Chapter 2
Robust Control for Constant Thrust Rendezvous

Yongqiang Qi and Minghui Ou

Abstract In this paper, a robust orbit design approach under constant thrust is proposed based on the relative motion dynamic model. First, the design problem is cast into a convex optimization problem by introducing a Lyapunov function subject to linear matrix inequalities. Next, the robust controllers satisfying the requirements can be designed by solving this optimization problem. At last, the proposed method has the advantage of saving fuel is proved and the actual constant thrust switch control laws are obtained through the isochronous interpolation method, an illustrative example is provided to show the effectiveness of the proposed control design method.

Keywords Rendezvous · Constant thrust · Linear matrix inequality · Robust controller

2.1 Introduction

New challenges arise from the International Space Station (ISS) and the numerous formation flight projects [1, 2]. The close range rendezvous phase is usually divided into two subphases: a preparatory phase leading to the final approach corridor and a final approach phase leading to the mating conditions. The objectives of the close range rendezvous phase are the reduction of the range to the target and the achievement of conditions allowing the acquisition of the final approach corridor [3]. This means that at the end of this phase the chaser is, concerning position, velocity, attitude, and angular rates, ready to start the final approach on the proper approach axis within the constraints of the safety corridor [4, 5].

The rendezvous problem has been extensively studied as an optimal control problem [3, 6]. Both impulsive thrust and the continuous thrust assumptions have

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been exploited through the Pontryagin's maximum principle, respectively. In actual practice, maneuver during rendezvous and docking operations cannot normally be considered as continuous thrust or impulsive maneuver. In our previous study, constant thrust fuel-optimal control for spacecraft rendezvous was studied according to the C-W equations and the analytical solutions. But the traditional open-loop control method used in our previous studies is not applicable while they are often utilized during the long-distance navigation process. To overcome this problem, a robust closed-loop control law for constant thrust rendezvous to enhance the orbital control accuracy is proposed in this paper. And the fuel consumption of constant thrust is less than that of the continuous thrust by using the method proposed in this paper.

2.2 Multiobjective Robust Controller Design

The relative motion coordinate system can be established as follows: first, the target spacecraft is assumed as a rigid body and in a circular orbit, and the relative motion can be described by Clohessy-Wiltshire equations. Then, the centroid of the target spacecraft O_T is selected as the origin of coordinate, the x-axis is opposite to the target spacecraft motion, the y-axis is from the center of the earth to the target spacecraft, and the z-axis is determined by the right-handed rule.

$$\begin{cases} \ddot{x} - 2\omega\dot{y} = \frac{F_x + \eta_x}{m} \\ \ddot{y} + 2\omega\dot{x} - 3\omega^2 y = \frac{F_y + \eta_y}{m} \\ \ddot{z} + \omega^2 z = \frac{F_z + \eta_z}{m} \end{cases} \qquad (2.1)$$

where ω represents the angular velocity. F_x, F_y, F_z represent the vacuum thrust of the chaser, η_x, η_y, η_z represent the sum of the perturbation and nonlinear factors in the three axes, respectively. m represents the mass of the chaser at the beginning of the rendezvous. Suppose that the maximum thrusts are $\widehat{F}_x, \widehat{F}_y, \widehat{F}_z$ and the theoretical thrusts are F_x^*, F_y^*, F_z^*. The range of the thrust angle in the x-axis θ_x is defined as shown in Fig. 2.1.

The goal of the rendezvous maneuver is to design a proper controller for the chaser, such that the chaser can be asymptotically maneuvered to the target position. Define the state error vector $x_e(t) = x(t) - x_t(t)$, and its state equation can be obtained as

$$\begin{cases} \dot{x}_e(t) = (A_1 + \Delta A)x_e(t) + (B_1 + \Delta B)u(t) \\ u(t) = Kx_e(t) \end{cases} \qquad (2.2)$$

