Chapter 14
Stabilization of Underactuated Surface Vessel Based on Backstepping Control Method

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Abstract To address the problem of stabilization of underactuated surface vessels with parameter uncertainties and external disturbances, a stabilization control based on backstepping control method is designed, First, the conditions can make the system globally stable and prove the correctness of existing conditions using diffeomorphism transformation. Then the original stabilization problem can be changed into an equal new one, and backstepping method is used to construct the controller of the stabilization function. The controller has global asymptotic stability. Eventually, simulations of the stabilization control for the underactuated surface vessel with disturbances or without disturbances are performed. Results validate the effectiveness of the proposed method.

Keywords Underactuated surface vessels · Stabilization control · Backstepping · Global asymptotic stabilization

14.1 Introduction

Control of underactuated systems has been one of the active research areas because of its intrinsic nonlinear nature and practical applications [1]. As a typical example of underactuated systems, control of an underactuated ship has attracted much attention recently. The main difficulty with control of underactuated ships is that they are not actuated in the sway axis. This configuration is by far the most common among surface ships. Furthermore, unlike underactuated systems with non-integrable constraints, the surface vessels under consideration are a class of underactuated systems with non-integrable dynamics and are not transformed into a driftness system [2]. Nevertheless, the stabilization controller for underactuated surface vessel is more difficult owing to the conclusion of general non-integrity

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system that cannot be applied to the underactuated system directly [3]. On the condition of parameter identification of ship's model being not accurate, to achieve stabilization control has important practical significance to underactuated surface vessel.

This paper proposes the stabilization controller based on backstepping for underactuated surface vessel. Diffeomorphism transformation is implemented in order to construct a new system. Additionally, the stabilization controller is designed according to backstepping for the system, and thereby ensure that the system is global asymptotic stabilization.

14.2 System Model

The underactuated surface vessel can be described by the following model [4].

$$\begin{cases} \dot{x} = u\cos(\psi) - v\sin(\psi) \\ \dot{y} = u\sin(\psi) + v\cos(\psi) \\ \dot{\psi} = r \\ m_{11}\dot{u} = m_{22}vr - d_{11}u + \tau_u \\ m_{22}\dot{v} = -m_{11}ur - d_{22}v \\ m_{33}\dot{r} = (m_{11} - m_{22})uv - d_{33}r + \tau_r \end{cases} \tag{14.1}$$

where $(x,\ y)$ represents inertial position and ψ is course in geodetic coordinate system, u, v, r describe surge, sway, and yaw rate of ship motion in the body fixed coordinate frame. d_{11}, d_{22}, d_{33}, m_{11}, m_{22}, m_{33} denote the hydrodynamic damping and ship inertia including added mass in surge, sway, and yaw. The available controls are the surge force τ_u and the yaw control moment τ_r. However, we do not have available control in the sway direction, thus the problem of controlling the ship in three degrees of freedom is an underactuated control problem.

Alternation of control input is defined as

$$\begin{cases} \tau_u = \dfrac{m_{22}}{m_{11}}vr - \dfrac{d_{11}}{m_{11}}u + \dfrac{1}{m_{11}}\tau_1 \\ \tau_r = \dfrac{m_{11} - m_{22}}{m_{33}}uv - \dfrac{d_{33}}{m_{33}}r + \dfrac{1}{m_{33}}\tau_3 \end{cases} \tag{14.2}$$

Assume A and B are given as follows: $A = m_{11}/m_{22}$, $B = d_{22}/m_{22}$. Then by (14.1) and (14.2), we obtain

$$\begin{cases} \dot{u} = \tau_u \\ \dot{v} = -Auv - Bv \\ \dot{r} = \tau_r \end{cases} \tag{14.3}$$

According to coordinate transformation $Z_2 = z_2 + v/b$. Then (z_1, Z_2) can be transformed as

$$\begin{cases} \dot{z}_1 = u + Z_2 r - \dfrac{v}{B} r \\ \dot{Z}_2 = -\dfrac{A}{B} ur - z_1 r \end{cases} \tag{14.4}$$

