

Rationality and Context-Dependent Preferences

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Abstract The standard theory of rational choice in economics considers an agent's choices to be rational if and only if the agent makes her choices in different choice situations on the basis of a fixed preference ordering defined over the set of all possible options. This implies that a rational agent's preferences cannot be context-dependent. This paper outlines a simple framework for defining context-dependence of preferences and for discussing relationships between context-dependent preferences and the notion of rationality.

Keywords Context-dependence • Context-independence • Preference • Rationality • Standard theory

1 Introduction

An important criticism of the economists' theory of rational choice is that it rules out the possibility that an agent's preferences, which constitute the basis of her choices, may be context-dependent. The purpose of this paper is to introduce a simple framework to define the concept of context-dependent preferences of an agent, to indicate how a number of phenomena described in the literature can be fitted in this framework, and to explore the significance of context-dependent preferences for the notion of rational choice in economics.

To see how the concept of context-independence of preferences is embedded in the concept of rational choice in economics, it may be useful to note that, methodologically, there are two distinct approaches in the economists' theory of choice, namely, the preference-based approach which starts with preference as a

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primitive concept in the model, and the choice-based approach (also widely known as the revealed preference approach), which starts with choice, but not preference, as the primitive concept. In this paper, we focus on the preference-based approach, but what we have to say about rational choice and context-dependent preferences in this approach can be readily adapted to the choice-based approach.

In the preference-based approach, an agent is said to be rational (or, equivalently, an agent is said to be choosing rationally) if she has a given preference ordering defined over the universal set of options (i.e., the set of all conceivable options) and if, for every possible set of feasible options that may confront her, she always chooses from that set of options¹ which are the best options in that set, “best” being defined in terms of her given preference ordering over the universal set of options. This notion of rational choice by an agent can be conceptually split into two components. The first component is the idea that a rational agent has a fixed binary weak preference relation (“at least as good as”) over the set of all conceivable options, which constitutes the sole basis of her choices from different sets of feasible options; the second component is the assumption that this fixed binary weak preference relation over the universal set, which serves as the basis of the agent’s choices in all contexts, is an ordering (i.e., it satisfies reflexivity, connectedness (or completeness), and transitivity). Intuitively, the existence of a fixed binary weak preference relation over the set of all conceivable options, which determines the agent’s choice(s) from every set of feasible options in all contexts, implies that the specific context in which the agent chooses from a set of feasible options does not matter at all for her preferences over the options in that feasible set and that all the information that she considers to be relevant for her preferences are already contained in the specification of the options at the outset; this feature has sometimes been called the axiom of context-independence [13]. In this paper, we are primarily concerned with the first constituent component, to wit, context-independence of preferences, of the economists’ conception of rational choice, though we also comment on the second constituent component, namely, the assumption that the fixed binary weak preference relation over the universal set of options is an ordering.

Section 2 develops a simple framework in which the notion of context-dependent preferences can be defined precisely and explicitly. Also, in Sect. 2, we present several examples that are well-known in the literature and show how they fit in our framework. In Sect. 3, we discuss the formal link between context-independence and the conventional concept of rational choice. Finally, in Sect. 4, we comment on the significance of context-dependent preferences for the economic theory of rational choice.

¹Following a fairly common practice in the literature, we are permitting the choice set for a given set of feasible options to have multiple elements.

2 The Framework

Let X be a given, non-empty universal set of *conventionally defined* options. For example, in a competitive consumer's choice problem in economics, X is the set of all commodity bundles, a commodity bundle being a specification of the quantities of all commodities under consideration. If the choice of an entree for dinner is the problem at hand, then the set of all possible alternative entrees constitutes the universal set of (conventionally defined) options. Y denotes a non-empty subclass of the class of all non-empty subsets of X , the elements of Y being denoted by A, B , etc. The interpretation of Y is that Y is the class of all possible alternative menus or sets of feasible conventionally defined options from which the agent may have to choose. Y is permitted, but not constrained, to be the class of all non-empty subsets of X . In the standard economic theory of rational choice, an agent makes her choices rationally if and only if: (1) she has a fixed binary weak preference relation \succeq over X , such that, for all $A \in Y$, the agent's set of chosen elements in A are given by $\{x \in A : \text{for all } y \in A, x \succeq y\}$; and (2) this fixed binary weak preference relation \succeq is an ordering. There are, however, numerous examples in the literature where the agent does not seem to make her choices in this fashion. In such examples, either the agent directly tells us that she strictly prefers an option x in X to another option y in X under certain circumstances but she considers option y to be at least as good as x under certain other circumstances or we observe the agent choosing in a fashion, which is not consistent with the assumption that she has a fixed binary weak preference relation \succeq over X , such that from every $A \in Y$, she chooses the \succeq -greatest elements in A .

