Chapter 12
Bi-level Programming for Competitive Strategic Bidding Optimization in Electricity Markets

We focus on the application of bi-level programming in electricity markets (power market) in this chapter. Competitive strategic bidding optimization of electric power plants (companies) is becoming one of the key issues in electricity markets. This chapter presents a strategic bidding optimization technique developed by applying the bi-level programming. By analyzing the strategic bidding behavior of power plants, we understand that this bidding problem includes several power plants and only one market operator respectively known as multiple leaders and single follower. The problem can be considered as a bi-level multi-leader optimization problem which is introduced in Chap. 5. We therefore build a *bi-level multi-leader* (BLML) decision (programming) model for this bidding problem in day-ahead electricity markets. In the BLML decision model, each power plant is allowed to choose its biddings to maximize its individual profit, and the market operator can find its minimum purchase electricity fare that is determined by the output power of each unit and the uniform marginal prices.

In this chapter, we first give the background of this bi-level programming application in Sect. 12.1. Section 12.2 conducts bidding strategy analysis in competitive electricity markets that is used for modeling. Section 12.3 presents a BLML competitive electricity markets model. A real data-based case study is shown in Sect. 12.4 to illustrate and test the bi-level programming model for competitive strategic bidding optimization in electricity markets. Experimental results on a strategic bidding problem for a day-ahead electricity market have demonstrated the validity of the proposed decision model. Section 12.5 summaries this application.

12.1 Background

Throughout the world, electric power industries are undergoing enormous restructuring from nationalized monopolies to individual organizations in a competitive market (Huang and Pai 2002) with the support of digital eco-systems. Because of the significance of electricity energy to national economies and society (Guerrero et al. 2008), electricity markets must be operated under extensive

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conditions of absolute security and stabilization. The research on electricity markets has attracted many researchers, owners and managers from electricity entities. The competitive mechanism of day-ahead markets is a very important research issue in electricity market studies, which can be described as follows: each power plant submits a set of hourly (half-hourly) generation prices and the available capacities for the following day. According to this data and an hourly (half-hourly) load forecast, a market operator allocates the generation output for each unit.

As no determinate operation model for electricity markets exists, the marketing procedure of electric power industries varies from country to country. Generally speaking, there are three kinds of running models in electricity markets: the power pool model, wholesale competitive model, and retail competitive model. These models adopt three kinds of electric power trading methods: long term contract, day-ahead market, and facility service. Among them, the day-ahead market is the most competitive and active imposing great influence on profits for each participant in the market. Specifically, each power plant submits a set of generation prices and other related data, based on which the market operator makes a generating plan for the following day. To optimize this procedure, many models and algorithms have been proposed.

This chapter applies the bi-level optimization approach for dealing with strategic bidding optimization in electricity markets. We propose both a strategic bidding model for power plants and a generation output dispatch model for a market operator in a day-ahead electricity market. Since there are several power plants considered as leaders; and there is only one market operator as the follower, this decision problem is a bi-level multi-leader optimization model which was introduced in Chap. 5. Based on these two models, a specific BLML decision model, which includes ramp rate constraints for competitive electricity markets, is proposed. A real data-based case study on competitive strategic bidding problem in an electricity market is then presented.

12.2 Bidding Strategy Analysis in Competitive Electricity Markets

In an auction-based day-ahead electricity market, each power plant will try to maximize its own profit by strategic bidding. Normally, each power plant submits a set of hourly (half-hourly) generation prices and available capacities for the following day. Based on this data and an hourly-load (half-hourly-load) forecast, a market operator will allocate generation output. In this section, under the analysis of bidding strategy optimization problems, we build a competitive strategic bidding model for power plants and a generation output dispatch model for a market operator in a day-ahead electricity market.

12.2.1 Strategic Pricing Model for Power Plants

In the upper level, each power plant is concerned with how to choose a bidding strategy, which includes generation price and available capacity. Many bidding functions have been proposed. For a power system, the generation cost function generally adopts a quadratic function of the generation output, i.e. the generation cost function can be represented as:

$$C_j(P_j) = a_j P_j^2 + b_j P_j + c_j \tag{12.1}$$

where P_j is the generation output of generator j, and a_j, b_j, c_j are coefficients of the generation cost function of generator j.