Then τ_α is selected as $B/A(z_1 + \alpha) - r(Z_2 - v/B) - \tau_u A/B$, Virtual control input α is adhibited, then

$$\begin{cases} \dot{z}_1 = -\frac{B}{A} z_1 - \frac{B}{A}\alpha + Z_2 r - \frac{v}{B} r \\ Z_2 = \alpha r \\ \dot{z}_3 = r \\ \dot{v} = -Bv + B(z_1 + \alpha) r \\ \dot{\alpha} = \tau_\alpha \\ \dot{r} = \tau_r \end{cases} \tag{14.5}$$

The same original system as the new system can prove stable after the diffeomorphism transform.

$$\begin{cases} f_1(t, x_1) + G(t, x) x_2 = \begin{bmatrix} -\frac{B}{A} z_1 - \frac{B}{A}\alpha - Z_2 r - \frac{v}{B} r \\ -Bv + B(z_1 + \alpha) r \end{bmatrix} \\ f_2(t, x_2, u) = \begin{bmatrix} \alpha r & r & \tau_\alpha & \tau_r \end{bmatrix}^T \end{cases} \tag{14.6}$$

Theorem *For given two cascaded subsystems* Σ_1 *and* $\Sigma_{2,}$ *the conditions of global asymptotic stabilization are as follows* [5]:

1. $f_1(t, x_1)$ *is global asymptotic stabilization.*
2. $G(t, x)$ *satisfy the following condition:*

$$\|G(t, x)\|_2 \le \theta_1 \|x_2\|_2 + \theta_2(\|x_2\|_2)\|x_1\|_2 \tag{14.7}$$

3. *The existence of a control law u in order to make* Σ_2 *stabilization.*

Proof System can be rewritten as

$$\dot{x}_1 = f_1(t, x_1) = \begin{bmatrix} -B & 0 \\ 0 & -\frac{B}{A} \end{bmatrix} x_1 = A x_1 \tag{14.8}$$

As subsystem x_1 is linear time invariant system and the matrix is nonsingular, $A + A^T$ is negative. $f_1(t, x_1)$ is global asymptotic stabilization. From Eq. (14.6), $G(t, x)$ can be chosen as

$$\begin{aligned} G(t,x) = {} & G_1(t,x_1) + G_2(t,x) \\ = {} & \begin{bmatrix} 0 & 0 & 0 & -\frac{v}{B} \\ 0 & 0 & 0 & Bz_1 \end{bmatrix} + \begin{bmatrix} r & 0 & -\frac{B}{A} & 0 \\ 0 & 0 & Br & 0 \end{bmatrix} \end{aligned} \tag{14.9}$$

where $\|G_1(t,x_1)\|_2 \leq \max(B, 1/B)\sqrt{z_1^2 + v^2} = \max(B, 1/B)\|x_1\|_2 = \theta_1\|x_1\|_2$

Thus $\|G(t,x)\|_2 \leq \|G_1(t,x_1)\|_2 + \|G_2(t,x)\|_2$ meet condition 2 in Theorem.

The control law u must be designed in order to make subsystem Σ_2 stable. Suppose α and r are virtual control input, from Eq. (14.6), it can be chosen as

$$\begin{cases} y_1 = Z_2 & y_2 = \alpha \quad y_3 = z_3 \\ u_1 = r & \dot{r} = \tau_r \quad u_2 = \tau_\alpha \end{cases} \tag{14.10}$$

In order to realize the desired exponential convergence to zero in Z_3, feedback control input $u_1 = -k_3Z_3$ of linear should be chosen. As discussed earlier, the feedback control law u_2 should be designed on the basis of backstepping.