2.1 Two Examples

It may be useful to start with two well-known examples, where the choice behavior of the agent could not have been generated by any fixed binary relation over the universal set of options (later we consider several other examples, which have been discussed in the literature and which have the same feature).

Example 1(a): Menu-Dependence

The first example here is a slightly modified version of an example due to Sen [17]. Consider the following choices of an individual. When a fruit basket on the dinner table contains many apples and many oranges, the agent, who is one of several guests at the dinner, chooses an apple rather than any of the oranges, but, had the fruit basket on the table contained the same oranges but exactly one apple,

the individual would have chosen an orange rather than the single apple. It is clear that there cannot be any fixed binary weak preference relation \succeq over the fruits that can induce the choice of an apple over the other fruits from the first fruit basket and the choice of an orange over the apple from the second fruit basket. The intuitive explanation given by Sen for such choices is that choosing the single apple from the second basket would be rude while no rudeness is involved in choosing one of several available apples.

Example 1(b): Menu-Dependence

Consider another example, which is due to Luce and Raiffa [14]. The waiter in a restaurant gives a customer, who does not know much about the restaurant, a menu for the day's dishes. The menu contains two items: steak and fish. The customer chooses fish. A little later, the waiter reports that, because of a mistake, frog's legs have been omitted from the day's menu but they are available. The customer then chooses steak from this expanded menu. Again, there cannot be any fixed binary weak preference relation \succeq over all conceivable dishes, such that, in terms of \succeq , fish is the most preferred dish in the initial menu consisting of fish and steak while steak is the most preferred dish in the expanded menu consisting of fish, steak, and frog's legs. The explanation that Luce and Raiffa [14, p. 288] give for the agent's choices is as follows. The customer would prefer steak to fish if she had reasonable assurance that the restaurant was good so that steak will be well prepared, but, in the absence of any information about the quality of the restaurant, she prefers fish to steak. The customer's past experience tells her that frog's legs are served only in good restaurants. So, when she knows that frog's legs are in the menu, she believes that the restaurant must be good and accordingly prefers steak to fish (she prefers steak to frog's leg in any case).

In Example 1(a), whether or not a particular fruit is the only one of its kind in the fruit basket is a relevant consideration for the agent, but no information about this is available in the description of the an apple or the description of an orange as such. Only when the context for the agent's choice is given in the form of a specific fruit basket in front of her, does she get additional information about whether an apple or an orange is the only fruit of its kind available to her. Similarly, in Example 1(b) the customer's preference over fish and meat depends on contextual information regarding the availability of frog's legs in the menu. In both cases, the relevance, for the agent, of the information, which is to be found in the contextual features but not in the descriptions of the options in X , causes problem for the standard theory of rational choice formulated in terms of a fixed binary weak preference relation defined over the options. Given this, one possibility may be to reformulate explicitly the theory so as to allow for the fact that, in assessing the relative desirability of any two options, the agent takes into account the contextual circumstances, in which the options are presented to her, besides the information contained in the specification

of those options themselves.² But what exactly is the demarcating line between the features which are parts of the specification of options and the features which constitute parts of the descriptions of contexts? If the theorist constructing the model of choice knows that certain features affect the relative desirability, for the agent, of options as specified by her (i.e., the theorist), then why should the theorist not re-specify the options so as to make these features a part of the descriptions of these re-specified options? These are questions which arise naturally. In fact, Savage's [16, Chap. 2] discussion of the largeness or the smallness of the "world" (i.e., "the objects about which the person is concerned") is essentially concerned with this issue. As one switches progressively from narrower conceptions of options to broader conceptions of options, more and more of the features which can be regarded as contextual features under narrower conceptions will be subsumed in the description of options conceived more broadly. There are clearly advantages in starting with a very broad notion of options (or a "sufficiently broad world", to use Savage's [16] terminology). But as Savage [16, p. 9] points out, "the use of modest little worlds, tailored to particular contexts is often a simplification, the advantage of which is justified by a considerable body of mathematical experience with related ideas". It seems to us that, in many areas of economics, conventional specifications of the agent's options are simplifications of this type. Though it may be known that certain features outside such narrow but simple specifications may be considered relevant by the agent, there may not be sufficient agreement about the significance of such features, and so the conventional and narrow specifications of options continue to be used. An example is the theory of consumers' behavior in economics, where commodity bundles are assumed to be the objects with which the consumer is concerned. While it may be recognized that a consumer's preferences over a pair of commodity bundles may depend on certain features of the "context" in which the consumer faces the task of comparing two commodity bundles, there may not be any general consensus about the importance of such contextual features, and, therefore, the conventional specification of options as commodity bundles continues to be used. Also, often the theorist may not know what matters for an agent beyond the features captured by a certain specification of options; in such cases, the contextual features will represent the gap between what features the agent perceives to be relevant for her own preferences and what the theorist initially perceives to be relevant for the agent. Once the theorist becomes aware of this gap, she has the option of re-specifying the options in her theory, incorporating the features missed out in her initial specification; we shall take up this issue again in Sect. 4.

