

# Chapter 22

## RBFNN-Based Path Following Adaptive Control for Underactuated Surface Vessels

Wei Meng and Chen Guo

**Abstract** A robust adaptive control strategy is developed to force an underactuated surface vessel to follow a reference path at a desired speed with the unknown parameters, despite the presence of environmental disturbances induced by wave, wind, and ocean current. The proposed controller is designed by using RBF neural networks and the backstepping techniques. The proposed control system allows for both low- and high-speed applications since linear and nonlinear damping terms were considered in the control design. Numerical simulation results are provided to demonstrate the effectiveness of the proposed controller design and the accuracy of stability analysis.

**Keywords** Underactuated surface vessels · Path following · RBF neural networks · Adaptive control

### 22.1 Introduction

Robust path following is an issue of vital practical importance to the ship industry. For the path following problem, the main challenge is that most ships are usually equipped with one or two main propellers for surge motion control, and rudders for yaw motion control of the ship. There are no side thrusters, so the sway axis is not actuated. This configuration is mostly used in the marine vehicles [1]. Meanwhile, another challenge of path following issue is the inherent nonlinearity of the ship dynamics and kinematics with the uncertain parameters and unstructured

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uncertainties including external disturbances and measurement noise, etc. To overcome these challenges, many different nonlinear design methodologies have been introduced to the underactuated ships. By applying the Lyapunov's direct method, two constructive tracking solutions were developed in Jiang [2]. In [3–5], the controllers were designed to force an underactuated surface vessel to follow a predefined path. The stability analysis was investigated relying on the Lyapunov's direct method. A robust adaptive control scheme was proposed for point-to-point navigation of underactuated ships by using a general backstepping technique [6]. In [7], a simple control law was presented by using the novel backstepping and feedback dominance. Furthermore, the control design was verified using a model ship in a tank. By using intelligent control, Liu proposed a stable adaptive neural network algorithm for the path following of underactuated ship with parameters uncertainties and disturbances [8].

Motivated by these recent developments in path following of underactuated surface vessels, this paper presents an adaptive RBF neural networks control law. The stability analysis is performed based on the Lyapunov theory. The proposed controller can guarantee that all signals of the underactuated system are bounded. Numerical simulations are provided to validate the effectiveness of the proposed path following controller.

## 22.2 Problem Statements

Consider the path following problem of an underactuated surface vessel. Generally, for path following, the vessel is moving in the horizontal plane, the heave, roll, and pitch are normally neglected. The mathematical model of the underactuated surface vessel moving in three degrees of freedom can be described as [9]:

$$\begin{cases} \dot{x} = u \cos \psi - v \sin \psi \\ \dot{y} = u \sin \psi + v \cos \psi \\ \dot{\psi} = r \\ \dot{u} = f_u(u, v, r) + \tau_u/m_{11} + b_u/m_{11} \\ \dot{v} = f_v(u, v, r) + b_v/m_{22} \\ \dot{r} = f_r(u, v, r) + \tau_r/m_{33} + b_r/m_{33} \end{cases} \quad (22.1)$$

with  $f_u = m_{22}vr/m_{11} - d_u u/m_{11} - \sum_{i=2}^3 d_{ui}|u|^{i-1}u/m_{11}$ ,  $f_v = -m_{11}ur/m_{22} - d_v v/m_{22} - \sum_{i=2}^3 d_{vi}|v|^{i-1}v/m_{22}$ ,  $f_r = (m_{11} - m_{22})uv/m_{33} - d_r r/m_{33} - \sum_{i=2}^3 d_{ri}|r|^{i-1}r/m_{33}$ ,  $[b_u, b_v, b_r]^T = R(\psi)^T [b_x, b_y, b_\psi]^T$ ,  $R(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .

where  $x$ ,  $y$ , and  $\psi$  are the surge displacement, sway displacement, and the yaw angle in the earth fixed frame, and  $u$ ,  $v$ , and  $r$  are the velocities in surge, sway, and yaw, respectively. The constant parameters  $m_{jj} > 0$ ,  $1 \leq j \leq 3$ , denote the ship's inertia and added mass effects. The positive terms  $d_u$ ,  $d_v$ ,  $d_r$ ,  $d_{ui}$ ,  $d_{vi}$ , and  $d_{ri}$ ,  $i = 2, 3$ , are given by the hydrodynamic damping in surge, sway, and yaw.  $\tau_u$  and  $\tau_r$  denote the available control inputs, respectively, the surge force and the yaw moment.  $b = [b_x, b_y, b_\psi]^T$  denote the low frequency interference in the earth fixed frame,  $\dot{b} = 0$ .