A nonnegative control Lyapunov function is chosen to analyze the stability of system, this function is formed as

$$V_1(y_1) = \frac{1}{2}y_1^2 \tag{14.11}$$

y_2 is virtual control input of y_1. In this paper, we assume that $y_2 = k_1y_1/k_3y_3$. The derivative of V_1 along the trajectory of system is

$$\dot{V}_1(y_1) = y_1\dot{y}_1 = -k_3y_1y_2y_3 = -k_1y_1^2, \quad k_1 \geq 0 \tag{14.12}$$

If you want to meet the system y_2 is stable. Define a new variable ζ_1, meanwhile $\zeta_1 = y_2 - \Psi_1(y_1, y_3)$ is Deviation between y_2 and virtual control input $\Psi_1(y_1, y_3)$. The Lyapunov function can be chosen as

$$V_2(y_1, y_2) = \frac{1}{2}\zeta_1^2 + V_1(y_1) \tag{14.13}$$

Then, it follows that

$$\begin{aligned} \dot{V}_2(y_1,y_2) &= \zeta_1\dot{\zeta}_1 + \dot{V}_1(y_1) \\ &= -k_1y_1^2 + \zeta_1(u_2 + k_1(\zeta_1 + \Psi_1(y_1,y_3)) - k_1y_1/y_3 - k_3y_1y_3) \end{aligned} \tag{14.14}$$

To prove the stability, control law should be defined as

$$\dot{V}_2(y_1, y_2) = -k_1 y_1^2 - k_2 \zeta_1^2 \leq 0, \quad k_2 \geq 0 \tag{14.15}$$

Therefore, we can write y_1 and ζ approaches 0 when t approach ∞.

If you want to meet the first order linear system which is stable, then define a new variable ζ_2, meanwhile $\zeta_2 = u_1 - \Psi_2(y_3)$ is deviation between u_1 and virtual control input $\Psi_2(y_3)$. The Lyapunov function can be chosen as

$$V_3(r) = \frac{1}{2}\zeta_2^2 + V_2(y_1, y_2) \tag{14.16}$$

Then, it follows that

$$\dot{V}_3(r) = \zeta_2\dot{\zeta}_2 + \dot{V}_2 = \zeta_2(\tau_r + k_3 r) - k_1 y_1^2 - k_2 \zeta_1^2 \tag{14.17}$$

For a control law τ_r, where

$$\tau_r = -k_3 r - k_4 \zeta_2 = -k_3 r - k_4(r + k_3 y_3) \tag{14.18}$$

Therefore, we can write $\dot{V}_3(r) < 0$, which completes the proof.

14.3 Simulation Results

To evaluate performance of proposed method, the computer simulation has been used. Consider an underactuated surface vessel with model parameters as $m_{11} = 200$ kg, $m_{22} = 250$ kg, $m_{33} = 80$ kg m^2, $d_{11} = 70$ kg s^{-1}, $d_{22} = 100$ kg s^{-1} and $d_{33} = 50$ kg m^2 s^{-1}. Choice of initial condition for the vessel system: $[x_0, y_0, \Psi_0, u_0, v_0, r_0]^T = [5\text{ m}, 5\text{ m}, \pi/4\text{ rad}, 0\text{ m/s}, 0\text{ m/s}, 0.5\text{ rad/s}]^T$.

14.3.1 Simulation Results in No Flow Conditions

The control parameters are taken as $k_1 = 0.8$, $k_2 = 1$, $k_3 = 0.4$, $k_4 = 1.55$ and the simulation time is set to 50 s.

Figure 14.1 displays the course of ship trajectory tracking under the proposed backstepping law. It is clearly seen that the ship follows the reference trajectory with high accuracy. Figure 14.2 shows the response of yaw angle and angular velocity of the surface vessel. System state average index of underactuated surface vessel can be converged to zero by controller designed by backstepping method, and better convergence curve. Figure 14.3 shows intermediate variables can be

