Chapter 2
Robust Covariance Intersection Fusion Steady-State Kalman Filter with Uncertain Parameters

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Abstract For the linear discrete time-invariant system with uncertain parameters and known noise variances, a robust covariance intersection (CI) fusion steady-state Kalman filter is presented by the new approach of compensating the parameter uncertainties by a fictitious noise. Based on the Lyapunov equation approach, it is proved that for the prescribed upper bound of the fictitious noise variances, there exists a sufficiently small region of uncertain parameters; such that its actual filtering error variances are guaranteed to have a less-conservative upper bound. This region is called the robust region. By the searching method, the robust region can be found. Its robust accuracy is higher than that of each local robust Kalman filter. A Monte-Carlo simulation example shows its effectiveness and the good performance.

Keywords Covariance intersection fusion · Robust Kalman filter · Uncertain parameters · Fictitious noise approach · Robust region

2.1 Introduction

Multisensor information fusion Kalman filtering has been applied to many fields, such as signal processing, data fusion, and target tracking. For Kalman filtering fusion, there are two basic fusion methods: The centralized and distributed fusion methods. For the distributed fusion method, the three-weighted state fusion approaches weighted by matrices, diagonal matrices, and scalars have been presented. In order to compute the weights, the cross-covariances among the local filtering errors are required. However, in many practical applications, the computation of the cross-covariance is very difficult [1]. In order to overcome this limitation, the covariance intersection fusion algorithm has been presented [2].

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In this paper, a robust CI fusion steady-state Kalman filter is presented for system with uncertain parameters and known noise variances. Two important approaches used to develop the robust Kalman filter are the Riccati equation approach [3] and the linear matrix inequality (LMI) approach [4]. More research references on this topic are using these two approaches; however, in this paper, a new approach is presented by compensating the uncertain parameters by a fictitious noise which converts the system with uncertain parameters into the system with noise variance uncertainties [5].

This paper extends the robust CI fusion Kalman filter with uncertain noise variances [5] to the robust CI fusion Kalman filter with uncertain parameters. Compared with the suboptimal Kalman filter without fictitious noise, the proposed robust Kalman filter can significantly improve the filtering performance, and its robust accuracy is higher than that of each local robust Kalman filter.

2.2 Local Robust Steady-State Kalman Filter

Consider the multisensor uncertain system with model parameters uncertainties

$$x(t+1) = (\Phi_e + \Delta\Phi)x(t) + \Gamma w(t) \tag{2.1}$$

$$y_i(t) = H_i x(t) + v_i(t),\ i = 1, \ldots, L \tag{2.2}$$

where t is the discrete time, $x(t) \in R^n$ is the state to be estimated, $y_i(t) \in R^{m_i}$ is the measurement of the ith subsystem, $w(t) \in R^r$, $v_i(t) \in R^{m_i}$ are uncorrelated white noises with zero means and known variances Q and R_i, respectively. Φ_e, Γ, and H_i are known constant matrices with appropriate dimensions. L is the number of sensors. $\Phi = \Phi_e + \Delta\Phi$ is uncertain transition matrix, $\Delta\Phi$ is the uncertain parameter disturbance. Assume that Φ and Φ_e are stable matrices.

$\xi(t)$ is a uncertain fictitious white noise with zeros mean and upper-bound variance $\Delta_\xi > 0$, which is used to compensate the uncertain model parameter error term $\Delta\Phi x(t)$ in (2.1), so that the systems (2.1) and (2.2) with uncertain model parameters can be converted into the following worst-case conservative system with known model parameters and noise variances Q, R_i, and Δ_ξ.

$$x_e(t+1) = \Phi_e x_e(t) + w_e(t),\ w_e(t) = \Gamma w(t) + \xi(t) \tag{2.3}$$

$$y_{ei}(t) = H_i x_e(t) + v_i(t),\ i = 1, \ldots, L \tag{2.4}$$

Assume that each conservative subsystem is completely observable and completely controllable. The conservative local steady-state optimal Kalman filters are given as

$$\hat{x}_{ei}(t|t) = \Psi_i \hat{x}_{ei}(t-1|t-1) + K_i y_{ei}(t) \tag{2.5}$$

$$\Psi_i = [I_n - K_i H_i]\Phi_e,\ K_i = \Sigma_i H_i^{\mathrm{T}}\left(H_i \Sigma_i H_i^{\mathrm{T}} + R_i\right)^{-1},\ P_i = [I_n - K_i H_i]\Sigma_i \tag{2.6}$$

where I_n is an $n \times n$ identity matrix, Ψ_i is a stable matrix, and Σ_i satisfies the steady-state Riccati equation

