Chapter 13 Bi-level Pricing and Replenishment in Supply Chains

Effective pricing and replenishment strategies in supply chain management are the keys to business success. Notably, with rapid technological innovation and strong competition in hi-tech industries such as computer and communication organizations, the upstream component price and the down-stream product cost usually decline significantly with time. As a result, effective pricing and replenishment decision models are very important in supply chain management. This chapter first establishes a bi-level pricing and replenishment strategy optimization model in hi-tech industry. Then, two bi-level pricing models for pricing problems, in which the buyer and the vendor in a supply chain are respectively designated as the leader and the follower, are presented. Experiments illustrate that bi-level decision techniques can solve problems defined by these models and can achieve a profit increase under some situations, compared with the existing methods.

This chapter is organized into four sections. After introducing the background in Sects. 13.1 and [13.2](#page-1-0) shows a case study about hi-tech collaborative pricing and replenishment strategy making. In Sect. [13.3,](#page-7-0) we use bi-level decision techniques to develop two bi-level pricing models within another case study, one considering the buyer as the leader who has priority in deciding, and the other taking the vendor as the leader. Finally, the summary of this chapter is given in Sect. [13.4](#page-11-0).

13.1 Background

Hi-tech products such as computers and communication consumer products have driven the need for globalization and massive customization, and have come to occupy a large section of the supply chain industry. Features of hi-tech products include short product life cycle time and quick response time. The lead-time from order to delivery is usually compressed from 955 (95 % order delivered within 5 days) to 1,002 (100 % order delivered within 2 days), and both component costs and product prices are declining at a rate of about 1% per week (Sern 2003). This implies that purchasing or selling one-week earlier or later will result in an approximate loss of 1 % (Lee 2002). As a result, hi-tech products require a more

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effective optimization method to support policy-making by both the buyer and the vendor in a supply chain.

In reality, the buyer and vendor in a hi-tech product supply chain are two echelons that need to achieve a win-win business solution. Under this principle of collaboration, some collaborative pricing and replenishment optimization models are developed and both the vendor and buyer aim to reduce/optimize the purchase cost and price respectively.

To reveal well and clearly reflect the interactive and internal relationship between a vendor and a buyer, we consider both sides to be well-optimized for the supply chain: the maximum optimization to one side, such as buyer, while still considering the profit achievement of the other side, such as vendor. In fact, neither the vendor nor buyer has direct control over the strategy/policy-making of the other, but their actions affect subsequent responses of each other. Therefore, the pricing and replenishment strategy problem is naturally a bi-level optimization problem where either the vendor or buyer can be the leader based on the requirement and goal of the decision support system.

In the following sections, we will present the formulation of the hi-tech collaborative pricing and replenishment strategy problem using non-linear bi-level programming in the first case study, in which the buyer is the leader. Then, we address how bi-level pricing models are developed by using bi-level programming in the second case study. In the second case study, one bi-level pricing model considers the buyer as the leader who has the privilege of deciding first, and the vendor as the follower who makes decisions after the buyer; the other bi-level pricing model takes the vendor's profit as priority and makes the vendor the leader and the buyer the follower. These two pricing models allow the buyer and vendor to make decisions in sequence, fully considering the mutual influence of each other. To obtain solutions from these non-linear bi-level decision models, the $Fuzzy$ Bi-level Decision Support System (FBLDSS) software developed in Chap. [11](http://dx.doi.org/10.1007/978-3-662-46059-7_11) is used. We also conduct experiments to illustrate the proposed models.

13.2 Case Study 1: Hi-tech Product Pricing and Replenishment Strategy Making

This section will handle hi-tech product pricing and replenishment strategy problem by bi-level decision techniques.

13.2.1 Problem Formulation

The formulation for the pricing and replenishment strategy problem is presented based on the assumptions of Yang et al. (2007):

- 1. Vendor and buyer's replenishment rates are instantaneous.
- 2. Component purchase cost and product price to the end consumer decline at a continuous rate per unit time.
- 3. Finite planning horizon and constant demand rate are considered.
- 4. Each replenishment time interval is the same.
- 5. No shortage is allowed.
- 6. Purchase lead-time is constant.

