protection levels



12.5 Conclusions

Positioning solutions with a single constellation GNSS (e.g. GPS) have been widely used. The performance in terms of accuracy, continuity and integrity of these solutions is compromised in difficult environments such as city canyons. Multiple GNSS constellations provide more satellites, frequencies, message types, and better signal design and geometry. These will enable a multi-constellation receiver having the potential to support many more applications. There are also challenges including the best way to handle differences (e.g. system quality) and to maximum the benefits of multiple constellations. However, due to the interest of the owner of a GNSS, one constellation may be mandated. The mandated constellation is referred to as "core constellation". Its performance may be enhanced by independent systems.

The positioning solution is based on Beidou as core constellation together with RAIM when it has enough measurements. In the meantime, it models the residuals of all GPS measurements. In case there are a limited number of Beidou measurements, the GPS measurements are used together with the predicted correction using the residual model. Test results with real data have shown that the proposed method can achieve the best performance both with limited number and enough number of visible satellites. It has the potential to be used in applications where one constellation is mandated. It can achieve a better accuracy without losing continuity and integrity in relatively difficult environments. This paper takes Beidou and GPS constellations as an example to verify the Beidou based positioning and integrity monitoring method. The method can be extended to multiple constellations cases.

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