Multi-criteria Methods 6

6.1 Introduction

For many investment decisions, the decision-maker wishes to pursue several targets, rather than a single target as the earlier chapters have assumed. Such a decision-making problem is typical in strategic investment decision-making as, for example, when installing a new plant in a new location, using new technology and/or manufacturing a new product.

The following chapter describes and discusses models and procedures developed to satisfy several target measures simultaneously, i.e. for multi-criteria decision-making. Multi-criteria decision-making (MCDM) may be divided into two groups. Decisions about alternative investment projects require multi-attribute decision-making (MADM); and decisions about alternative programmes require multi-objective decision-making (MODM). The first of these is explored in this chapter.

Multi-criteria decision-making affects all phases of the planning process. Initially, an extensive analysis is required to ascertain targets, their significance and likely conflicts. Where several targets exist, as assumed here, the decision-maker's preferences play a decisive role and must be investigated in detail. The MADM procedures discussed in this chapter support these processes of goal setting and decision-planning.

For a clear understanding of MADM procedures, some basic knowledge of utility theory is required.

First, an appropriate scale is necessary to measure targets quantitatively, in order to assess options as, in this case, alternative investment projects. The various types of available scales differ in the degree of measurability they imply.

A *nominal scale* is used to assign outcomes of a target criterion to different classes without ambiguity. No measurable relationship exists between the nominal classes and, therefore, no arithmetic operations are possible. Bank account numbers would be an example of the use of a nominal scale.

An *ordinal scale* allows statements about relationships, like 'smaller than' or 'bigger than'. Although differences between points on this scale cannot be

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measured, comparisons can be made. One example of an ordinal scale is the sequential list of place-getters in a competition.

The measurement of differences between points on the scale, and mathematical operations like addition, subtraction and averaging, may all be performed using an *interval scale*. A baseline may be fixed arbitrarily but, since a natural neutral point does not exist for an interval scale, the calculation of quotients is not meaningful. Examples of the use of interval scales include times and dates.

A *relational scale* differs from an interval scale in that a natural baseline exists and quotients can be calculated. Examples of values measured with a relational scale are lengths and weights.

For an *absolute scale* the scale unit is defined and consists of real numbers only, the values measured being dimensionless. Examples are absolute frequencies or probabilities. This scale type shows the highest level of measurability.

Interval, relational and absolute scales are all referred to as *cardinal scales*.

Having described the various scales that can be used to measure targets, it is now relevant to consider, first, the preference relationships and, second, the orders of preference that might apply to the alternatives being assessed.

From the relevant elements (alternatives) of a set A in a decision problem, a set of all possible ordered pairs (a, b) may be derived. This set is expressed as:

$$A \times A = \{(a,\ b) \mid a \in A,\ b \in A\} \tag{6.1}$$

Preference (or priority) relationships R are determined for pairs of alternatives belonging to A. One single preference relationship R is a partial set of $A \times A$, so the relationship is not necessarily valid for all pairs of alternatives. If R links a given pair (a, b), e.g. (a, b) is an element of R, this is symbolised as ‘aRb’. Characteristic features of relationships between pairs might be:

- *Completeness*: For all pairs (a, b) from the elements of a set A at least one of the relationships aRb or bRa exists and, therefore, all elements can be compared with one another.
- *Transitiveness*: The relationships aRb and bRc, for all elements $a, b, c \in A$, determine the relationship aRc. This, for example, is the case for the greater than relation ($a > b$ and $b > c$, therefore: $a > c$).
- *Reflexivity*: For all $a \in A$, the relationship aRa is valid. The greater than or equality relation is reflexive, e.g. $a \geq a$.
- *Irreflexivity*: In a set A, for all $a \in A$ the relationship aRa is not valid. For example, the greater than relation (e.g. $a > a$) is not valid.
- *Symmetry*: In a set A, from the relationship aRb it follows that bRa. For example, the equality relationship is symmetrical (i.e. from $a = b$, $b = a$ may be derived).
- *Asymmetry*: From aRb it follows that bRa is not valid. This is the case in a greater than relation, because if $a > b$, then $b > a$ cannot be correct.
- *Anti-symmetry*: In a set A, for all $a, b \in A$, the relationships aRb and bRa imply that $a = b$. For example, in a greater than or equality relationship, if $a \geq b$ and $b \geq a$ it follows that $a = b$.

To further characterise relationships in general, and especially the preference (or priority) relationships relevant here, the relationships mentioned above may be combined to obtain so-called *preference orders*. One kind of preference order is the *indifference order*, for which the characteristic features of transitiveness, reflexivity and symmetry are valid. According to this order, two alternatives are essentially equivalent, as symbolised by ~ (a ~ b indicates that a and b are regarded as equivalent). A *strict preference order* is characterised by completeness, transitiveness and asymmetry. This order is symbolised by $\succ$ (a $\succ$ b indicates a preference for a over b). For a *weak preference order,* completeness, transitiveness, reflexivity and anti-symmetry are characteristic. Here, the symbol a $\gtrsim$ b indicates that a is either better than or equivalent to b.

A *weak preference order* may be represented by a quantitative utility function (also called a preference or value function). The utility function transforms the preferential relationships '$\succ$' and '~' into the numerical relationships '$>$' and '$=$' concerning the utility (U) of alternatives. For all alternatives a, b $\in$ A it is valid that:

$$\mathrm{a} \succ \mathrm{b} \Leftrightarrow \mathrm{U}(\mathrm{a}) > \mathrm{U}(\mathrm{b}) \tag{6.2}$$

$$\mathrm{a} \sim \mathrm{b} \Leftrightarrow \mathrm{U}(\mathrm{a}) = \mathrm{U}(\mathrm{b}) \tag{6.3}$$

To conclude this introductory section, an overview of the methods applicable for MADM is given. These may be classified in various ways. For example, a distinction might be made between methods that assume a weak preference order (so that all alternatives can be ordered transitively and entirely and, thus, the optimum decision/action can be defined unambiguously), and the so-called decision technology-based procedures, like the fuzzy set approaches, which are not based on this assumption. Another classification suggested by Hwang and Yoon (1981) is based on the types of information that are used. This is summarised in Fig. 6.1.

A discussion of all the methods shown in Fig. 6.1 would exceed the scope of this book. However, the most important multi-criteria decision-making methods are described and discussed using examples related to a location decision—a typical strategic investment decision with multiple target criteria.

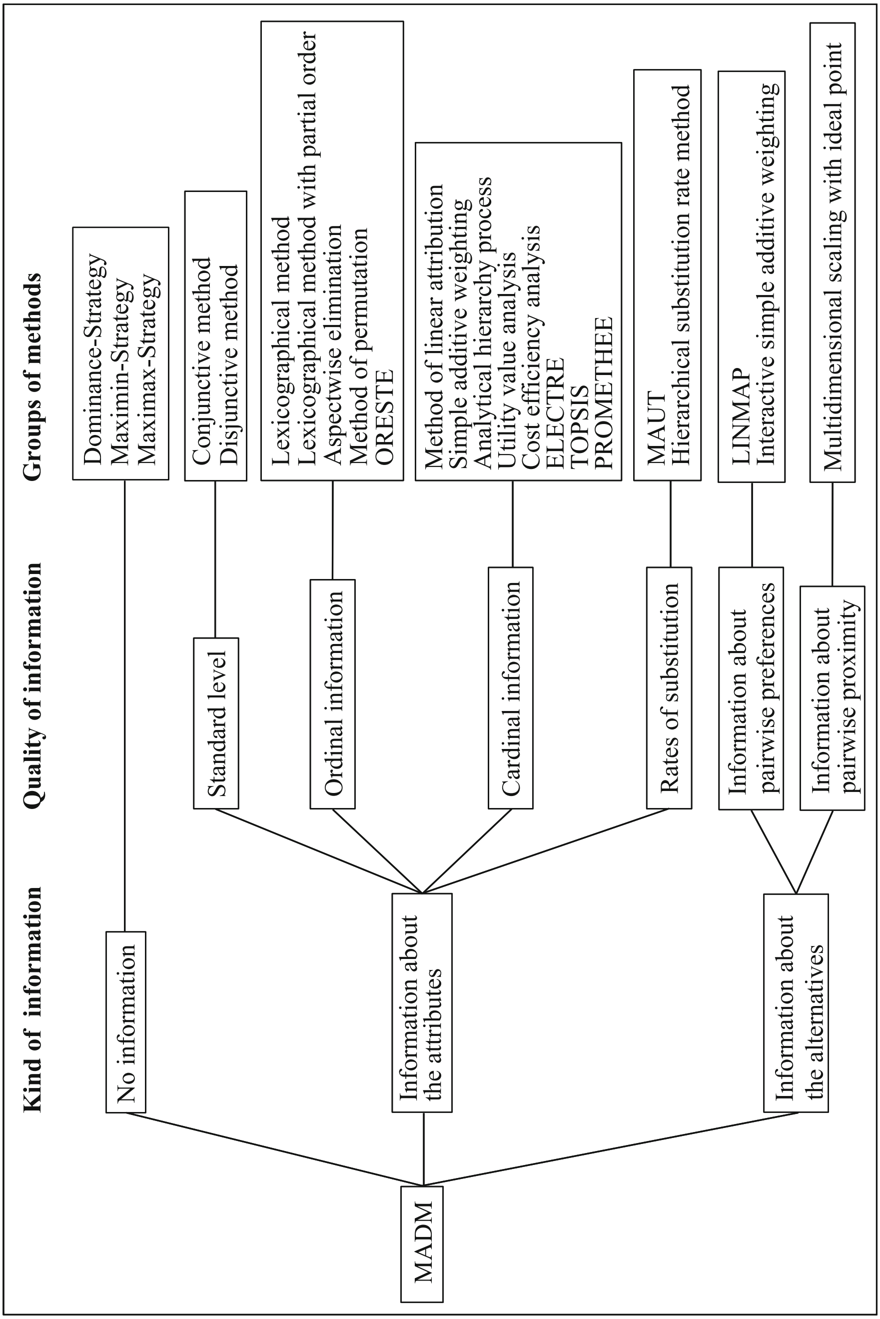

Fig. 6.1 MADM methods according to the type of information

6.2 Utility Value Analysis

Description of the method

This method seeks to analyse a number of complex alternatives, with the aim of ordering them according to the preferences of the decision-maker in a multi-dimensional target system. The ordering is carried out by calculating so-called *utility values* for the alternatives.

In utility value analysis, multiple target criteria are weighted according to their importance to the decision-maker. The ability of the different alternatives (here, the investment projects) to fulfil each target is measured and a corresponding partial utility value is given. The weighted partial utility values are summed to obtain a total value for every alternative—the utility value. For any one alternative, the aggregation of (weighted) partial utility values allows unfavourable results on one target measure to be compensated by better results on others. If certain criteria have minimum requirements, those must be fulfilled before carrying out a utility value analysis.

The utility value analysis consists of the following steps:

1. Determination of target criteria.
2. Weighting of each target criterion.
3. Calculation of partial utility values.
4. Calculation of (total) utility values.
5. Assessment of profitability.

In the first step of the utility value analysis, the *determination of target criteria*, a measurement scale (which may be nominal, ordinal or cardinal) is required for every criterion. The consideration of project attributes should not be duplicated by applying more than one criterion per attribute, and the extent to which an investment project fulfils one target criterion should be measured independently of the assessments made for other criteria. Monetary criteria are not normally included in a utility value analysis, since cash inflows and outflows, or yields and expenditures, are typically affected by many characteristics of investment projects that fall under some of the other criteria. Determining the target criteria requires a careful structuring and analysis of the target system. In complex decision problems, it is often worthwhile to split target measures into a multi-level hierarchy.

In the second step of the utility value analysis, a *weighting* w_c is determined for each criterion c in order to rank its importance to the decision-maker. The weightings should total 1 or 100 in order to simplify the interpretation of analysis results.

In the third step, the alternative projects are evaluated with respect to each criterion using, as appropriate, a nominal, ordinal or cardinal scale. Then, the results are transformed into partial utility values u_{ic} for each alternative i and for each criterion. The *partial utility values* are measured using a uniform cardinal scale, preferably with a range of 0–1, or 0–100.

In the fourth step, a (total) *utility value* U_{Ui}, is calculated as follows:

$$U_{Ui} = \sum_{c=1}^{C} u_{ic} \cdot w_c \tag{6.4}$$

Finally, an *assessment of profitability* is made using the following definitions:

Key Concept
Absolute profitability is achieved if an investment project's utility value is higher than a given target value.
Relative profitability: an investment project is preferred if its utility value is higher than that of any alternative project.

In some situations the utility value is not the only result of model analyses used for profitability assessment. As mentioned above, monetary target measures (e.g. net present value) should not be included in a utility value analysis, but considered separately. In such situations, goal conflicts are possible and a new multi-criteria problem can arise.