The marginal cost of generator j is calculated by:

$$\lambda_j = 2a_j P_j + b_j \tag{12.2}$$

It is a linear function of its generation output P_j. The rule in a goods market may expect each power plant to bid according to its own generation cost. Therefore, we adopt this linear bid function. Suppose that the bidding for the jth unit at time t is:

$$R_{tj} = \alpha_{tj} + \beta_{tj} P_{tj} \tag{12.3}$$

where $t \in T$ is the time interval, T is time interval number, j represents the unit number, P_{tj} is the generation output of unit j at time t, and α_{tj} and β_{tj} are the bidding coefficients of unit j at time t.

According to the justice principle of *the same quality, the same network, and the same price*, we adopt a uniform marginal price (UMP) as the market clearing price. Once the energy market is cleared, each unit will be paid according to its generation output and UMP. The payoff of the ith power plant is:

$$F_i = \sum_{t=1}^{T} \Big(\sum_{j \in G_i} UMP_t P_{tj} - \sum_{j \in G_i} (a_j P_{tj}^2 + b_{tj} P_{tj} + c_{tj}) \Big) \tag{12.4}$$

where G_i is the suffix set of the units belonging to the ith power plant. Each power plant wishes to maximize its own profit F_i. In fact, F_i is the function of P_{tj} and UMP_t, and UMP_t is the function of all units' bidding α_{tj}, β_{tj} and output power P_{tj}, which will impact on each other.

Therefore, we establish a *strategic pricing model* for power plants as follows:

$$\begin{aligned} \max_{\alpha_{tj}, \beta_{tj} \in G_i} F_i &= F_i(\alpha_{t1}, \beta_{t1}, \ldots, \alpha_{tN}, \beta_{tN}, P_{t1}, \ldots, P_{tN}) \\ &= \sum_{t=1}^{T} \left(UMP_t P_{ti} - \sum_{j \in G_i} \left(a_j P_{tj}^2 + b_{tj} P_{tj} + c_{tj} \right) \right) \\ & \qquad i = 1, \ldots, L \end{aligned} \tag{12.5}$$

where L is the number of power plants, $P_{ti} = \sum_{j \in G_i} P_{tj}$, $t = 1, \ldots, T$.

The profit calculated for each power plant will consider both P_{ti} and UMP_t, which can be computed by a market operator, according to the market clearing model.

12.2.2 Generation Output Dispatch Model for Market Operator

A market operator actually represents the consumer electricity purchase from power plants, under the conditions of security and stabilization. The objective of a market operator is to minimize the total purchase fare, while encouraging power plants to use a bid price as low as possible. It is reasonable that the lower the price, the more the output. Thus, the function value of a market operator's objective will be calculated according to the bidding price. Most previous strategic bidding models do not include ramp rate constraints, without which the solution for generating dispatch may not be a truly optimal one. We should consider the ramp rate constraints in the real world when modeling a generating dispatch. However, if a model includes ramp rate as a constraint, the number of decision variables involved in the problem will increase dramatically, which requires a more powerful solution algorithm. Based on the analysis above, we build a market operator's *generation output dispatch model* as follows:

$$\begin{aligned} \min_{P_{tj}} \quad & f = f(\alpha_{t1}, \beta_{t1}, \ldots, \alpha_{tN}, \beta_{tN}, P_{t1}, \ldots, P_{tN}) = \sum_{t=1}^{T} \sum_{j=1}^{N} R_{tj} P_{tj} \\ \text{s.t.} \quad & \sum_{j=1}^{N} P_{tj} = P_{tD} \\ & P_{jmin} \le P_{tj} \le P_{jmax} \\ & -D_j \le P_{tj} - P_{t-1,j} \le U_j, \quad t = 1, 2, \ldots, T \end{aligned} \tag{12.6}$$

where $t \in T$ is the time interval, T is the time interval number, j represents the unit number, P_{tj} is the generation output of unit j at the time t, and α_{tj} and β_{tj} are the bidding co-efficients of unit j at the time t, P_{tD} is the load demand at the time t, $P_{j\min}$ is the minimum output power of the jth unit, $P_{j\max}$ is the maximum output power of the jth unit, D_j is the maximum downwards ramp rate of the jth unit, and U_j is the maximum upwards ramp rate of the jth unit.

