Chapter 14
Bi-level Decision Making in Railway Transportation Management

Transportation management is an important application field of bi-level decision-making. For example, transportation facilities, resources planning and moving, as well as staff relocation all involve sub-optimization and optimization problems, that is, the decision entities are often at two decision levels. This chapter presents two real applications of the bi-level decision techniques in railway transportation management.

This chapter is organized by two case studies. Section 14.1 presents a case study about a bi-level decision model which is established for a railway train set organization. Section 14.2 shows a bi-level decision model for a railway wagon flow management problem. Experiments are carried out in each section to further illustrate the applications of these two bi-level decision models on them in railway transportation management.

14.1 Case Study 1: Train Set Organization

In this section, a decision model for *train set organization* (TSO) is developed by bi-level programming techniques. We first analyze the bi-level optimization nature of the management on TSO. A bi-level decision model for TSO is then developed, and applied in a real-world railway station to illustrate the bi-level decision model.

14.1.1 Background

Railway transportation, as one of the most important ways of transportation, has always been playing an irreplaceable role in social economics. For railway freight transportation, about 80 % of the whole transportation time is allotted to the operations of loading/unloading, transferring, and overhauling in railway technical stations (Li and Du 2002). The working state of technical stations, therefore, will

G. Zhang et al., *Multi-Level Decision Making*,
Intelligent Systems Reference Library 82, DOI 10.1007/978-3-662-46059-7_14

influence the whole overpass ability of the railway network. Thus the research on the railway transportation optimization will be bound to focus on the operation of technical stations.

Train set organization, aiming at arranging the train set in railway freight transportation and with extraordinary professional and technical specialties, is one of the main subjects in railway transportation management. The objectives of TSO include: to make the transportation efficient and even; to use the transporting device reasonably and to promote the cooperation among different departments involved in the freighting procedure. The term of *organizing* here means arranging, deciding and managing, while *train set organizing* acts to arrange the train set, make decisions on related issues, and manage the procedure in railway transportation.

There exist multiple levels among the running of TSO: (1) the *national railway network level* the top, (2) the *local bureau railway network level* the second, (3) the *stations* the third, and (4) the *operating group* the bottom. However, as the operating objects of both the national railway network and the local bureau railway network are train sets, while those of the two lower levels are trains, the organization of TSO can be generalized into two levels: the railway network as the leader and the stations as the followers. Thus bi-level programming techniques can be used to analyze the problem.

The main concerns of the railway network are to decide the train type (pick-up-and-drop-train, district-train, transit-train, or through-train), the train constitution, the train number, and the detailed route of the departing train set. The objectives of the railway network include: improving the transportation capacity and service speed, reducing the cost, balancing the working rhythm among divisions, and assigning the break-up and make-up jobs among different stations rationally.

The tasks assigned to a station are to constitute a normative train set required by the railway network from all kinds of freight wagons that stop by this station. Involved with these tasks, there also include a series of relevant operations, such as: collecting or delivering, shunting, loading/unloading, and wagon checking.

The main concerns of stations include: making the operating efficient, economical and safe; rationally using the transportation devices such as track, shunting locomotive, and hump; deciding the operation steps together with its schedules; and cooperating among steps within the schedule-frame of the railway network.

The TSO can be divided into two levels, even though the separate levels still share intrinsic consistency. For the upper level, when making a TSO plan, the railway network must consider the influence from the specific operating ability and device conditions of stations, while calculating the influence factors from itself such as the amount and destinations of trains and the track conditions. For stations located at the lower level, when implementing the working goals, they should try their best to harmonize between their own operation abilities and the working arrangement from their top counterpart.

Railway stations can be grouped into two classes: *through stations* and *technical stations*. Compared with *technical stations*, *through stations* are small sized and their daily works, mainly on helping trains go through or two train set from opposite directions meet, are simple and the workload is small. Except for all the functions of *through stations*, *technical stations* are to make a new train set by breaking up the old ones and adding transship trains and trains originated there. Related tasks also include: arrival/departure operating, collection-and-delivering operating, shunting, loading/unloading, and wagon checking. We generalize these operations at *technical stations* as *shunting and transship operations*.

For the reason of facilitating the modeling, we simplify the tasks of TSO by the following assumptions:

1. The railway transportation supply is less than the demand; the aim of the TSO is to fully use the transportation ability to provide as much transportation as possible.
2. The topological structure of a railway network is a circle formed by train lines. This is to embody the continuous nature of the net and transportation circulation.
3. The main line is double-track with every track direction fixed, which means there allows two train sets running in opposite directions between two stations simultaneously. This is to avoid the meeting problem of two train set with opposite running directions.
4. Within a railway network, there are located only *technical stations*, and only one type of trains run, the *district trains*, which are from one *technical station* A and to another *technical station* B. Between *technical stations* A and B there are no other *technical stations*.
5. The unit workload of *shunting and transship operation* for all *technical stations* are the same. In other words, every *technical station* shares the identical amount of operating time for the same train set.

Based on these assumptions above, the decision maker on a railway network wishes that the density of train sets (calculated by the time intervals between any two side-by-side running train sets) and the length of a train set (the number of trains of any train set) as large as possible to obtain the maximal transport capacity. However, for the sake of safety, the density has its upper limit set by the railway network. Restricted by the motive power of the locomotive and the useful length limit of arrival-departure track, the train set length has its upper limit as well.

Ignoring the constraints by a railway network, the stations, on one hand, wish the length of train set to be large because the larger the length, the more efficient the operating and the lower the unit operating cost. The operating efficiency is the amount of trains shunted and transshipped per unit time, while the unit operating cost is the cost for every single train. On the other hand, the operating time for shunting and transshipping, which influences the cost, will increase if the length of the train set increases. However, the overall effect of the train set length is that the general unit operating cost will decrease with the increase of the train set length.