Example 6.1
In the following example, a utility value analysis is carried out in order to assess the relative profitability of three location alternatives: A_1, A_2 and A_3.

As a first step, the targets shown in Fig. 6.2 have been determined. The main target, selection of the optimum location, is split into sub-targets as illustrated. The weightings, which are determined in the second step, also appear in Fig. 6.2. The third step, the calculation of partial utility values is illustrated using the criterion 'size of land' in Fig. 6.3. The alternatives under consideration have sizes of 60,000 m^2 (A_1), 42,500 m^2 (A_2) and 35,000 m^2 (A_3).

In accordance with this function, the partial utility values of the alternatives for this criterion are: 1 (A_1), 0.2 (A_2) and 0 (A_3). For the other criteria, partial utility values have been determined as follows:

Table 6.1 Partial utility values of the alternatives A_1, A_2 and A_3

	Target criteria										
Alternative	S	P	D	LP	LC	T	FC	DP	FS	AA	MF-TT
A_1	1	0.4	1	0.2	0.4	0.6	0.4	0.6	0.8	0.4	0.6
A_2	0.2	0.4	0.2	0.6	0.8	0.4	0	1	0.8	0.8	1
A_3	0	0.6	0.8	0.9	1	0.8	1	0.2	0.8	0.4	0.4

In the fourth step, the total utility values are calculated. The weighted partial utility values are determined by multiplying the partial utility values by the weightings of the associated criterion and sub-target. For alternative A_1 and the criterion 'size of land', for example, it is:

$$1 \cdot 0.3 \cdot 0.2 = 0.06$$

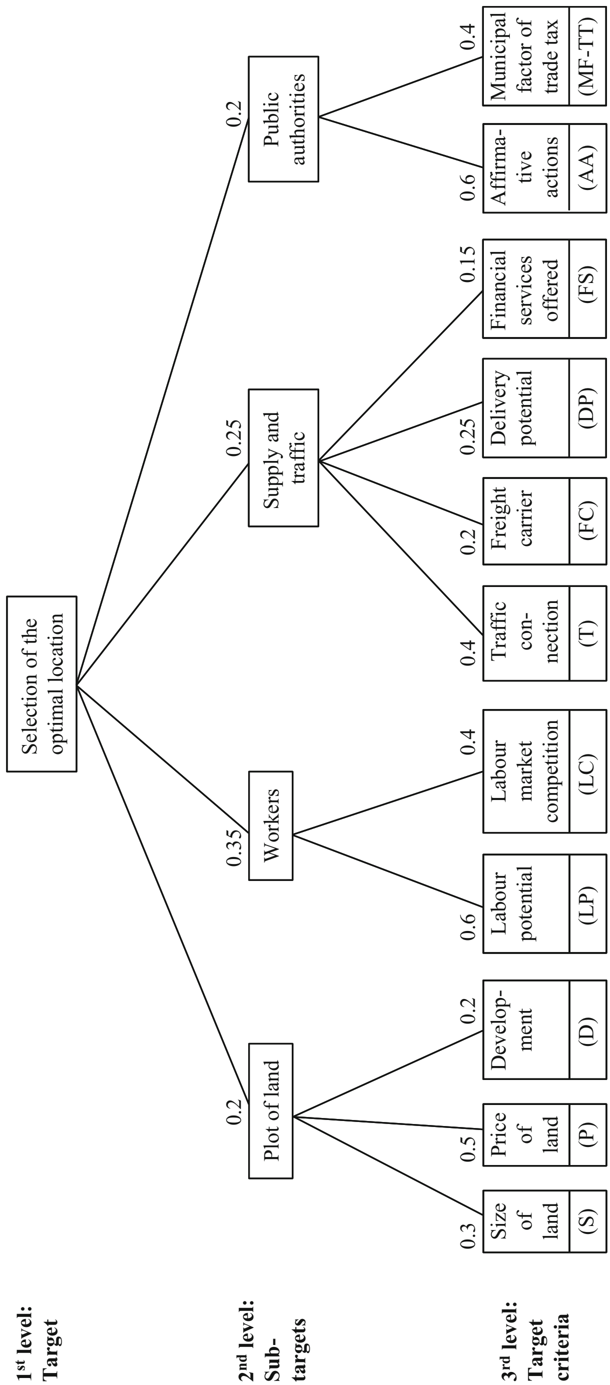

Fig. 6.2 Hierarchy of targets

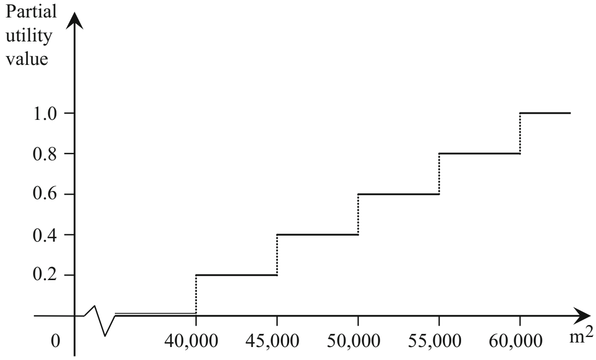

Fig. 6.3 Transformation function for the criterion 'size of land'

This value indicates the contribution of the criterion 'size of land' to the fulfilment of the highest-level target. By multiplying other partial values by their weightings, and adding the resulting weighted partial utility values, (total) utility values U_{Ui} for the three alternatives A_i can be determined:

$$U_{U1} = 0.48 \quad U_{U2} = 0.61 \quad U_{U3} = 0.67$$

It can be concluded, that Alternative A_3 is relatively profitable because it has the highest utility value.

Assessment of the method

Utility value analysis is a comparatively simple method for multi-criteria decision-making. It is easily comprehended and requires only minor computational effort. Also, its application encourages systematic structuring of the underlying problem.

The results of a utility value analysis can be easily interpreted, especially if standardised scales are used for the weightings and partial utility values, as proposed above. Then, a utility value of 1 or 100 is the maximum attainable and the utility value of an alternative can be interpreted as a proportion or percentage of this maximum value. Perhaps for these reasons, utility value analysis is a popular method in practice.

However, data collection can be problematic as target criteria, weightings and partial utility values must be determined and, for the latter two, cardinal measuring scales are required. The target criteria, target weightings and transformation into partial utility values must be based on personal, subjective judgements and estimates, often requiring extensive effort. It might also be questionable whether these criteria, weightings and transformations fully reflect the preferences of the decision-maker, whether target criteria are completely independent, and whether each project characteristic is examined under only one criterion. Effects caused by uncertainty and subjectivity of data, and deviations from assumptions, may by analysed by combining utility value analysis with appropriate procedures for investment methods under uncertainty (especially sensitivity analysis and risk analysis, as described in Chap. 8).

Some other reservations concern the weightings used. These represent overall statements about the relative importance of targets only, i.e. the relationship between two weightings must not be interpreted as a substitution rate for the outcomes of these two targets. Therefore, the utility function is not necessarily additive as this method implies. These aspects are reconsidered in Sect. 6.4, in the context of multi-attribute utility theory.

Another method for multi-criteria decision-making is now described: the analytic hierarchy process.

6.3 Analytic Hierarchy Process

Description of the method

The analytic hierarchy process (AHP) was developed by Saaty (1990a) in the early 1970s to structure and analyse complex decisions. One important application of the method is the support of decision-making involving multiple objectives.

The AHP splits the decision process into partial problems in order to structure and simplify it. A hierarchy containing multiple target levels, such that the main target is broken down into sub-targets. At the lowest level(s) of the hierarchy, the alternatives (here, the investment projects) are included.

Using the AHP, both qualitative and quantitative criteria can be considered. In each case, the relative importance (weightings) of the different criteria, and the relative profitability of alternatives, is determined with respect to each element of the higher level by using pair comparisons. Then, a total value is calculated for sub-targets to determine their relative importance for the whole hierarchy, and, ultimately, to assess the overall profitability of the alternative investment projects.

The AHP is carried out using the following steps:

1. Formation of the hierarchy.
2. Determination of the priorities.
3. Calculation of local priority vectors (weighting factors).
4. Examination of the consistency of the priority assessments.
5. Determination of (global) priorities for the sub-targets and alternatives with respect to the whole hierarchy.

Under certain circumstances some of these steps must be repeated, particularly where priority estimations are inconsistent. Evaluation of the subjective priority assessments for consistency is another characteristic feature of the method.

The initial *formation of the hierarchy* requires segmentation and hierarchical structuring of the decision problem. In this step, an unambiguous demarcation must be drawn between different alternatives and sub-targets. Relevant relationships should exist between the elements of successive levels only. This implies that no (or only minor) relationships exist between the elements of a single level. In addition, the elements of a single level should be comparable and belong to the same category of importance. Finally, assessments should be independent of other assessments at the same and other levels. Usually, it is also assumed that all relevant alternatives and target measures will be considered. The measurability of target criteria has not to be considered in this step of the AHP.

The second step is the *determination of priorities* for all elements of the hierarchy. This involves estimating and quantifying the relative importance of every element in relation to each element of the hierarchy immediately above. This is done using pair comparisons with other elements at the same level. Thus, each element's relative importance for fulfilling target criteria is ranked at each level, as a contribution to the fulfilment of the overall target. For alternative investment projects, this relative importance represents a degree of profitability.

With regard to the pair comparisons, it is assumed that the decision-maker is able to determine values v_{ic} for all pairs i and c from the set A (target criteria or alternatives) on a relational scale. This will indicate, for an element at the next level up, the relative importance of i and c, and must be estimated for all elements of the higher level and for all levels. Reciprocity should apply for the estimated values. That is, the comparative value of i relative to c must equal the reciprocal of the comparison between c and i. Then, for an element at the next level up it applies:

$$v_{ic} = \frac{1}{v_{ci}} \quad \text{for all } i, c \in A \tag{6.5}$$

Moreover, a comparative value v_{ic} should never be infinite. An infinite relative importance would mean the target criteria or alternatives regarded were not comparable, and a renewed target and problem analysis would be required.

For the pair comparisons, the nine-point scale suggested by Saaty (1990a) and illustrated in Fig. 6.4 may be used.

Scale value	Definition	Interpretation
1	Equal importance	Both compared elements have the same importance for the next higher element.
3	Slightly greater importance	Experience and estimation suggest a slightly greater importance of one element in comparison with the other element.
5	Considerably greater importance	Experience and estimation suggest a considerably greater importance of one element in comparison with the other element.
7	Very much greater importance	The very much greater importance of one element in comparison with the other element has been shown clearly in the past.
9	Absolutely dominating	The maximum difference of importance between two elements.
2,4,6,8	Intermediate values	

Fig. 6.4 Saaty's nine-point scale for pair comparisons

This scale has the advantage of converting verbal comparisons into numerical values, so that measurability on a relational scale is possible. A more detailed scale is not regarded as meaningful. Using this scale, comparisons can yield only values between one and nine, or their reciprocals (which apply where an element is of lesser importance than the other element).

The results of pair comparisons related to an element of the next level up may be shown in the form of a $C \times C$ matrix [denoted 'V'] with C elements being compared. The values along the main diagonal of this pair comparison matrix are always 1.

To obtain a pair comparison matrix for C elements being compared, $0.5 \cdot C \cdot (C - 1)$ pair comparisons must be made, since the values across the main diagonal are 1 and reciprocity is assumed. Therefore, the determination of a comparative value v_{ic} is not required if the reciprocal value v_{ci} is known. The required number of pair comparisons increases steeply with an increasing number of elements at a single level; this should be considered when determining a hierarchy.

A perfect (i.e. consistent) execution of all pair comparisons has been made if, for every matrix element v_{ic}, and all elements j different to i and c, the following equation is valid:

$$v_{ic} = v_{ij} \cdot v_{jc} \tag{6.6}$$

If such a consistent execution of the pair comparisons can be assumed, some values can be derived from prior assessments, and the required number of pair comparisons may be reduced to $C - 1$.

In the third step, *local priority vectors (weighting factors)* are calculated for every pair comparison matrix. From the totality of the pair comparisons, the relative importance of the elements (alternatives, target criteria) is determined and summarised in the form of a priority vector. Accordingly, every component of this vector indicates the relative importance of its associated element to the relevant element at the next level up.