We now define the path following errors in a frame attached to the path as follows [8]:

$$(x_e, y_e, \psi_e)^T = R^T(\psi)(x - x_d, y - y_d, \psi - \psi_d)^T, \quad (22.2)$$

where  $\psi_d$  represents the desired yaw angle and was defined as  $\psi_d = \arctan(y'_d(s)/x'_d(s))$ ,  $x'_d = \partial x_d / \partial s$ ,  $y'_d = \partial y_d / \partial s$ ;  $x_d$  and  $y_d$  denote the desired displacement in path of the vessel.

**Assumption 22.1** The parameters of underactuated surface vessels such as  $m_{jj}$ ,  $d_u$ ,  $d_v$ ,  $d_r$ ,  $d_{ui}$ ,  $d_{vi}$ , and  $d_{ri}$ ,  $1 \leq j \leq 3$ ,  $i = 2, 3$ , are known.

**Assumption 22.2** The reference path is regular,  $x_d$ ,  $\dot{x}_d$ ,  $\ddot{x}_d$ ,  $y_d$ ,  $\dot{y}_d$ ,  $\ddot{y}_d$ ,  $\dot{\psi}_d$  and  $\ddot{\psi}_d$  are all bounded.

**Control objective:** Under Assumptions 22.1 and 22.2, the objective of this paper is to seek the adaptive control laws  $\tau_u$  and  $\tau_r$  that force the vessel from the initial position and orientation to follow a reference path  $\Omega$ .

## 22.3 Control Design

In this section, we develop an adaptive control law for underactuated surface vessels (22.1) with uncertain dynamics.

From (22.2), we have

$$\begin{cases} \dot{x}_e = u - u_d \cos(\psi_e) + r y_e \\ \dot{y}_e = v + u_d \sin(\psi_e) - r x_e \\ \dot{\psi}_e = r - r_d \end{cases} \quad (22.3)$$

where  $u_d = \bar{u}_d \dot{s}$ ,  $\bar{u}_d = \sqrt{x_d'^2(s) + y_d'^2(s)}$ ,  $r_d = \frac{x_d'^2(s)y_d''^2(s) - x_d''^2(s)y_d'^2(s)}{x_d'^2(s) + y_d'^2(s)} \dot{s}$ .

We define

$$u_e = u - \alpha_u, \bar{\psi}_e = \psi_e - \alpha_{\psi_e} \quad (22.4)$$

where  $\alpha_u$  and  $\alpha_{\psi_e}$  are virtual controls of  $u$  and  $\psi_e$ . Substituting (22.4) into (22.3) results in

$$\begin{cases} \dot{x}_e = \alpha_u + u_e - u_d \cos(\psi_e) + \Delta_1 + ry_e \\ \dot{y}_e = v + u_d \sin(\psi_e) + \Delta_2 - rx_e \end{cases} \quad (22.5)$$

where  $\Delta_1 = -u_d((\cos(\bar{\psi}_e) - 1) \cos(\alpha_{\psi_e}) - \sin(\bar{\psi}_e) \sin(\alpha_{\psi_e}))$ ,  $\Delta_2 = u_d \sin(\bar{\psi}_e) \cos(\alpha_{\psi_e}) + (\cos(\bar{\psi}_e) - 1) \sin(\alpha_{\psi_e})$ .