Fig. 2.1 Variable thrust angle thrusters
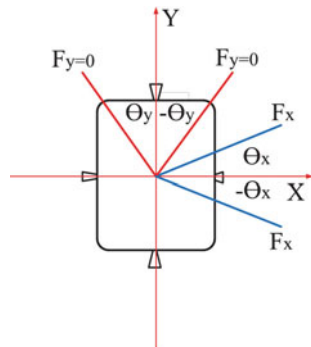
Lyapunov function is defined as follows:

$$V = x_e^T(t)Px_e(t) \tag{2.3}$$

where P is a positive definite symmetric matrix. According to the system stability theory, the necessary and sufficient conditions for robust stability of the system (2.3) are as follow:

$$A^T P + PA < 0 \tag{2.4}$$

Then a multiobjective controller design strategy is proposed by translating a multiobjective controller design problem into a convex optimization problem. And the control input constraints can be met simultaneously. Assuming the initial conditions satisfy the following inequality, where ρ is a given positive constant.

$$x^T(0)Px(0) < \rho \tag{2.5}$$

Theorem 2.1 *If there exist a corresponding dimension of the matrix L, a symmetric positive definite matrix X, and two parameters $\varepsilon_1 > 0, \varepsilon_2 > 0$, then the sufficient condition for robust stability is there exist a state feedback controller K which can meet the following conditions simultaneously.*

$$\begin{pmatrix} \Sigma & X & L \\ X & -\varepsilon_1 & 0 \\ L^T & 0 & -\varepsilon_2 \end{pmatrix} < 0, \quad \begin{pmatrix} \rho I & x^T(0) \\ x(0) & X \end{pmatrix} < 0, \tag{2.6}$$

where $\Sigma = XA_0^T + A_0X + L^T B_0 + B_0 L + \varepsilon_1 \alpha^2 I + \varepsilon_2 \beta^2 I$, then the theoretical state feedback controller K can be calculated as follows:

$$K = LX^{-1} = \begin{pmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \end{pmatrix} \tag{2.7}$$

Then the following results can be obtained.

$$\begin{cases} \dfrac{L_x}{N_x} \hat{F}_x \cos\theta_x + \dfrac{L_y}{N_y} \hat{F}_y \sin\theta_y = k_{11}x_e(t) + k_{12}y_e(t) + k_{13}\Delta V_x + k_{14}\Delta V_y \\ \dfrac{L_x}{N_x} \hat{F}_x \sin\theta_x + \dfrac{L_y}{N_y} \hat{F}_y \cos\theta_y = k_{21}x_e(t) + k_{22}y_e(t) + k_{23}\Delta V_x + k_{24}\Delta V_y \end{cases} \tag{2.8}$$

Then the thrust angle control lows θ_x, θ_y which satisfy the robust stability of the in plane motion can be obtained from Eq. (2.8).

2.3 Compare Fuel Consumption and Calculate the Control Law

Constant thrust fitting is proposed by using the impulse compensation method as follow. Suppose that the thrusters in the z-axis can provide different sizes of constant thrust to meet different thrust requirements. If the theoretical working time of z-axis thruster in the ith thrust arc $t_z^* = \Delta T < T_i$ and t_z^* can be any one of M_i shortest switching time interval in the ith thrust arc. Without loss of generality, suppose that t_z^* is the first shortest switching time interval and the impulse error in the z-axis in the ith thrust arc ΔI_{zi} can be calculated as follows:

There are $N_z + 1$ thrust levels that can be selected and the level of the constant thrust can be calculated as follows:

$$L_z = \left[\frac{N_z \int_{T_i}^{T_i + \Delta T} |F_z^*(t)| \mathrm{d}t}{\hat{F}_z \Delta T} \right] \tag{2.9}$$

Calculate the impulse error.

$$\Delta I_{zi} = \mathrm{sgn}(F_z^*(t)) \left| \int_{T_i}^{T_{i_i} + \Delta T} |F_x^*(t)| \mathrm{d}t - \frac{L_z \hat{F}_z \Delta T}{N_z} \right| \tag{2.10}$$