$$\Sigma_i = \Phi_e\left[\Sigma_i - \Sigma_i H_i^{\mathrm{T}}(H_i \Sigma_i H_i^{\mathrm{T}} + R_i)^{-1} H_i \Sigma_i\right]\Phi_e^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} + \Delta_\xi \tag{2.7}$$

where the symbol T denotes the transpose. Define $\tilde{x}_{ei}(t|t) = x_e(t) - \hat{x}_{ei}(t|t)$, applying (2.3) and (2.5), we have

$$\begin{aligned} \tilde{x}_{ei}(t|t) = \Psi_i \tilde{x}_{ei}(t-1|t-1) + [I_n - K_i H_i]\Gamma w(t-1) \\ + [I_n - K_i H_i]\xi(t-1) - K_i v_i(t) \end{aligned} \tag{2.8}$$

Applying (2.8) yields that the conservative local filtering error variances P_i and cross-covariance P_{ij} satisfy the conservative Laypunov equation

$$\begin{aligned} P_{ij} = \Psi_i P_{ij} \Psi_j^{\mathrm{T}} + [\mathrm{I}_n - \mathrm{K}_i \mathrm{H}_i]\Gamma Q \Gamma^{\mathrm{T}}\left[I_n - K_j H_j\right]^{\mathrm{T}} \\ + [\mathrm{I}_n - \mathrm{K}_i \mathrm{H}_i]\Delta_\xi\left[I_n - K_j H_j\right]^{\mathrm{T}} + K_i R_{ij} K_j^{\mathrm{T}} \delta_{ij}, \quad i,j = 1,\ldots,L \end{aligned} \tag{2.9}$$

where δ_{ij} is the Kronecker δ function, $\delta_{ii} = 1$, $\delta_{ij} = 0$ (i ≠ j).

Remark 2.1 Notice that in (2.5), the conservative measurements $y_{ei}(t)$ are unavailable, only the actual measurements $y_i(t)$ are known. Therefore, replacing the conservative measurements $y_{ei}(t)$ with the known actual measurements $y_i(t)$, we obtain the actual local Kalman filters as

$$\hat{x}_i(t|t) = \Psi_i \hat{x}_i(t-1|t-1) + K_i y_i(t) \tag{2.10}$$

From (2.10) and (2.11) we have

$$\begin{aligned} \tilde{x}_i(t|t) = \Psi_i \tilde{x}_i(t-1|t-1) + [I_n - K_i H_i]\Delta\Phi x(t-1) \\ + [I_n - K_i H_i]\Gamma w(t-1) - K_i v_i(t) \end{aligned} \tag{2.11}$$

So the actual local filtering error variances and cross-covariance are given as

$$\begin{aligned} \bar{P}_{ij} = \Psi_i \bar{P}_{ij} \Psi_j^{\mathrm{T}} + [\mathrm{I}_n - \mathrm{K}_i \mathrm{H}_i]\Delta\Phi X \Delta\Phi^{\mathrm{T}}\left[I_n - K_j H_j\right]^{\mathrm{T}} \\ + [\mathrm{I}_n - \mathrm{K}_i \mathrm{H}_i]\Gamma Q \Gamma^{\mathrm{T}}\left[I_n - K_j H_j\right]^{\mathrm{T}} + [\mathrm{I}_n - \mathrm{K}_i \mathrm{H}_i]\Delta\Phi C_j \Psi_j^{\mathrm{T}} \\ + \Psi_i C_i^{\mathrm{T}} \Delta\Phi^{\mathrm{T}}\left[I_n - K_j H_j\right]^{\mathrm{T}} + K_i R_{ij} K_j^{\mathrm{T}} \delta_{ij} \end{aligned} \tag{2.12}$$

where $X = \mathrm{E}\left[x(t)x^{\mathrm{T}}(t)\right], C_i = \mathrm{E}\left[x(t)\tilde{x}_i^{\mathrm{T}}(t|t)\right]$. From (2.1), X satisfies the following Lyapunov equation

$$\mathrm{X} = \Phi \mathrm{X} \Phi^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}} \tag{2.13}$$

Applying (2.1) and (2.11), we have the Lyapunov equation

$$C_i = \Phi C_i \Psi_i^{\mathrm{T}} + \Phi X \Delta\Phi^{\mathrm{T}}[I_n - K_i H_i]^{\mathrm{T}} + \Gamma Q \Gamma^{\mathrm{T}}[I_n - K_i H_i]^{\mathrm{T}} \tag{2.14}$$

Lemma 2.1 [6] Consider the Lyapunov equation with U to be a symmetric matrix

$$P = FPF^{\mathrm{T}} + U \tag{2.15}$$

If the matrix F is stable (all its eigenvalues are inside the unit circle) and U is positive (semi)definite, then the solution P is unique, symmetric, and positive (semi-)definite.