The calculation of the priority vectors [denoted 'W'] may be carried out by means of the *eigenvector* method, as explained below. Based on the pair comparison matrix V, and (temporarily) assuming that the estimations are perfect and the relative importance w_c of all the separate elements of c is known, the matrix elements v_{ic} can be calculated as follows:

$$v_{ic} = \frac{w_i}{w_c} \quad \text{for all } i, c \in A. \tag{6.7}$$

Moreover, on account of the reciprocity condition:

$$v_{ic} = \frac{1}{v_{ci}} = \frac{1}{\frac{w_c}{w_i}} \quad \text{for all } i, c \in A \tag{6.8}$$

Or:

$$v_{ic} \cdot \frac{w_c}{w_i} = 1 \quad \text{for all } i, c \in A \tag{6.9}$$

Additionally:

$$\sum_{c=1}^{C} v_{ic} \cdot \frac{w_c}{w_i} = \sum_{c=1}^{C} \frac{w_i}{w_c} \cdot \frac{w_c}{w_i} = C \quad \text{is valid for all } i \in A \tag{6.10}$$

And also:

$$\sum_{c=1}^{C} v_{ic} \cdot w_c = C \cdot w_i \quad \text{for all } i \in A \tag{6.11}$$

Because this relationship applies to all lines i (i = 1, . . . , C) of the pair comparison matrix, the following system of C equations can be formulated:

$$\begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1C} \\ v_{21} & v_{22} & \cdots & v_{2C} \\ \vdots & \vdots & & \vdots \\ v_{C1} & v_{C2} & \cdots & v_{CC} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_C \end{pmatrix} = C \cdot \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_C \end{pmatrix} \tag{6.12a}$$

Or:

$$V \cdot W = C \cdot W \tag{6.12b}$$

This system of equations represents a specific so-called *eigenvalue problem*. Such a mathematical problem is generally defined as follows: for a C × C matrix (B), real numbers L and corresponding vectors X must be found which fulfil the following system of equations:

$$B \cdot X = L \cdot X \tag{6.13}$$

The numbers (L) are called *eigenvalues* of B, and the assigned vectors (X) are called *eigenvectors*. The sum of the eigenvalues in an eigenvalue problem equals the sum formed by the elements of the main diagonal. As for the pair comparison matrices considered here, these elements are each equal to 1 and so the sum of the eigenvalues is the same as the dimension (C) of the matrix. If all assessments are consistent, there is only one positive eigenvalue with the value C.

However, in a multi-criteria decision problem priority estimates are often inconsistent and the weighting vectors are not known. Therefore, in the following discussion the corresponding assumptions must be abandoned. If priority estimates are inconsistent, several eigenvalues and eigenvectors will result. Thus, the maximum eigenvalue L_{max} of the pair comparison matrix and the associated eigenvector must be determined. The latter should be standardised so that the sum of its components is 1, then it can be regarded as the weighting vector W. The calculation of such a weighting vector is meaningful, even with an inconsistent pair comparison matrix, as small inconsistencies will show only a slight effect on the weighting vector.

To determine the maximum eigenvalue and the weighting vector, the following eigenvalue problem must be solved:

$$V \cdot W = L \cdot W \quad \text{or} \quad (V - L \cdot U) \cdot W = 0 \tag{6.14}$$

Here, U represents a $C \times C$ unit matrix. For the eigenvalues L in this problem, the determinant of the matrix $(V - L \cdot U)$ is zero, i.e.:

$$\det\left|V - L \cdot U\right| = 0 \tag{6.15}$$

The maximum value L fulfilling this condition is the maximum eigenvalue L_{max}. By inserting this value in the equation system given above, the eigen- or weighting vector may be calculated. For this vector it applies:

$$(V - L_{max} \cdot U) \cdot W = 0 \tag{6.16}$$

And:

$$\sum_{c=1}^{C} w_c = 1 \tag{6.17}$$

The calculation of the maximum eigenvalue and weighting vector involves substantial computational effort. Therefore, approximations are suggested, e.g. the weighting vector can be approximated from the pair comparison matrix V by using the following arithmetical rule to generate matrix products gradually:

$$V \cdot U;\ V^2 \cdot U;\ V^3 \cdot U; \ldots;\ V^o \cdot U \tag{6.18}$$

Where:

$V = C \times C$ (pair comparison matrix)
$U = C \times 1$ (unity vector)

Matrix dimension	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Average value (RI)	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Fig. 6.5 Average values of indices of consistency

With a sufficiently high value o, the vector $V^o \cdot U$ is a good approximation for the eigenvector.

An *examination of the consistency of priority assessments* takes place in the fourth step of the AHP for all pair comparison matrices. This step is necessary because the consistency of all estimates cannot be taken for granted.

If all the assessments are totally consistent, the maximum eigenvalue is C. Where there are inconsistencies, however, a higher eigenvalue L_{max} arises. This value L_{max} might not be known exactly if, in the third step, the eigenvectors were calculated using an approximation. Then, L_{max} can only be approximated (e.g. using the well known Newton method to determine zero points). The difference between L_{max} and C increases with increasing inconsistency, so it provides an indication of the consistency of the estimates. An index of consistency (IOC) can be formulated using an additional calculation:

$$\mathrm{IOC} = \frac{L_{max} - C}{(C - 1)} \tag{6.19}$$

In assessing consistency, the matrix dimension should also be taken into account, since it influences the extent of typical inconsistencies. To do this, a *value of consistency* (VOC) is calculated. The VOC indicates the relationship between the index of consistency (IOC) and an average value of indices of consistency (RI) derived from reciprocal matrices of the same size, which are produced randomly based on SAATY's nine-point scale:

$$\mathrm{VOC} = \frac{\mathrm{IOC}}{\mathrm{RI}} \tag{6.20}$$

Figure 6.5 shows the average values, calculated by SAATY, in dependance on the matrix dimension.

SAATY suggests 0.1 as a critical limit for the value of consistency. Accordingly, pair comparison matrices with a consistency value VOC ≤ 0.1 are regarded as being sufficiently consistent, while matrices with VOC > 0.1 require an examination and revision of the pair comparisons.

Up to this point in the analysis, each estimated priority has been related to only one element at the next level up the hierarchy. In the fifth step of the AHP, the *determination of target and alternative priorities for the whole hierarchy*, the weighting vectors are aggregated with respect to all elements in the next level up and all other higher levels. This facilitates the assessment of both the global priority (or relative importance) of each target criterion and the ultimate profitability of alternatives.

As a result of the pair comparisons for the second level of the hierarchy, a weighting vector is generated. This indicates the importance of target criteria at this level relative to the overall target, thereby showing both the local and global priority of the targets. The weighting vector is a starting point for the calculation of global priorities for the elements of each subsequent level. It is multiplied by a weighting matrix, which incorporates the weighting vectors of the level subsequent to it. The product is also a weighting vector, whose components represent the global priorities of the elements of the subsequent level. The successive continuation of this step leads to the calculation of the global priority for the alternatives at the lowest level of the hierarchy.

This procedure for determining global priorities for alternatives may also be interpreted as the additive calculation of a utility measure U_{Ai} for each alternative A_i with the formula:

$$U_{Ai} = \sum_{c=1}^{C} w_c \cdot u_{ic} \tag{6.21}$$

The index c refers to the elements of the next level up, which here represents target criteria. The symbol w_c indicates the global priority of these target criteria, and u_{ic} is the relative importance (profitability) of the alternative i concerning the criterion c. Therefore, the global priority (as for the utility value analysis described in the previous subchapter) is calculated as a sum of weighted partial priorities.

The global priorities determined in this step represent weightings of the target criteria. Concerning the alternatives under consideration, they estimate the contribution made to the fulfilment of the overall target. In assessing the relative profitability of (investment) alternatives when the overall target is to be maximised, the following key concept applies:

Key Concept
Relative profitability: an investment project is preferred if its global priority is higher than that of every other project under consideration.

The isolated assessment of absolute profitability by AHP is not possible, as the procedure is based on pair comparisons and, therefore, assessment of one alternative depends on the other alternatives selected. However, the alternative of *not* investing may be included in the procedure. In that case, an estimate of absolute profitability can be made by comparing the global priority of not investing with that of the remaining alternatives.

Example 6.2
The following example draws on Example 6.1.

The first step of the AHP is the formation of the hierarchy. In this example, the target system is drawn from the previous section. Figure 6.6 depicts this target system and contains, in addition to the previous example, the location alternatives A_1, A_2 and A_3 as elements of the lowest hierarchy level.

The second, third and fourth step of the AHP (the determination of priorities, the calculation of local priority vectors or weighting factors, and the examination of the priority assessments for consistency) are now presented together. First, the level of the alternatives is considered. For the criterion 'size of land', the following pair comparison assessments are obtained with regard to the profitability of the alternatives.

$$V = \begin{pmatrix} 1 & 4 & 5 \\ \frac{1}{4} & 1 & 3 \\ \frac{1}{5} & \frac{1}{3} & 1 \end{pmatrix}$$

To determine the exact weighting vector, the maximum eigenvalue L_{max} of the pair comparison matrix V must first be calculated. For all eigenvalues L of the matrix, the determinant of the matrix $(V - L \cdot U)$ represented below is zero.

$$(\text{V-L} \cdot \text{U}) = \begin{pmatrix} 1\text{-L} & 4 & 5 \\ \frac{1}{4} & 1\text{-L} & 3 \\ \frac{1}{5} & \frac{1}{3} & 1\text{-L} \end{pmatrix} \begin{matrix} 1\text{-L} & 4 \\ \frac{1}{4} & 1\text{-L} \\ \frac{1}{5} & \frac{1}{3} \end{matrix}$$

The determinant of a 3×3 matrix can be calculated using the Sarrus rule. For this, the first and second columns of the matrix are repeated after the third column. Then, the products of the elements of (i) the main diagonal of the original matrix and (ii) the components of the diagonals which lie parallel to it, are calculated and summed.

The determinant is this sum, less the products of the elements of the side diagonal and its parallel diagonals. In the example it is:

$$\det\left|V - L \cdot U\right| = (1-L)^3 + 4 \cdot 3 \cdot \frac{1}{5} + 5 \cdot \frac{1}{4} \cdot \frac{1}{3} - \frac{1}{5} \cdot (1-L) \cdot 5 - \frac{1}{3} \cdot 3 \cdot (1-L) - (1-L) \cdot \frac{1}{4} \cdot 4$$

$$\det\left|V - L \cdot U\right| = (1-L)^3 - 3 \cdot (1-L) + 2.8167$$

Based on the necessary condition

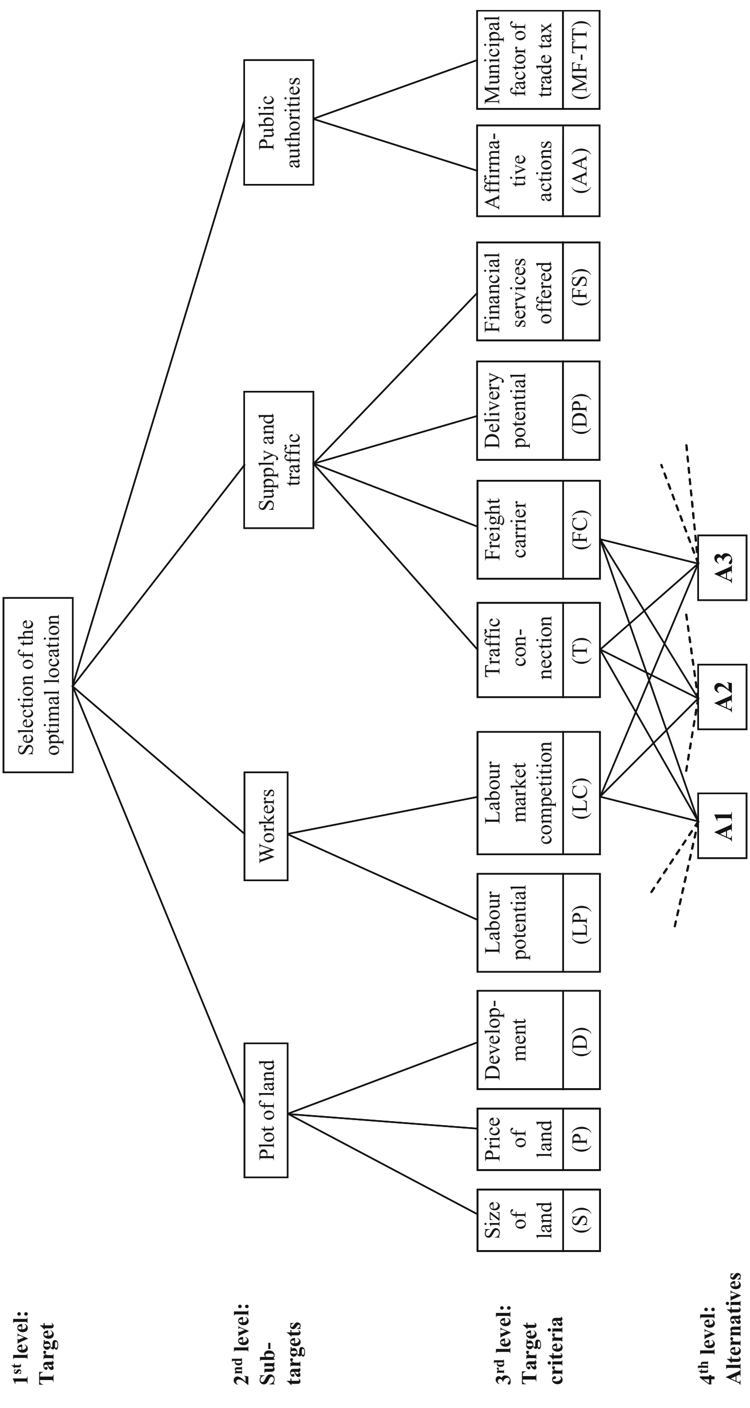

Fig. 6.6 Decision hierarchy

$$\det\left|V - L \cdot U\right| \stackrel{!}{=} 0$$

The maximum eigenvalue (L_{max}) can be determined using a suitable procedure such as the NEWTON procedure:

$$L_{max} = 3.0858$$

The associated eigen- or weighting vector can be determined using the equation system:

$$(V - L_{max} \cdot U) \cdot W = 0$$

Or:

$$(1\text{-}\ 3.0858) \cdot w_1 + 4 \cdot w_2 + 5 \cdot w_3 = 0$$

$$\frac{1}{4} \cdot w_1 + (1\text{-}\ 3.0858) \cdot w_2 + 3 \cdot w_3 = 0$$

$$\frac{1}{5} \cdot w_1 + \frac{1}{3} \cdot w_2 + (1\text{-}\ 3.0858) \cdot w_3 = 0$$

First, the relationship between the weighting factors is derived. Then, the (local) weighting factors are calculated using the condition $w_1 + w_2 + w_3 = 1$. Here, these factors are:

$$w_1 = 0.6738 \quad w_2 = 0.2255 \quad w_3 = 0.1007$$

They indicate the profitability (local priority) of the location alternatives A_1, A_2 and A_3 in regard to the criterion ‘size of land’.