Determine the value of the impulse compensation threshold. Suppose that the value of the impulse compensation threshold is a positive constant $\gamma > 0$, if the impulse error ΔI_{zi} satisfies the following condition,

$$\left| \int_{T_i}^{T_{i_i}+\Delta T} \left| F_x^*(t) \right| dt - \frac{\hat{F}_z \Delta T}{N_z} \left[\frac{N_z \int_{T_i}^{T_i+\Delta T} \left| F_z^*(t) \right|}{\hat{F}_z \Delta T} \right] \right| \leq \gamma \tag{2.11}$$

the actual constant thrust of the chaser in the z-axis can be calculated as follows:

$$F_z = \text{sgn}(F_z^*(t)) \frac{\hat{F}_z \Delta T}{N_z} \left[\frac{N_z \int_{T_i}^{T+\Delta T_i} \left| \hat{F}_z^*(t) \right| dt}{\hat{F}_z \Delta T} \right] \tag{2.12}$$

then the chaser will not carry out impulse compensation. The fuel savings in the x-axis in the ith thrust arc can be calculated as follows:

$$\Delta P_{zi} = \sum_{j=0}^{M_1} \int_{T_i+j\Delta T}^{T_i+(j+1)\Delta T} \left\{ \frac{p_0 N_z \left| F_z^*(t) \right|}{\hat{F}_z} - \text{sgn}(F_z^*(t)) p_0 \left[\frac{N_z \int_{T_i+j\Delta T}^{T+(j+1)\Delta T} \left| \hat{F}_z^*(t) \right| dt}{\hat{F}_z \Delta T} \right] \right\} dt \tag{2.13}$$

Then the total number of the accelerating time intervals and the decelerating time intervals is $M_1 - m_1$ and the number of zero-thrust time intervals is $M_1 - M_1 + m_1$. The position of the three types of time intervals is decided by the curve of the theoretical continuous thrust.

At last, the switch control laws for the rendezvous maneuver can be given in three axes. For convenience, let us take the time intervals in the ith thrust arc in the x-axis for example:

$$S_{zi} = \left\{ \left(T_i + j\Delta T, \text{sgn}(F_z^*(t)) \frac{\hat{F}_z \Delta T}{N_z} \left[\frac{N_z \int_{T_i+j\Delta T}^{T+(j+1)\Delta T} \left| \hat{F}_z^*(t) \right| dt}{\hat{F}_z \Delta T} \right] \right) \right\} \tag{2.14}$$

2.4 Simulation Example

The height of target spacecraft is assumed to be 356 km in a circular orbit, then the mean angular velocity is $\omega = 0.0654 \times 10^{-3}$ rad/s and the uncertainty parameters is assumed as $\Delta\omega = \pm 1 \times 10^{-3}$ rad/s. The initial mass of the chaser is assumed to be 300 kg at the beginning of rendezvouse maneuver. The size of thrusts are assumed to be $\pm 3,000$ N in three axes and the shortest switching time is $\Delta T = 1$ s. The initial position and velocity of the chaser are assumed to be (2000, 100, -500 m) and (-20, 10, 5 m/s). Suppose that the thrusters in three axes can provide

Fig. 2.2 The change of
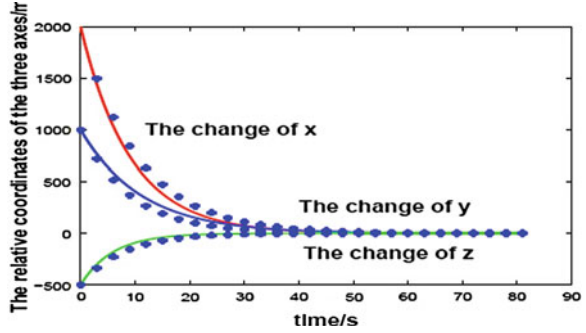position during rendezvous
maneuver