²See Baigent [1], Baigent and Gaertner [2], Bhattacharyya et al. [5], Bossert and Suzumura [6, 7], Gaertner and Xu [8], and Xu [19] for some axiomatic studies along this line of research.

2.2 Context-Dependent Preferences

Let O be a set of all (mutually exclusive) contextual features (or, simply, contexts), which may affect the relative desirability or undesirability of conventionally defined options for the agent under consideration but which do not constitute a part of the descriptions of the options in X . Given X and O , we now introduce a simple but general framework which permits (without making it mandatory) context-dependence of the agent's preferences. For all $A \in Y$, let $O(A)$ be the (non-empty) set all contexts $o \in O$, such that the agent may have to choose from A given the context o (note that there may be many different contexts in which x may be chosen from A , so that $O(A)$ may have more than one element). Let S be the set of all (x, o) in $X \times O$, such that, for some A in Y , $x \in A$ and $o \in O(A)$. It is possible that there may be $x \in X$ such that, for all $A \in Y$, $x \notin A$; let X' be the set of all such x .

Let R be a binary relation over S . For all $(x, o), (y, o') \in S$, $(x, o)R(y, o')$ will be interpreted as "the agent considers x in the context o to be at least as desirable as y in the context o' ". We will use I and P , respectively, to denote the symmetric and asymmetric parts of R , that is, for all $(x, o), (y, o') \in S$, $[(x, o)I(y, o') \text{ iff } ((x, o)R(y, o') \text{ and } (y, o')R(x, o))]$ and $[(x, o)P(y, o') \text{ iff } ((x, o)R(y, o') \text{ and not}(y, o')R(x, o))]$. For all $A \in Y$ and all $o \in O(A)$, when the agent chooses from A given the context o , the set of her chosen options is $\{x \in A : (x, o)R(y, o) \text{ for all } (y, o) \in A \times \{o\}\}$. Essentially, this simple framework augments the informational basis of the theory of choice by introducing explicitly into the formal framework information about the different contexts in which the agent may have to make her choices. For the sake of convenience, we shall refer to this framework as the *context-inclusive framework*.

We now introduce three notions of context-independence of the preferences represented by R in our context-inclusive framework.

Context-independence (I). For all $(x, o), (y, o'), (x, \bar{o}), (y, \bar{o}') \in S$, $[(x, o)R(y, o') \text{ iff } (x, \bar{o})R(y, \bar{o}')] \text{ and } [(y, o')R(x, o) \text{ iff } (y, \bar{o}')R(x, \bar{o})]$.

The following is a somewhat weaker notion of context-independence, which stipulates that the ranking of any two options, x and y given the contextual features o is exactly analogous to the ranking of x and y given any other contextual features o' .

Context-independence (II). For all $(x, o), (y, o), (x, o'), (y, o') \in S$, $[(x, o)R(y, o) \text{ iff } (x, o')R(y, o')] \text{ and } [(y, o)R(x, o) \text{ iff } (y, o')R(x, o')]$.

Our last formulation of context-independence stipulates that the desirability of an option does not depend on the context in which it is chosen.

Context-independence (III). For all $(x, o), (x, o') \in S$, $(x, o)I(y, o')$.

We say that R is *context-dependent (I)* (resp. *context-dependent (II)*, resp. *context-dependent (III)*) iff R does not satisfy context-independence (I) (resp. context-independence (II), resp. context-independence (III)).

Our first result examines the relationship between these notions of context-independence.

