

Chapter 4

Robust Weighted Measurement Fusion Kalman Filter with Uncertain Parameters and Noise Variances

Chunshan Yang and Zili Deng

Abstract For the multisensor time-invariant system with both the uncertainties noise variances and parameters, by introducing a fictitious white noise to compensate the uncertain parameters, based on the minimax robust estimation principle and the Lyapunov equation method, a robust weighted measurement fusion Kalman filter is presented. It is proved that for prescribed upper bound variance of fictitious noise, there exists a sufficiently small robust region of uncertain parameter perturbances, such that its actual filtering error variances are guaranteed to have a conservative upper bound. A simulation example shows how to search the robust region, and shows its good performances.

Keywords Uncertain parameters · Uncertain noise variances · Fictitious white noise · Weighted measurement fusion · Minimax robust Kalman filter

4.1 Introduction

Multisensor information fusion Kalman filtering has been applied to many fields, [1, 2]. One of the key assumptions in Kalman filtering is that the model parameters and noise variances are exactly known. But in many applications, this condition cannot always hold, thus the performance of the Kalman filter may degraded or an

C. Yang · Z. Deng (✉)

Department of Automation, Heilongjiang University, XueFu Road 74, Heilongjiang University, Electronic and Engineering College 130, Harbin 150080, China
e-mail: dengzili891@163.com

C. Yang
e-mail: dzl@hlju.edu.cn

C. Yang
Department of Computer Science and Technology, ChengDong College of Northeast Agricultural University, Harbin 150025, China

inexact model may cause the filter divergence [3]. This has stirred up many studies on robust Kalman filter design.

So far, robust Kalman filters for systems with uncertain parameters have been designed, two important approaches are the Riccati equation approach [3] and linear matrix inequality (LMI) approach [4]. The robust Kalman filters for systems with uncertain noise variances have been designed [5, 6], a Lyapunov equation approach is presented to prove the robustness of the proposed robust Kalman filters. Up to now, the robust Kalman filters for uncertain systems both in noise variances and model parameters are seldom considered.

In this paper, we consider these two uncertainties for multi-sensor invariant system. By introducing a fictitious white noise to compensate the uncertain model parameter, the uncertain system can be converted into the worse-case conservative system with known parameters and uncertain noise variance. Using the minimax robust estimation principle, weighted least squares method, a robust weighted measurement fusion Kalman filter is presented based on the worst-case conservative system with the conservative upper bounds of noise variances. Furthermore, the robustness of the proposed robust Kalman filters is proved by Lyapunov equation approach.

4.2 Weighted Measurement and Local Robust Steady-State Kalman Filter

Consider the true discrete system with uncertain noise variances and uncertain model parameters.

$$x(t+1) = (\Phi_e + \Delta\Phi)x(t) + \Gamma w(t) \quad (4.1)$$

$$y_i(t) = H_i x(t) + v_i(t), \quad i = 1, \dots, L \quad (4.2)$$

where state $x(t) \in R^n$, measurement of the i th subsystem $y_i(t) \in R^{m_i}$, $w(t) \in R^r$ and $v_i(t)$ are uncorrelated white noises with zero means and uncertain actual variances \bar{Q} and \bar{R}_i , respectively. Assume that Q and R_i are conservative upper bounds of \bar{Q} and \bar{R}_i , i.e.,

$$\bar{Q} \leq Q, \quad \bar{R}_i \leq R_i, \quad i = 1, \dots, L \quad (4.3)$$

Φ_e , Γ , H_i are known constant matrices. $\Phi = \Phi_e + \Delta\Phi$ is the true transition matrix. $\Delta\Phi$ is the uncertain perturbances of model parameter matrix and satisfies that

$$\Delta\Phi \in \mathfrak{R}_{\Delta\Phi} \quad (4.4)$$

And each subsystem is completely observable and completely controllable.