Theorem 2.1 *For uncertain systems* (2.1) *and* (2.2) *with uncertain parameters, the actual local steady-state Kalman filter* (2.10) *is robust in the sense that there exists a region* $\Re_{\Delta\Phi}^{(i)}$*, such that for all admissible uncertain model parameter* $\Delta\Phi \in \Re_{\Delta\Phi}^{(i)}$*, the corresponding actual filtering variances* $\bar{P}_i$ *have the upper-bound* P_i*, i.e.,*

$$\bar{P}_i < P_i \tag{2.16}$$

and $\Re_{\Delta\Phi}^{(i)}$ *is called the robust region of the local robust Kalman filter* (2.10).

Proof Define $\Delta P_i = P_i - \bar{P}_i$, subtracting (2.12) from (2.9) yields

$$\Delta P_i = \Psi_i \Delta P_i \Psi_i^{\mathrm{T}} + U_i(\Delta\Phi) \tag{2.17}$$

$$\begin{aligned} U_i(\Delta\Phi) = {} & [\mathrm{I}_n - \mathrm{K}_i\mathrm{H}_i]\Delta_\xi[I_n - K_i H_i]^{\mathrm{T}} - [\mathrm{I}_n - \mathrm{K}_i\mathrm{H}_i]\Delta\Phi X \Delta\Phi^{\mathrm{T}}[I_n - K_i H_i]^{\mathrm{T}} \\ & - [\mathrm{I}_n - \mathrm{K}_i\mathrm{H}_i]\Delta\Phi C_i \Psi_i^{\mathrm{T}} - \Psi_i C_i^{\mathrm{T}} \Delta\Phi^{\mathrm{T}}[I_n - K_i H_i]^{\mathrm{T}} \end{aligned} \tag{2.18}$$

From (2.6) yields $I_n - K_i H_i = P_i \Sigma_i^{-1}$, so we have $\det[I_n - K_i H_i] = \det P_i \det \Sigma_i^{-1} \neq 0, [I_n - K_i H_i]$ is invertible. Since $\Delta_\xi > 0$, then $U_{0i} = [\mathrm{I}_n - \mathrm{K}_i\mathrm{H}_i]\Delta_\xi[I_n - K_i H_i]^{\mathrm{T}} > 0$. According to the property of the continuous function, as $\Delta\Phi \longrightarrow 0$, we have $U_i(\Delta\Phi) \rightarrow U_{0i} > 0$. Hence there exists a sufficiently small region $\Re_{\Delta\Phi}^{(i)}$, such that for all admissible $\Delta\Phi \in \Re_{\Delta\Phi}^{(i)}$, we have $U_i(\Delta\Phi) > 0$. Applying Lemma 2.1 yields $\Delta P_i > 0$, i.e., (2.16) holds, and $\Re_{\Delta\Phi}^{(i)}$ is called the robust region of uncertain parameters for the local robust Kalman filter (2.10). The proof is completed.

2.3 Robust CI Fusion Steady-State Kalman Filter

For multisensor uncertain time-invariant systems (2.1) and (2.2), the robust steady-state CI-fused Kalman filter is presented as

$$\hat{x}_{CI}(t|t) = P_{CI}\sum_{i=1}^{L}\omega_i P_i^{-1}\hat{x}_i(t|t) \tag{2.19}$$

$$P_{CI} = \left[\sum_{i=1}^{L}\omega_i P_i^{-1}\right]^{-1}, \ \sum_{i=1}^{L}\omega_i = 1, \ \omega_i \geq 0 \tag{2.20}$$

The optimal weighting coefficientsω_i are obtained by minimizing the performance index

$$J = \min_{\omega_i} \mathrm{tr} P_{CI} = \min_{\substack{\omega_i \in [0,1] \\ \omega_1 + \cdots + \omega_L = 1}} \mathrm{tr}\left\{\left[\sum_{i=1}^{L}\omega_i P_i^{-1}\right]^{-1}\right\} \tag{2.21}$$

The actual error variances are given as

$$\bar{P}_{CI} = P_{CI}\left[\sum_{i=1}^{L}\sum_{j=1}^{L}\omega_i P_i^{-1}\bar{P}_{ij}P_j^{-1}\omega_j\right]P_{CI} \tag{2.22}$$

It is proved that [7] the local robustness (2.16) yields the robustness of the CI fuser for all $\Delta\Phi \in \Re_{\Delta\Phi}^{CI} = \bigcap_{i=1}^{L}\Re_{\Delta\Phi}^{(i)}$

$$\bar{P}_{CI} \leq P_{CI} \tag{2.23}$$

where the symbol $\cap$ denotes the intersection of sets.

