Non-Preference Based Pruning Algorithm for Multi-Objective Redundancy Allocation Problem

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Abstract. A non-preference based pruning algorithm is proposed to rank the Pareto-optimal solutions according to the cost and reliability trade-off for solving multi-objective redundancy allocation problem. The proposed method demonstrates on multi-objective redundancy allocation problem with mixing of non-identical component types in each subsystem. The objectives of system design are to maximize system reliability and minimize system cost simultaneously while satisfying system requirement constraints. Non-dominated sorting genetic algorithm-II (NSGA-II) finds an approximation of Pareto-optimal solutions. After obtaining the approximation of Pareto-optimal solutions by NSGA-II, K-means clustering is used to cluster the approximation of Pareto-optimal solutions in to some trade-off regions. Thereafter, the Pareto-optimal solutions are ranked based on the cost and reliability trade-off compare to the centroid solution of each cluster. The results show that the proposed method is able to identify the most-compromised solution.

Keywords: multi-objective optimization · redundancy allocation problem · pruning algorithm · non-preference based

1 Introduction

Generally, a multi-objective optimization problem has a large set of trade-off solutions. The set of non-dominated solutions or Pareto-optimal solutions have trade-off between the objective functions in which a gain in one objective causes sacrifices in the other objective. Therefore, the most-compromised solution is difficult to identify. This can be challenging for selecting one Pareto-optimal solution that can be practically implemented and compromised between the objectives as the system is designed.

The redundancy allocation problem (RAP) is NP-hard problem. The RAPs have been researched for finding the approximation of Pareto-optimal solutions by using NSGA-II [2], which is a well-known algorithm and efficient in searching the Pareto-optimal solutions [3 and 4] for multi-objective optimization problem.

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K. Deb et al [5] proposed the reference pointed based non-dominated sorting genetic algorithm-II (R-NSGA-II). The decision maker (DM) specifies the reference points of all objective function. After that, NSGA-II ranks the non-dominated solutions and search for the optimal solutions that close to the reference points in objective space. J. Branke et al. [6] presented a method that modified the definition of dominance. The DM needs to specify the minimum and maximum acceptable trade-offs for each pair of objectives, which represented by slope of straight line. The dominated areas are expanding while compare to traditional definition of dominance. Therefore, some non-dominated solutions are pruned. As the number of objectives increases, specify minimum and maximum trade-offs is need to specify the trade-off values for all pair of objectives. Tilahun and Ong [7] proposed the fuzzy preferences incorporate with genetic algorithm (GA) for multiple DMs. This method collected preferences as fuzzy conditional trade-offs then formulated the acceptability of preference membership functions. GA generates weight values for objective functions according to the DM's trade-off values. This method provided flexible trade-off however it is difficult to specify trade-off for every alternative solution.

This paper proposed a non-preference based pruning algorithm for ranking the optimal system design of RAP according to cost and reliability trade-off. In this research, the RAP considers series-parallel system with mixing of non-identical component types. NSGA-II is applied to find the approximation of Pareto-optimal solutions. K-means clustering is used to cluster the approximation of Pareto-optimal solutions in to some trade-off regions. After clustering, the proposed method ranks the approximation of Pareto-optimal solutions according to cost and reliability trade-off. Therefore, the DM is able to select the final system design from a large size of the solutions.

The remaining of the paper is organized as follows. In Section II, multi-objective RAP is described. In Section III, the non-preference based pruning algorithm is presented. In Section IV, the experimental results and discussion are provided. Finally, the conclusion is in Section V.

2 Multi-Objective Redundancy Allocation Problem

The RAP is to determine the optimal design configuration from the redundant alternatives. The subsystems are connected in series, while the redundant components connected in parallel in each subsystem. The RAP with a series-parallel structure is shown in Fig. 1. The redundant components improve system reliability, while system cost and weight is increasing. Due to mixing of non-identical component type is allowed, the problem can be very complex and the search space is extended to large size. Therefore, it is difficult to find the Pareto-optimal solutions and identify the selected solutions.

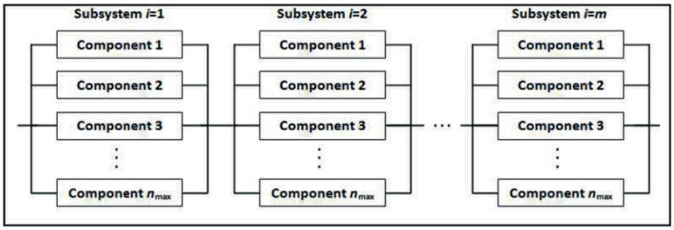

Fig. 1. General series-parallel redundancy system [8]