A fictitious white noise $\zeta(t)$ with zero mean and upper bound variance $\Delta_\xi > 0$ is used to compensate the uncertain model parameter error term $\Delta\Phi x(t)$ in (4.1), then the system (4.1) and (4.2) is transformed into the following worse-case conservative system with known parameters and uncertain noise variance

$$x(t+1) = \Phi_e x(t) + \Gamma w(t) + \zeta(t) \quad (4.5)$$

$$y_i(t) = H_i x(t) + v_i(t), \quad i = 1, \dots, L \quad (4.6)$$

Assume that each measurement matrix H_i has a common $m \times n$ right factor H [7], i.e., $H_i = M_i H$, $i = 1, \dots, L$ and define $M^{(0)} = [M_1^T, \dots, M_L^T]^T$, where the symbol T denotes the transpose. Assume that $M^{(0)}$ is of full-column rank. The centralized fusion measurement equation is given as

$$y_c(t) = H_c x(t) + v_c(t) \quad (4.7)$$

$$H_c = [H_1^T, \dots, H_L^T]^T, \quad v_c(t) = [v_1^T(t), \dots, v_L^T(t)]^T \quad (4.8)$$

and $v_c(t)$ has the conservative and actual variance matrix

$$R_c = \text{diag}(R_1, \dots, R_L), \quad \bar{R}_c = \text{diag}(\bar{R}_1, \dots, \bar{R}_L) \quad (4.9)$$

Applying the WLS method, Eq. (4.7) can be converted into

$$y_M(t) = Hx(t) + v_M(t) \quad (4.10)$$

where $y_M(t)$ is the conservative weighted fusion measurement, $v_M(t)$ is the fused measurement white noise, such that

$$y_M(t) = \sum_{i=1}^L [M_i^T R_i^{-1} M_i]^{-1} \sum_{i=1}^L M_i^T R_i^{-1} y_i(t) \quad (4.11)$$

$$v_M(t) = \sum_{i=1}^L [M_i^T R_i^{-1} M_i]^{-1} \sum_{i=1}^L M_i^T R_i^{-1} v_i(t) \quad (4.12)$$

$v_M(t)$ has the conservative and actual variances matrix [5]

$$R_M = [M^{(0)T} R_c^{-1} M^{(0)}]^{-1} \quad (4.13)$$

$$\bar{R}_M = \sum_{i=1}^L [M_i^T \bar{R}_i^{-1} M_i]^{-1} \sum_{i=1}^L M_i^T \bar{R}_i^{-1} \bar{R}_i R_i^{-1} M_i \sum_{i=1}^L [M_i^T R_i^{-1} M_i]^{-1} \quad (4.14)$$

For the conservative system (4.5) and (4.11), we have the conservative weighted measurement fusion steady-state Kalman filter

$$\hat{x}_M(t|t) = \Psi_M \hat{x}_M(t-1|t-1) + K_M y_M(t) \quad (4.15)$$

$$\Psi_M = [I_n - K_M H] \Phi_e, \quad K_M = \Sigma_M H^T [H \Sigma_M H^T + R_M]^{-1} \quad (4.16)$$

Σ_M satisfies the steady-state Riccati equation

$$\Sigma_M = \Phi_e \left[\Sigma_M - \Sigma_M H^T (H \Sigma_M H^T + R_M)^{-1} \right] \Phi_e^T + \Gamma Q \Gamma^T + \Delta_\xi \quad (4.17)$$

From (4.5), (4.11), and (4.15), we have

$$\tilde{x}_M(t|t) = \Psi_M \tilde{x}_M(t-1|t-1) + [I_n - K_M H] [\Gamma w(t-1) + \xi(t-1)] - K_M v_M(t) \quad (4.18)$$

So we have the conservative filtering error variance

$$P_M = \Psi_M P_M \Psi_M^T + [I_n - K_M H] (\Gamma Q \Gamma^T + \Delta_\xi) [I_n - K_M H]^T + K_M R_M K_M^T \quad (4.19)$$

Now we find the actual filter error variance

$$\tilde{x}_M(t|t) = x(t) - \hat{x}_M(t|t) \quad (4.20)$$

where $x(t)$ is the true state given in (4.1), $\hat{x}_M(t|t)$ is the actual Kalman filter (4.15) with $y_M(t)$ is the actual fused measurement (4.11) with $y_i(t)$ is the actual $y_i(t)$ are define by (4.1) and (4.2).