After receiving all power plants' bid data, a market operator determines the output power of each unit and UMP_t in time slot t. UMP_t can be calculated according to the following steps:

[Begin]
Step1: calculate output power of each unit j for all time slots t using formula (12.6);
Step2: compute bidding R_{tj} corresponding to the generation output P_{tj};
Step3: account $UMP_t = \max_{j=1}^{N} R_{tj}$.
[End]

12.3 BLML Decision Model in Competitive Electricity Markets

From the analysis above, we know that in an auction-based day-ahead electricity market, each power plant tries to maximize its own profit by strategic bidding, and each market operator tries to minimize its total electricity purchase fare. The decision of one will influence the other. This is a typical bi-level decision problem, which has multiple leaders and only one follower, with power plants as leaders and a market operator as a follower.

By combining the strategic pricing model defined in (12.5) with the generation output dispatch model defined in (12.6), we establish a BLML decision model for competitive strategic bidding-generation output dispatch in an auction-based day-ahead electricity market as follows:

$$
\begin{aligned}
\max_{\alpha_{tj},\beta_{tj}\in G_i} \quad & F_i = F_i(\alpha_{t1},\beta_{t1},\ldots,\alpha_{tN},\beta_{tN},P_{t1},\ldots,P_{tN}) \\
& = \sum_{t=1}^{T}\left(UMP_t P_{ti} - \sum_{j\in G_i}\left(a_j P_{tj}^2 + b_{tj}P_{tj} + c_{tj}\right)\right) \\
\text{s.t.} \quad & \alpha_{t\min} \le \alpha_{tj} \le \alpha_{t\max}, \\
& \beta_{t\min} \le \beta_{tj} \le \beta_{t\max}, \\
& t = 1,2,\ldots,\ T, \quad j = 1,2,\ldots,N, \quad j = 1,2,\ldots,L \\
& \min_{P_{tj}} \quad f = f(\alpha_{t1},\beta_{t1},\ldots,\alpha_{tN},\beta_{tN},P_{t1},\ldots,P_{tN}) = \sum_{t=1}^{T}\sum_{j=1}^{N} R_{tj}P_{tj} \\
& \text{s.t.} \quad \sum_{j=1}^{N} P_{tj} = P_{tD}, \\
& \qquad P_{j\min} \le P_{tj} \le P_{j\max}, \\
& \qquad -D_j \le P_{tj} - P_{t-1,j} \le U_j, \\
& \qquad t = 1,2,\ldots,\ T.
\end{aligned}
\tag{12.7}
$$

where α_{tj} and β_{tj} are the bidding coefficients of unit j at time t, $\alpha_{t\min}$, $\alpha_{t\max}$, $\beta_{t\min}$, $\beta_{t\max}$ are the lower and upper limits for α_{tj} and β_{tj} respectively, L is the number of power plants, $P_{ti} = \sum_{j\in G_i} P_{tj}$, $P_{j\min}$ is the minimum output power of the jth unit,

$P_{j\max}$ is the maximum output power of the jth unit, D_j is the maximum downwards ramp rate of the jth unit, and U_j is the maximum upwards ramp rate of the jth unit.

This model describes strategic bidding problems in competitive electricity markets from a bi-level angle. In this model, there are multiple leaders (power plants) but only one follower (a market operator). This kind of problem has been studied in Chap. 5, and we will use the developed BLML-PSO algorithm proposed in Sect. 5.6 of Chap. 5 to solve it.

12.4 A Case Study

In this section, we will use a real world competitive strategic bidding example to illustrate the application of bi-level decision technology on an electricity market.

12.4.1 Test Data

In order to test the effectiveness of the proposed BLML decision model and the BLML-PSO algorithm when solving the model defined by (12.7), a typical competitive strategic bidding case consisting of three companies with six units and twenty-four time intervals is chosen. The generation cost function can be calculated by using formula (12.1), where the cost coefficients a_j, b_j, c_j of unit j and other technical data are given in Table 12.1. The load demands for each time interval t are given in Table 12.2.

In Table 12.1, Units 1 and 2 belong to the first power plant, Units 3 and 4 belong to the second power plant, and Units 5 and 6 belong to the third power plant.