From the analysis above, we can conclude that:

1. For the variable of the length of a train set, the two levels share the same objective: the larger the length of the train set the better.
2. For the variable of the density of a train set, the decision makers at the upper level pursue its minimum while those at the lower level wish it to change with the train set length in the same direction of travel.

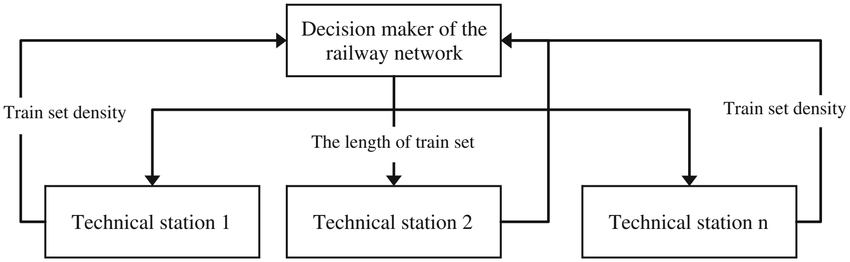

Fig. 14.1 The relationship between the railway network and technical stations

Generally speaking, the shunting and transshipping time in stations is larger than the safe time intervals of any two side-by-side running train sets, so the variable of the density of train set is determined by the lower level, the stations, while the variable of the length of train set is controlled by the top level, the railway network. The relationship between decision makers of the railway network and technical stations is illustrated in Fig. 14.1.

14.1.2 Problem Formulation

Based on the analysis above, a bi-level decision model of TSO is built as:

For $x = (x_1, x_2, \ldots, x_n) \in X \subset R^n$, $y \in Y \subset R^m$, $F, f : X \times Y \to F(R)$, Leader: decision-maker of the railway network

$$\max_{x \in X} \quad F(x,y) = \max_{x \in X} \quad F(x,y) = \frac{a_1 \cdot \sum_{i=1}^{n} w_i \cdot x_i}{\sum_{i=1}^{n} w_i \cdot y_i} \tag{14.1a}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} w_i \cdot x_i < m, \tag{14.1b}$$

$$\sum_{i=1}^{n} w_i \cdot y_i < c_{1,} \tag{14.1c}$$

The *i*th follower: the *i*th technical station

$$\min_{y_i} \quad f_i(x_i, y_i) = -b_1 \cdot x_i - b_2 \cdot y_i \tag{14.1d}$$

$$\text{s.t.} \quad c_2 \leq \frac{x_i}{y_i} \leq c_3, \tag{14.1e}$$

$$y_i > c_{4.} \tag{14.1f}$$

Explanation:

1. Variables:

   x_i: the length of a train set for the ith station, which is the number of trains of any train set controlled by the leader, the decision maker of the railway network.

   y_i: the density of train sets for the ith station, which is the time interval between any two side-by-side running train sets, controlled by the ith follower, the ith technical station.
2. Coefficients and constants:

   n: the number of *technical stations* in the railway network.

   w_i: the relative weight for the ith station in the railway network.

   a_1: the time interval. If $a_1 = 24$, then $a_1 \big/ \sum_{i=1}^{n} w_i \cdot y_i$ means the number of train sets going through the network within 24 h. $a_1 \cdot \sum_{i=1}^{n} w_i \cdot x_i \big/ \sum_{i=1}^{n} w_i \cdot y_i$ is the number of trains going through the network per day, and $a_1 > 0$.

   m: the maximum number of trains of any train set regulated by the *Safety Terms*. When the trains are empty, the main concern is not to exceed the length limit. When the trains are loaded, the weight limit becomes the decisive factor. However, for the sake of safety, when computing, both the length and weight must meet the requirements. No matter whether it is the weight or length, the ultimate limit is put on the number of trains.

   c_1: the minimum time interval between any trains list regulated by the *Safety Terms.*

   b_1 and b_2: the weights set for the influencing power by the length and density to the unit cost.

   c_2 and c_3: the lower and upper number limits of the trains for *technical stations* to shunt and transship per time unit.

   c_4: the least time for the technical stations to complete the shunting and transshipping.
3. Formula:

   (14.1a) means that the leader aims at obtaining the maximum throughput capacity within a certain period of time. $\sum_{i=1}^{n} w_i \cdot x_i \big/ \sum_{i=1}^{n} w_i \cdot y_i$ means the number of trains shunted and transshipped per time unit.

   (14.1b) means that the length of a train set has its upper limit imposed by the locomotive's motive power and the arrival-departure track's useful length. When the trains are loaded, except for the length limit, there is still weight restriction set upon the train set, which means, the weights of goods loaded together with the weights of trains cannot exceed its upper limit.

   (14.1c) means that any two adjacent running train sets cannot be too close for the sake of safety.

   (14.1d) means that the followers wish that the cost is as low as possible. The first part of (14.1d) means that the more the length of trains sets results in the more efficient of the shunting and the lower the unit cost. The second part means that the longer of the time the train set remains in the station the higher the cost.

(14.1e) means that *technical stations* have their own lower and upper time limits to shunt and transship trains.

(14.1f) means that there exists a least period of time for the *technical station* to complete the operation.

14.1.3 Experiments

In this section, we take the railway freight operation in a railway station: Station A into consideration. Station A is a railway *technical station* with the duty of managing both passenger transportation and freight transportation within the precinct of its Railway Bureau. The data collected from Station A cover the duration between November 1, 2006 and December 31, 2006.

Suppose that the trains shunted and transshipped are to the direction of Station B, which is another station located next to Station A along its downlink. The weight distribution of trains is listed in Table 14.1, with the locomotive being SS1 (137 ton, 1.9 unit length).