The index of consistency (IOC) arises from the maximum eigenvalue (L_{max}):

$$IOC = \frac{3.0858}{3 - 1} = 0.0429$$

The value of consistency (VOC) amounts to:

$$VOC = \frac{0.0429}{0.58} = 0.0740$$

Because the VOC is below 0.1, the assessment from this pair comparison matrix can be regarded as sufficiently consistent.

In the same manner, pair comparison matrices can also be formulated and evaluated to compare alternatives concerning the other target criteria. Figure 6.7 shows these matrices as well as the maximum eigenvalues, the weighting vectors,

Price of land

	A_1	A_2	A_3
A_1	1	3	1/4
A_2	1/3	1	1/6
A_3	4	6	1

Maximum eigenvalue: 3.0536
Weighting vector: (0.2176; 0.0914; 0.6910)
Value of consistency: 0.0462

Development

	A_1	A_2	A_3
A_1	1	7	3
A_2	1/7	1	1/4
A_3	1/3	4	1

Maximum eigenvalue: 3.0323
Weighting vector: (0.6586; 0.0786; 0.2628)
Value of consistency: 0.0278

Labour potential

	A_1	A_2	A_3
A_1	1	1/4	1/9
A_2	4	1	1/5
A_3	9	5	1

Maximum eigenvalue: 3.0713
Weighting vector: (0.0633; 0.1939; 0.7428)
Value of consistency: 0.0615

Labour market competition

	A_1	A_2	A_3
A_1	1	1/5	1/7
A_2	5	1	1/3
A_3	7	3	1

Maximum eigenvalue: 3.0649
Weighting vector: (0.0719; 0.2790; 0.6491)
Value of consistency: 0.0559

Traffic connection

	A_1	A_2	A_3
A_1	1	1/2	1/2
A_2	2	1	1
A_3	2	1	1

Maximum eigenvalue: 3
Weighting vector: (0.2000; 0.4000; 0.4000)
Value of consistency: 0

Forwarding agents

	A_1	A_2	A_3
A_1	1	8	4
A_2	1/8	1	1/3
A_3	1/4	3	1

Maximum eigenvalue: 3.0183
Weighting vector: (0.7167; 0.0782; 0.2051)
Value of consistency: 0.016

Delivery potential

	A_1	A_2	A_3
A_1	1	8	4
A_2	1/8	1	1/3
A_3	1/4	3	1

Maximum eigenvalue: 3.0093
Weighting vector: (0.7166; 0.0783; 0.2051)
Value of consistency: 0.008

Bank facility offer

	A_1	A_2	A_3
A_1	1	3	3
A_2	1/3	1	1
A_3	1/3	1	1

Maximum eigenvalue: 3
Weighting vector: (0.6000; 0.2000; 0.2000)
Value of consistency: 0

Affirmative actions

	A_1	A_2	A_3
A_1	1	1/7	1/2
A_2	7	1	6
A_3	2	1/6	1

Maximum eigenvalue: 3.0324
Weighting vector: (0.0905; 0.7583; 0.1512)
Value of consistency: 0.0279

Municipal trade tax rate

	A_1	A_2	A_3
A_1	1	6	4
A_2	1/6	1	1/3
A_3	1/4	3	1

Maximum eigenvalue: 3.0536
Weighting vector: (0.6910; 0.0914; 0.2176)
Value of consistency: 0.0462

Fig. 6.7 Pair comparison assessments for the alternatives and their evaluation

Plot of land

	S	P	D
S	1	1/3	4
P	3	1	9
D	1/4	1/9	1

Maximum eigenvalue: 3.0093
Weighting vector: (0.2499; 0.6813; 0.0688)
Value of consistency: 0.008

Workers

	LP	LC
LP	1	5
LC	1/5	1

Maximum eigenvalue: 2
Weighting vector: (0.8333; 0.1667)
Value of consistency: 0

Supply and traffic

	T	FC	DP	FS
T	1	7	5	9
FC	1/7	1	1/4	3
DP	1/5	4	1	5
FS	1/9	1/3	1/5	1

Maximum eigenvalue: 4.2314
Weighting vector: (0.6474; 0.0899; 0.2165; 0.0462)
Value of consistency: 0.0857

Public authorities

	AA	MF-TT
AA	1	3
MF-TT	1/3	1

Maximum eigenvalue: 2
Weighting vector: (0.7500; 0.2500)
Value of consistency: 0

Upper target: Optimal location

	PL	W	SP	PA
PL	1	1/8	1/3	2
W	8	1	4	6
SP	3	1/4	1	5
PA	1/2	1/6	1/5	1

Maximum eigenvalue: 4.1670
Weighting vector: (0.0871; 0.6238; 0.2281; 0.0610)
Value of consistency: 0.0619

Fig. 6.8 Pair comparison assessments for the target criteria and sub-targets, and their evaluation

and the indices and values of consistency determined for each of the different target criteria.

Analogous assessments and calculations are also made for the higher levels of the decision hierarchy. Figure 6.8 shows the results of the pair comparison assessments and their evaluations for the target criteria (with regard to the sub-targets) and the sub-targets (with regard to the overall target).

As the consistency values of all pair comparison matrices of the hierarchy are smaller than 0.1, sufficient consistency may be assumed.

The fifth step of the AHP consists of determining target and alternative priorities for the whole hierarchy, which can be done in a way similar to the corresponding step of utility value analysis. The contribution that the alternative A_1 makes to fulfilling the overall target via the criterion 'size of land' can be calculated by multiplying the local priority of the alternative (0.6738) by the local priority of this criterion (0.2499) and that of the associated sub-target 'plot of land' (0.0871). This contribution amounts to 0.0147. By calculating results for all other criteria in the same way and adding them up, the global priority of the alternative under consideration is determined. Here, the global priorities of the three alternatives are:

$$U_{A1} = 0.172 \quad U_{A2} = 0.244 \quad U_{A3} = 0.584$$

Alternative A_3 shows the highest global priority and is, therefore, relatively the most profitable.

Assessment of the method

The assessment of the AHP method focuses on the effort it requires and its underlying assumptions.

The computational effort is high compared with utility value analysis. With a high number of elements at a single level, approximation procedures must be applied. Also, the data collection is relatively complicated because, for all pairs of elements at a given level, pair comparisons are needed with regard to every element at the next level up. For these pair comparisons it is assumed that a relational scale measurement is possible. Fundamentally, this sets high requirements for measurability, although the use of Saaty's nine-point scale allows the comparison of attributes of lower measurability. However, the nine-point scale has some problems of its own. Unlike a true relational scale, it has no natural neutral point. This can produce errors in the pair comparison judgements. Generally, it is doubtful whether a decision-maker is able to differentiate between statements like 'considerably greater' (scale value 5) and 'very much greater' (scale value 7), and additionally may consider their intermediate values. In addition, the nine-point scale can lead to inconsistencies. If, for example, the scale value 7 is assigned to an element C_1 compared to C_2, as well as to C_2 compared to C_3, the priority of C_1 compared to C_3 would have to be represented by the scale value 49. This, however, is not possible, because the scale value 9 is the upper limit.

A crucial point is the assumption that all relevant alternatives have been considered. Since it makes pair comparisons, the ranking determined using the AHP depends on the choice of alternatives. The consideration of additional alternatives can lead to changes in the ranking, so the ranking is not stable and the result of the AHP is valid only amongst the alternatives included in the comparison. For this set of alternatives, in spite of any inaccuracies caused by approximations, the preferences of the decision-maker are quite accurately represented. The examinations of consistency, which are an essential component of the procedure, support this claim.

The condition that judgements must be independent is restrictive, as it is with utility value analysis. In general, the AHP procedure resembles utility value analysis in its structuring of the decision problem, the utility function, and the interpretation of the criteria weightings. Thus, a combination of both procedures is possible.

Within the framework of the AHP it is also possible to include elements of uncertainty by creating a level in the hierarchy that reflects possible environmental conditions or scenarios. Uncertainties about the preferences expressed in the pair comparison judgements can be examined with the help of sensitivity analysis.

A central criticism of the AHP is that it is not based on an additive utility function. This criticism was also noted for the utility value analysis method described earlier. The weightings merely represent overall statements about the importance of the targets, and an additive utility function cannot be taken for granted.

A method more soundly based on utility theory is described in the following section.

6.4 Multi-attribute Utility Theory

Description of the method

The multi-attribute utility theory (MAUT) was originally developed for the analysis of multi-criteria problems under uncertain conditions, but it can also be applied in more predictable conditions (of certainty), as assumed here. A characteristic feature of the method is that a multi-criteria problem is solved using cardinal utility functions (or ‘preference functions’) based on substitution rates between the attributes.

Using MAUT, cardinal utility functions are assigned to each attribute according to the preferences of the decision-maker (called *individual utility functions* in the following). The total utility (value) U_M then arises as a function of the individual utilities u_c assigned to the outcomes o_c ($c = 1, \ldots, C$) of the target criteria:

$$U_M(o_1, o_2, \ldots, o_C) = f(u_1(o_1), u_2(o_2), \ldots, u_C(o_C)) \tag{6.22}$$

Because each separate criterion is analysed, specific value assessments can be made for them, and exchange relationships between them can be explicitly considered. It is assumed that the criteria are interchangeable, i.e. all changes to the fulfilment of a target criterion can be balanced by changes in other target criteria. This requires that the outcomes of the different alternatives lie close to each other, a prerequisite that can only be fully achieved with an unlimited number of alternatives. Furthermore, it is assumed that the substitution rate (i.e. the relationship between the utility changes that lead to a utility balance between two attributes) can be quantified.

The determination of total utilities requires criteria whose fulfilment is clearly independent of the fulfilment of the other criteria. Depending on the type of independence, different total utility functions may be used. For the multi-criteria decisions under conditions of certainty discussed here, an additive total utility function of the following form may be applied:

$$U_M = \sum_{c=1}^{C} w_c \cdot u_c \tag{6.23}$$

Where:

$w_c =$ weighting factor for target criterion c

In addition to interchangeability of the attributes, this approach assumes that:

- For the alternatives, a weak order of priority can be formed.
- The decision-maker regards the attributes as mutually preference independent.

Mutual preferential independence can be said to occur if every subset of the set of all criteria has a preference assessment for its criteria outcomes that is independent of the outcomes of the remaining criteria in the target system.

In the following discussion, it is assumed that the conditions specified above are fulfilled, and only an additive utility function as shown above is analysed. MAUT resembles utility value analysis in this regard except that, in MAUT, the utility theory assumptions indicated above are taken into consideration. Moreover, both the individual utility functions u_c and the weighting factors w_c are determined using attribute comparisons in a consistent format.

A multi-criteria problem under certainty is solved with the MAUT using the following steps:

1. Choice of the attributes or criteria.
2. Examination of the independence of the criteria.
3. Determination of an individual utility function for each attribute.
4. Determination of a weighting factor for each criterion.
5. Calculation of the total utility for each alternative.

In the first step, *the choice of criteria*, the overall target is split hierarchically into sub-targets. The lowest target level contains the attributes that measure the achievement of objectives (targets) by the alternatives. These may be quantitative or qualitative. In the case of qualitative criteria, an appropriate measurement scale must be chosen, depending on the attributes (in contrast to AHP no generally applicable scale is suggested).