Proposition 1

- (1.i) *If R is context-independent (I), then it is context-independent (II), and, if R reflexive and context-independent (I), then it is context-independent (III).*
 (1.ii) *If R is transitive, then context-independence (III) of R implies context-independence (I) of R and context-independence (II) of R*

Proof

- (1.i) It is obvious that, if R is context-independent (I), then R is context-independent (II). We show that, if R is reflexive and context-independent (I), then R is also context-independent (III). Suppose R is reflexive and context-independent (I). Let $(x, o), (x, o') \in S$. By context-independence (I), for all $(z, o^1), (w, o^2), (z, o^3), (w, o^4) \in O$, $[(z, o^1)R(w, o^2) \text{ iff } (z, o^3)R(w, o^4)]$ and $[(w, o^2)R(z, o^1) \text{ iff } (w, o^4)R(z, o^3)]$. Then, letting $z = w = x, o^1 = o^2 = o^3 = o$, and $o^4 = o'$, and noting $(x, o)I(x, o)$, which follows from reflexivity of R , we have $(x, o)I(x, o')$.
- (1.ii) Suppose R is transitive and context-independent (III). Then we show that R is context-independent (I). Let $(x, o), (y, o'), (x, \bar{o})$, and (y, \bar{o}') be in S . Suppose $(x, o)R(y, o')$. By context-independence(III) of R , we have $(x, o)I(x, \bar{o})$ and $(y, o')I(y, \bar{o}')$. Then, by the transitivity of R , we obtain $(x, \bar{o})R(y, \bar{o}')$. Similarly, we can show that, if $(x, \bar{o})R(y, \bar{o}')$ then $(x, o)R(y, o')$, if $(y, o')R(x, o)$ then $(y, \bar{o}')R(x, \bar{o})$, and, if $(y, \bar{o}')R(x, \bar{o})$ then $(y, o')R(x, o)$. Therefore, we have $[(x, o)R(y, o') \text{ iff } (x, \bar{o})R(y, \bar{o}')] \text{ and } [(y, o')R(x, o) \text{ iff } (y, \bar{o}')R(x, \bar{o})]$. Thus, R is context-independent (I).

Since, in the presence of transitivity, context-independence (III) implies context-independence (I), by Proposition (1.i), it follows that, in the presence of transitivity, context-dependence (III) implies context-dependence (II). ■

As a corollary to Proposition 1, we have the following result.

Corollary 1 *Suppose R is reflexive and transitive. Then, (i) context-independence (I) of R and context-independence (III) of R are equivalent; and (ii) context-independence (III) of R , as well as context-independence (I) of R , implies context-independence (II) of R .*

In the presence of reflexivity and transitivity of R , context-independence (II) of R is strictly weaker than each of context-independence (I) of R and context-independence (III) of R as shown by the example in the following remark.

Remark 1 Suppose $O = \{o_1, \dots, o_n\}$ with $n \geq 2$ and $S = X \times O$. Let \succeq over X be an ordering, and define R^* over S such that, for all $(x, o_i), (y, o_j) \in S$, if $i > j$ then $(x, o_i)P^*(y, o_j)$, and if $i = j$ then $(x, o_i)R^*(y, o_j) \text{ iff } x \succeq y$. Then, R^* is context-independent (II) since, for all $(x, o_i), (y, o_i), (x, o_j), (y, o_j) \in S$, from the definition of R^* , we have $[(x, o_i)R^*(y, o_i) \text{ iff } x \succeq y \text{ and } x \succeq y \text{ iff } (x, o_j)R^*(y, o_j)]$ implying that $(x, o_i)R^*(y, o_i) \text{ iff } (x, o_j)R^*(y, o_j)$. That R^*

is an ordering follows from the fact that \succeq is an ordering and the definition of R^* . On the other hand, R^* is not context-independent (III) by noting, for example, $(x, o_1)P(x, o_2)$. Given that R^* is an ordering, that R^* is not context-independent (I) follows from Corollary 1 and that R^* is not context-independent (III). Thus, the intuition that context-independence (II) of R is strictly weaker than either context-independence (I) or (III) of R seems to point to the fact that, in context-independence (II) of R , contexts may contribute to R independently of the outcomes, while such possibly independent values of contexts do not exist in context-independence (I) or (III) of R .

Remark 2 Proposition 1, Corollary 1, and Remark 1 spell out the formal relations between the three concepts of context-independence. It may, however, be helpful to comment on the intuitive difference between these concepts. Intuitively, what context-independence (I) says is that, to compare two outcomes x and y , the agent does not need any information about the context where x is to be chosen and the context where y is to be chosen. Context-independence (II) says something significantly weaker: it says that, so long as the agent is comparing outcomes x and y in a fixed and unchanging context, the agent's comparison of outcomes is not affected by any information about what that fixed context happens to be. Context-independence (III) is concerned with the agent's assessment of the desirability of a given outcome in two different contexts: it says that the desirability of a given outcome x remains exactly the same even if the context changes. By Proposition 1.i, if R is reflexive, then context-independence (I) implies context-independence (III); and, by Proposition 1.ii, if R is transitive, then context-independence (III) implies context-independence (I). It is difficult to think of any circumstance where R may not be reflexive. So, intuitively, the difference between context-independence (I) and context-independence (III) matters only in the absence of transitivity of R .