The terms in Table 14.1 are explained as below:

1. Wagon Type: the type of wagon used.
2. Wagon Suttle: the weight of the empty wagon.
3. Load: the weight of the goods loaded.
4. Equivalent Length: the equivalent length of a wagon is calculated from the front clasp to the rear clasp, with the unit length as 11 m. If the equivalent length is 1.1, then its actual length is 11 × 1.1 = 12.1 m.

According to the model defined by (14.1a–14.1f), the coefficients are calculated and discussed below:

1. a_1: as the computation is within the *Basal Daily Working Plan*, which is to arrange wagon assignment and schedule necessary operations based on the *Trains Running Chart*, *Trains Shunting Plan*, *Detailed Rules on Technical Station Management*, and constraints set by operating spots; the computing of the freighting wagon organization is limited within a working day of 24 h. So a_1 is set to 24.

Table 14.1 Train set distribution

Wagon type	Wagon Suttle (ton)	Load (ton)	%	Equivalent length
B23	38	40	3	2.1
P64A	26	58	3	1.5
G70	23	58	9	1.1
G60	23	50	59	1.1
G70	23	55	35	1.1

2. m: limited by the pulling ability of the locomotive and the territorial landform, such as grading, within Station A's precinct, the weight of the train set must not be larger than 3,500 tons. The departure track used for train sets to the direction to Station B is Track IV, Filed II, whose effective length is 890 m. By 30 m of braking distance, which is left for trains to stop safely, the maximum length for the trains sets is 860 m.

   Taking the constitution of the trains listed in Table 14.1, we set 1 *unit train* as a virtual train whose equivalent length, denoted by l_1 (meters), and weight, denoted by w_1 (ton), are calculated below:

$$\begin{aligned} l_1 &= 2.1 \times 0.03 + 1.5 \times 0.03 + 1.1 \times 0.09 \\ &\quad + 1.1 \times 0.5 + 1.1 \times 0.35 = 1.142 \\ w_1 &= (38 + 40) \times 0.03 + (26 + 58) \times 0.03 + (23 + 58) \times 0.09 \\ &\quad + (23 + 50) \times 0.5 + (23 + 55) \times 0.35 = 66.95 \end{aligned}$$

   The maximum number of such empty *unit train*, denoted by m_e, is $(860 - 1.9 \times 11)/(1.142 \times 11) = 66$, and the maximum number of such loaded *unit train*, denoted by m_l, is $(3{,}500 - 137)/66.95 = 50$.

   From above analyzing and computing, we obtain:

$$m = \min(m_e, m_l) = \min(66, 50) = 50.$$

3. c_1: for the sake of safety, the pursuing distance, the minimum distance interval between any side-by-side running trains list, is 10 km, which costs about 0.2 h in the journey from Station A to Station B. So c_1 is set to 0.2.
4. b_1 and b_2: we set the weights of length and density of trains set on the cost of the station as 0.4 and 0.6 respectively.
5. c_2 and c_3: the least number of trains Station A can shunt and transship is 30 per hour, while the max number is 150.
6. c_4: the least time for Station A to complete the shunting and transshipping for a train set is 0.68 h.

Thus, the bi-level decision problem defined by (14.1a–14.1f) is specialized as (14.2a–14.2f) in Station A.

Leader: decision-maker of the railway network

$$\max_x \quad F(x, y) = \frac{24x}{y} \tag{14.2a}$$

$$\text{s.t.} \quad x < 50, \tag{14.2b}$$

$$y > 0.2. \tag{14.2c}$$

Follower: Station A

$$\min_{y} \quad f(x,y) = -0.4x - 0.6y \tag{14.2d}$$

$$\text{s.t.} \quad 30 \le \frac{x}{y} \le 150, \tag{14.2e}$$

$$y > 0.68. \tag{14.2f}$$

To solve the problem in (14.2a–14.2f), we use the *fuzzy bi-Level decision support system* (FBLDSS) software presented in Chap. 11, and come to the solutions of $(x^{*}, y^{*}) = (50, 1.67)$ with $F^{*} = 718.6$ and $f^{*} = -21.002$, which means, the railway network will obtain its maximum throughput capacity of 718.6 trains per day, if the decision makers of the railway network set the average number of trains to 50, followed by Station A setting the time interval between every two side-by-side train sets to 1.67 h.

14.2 Case Study 2: Railway Wagon Flow Management

This section presents a bi-level decision model for *railway wagon flow management* (RWFM). We first analyze the multi-level nature of RWFM and then develop a bi-level decision model for it. Experiments are then carried out to illustrate its applications.

14.2.1 Background

Railway wagon flow management (RWFM) is to arrange wagon flows in railway freight transportation. One of the key issues faced by RWFM is how to arrange wagons generated or transferred in *technical stations* to form new wagon flows, while aiming at making transportation cost minimum and under constraints from both technical stations and rail tracks. An optimal solution to this problem can not only ensure freight to be sent to the destinations economically, but also make full use of all transportation facilities, thus reduce jamming probabilities and improve the transportation ability as a whole.

Due to the difficulties arising from both wagon routing and marshalling plan optimization, it is even more difficult to integrate these two issues. The most popular way is to choose wagon rout first, and then optimize the marshalling plans in every *technical station*. Although this strategy can decrease the problem solving difficulties, it still cannot reach global solutions as the benefits from the best routing can be offset by some extra workload brought in stations (Lin and Zhu 1996).

Most current research by bi-level decision techniques on traffic controlling focuses on the transformation network design and layout (Feng and Wen 2005). Little research has been conducted towards wagon flow management problems from the multi-level angle. In this section, we use a bi-level method to study the problem of RWFM.

14.2.2 Problem Formulation

Before establishing the bi-level decision model for RWFM, we list some terms used in following content.