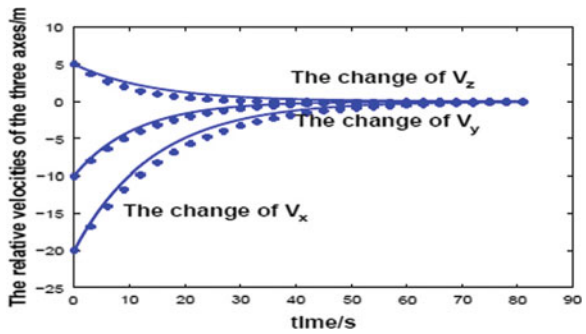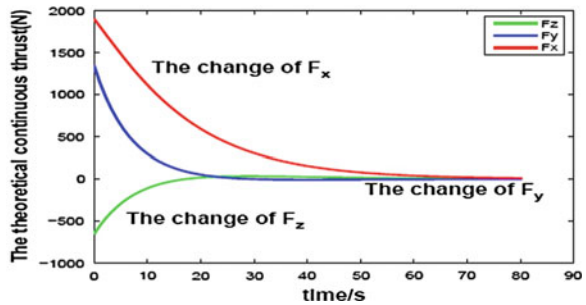**Fig. 2.3** The change of
velocity during rendezvous
maneuver

Fig. 2.4 The change of thrust
during rendezvous maneuver

10 different sizes of constant thrust. Suppose that the value of the impulse compensation threshold is a positive constant $\gamma = 300$ Ns.

Figure 2.2 shows the change of x, y, and z during rendezvous maneuver.

The results in Fig. 2.3 show the change of V_x, V_y, V_z during rendezvous maneuver.

The results in Fig. 2.4 show the change of F_x, F_y, F_z during rendezvouse maneuver.

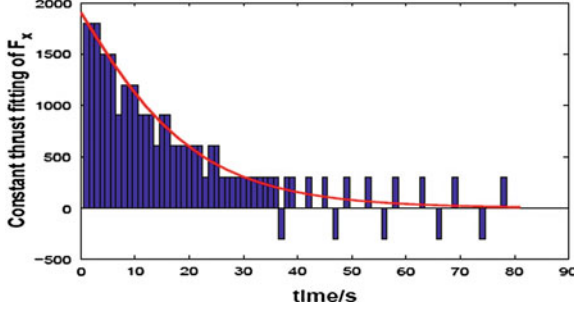The results in Fig. 2.5 show the change of F_z during rendezvous maneuver.

Fig. 2.5 The constant thrust fitting of F_x during rendezvous maneuver
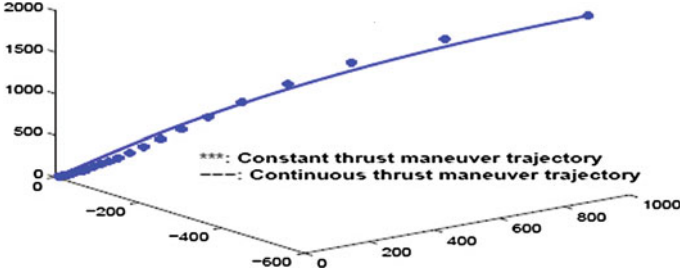

Fig. 2.6 The trajectory of the chaser during rendezvouse maneuver

The result in Fig. 2.6 shows the trajectory of chaser during rendezvous maneuver. It shows that with the switch control control, the chaser can get to the 27 target positions smoothly. The solid lines represent the curves caused by theoretical thrust and the dotted lines represent the curves caused by constant thrust.

The fuel savings in the x-axis in the ith thrust arc can be calculated as follows:

$$\Delta P_{zi} = \sum_{i=1}^{27} \sum_{j=1}^{3} \int_{T_i}^{T_i+j} \left\{ \frac{\left| F_z^*(t) \right|}{30} - 10\mathrm{sgn}(F_z^*(t)) \left[\frac{N_z \int_{T_i}^{T+j} \left| \hat{F}_z^*(t) \right| \mathrm{dt}}{300} \right] \right\} \mathrm{dt} = 19.9158g$$

(2.15)

At last, the switch control laws for the ith thrust arc in the z-axis for example:

$$S_{zi} = \{1{,}000 \text{ m}; -10 \text{ m/s}; (1 \text{ s}, 1{,}800 \text{ N}); \ldots (77 \text{ s}, -300 \text{ N}); \ldots\} \qquad (2.16)$$

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