The *examination of independence* follows in the second step, as this independence is a prerequisite for the meaningful aggregation of the individual utilities assigned to single criteria to find an alternative's total utility. Using an additive utility function, mutual preferential independence is assumed; this must be proven for the present system of attributes and their outcomes.

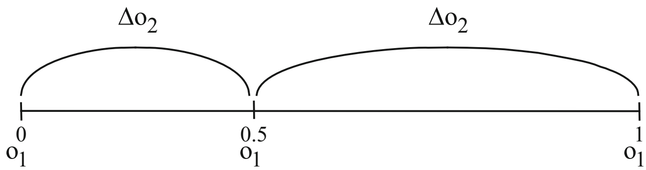

Fig. 6.9 Utility measurement by attribute comparison

In the third step, *individual utility functions u_c for the separate attributes c are formulated* to assign cardinal utility measures to the attributes. This requires knowledge of the relevant possible outcomes for the attributes. The individual utility functions are standardised so that their values u_c are restricted to the interval [0;1], for example by assigning the individual utility value of zero $\left(u_c\left(o_c^0\right) = 0\right)$ to the worst outcome o_c^0 for criterion c and the utility value of one $\left(u_c\left(o_c^1\right) = 1\right)$ to the best outcome o_c^1.

The individual utility functions may take different forms—they can be linear, concave or convex. Their course can be determined using a sequence of queries in accordance with the so-called mid-value splitting technique. Using this approach, an attribute C_1 with o_1^0 and o_1^1 is assigned a 'midvalue' $o_1^{0.5}$ that represents the outcome for which the increase in utility achieved by the change from o_1^0 to $o_1^{0.5}$ equals the utility increase resulting from the change from $o_1^{0.5}$ to o_1^1. Then, an individual utility of 0.5 is assigned to this outcome $o_1^{0.5}$, e.g. $u_1\left(o_1^{0.5}\right) = 0.5$. To determine $o_1^{0.5}$, a second attribute C_2 is used in successive querying so that, starting from a level of $o_2^{'}$, the change Δo_2 that balances the step from o_1^0 to $o_1^{0.5}$ with the step from $o_1^{0.5}$ to o_1^1 can be identified.

Accordingly, the following indifference judgments must apply:

$$\left(o_1^0, o_2^{'}\right) \sim \left(o_1^{0.5}, o_2^{'} - \Delta o_2\right) \tag{6.24}$$

$$\left(o_1^{0.5}, o_2^{'}\right) \sim \left(o_1^1, o_2^{'} - \Delta o_2\right) \tag{6.25}$$

This procedure is illustrated in Fig. 6.9.

Additional querying for the partial intervals $[o_1^0; o_1^{0.5}]$ and $[o_1^{0.5}; o_1^1]$ will determine the mid-values ($o_1^{0.25}$ and $o_1^{0.75}$). These values often allow a sufficient approximation of the individual utility function u_1, especially if their type is known (e.g. a linear function). However, additional values for the individual utility function u_1 may be calculated in the same way. An example showing the determination of an individual utility function is given in Fig. 6.10.

Individual utility functions ($u_2, \ldots, u_C$) can be determined for the remaining criteria in the same way. In each case, a consistency examination should be carried out—e.g. the value $o_1^{0.5}$ may be verified by re-calculating it as the mid-value of the

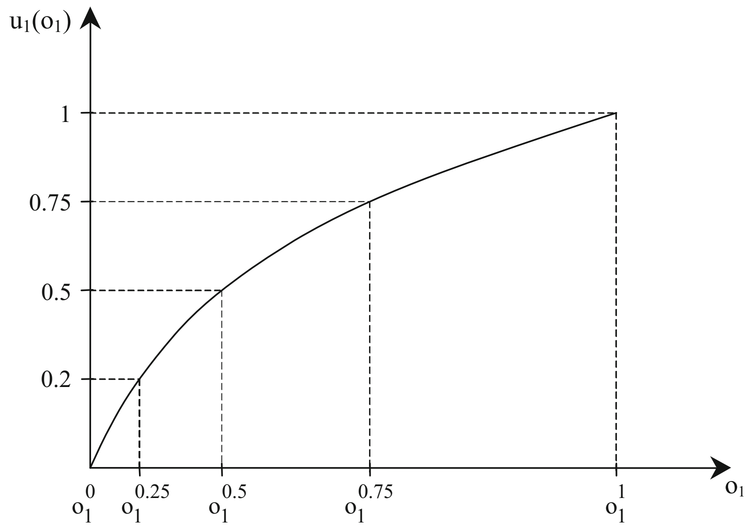

Fig. 6.10 Determination of an individual utility function

interval $[o_1^{0.25}; o_1^{0.75}]$, and repeated determination of an individual utility function can be performed using different other attributes.

If it is sufficient to know the individual utility values only for specific attributes of the relevant alternatives, it is not necessary to ascertain the full individual utility functions.

Determination of the weighting factors for the criteria, the fourth step, is achieved using the relationship between the weighting factors (also known as scale factors) of two attributes, which can be interpreted as substitution rates and derived from the indifference judgments made in the third step. To help explain this, the case of two target measures (C = 2) is considered first. Then the linear and additive total utility function is:

$$U_M = w_1 \cdot u_1 + w_2 \cdot u_2 \tag{6.26}$$

For a given utility level $\overline{U}_M$ the following equation applies:

$$\overline{U}_M = w_1 \cdot u_1 + w_2 \cdot u_2 \tag{6.27}$$

This relationship can be represented in a u_1/u_2 diagram in the form of a straight line representing the utility combinations u_1 and u_2 that lead to the same total utility $\overline{U}_M$. This may be interpreted as an indifference curve and, together with other indifference curves embodying different levels of the total utility, it is presented in Fig. 6.11.

The slope of the straight lines $\frac{du_2}{du_1}$ equals the substitution rate between u_1 and u_2. It specifies how many units u_2 must be reduced by in order to gain the same utility with one more unit of u_1. The slope, or substitution rate, can be derived from the equation for the indifference curve as follows:

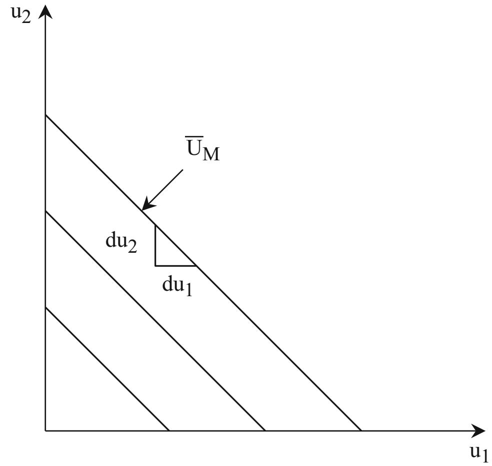

Fig. 6.11 Indifference curves

$$\frac{du_2}{du_1} = -\frac{w_1}{w_2} \tag{6.28}$$

The substitution rate equals the negative reciprocal quotient of the weighting factors of two attributes. Therefore, the relationship between two attributes is also characterised by:

$$\left|\Delta u_2\right| \cdot w_2 = \left|\Delta u_1\right| \cdot w_1 \tag{6.29}$$

The value changes Δu_1 and Δu_2 can be derived from the indifference estimates obtained in order to determine their mid-values:

$$\left(o_1^0, o_2'\right) \sim \left(o_1^{0.5}, o_2' - \Delta o_2\right) \tag{6.30}$$

$$\left(o_1^{0.5}, o_2'\right) \sim \left(o_1^1, o_2' - \Delta o_2\right) \tag{6.31}$$

The difference Δu_1 between $u_1(o_1^0)$ and $u_1(o_1^{0.5})$ is known as $\Delta u_1 = 0.5$. The value difference Δu_2 between $u_2(o_2')$ and $u_2(o_2') - \Delta o_2$ can be derived from the individual utility function $u_2(o_2)$. Then Δu_1 and Δu_2 can be inserted in the equation given above to determine the numeric relationship between the weighting factors w_1 and w_2:

$$w_1 = \frac{|\Delta u_2|}{|\Delta u_1|} \cdot w_2 \tag{6.32}$$

Since the mutual preferences are independent, the procedure presented here can be used where there are several target measures. Relationships between w_1 and the remaining weighting factors ($w_3, \ldots, w_C$) can be determined in the same way. Then, using these relationships and the condition (6.17),

$$\sum_{c=1}^{C} w_c = 1$$

A system of equations can be formulated and used to determine the weighting factors w_c.

In the fifth step, *calculation of the total utilities of the alternatives*, individual utility functions are used to convert the attributes of the alternatives into individual utilities. Then, taking account of the weighting factors, they are summed to obtain a total utility (using Formula (6.23)). The maximum achievable total utility is 1. The following conditions for profitability then apply:

Key Concept
Absolute profitability is achieved if an investment project's total utility is higher than a given target value.
Relative profitability: an investment project is preferred if its total utility is higher than those of every other project under consideration.

Example 6.3
The example considered in the previous sections (a location decision) is used again here, assuming that the prerequisites for an additive utility function apply.

The choice of attributes, the first step, draws on the target criteria list given above. Using MAUT, the lowest level criteria serve as indicators for analysing the achievement of objectives. In this example, for reasons of complexity only 4 of the 11 lowest level criteria will be considered: one from each criteria group. Accordingly, it is assumed that only the attributes 'size of land (S)' (in m^2), 'labour potential (LP)', 'freight carrier (FC)' and 'municipal factor of trade tax (MF)' (in %) are relevant. The 'labour potential' is measured on the basis of the available workers, and the criterion 'freight carrier' on the number of carriers resident in the locality.

For the location alternatives A_1, A_2, A_3 the following data are available:

Table 6.2 Data for the location alternatives A_1, A_2 and A_3

Alternative	S	LP	FC	MF
A_1	60,000	800	15	350
A_2	42,500	1,100	12	250
A_3	35,000	1,300	25	450

In the second step, the criteria are examined for independence and in this instance it is assumed that they are mutually preference independent, thus an additive utility function may be applied.

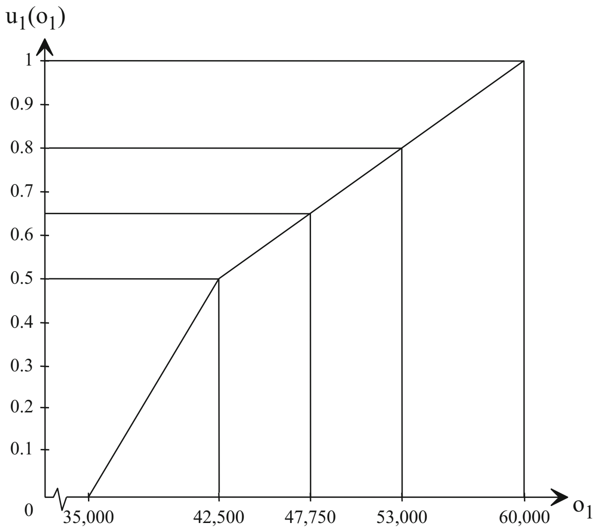

Fig. 6.12 Individual utility function for the attribute 'size of land'

Steps 3 and 4 (determinations of the individual utility functions u_c and the weighting factors w_c) can, because of the relationships described above, be presented together.

The minimum and maximum outcomes for the attributes can be read from the data given. Their individual utility values are fixed at 0 and 1: thus, the lowest outcome should score an individual utility of 1 if the aim is to minimise the attribute (e.g. tax) or 0 if the aim is to maximise the attribute (e.g. labour potential):

$$\begin{array}{llll} u_1\,(35{,}000) & = 0 & \qquad u_1\,(60{,}000) & = 1 \\ u_2\,(800) & = 0 & \qquad u_2\,(1{,}300) & = 1 \\ u_3\,(12) & = 0 & \qquad u_3\,(25) & = 1 \\ u_4\,(450) & = 0 & \qquad u_4\,(250) & = 1 \end{array}$$

Furthermore, it is assumed that the individual utility function has already been determined for attribute C_1 (size of land) with the help of the mid-value splitting technique and corresponding indifference estimates. Figure 6.12 shows this individual utility function: increasing the size of land from 35,000 m^2 initially results in a relatively high increase in the utility, but after reaching 42,500 m^2 the utility increases diminish.