1. *Local wagon flow*: wagons that are loaded/uploaded or repaired in one *technical station* are called local wagon flow for this station.
2. *Local district wagon flow*: some wagons are loaded/uploaded or repaired in intermediate stations between two *technical stations*. This kind of wagon flow is called *local district wagon flow* for the two *technical stations*.
3. *Long-distance wagon flow*: for a *technical station*, if a wagon flow is not its *local wagon flow* or *local district wagon flow* but belongs to another *technical station* (*local wagon flow* or *local district wagon flow*), this wagon flow is called *long-distance wagon flow* for this *technical station*.
4. *Service operation*: to assist on the marshalling operation within one station, some auxiliary operations must be made, including: taking-out and placing-in of cars, picking-up and dropping trains, loading/uploading goods, and repairing. We call this kind of auxiliary operation as *service operation*.

Railway wagon flow management characterized by monopolization, is usually run by three levels, i.e. railway ministry level, railway bureau level, and station level. However, when carrying out tasks assigned by its corresponding superior, a lower level can arrange its own resources to achieve as much profit as possible. The communication among levels is through marshalling plans which are designed by the upper level but implemented by the lower counterparts. Marshalling plans are regulations on organizing vans which may be destined to different destinations to form van lists. Optimization on marshalling plans aims at minimizing the time spent for centralizing and detention in *technical stations*.

In this section, we take *railway bureaus as leaders and stations as followers*. A *railway bureau* controls the workload and working rhythm of the stations in its administration area. A station, while controlling its own producing resources, decides which specific method it will use to achieve tasks to be carried in this station. Thus, the cost in a station is determined by both the station and its upper administrator, the *railway bureau*. However, the optimal cost level for a station does not necessarily produce the most ideal cost status for the *railway bureau* who seeks equilibrium with traffic and cost. Although the *railway ministry* is located above *railway bureaus* and *technical stations*, this study, while not focusing on the reciprocal decision relation between a *railway ministry* and its *bureaus*, only takes the decision from the *railway ministry* as input constraints.

Once a *railway bureau* selects a marshalling plan, it means two kinds of data are determined. One is the *technical station sequences* where some marshalling operation will be carried for every *long-distance wagon flow*. The other is the number of vans to be marshaled in every station. For the leader, a decision involves whether accepting a carriage and the way to deliver it. The marshalling plans made by a *railway bureau* involve only *long-distance wagons*. In some *technical stations*, some *long-distance wagons* should be merged or separated to decrease cost in stations and increase traffic efficiencies.

Technical stations perform marshalling operations as well as relevant following services, such as collecting, delivering, shunting, loading/unloading, and wagon checking. The facilities of these services depend on the quality of the marshalling operation which is performed beforehand. Having *local wagons*, *local district wagons* and *long-distance wagons* as three kinds of marshalling objectives, marshalling operations with *local wagons* and *local district wagons* are flexible in *technical stations*. Stations can determine the extent and depth of the marshalling operation for *local wagons* and *local district wagons*. The better performance of marshalling operation results in the easier the following services and the lower the cost. With the objective of making the costs as low as possible, *technical stations* reasonably marshal *local wagons* and *local district wagons* as thoroughly as possible. However, profound marshalling operation will inevitably raise the cost and the time allocated for marshalling in a *technical station* within some limitations. Thus a *technical station* will seek a best point where its marshalling operations can bring itself the lowest cost. The decision on how to marshal local wagons and local district wagons becomes key content for *technical stations*.

Among 1,440 min a day, some time is allocated for operations other than marshalling. Also, some marshalling operations are fixed so that a *technical station* cannot adjust it. Thus, a station can only decide on flexible wagon flows within available working time. A station needs first to distribute working time between *local wagons* and *local district wagons*, then divide it among different sections of a *local district wagon flow*. Based on this distribution, a station will decide the amount of marshalling a day, the amount of wagons and time for every marshalling. Generally speaking, *technical stations* make decisions from the following aspects.

1. Marshalling percentage: Influenced by time limitation, some marshalling operations can be executed to only some wagons while others must be treated as if they had the same sequence number (the same destination station) to reduce marshalling load. Thus, the percentage of wagons which will be marshaled is a decision made by a *technical station*.
2. Shunting choice: within limited working time, a *technical station* can decrease the shunting precision to finish a marshalling operation on time. Different shunting precisions occur in both *sort-shunting* and *group-shunting*. For *sort-shunting*, every wagon should be placed sequentially by their destinations. For *group-shunting*, marshalling is supposed to be finished as long as wagons with

the same destination are placed together. *Group-shunting* takes less working time than *sort-shunting*.

3. Marshalling precision: marshalling can be divided into different precise degrees. Actually, the destination of a wagon can be defined from generality to specificity by stations, operation areas, operation lines, or operation spots. The more precise, the more working time will be needed.

To facilitate modeling the RWFM problem, we have the following assumptions:

1. Marshalling difficulty is decided by the disorder degree of the wagon flow to be marshaled. Disorder degree depends on the destination stations of every wagon and the relationship among them, which occurs randomly. In this research, we hold that the disorder degrees for wagon flows have no difference.
2. Marshalling costs from two train flows, one of which is from Station A to Station B and the other is from Station B to Station A, may have trivial difference on marshalling cost. When making plans and calculating the cost, decision makers sometimes need to consider the influence from these differences. However, compared with other influencing factors, the influence from different directions is trivial and can be ignored. In this research, we hold that the marshalling costs with two train flows with different directions are exactly the same.