Notice that the actual system (4.1)–(4.2) and conservative system (4.5)–(4.6) have the same weighted measurement fusion equation as (4.10)–(4.11).

For the actual system, $x(t)$ is defined by (4.1) and $y_M(t)$ is defined by the actual measurement based on (4.1) and (4.2). Hence we have the actual error

$$\tilde{x}_M(t|t) = \Phi_e x(t-1) + \Delta \Phi x(t-1) + \Gamma w(t-1) - \hat{x}_M(t|t) \quad (4.21)$$

From (4.1), (4.2) and (4.15) we have

$$\begin{aligned} \tilde{x}_M(t|t) &= \Psi_M \tilde{x}_M(t-1|t-1) + [I_n - K_M H] \Gamma w(t-1) \\ &\quad + [I_n - K_M H] \Delta \Phi x(t-1) - K_M v_M(t) \end{aligned} \quad (4.22)$$

So we have the actual filtering error variance

$$\begin{aligned} \bar{P}_M &= \Psi_M \bar{P}_M \Psi_M^T + [I_n - K_M H] (\Gamma \bar{Q} \Gamma^T + \Delta \Phi X \Delta \Phi^T) [I_n - K_M H]^T \\ &\quad + [I_n - K_M H] \Delta \Phi C \Psi_M^T + \Psi_f C^T \Delta \Phi^T [I_n - K_f H]^T + K_M \bar{R}_M K_M^T \end{aligned} \quad (4.23)$$

where $X = E[x(t)x^T(t)]$, $C = E[x(t)\tilde{x}^T(t|t)]$, X and C satisfy the following equation, respectively

$$X = \Phi X \Phi^T + \Gamma \bar{Q} \Gamma^T \quad (4.24)$$

$$C = \Phi C \Psi_M^T + \Phi X \Delta \Phi^T [I_n - K_M H]^T + \Gamma \bar{Q} \Gamma^T [I_n - K_M H]^T \quad (4.25)$$

Lemma 4.1 [8] *Consider the Lyapunov equation with U being a symmetric matrix*

$$P = F P F^T + U \quad (4.26)$$

If the matrix F is stable and U is positive (semi-)definite, then the solution P is unique, symmetric, and positive (semi-)definite.

Theorem 4.1 *For multisensor uncertain system (4.1) and (4.2) with uncertain model parameters and uncertain noise variances, the actual steady-state Kalman filter are robust in the sense that for all admissible model parameters $\Delta \Phi$ satisfying (4.4) with δ_φ being a sufficiently small position number, we have*

$$\bar{P}_M < P_M \quad (4.27)$$

Proof Define $\Delta P_M = P_M - \bar{P}_M$, subtracting (4.23) from (4.19) yields the Lyapunov equation $\Delta P_M = \Psi_M \Delta P \Psi_M^T + V_M$ with the definition

$$V_M = [I_n - K_M H] \Gamma (Q - \bar{Q}) \Gamma^T [I_n - K_M H]^T + K_M (R - \bar{R}_M) K_M^T + \bar{V}_M \quad (4.28)$$

$$\begin{aligned} \bar{V}_M = & [I_n - K_M H] \Delta_\xi [I_n - K_M H]^T - [I_n - K_M H] \Delta \Phi X \Delta \Phi^T [I_n - K_M H]^T \\ & - [I_n - K_M H] \Delta \Phi C \Psi_M^T - \Psi_M C^T \Delta \Phi^T [I_n - K_M H]^T \end{aligned} \quad (4.29)$$