It is assumed that the purchase cost of the vendor and the market price to the end-consumer are fixed. To maximize profit through increased sales, the vendor offers a price discount rate of r_b to the buyer.

The related parameters included in our model are listed in Table 13.1.

If the vendor's and buyer's costs decline at continuous rates of r_v and r_b respectively, their purchase costs are:

$$
P_{\nu}(t) = P_{\nu 0} (1 - r_{\nu})^t, \quad 0 \le t \le H \tag{13.1}
$$

Parameter	Description
\boldsymbol{n}	The number of orders that a vendor places for the item from a supplier in the
	planning horizon
\boldsymbol{m}	The number of buyer's lot size deliveries per vendor's lot size
ϱ	The buyer's lot size
r_b	The weekly decline-rate of the buyer's purchase cost
D	The weekly demand rate
r_{v}	The weekly decline-rate of the vendor's purchase cost
r_m	The weekly decline-rate of market price to the end-consumer
H	The weekly length of the planning horizon
F_v	The vendor's holding cost per dollar per week
F_b	The buyer's holding cost per dollar per week
C_v	The vendor's ordering cost per order
C_h	The buyer's ordering cost per order
P_{v0}	The vendor's unit purchase cost at the initial time
P_{b0}	The buyer's unit purchase cost at the initial time
P_{m0}	Market price to the end consumer at the initial time
$P_{v}(t)$	The vendor's unit purchase cost in week t
$P_b(t)$	The buyer's unit purchase cost in week t
$P_m(t)$	The market price to the end consumer in week t
NP_v	The vendor's net profit in the planning horizon
NP _b	The buyer's net profit in the planning horizon
NP	The joint net profit of both the vendor and the buyer in the planning horizon

Table 13.1 Related parameters

and

$$
P_b(t) = P_{b0}(1 - r_b)^t, \quad 0 \le t \le H \tag{13.2}
$$

respectively.

If the market price declines at a continuous rate of r_m , the unit market price to the end-consumer is

$$
P_m(t) = P_{m0}(1 - r_m)^t, \quad 0 \le t \le H.
$$
 (13.3)

The buyer's average inventory level is $Q/2$, that is, one half of the buyer's lot size. The unit purchase cost is

$$
P_{b0}, P_{b0}(1-r_b)^{\frac{H}{mn}}, P_{b0}(1-r_b)^{\frac{2H}{mn}}, \ldots, P_{b0}(1-r_b)^{(\frac{n-1+\frac{m-1}{m})H}{n}}.
$$

The buyer's holding cost in the planning horizon is

$$
HC_b = \frac{F_b H}{mn} \sum_{i=0}^{n-1} \sum_{j=1}^{m-1} \frac{P_{b0} (1 - r_b)^t Q}{2} = \frac{F_b H P_{b0} Q}{2mn} \frac{1 - (1 - r_b)^H}{1 - (1 - r_b)^{\frac{H}{mn}}},
$$
(13.4)

where $t = (i + \frac{j}{m})H/n$, $i = 0, 1, ..., n - 1$, $j = 1, 2, ..., m - 1$.

Note that m and n are positive integers, t is a continuous real number and is discrete valued in the analytical steps for ease of analysis.

Since the vendor-buyer-combined average inventory level is $mQ/2$, the vendor's average inventory level is $Q(m-1)/2$ in the collaborative system. The vendor's holding cost in the planning horizon is

$$
HC_{\nu} = \frac{F_{\nu}H}{n} \sum_{i=0}^{n-1} \frac{P_{\nu 0}(1 - r_{\nu})^t Q(m-1)}{2} = \frac{F_{\nu}H P_{\nu 0}Q(m-1)}{2n} \frac{1 - (1 - r_{\nu})^H}{1 - (1 - r_{\nu})^{\frac{H}{n}}},
$$
(13.5)

where $t = \frac{iH}{n}$, $i = 0, 1, 2, ..., n - 1$.

