Chapter 3
Robust Centralized Fusion Steady-State Kalman Predictor with Uncertain Parameters

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Abstract For multisensor time-invariant systems with uncertain parameter and known noise variances, the centralized fusion robust steady-state Kalman predictor based on the minimax robust estimation principle is presented by a new approach of compensating the parameter uncertainties by fictitious noise. Using the Lyapunov equation, it is proved that the variances of its actual prediction error variances have a conservative upper bound when the uncertainty of parameters is restricted in a sufficiently small region, which is called the robust region of the parameter uncertainties. It is also proved that the robust accuracy of the centralized fuser is higher than that of each local robust Kalman predictor. A simulation example shows how to search the robust region and shows its good performances.

Keywords Robust · Kalman predictor · Uncertain parameters · Centralized fusion · Lyapunov equation approach

3.1 Introduction

Multisensor information fusion has been applied to many fields, including military affairs, navigation, guidance, remote sensing, signal processing, target tracking. There exist two basic fusion methods: one is the centralized fusion approach [1],

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which can give a globally optimal state estimate by directly combing the local measurement equations to obtain an augmented measurement equation. The other is the distributed fusion approach [2–5], which can combine or weight the local Kalman estimators to obtain a global optimal or suboptimal state estimator.

The standard Kalman filtering is only suitable for the systems with exactly known model. For uncertain systems with the uncertainties of model parameters and/or noise variances, the performance of the Kalman filter will degrade or the filter may be divergent [6]. However, since the system model is usually an approximation to a physical situation in many applications, the research on robust Kalman filters for uncertain systems received great attention. An important class of robust Kalman filtering problems is to find a Kalman filter such that its actual filtering error variances yielded by all admissible uncertainties are guaranteed to have a minimal upper bound [7]. Such a Kalman filter is called robust Kalman filter, and such property is called robustness. There are two main approaches to solve this problem, i.e., the Riccati equation approach [8] and the linear matrix inequality (LMI) approach [9]. The limitation of the above robust Kalman filters is that only model parameters are assumed to be uncertain, while the noise variances are assumed to be exactly known.

Centralized fusion steady-state robust Kalman filter [10] for multisensor systems with uncertainty of noise variances, the local and centralized fusion robust steady-state Kalman filter are presented.

In this paper, we consider the problem of designing the local and centralized fusion robust steady-state Kalman predictors for systems with uncertain parameters and known noise variances by a fictitious noise-based compensation technique. The uncertainty of parameters is compensated by introducing a fictitious noise with upper bound variance. Further, we can obtain the robust region by the searching method. Finally, it is proved that the robust accuracy of the centralized fuser is higher than that of the local robust Kalman predictor.

3.2 Local and Centralized Fusion Robust Kalman Predictors

Consider the multisensor system with uncertain parameters

$$x(t+1) = (\Phi_e + \Delta\Phi)x(t) + \Gamma\omega(t) \tag{3.1}$$

$$y_i(t) = H_i x(t) + v_i(t),\ i = 1, \ldots, L \tag{3.2}$$

$$\Phi = \Phi_e + \Delta\Phi \tag{3.3}$$

where t is the discrete time, $x(t) \in R^n$ is the state to be estimated, $y_i(t) \in R^{m_i}$ is the measurement of the ith subsystem, $\omega(t) \in R^r$ is the input noise, $v_i(t) \in R^{m_i}$ is the measurement noise, and they are mutually uncorrelated white noises with zero means and known variances Q and R_i. Φ is the true transition matrix, Φ_e is a known estimate of Φ and $\Delta\Phi$ is the uncertain parameter disturbance matrix. Γ, H_i, Q and R_i are known constant matrices with appropriate dimensions.

$$\Delta\Phi \in \Re_{\Delta\Phi} \tag{3.4}$$

$\Re_{\Delta\Phi}$ can be found or prescribed.

From (3.1)

$$x(t+1) = \Phi_e x(t) + \Delta\Phi x(t) + \Gamma\omega(t) \tag{3.5}$$

Introducing a fictitious white noise $\xi(t)$ with zero mean and known upper bound variance Δ_ξ of variances, which compensates the model error term $\Delta\Phi x(t)$, then we have the worst-case conservative multisensor system

$$x(t+1) = \Phi_e x(t) + \xi(t) + \Gamma\omega(t) \tag{3.6}$$

$$y_i(t) = H_i x(t) + v_i(t),\ i = 1, \ldots, L \tag{3.7}$$

where $\omega(t)$ and $v_i(t)$ have the known true variances Q and R_i.