Theorem 2.2 [8] *The local and CI fusion robust Kalman filters have the following robust accuracy relations*

$$\mathrm{tr}\bar{P}_i < \mathrm{tr}P_i, i = 1, \ldots, L \tag{2.24}$$

$$\mathrm{tr}\bar{P}_{CI} \leq \mathrm{tr}P_{CI} \leq \mathrm{tr}P_i, i = 1, \ldots, L \tag{2.25}$$

Remark 2.2 Taking the trace operation for (2.16), we have $\mathrm{tr}\bar{P}_i < \mathrm{tr}P_i, i = 1, \ldots, L$. The trace $\mathrm{tr}P_i$ is called robust accuracy or global accuracy of a robust Kalman filter, the trace $\mathrm{tr}\bar{P}_i$ is called its actual accuracy. Theorem 2.1 shows that the robust accuracy of the CI fusion Kalman filter is higher than that of each local robust Kalman filter. The actual accuracy of the local or CI fuser is higher that its robust accuracy.

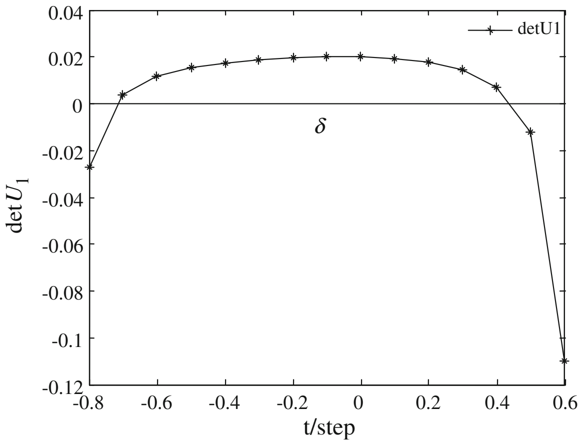

Fig. 2.1 The robust region of the local robust Kalman filter $\hat{x}_1(t|t)$

2.4 Simulation Example

Consider a 2-sensor time-invariant system with uncertain model parameters

$$x(t+1) = (\Phi_e + \Delta\Phi)x(t) + \Gamma w(t) \tag{2.26}$$

$$y_i(t) = H_i x(t) + v_i(t),\ i = 1, 2 \tag{2.27}$$

In the simulation, we take $\Phi_e = \begin{bmatrix} 0.43 & 0.32 \\ 0.56 & 0 \end{bmatrix}, \Delta\Phi = \begin{bmatrix} 0 & 0 \\ 0 & \delta \end{bmatrix}, \Gamma = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, H_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $H_2 = I_2, Q = 1, R_1 = 1, R_2 = \mathrm{diag}(6, 0.36), \delta$ is the uncertain parameter. The simulation results are shown in the following. From Figs. 2.1 and 2.2, the necessary and sufficient condition of $U_i(\delta) > 0$ is that $\det U_i(\delta) > 0$, so we can

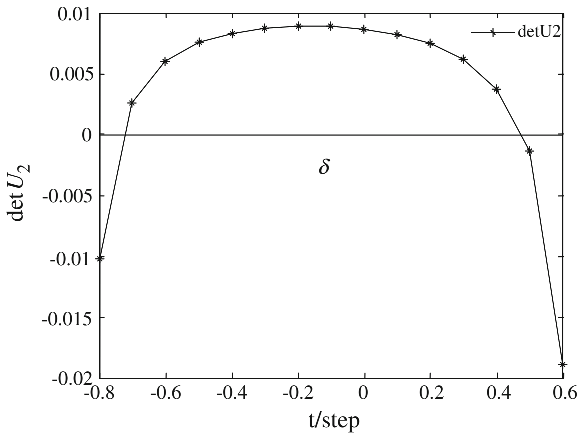

Fig. 2.2 The robust region of the local robust Kalman filter $\hat{x}_2(t|t)$

Table 2.1 The robust and actual accuracy comparison of $\mathrm{tr}P_i$ and $\mathrm{tr}\bar{P}_i$, i = 1, 2, CI

$\mathrm{tr}P_1(\mathrm{tr}\bar{P}_1)$	$\mathrm{tr}P_2(\mathrm{tr}\bar{P}_2)$	$\mathrm{tr}P_{CI}(\mathrm{tr}\bar{P}_{CI})$
1.1477 (0.7412)	1.4713 (1.1673)	1.1059 (0.6299)