We choose the virtual control  $\alpha_u$  as

$$\alpha_u = -k_1 x_e + u_d \cos(\alpha_{\psi_e}) \quad (22.6)$$

where  $k_1 > 0$ . The derivative of the path parameter  $s$  satisfies

$$\dot{s} = \sqrt{u_{d0}^2 + (k_2 y_e + v_d)^2} / \bar{u}_d \quad (22.7)$$

where  $k_2 > 0$ ,  $v_d$  is the filter of  $v$ ,  $v_e = v - v_d$ . From (22.7), we have

$$u_d = \sqrt{u_{d0}^2 + (k_2 y_e + v_d)^2} \quad (22.8)$$

We choose the virtual control  $\alpha_{\psi_e}$  as

$$\alpha_{\psi_e} = -\arctan((k_2 y_e + v_d)/u_{d0}) \quad (22.9)$$

Substituting (22.6), (22.7), and (22.9) into (22.5), we have

$$\begin{cases} \dot{x}_e = -k_1 x_e + u_e + \Delta_1 + ry_e \\ \dot{y}_e = -k_2 y_e + v_e + \Delta_2 - rx_e \end{cases} \quad (22.10)$$

And substituting (22.9) into (22.6), we have

$$\alpha_u = -k_1 x_e + u_{d0} \quad (22.11)$$

The time derivative of (22.4) using (22.3) and (22.9) can be derived as

$$\dot{\bar{\psi}}_e = r - r_d + \{[k_2(-k_2 y_e - rx_e + \Delta_2 + v_e) + \dot{v}_d]u_{d0} - (k_2 y_e + v_d)\dot{u}_{d0}\} / u_d^2 \quad (22.12)$$

We define the  $r_e$  as

$$r_e = r - \alpha_r \quad (22.13)$$

Substituting (22.13) into (22.12), we have

$$\dot{\bar{\psi}}_e = -k_3 \bar{\psi}_e + f_x r_e + k_2 u_{d0} v_e / u_d^2 \quad (22.14)$$

where  $k_3 > 0$ ,  $f_x = 1 - k_2 x_e u_{d0} / u_d^2$ .

Differentiating  $v_e$ , and substituting (22.1) into it, we have

$$v_e = g_v - \dot{v}_d \quad (22.15)$$

where  $g_v = f_v(u, v, r) + b_v / m_{22}$ .

According to the approximation property of NNs, the smooth function  $g_v$  can be approximated by RBF neural networks as follows

$$g_v = W_v^T \sigma(\eta) + \varepsilon_v \quad (22.16)$$

where  $W_v$  is the idea weight matrix,  $\varepsilon_v$  is the approximation error,  $|\varepsilon_v| \leq \varepsilon_{vM}$ ,  $\eta = [x, y, \psi, u, v, r]^T$ .

Let  $\hat{W}_v$  be the estimations of the weights  $W_v$ ,  $\hat{g}_v$  is the estimation of the  $g_v$ , and can be defined as

$$\hat{g}_v = \hat{W}_v^T \sigma(\eta) \quad (22.17)$$

In order to stabilize the  $v_e$ , the  $\dot{v}_d$  can be chosen as

$$\dot{v}_e = \hat{W}_v^T \sigma(\eta) - k_6 v_e - k_2 u_{d0} \bar{\psi}_e / u_d^2 + \varepsilon_v \quad (22.18)$$

The time derivative of (22.4) can be derived

$$\dot{u}_e = g_u + \tau_u / m_{11} - \dot{\alpha}_u \quad (22.19)$$

with  $g_u = f_u(u, v, r) + b_u / m_{11}$ ,  $\dot{\alpha}_u = \frac{\partial \alpha_u}{\partial x_e} \dot{x}_e + \frac{\partial \alpha_u}{\partial u_{d0}} \dot{u}_{d0}$ .

The smooth function  $g_u$  can also be approximated by RBF neural networks as follows

$$g_u = W_u^T \sigma(\eta) + \varepsilon_u \quad (22.20)$$

where  $W_u$  is the idea weight matrix,  $\varepsilon_u$  is the approximation error,  $|\varepsilon_u| \leq \varepsilon_{uM}$ ,  $\eta = [x, y, \psi, u, v, r]^T$ .

Let  $\hat{W}_u$  be the estimations of the weights  $W_u$ ,  $\hat{g}_u$  is the estimation of the  $g_u$ , and can be defined as

$$\hat{g}_u = \hat{W}_u^T \sigma(\eta) \quad (22.21)$$

The time derivative of (22.13) can be derived as

$$\dot{r}_e = g_r + \tau_r/m_{33} - \dot{\alpha}_r \quad (22.22)$$

where  $g_r = f_r(u, v, r) + b_r/m_{33} + \dot{g}_v$ .