2.3 Some Further Examples

The literature on the theory of preference and choice discusses several types of choice behavior which are clearly incompatible with the idea that the agent has a fixed binary weak preference relation, which constitutes the basis of her choices in different choice situations. In what follows, we will consider several such examples and interpret them in terms of our formal framework.

First consider Examples 1(a) and 1(b) above. In Example 1(a), let x be the physically specified option of having an apple and y be the physically specified option of having an orange. The two contexts under consideration in this example can be specified as $o =$ "the fruit basket contains more than one apple and more than one orange" and $o' =$ "the fruit basket contains exactly one apple and several oranges". Consider (x, o) , (x, o') , (y, o) , and (y, o') in S . Suppose the agent has the binary relation R over S , such that $(x, o)P(y, o)$ and $(y, o')P(x, o')$. It is then clear that: (1) R violates context-independence (II) and, hence, context-independence

(I); and (2) given the set $\{x, y\}$, the agent will choose only x if the context is o and she will choose only y if the context is o' . In Example 1(b), the options are $x = \text{fish}$, $y = \text{steak}$, and $z = \text{frog's legs}$, and the contexts are $o = \text{"frog's legs are available in the restaurant"}$ and $o' = \text{"frog's legs are not available in the restaurant"}$. R over S is such that $(x, o)P(y, o)$, $(y, o')P(x, o')$, and $(y, o')P(z, o')$. Then R is context-dependent (II), and, hence, context-dependent (I). Given the context o , the agent chooses only x from $\{x, y\}$, and, given the context o' , the agent chooses only y from $\{x, y, z\}$.

Example 2: Procedural Considerations

An individual expresses a preference for reading the government's official newspaper when a spectrum of newspapers, from the left to the right and from pro-government to anti-government, is available. However, after the government cracks down on dissenting newspapers by allowing only pro-government newspapers to remain published, the individual changes his preference and now prefers not to read any newspaper [9, 10]. The change of the individual's preference is due to the individual's concerns about procedures that have brought about the change in the availability of newspapers. Contexts in this example are possible procedures that are used to bring about the options under consideration. R is such that:

(reading the official newspaper; there is no government interference with the publication of newspapers) P (not reading any newspaper; there is no government interference with the publication of newspapers)

and

(not reading any newspaper; the government has shut down the dissenting newspapers) P (reading the official newspaper; the government has shut down the dissenting newspapers).

Example 3: States of Nature as Contexts

A consumer expresses a preference for a scoop of ice cream over a cup of chicken noodle soup when the weather is hot, while she prefers a cup of chicken noodle soup to a scoop of ice cream when it is cold [3, 4]. It is easy to interpret this problem in our framework by treating hot weather and cold weather as the two contexts. The consumer's weak preference relation R in our framework will be as follows:

(a scoop of ice cream; hot weather) P (a cup of chicken noodle soup; hot weather),
and (a cup of chicken noodle soup; cold weather) P (a scoop of ice cream; cold weather).

3 Context-Independence of Preferences and Rationality

In this section, we examine the formal connection between our concepts of context-independence and the notion of rational choice in standard economic theory.

In the standard economic theory, an agent's observed choices are said to be *rationalizable in terms of a binary weak preference relation* if there exists a binary weak preference relation, \succeq , over X such that, for all $A \in Y$, the set of \succeq -greatest elements in A coincides with the observed set of options chosen by the agent from A , and the agent's observed choices are said to be rational if they are rationalizable in terms of a preference ordering \succeq over X . Adapting this terminology, we shall say that, an agent's observed choices are *rationalizable in terms of a binary relation, R* , over S in the context-inclusive framework if, for all $A \in Y$ and all $o \in O(A)$, the set $\{x \in A : (x, o) \text{ is } R\text{-greatest in } A \times \{o\}\}$ coincides with the observed set of options chosen by the agent from A ; and we say that the agent's observed choices are *rational* in the context-inclusive framework if they are rationalizable in terms of an ordering R over S in that framework.