2.1 Problem Formulation

The model of RAPs has been proposed by previous researches [1, 8, 9 and 10]. The system consists of m subsystems that connected in series. The number of component type j allocated in subsystem i is x_{ij}, which represented in vector by $\mathbf{x}$ or $\mathbf{x}_i$. The reliability in subsystem i is R_i. The optimal configuration of system design with mixing of non-identical components has to be determined from t_i different component types in subsystem i^{th}. In this research, the objective functions are to maximize system reliability, R_{sys} and minimize system cost, C_{sys} simultaneously while satisfying system weight constraint, W_{sys_con}. The mathematical model formulation is presented as follows:

$$\max R_{sys}(\mathbf{x}) = \prod_{i=1}^{m} R_i(\mathbf{x}_i) \quad (1)$$

$$\min C_{sys}(\mathbf{x}) = \sum_{i=1}^{m}\sum_{j=1}^{t_i} c_{ij}x_{ij} \quad (2)$$

$$\text{s.t.} \quad \sum_{i=1}^{m}\sum_{j=1}^{t_i} w_{ij}x_{ij} \le W_{sys\_con}$$

$$1 \le \sum_{j=1}^{t_i} x_{ij} \le n_{max}$$

$$R_i(\mathbf{x}) = 1 - \prod_{j=1}^{t_i}\left(1 - R_{ij}(\mathbf{x})\right)^{x_{ij}}$$

where $x_{ij} \in \{0, 1, 2, \ldots, n_{\max}\}$, $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, t_i$. The j^{th} component in subsystem i has reliability (r_{ij}), cost (c_{ij}) and weight (w_{ij}). The maximum number of components in subsystem i is n_{max}.

2.2 Problem Assumption

The alternative component types for each subsystem have different and independent component reliability, cost and weight. The system design is possible to mix non-identical component types for each subsystem. The states of the components and the system include work and failure. The component failure is statistically independent.

3 The Non-Preference Based Pruning Algorithm

The non-preference based pruning algorithm aim to obtain the ranking of Pareto-optimal solutions for multi-objective RAP. The steps of this method are following:

1. NSGA-II searches the approximation of Pareto-optimal solutions.
2. The DM specifies the number of clusters, *k*.
3. K-means clustering finds a centroid of each cluster. The Pareto-optimal solution that is closest to each centroid is obtained and called centroid solution.
4. The cost and reliability trade-off between the non-dominated solution and its centroid solution in each cluster is calculated using the following equation.

$$\text{Trade} - \text{off}_{ij} = \frac{\left| f_{centroid_j}^{cost} - f_{ij}^{cost} \right|}{\left| f_{centroid_j}^{reliability} - f_{ij}^{reliability} \right|} \tag{3}$$

   where $f_{centroid_j}^{cost}$ and $f_{centroid_j}^{reliability}$ are the system cost and system reliability of centroid solution in cluster *j,* respectively. $f_{ij}^{cost}$and $f_{ij}^{reliability}$ are the system cost and system reliability of solution *i* in cluster *j* where *j* = 1, 2, …, *k*. The trade-off$_{ij}$ is a ratio of the absolute difference between the system cost of the centroid solution in cluster *j* and the system cost of solution *i* in cluster *j* to the absolute difference between the system reliability of the centroid solution in cluster *j* and the system cost of solution *i* in cluster *j*. The trade-off$_{ij}$ represents sacrificing units in the objective to order to gain one unit in the other objective.
5. In each cluster, the non-dominated solutions are sorted according to the cost and reliability trade-off value in ascending order. The alternative that has the smallest cost and reliability trade-off value is rank 1 which is the least amount to sacrifice in one objective when compare to its centroid solution.
6. The ranking of Pareto-optimal solutions with the cost and reliability trade-off values are presented to the DM. The cost and reliability trade-off value indicates the cost-effective solutions. The solution with low trade-off value is preferred to the other solutions.

4 Experimental Results and Discussion

In our experiment, the system configuration of RAP [6] consists of 7 subsystems, with different component types, as presented in Table 1. The objectives are to maximize system reliability and minimize system cost subject to a weight constraint. NSGA-II is used as the searching algorithm. In order to achieve the optimal solutions, NSGA-II requires parameter tunings including a population size, a mutation probability, a crossover probability and a max generation. After significant trial and error experiments, we obtain the optimal parameter settings of NSGA-II as shown in Table 2. The binary tournament selection, simulated binary crossover (SBX) [11] and polynomial mutation operators [12] are used in NSGA-II.

Table 1. Component input data
Note: Sub = subsystem, Comp = component, The symbol "-" means that design alternative is not available.