From the analysis above, a bi-level decision model for RWFM is built as follows:

For $x = (x_1, x_{21}, x_{22}, \ldots, x_{2m}) \in X \subset R^{m+1}, y_i = (\eta_{li}, \eta_{di}, y_{1Gil}, y_{1Sil}, y_{1Sid}, y_{wGil}, y_{2Sil}, y_{2Gid}, y_{2Sid}, y_{2ik}) \in Y_i \subset R^{10}, \quad i = 1, 2, \ldots,$

$$\max_{x \in X} \sum_{p_j, l_j \in D} (p_j \times l_j)\left[J_w(1 - r_j) - \bar{C}_{wj}(x)\right] + \sum_{s_i \in S} [q_{di}(x_1) \times \xi_i \times J_s - C_{si}(x, y_i)] \tag{14.3a}$$

$$\text{s.t.} \quad p_{ju\_min} \le p_{ju} \le p_{ju\_max}, \quad j = 1, 2, \ldots, m \tag{14.3b}$$

$$p_{jd\_min} \le p_{jd} \le p_{jd\_max}, \quad j = 1, 2, \ldots, m \tag{14.3c}$$

$$m_{j\_min} \le x_{2j} \le m_{j\_max}, \quad j = 1, 2, \ldots, m \tag{14.3d}$$

$$m_{j\_min} = \max\left\{I_{ju} \times t_{T_jd}, I_{jd} \times t_{T_jd}\right\}, \quad j = 1, 2 \tag{14.3e}$$

$$0 \le q_{di} \le v_i \tag{14.3f}$$

$$0 \le q_{2i} + q_{di} \le u_i \tag{14.3g}$$

$$\min_{y_i \in Y_i} \sum_{s_i \in S} \left[\left(C'_{z1i} + C''_{z1i} + C'''_{z1i} + C'_{z2i} + C''_{z2i} + C_{z3i} \right) \left(1 + A_i \sigma_i^{B_i} \right) + \Delta C_{2i} \right] \tag{14.4a}$$

$$\text{s.t. } q_i = q'_{d_i} + \sum_{d_k \in D_i} q''_{dik} + q_{zi} \tag{14.4b}$$

$$T_{il\_min} \le \eta_{li} \times q_{di} \times (y_{1Gil} \times y_{2Gil} \times T_{Gi} + y_{1Sil} \times y_{2Sil} \times T_{Si}) + q_{di} \times S_{il} \times T_{si} \le T_{il\_max} \tag{14.4c}$$

$$T_{id\_min} \le \eta_{di} \times q_{di} \times (y_{1Gil} \times y_{2Gil} \times T_{Gi} + y_{1Sil} \times y_{2Sil} \times T_{Si}) + q_{di} \times S_{id} \times T_{si} \le T_{id\_max} \tag{14.4d}$$

$$i = 1, 2, \ldots, m \tag{14.4e}$$

where

$$\begin{aligned}
&\bar{C}_{wj}(x) = \bar{C}_{w0} + \frac{\bar{C}_{wj}}{p_j} + \frac{\bar{C}_{2wj}}{x_{2j}} + \Delta\bar{C}_{wj}, \\
&\Delta\bar{C}_{wj} = \left| p_{ju} - p_{jd} \right| \times \frac{\bar{C}_{w2j}}{x_{2j} \times p_j}, \\
&I_{ju} = \frac{p_{ju}}{1{,}440 \ \times \ \psi_{ju}}, \\
&I_{jd} = \frac{p_{jd}}{1{,}440 \ \times \ \psi_{jd}}, \\
&C'_{z1i} = \bar{C}'_{z1i} \times \bar{q}'_{d_i}, \\
&\bar{C}'_{z1i} = z'_{10i} + \frac{Z_{11i}}{a'_{1i} \times \bar{q}_{d_i}^{\mathrm{b}'_{1i}}} + Z_{12i} \times \left((y_{1Gil} + 2 \times y_{1Sil} + y_{1i})^{b'_{2i}} \right. \\
&\qquad \left. + \left(y_{2Gil} + y_{2Sil} \right)^{b'_{3i}} + \eta_{li}^{b'_{4i}} \right), \\
&C''_{z1i} = \bar{C}''_{z1i} \times \bar{q}''_{d_i}, \\
&\bar{C}''_{z1i} = z''_{10i} + \frac{Z_{11i}}{\sum_{d_k \in D_i} a''_{1ik} \times x_{2ik}^{b''_{1ik}}} \\
&+ Z_{12i} \times \sum_{d_k \in D_i} a''_{1ik} \\
&\qquad \times \left[a_{2ik} \times \left((y_{1Gil} + 2 \times y_{1Sil} + y_{1i})^{b''_{2ik}} + y_{2ik}^{b''_{3ik}} + \eta_{dik}^{b''_{4ik}} \right) \right]
\end{aligned}$$

$$\begin{aligned}
C'''_{z1i} &= \bar{C}'''_{z1i} \times q_{zi},\\
\bar{C}'''_{z1i} &= z'''_{10i} + \frac{Z_{11i}}{\sum_{d_k \in D_i} a'''_{1ik} \times x_{2ik}^{b'''_{1ik}}},\\
C'_{z2i} &= \bar{C}'_{z2i} \times \bar{q}'_{d_i},\\
\bar{C}'_{z2i} &= z'_{20i} + \frac{Z'_{21i}}{\alpha'_{21i} \times ((y_{1Gil} + y_{1Sil} + y_{1i})^{\beta'_{21i}} + (y_{2Gil} + y_{2Sil})^{\beta'_{22i}} + \eta_{li}^{\beta'_{23i}})},\\
C''_{z2i} &= \bar{C}''_{z2i} \times \bar{q}''_{d_i},\\
\bar{C}''_{z2i} &= z''_{20i} + \frac{Z''_{21i}}{\sum_{d_k \in D_i} \times \alpha''_{21i} \times ((y_{1Gil} + 2 \times y_{1Sil} + y_{1i})^{\beta''_{21i}} + y_{2ik}^{\beta''_{22i}} + \eta_{dik}^{\beta''_{23i}})},\\
C_{z3i} &= N_i \times \bar{C}_{cx},\\
N_i &= \sum_{d_k \in D_i} [(C_{di} + C_{cHi} \times \varsigma_{ik}) \times x_{2ik} + C''_{fik} \times q''_{dik}] + C'_{fi} \times q'_{di},\\
C'_{fi} &= \frac{Z'_{31i}}{\alpha'_{31i} \times ((y_{1Gil} + y_{1Sil} + y_{1i})^{\beta'_{31i}} + (y_{2Gil} + y_{2Sil})^{\beta'_{32i}} + \eta_{li}^{\beta'_{33i}})},\\
C''_{fi} &= \frac{Z''_{31i}}{\alpha''_{3iki} \times ((y_{1Gil} + y_{1Sil} + y_{1i})^{\beta''_{31ik}} + y_{2ik}^{\beta''_{32ik}} + \eta_{dik}^{\beta''_{33ik}})},\\
\Delta C_{2i} &= \left(\sum_{d_k, d_l \in D_{i,k \neq i}} |x_{2ik} - x_{2il}| \right) \times Z_{4i},\\
Z_{4i} &= Z_{40i} + \frac{Z_{41i}}{\sum_{d_k, d_l \in D_{i,k \neq i}} \alpha_{4ik} |x_{2ik} - x_{2il}|}.
\end{aligned}$$