Proposition 2

- (2.i) *Suppose, in the context-inclusive framework, an agent's observed choices are rationalizable in terms of an ordering, R , over S , such that R is context-independent (I) or context-independent (III). Then the agent's observed choices are rational in the sense of standard economic theory.*
- (2.ii) *Suppose, in the context-inclusive framework, an agent's observed choices are rationalizable in terms of an ordering, R , over S , such that R is context-independent (II). Then the agent's observed choices are rational in the sense of standard economic theory.*
- (2.iii) *If an agent's observed choices are rational in the sense of standard economic theory, then, in the context-inclusive framework, the agent's observed choices are rationalizable in terms of an ordering R over S , such that R is context-independent (I), context-independent (II), and context-independent (III).*

Proof

- (2.i) By Proposition 1, if a binary relation R over S is an ordering, then context-independence (I) of R is equivalent to context-independence (III) of R . So we give the proof only for the case where R is context-independent (I).

Suppose, in the context-inclusive framework, the agent's observed choices are rationalizable in terms of an ordering R over S , such that R is context-independent (I). Define a binary weak preference relation \succeq over X as follows:

- (i) For all $x \in X - X'$ and all $y \in X'$, $x \succ y$, and, for all $y, z \in X'$, $y \sim z$, where \succ and \sim are, respectively, the strict preference relation and indifference relation corresponding to \succeq (that is, for all $a, b \in X$, $[a \succ b$ iff $a \succeq b$ and not($b \succeq a$)] and $[a \sim b$ iff $a \succeq b$ and $b \succeq a]$);
- (ii) For all $x, y \in X - X'$, $x \succeq y$ iff $(x, o)R(y, o')$ for some $o, o' \in O$.

Note that, by context independence (I) of R , for all $(x, o), (y, o'), (x, \bar{o}), (y, \bar{o}') \in S$, $[(x, o)R(y, o') \text{ iff } (x, \bar{o})R(y, \bar{o}')] \text{ and } [(y, o')R(x, o) \text{ iff } (y, \bar{o}')R(x, \bar{o})]$. Therefore, by (ii), $x \succeq y$ is well-defined for all $x, y \in X - X'$. Given (i), it then follows that $x \succeq y$ is well-defined for all $x, y \in X$.

Given that R is reflexive, connected, and transitive, it can be easily checked that, by (ii), the restriction of \succeq to $X - X'$ is reflexive, connected, and transitive over $X - X'$. Hence, noting (i), \succeq is an ordering over X .

R being context independent (I), for all $A \in Y$, the observed set of options chosen by the agent from A is the same as $\{x \in A : (x, o)R(y, o) \text{ for all } (y, o) \in A \times \{o\}\}$ for all $o \in O(A)$. From the definition of \succeq , it follows that, for all $A \in Y$ and all $o \in O(A)$, $\{x \in A : (x, o)R(y, o) \text{ for all } (y, o) \in A \times \{o\}\} = \{x \in A : x \succeq y \text{ for all } y \in A\}$. Therefore, for all $A \in Y$, $\{x \in A : x \succeq y \text{ for all } y \in A\}$ is the same as the observed set of options chosen by the agent from A .

Thus, the observed choices of the agent are rationalizable in terms of the ordering \succeq in the standard economic framework.

(2.ii) Suppose, in the context-inclusive framework, the agent's observed choices are rationalizable in terms of an ordering R over S , such that R is context-independent (II). Define a binary weak preference relation \succeq over X as follows:

- (iii) For all $x \in X - X'$ and all $y \in X'$, $x \succ y$, and, for all $y, z \in X'$, $y \sim z$, where \succ and \sim are, respectively, the strict preference relation and indifference relation corresponding to \succeq ;
- (iv) For all $x, y \in X - X'$, $x \succeq y$ iff $(x, o)R(y, o)$ for some $o \in O$.

Note that, by context independence (II) of R , for all $(x, o), (y, o), (x, o'), (y, o') \in S$, $[(x, o)R(y, o) \text{ iff } (x, o')R(y, o')] \text{ and } [(y, o)R(x, o) \text{ iff } (y, o')R(x, o')]$. Therefore, by (iv), $x \succeq y$ is well-defined for all $x, y \in X - X'$. Given (iii), it then follows that $x \succeq y$ is well-defined for all $x, y \in X$.

Given that R is reflexive, connected, and transitive, it can be easily checked that, by (iv), the restriction of \succeq to $X - X'$ is reflexive, connected, and transitive over $X - X'$. Hence, noting (i), \succeq is an ordering over X .