Sub i	Design Alternative j											
	Comp Type 1			*Comp Type 2*			*Comp Type 3*			*Comp Type 4*		
	r_{ij}	c_{ij}	w_{ij}	r_{ij}	c_{ij}	w_{ij}	r_{ij}	c_{ij}	w_{ij}	r_{ij}	c_{ij}	w_{ij}
1	0.90	1	3	0.93	1	4	0.91	2	2	0.95	4	5
2	0.95	4	8	0.94	2	10	0.93	1	9	-	-	-
3	0.85	2	7	0.90	3	5	0.87	1	6	0.92	4	4
4	0.83	3	5	0.87	4	6	0.85	5	4	-	-	-
5	0.94	2	4	0.93	2	3	0.95	5	5	0.94	2	4
6	0.99	6	5	0.98	4	4	0.97	2	5	0.96	2	4
7	0.91	4	7	0.92	4	8	0.94	5	9	-	-	-

Table 2. Parameter setting for NSGA-II

Parameter	Value
Population size	100
Mutation probability	0.07
Crossover probability	0.9
Max generation	1000

We consider k-means clustering $k = 3$, so that 3 clusters are obtained representing groups of solutions with low system reliability, medium system reliability and high system reliability, respectively. Two test cases are considered as follows.

Case 1: Two objectives without system weight constraint are considered. The maximum number of components is 8 for each subsystem. The approximation of Pareto-optimal solutions is shown in Fig. 2.
Case 2: Two objectives with system weight constraint, 100 are considered. The maximum number of components is 4 in each subsystem. The approximation of Pareto-optimal solutions is shown in Fig. 3.

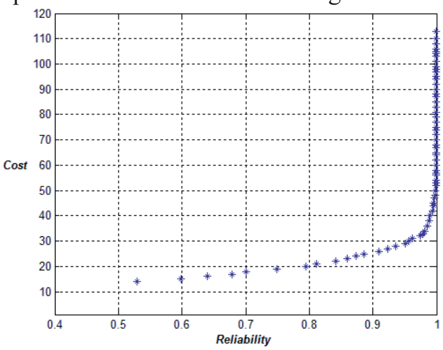

Fig. 2. The solutions of the system reliability and cost for case 1

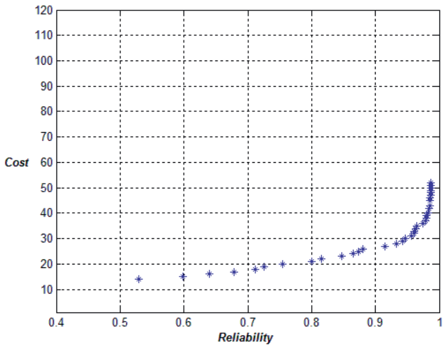

Fig. 3. The solutions of the system reliability and cost for case 2

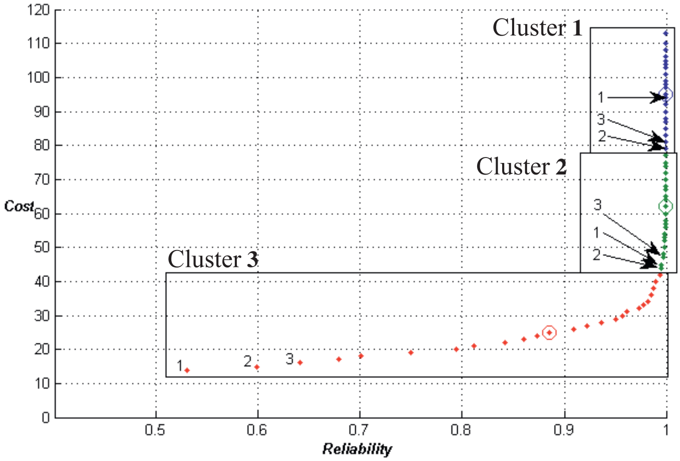

Fig. 4. The ranking and clustering solutions of the system reliability and cost for case 1

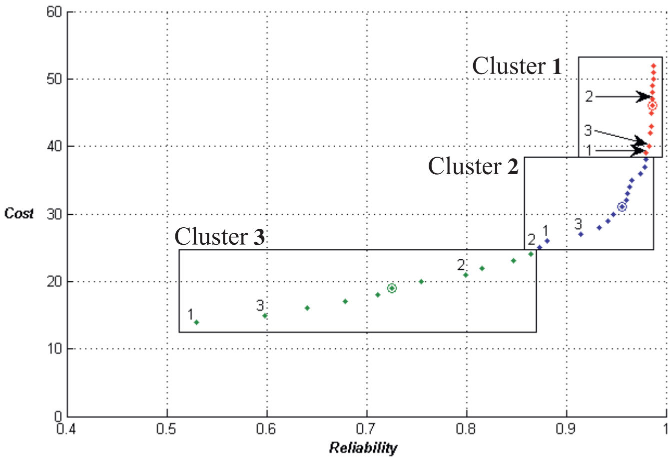

Fig. 5. The ranking and clustering solutions of the system reliability and cost for case 2

From the experiments, the approximation of Pareto-optimal solutions is clustered into 3 regions including high reliability (cluster 1), medium reliability (cluster 2) and low reliability (cluster 3). In each cluster, solution with rank 0 surrounded with a circle shown in Figs. 4 and 5 represent the centroid solution. In Figs. 4 and 5, the alternative solutions are ranked according to system cost and reliability trade-off value compared with their centroid solution. The solution with rank 1 has the least sacrificing amount of one objective in order to improve in the other objective when compare to its centroid solution.