Next, the individual utility function for the second attribute is determined using mid-values. First, the outcome $o_2^{0.5}$ is identified, which leads to an individual utility of 0.5. Using the first criterion and starting at $o_2^{'} = 42{,}500$, the change Δo_1 for the step from o_2^0 to the required $o_2^{0.5}$ and from this to o_2^1 is estimated. In the example this would be $\Delta o_1 = 7{,}500$, as the following indifference assessments demonstrate:

$$\left(o_1^{'}, o_2^{0}\right) \sim \left(o_1^{'} - \Delta o_1, o_2^{0.5}\right) \Rightarrow (42,500; 800) \sim (35,000; 1,100)$$

$$\left(o_1^{'}, o_2^{0.5}\right) \sim \left(o_1^{'} - \Delta o_1, o_2^{1}\right) \Rightarrow (42,500; 1,100) \sim (35,000; 1,300)$$

Therefore, $o_2^{0.5}$ is found at 1,100 and the utility value $u_2(1{,}100)$ is 0.5. As all individual utility values for all outcomes of the three alternatives are now known, a further analysis of the individual utility function u_2 is not required.

From the indifference estimates, the relationship between the weighting factors w_1 and w_2 may also be derived. For the first criterion, the difference between the individual utilities (resulting from the increase from 35,000 to 42,500) is $\Delta u_1 = 0.5$, as shown in Fig. 6.12.

Since the variation of Δu_2 (the second criterion's individual utility that is compensated by this difference) is also 0.5, the following applies:

$$\begin{aligned} |\Delta u_2| \cdot w_2 &= |\Delta u_1| \cdot w_1 \\ 0.5 \cdot w_2 &= 0.5 \cdot w_1 \\ w_2 &= w_1 \end{aligned}$$

Therefore, the first and second criteria are weighted identically.

To determine the individual utility function u_3 and its weighting factor w_3 the first criterion is used again. The following indifference estimates may be considered for the outcomes for the first and third criteria:

$$(53,000;\ 12) \sim (42,500;\ 17)$$

$$(53,000;\ 17) \sim (42,500;\ 25)$$

This means:

$$u_3(17) = 0.5$$

The relationship between the weighting factors w_1 and w_3 can now be determined as:

$$\begin{aligned} |\Delta u_3| \cdot w_3 &= |\Delta u_1| \cdot w_1 \\ 0.5 \cdot w_3 &= 0.3 \cdot w_1 \\ w_3 &= 0.6 \cdot w_1 \end{aligned}$$

To determine the individual utility from $o_3 = 15$, which is necessary to assess the first alternative, other indifference assessments must be included:

$$(47,750;\ 12) \sim (42,500;\ 15)$$

$$(47,750;\ 15) \sim (42,500;\ 17)$$

Therefore, that individual utility is:

$$u_3(15) = 0.25$$

Determination of the individual utility function u_4 and the relationship between w_4 and w_1 will not be presented here. It is simply assumed that the relevant value $o_4 = 350$ results in an individual utility $u_4(350)$ of 0.5. The relationship between w_4 and w_1 is: $w_4 = 0.4 \cdot w_1$.

All relevant individual utility functions or values are now known. With the help of the standardisation condition:

$$w_1 + w_2 + w_3 + w_4 = 1$$

The weighting factors may be determined as well:

$$w_1 = \frac{1}{3} \quad w_2 = \frac{1}{3} \quad w_3 = \frac{1}{5} \quad w_4 = \frac{2}{15}$$

In the fifth step, the total utility U_M of the alternatives is calculated. The following additive total utility function is used for this:

$$U_M = \frac{1}{3} \cdot u_1(o_1) + \frac{1}{3} \cdot u_2(o_2) + \frac{1}{5} \cdot u_3(o_3) + \frac{2}{15} \cdot u_4(o_4)$$

Finally, by inserting the relevant outcomes for the three alternatives, the following total utilities can be determined. Their comparison shows, that location alternative A_3 is relatively profitable:

Table 6.3 Total utilities of the alternatives A_1, A_2 and A_3

Alternatives	A_1	A_2	A_3
Total utilities	9/20	7/15	8/15

Assessment of the method

The MAUT approach is quite similar to utility value analysis and, where an additive total utility function is assumed, it also corresponds in this regard with the AHP. However, MAUT has stronger utility theory foundations and a framework in which individual utility functions and criterion weightings can be determined in a consistent way, taking into account the conditions that must be considered for particular total utility functions. For an additive total utility function these are, as mentioned, the existence of a weak order, interchangeability, and mutual preference independence. Interchangeability of criteria, as shown, suggests that the alternatives are similar. However, this condition can be fully achieved only in the unrealistic case of an unlimited number of alternatives. Furthermore, it is assumed that the relationship between the utility changes leading to a utility balance between two attributes can be quantified.

These are relatively strict conditions, which will not be fulfilled in all decision situations and tend to impose high demands on the decision-maker. Since, in reality, only a limited number of alternatives will be available, interchangeability is usually not possible and the decision-maker may be forced to include hypothetical alternatives in order to find substitution rates.

The requirement for mutual preferential independence (with an additive total utility function) restricts the range of applications for MAUT in comparison to utility value analysis and AHP, as these require less strict independence conditions. Moreover, it is difficult to examine the mutual independence of preferences and this requires considerable effort. In fact, the MAUT can also be applied on a utility theory basis assuming a weaker independence condition, but then other forms of total utility function must be used.

The data collection requirements for MAUT present another particularly serious problem. Individual utility functions and weighting factors must be determined with the help of indifference estimates, and the effort involved is a considerable disadvantage of the procedure.

The relationships between the weightings of the attributes may be interpreted as substitution rates between the scale units of the criteria. However, this assumes the use of an interval scale to measure the individual utility value for all attributes. For qualitative attributes in particular, it is difficult to find a suitable scale. Additionally, it may be difficult to decide which outcomes the individual utility values of 1 and 0 should be assigned to. Apart from the worst and the best outcomes from the set of alternatives (as in the example), other outcomes may also be used for this standardisation, e.g. the best or worst conceivable outcomes, or maximum or minimum outcomes.

A consistency examination of the estimations may also be carried out within MAUT, as shown. The effects of possibly incorrect assessments can be examined with the help of sensitivity analysis. As well, uncertain environmental conditions in the future can be explicitly considered with MAUT, as mentioned, since the procedure was originally developed for uncertain conditions.

Compared with AHP, one advantage of MAUT is that it always leads to a stable ranking of the alternatives.

The MAUT approach represents a utility theory-based method for multi-criteria decision-making. Its theoretical foundation is an advantage over utility value analysis and AHP, but strict conditions and high data collection requirements limit the realisation of that advantage. The method described in the next section, PROMETHEE, requires far less strict conditions.

6.5 PROMETHEE

Description of the method

PROMETHEE (preference ranking organisation method for enrichment evaluations) is one of the so-called *outranking methods* (also called prevalence methods), along with ELECTRE (elimination et choix traduisant la realité) and

ORESTE (organisation, rangement et synthèse de données relationelles). These procedures differ from the classic methods of multi-criteria decision-making in their basic assumptions about the decision-maker. In contrast to the classic methods, the outranking procedures' starting point is that a decision-maker does not have access to the information needed to form at least a weak order and make an optimum choice. The classic procedure assumptions that (a) a complete compensation or balancing between the attributes is possible and (b) an unambiguous estimate of the indifference or preference between two alternatives can be made are often unrealistic in multi-criteria problems. The assumptions underpinning outranking procedures differ on these points. Using PROMETHEE, graded preference estimations are permitted when assessing two alternatives, as well as strict preference and indifference judgements. Critical values, which indicate the difference in a criterion outcome at which a preference emerges, can also be included. Incomparability of alternatives caused by an inability to compensate may be considered as well, so often neither strong nor weak orders can be formed and no full ranking can be determined. However, the determination of an optimum alternative is not the purpose of the outranking procedures. Rather, they aim to support problem solving and contribute to identifying suitable alternatives.

To describe differentiated preference situations, the outranking procedures use a graduated relationship, the so-called outranking relationship (or prevalence relationship). This indicates the likelihood π_{ij}, that the decision-maker estimates alternative i to be at least as good as alternative j. It must be formulated for every possible alternative pair. Pair comparisons between the alternatives are, as with the AHP, an essential feature of outranking procedures, so this approach is primarily suited to the assessment of relative profitability.

An evaluation of outranking relationships should help to solve any problem that is defined as the selection, arranging or ordering of alternatives. Since PROMETHEE has been primarily developed to determine rank orders, it aims to do so in the form of so-called *pre-orders,* for some or all alternatives. A pre-order is a specific order to which transitivity must not apply, and via which non-comparable factors can be incorporated.

Another fundamental characteristic of PROMETHEE is the use of *generalised criteria*. These consist of a typical series of so-called preference functions, which indicate the intensity of the preference for one alternative against another regarding a particular criterion. On the basis of the preferences determined using these functions, an outranking relationship and an outranking graph are produced.

This can be illustrated for a multi-criteria problem of the form:

$$\text{Max}\ \{f_1(A_i),\ f_2(A_i),\ \ldots,\ f_c(A_i),\ \ldots,\ f_C(A_i)\} \quad \text{with}:\ A_i \in A \tag{6.33}$$

$A = \{A_1,\ A_2,\ \ldots,\ A_i,\ \ldots,\ A_I\}$ represents the set of all alternatives and $f_c(A_i)$ represents A in real numbers in each case. Accordingly, $f_c(A_i)$ indicates the cardinally measured outcome of an alternative A_i with regard to the criterion c. This formulation of the multi-criteria problem assumes that all target measures are to be maximised.

Therefore, criteria that are minimised must be transformed into a maximisation task (e.g. by multiplying by −1).

In PROMETHEE, a pair-wise comparison of all alternatives takes place for every criterion c. Thus, for an alternative $A_i \in A$, a preference against the alternative $A_j \in A$ can be determined by calculating the difference d_c between the outcomes $f_c(A_i)$ and $f_c(A_j)$ and converting this difference into a preference value using the preference function. For the preference function $p_c(A_i, A_j)$:

$$p_c(A_i, A_j) = p_c(f_c(A_i) - f_c(A_j)) = p_c(d_c(A_i, A_j)) \tag{6.34}$$

The preference value $p_c(A_i, A_j)$ indicates the level of dominance of alternative A_i over alternative A_j in regard to criterion c, and may have values between 0 and 1. For $d_c \leq 0$, i.e. indifference or negative preference of A_i over A_j, a value of 0 is assigned to $p_c(A_i, A_j)$. For a strict preference for A_i over A_j, $p_c(A_i, A_j)$ amounts to 1. With PROMETHEE it is possible to consider preference estimations (preference intensities) lying anywhere between indifference and strict preference. These are represented by preference values between 0 and 1. The higher the preference value, the more intense the preference: the increased intensity being the result of increasing differences d. The flexible means of assigning preference values p_c to value differences using preference functions is another characteristic of PROMETHEE. Critical values can be included, as mentioned, for indifference and/or preference.

For most practical applications, six typical kinds of preference functions (the 'generalised criteria' mentioned above) are sufficient. Figure 6.13 shows these generalised criteria.

The *usual criterion* represents the classic case in decision theory, with a strict division between indifference ($p(d) = 0$, if $d \leq 0$ or $f(A_i) \leq (A_j)$) and strict preference ($p(d) = 1$, if $d > 0$ or $f(A_i) > f(A_j)$). The intensity, or degree, of preference is not considered.

The *quasi-criterion* differs from the usual criterion in that it includes a critical value (q) for indifference. This critical value equals the highest value of d at which indifference still exists. Small differences are then irrelevant. Strict preference, with $p(d) = 1$, applies to all values of $d > q$.

For a *criterion with linear preference,* a critical value for preference (s) is included, which represents the smallest value of the difference for which a strict preference exists. In the range between 0 and this critical value, preferences rise linearly, i.e. there is a proportionate relationship between differences and preference intensities.

For a *step-criterion,* critical values are considered for both indifference (q) and preference (s). Differences of $d \leq q$ lead to indifference, differences above s indicate strict preference. In the range between q and s (including s), a weak preference with $p(d) = 0.5$ can be assumed. Alternatively, other preference values between 0 and 1, and more than two gradations, can be included as well.

A *criterion with a linear preference and an indifference area* also uses two critical values. This criterion represents a combination of the two previous criteria.