The smooth function  $g_u$  can be approximated by RBF neural networks as follows

$$g_r = W_r^T \sigma(\eta) + \varepsilon_r \quad (22.23)$$

where  $W_r$  is the idea weight matrix,  $\varepsilon_r$  is the approximation error,  $|\varepsilon_r| \leq \varepsilon_{rM}$ ,  $\eta = [x, y, \psi, u, v, r]^T$ .

Let  $\hat{W}_r$  be the estimations of the weights  $W_r$ ,  $\hat{g}_r$  is the estimation of the  $g_r$ , and can be defined as

$$\hat{g}_r = \hat{W}_r^T \sigma(\eta) \quad (22.24)$$

From (22.19) and (22.22), the adaptive NNs surge control law  $\tau_u$  and the yaw moment control law  $\tau_r$  can be presented as

$$\tau_u = m_{11}(-\hat{g}_u - k_4 u_e + \dot{\alpha}_u), \quad k_4 > 0 \quad (22.25)$$

$$\tau_r = m_{33}(-\hat{g}_r - k_5 r_e + \dot{\alpha}_r - f_x \bar{\psi}_e), \quad k_5 > 0 \quad (22.26)$$

The adaptive laws are given by

$$\dot{\hat{W}}_u = \Gamma_u [\sigma(\eta) u_e - k_u \hat{W}_u] \quad (22.27)$$

$$\dot{\hat{W}}_v = \Gamma_v [\sigma(\eta) v_e - k_v \hat{W}_v] \quad (22.28)$$

$$\dot{\hat{W}}_r = \Gamma_r [\sigma(\eta) r_e - k_r \hat{W}_r] \quad (22.29)$$

where  $\Gamma_u = \Gamma_u^T > 0$ ,  $\Gamma_v = \Gamma_v^T > 0$ ,  $\Gamma_r = \Gamma_r^T > 0$  are constant design parameters.

## 22.4 Stability Analysis

**Theorem 22.1** Assume that the Assumptions 1–2 hold, the adaptive NNs surge control law  $\tau_u$  and the yaw moment control law  $\tau_r$  are derived as (22.25) and (22.26), and adaptation laws are given by (22.27–22.29), the control objective of

path following for underactuated surface vessels in the presence of uncertain parameters and unstructured uncertainties is solved, and the systems (22.1) are asymptotic stability.

*Proof* From (22.29) and (22.30), we have

$$\begin{cases} \dot{Z}_1 = f_1(Z_1, Z_2) \\ \dot{Z}_2 = f_2(Z_2) \end{cases}, \quad (22.30)$$

with  $Z_1 = [x_e, y_e]^T$ ,  $Z_2 = [\bar{\psi}_e, u_e, v_e, r_e, \tilde{W}_u, \tilde{W}_v, \tilde{W}_r]^T$

$$\begin{aligned} f_1 &= [-k_1 x_e + u_e + \Delta_1 + r y_e, -k_2 y_e + v_e + \Delta_2 - r x_e]^T, \\ f_2 &= [-k_3 \bar{\psi}_e + f_x r_e + k_2 u_{d0} v_e / u_d^2, \tilde{W}_u^T \sigma(\eta) - k_4 u_e + \varepsilon_u, \tilde{W}_v^T \sigma(\eta) \\ &\quad - k_6 v_e - k_2 u_{d0} \bar{\psi}_e / u_d^2 + \varepsilon_v, \tilde{W}_r^T \sigma(\eta) - k_5 r_e - f_x \bar{\psi}_e + \varepsilon_r, \\ &\quad -\Gamma_u [\sigma(\eta) u_e - \sigma_u \hat{W}_u], -\Gamma_v [\sigma(\eta) v_e - \sigma_v \hat{W}_v], -\Gamma_r [\sigma(\eta) r_e - \sigma_r \hat{W}_r]]^T. \end{aligned}$$

To investigate stability of this subsystem, we consider the following Lyapunov function:

$$V_1 = \frac{1}{2} \bar{\psi}_e^2 + \frac{1}{2} u_e^2 + \frac{1}{2} r_e^2 + \frac{1}{2} \tilde{W}_u^T \Gamma_u^{-1} \tilde{W}_u + \frac{1}{2} \tilde{W}_v^T \Gamma_v^{-1} \tilde{W}_v + \frac{1}{2} \tilde{W}_r^T \Gamma_r^{-1} \tilde{W}_r, \quad (22.31)$$