R being context independent (II), for all $A \in Y$, the observed set of options chosen by the agent from A is the same as $\{x \in A : (x, o)R(y, o) \text{ for all } (y, o) \in A \times \{o\}\}$ for all $o \in O(A)$. From the definition of \succeq , it follows that, for all $A \in Y$ and all $o \in O(A)$, $\{x \in A : (x, o)R(y, o) \text{ for all } (y, o) \in A \times \{o\}\} = \{x \in A : x \succeq y \text{ for all } y \in A\}$. Therefore, for all $A \in Y$, $\{x \in A : x \succeq y \text{ for all } y \in A\}$ is the same as the observed set of options chosen by the agent from A .

Thus, the observed choices of the agent are rationalizable in terms of the ordering \succeq in the standard economic framework.

(2.iii) Suppose an agent's observed choices are rational in the sense of standard economic theory. Then, there exists a preference ordering \succeq over X such that, for all $A \in Y$, the observed set of options chosen by the agent from A coincides with the set $\{x \in A : x \succeq y \text{ for all } y \in A\}$. Define a binary relation R over S as follows: for all $(x, o), (y, o') \in S$, $(x, o)R(y, o')$ iff $x \succeq y$. Clearly, R is well-defined. Since \succeq is reflexive, R is reflexive. Similarly, the connectedness

and transitivity of R follow from the connectedness and transitivity of \succeq . It is clear that R is context-independent (I). Then, by Proposition (1.i), R is context-independent (II), and, noting reflexivity of R , by Proposition (1.i) again, R is context-dependent (III). Finally, note that, by the construction of R , for all $A \in Y$ and all $o \in O(A)$, $\{x \in A : (x, o)R(y, o) \text{ for all } (y, o) \in A \times \{o\}\}$ coincides with $\{x \in A : x \succeq y \text{ for all } y \in A\}$, which, in turn, coincides with the observed set of options chosen by the agent from A .

Thus, the agent's observed choices are rationalizable in terms of the ordering R over S in the context-inclusive framework and R is context-independent in each of the three senses under consideration. ■

Remark 3 Proposition 2 formally confirms that the following two concepts can be thought of as being exact counterparts of each other: (1) the concept of rational choice in standard economic theory; and (2) the concept, in the context-inclusive framework, of observed choices being rationalizable in terms of an ordering R , which is defined over S and which satisfies any of the three types of context-independence introduced earlier in this paper.

Remark 4 Should context-dependence of R in the context-inclusive framework be regarded as an indication of irrationality? If the term "rationality" is being used in a normative fashion,³ then we do not find anything particularly irrational in the context-dependence of R in Examples 1(a), 1(b), 2, and 3. We do not, however, rule out the possibility that some other specific manifestations of context-dependence of R may be judged "irrational". Thus, if a person living in Los Angeles and ordering her lunch to be eaten in Los Angeles, prefers ice cream to a cup of chicken noodle soup when it is hot in Delhi and prefers a cup of chicken noodle soup to ice cream when it is cold in Delhi, the person's preferences can be modeled as context-dependent preference, the context being the specific weather prevailing in Delhi (cf. Example 3 above). But, in the absence of any known connection between the weather in Delhi and the state of affairs in Los Angeles, we believe that such context-dependence will be considered irrational by most people. Therefore, it seems to us that, in discussing the normative concept of rational choice, there is some advantage in using the context-inclusive framework, which permits context-dependence of R and allows us to discuss which forms of context-dependence may or may not be "reasonable". The question may arise whether one can lay down some general criteria to judge whether a particular manifestation of context-dependence is "unreasonable". Saying that certain manifestations of context-dependent preferences are reasonable or unreasonable involves a value judgment and value judgments are not beyond the boundaries of systematic reasoning.⁴ But, though one can outline the formal structure of the type of reasoning that can be used to question or support value judgments, the intuitive content of such reasoning has to depend on the

³In the following section, we elaborate the distinction between normative and non-normative uses of the term "rationality".

⁴See, among others, Hare [11, 12], Sen [18], and Pattanaik [15, Chap. 2].

specific value judgment under consideration. Thus, it would seem difficult, if not impossible, to formulate *a priori* standards for judging which types of context-dependent preferences are unreasonable.

Remark 5 Note that, in the context-inclusive framework, if O has exactly one element and the agent's observed choices are rationalizable in terms of an ordering R over S , then the agent's choices are rational in the standard economic framework. This is because that, when $O = \{o\}$ and R is an ordering, R is trivially context-independent (I), (II), and (III). Therefore, by Proposition 2, the agent's observed choices are rational in the standard economic framework.