Case 1: Table 3 shows the detail of the solutions including component allocation. In detail at solution rank 0 (cluster 1), subsystem 1 has 2 components of type 1 and 4 components of type 2, while subsystem 2 has 5 components of type 2 and so on. From the table, we can see that component mixing is obtained for all the solutions. In cluster 1, the system reliability of solution rank 1 is less than the system reliability of the centroid solution, while the system cost is lower.

Case 2: The component allocation obtained from case 2 is presented in Table 4. In cluster 1, the system reliability of solution rank 1 is less than the system reliability of the centroid solutions, while the system cost is lower.

The traditional k-means method identifies only the centroid solutions. On the other hand, our method suggests the most-compromised solution according to the system cost and reliability trade-off value besides the centroid solution. Our method is suitetable for multi-objective optimization problems where the DM has no preference in any objectives.

Table 3. Component allocation for case 1
Note: Sub = subsystem, *R* = reliability, *C* = cost, *W* = weight

Rank	R	C	W	Cluster	Sub 1				Sub 2			Sub 3				Sub 4			Sub 5			Sub 6				Sub 7		
					1	2	3	4	1	2	3	1	2	3	4	1	2	3	1	2	3	1	2	3	4	1	2	3
0	0.999995692	95	237	1	2	4	0	0	0	5	0	1	1	5	0	3	3	1	0	2	3	0	3	1	0	0	6	0
1	0.999993263	94	235	1	2	4	0	0	0	5	0	1	1	5	0	3	3	1	1	1	4	0	3	1	0	0	5	0
0	0.999763233	62	173	2	0	4	0	0	0	4	0	0	0	5	0	3	2	0	1	2	1	0	1	1	1	0	4	0
1	0.995507911	45	134	2	0	3	0	0	0	3	0	0	0	5	0	0	3	0	1	2	0	0	0	2	0	0	3	0
0	0.885793781	25	77	3	0	2	0	0	0	2	0	0	0	2	0	2	0	0	2	0	0	0	0	2	0	0	0	1
1	0.529536088	14	42	3	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1	0

Table 4. Component allocation for case 2.
Note: Sub = subsystem, *R* = reliability, *C* = cost, *W* = weight

Rank	R	C	W	Cluster	Sub 1				Sub 2			Sub 3				Sub 4			Sub 5			Sub 6				Sub 7		
					1	2	3	4	1	2	3	1	2	3	4	1	2	3	1	2	3	1	2	3	4	1	2	3
0	0.98599356	46	98	1	1	2	0	0	0	2	0	0	0	1	2	1	1	1	2	0	0	0	0	0	2	2	1	0
1	0.979962339	39	96	1	1	2	0	0	0	2	0	0	1	2	0	1	1	1	2	0	0	1	0	1	0	1	1	0
0	0.955865976	31	91	2	0	2	0	0	0	2	0	1	0	2	0	1	1	0	2	0	0	0	0	2	0	1	1	0
1	0.88067852	26	75	2	0	2	0	0	0	2	0	0	0	2	0	1	1	0	1	0	0	1	0	0	0	1	1	0
0	0.726052908	19	63	3	0	2	0	0	0	2	0	0	0	2	0	0	1	0	1	0	0	1	0	0	0	0	1	0
1	0.529536088	14	42	3	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	1	0

5 Conclusion

The proposed method aims to solve the multi-objective RAPs and rank the most-compromised solutions among the large set of the optimal solutions. The final pruned solutions are presented to determine choices of system design for the RAP. After the approximation of Pareto-optimal solutions is obtained by NSGA-II, K-means clustering is used to cluster the Pareto-optimal solutions in to some trade-off regions. Then, this algorithm ranks the Pareto-optimal solutions that emphasizes on the cost and reliability trade-off. The alternatives are ranked from the lowest to the highest trade-off value. The cost and reliability trade-off value indicates the least amount of one objective to sacrifice in order to improve in the other objective when compare to its centroid solution. The alternative that has the smallest cost and reliability trade-off value is preferred. This is a simple method that can provide most cost-effective solutions to the decision maker.

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