 λ

The buyer's net income (sales revenue minus purchase cost) is denoted by NI_b as follows:

$$
NI_b = \int_{0}^{H} P_{m0} (1 - r_m)^t Ddt - \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} P_{b0} (1 - r_b)^t Q
$$

=
$$
\frac{P_{m0} D}{\ln(1 - r_m)} \left(e^{H \ln(1 - r_m)} - 1 \right) - \frac{P_{b0} Q (1 - (1 - r_b)^H)}{1 - (1 - r_b)^{\frac{H}{mn}}}
$$
(13.6)

The vendor's net income (sales revenue minus purchase cost) is denoted by NI_v as follows:

$$
NI_{\nu} = \sum_{i=0}^{n-1} \sum_{j=0}^{m-1} P_{b0} (1 - r_b)^t Q - \sum_{i=0}^{n-1} P_{\nu 0} (1 - r_{\nu})^t m Q
$$

=
$$
\frac{P_{b0} Q (1 - (1 - r_b)^H)}{1 - (1 - r_b)^{\frac{H}{mn}}} - \frac{P_{\nu 0} m Q (1 - (1 - r_{\nu})^H)}{1 - (1 - r_{\nu})^{\frac{H}{n}}},
$$
(13.7)

where $1 > r_b > 0, 1 > r_v > 0.$

The buyer's net profit (formula (13.6) (13.6) (13.6) minus formula (13.4) and the ordering cost) is denoted by NP_b as follows:

$$
NP_b = \frac{P_{m0}D}{\ln(1 - r_m)} \left(e^{H \ln(1 - r_m)} - 1 \right) - \frac{P_{b0}Q \left(1 - \left(1 - r_b \right)^H \right)}{1 - \left(1 - r_b \right)^{\frac{H}{mn}}} - \frac{F_bHP_{b0}Q}{2mn} \frac{1 - \left(1 - r_b \right)^H}{1 - \left(1 - r_b \right)^{\frac{H}{mn}}} - mnC_b.
$$
\n(13.8)

where $1 > r_b > 0, 1 > r_m > 0.$

The vendor's net profit (formula (13.7) minus formula (13.5) and the ordering cost) is as follows:

$$
NP_v = \frac{P_{b0}Q(1 - (1 - r_b)^H)}{1 - (1 - r_b)^{\frac{H}{mn}}} - \frac{P_{v0}mQ(1 - (1 - r_v)^H)}{1 - (1 - r_v)^{\frac{H}{n}}}
$$

$$
- \frac{F_vHP_{v0}(m - 1)Q}{2n} \frac{1 - (1 - r_v)^H}{1 - (1 - r_v)^{\frac{H}{n}}} - nC_v.
$$
 (13.9)

The joint net profit for both vendor and buyer, the sum of formula (13.8) and formula (13.9) , denoted by NP, is

$$
NP = NP_b + NP_v. \tag{13.10}
$$

The relationship between the lot size and the number of deliveries is

$$
Q = \frac{HD}{mn} \tag{13.11}
$$

The value of r_b is dependent on the net profit sharing between the two players. The relationship between the vendor's net profit and buyer's net profit is defined as

$$
(NP_v) = \alpha(NP_b) \tag{13.12}
$$

where α is a negotiation factor.

When $\alpha = 0$, it means all net profit sharing is accrued by the buyer; when $\alpha = 1$, it implies that all net profit sharing is equally distributed. A large α means that all net profit is accrued mainly by the vendor. The optimization problem is a constrained non-linear programming problem, stated as

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$$
\begin{array}{ll}\n\max & NP = NP_v + NP_b \\
\text{s.t.} & (NP_v) = \alpha (NP_b), \ \alpha \ge 0, \\
Q = \frac{HD}{mn}\n\end{array} \tag{13.13}
$$

When there is no cost/price reduction (i.e., $r_v = 0$, $r_b = 0$ and $r_m = 0$), formulas [\(13.8\)](#page-4-0) and [\(13.9\)](#page-4-0) are undefined. Using L'Hospital's rule to take the derivatives of both the numerator and the denominator (Yang et al. 2007), a buyer's and a vendor's net profits in formulas [\(13.8\)](#page-4-0) and [\(13.9\)](#page-4-0) are derived as

$$
NP_b = P_{m0}DH - P_{b0}Qmn - \frac{P_{b0}F_bHQ}{2} - mnC_b \tag{13.14}
$$

and

$$
NP_v = P_{b0}Qmn - P_{v0}mnQ - \frac{P_{v0}F_vH(m-1)Q}{2} - nC_v
$$
 (13.15)

respectively.