4 Discussion

In this paper, we have explored a component of the standard economic conception of a rational agent and rational choices. This component requires that for an agent to be rational, she should have a fixed weak-preference relation over the universal set of all possible options and she should make her choices from different sets of feasible options on the basis of this fixed weak preference relation over the universal set of options. This, of course, implies that the preferences of the agent should be independent of the context in which she makes her choice. Over the last few decades, the literature on choice and preference has highlighted many instances where an individual's preferences are context-dependent. We have discussed some of these examples. What are their implications for the standard theory of rational choice? To see this, it will be useful to distinguish between two distinct ways in which the theory of rational choice in economics can be viewed. First, it can be viewed as an attempt to articulate what we intuitively mean when we say that an agent is rational or is choosing options in a rational fashion. Viewed in this way, the concept of a rational agent has normative content: a person whose choices in different choice situations are not based on a fixed preference ordering defined over the universal set of options is then regarded as falling short of fulfilling our intuitive standards for judging rationality. One can, however, take a second, and alternative, view of the theory of rational choice in economics. From this perspective, the term "choosing rationally" is simply a technical shorthand expression for "making choices in different choice situations on the basis of a fixed preference ordering defined over the universal set of options", and it does not seek to articulate any deep intuition about rationality. Under this interpretation, economists' theory of rational choice can be viewed as an exploration of the implications of the empirically testable hypothesis that an agent always makes her choices in different choice situations on the basis of a fixed preference ordering defined over the universal set of options (this central hypothesis, however, is typically combined with other empirical hypotheses to construct a theoretical model).

Irrespective of which of the two interpretations of the economists' theory of rational choice we adopt, the concept of context-independent preferences constitutes

an integral part of the notion of rational choice in this theory. Earlier, we have discussed various examples of context-dependent preferences. Our reaction to these examples will depend on our specific interpretation of the economic theory of rational choice. Consider first the case where we interpret the definition of rational choice as an attempt to articulate our intuitive notion regarding the “rationality” of choices so that the definition is taken to have a normative significance. Given this interpretation of the definition of rational choice, it seems reasonable to maintain that there is nothing irrational about the choices of the agents in our Examples 1(a), 1(b), 2, and 3. Thus, the agent in Example 1(a) can be considered to be choosing rationally, despite his context-dependent preferences, since he has good reasons for choosing an orange over an apple in one context and choosing an apple over an orange in a different context. In the case of this example, we feel that: (1) if the options are specified simply as alternative fruits, then there are good reasons not to insist on treating context-independence of preferences as a condition for rational choice of options specified in this fashion; and (2) if, at all, we want to have context-independence as a condition for rational choice of options, we should redefine the options so that the redefined options are “picking up an apple in a dinner party when the apple happens to be the only apple in the available fruit basket”, “picking up the apple in a dinner party when there are several apples in the available fruit basket”, and so on. In contrast, while, in the example discussed in Remark 4 (where the choice between chicken noodle soup and ice cream in Los Angeles is contingent on the weather in Delhi), one can model the agent’s preferences as being context-dependent, such context-dependence and the resultant choices would seem irrational, in a normative sense of the word “irrational”, to most people. Now consider the second interpretation of the theory of rational choice which is devoid of normative content and under which saying that all agents make their choices rationally amounts to an empirical hypothesis that every agent makes her choices in different choice situations on the basis of a fixed preference ordering defined over a universal set of options. In this case, manifestations of context-dependent preferences, including those in the examples that we have considered in this paper, falsify that hypothesis given the specification of options that the theorist uses in his model. Faced with such falsification, the theorist may decide to take any one of the following routes:

- (a) The theorist may re-specify the options so as to incorporate in the new specification of options certain features of what was called a context under the original specification of options (the phenomenon of context-dependent preferences that arose given the original specification of options, may disappear after such re-specification).
- (b) The theorist may retain her original specification of options and continue to work with the falsified empirical hypothesis of rational choice by agents on the ground that, though, given her specification of the options, the hypothesis is contradicted by some empirical evidence, the hypothesis is still useful since it “works most of the time” or at least “works most of the time for the class of choice problems under consideration”.

- (c) The theorist may retain her specification of options, abandon altogether the empirical hypothesis of rational choice (at least for the type of choices under consideration), and proceed to explore the implications of alternative models which permit context-dependent preferences.

We believe that, when, despite some evidence of context-dependent preferences, most economists continue to retain the standard theory of rational choice, they are really opting for route (b) or (c) above.

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