From $[I_n - K_M H] = P_M \Sigma_M^{-1}$, we have that $\det[I_n - K_M H] = \det(P_M \Sigma_M^{-1}) \neq 0$, so that $[I_n - K_M H]$ is invertible, and

$$V_0 = [I_n - K_M H] \Delta_\xi [I_n - K_M H]^T > 0 \quad (4.30) \quad \square$$

From (4.29) and (4.30), when $\Delta \Phi \rightarrow 0$, then $\bar{V}_M \rightarrow V_0$. Hence there exists a sufficiently small robust region $\mathfrak{R}_{\Delta \Phi}$ of uncertain $\Delta \Phi$, such that for all $\Delta \Phi \in \mathfrak{R}_{\Delta \Phi}$, it follows that $\bar{V}_M > 0$. Applying Lemma 4.1 to Lyapunov equation $\Delta P_M = \Psi_M \Delta P \Psi_M^T + V_M$ yields $\Delta P_M \geq 0$, i.e., $\bar{P}_M < P_M$. The proof is completed.

Similarly, for the local subsystem (4.1) and (4.2) we can obtain the robust local steady-state Kalman filter $\hat{x}_i(t|t)$ with the conservative and actual variances P_i and \bar{P}_i , and with the robustness $\bar{P}_i < P_i$, $P_M < P_i$, $i = 1, \dots, L$.

Corollary 4.1 *We have the accuracy relations*

$$\bar{P}_i < P_i, \bar{P}_M < P_M \leq P_i, i = 1, \dots, L \quad (4.31)$$

$$\bar{P}_c < P_c, \text{tr}\bar{P}_M < \text{tr}P_M < \text{tr}P_i, i = 1, \dots, L \quad (4.32)$$

Proof For the conservative system, we have $P_M = P_c$, where P_c is conservative variance of the centralized fusion Kalman filter for system (4.5) and (4.7). From $P_c \leq P_i$, we have $P_M \leq P_i$, $i = 1, \dots, L$. The proof is completed. \square

4.3 Simulation Example

Consider the 2-sensor invariant tracking system (4.1) and (4.2) with $\Phi_e = \begin{bmatrix} 0.8 & 0.3 \\ 0.5 & 0 \end{bmatrix}$, $\Delta\Phi = \begin{bmatrix} 0 & 0 \\ 0 & \delta \end{bmatrix}$, $\Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $H_1 = [1 \ 0]$, $Q = 1.5$, $R_1 = 2.5$, $H_2 = [0 \ 1]$, $\bar{Q} = 1.0$, $\bar{R}_1 = 2.0$, $R_2 = 4.5$, $\bar{R}_2 = 3.8$. The simulation results are given in the following. δ is uncertain perturbances of parameter.

The common right factor we select is $H = I_2$. Taking the conservative upper bound of the compensating fictitious noise variance as $\Delta\xi = 0.5I_2$. The values of determinant \bar{V}_M changed with the uncertainty δ are shown in Fig. 4.1. From Fig. 4.1, the robust region of uncertainty δ is $\mathfrak{R}_{\Delta\Phi} = \{\delta \mid \det \bar{V}_M > 0\} = (-0.806, 0.171)$, which ensures $\Delta P_M > 0$, i.e., $\bar{P}_M < P_M$.