The explanations for the above formulas are listed below:

1. Controlling variables:

   x_1: Assignment of wagons which will go through the area administrated by a *railway bureau* and have more than one shunting operation in some *technical station* in this area.

   x_{2j}: The number of vans within a shunted wagon list from the *j*th section, which is from one *technical station* to another in a *railway bureau.*

   x_{2ik}: The number of wagons in a wagon list which is to the *k*th section in the *i*th station.

   η_{li}: The percentage of wagons to be marshaled for local wagon list in the *i*th station.

   η_{di}: The percentage of wagons to be marshaled for *local district wagon list* in the *i*th station.

η_{dik}: The percentage of wagons to be marshaled for *local district wagon list* to the *k*th direction in the *i*th station.

y_{1Gil}: The percentage of wagons to be marshaled by *group-shunting* of *local wagons* in the *i*th station.

y_{1Sil}: The percentage of wagons to be marshaled by *sort-shunting* of *local wagons* in the *i*th station.

y_{1Gid}: The percentage of wagons to be marshaled by *group-shunting* of *local district wagons* in the *i*th station.

y_{1Sid}: The percentage of wagons to be marshaled by *sort-shunting* of *local district wagons* in the *i*th station.

y_{2Gil}: The shunting precision for *local wagons* to be marshaled by *group-shunting* in the *i*th station.

y_{2Sil}: The shunting precision for *local wagons* to be marshaled by *sort-shunting* in the *i*th station.

y_{2Gid}: The shunting precision for *local district wagons* to be marshaled by *group-shunting* in the *i*th station.

y_{2Sid}: The shunting precision for *local district wagons* to be marshaled by *sort-shunting* in the *i*th station.

y_{2ik}: The shunting precision for *local district wagon flow* marshaled in the *i*th station to the *k*th direction.

2. Other variables: while decision makers from both the upper and lower levels directly control variables of x and y, there are some other variables whose values are influenced by x and y directly or indirectly. These variables are summarized below:

Variables influenced by x_1:

p_j: The average wagon flow in the *j*th section, $p_j = p_{ju} + p_{jd}$.

p_{ju}: The average wagon flow in the *j*th section in the up-direction, which fluctuates with the change of $x_{1.}$

p_{jd}: The average wagon flow in the *j*th section in the down-direction, which fluctuates with the change of $x_{1.}$

q_{di}: The number of *local district wagons* and *local wagons* operated per day in the *i*th station.

q'_{d_i}: The number of local wagons operated in the *i*th station.

q''_{d_ik}: The number of local district wagons operated to the *k*th direction in the *i*th station.

q''_{d_i}: The number of local district wagons operated in the *i*th station.

q_{2i}: The number of long-distance wagons operated in the *i*th station.

ξ_i: The loading percentage in the *i*th station.

r_j: The percentage of empty to loaded wagon kilometers in the *j*th section. It fluctuates with the change of $x_{1.}$

Variables influenced by *x*:

q_{zi}: The number of wagons marshaled in the *i*th station a day.

ς_{ik}: The number of *long distance wagons* to the *k*th direction in the *i*th station.

n_{kl}: The number of wagon lists some of whose wagons have been added/removed from the *k*th section to the *l*th section.

Variables influenced by *y*:

y_{li}: Marshalling degree, which is determined by different operating depth, such as *group shunting*, *sort-shunting* and the fit degree of the regulation of *Safety terms*, for the *local district wagon* in the *i*th station.

σ_i: The average time difference among the operations for *local wagon flow*, *local district wagon flow*, and *long-distance wagon flow* in the *i*th station.

Variables influenced by *x* and *y*:

q_{i}: The number of wagons operated in the *i*th station.

3. Coefficients and constants:

$S = \{s_i,\ i = 1, 2, \ldots, n\}$: The set of the *technical stations* administrated by a *railway bureau.*

$D = \{d_j,\ j = 1, 2, \ldots, m\}$: The set of the train running sections administrated by a *railway bureau.*

$D_i = \{d_k,\ k = 1, 2, \ldots, l\}$: The set of train running sections which are adjacent to the *i*th station.

l_j: The hauling distance in the *j*th section, which is a constant.

J_w: Railway average tariff, which is a constant.

$\bar{C}_{w0}$: Freight traffic fixed unit cost.

$\bar{C}_{w1j}$: The freight traffic unit cost in the *j*th section per day per kilometer.

$\bar{C}_{w2j}$: Hauling cost in the *j*th section per wagon per kilometer.

$\bar{C}'_{z2j}$: The locomotive cost in the *j*th section per kilometer when there is no wagon hauled by the locomotive.