The conservative centralized fused system is given as

$$x(t+1) = \Phi_e x(t) + \xi(t) + \Gamma\omega(t) \tag{3.8}$$

$$y_c(t) = H_c x(t) + v_c(t) \tag{3.9}$$

$$y_c(t) = \left[y_1^{\mathrm{T}}(t), \ldots, y_L^{\mathrm{T}}(t)\right]^{\mathrm{T}},\ H_c = \left[H_1^{\mathrm{T}}, \ldots, H_L^{\mathrm{T}}\right]^{\mathrm{T}},\ v_c(t) = \left[v_1^{\mathrm{T}}(t), \ldots, v_L^{\mathrm{T}}(t)\right]^{\mathrm{T}} \tag{3.10}$$

where the symbol T denotes the transpose. $v_c(t)$ is the conservative fused noise, and has the variance $R_c = diag(R_1, \ldots, R_L)$.

The conservative centralized fusion Kalman predictor is given as

$$\hat{x}(t+1|t) = \Psi_c \hat{x}(t|t-1) + \mathrm{K}_c y_c(t) \tag{3.11}$$

where $y_c(t)$ is conservative measurement, and

$$\Psi_c = \Phi_e - K_c H_c,\ K_c = \Phi_e \Sigma_c H_c^{\mathrm{T}}\left(H_c \Sigma_c H_c^{\mathrm{T}} + R_c\right)^{-1} \tag{3.12}$$

where Ψ_c is stable, the conservative prediction error variance Σ_c satisfies the Riccati equation

$$\Sigma_c = \Phi_e\left[\Sigma_c - \Sigma_c H_c^{\mathrm{T}}\left(H_c \Sigma_c H_c^{\mathrm{T}} + R_c\right)^{-1} H_c \Sigma_c\right]\Phi_e^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} + \Delta_\xi \tag{3.13}$$

From (3.6), (3.9), and (3.11), we easily obtain the conservative prediction error system

$$\tilde{x}(t+1|t) = \Psi_c \tilde{x}(t|t-1) + \Gamma\omega(t) + \xi(t) - K_c v_c(t) \tag{3.14}$$

where $\tilde{x}(t+1|t) = x(t+1) - \hat{x}(t+1|t)$, $x(t+1)$ is the conservative state in (3.6), $\hat{x}(t+1|t)$ is the conservative Kalman predictor in (3.11). $v_c(t)$ is the conservative fused noise with variance R_c.

This yields the conservative variance Σ_c satisfies the Lyapunov equation

$$\Sigma_c = \Psi_c \Sigma_c \Psi_c^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} + \Delta_\xi + K_c R_c K_c^{\mathrm{T}} \tag{3.15}$$

Now we find the actual prediction error

$$\tilde{x}(t+1|t) = x(t+1) - \hat{x}(t+1|t) \tag{3.16}$$

where $x(t+1)$ is the true state given by (3.1), and $\hat{x}(t+1|t)$ is the actual Kalman predictor (3.11) with $y_c(t)$ is the actual measurement, i.e., $y_c(t) = \left[y_1^{\mathrm{T}}(t), \ldots, y_L^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, where $y_i(t)$ is the actual measurement, which is available, and which is yielded from (3.1) and (3.2). Hence from (3.1), (3.9) and (3.11) we obtain

$$\tilde{x}(t+1|t) = \Psi_c \tilde{x}(t|t-1) + \Delta\Phi x(t) + \Gamma\omega(t) - K_c v_c(t) \tag{3.17}$$

where $v_c(t)$ is the actual fused noise with variance $\bar{R}_c = diag(\bar{R}_1, \ldots, \bar{R}_L)$.