Differentiating (22.32) along with (22.27–22.30), we have

$$\begin{aligned} \dot{V}_1 &\leq -k_3 \bar{\psi}_e^2 - k_4 u_e^2 - k_5 r_e^2 - k_6 v_e^2 + \sigma_u \tilde{W}_u^T \hat{W}_u + \sigma_v \tilde{W}_v^T \hat{W}_v + \sigma_r \tilde{W}_r^T \hat{W}_r \\ &\quad + u_e \varepsilon_u + v_e \varepsilon_v + r_e \varepsilon_r \end{aligned} \quad (22.32)$$

The (22.33) can be described as

$$\begin{aligned} \dot{V}_1 &\leq -k_3 \bar{\psi}_e^2 - \left(k_4 - \frac{1}{4}\right) u_e^2 - \left(k_5 - \frac{1}{4}\right) v_e^2 - \left(k_6 - \frac{1}{4}\right) r_e^2 \\ &\quad - \frac{1}{2} \sigma_u \|\tilde{W}_u\|^2 - \frac{1}{2} \sigma_v \|\tilde{W}_v\|^2 \\ &\quad - \frac{1}{2} \sigma_r \|\tilde{W}_r\|^2 + \varepsilon_u^2 + \varepsilon_v^2 + \varepsilon_r^2 + \frac{1}{2} \sigma_u \|W_u\|^2 + \frac{1}{2} \sigma_v \|W_v\|^2 + \frac{1}{2} \sigma_r \|W_r\|^2 \\ &\leq -\mu V_1 + \rho \end{aligned} \quad (22.33)$$

with

$$\begin{aligned} \mu &:= \min \left\{ 2k_3, 2\left(k_4 - \frac{1}{4}\right), 2\left(k_5 - \frac{1}{4}\right), 2\left(k_6 - \frac{1}{4}\right), \min \left( \frac{\sigma_u}{\lambda_{\max}(\Gamma_u^{-1})}, \min \left( \frac{\sigma_v}{\lambda_{\max}(\Gamma_v^{-1})}, \right. \right. \right. \\ &\quad \left. \left. \min \left( \frac{\sigma_r}{\lambda_{\max}(\Gamma_r^{-1})} \right) \right) \right\}, \rho := \varepsilon_u^2 + \varepsilon_v^2 + \varepsilon_r^2 + \frac{\sigma_u}{2} \|W_u\|^2 + \frac{\sigma_v}{2} \|W_v\|^2 + \frac{\sigma_r}{2} \|W_r\|^2 \end{aligned}$$

Let  $\Phi = \frac{\rho}{\mu}$ , the (22.34) can be rewritten as

$$0 \leq V(t) \leq \Phi + [V(0) - \Phi]e^{-\mu t} \quad (22.34)$$

Hence, all signals of the closed-loop system are uniformly ultimately bounded. The path following errors will converge to a small neighborhood of zero, and can be adjusted by the design parameters  $k_3, k_4, k_5, k_6, \sigma_u, \sigma_v, \sigma_r$ .

## 22.5 Numerical Simulations

In this section, some numerical simulations are provided to demonstrate the effectiveness of the proposed control laws and the accuracy of stability analysis. In this paper, we use a monohull ship with the length of 38 m, mass of  $118 \times 10^3$  kg, the numerical values of the vessel are adapted from [6].

In the simulation, the reference path is generated by a virtual ship as follows:

$$\begin{cases} \dot{x}_d = u_d \cos(\psi_d) - v_d \sin(\psi_d) \\ \dot{y}_d = u_d \sin(\psi_d) + v_d \cos(\psi_d) \\ \dot{\psi}_d = r_d \\ \dot{v}_d = -\frac{m_{11}}{m_{22}}u_d r_d - \frac{d_{22}}{m_{22}}v_d - \sum_{i=2}^3 \frac{d_{vi}}{m_{22}}|v_d|^{i-1}v_d \end{cases}$$