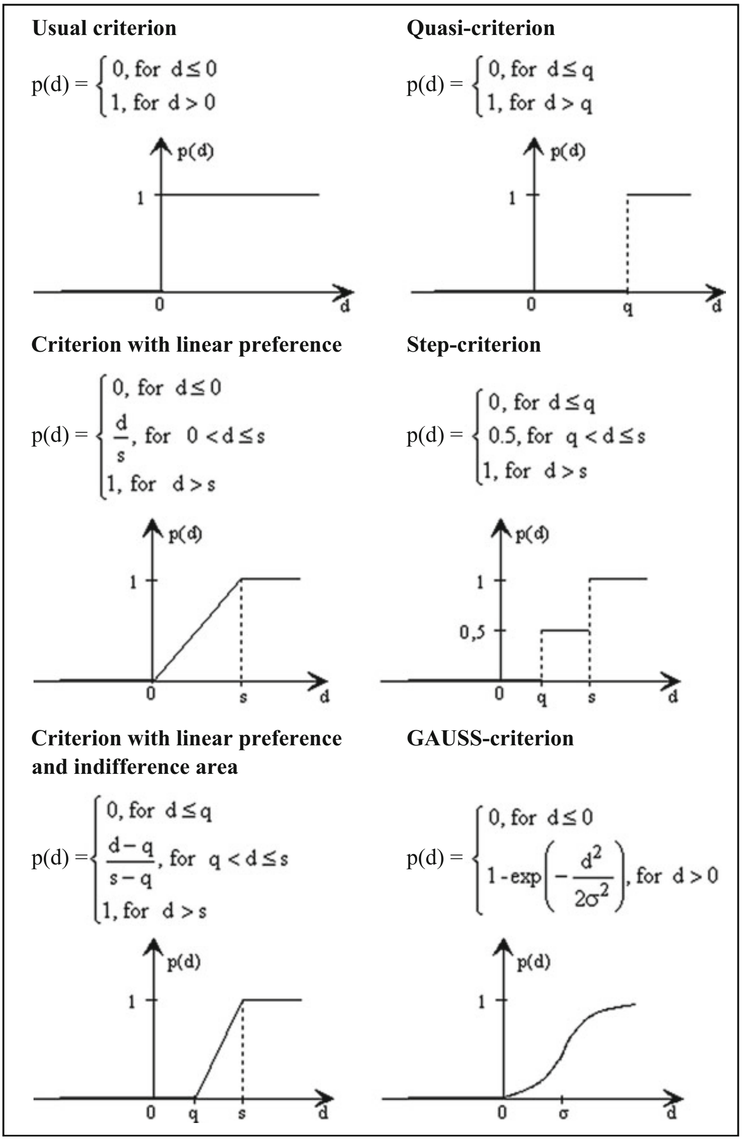

Fig. 6.13 Generalised criteria with PROMETHEE

It differs from the step-criterion in that a linear preference function is assumed to exist between the critical values.

Using the *Gauss-criterion*, preference is strictly increasing with the difference d, beginning at $d = 0$. Even for high values of d, a strict preference ($p(d) = 1$) is not fully reached. With this criterion, a parameter σ, which determines the turning point of the preference function, must be identified. The Gaussian distribution is included in the generalised criteria since the preference function based on it is quite stable, i.e. small changes in σ result in only slight changes in preference.

PROMETHEE is carried out using the following steps:

1. Determination of the target criteria and data collection.
2. Selection of generalised criteria and definition of preference functions.
3. Determination of an outranking relationship.
4. Evaluation of the outranking relationship.

The first step, *definition of the target criteria*, requires a detailed analysis of the target system, as in all multi-criteria methods. After defining the targets, the possible outcomes for the available alternatives must be assigned cardinal numbers with respect to each criterion.

The second step, *selection of generalised criteria and definition of preference functions* is performed for every criterion and includes, if necessary, the specification of the generalised criteria by determining the associated parameters (s, q, σ). This second step implies the assumption that the preference functions accurately reflect the preferences of the decision-maker in regard to the outcomes, or more precisely outcome differences, of each criterion.

To *determine an outranking relationship* (the third step), value differences must be calculated for all criteria and alternative pairs. Then, using the preference functions, the preference values are derived from the value differences. For every pair of alternatives (A_i, A_j) and every criterion *two* preference values are determined: a preference value indicating the preference for A_i against A_j; as well as one indicating the preference for A_j against A_i. One of the two values is always zero.

The relative importance of the criteria must also be fixed in this step. This is achieved using cardinally measured weighting factors w_c for all criteria c. As with other methods, the weighting factors must fulfil the Condition (6.17):

$$\sum_{c=1}^{C} w_c = 1$$

Then, for the preference of every alternative A_i against another A_j, an outranking relationship can be determined using the weighted means of all criteria-specific preference values $p_c(A_i, A_j)$.

$$\pi(A_i, A_j) = \sum_{c=1}^{C} w_c \cdot p_c(A_i, A_j) \tag{6.35}$$

The values of the outranking relationship can be interpreted as preference indications that reflect the level of preference for A_i against A_j. After including all criteria, they can be interpreted similarly to the values $p_c(A_i, A_j)$ for a criterion c, that is $\pi = 0$ indicates indifference and $\pi = 1$ indicates strict preference. Between 0 and 1, the degree of preference rises with increasing values of π. For each alternative pair A_i, A_j, two values of the outranking relationship are determined (as for a single criterion).

The outranking relationships identified may be summarised as a square matrix. The elements of the main diagonal of this matrix represent the values $\pi(A_i, A_i)$ at zero. Alternatively, the outranking relationship may be illustrated in the form of a graph. The nodes of the graph correspond to the alternatives, and the arrows correspond to the values of the outranking relationship between alternatives. Because, for two alternatives A_i and A_j, two outranking values are calculated, the graph contains two arrows between two nodes.

The fourth step of PROMETHEE is the *evaluation of the outranking relationship*. Based on the outranking graph, two flow measures can be determined for every node and every alternative. The outflow measure of a node (F^+) is the sum of the assessments of all arrows (values of the outranking relationship) starting at the node:

$$F_i^+ = \sum_{j=1}^{I} \pi\left(A_i, A_j\right), \quad \text{for all } i,\ i = 1, \ldots, I \tag{6.36}$$

It indicates the level of preference for one alternative against all others. The greater it is, the more preferable that alternative.

The inflow measure of a node (F^-) is determined in the same way, as the sum of the estimates of all arrows flowing into the node:

$$F_i^- = \sum_{j=1}^{I} \pi\left(A_j, A_i\right), \quad \text{for all } i,\ i = 1, \ldots, I \tag{6.37}$$

The inflow measure shows the extent to which an alternative is dominated by other alternatives. The higher it is, the greater the dominance by other alternatives.

Now, to set up a rank order of alternatives, each alternative is evaluated on the basis of the inflow and outflow measures. A suitable pre-order can be formulated to assess relative profitability. As a basis for this, an entire (pre)order is derived from both measures.

The pre-order resulting from the outflow measures, characterised by the symbols P^+ (preference) and I^+ (indifference), contains the following statements:

A_i is preferred to A_j ($A_iP^+A_j$), if $F^+(A_i) > F^+(A_j)$
A_i is indifferent to A_j ($A_iI^+A_j$), if $F^+(A_i) = F^+(A_j)$

Accordingly, a pre-order based on the inflow measures (with the symbols P^- and I^-) may be formed:

A_i is preferred to A_j ($A_iP^-A_j$), if $F^-(A_i) < F^-(A_j)$
A_i is indifferent to A_j ($A_iI^-A_j$), if $F^-(A_i) = F^-(A_j)$

After simultaneous inclusion of outflow and inflow measures, a pre-order of the following form can be produced to assess profitability (with the symbols P, I and U):

A_i is preferred to A_j (A_iPA_j),
if ($A_iP^+A_j$ and $A_iP^-A_j$)
or ($A_iP^+A_j$ and $A_iI^-A_j$)
or ($A_iI^+A_j$ and $A_iP^-A_j$)
A_i is indifferent to A_j (A_iIA_j),
if $A_iI^+A_j$ and $A_iI^-A_j$
A_i and A_j cannot be compared (A_iUA_j),
if not A_iPA_j *and* not A_iIA_j

If the relationship A_iPA_j is valid, the alternative A_i is clearly preferable to A_j—i.e. 'A_i outranks A_j'. For A_iIA_j the decision-maker is indifferent between these options, and for A_iUA_j the alternatives are not comparable. A pre-order derived in this way is always a so-called *partial pre-order* when the alternatives (U) are not comparable. This is another difference between PROMETHEE and the methods discussed previously.

Example 6.4

Now the MAUT example is reconsidered. As in all outranking procedures, PROMETHEE is particularly suitable for decisions involving many alternatives. Therefore, the example is extended by a further two alternatives (A_4, A_5).

In the first step of PROMETHEE, the determination of target criteria and data collection, the following data are recorded for four target criteria (size of land (S), labour potential (LP), freight carrier (FC) and municipal factor of trade tax (MF)):

Table 6.4 Data for the five alternatives

	Target criteria			
Alternative	S	LP	FC	MF
A_1	60,000	800	15	350
A_2	42,500	1,100	12	250
A_3	35,000	1,300	25	450
A_4	35,000	900	14	300
A_5	40,000	1,000	17	400

The second step involves selecting generalised criteria and defining preference functions for the four target criteria. Figure 6.14 contains the relevant generalised criteria and preference functions. It is assumed that they reflect the preferences of the decision-maker.

In the third step, the outranking relationship is determined, with the weightings w_c being assigned first. In this example they are:

Criterion	Generalised criterion and preference function
Size of land (Criterion 1)	Quasi-criterion with parameter q = 5,000 $p_1(d_1) = \begin{cases} 0, \text{ for } d_1 \leq 5{,}000 \\ 1, \text{ for } d_1 > 5{,}000 \end{cases}$
Labour potential (Criterion 2)	Step-criterion with parameters q = 50 and s = 200 $p_2(d_2) = \begin{cases} 0, \text{ for } d_2 \leq 50 \\ 0.5, \text{ for } 50 < d_2 \leq 200 \\ 1, \text{ for } d_2 > 200 \end{cases}$
Freight carrier (Criterion 3)	Criterion with linear preference and indifference area; parameters q = 1 and s = 4 $p_3(d_3) = \begin{cases} 0, \text{ for } d_3 \leq 1 \\ \frac{d_3 - 1}{3}, \text{ for } 1 < d_3 \leq 4 \\ 1, \text{ for } d_3 > 4 \end{cases}$
Municipal factor of trade tax (Criterion 4)	Criterion with linear preference, parameter s = 100 $p_4(d_4) = \begin{cases} 0, \text{ for } d_4 \leq 0 \\ \frac{d_4}{100}, \text{ for } 0 < d_4 \leq 100 \\ 1, \text{ for } d_4 > 100 \end{cases}$

Fig. 6.14 Generalised criteria and preference functions in the example

$$w_1 = 0.3 \quad w_2 = 0.35 \quad w_3 = 0.2 \quad w_4 = 0.15.$$

Substituting into the Formula (6.35) for the value of the outranking relationship $\pi(A_1, A_2)$ for an alternative A_1 compared to alternative A_2:

$$\pi(A_1, A_2) = \sum_{c=1}^{C} w_c \cdot p_c(A_1, A_2)$$

The following is obtained:

$$\pi(A_1, A_2) = 0.3 \cdot p_1(A_1, A_2) + 0.35 \cdot p_2(A_1, A_2) + 0.2 \cdot p_3(A_1, A_2) + 0.15 \cdot p_4(A_1, A_2).$$

By inserting the outcome differences between A_1 and A_2 in the preference functions and, subsequently, transforming the preference values, the following can be determined:

	A_1	A_2	A_3	A_4	A_5	F^+
A_1	0	0.43	0.45	0.30	0.38	**1.56**
A_2	0.50	0	0.45	0.55	0.33	**1.83**
A_3	0.55	0.38	0	0.55	0.55	**2.03**
A_4	0.25	0.07	0.15	0	0.15	**0.62**
A_5	0.24	0.20	0.08	0.31	0	**0.83**
F^-	**1.54**	**1.08**	**1.13**	**1.71**	**1.41**	

Fig. 6.15 The outranking relationship

$$\begin{aligned}\pi(A_1, A_2) &= 0.3 \cdot p_1(60{,}000\text{-}42{,}500) + 0.35 \cdot p_2(800\text{-}1{,}100) + 0.2 \cdot p_3(15\text{-}12) \\ &\quad + 0.15 \cdot p_4(\text{-}350\text{-}(\text{-}250))\end{aligned}$$

$$\pi(A_1, A_2) = 0.3 \cdot p_1(17{,}500) + 0.35 \cdot p_2(\text{-}300) + 0.2 \cdot p_3(3) + 0.15 \cdot p_4(\text{-}100)$$

$$\pi(A_1, A_2) = 0.3 \cdot 1 + 0.35 \cdot 0 + 0.2 \cdot \frac{2}{3} + 0.15 \cdot 0$$

$$\pi(A_1, A_2) = 0.43$$

In the same way, the value $\pi(A_2, A_1)$ can be calculated:

$$\pi(A_2, A_1) = 0.3 \cdot p_1(\text{-}17{,}500) + 0.35 \cdot p_2(300) + 0.2 \cdot p_3(\text{-}3) + 0.15 \cdot p_4(100)$$

$$\pi(A_2, A_1) = 0.3 \cdot 0 + 0.35 \cdot 1 + 0.2 \cdot 0 + 0.15 \cdot 1$$

$$\pi(A_2, A_1) = 0.50$$

The other values of the outranking relationship can be determined in the same way. The matrix in Fig. 6.15 shows the entire outranking relationship.