Thus we obtain the actual predictor error variance $\bar{\Sigma}_c$ satisfies the Lyapunov equation

$$\bar{\Sigma}_c = \Psi_c \bar{\Sigma}_c \Psi_c^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} + K_c R_c K_c^{\mathrm{T}} + \Delta\Phi X \Delta\Phi^{\mathrm{T}} + \Delta\Phi \underline{C} \Psi_c^{\mathrm{T}} + \Psi_c \underline{C}^{\mathrm{T}} \Delta\Phi^{\mathrm{T}} \tag{3.18}$$

where we defind the steady-state cross-covariance

$$\underline{C} = E\left[x(t)\tilde{x}^{\mathrm{T}}(t|t-1)\right] = E\left[x(t+1)\tilde{x}(t+1|t)^{\mathrm{T}}\right] \tag{3.19}$$

From (3.1), (3.17) and (3.19) we obtain the Lyapunov equation

$$\underline{C} = \Phi \underline{C} \Psi_C^{\mathrm{T}} + \Phi X \Delta\Phi^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} \tag{3.20}$$

with the definition $X = E[x(t)x^{\mathrm{T}}(t)]$. From (3.1) we have

$$X = \Phi X \Phi^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} \tag{3.21}$$

$$\Phi = (\Phi_e + \Delta\Phi) \tag{3.22}$$

Theorem 3.1 *For multisensor system (3.1, 3.2 and 3.3) with uncertain parameters and known noise variances, the actual centralized fusion steady-state Kalman predictor (3.11) with the actual fused measurement $y_c(t)$, is robust in the sense that for the prescribed upper bound $\Delta_\xi > 0$ of fictitious noise variances, there exists a sufficiently small region $\Re_{\Delta\Phi}$, such that for all admissible uncertain disturbance $\Delta\Phi \in \Re_{\Delta\Phi}$, we have*

$$\bar{\Sigma}_c < \Sigma_c \tag{3.23}$$

which is called the robustness of robust Kalman predictor.

Proof Letting $\Delta\Sigma_c = \Sigma_c - \bar{\Sigma}_c$, from (3.15) and (3.18) we have the Lyapunov equation

$$\Delta\Sigma_c = \Psi_c \Delta\Sigma_c \Psi_c^{\mathrm{T}} + \Delta_\xi - \Delta\Phi X \Delta\Phi^{\mathrm{T}} - \Delta\Phi \underline{C} \Psi_c^{\mathrm{T}} - \Psi_c \underline{C}^{\mathrm{T}} \Delta\Phi^{\mathrm{T}} \tag{3.24}$$

Defining

$$U = \Delta_\xi - \Delta\Phi X \Delta\Phi^{\mathrm{T}} - \Delta\Phi \underline{C} \Psi_c^{\mathrm{T}} - \Psi_c \underline{C}^{\mathrm{T}} \Delta\Phi^{\mathrm{T}} \tag{3.25}$$

Since $\Delta\Phi \to 0$, $U \to \Delta_\xi > 0$, hence there exists a sufficiently small region $\Re_{\Delta\Phi}$, such that for all $\Delta\Phi \in \Re_{\Delta\Phi}$,we have

$$U > 0 \tag{3.26}$$

Form (3.24–3.26) we obtain $\Delta\Sigma_c > 0$, i.e.,

$$\bar{\Sigma}_c < \Sigma_c \tag{3.27}$$

The proof is completed. □

Remark 3.1 Similar to the derivation of the centralized fusion robust Kalman predictor, for the system (3.1–3.3), we can also obtain the corresponding local robust Kalman predictors $\hat{x}_i(t+1|t), i = 1, \ldots, L$ with the actual variance $\bar{\Sigma}_i$ and the conservative upped bounds Σ_i, and similar to the derivation of Theorem 3.1, we have the robustness $\bar{\Sigma}_i < \Sigma_i$.

Theorem 3.2 *The local and centralized robust Kalman predictors have the robust accuracy relation*

$$\Sigma_c < \Sigma_i, i = 1, \ldots, L \tag{3.28}$$

$$\mathrm{tr}\bar{\Sigma}_c < \mathrm{tr}\Sigma_c < \mathrm{tr}\Sigma_i, i = 1, \ldots, L \tag{3.29}$$

where the symbol tr *denote the trace of matrix.*

Proof For the worst-case conservative system (3.6) and (3.9), applying [10] yields (3.28) to hold. Taking the trace operations to (3.23) and (3.28) yield (3.29).The proof is completed. □

Remark 2 The trace $\mathrm{tr}\Sigma_c$ is called as robust accuracy, and the trace $\mathrm{tr}\bar{\Sigma}_c$ is called as actual accuracy. This shows that the actual accuracy of a robust Kalman predictor is higher than its robust accuracy. The robust accuracy of the centralized fuser is higher than that of each local robust Kalman predictor.