To simplify computation, the limit of strategic bidding coefficients does not vary by different time slots and we suppose:

$$\alpha_{t\min} = 7, \alpha_{t\max} = 9, \beta_{t\min} = 0.0002, \beta_{t\max} = 0.007,$$
$$t = 1, 2, \ldots T, \quad j = 1, 2, \ldots, N.$$

Table 12.1 Technical data of units

Unit no.	a_j	b_j	c_j	$p_{\min}$ (MW)	$p_{\max}$ (MW)	D_j (MW/h)	U_j (MW/h)
1	0.00028	4.10	150	50	680	80	85
2	0.00312	4.50	80	30	150	45	60
3	0.00048	4.10	109	50	360	60	65
4	0.00324	3.74	125	60	240	45	80
5	0.00056	3.82	130	60	300	70	80
6	0.00334	3.78	100	40	160	55	40

Table 12.2 Load demands in different time intervals

t	1	2	3	4	5	6
P_{tD}	1,033	1,000	1,013	1,027	1,066	1,120
t	7	8	9	10	11	12
$P_{t\mathrm{D}}$	1,186	1,253	1,300	1,340	1,313	1,313
tt	13	14	15	16	17	18
$P_{t\mathrm{D}}$	1,273	1,322	1,233	1,253	1,280	1,433
tt	19	20	21	22	23	24
P_{tD}	1,273	1,580	1,520	1,420	1,300	1,193

Table 12.3 Running results for α_{tj} from the example

t	j					
	1	2	3	4	5	6
1	7.37	7.61	7.32	7.03	7.27	8.98
2	8.76	7.74	8.71	7.15	8.13	7.10
3	7.18	8.89	8.60	8.84	8.55	8.26
4	7.36	8.33	7.31	8.28	8.73	7.70
5	7.10	8.80	8.51	8.75	8.46	8.17
6	8.59	7.57	8.54	8.98	7.96	8.93
7	7.11	7.35	7.06	8.77	7.01	8.72
8	8.45	8.90	7.87	8.85	7.82	8.80
9	8.29	8.00	8.24	7.95	7.66	7.37
10	8.42	7.39	8.37	8.81	7.79	8.76
11	7.57	7.28	7.52	7.23	8.94	7.18
12	7.02	7.99	8.97	7.94	8.39	7.36
13	7.49	7.20	7.44	7.15	8.86	7.10
14	8.25	7.22	8.20	8.64	7.62	8.59
15	8.04	7.74	7.98	7.69	7.40	7.11
16	8.11	8.56	7.53	8.51	7.48	8.45
17	8.68	8.92	8.63	8.34	8.58	8.29
18	8.08	7.05	8.03	8.47	7.45	8.42
19	8.50	8.21	7.92	8.16	7.86	7.57
20	8.68	7.65	8.63	7.07	8.04	7.02
21	8.41	8.12	7.83	8.07	7.78	7.49
22	7.91	8.88	7.33	8.30	7.27	8.25
23	8.43	8.67	8.38	8.09	8.33	8.04
24	7.24	8.21	7.19	8.16	7.14	8.11

12.4.2 Experiment Results

This example is run by the BLML-PSO algorithm developed in Sect. 5.6. The running results are listed in Tables 12.3, 12.4, 12.5, 12.6 and 12.7, where α_{tj} and β_{tj}