The results of formulas (13.14) and (13.15) are the same as the case for static cost and price.

In the vendor–buyer pricing system, both the vendor and buyer aim to maximize their profits but their decisions are related to each other in a hierarchical way: the buyer as the leader and the vendor as the follower, or vice versa. When making the pricing strategy, if we take the buyer's point of view as having priority over a vendor, we can set the buyer as the leader and the vendor as the follower. By combining formulas (13.14) and (13.15), we can establish a bi-level pricing and replenishment strategy optimization model in a supply chain as follows:

$$
\max_{m \in M} NP_b(m) = P_{m0}DH - P_{b0}Qmn - \frac{P_{b0}F_bHQ}{2} - mnC_b
$$

s.t. $m > 0$,

$$
\max_{n \in N} NP_v(n) = P_{b0}Qmn - P_{v0}mnQ - \frac{P_{v0}F_vH(m-1)Q}{2} - nC_v
$$

s.t. $n > 0$. (13.16)

In this non-linear bi-level programming model, both the buyer and vendor adjust their own controlling variables, wishing to maximize their own profits under their specific constraints, but the buyer's objective is also subject to the vendor's optimized objective function value. That is, the buyer is the leader, who makes a decision first; and the vendor is the follower, who makes a decision based on the possible strategy of the buyer.

13.2.2 Experiments

The bi-level pricing and replenishment strategy optimization model can be illustrated by the following example from Yang et al. (2007).

Example 13.1

The demand rate per week, $D = 400$ units; The vendor's unit purchase cost at the initial time, $P_{v0} = $4;$ The buyer's unit purchase cost at the initial time, $P_{b0} = $5;$ The market price to the end consumer from the buyer at the initial time, $P_{m0} = $6;$ The buyer's ordering cost per order, $C_b = $30;$ The vendor's ordering cost per order, $C_v = $1,000$; The buyer's holding cost per dollar per week, $F_b = 0.004$; The vendor's holding cost per dollar per week, $F_v = 0.004$; The time horizon considered, $H = 52$ weeks; The negotiation factor, $\alpha = 1$.

After substituting the above parameters into formula (13.16) (13.16) , we have the following simplified bi-level pricing and replenishment strategy optimization model:

$$
\max_{n \in N} NP_b(n, m) = 20,800 - \frac{10,816}{mn} - 30mn
$$
\ns.t. $n > 0$
\n
$$
\max_{m \in M} NP_v(n, m) = 20,800 - \frac{8,652.8(m - 1)}{mn} - 1,000n
$$
\ns.t. $m > 0$. (13.17)

We use the developed FBLDSS in Chap. [11](http://dx.doi.org/10.1007/978-3-662-46059-7_11) to solve problem (13.17). To obtain a solution, we first input the objective functions and constraints of both the leader (buyer) and follower (vendor). We then run the software and obtain a solution $(m, n) = (6.9743, 2.7225), (NP_v, NP_b) = (19,660.7371, 15,354.9525).$

Through comparison of objective values when $m = 6$ and $m = 7$, we select $m = 6$ since it results in a bigger objective function value for the buyer in the problem. Similarly, $n = 3$ results in a bigger objective value for the vendor.

We then use the model and solution method from Yang et al. (2007) to obtain a solution for the same problem when the negotiation factor α is set as one. It should be noted that since both m and n can only be positive integers, the buyer's net profit and the vendor's net profit cannot be completely equal in most situations. In this example, the buyer's net profit and the vendor's net profit will be the closest and maximized under $m = 1$ and $n = 3$. Meanwhile, further experiments are carried out by adjusting the negotiation factor α in a wider range.