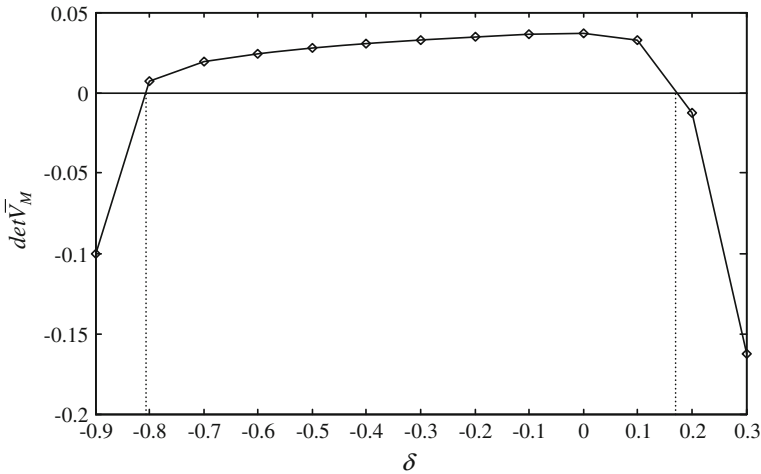


Fig. 4.1 The robust region of the fused robust Kalman filter

When \bar{Q} varies from 0 to Q , the changes of robust region of the uncertainty with \bar{Q} are given in Fig. 4.2. From Fig. 4.2, we can obtain that when \bar{Q} varies from 0 to Q , the robust region of the fused Kalman filter narrows.

A three-dimensional figure of the robust region of the Kalman filter is given in Fig. 4.3. From Fig. 4.3, we can see that the robust region of the fused robust Kalman filter how changes over \bar{Q} and δ .

Taking $\Delta_\xi = 0.5I_2$, $\delta = 0.1$ in the robust region, the comparisons of filtering performance among the weighted measurement fusion optimal, robust, and suboptimal Kalman filters are given in Fig. 4.4. From Fig. 4.4, we can see that the performance of suboptimal Kalman filter is clearly worse than that of the other two filters, because it does not consider the uncertainty of model parameter and noise variances, so suboptimal Kalman filter leads to serious performance loss.

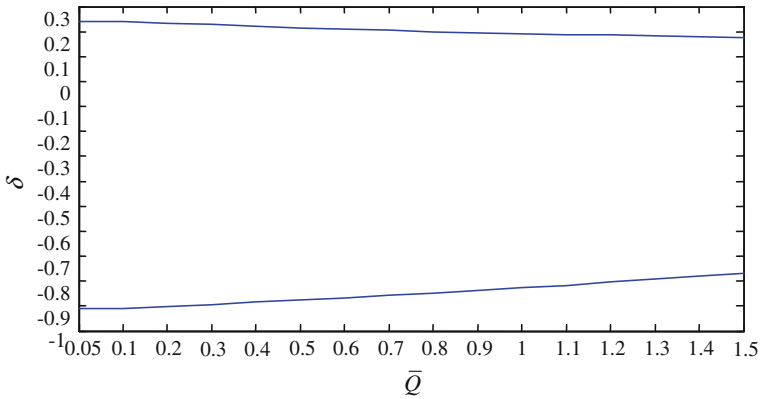


Fig. 4.2 The robust region of the fused robust Kalman filter changes over \bar{Q}

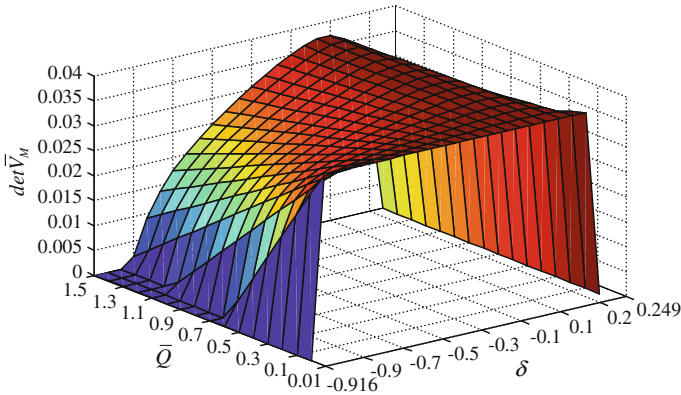


Fig. 4.3 The robust region of the fused robust Kalman filter changes over \bar{Q} and δ