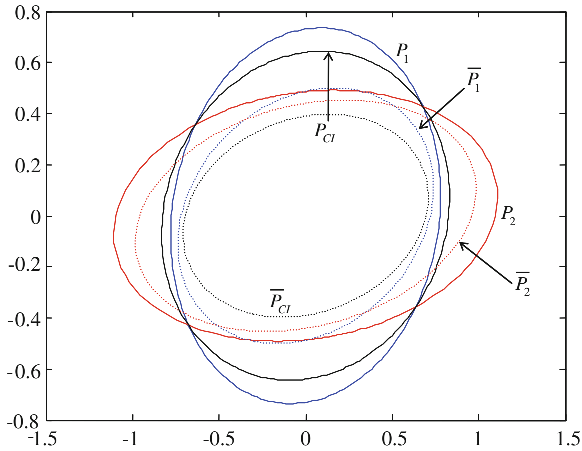

Fig. 2.3 The covariance ellipses of the local and CI fusion robust Kalman filters

obtain that the robust region of $\hat{x}_1(t|t)$ is $\Re_\delta^{(1)}: -0.72 < \delta < 0.43$, the robust region of $\hat{x}_2(t|t)$ is $\Re_\delta^{(2)}: -0.73 < \delta < 0.48$, so the robust region of CI fuser is $\Re_\delta^{CI} = \Re_\delta^{(1)} \cap \Re_\delta^{(2)}: -0.72 < \delta < 0.43$.

When δ = 0.2 in the robust region, the traces comparisons of the conservative and actual filtering error variances are given in Table 2.1, which verify the accuracy relations (2.24) and (2.25).

In order to give a geometric interpretation of the matrix accuracy relations, the covariance ellipse of variance $P$is defined as the locus of points $\{x : x^{\mathrm{T}}P^{-1}x = c\}$, where P is $n \times n$ the variance matrix and $x \in R^n$ and $c$is a constant. Generally, we select c = 1 without loss of generality. It has been proved in [8] that $P_1 \le P_2$ is equivalent to that the covariance ellipse of P_1 is enclosed in that of P_2.

The matrix accuracy relations are given based on the covariance ellipses as shown in Fig. 2.3. From Fig. 2.3, we see that the ellipse of $\bar{P}_i$ is enclosed in that of P_i, the ellipse of $\bar{P}_{CI}$ is enclosed in that of P_{CI}. These verify that the accuracy relations (2.16) and (2.23) hold.

In order to verify the above theoretical results for the accuracy relation, taking the Monte-Carlo simulation with 1,000 runs, the mean-square error (MSE) curves of the local and CI-fused Kalman filters are shown in Fig. 2.4; we see that the values of the $\mathrm{MSE}_i(t)$, i = 1, 2, CI are close to the corresponding $\mathrm{tr}\bar{P}_i$ and the accuracy relations (2.24–2.25) hold.

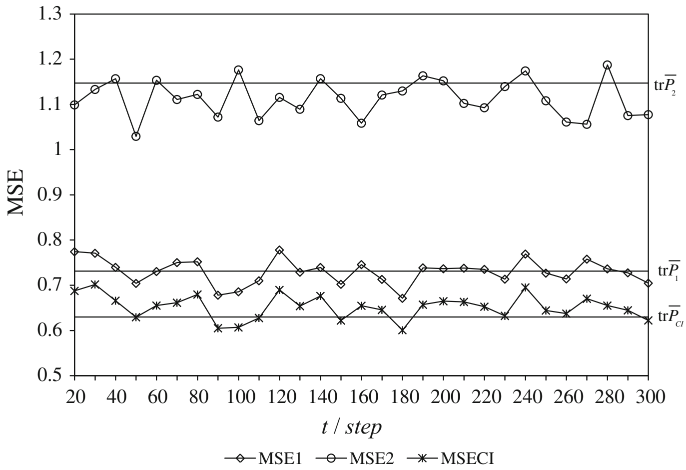

Fig. 2.4 The MSE curves of the local and CI fusion robust Kalman filters

2.5 Conclusion

For multisensor systems with uncertain parameters and known noise variances, the local and CI-fused steady-state Kalman filters are presented by the new approach of compensating the parameters uncertainties by a fictitious noise. It is proved that the local and CI-fused Kalman filters are robust for all admissible uncertain parameters in the robust region, this is, the actual filtering error variances have a less-conservative upper bound, and the robust accuracy of the CI fuser is higher than those of the local robust Kalman filters. When the fictitious noise variance is prescribed, by the searching method, the robust region can be found.

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