J_s: Fees charged per wagon.

p_{ju_min}: The minimum wagon flow which can meet the requisite traffic demand required for the *j*th section in the up-direction.

p_{ju_max}: The maximum wagon flow which can be run for the *j*th section in the up-direction.

p_{jd_min}: The minimum wagon flow which can meet the requisite traffic demand required for the *j*th section in the down-direction.

p_{jd_max}: The maximum wagon flow which can be run for the *j*th section in the down-direction.

m_{j_max}: The maximum number of wagons to form a wagon list in the *j*th section. It is determined by the locomotive hauling limit and the useful length of the receiving and departure tracks in the *j*th section.

v_i: The maximum possible number of wagons that can be operated by *service operation* in the *i*th station.

u_i: The maximum possible number of wagons that can be marshaled in the *i*th station.

φ_{ju}: The percentage of time that can be used a day (1,440 min) for freight transportation in the *j*th section in the up direction.

φ_{jd}: The percentage of time that can be used a day (1,440 min) for freight transportation in the *j*th section in the down direction.

t_{T_ju}: The minimum time interval between two wagon lists of the *j*th section in the up direction regulated by the train working diagram.

t_{T_jd}: The minimum time interval between two wagon lists of the *j*th section in the down direction regulated by the train working diagram.

Z'_{10i}: The minimum cost for marshalling one *local wagon* in the *i*th station. This cost happens in an ideal situation when the number of wagons to be marshaled is large enough and the marshalling degree is deep enough for one marshalling operation.

Z_{11i}: Coefficient for the effect from centralized marshalling operation.

$\bar{q}_{di}$: The number of local wagons to be marshaled for one marshalling operation. It is determined by the loading/uploading capacity in the *i*th station.

Z_{12i}: Coefficient for the effect from deepened marshalling operation. It is an average additional cost spent for one wagon for marshalling operation.

$A_i, B_i, a'_{1i}, b'_{1i}, b'_{2i}, b'_{3i}, b'_{4i}, a''_{1ik}, b''_{1ik}, \alpha'_{21i}, \beta'_{21i}, \beta'_{22i}, \beta'_{23i}, \alpha''_{21i}, \beta''_{21i}, \beta''_{22i}, \beta''_{23i}, \alpha'_{31k}, \beta'_{31k}, \beta'_{32k}, \beta'_{33k}, \alpha''_{31k}, \beta''_{31k}, \beta''_{32k}, \beta''_{33k}$: Coefficients which are to be obtained through statistic data.

Z''_{10i}: The minimum cost for marshalling one *local district wagon* in the *i*th station. This cost happens in an ideal situation when the number of wagons to be marshaled is large enough and the marshalling degree is deep enough for one marshalling operation.

Z'''_{10i}: The minimum cost for marshalling one *long distance wagon* in the *i*th station. This cost happens in an ideal situation when the number of wagons to be marshaled is large enough and the marshalling degree is deep enough for one marshalling operation.

Z'_{20i}: The minimum cost for *service operation* for one local wagon in the *i*th station. This cost happens in an ideal situation when marshalling is deep enough such that service operation can be operated easily and conveniently.

Z'_{21i}: The additional cost for service operation for one *local wagon* in the *i*th station. This cost happens when marshalling is superficial thus service operation become unhandy.

Z''_{20i}: The minimum cost for service operation for one *local district wagon* in the *i*th station. This cost happens in an ideal situation when marshalling is deep enough such that service operation can be operated easily and conveniently.

Z''_{21i}: The additional cost for service operation for one local district wagon in the *i*th station. This cost happens when marshalling is superficial thus service operation become unhandy.

C_{di}: The coefficient on centralisation and detention for *local wagon flow*, which is a number between eight and twelve. The number is decided by specialties from different wagon flows.

C_{cHi}: The coefficient on centralisation and detention for *long distance wagon flow*, which is a number between eight and twelve.

Z'_{3i}: Coefficient on service facilitation for *local wagons*, which equals the additional halting time when service operation is totally inconvenient.

Z''_{3i}: Coefficient on service facilitation for *local district wagons*, which equals the additional halting time when service operation is totally inconvenient.

Z_{40i}: The basic cost for adding/removing one wagon to/from a wagon list in the ith station. This cost happens when the number of wagons to be added/removed is large enough.

Z_{41i}: The additional cost for adding/removing one wagon to/from a wagon list in the ith station. This cost happens when the number of wagons to be added/removed is small enough (equaling one).

$\bar{C}_{cx}$: The cost for one wagon to halt one hour, which is caused by wagon depreciation.

S_{il}: The percentage of wagons which have special safety requirement of *local wagons* in the ith station.

S_{id}: The percentage of wagons which have special safety requirement of *local district wagons* in the ith station.

T_{si}: Average time to marshal a wagon which has special safety requirement in the ith station.

T_{il_min}: The least time for marshalling one *local wagon list* in the ith station.

T_{il_max}: The time spent for marshalling one *local wagon list* with the highest specification and completeness in the ith station.

T_{id_min}: The least time for marshalling one *local district wagon list* in the ith station.

T_{il_max}: The time spent for marshalling one *local district wagon list* with the highest specification and completeness in the ith station.

4. Formula:

(14.3a) describes a *railway bureau*'s objective which aims at achieving the maximum profit for freight operation administrated by this *railway bureau*. It has two parts: the profit from the railway network and technical stations in this bureau.

(14.3b) and (14.3c) mean that wagon flows in both the up-direction and down-direction have their minimum and maximum limits. Thus the total number of vans from one station to another has its limits too.

(14.3d) and (14.3e) tell how the limits set for wagon flows in the up-direction and down-direction are determined and calculated.

(14.3f) and (14.3g) mean the number of long-distance wagon flows to be marshaled in *technical stations* cannot exceed their operating abilities.