Fig. 14.1 Trajectory tracking of underactuated surface vessel

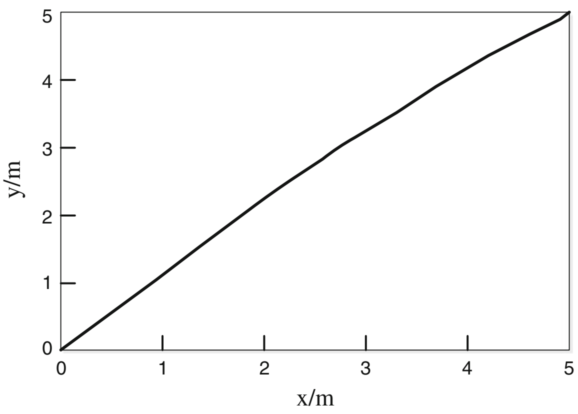


Fig. 14.2 Response of yaw angle and angular velocity

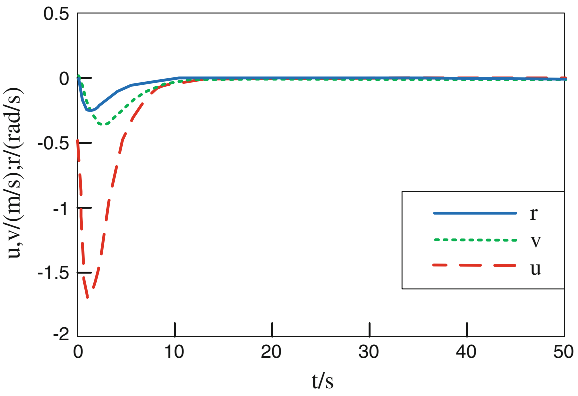
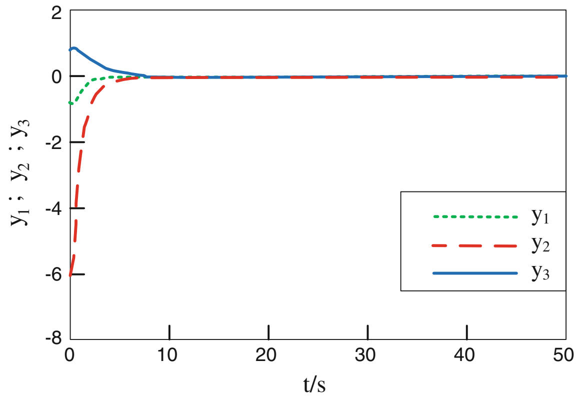


Fig. 14.3 Curve of intermediate variable



converged within 15 s and controller is stabilized to the origin. Curve convergence rate of y_3 is smaller than y_1 and y_2 due to controller is designed, but time of convergence is longer.

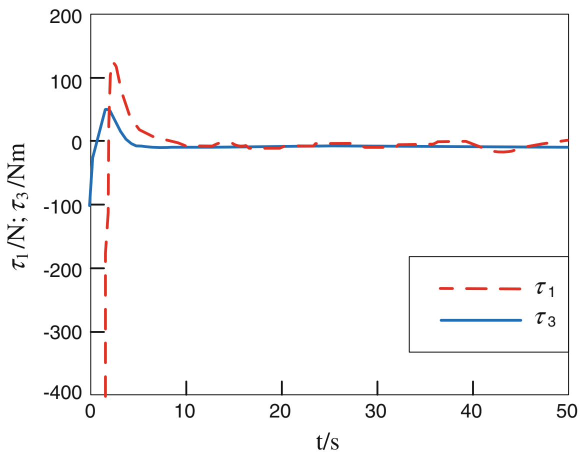


Fig. 14.4 Curve of forces applied to the vessel

14.3.2 Simulation Results in Flow Conditions

The disturbance torque is caused by the flow of the external wind and wave as [6].

The control parameters are taken as $k_1 = 0.8$, $k_2 = 1$, $k_3 = 0.4$, $k_4 = 1.55$ and the simulation time is set at 50 s.

Figure 14.4 displays the forces applied to the vessel. If stabilized at the origin, it should provide negative thrust and torque because the initial position is positive. When underactuated surface vessel is disturbed, the control force and torque vibrates in the vicinity of the equilibrium point. Figure 14.5 shows the longitudinal velocity and rate of heading. Figure 14.6 shows the strong robustness and stability in the vicinity of the origin for the underactuated surface ship in the presence of disturbances, nevertheless ship trajectory tracking is unideal. Figure 14.7 shows the