The results of our bi-level pricing and replenishment strategy optimization model and the model of Yang et al. (2007) are compared, as shown in Table [13.2](#page-7-0).

	A	m	n	NP _b	NP_{v}	NP
Yang et al. (2007)	$\alpha=1$		3	\$17,104	\$17,800	\$34,904
	$\alpha > 2$			\$9,954	\$19,800	\$29,754
	$1.5 < \alpha < 2$			\$9,954	\$19,800	\$29,754
	$1 < \alpha < 1.5$		3	\$17,105	\$17,800	\$34,904
	$0.5 \leq \alpha \leq 1$	3	3	\$19,328	\$15,877	\$35,205
	α < 0.5	2	10	\$19,659	\$10,367	\$30,026
Our bi-level optimization model 6			3	\$19,659	\$15,396	\$35,016

Table 13.2 Summary of results for Example [13.1](#page-6-0)

From the experimental results, it is noted that with our bi-level pricing and replenishment strategy optimization model, compared with Yang et al.'s (2007) original model (i.e., $\alpha = 1$), the profit for the buyer increases by about 15 % (from \$17,104 to \$19,659) and the profit for the vendor decreases by about 13.5 $%$ (from \$17,800 to \$15,396). The total percentage increase for the buyer and the vendor is about 3.2 % (from \$34,904 to \$35,016) when compared with the results from Yang et al. (2007). When α is adjusted in a wider range, the buyer still achieves more profit in all situations of α with our bi-level strategy optimization model. Even if the vendor as the follower loses some profit when $\alpha \geq 0.5$, the profit sum of both the buyer and vendor are still higher in our bi-level model under most choices of a.

The proposed bi-level pricing and replenishment optimization model can achieve more profit for a buyer in a supply chain at the price of some profit loss for the vendor. This is understandable, as bi-level optimization models always take the leader's interest as priority. The reason our results outperform others is that our bilevel pricing and replenishment strategy optimization model gives the buyer or the vendor the freedom to optimize their choices, without having to obey the heavy restrictions faced in the model by Yang et al. (2007).

13.3 Case Study 2: Hi-tech Product Pricing and Replenishment Strategy Making with Weekly Decline-Rates

This section takes the hi-tech product pricing and replenishment strategy problem again, where weekly demand rate and weekly decline-rates are added as extra decision variables, to carry out the second case study by bi-level decision techniques.

13.3.1 Problem Formulation

In this section, by switching the leader and follower roles, respectively, between a buyer and a vendor, we develop two bi-level pricing models in a supply chain.

The buyer's net profit in a buyer-vendor system can be calculated by:

$$
NP_b = \frac{P_{m0}D}{\ln(1 - r_m)} \left(e^{H \ln(1 - r_m)} - 1 \right) - \frac{P_{b0}Q \left(1 - (1 - r_b)^H \right)}{1 - (1 - r_b)^{\frac{H}{mn}}} - \frac{F_b H P_{b0} Q}{2mn} \frac{1 - (1 - r_b)^H}{1 - (1 - r_b)^{\frac{H}{mn}}} - m n C_b.
$$
\n(13.18)

The vendor's net profit can be calculated by:

$$
NP_v = \frac{P_{b0}Q(1 - (1 - r_b)^H)}{1 - (1 - r_b)^{\frac{H}{mn}}} - \frac{P_{v0}mQ(1 - (1 - r_v)^H)}{1 - (1 - r_v)^{\frac{H}{n}}}
$$

$$
- \frac{F_vHP_{v0}(m - 1)Q}{2n} \frac{1 - (1 - r_v)^H}{1 - (1 - r_v)^{\frac{H}{n}}} - nC_v.
$$
 (13.19)

In (13.18) , the buyer controls m, the number of the buyer's lot size deliveries per vendor's lot size; and r_m , the weekly decline-rate of market price to an end-consumer. In (13.19) , the vendor controls *n*, the number of the orders that the vendor places for the item from a supplier in the planning horizon; r_b , the weekly declinerate of the buyer's purchase cost; and r_v , the weekly decline-rate of the vendor's purchase cost. All other parameters defined in the problem are constants, which may change if other specific problems are introduced. The explanations of symbols used in the above two formulas are listed in Table [13.1.](#page-2-0)