In the simulation we select  $u_d = 5$ ,  $r_d = 0.015$ ; the control parameters selected for the simulation are:  $k_1 = 15$ ,  $k_2 = 7.5$ ,  $k_3 = 12$ ,  $k_4 = 10$ ,  $k_5 = 10$ ,  $k_6 = 10$ ,  $\Gamma_u = 10$ ,  $\Gamma_v = 30$ ,  $\Gamma_r = 0.5$ ,  $\sigma_u = \sigma_v = \sigma_r = 0.01$ ,

The initial conditions are chosen as:

$$[x(0), y(0), \psi(0), u(0), v(0), r(0)] = [-100, 0, 0, 0, 0, 0].$$

The simulation results of ship path following control are plotted in Figs. 22.1 and 22.2. Figure 22.1 shows the position and the orientation of the vessel in the  $xy$  plane, and the control inputs  $\tau_u$  and  $\tau_r$  are plotted. The path following position errors are plotted in Fig. 22.2. It can be seen from these figures that all the signals of the closed-loop system are bounded. From Fig. 22.2, the path following position errors  $x_e, y_e$ , the velocity errors  $u_e, r_e$ , and the orientation error  $\psi_e$  converge to zero while the sway motion error  $v_e$  converges to a small value, since the reference path is generated by a virtual ship, the sway velocity error is always a constant value.



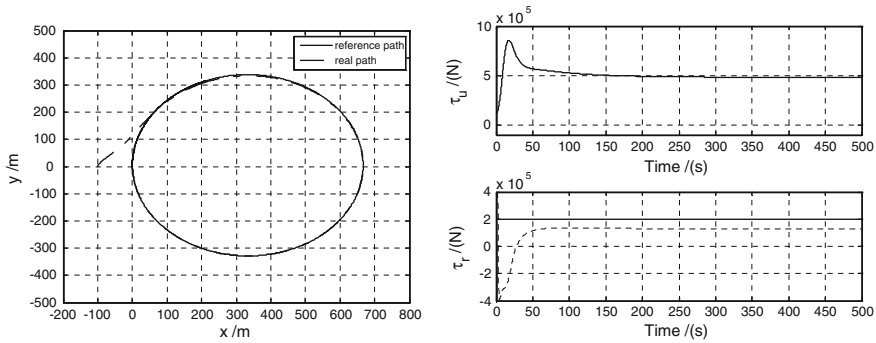


Fig. 22.1 Position and orientation and the inputs of the vessel

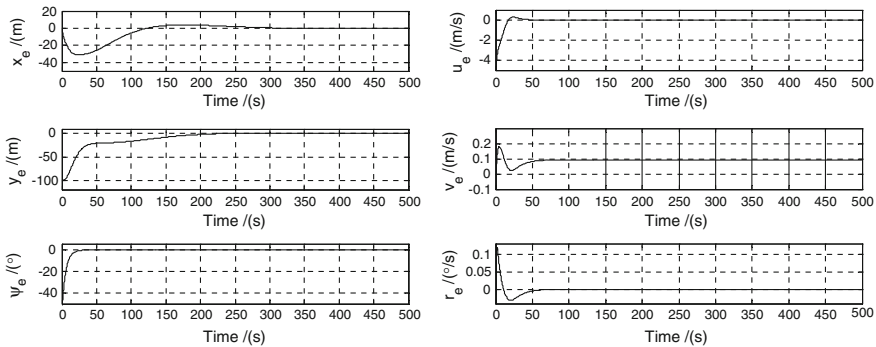


Fig. 22.2 Position, orientation, and velocity errors of the vessel

## 22.6 Conclusions

In this paper, we present an adaptive RBF neural networks scheme for path following of underactuated surface vessels with uncertain parameters and unstructured uncertainties including exogenous disturbances and measurement noise, etc. The proposed controller is designed by using RBF neural networks and the backstepping techniques. It is noted that the proposed control system allows for both low- and high-speed applications since linear and nonlinear damping terms were considered in the control design. The stability analysis is performed based on the Lyapunov theory. The effectiveness of the designed controller is also validated by the numerical simulations. Based on the ideas of this paper, the future work will consider the rudder saturation and rate limits.

**Acknowledgments** The work for this paper was financially supported by the National Natural Science Foundation of China (NSFC, Grant No: 61374114) and the Fundamental Research Funds for the Central Universities (Grant No: 3132014321).

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