3.3 Simulation Example

Consider two-dimensional 2-sensors time-invariant system (3.1–3.3) with uncertain parameter, where $x(t) = [x_1(t), x_2(t)]^T$ is the state. In the simulation, we take $\Phi_e = \begin{bmatrix} 0.3 & -0.5 \\ 1 & 0.5 \end{bmatrix}, \Delta\Phi = \begin{bmatrix} \delta & 0 \\ 0 & 0 \end{bmatrix}, \Gamma = \begin{bmatrix} -6 \\ 1 \end{bmatrix}, H_1 = [1 \quad 1], H_2 = [1 \quad 1], Q = 1.5,$ $R_1 = 20, R_2 = 3.5$.

The conservative prediction error variances of the local robust and centralized robust fused steady-state Kalman predictors are given in Table 3.1.

When the fictitious noise variance $\Delta_\xi = \alpha I_2$ is prescribed, the robust region of uncertainty in the state matrix can be obtained by the searching method. When $\alpha = 3.6$, from Table 3.2, we can obtain that the robust region of centralized robust fused Kalman prediction is $-0.3 < \delta < 0.1$, which ensures $\det \Delta\Sigma_c > 0$, which yields $\Delta\Sigma_c > 0$. Similarly, the robust regions of the local robust Kalman prediction are

Table 3.1 The conservative prediction error variances of the local and centralized robust steady-state Kalman predictors

Σ_1	Σ_2	Σ_c
$\begin{bmatrix} 68.6106 & -0.6727 \\ -0.6727 & 21.9944 \end{bmatrix}$	$\begin{bmatrix} 64.1738 & -4.2876 \\ -4.2876 & 10.6171 \end{bmatrix}$	$\begin{bmatrix} 63.9180 & -4.5165 \\ -4.5165 & 10.0066 \end{bmatrix}$

Table 3.2 The determinants of $\Delta\Sigma_\theta$, $\theta = 1, 2, c$ with respect to δ

δ	det $\Delta\Sigma_1$	det $\Delta\Sigma_2$	det $\Delta\Sigma_c$
−0.5	−54.2566	−28.7639	−28.5661
−0.4	−24.2111	−8.1500	−7.9089
−0.3	−1.3032	7.5152	7.7730
−0.2	14.5833	18.3569	18.6248
−0.1	22.5712	23.8877	24.1846
0.0	20.4353	22.7732	23.1547
0.1	3.8500	12.2712	12.8509
0.2	−35.1000	−13.0677	−12.0807

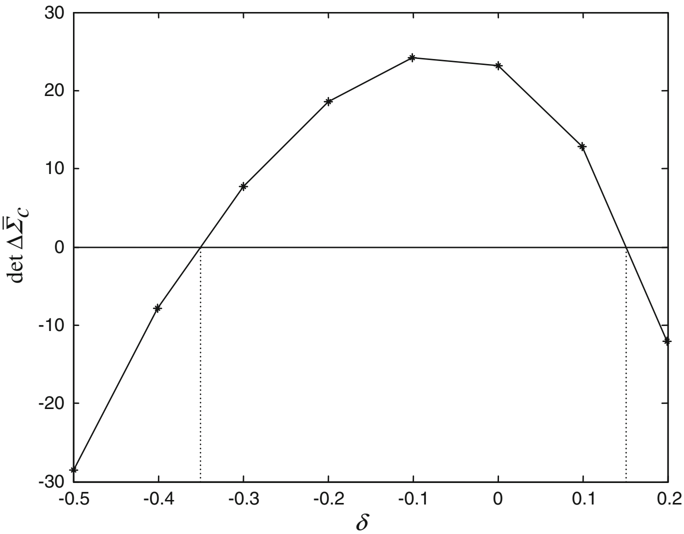

Fig. 3.1 Robust region of the centralized robust Kalman predictor

$-0.2 < \delta < 0.1$ and $-0.3 < \delta < 0.1$ respectively, which ensures $\det \Delta\Sigma_i > 0$, which yields $\Delta\Sigma_i > 0$.