When making the pricing strategy, if we take the buyer's point of view to make its profit a priority over the vendor, we can designate the buyer as the leader and the vendor as the follower. By combining Formulas (13.18) and (13.19), we establish a bi-level pricing model in a supply chain as follows:

$$
\max_{m,r_m} NP_b(m, r_m, n, r_b, r_v)
$$
\n
$$
= \frac{P_{m0}D}{\ln(1 - r_m)} \left(e^{H \ln(1 - r_m)} - 1 \right) - \frac{P_{b0}Q(1 - (1 - r_b)^H)}{1 - (1 - r_b)^{\frac{H}{mn}}}
$$
\n
$$
- \frac{F_bHP_{b0}Q}{2mn} \frac{1 - (1 - r_b)^H}{1 - (1 - r_b)^{\frac{H}{mn}}} - mnC_b
$$
\n
$$
\text{s.t. } m > 0,
$$
\n
$$
0.0001 \le r_m \le 0.5,
$$
\n
$$
\max_{b, r_v} NP_v(m, r_m, n, r_b, r_v) = P_{b0}Q \frac{1 - (1 - r_b)^H}{1 - (1 - r_b)^{\frac{H}{mn}}} - P_{v0}mQ \frac{1 - (1 - r_v)^H}{1 - (1 - r_v)^{\frac{H}{n}}}
$$
\n
$$
- \frac{F_vHP_{v0}(m - 1)Q}{2n} \frac{1 - (1 - r_v)^H}{1 - (1 - r_v)^{\frac{H}{n}}} - nC_v
$$

s.t. $n > 0$, $0.0001 \le r_b \le 0.5$, $0001 \le r_v \le 0.5$.

m $n.r$

 (13.20)

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In this model, both the buyer and vendor adjust their own decision variables respectively, wishing to maximize their own profits, under specific constraints. The buyer is the leader, who makes a decision first; and the vendor is the follower, who makes a decision after the buyer.

If we take the point of view of the vendor to make its profit a priority over the buyer, we can designate the vendor as the leader and the buyer as the follower. By combining formulas ([13.18](#page-8-0)) and [\(13.19\)](#page-8-0), we establish another bi-level pricing model in a supply chain as follows:

 $\ddot{}$

$$
\max_{n,r_b,r_v} NP_v(m,r_m,n,r_b,r_v) = P_{b0}Q \frac{1 - (1 - r_b)^H}{1 - (1 - r_b)^{\frac{H}{mn}}} - P_{v0}mQ \frac{1 - (1 - r_v)^H}{1 - (1 - r_v)^{\frac{H}{n}}}
$$
\n
$$
- \frac{F_v H P_{v0}(m-1)Q}{2n} \frac{1 - (1 - r_v)^H}{1 - (1 - r_v)^{\frac{H}{n}}} - nC_v
$$
\n
$$
\text{s.t. } n > 0,
$$
\n
$$
0.0001 \le r_b \le 0.5,
$$
\n
$$
0.0001 \le r_v \le 0.5.
$$
\n
$$
\max_{m,r_m} NP_b(m,r_m,n,r_b,r_v)
$$
\n
$$
= \frac{P_{m0}D}{\ln(1 - r_m)} \left(e^{H \ln(1 - r_m)} - 1 \right) - \frac{P_{b0}Q(1 - (1 - r_b)^H)}{1 - (1 - r_b)^{\frac{H}{mn}}}
$$
\n
$$
- \frac{F_b H P_{b0}Q}{2mn} \frac{1 - (1 - r_b)^H}{1 - (1 - r_b)^{\frac{H}{mn}}} - m nC_b
$$
\n
$$
\text{s.t. } m > 0,
$$
\n
$$
0.0001 \le r_m \le 0.5.
$$
\n
$$
(13.21)
$$

In this model, both the buyer and vendor adjust their own decision variables respectively, wishing to maximize their own profits, under specific constraints. The vendor is the leader, who makes the first decision; and the buyer is the follower, who makes a decision after the buyer.