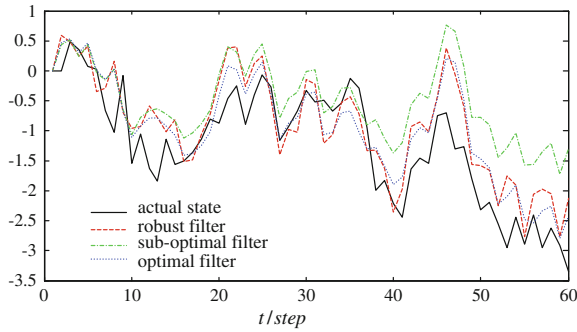


Fig. 4.4 The comparison among the optimal, robust, and suboptimal Kalman filters

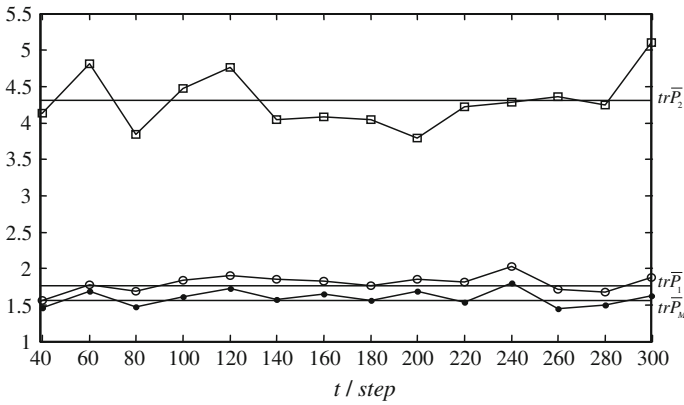


Fig. 4.5 The MSE curves of local and fused robust Kalman filters

In order to verify the above theoretical accuracy relations, Fig. 4.5 gives the mean square error (MSE) curves with $\rho = 200$ Monte Carlo simulation runs. According to the ergodicity [9], we have

$$MSE_{\theta}(t) \rightarrow tr\bar{P}_{\theta}, \text{ as } t \rightarrow \infty, \rho \rightarrow \infty, (\theta = 1, 2, M) \quad (4.33)$$

From Fig. 4.5, we can see that when $t \rightarrow \infty$, the values of $MSE(t)$ are close to the corresponding theoretical values $tr\bar{P}_{\theta}$, which verifies the robust accuracy relation (4.31).

4.4 Conclusion

For the multisensor system with uncertain parameters and noise variances, using a fictitious noise approach to compensate parameter uncertainties, a robust weighted measurement fusion Kalman filter has been presented based on the worst-case conservative system with the conservative upper bounds of noise variances. Based on the Lyapunov equation approach, its robustness is proved, and their robust accuracy is higher than that of each local robust Kalman filter. A search approach for finding the robust region is given.

Acknowledgments This work is supported by the Natural Science Foundation of China under grant NSFC-60874063 and NSFC-60374026.

References

1. Shalom YB, Li XR, Kirubarajan T (2001) Estimation with applications to tracking and navigation. Wiley, New York
2. Sun S, Deng Z (2004) Multi-sensor optimal information fusion Kalman filter. *Automatica* 40 (6):1017–1023
3. Sriyananda H (1972) A simple method for the control of divergence in Kalman filter algorithms. *Int J Control* 16(6):1101–1106
4. Ebinara Y, Hagivara T (2005) A dilated LMI approach to robust performance analysis of linear time-invariant uncertain systems. *Automatica* 41(11):1933–1941
5. Qi W, Zhang P, Deng Z (2014) Robust weighted fusion Kalman filters for multisensor time-varying systems with uncertain noise variances. *Sig Process* 99:185–200
6. Qi W, Zhang P, Nie G-H, Deng Z (2014) Robust weighted fusion Kalman predictors with uncertain noise variances. *Digit Signal Proc* 30:37–54
7. Ran CJ, Hu YS, Gu L, Deng ZL (2008) Correlated measurement fusion steady-state Kalman filtering algorithms and their optimality. *Acta Automatica Sinica* 34:233–239
8. Kailath T, Sayed AH, Hassibi B (2000) *Linear estimation*. Prentice Hall, New York
9. Ljung L (1999) *System identification. Theory for user*, 2nd edn. Prentice Hall PTR, Tsinghua University Press, Beijing