From Fig. 3.1, we have a parabola going downwards. It can obtain a more precise robust region of centralized fusion robust Kalman predictor $-0.3534 < \delta < 0.1607$ by dichotomy, so that $\det \Delta\bar{\Sigma}_c > 0$ and $\bar{\Sigma}_c < \Sigma_c$ in this robust region.

The 1,000 Monte Carlo runs are performed. The MSE curves of the local and centralized robust steady-state Kalman predictors are shown in Fig. 3.2.

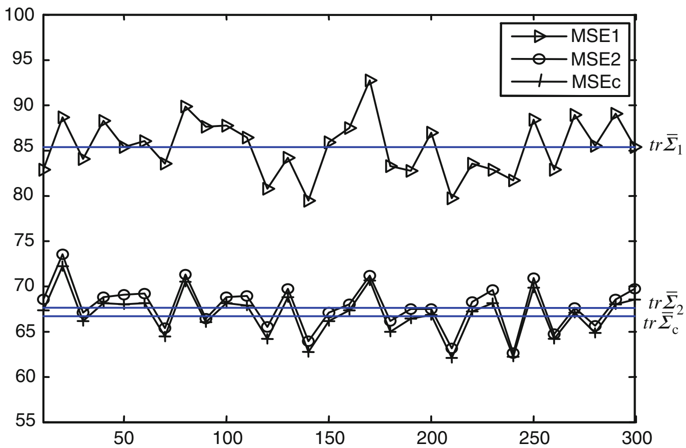

Fig. 3.2 MSE curves of the local robust and centralized robust fused Kalman predictors

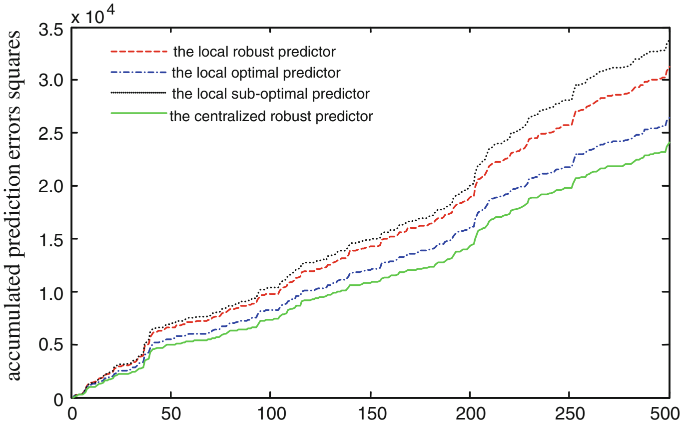

Fig. 3.3 The curves of accumulated prediction errors squares for component $x_1(t)$

From Fig. 3.2, we see that the values of $\mathrm{MSE}_\theta(t)$ are close to the corresponding $\mathrm{tr}\,\Sigma_\theta$, $\theta = 1, 2, c$ and the accuracy relation (3.29) holds. The curves of accumulated prediction errors squares for component $x_1(t)$ are shown in Fig. 3.3. From Fig. 3.3, we can see that the actual accuracy of centralized robust fused steady-state Kalman predictor is superior to others.

3.4 Conclusion

For multisensor systems with uncertainty parameters, a new robust Kalman prediction approach of compensating parametric uncertain by fictitious noise was presented. The problem is converted into the robust Kalman prediction problem for the system with uncertain noise variances. The local and centralized robust steady-state Kalman prediction algorithms are presented. Based on the Lyapunov equation, it is proved that the robustness of local and centralized robust fusion Kalman predictor, i.e., the actual predictor error variance have a conservative upper bound for all the admissible uncertainties. This approach is different from the Riccati equation approach and the linear matrix inequality (LMI) approach. The simulation results show that actual accuracy of centralized robust fusion Kalman predictor is higher than those of the local optimal Kalman predictor and the local suboptimal predictor. The simulation shows at how to search the robust region and shows its good performances.

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