We will use the FBLDSS to solve problems defined by the above two bi-level pricing models.

13.3.2 Experiments

In this section, we illustrate the bi-level pricing models in Sect. [13.3.1](#page-7-0) by the following numerical example where the parameters are given as follows:

Example 13.2

- 1. The demand rate per week, $D = 400$ units;
- 2. The vendor's unit purchase cost at the initial time, $P_{v0} = $4;$
- 3. The buyer's unit purchase cost at the initial time, $P_{b0} = 5 ;
- 4. The market price to the end consumer from the buyer at the initial time, $P_{m0} = $6;$
- 5. The buyer's ordering cost per order, $C_b = $30;$
- 6. The vendor's ordering cost per order, $C_v = $1,000;$
- 7. The buyer's holding cost per dollar per week, $F_b = 0.004$;
- 8. The vendor's holding cost per dollar per week, $F_v = 0.004$;
- 9. The time horizon considered, $H = 52$ weeks.

To deal with this problem, we relax the constraint of equal profit, and add r_m , r_b , and r_v as decision variables. By using the FBLDSS developed in Chapter 11 to solve problems defined by Formulas [\(13.20\)](#page-8-0) and ([13.21](#page-9-0)), we obtain solutions for both the buyer and the vendor. To evaluate the results of this research, we compare these results with the results from the original model by Yang et al. (2007) under a different negotiation factor α , which is defined as $\alpha = NP_v/Np_b$. To make the comparison fair and reasonable, besides m and n, we add r_m , r_b , and r_v as decision variables to be changeable to maximize the profit in Yang et al.'s (2007) model. Table 13.3 lists solutions from this research and solutions from the model by Yang et al. (2007).

From Table 13.3, we can see that, using the bi-level pricing model (the buyer as the leader) developed in this section, the buyer's profit will increase compared with Yang's model when $\alpha \geq 1.5$. If the vendor is taken as the leader, he or she can achieve a profit increase when $\alpha < 2$, which is true for most pricing problems in a supply chain. As the follower, the vendor or buyer is bound to lose, despite the range of the negotiation factor α . This is understandable, because in a bi-level decision situation, we always take the leader's interest as priority.

	m	r_m	N	r _b	r_{v}	NP _b	NP_{v}		
Yang et al. (2007) ($\alpha \ge 2$)	2	0.0001	9	0.0068	0.5	35,008	69.946		
Yang et al. (2007) $(1.5 \le \alpha \le 2)$	2	0.0001	9	0.01	0.5	41,280	63,710		
Yang et al. (2007) ($1 \le \alpha \le 1.5$)		0.0001	9	0.017	0.5	52,990	52,068		
Yang et al. (2007) (0.5 $\leq \alpha \leq 1$)		0.0001	9	0.032	0.5	68,548	36,605		
Yang et al. (2007) (α < 0.5)		Not applicable							
This study (buyer as leader)		0.0071	6	0.0372	0.0753	52.399	16.866		
This study (vendor as leader)		0.0015		0.0026	0.0767	21,359	64,165		

Table 13.3 Summary and comparison of running results for Example 13.2

These results reveal that when applying bi-level decision techniques on pricing problems in supply chains, some improvements can be achieved for a player (a buyer or a vendor) if it is the leader.

13.4 Summary

In this chapter, the pricing and replenishment strategy making problem for hi-tech products proposed by Yang et al. (2007) is remodeled by bi-level programming. To solve problems defined by these bi-level programming models, the FBLDSS is used. Experimental results show that the bi-level pricing models can achieve profit improvements for both the buyer and vendor. In the two-stage vendor-buyer inventory system, our experimental data show that the vendor, as the leader, outperforms the buyer as the leader. This is because the vendor, as the leader, improves the actual consumption rates; the vendor making the first decision ensures that production matches demand more closely, reduces inventory and improves business performance. This is why the vendor managed inventory has become very popular in recent years.