(14.4a) is the objective for a *technical station*, which aims at lowering its operation cost. The operation cost is from three parts: *local wagon flow*, *local district wagon flow*, and *long-distance wagon flow*.

(14.4b) denotes how the operation cost for *local wagon flow*, *local district wagon flow*, and *long-distance wagon flow* are calculated.

(14.4c) and (14.4d) mean that there exist minimum and maximum limits for marshalling both *local wagon flow* and *local district wagon flow*.

5. Symbols:

   F_1: The economical benefit of the railway network within the area administrated by a *railway bureau*.

   F_2: The economical benefit obtained by all of the *technical stations* administrated by the *railway bureau*.

   $\bar{C}_{w_j}$: The freight traffic unit cost in the *j*th section.

   $\Delta\bar{C}_{w_j}$: Additional unit cost in the *j*th section.

   C_{s_i}: The operating cost in the *i*th station. It fluctuates with the change of controlling variables from both the leader and the followers.

   I_{ju}: The number of wagons that go through the *j*th section in the up direction per minute.

   m_{j_min}: The minimum number of wagons to form a wagon list in the *j*th section.

   I_{jd}: The number of wagons that go through the *j*th section in the down direction per minute.

   C'_{z1i}: Daily cost spent for marshalling *local wagon flow* for the *i*th station.

   C''_{z1i}: Daily cost spent for marshalling *local district wagon flow* for the *i*th station.

   C'''_{z1i}: Daily cost spent for marshalling *long-distance wagon flow* for the *i*th station.

   C'_{z2}: Daily cost spent for *service operation* made for *local wagon flow* for the *i*th station.

   C''_{z2i}: Daily cost spent for *service operation* made for *local district wagon flow* for the *i*th station.

   C_{z3i}: Daily cost spent for centralizing and detention of wagons in the *i*th station.

   ΔC_{2i}: Different sections may have different requests on the number of wagon lists to be run in that section. Thus adding/reducing wagons may be needed in *technical stations* to meet the requirements of its adjacent sections. ΔC_{2i} is the daily cost spent for adding/reducing wagons in the *i*th station.

   $\bar{C}'_{z1i}$: Average daily cost spent for marshalling one *local wagon* for the *i*th station.

   $\bar{C}''_{z1i}$: Average daily cost spent for marshalling one *local district wagon* for the *i*th station.

   $\bar{C}'_{z2i}$: Average daily cost spent for *service operation* made for *local wagon flow* for the *i*th station.

   $\bar{C}''_{z2i}$: Average daily cost spent for *service operation* made for *local district wagon flow* for the *i*th station.

   N_i: The total hours spent by all wagons which are halted in the *i*th station.

   C'_{fi}: The number of additional hours spent for *service operation* for a *local wagon* in the *i*th station.

   C''_{fik}: The number of additional hours spent for *service operation* for a *local district wagon* to the *k*th direction in the *i*th station.

   Z_{4i}: The cost spent for adding/removing one wagon.

14.2.3 Experiments

In this section, we consider the RWFM problem in a railway bureau: Bureau U. Within the area administrated by Bureau U, there are three *technical stations*: Station A, Station B, and Station C. Connecting these stations, we have three sections: Section 1 that connects Station A and Station B, Section 2 that connects Station B and Station C, and Section 3 that connects Station C and Station A. We list the values of some of the main coefficients, which are used to build the bi-level decision model for this RWFM problem in Tables 14.2 and 14.3.

To help the decision maker in Bureau U make an optimal RWFM plan, we use the FBLDSS software presented in Chap. 11 to reach a solution. By 342 s running, the solutions for Bureau U are reached and summarized in Table 14.4.

To test the stability of the FBLDSS, this example has been run six times by the FBLDSS software. The solution variances are summarized in Table 14.5.

Table 14.2 Summary of the coefficient values in the experiments 1

Station	p_{ju_min}	p_{ju_max}	p_{jd_min}	p_{jd_max}	u_i	v_i
A	10	29	10	29	19	19
B	10	29	10	29	19	19
C	10	29	10	29	19	19

Table 14.3 Summary of the coefficient values in the experiments 2

Section	T_{il_min} (min)	T_{il_max} (min)	T_{id_min} (min)	T_{id_max} (min)
1	15	20	15	22
2	15	23	15	25
3	15	22	15	21

Table 14.4 Summary of the solutions for Bureau U and the stations

Station	P_{ju}	P_{jd}	$q_{di} + q'_{di}$	q_{2i}	x_{2k}	y_{1Gil}	y_{1Sil}	y_{1i}
A	24	28	42	10	18	0.2	0.78	0.41
B	27	23	38	12	11	0.55	0.67	0.35
C	24	17	25	16	7	0.53	0.98	0.48

Table 14.5 Summary of the solutions for Bureau U and the stations

Station	P_{ju}	P_{jd}	$q_{di} + q'_{di}$	q_{2i}	x_{2k}	y_{1Gil}	y_{1Sil}	y_{1i}
A	0.002	0.12	0.1	0.05	0.37	0.16	0.00043	0.1
B	0.04	0.29	0.02	0.27	0.42	0.33	0.01	1.23
C	0.17	0.37	0.42	0.07	0.57	0.02	0.27	0.09

In Table 14.5, we can see that, there is no very large diversion among the solutions obtained. For every running, the solution has been obtained within 400 s. Thus, we can come to the conclusion that the FBLDSS could explore veracious solutions for RWFM problems with quite effective and stable performance.

14.3 Summary

In this chapter, the bi-level optimization natures in *train set organization* (TSO) and *railway wagon flow management* (RWFM) have been put forward by abstracting and simplifying railway trains management. First, two bi-level decision models are established for the two problems respectively. Then, these two decision models are applied to technical stations for real case studies. The experiment results obtained from these two case studies could be helpful for the tasks of train set organization and railway wagon flow management.