Table 12.4 Running results for β_{tj} from the example

t	*j*					
	1	2	3	4	5	6
1	0.00089	0.00205	0.00321	0.00438	0.00554	0.00670
2	0.00208	0.00074	0.00440	0.00307	0.00673	0.00539
3	0.00641	0.00077	0.00193	0.00310	0.00426	0.00542
4	0.00278	0.00644	0.00510	0.00377	0.00063	0.00609
5	0.00048	0.00164	0.00280	0.00397	0.00513	0.00629
6	0.00382	0.00249	0.00615	0.00481	0.00348	0.00034
7	0.00350	0.00467	0.00583	0.00699	0.00135	0.00251
8	0.00022	0.00568	0.00435	0.00121	0.00667	0.00353
9	0.00687	0.00123	0.00420	0.00536	0.00653	0.00089
10	0.00556	0.00423	0.00109	0.00655	0.00522	0.00208
11	0.00060	0.00176	0.00292	0.00408	0.00525	0.00641
12	0.00626	0.00493	0.00179	0.00045	0.00411	0.00278
13	0.00147	0.00263	0.00379	0.00496	0.00612	0.00048
14	0.00051	0.00597	0.00464	0.00150	0.00696	0.00382
15	0.00449	0.00565	0.00682	0.00118	0.00234	0.00350
16	0.00551	0.00237	0.00103	0.00469	0.00336	0.00022
17	0.00106	0.00222	0.00339	0.00455	0.00571	0.00687
18	0.00406	0.00092	0.00638	0.00324	0.00191	0.00556
19	0.00658	0.00094	0.00391	0.00507	0.00624	0.00060
20	0.00295	0.00162	0.00527	0.00394	0.00080	0.00626
21	0.00065	0.00362	0.00478	0.00595	0.00031	0.00147
22	0.00580	0.00266	0.00132	0.00498	0.00365	0.00051
23	0.00368	0.00484	0.00600	0.00036	0.00333	0.00449
24	0.00220	0.00586	0.00452	0.00138	0.00684	0.00551

Table 12.5 Running results for UMP_t from the example

t	1	2	3	4	5	6	7	8
UMP	17.81	8.62	1.49	8.19	2.77	4.35	14.31	4.43
t	9	10	11	12	13	14	15	16
UMP	8.75	6.40	13.45	12.30	1.06	9.47	18.93	15.88
t	17	18	19	20	21	22	23	24
UMP	14.39	18.87	5.42	11.10	19.35	14.60	18.23	7.34

are the bidding coefficients of unit j at time t, P_{tj} is the generation output of unit j at time t, UMP_t is the uniform marginal price at time t.

Under these solutions, the objective values for both the leaders and the follower are listed in Table 12.7.

Table 12.6 Running results for P_{tj} from the example

t	j					
	1	2	3	4	5	6
1	493	150	50	240	60	40
2	445	145	63	232	70	45
3	443	140	76	224	80	50
4	442	135	89	216	90	55
5	466	130	102	208	100	60
6	505	125	115	200	110	65
7	556	120	128	192	120	70
8	608	115	141	184	130	75
9	640	110	154	176	140	80
10	665	105	167	168	150	85
11	623	100	180	160	160	90
12	593	110	193	152	170	95
13	538	105	206	144	180	100
14	478	108	231	165	210	130
15	412	109	252	150	200	110
16	333	130	285	180	210	115
17	275	130	320	210	220	125
18	268	150	360	240	260	155
19	322	120	300	200	211	120
20	390	150	360	240	290	150
21	326	150	355	230	299	160
22	266	143	356	240	270	145
23	191	120	320	239	280	150
24	207	100	300	200	254	132

Table 12.7 Objective values for the decision makers

The 1st power plant	The 2nd power plant	The 3rd power plant	The market operator
73,313	65,799	46,376	225,272

12.4.3 Experiment Analysis

By the BLML-PSO algorithm developed in Sect. 5.6, solutions are reached for both the power plants and the market operator to help them make strategic decisions. We conclude the BLML decision model and the BLML-PSO algorithm in the experiment as follows:

1. The BLML decision model can effectively model strategic bidding problems from electricity markets. By considering the gaming and bi-level relationships between several power plants and a market operator, the BLML decision model

can better reflect the features of such real-world strategic bidding problems in electricity markets and format these problems practically.

2. The BLML-PSO algorithm is quite effective for solving strategic bidding problems defined by the BLML decision model. By making several power plants and a market operator decide sequentially, the hierarchical relation between them is fully considered. By moving the choice by power plants as close as possible to their rational reactions, the Nash equilibrium solution can be obtained.

12.5 Summary

Competitive strategic bidding optimization of power plants in electricity markets is in a practical sense important and it is technically implementable. This chapter applies a BLML decision model and BLML-PSO algorithm to handle the competitive strategic bidding decision-making problem in electricity markets. The proposed solution method can achieve a generalized Nash equilibrium for the BLML decision problem in an electricity market by providing power plants with competitive strategic bidding within the prevailing network security constraints.