The fourth step is the evaluation of the outranking relationship. Firstly, flow measures are determined. The outflow measure F^+ results from adding the values of the columns for each alternative; the inflow measure F^- results from summing up the values of the rows (see Fig. 6.15). By simultaneously considering outflow and inflow measures, the partial pre-order shown in Fig. 6.16 can be formulated.

In the matrix above it can be seen that the alternative A_1 is preferable to A_4 (A_1PA_4, there is: $F_1^+ > F_4^+$ and $F_1^- < F_4^-$); A_2 is preferable to A_1 (A_2PA_1, indicated by: $F_2^+ > F_1^+$ and $F_2^- < F_1^-$). The alternatives A_1 and A_5 are not comparable (A_1UA_5) because: $F_1^+ > F_5^+$ and $F_5^- < F_1^-$.

This result can also be presented in the form of a directional graph. In this graph, the nodes represent the alternatives. An arrow from A_i towards A_j indicates that alternative i is preferable to alternative j. Indifference is expressed by lines without

	A_1	A_2	A_3	A_4	A_5
A_1	x	-	-	A_1PA_4	A_1UA_5
A_2	A_2PA_1	x	A_2UA_3	A_2PA_4	A_2PA_5
A_3	A_3PA_1	A_3UA_2	x	A_3PA_4	A_3PA_5
A_4	-	-	-	x	-
A_5	A_5UA_1	-	-	A_5PA_4	x

Fig. 6.16 The partial pre-order

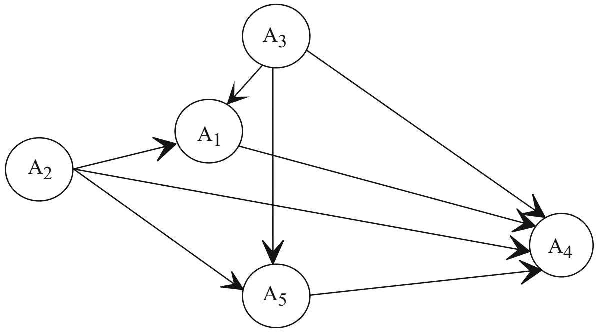

Fig. 6.17 Graphical presentation of the partial pre-order

arrows drawn between the nodes. No connection between two nodes signifies a lack of comparability, i.e. no preference can be stated for either alternative (Fig. 6.17).

From this analysis it is obvious that the alternatives A_4, A_1 and A_5 are not relatively profitable (A_4 is dominated by all the other alternatives; A_1 and A_5 are dominated by A_2 and A_3). Accordingly, either A_2 or A_3 should be selected; for these alternatives no preference can be stated, since the diagram shows no connection between the two (indicating a lack of comparability).

Assessment of the method

PROMETHEE (like the other outranking methods) can deal with a lack of comparability and incomplete information. In addition, critical values for preferences and preference intensities can be included in the profitability analysis.

The required computational effort is relatively low, and the data collection slightly simplified by the possibility of using generalised criteria. However, the preference functions, outcomes and weightings must be determined for each criterion. The measurements must be cardinal, which restricts the consideration of qualitative attributes.

The limitation to six generalised criteria, although not compulsory, might also be regarded as a problem. In general, there is doubt as to whether the preferences of the decision-maker can be encapsulated by generalised criteria, preference functions, and value differences (rather than absolute values). Again, the effects of uncertainty may be examined using sensitivity analysis.

In regard to the outranking relationship and the flow measures that form the basis of profitability assessments, it is assumed that target weightings can be assigned on a cardinal scale. The weighted means of all preference values (additive functions) as stated in the outranking relationship are purported to give an adequate comparison of alternatives. This also assumes—similar to the AHP and utility value analysis—that completely independent judgements are being made on each criterion. Using flow measures, it is assumed that preferences over other alternatives (outflow measures) as well as the 'domination' by other alternatives (inflow measures) will enable the formulation of a ranking. One weakness is that, as with the AHP, the pair comparisons depend upon the available alternatives and so the ranking obtained is unstable.

The inclusion of outflow and inflow measures is specific to the method. Due to the inclusion of inflow measures, PROMETHEE only allows limited compensation for unfavourable outcomes.

An order formed with PROMETHEE will reflect the preferences of the decision-maker only if the assumptions described above are fully met. Yet, such a preference statement is not the principal purpose of the procedure. Rather, and this is more important than with the other methods, decision support via preference and problem structuring is the main purpose of the PROMETHEE method.

To conclude the examination of multi-criteria methods, it should be pointed out that they share some similarities, in that they all operate by partitioning a problem. In each method the separate elements and target criteria must be determined and weighted, transformed into individual utility values or comparable values (partial utility values, local priorities, preference values) and, finally, summed taking the individual weightings into account.

Common features of utility value analysis and the AHP are primarily the step sequence and the additive total utility function. The AHP requires more effort, but has the advantage of examining the subjective estimates for consistency.

The MAUT differs from utility value analysis and the AHP in that it has a utility theory foundation and corresponding preconditions. Apart from that, the procedure is very similar to utility value analysis.

The PROMETHEE method has some similarity to the AHP, since it is based on the execution of pair comparisons. However, it offers decision support rather than a procedure for determining an optimum solution. In this regard, it differs from the other methods.

All procedures discussed in this chapter have specific advantages and disadvantages. It is therefore not possible to give a general recommendation for any one procedure; the choice of method depends on the problem being considered. A combination of methods, or elements of methods, may be useful—e.g. the target criteria weighting used with the AHP and MAUT may be applied within the framework of a utility value analysis.

Assessment Material

Exercise 6.1 (Utility Value Analysis)

The copiers in a department are due for renewal. There is a choice between two types of copier that have the same basic technical functions. A financial profitability analysis shows no significant difference between the two. Carry out a utility value analysis with the following list of target criteria:

Target criteria	Criteria weightings (%)
1. User-friendliness	30
1.1. Handling of the operating parts	10
1.2. Handling of paper loading	10
1.3. Frequency of faults	50
1.4. Finding and remedying faults	30
2. Service from supplier	30
2.1. Term of guarantee	30
2.2. Distance from customer service	30
2.3. Maintenance performance	40
3. Quality of copies	40
3.1. Copies on paper	60
3.2. Copies on slides	10
3.3. Copies on paper when in constant use	30

Over 4 weeks of testing, staff members obtain the following results:

	Outcomes	
Target criteria	Copier A	Copier B
1.1.	Simple	Simple
1.2.	Moderately simple	Simple
1.3.	3 faults per 1,000 copies	7 faults per 1,000 copies
1.4.	Complicated	Very simple
2.1.	6 months	1 year
2.2.	200 km	10 km
2.3.	Very good	Good
3.1.	Very good	Good
3.2.	Good	Good
3.3.	Satisfactory	Good

Use the following tables to transform the results into partial utility values:
For criteria 1.1, 1.2 and 1.4:

Result	Partial utility values
Very simple	1.00
Simple	0.80
Moderately simple	0.60
Moderately complicated	0.40
Complicated	0.20
Very complicated	0.00

For criterion 1.3:

Result	Partial utility values
Up to 1 fault	1.00
2–4 faults	0.80
5–8 faults	0.60
9–15 faults	0.40
16–30 faults	0.20
More than 30 faults (per 1,000 copies)	0.00

For criterion 2.1:

Result	Partial utility values
6 months	0.00
1 year	0.50
2 years	1.00

For criterion 2.2:

Result	Partial utility values
0–50 km	1.00
51–250 km	0.50
More than 250 km	0.00

For criteria 2.3, 3.1, 3.2 and 3.3:

Result	Partial utility values
Unsatisfactory	0.00
Sufficient	0.25
Satisfactory	0.50
Good	0.75
Very good	1.00

(a) Prepare the decision using utility value analysis.
(b) Describe briefly the various steps of a utility value analysis.
(c) What are the assumptions underlying a utility value analysis?

Exercise 6.2 (Analytical Hierarchy Process)

A company would like to use the analytical hierarchy process in planning its strategic investments. There are three strategies (alternatives) to choose from: Strategy A (growth), B (growth combined with a strategic alliance) and C (consolidation). The system of targets consists of three targets: 'company growth' (CG), 'securing the company's independence' (SI) and 'long-term profit maximisation' (LP). It is assumed here that these suffice to meet the requirements demanded of a system of targets within the scope of the AHP (see also Sect. 6.3).

The decision-makers have given the following assessments, using pair comparisons, of the relative importance of (a) the target criteria and (b) the alternatives:

Table 6.5 Pair comparisons for the target criteria

	CG	SI	LP
CG	1	1	1/3
SI	1	1	1/3
LP	3	3	1

Pair comparisons for the alternative strategies in relation to each target criterion:

Table 6.6 'Company growth'

	A	B	C
A	1	1	5
B	1	1	5
C	1/5	1/5	1

Table 6.7 'Securing independence'

	A	B	C
A	1	3	1/3
B	1/3	1	1/6
C	3	6	1

Table 6.8 'Long-term profit maximisation'

	A	B	C
A	1	1/3	1
B	3	1	2
C	1	1/2	1

(a) Determine the weighting vectors of the pair comparison matrices. Are the assessments sufficiently consistent?
(b) Calculate the global priority of the alternatives and assess their relative profitability.
(c) Assess the AHP in regard to the assumptions made in connection with its application.

Exercise 6.3 (Multi-attribute Utility Theory)

Now, the investment issue in Exercise 6.2 is reconsidered. It is assumed that it is possible to measure 'company growth' in terms of the number of employees (NE), and 'securing independence' in terms of the amount of outside capital required (OC). For these items and for the long-term profit (LP) it is assumed that preferences are mutually preferential independent and that the following data for the three Alternatives A, B and C can be forecasted with certainty.

Table 6.9 Data for alternatives A, B and C

Alternative	Criterion 1 (NE)	Criterion 2 (OC [€])	Criterion 3 (LP [€ p.a.])
A	15,000	5,000,000	3,000,000
B	12,000	2,500,000	4,000,000
C	10,000	1,000,000	3,200,000

The following individual utility values were determined for these outcomes of the target criteria:

$$\begin{array}{llll} u_1(10{,}000) & = 0 & u_1(15{,}000) & = 1 \\ u_2(5{,}000{,}000) & = 0 & u_2(1{,}000{,}000) & = 1 \\ u_3(3{,}000{,}000) & = 0 & u_3(4{,}000{,}000) & = 1 \end{array}$$

Then, the following indifference assessments were made, in order to ascertain the relevant additional points for each of the functions u_1, u_2 und u_3:

Function u_1:

To determine $o_1^{0.5}$ using the third criterion:

$$(10{,}000;\ 3{,}200{,}000) \sim (12{,}000;\ 3{,}000{,}000)$$

$$(12{,}000;\ 3{,}200{,}000) \sim (15{,}000;\ 3{,}000{,}000)$$

Function u_2:

To determine $o_2^{0.5}$ using the third criterion:

$$(5{,}000{,}000;\ 3{,}400{,}000) \sim (3{,}000{,}000;\ 3{,}000{,}000)$$

$$(3{,}000{,}000;\ 3{,}400{,}000) \sim (1{,}000{,}000;\ 3{,}000{,}000)$$

Assume that the function u_2 is linear for the interval $[o_2^{0.5}, o_2^{1}]$.

Function u_3:
To determine $o_3^{0.5}$ using the first criterion:

$$(15,000;\ 3,000,000) \sim (10,000;\ 3,400,000)$$

$$(15,000;\ 3,400,000) \sim (10,000;\ 4,000,000)$$

Once again, assume linearity, here for the interval $[o_3^0, o_3^{0.5}]$.

(a) Calculate the total utilities of each of the three alternatives and assess their relative profitability.
(b) Discuss the advantages and disadvantages of MAUT.

Exercise 6.4 (PROMETHEE)

Look again at the strategic investment issue in Exercises 6.2 and 6.3. This time, use the PROMETHEE method for decision support. Take as valid all of the alternatives, target criteria and outcomes from Exercise 6.3. Instead of the indifference judgements from 6 to 3, the following generalised criteria and preference function parameters for the target criteria should be used in decision-making with PROMETHEE.

Number of employees (NE):
Criterion with linear preference, parameter: $s = 3{,}000$
Outside capital (OC):
Step-criterion with parameters $q = 1{,}000{,}000$ and $s = 2{,}000{,}000$
Long-term profit (LP):
Step-criterion with parameters $q = 100{,}000$ and $s = 800{,}000$

(a) Determine the preference functions for the criteria.
(b) Calculate the outranking relationship, as well as the inflow and outflow measures. In so doing, assume the following weightings: NE: 1/5; OC: 1/5; LP: 3/5.
(c) Formulate an order of preference for the alternatives.
(d) Discuss the advantages and disadvantages of PROMETHEE.

Further reading: see recommendations at the end of this part.