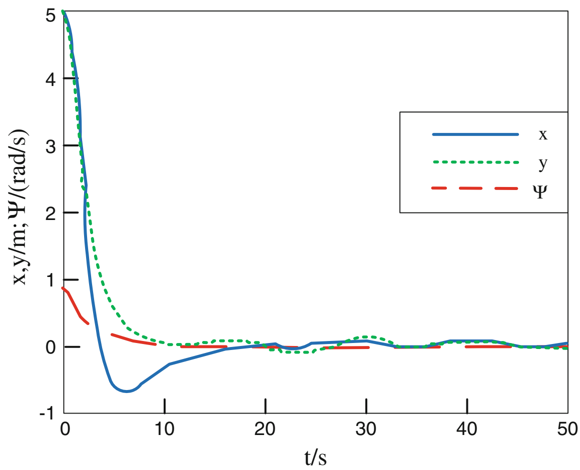


Fig. 14.5 Curve of location variable

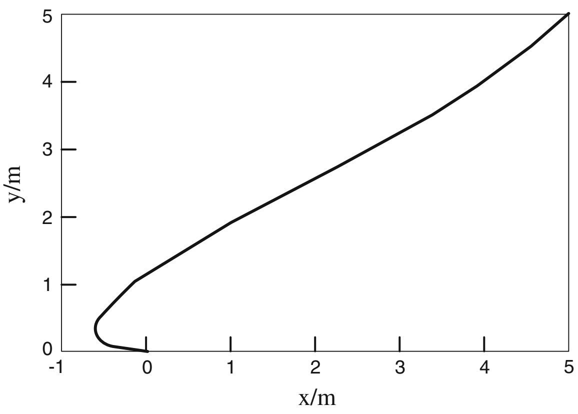


Fig. 14.6 Trajectory tracking of underactuated surface vessel

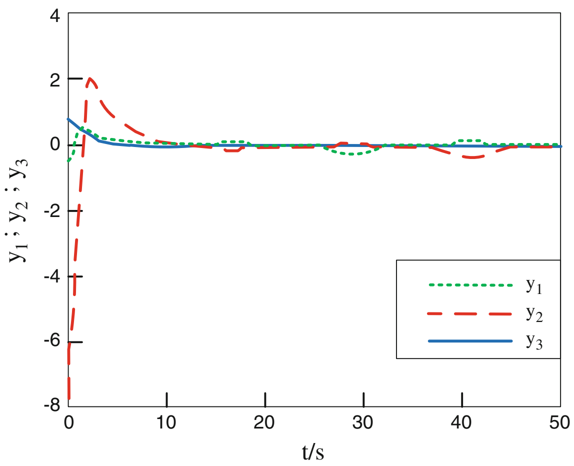


Fig. 14.7 Curve of intermediate variable

anti-jamming ability of y_3 is poor, furthermore, the amplitude of vibration is slightly larger than the other variables. Because the disturbances are positive, y_3 vibrates in the vicinity of 0.08.

14.4 Conclusion

In this paper, we proposed the new approach to stabilization of underactuated surface vessel based on backstepping control. Robustness and high performance can be achieved with the proposed scheme. Simulation results are included to demonstrate that the maneuvering motion of underactuated surface vessel is stabilized to the origin of the suggested approach, and is applied to engineering systems of underactuated vessel.

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References

1. Ghommam J, Mnif F, Derbel N (2010) Global stabilisation and tracking control of underactuated surface vessels. IET Control Theor Appl 4(1):71–88
2. Do KD, Pan J (2006). Global robust adaptive path following of underactuated ships. Automatica 42(10):1713–1722
3. Yongping PAN, Daoping HUANG, Zonghai SUN (2011) Backstepping adaptive fuzzy control for track-keeping of underactuated surface vessels. Control Theor Appl 28(7):907–914
4. Tang XH, Lu JA, Zhang WW (2007) The FPS of chaotic system using backstepping design. China Dyn Control 0705:216
5. Wang W, Yi J, Zhao D et al (2004) Design of a stable sliding-mode controller for a class of second-order underactuated systems. IEE Proc Control Theor Appl 151(6):683–690
6. Yu R, Zhu Q, Xia G, Liu Z (2011). Sliding mode tracking control of an underactuated surface vessel. IET Control Theor Appl 6(3):461–466