An Improved Iterative Closest Point Algorithm for Rigid Point Registration

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Abstract. Iterative Closest Point (ICP) is a popular rigid point set registration method that has been used to align two or more rigid shapes. In order to reduce the computation complexity and improve the flexibility of ICP algorithm, an efficient and robust subset-ICP rigid registration method is proposed in this paper. It searches for the corresponding pairs on subsets of the entire data, which can provide structural information to benefit the registration. Experimental results on 2D and 3D point sets demonstrate the efficiency and robustne*ss of the proposed method.*

Keywords: *Itera*tive Closest Point (ICP) · Point registration · Rigid transformation · Subset · Alignment

1 Introduction

Point set registration is an important research topic in computer vision and image processing. It has been widely applied to pattern recognition, shape reconstruction, motion tracking and stereo matching, etc. The task of point set registration is to recover an optimal transformation according to the current locations of two point sets and map one point set onto another to make them overlap as much as possible. Simultaneously, the shapes described by the points are aligned very well as shown in Figure 1.

Iterative Closest Point (ICP) algorithm [1, 2] is one of the most popular rigid point set registration methods. Expectation Maximization (EM) scheme [3] is often used as the alternating update procedure to search for the solution, whose E step and M step can be viewed as updating correspondences and recovering transformation of ICP respectively. An EM-ICP [4] was proposed to handle Gaussian noise in rigid registration of large points set. It was a coarse-to-fine approach based on an annealing scheme to balance the robustness and the accuracy of ICP. Liu [5] combined the soft-assign and EM-ICP algorithms for the automatic registration of overlapping 3D point clouds. When the transformation (M step) is required to be determined with fix correspondences, multiple layer feed-forward neural network [6] is an alternative rigid point set registration method. Combined with Principal Component Analysis (PCA) feature extraction, neural network can be used to align two rigid 2D gray images [7].

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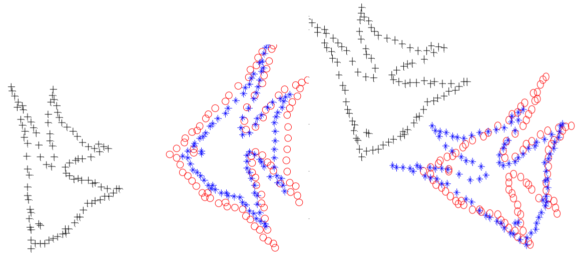

(a) (b)

Fig. 1. Two registration results. (a) is successful because the distance measurement is small and the shapes are aligned. (b) is fail in spite of the smaller distance measurement but poor shape alignment.

Point set registration can be considered as a probability density estimation problem. Many probabilistic methods, such as Gaussian Mixture Model (GMM) [8, 9] and Robust Point Matching (RPM) [10], are developed to acquire better registration result in the presence of noise and outliers at the cost of computation complexity.

In this paper, an efficient and robust rigid point set registration method, named subset-ICP, is presented, which is an improved ICP method. In this method, a whole target set is divided into several subsets, and the same process is done to the source set. For each pair of the target subset and source subset, standard ICP is conducted to find the optimal transformation, which is used to map the entire source set to the target set.

The proposed method can deal with larger rotation very well. Partial data instead of entire data set can implicitly provide structural information. Under the viewpoint of optimization, with basic matrix theory, the induction procedure of the rigid transformation parameters (scaling, rotation and translation) is provided in this paper. It is easier to understand the proof procedure than that of the quaternion method based on PCA.

The rest of this paper is organized as follows. Fundamental knowledge is briefly described in section 2. Section 3 presents the proof of the unknown variables of rigid transformation, and introduces our subset-ICP method. Experiments are shown in section 4. Finally, conclusions are given in section 5.

2 Preliminary

In this section, the fundamental steps of standard ICP is introduced firstly and then some basic concepts about matrix theory are recalled to easily understand the induction procedure of section 3.

2.1 Standard ICP

Given two point sets $X = \{x_1, x_2, \cdots, x_M\}$ and $Y = \{y_1, y_2, \cdots, y_N\}$, where x_i, $y_j \in R^n$ (each element is defined in n-dimensional Euclidean space), M,N are the numbers of points in X and Y respectively. The main steps of the standard ICP are the correspondences and the transformation, which are updated till the terminate conditions are satisfied.

- For each point y_j $(\mathrm{j} = 1,2,\cdots,\mathrm{N})$ of set Y, search for its closest point x_i from set X to form the correspondences set $\mathrm{N_Y(X)} = \{\, \mathrm{x_i} \mid d(y_j, x_i) = argmin_{x\in X} d(y_j, x)\}$;
- For sets $N_Y(\mathrm{X})$ and Y, compute the rotation matrix $\mathrm{R^k}$ and translation vector $\mathrm{t^k}$ using statistics technique PCA.
- Apply transformation $(\mathrm{R^k},\ \mathrm{t^k})$ to update set Y and accumulate rotation matrix R and translation vector t.

$$\mathrm{Y} = \mathrm{R^k} \cdot \mathrm{Y} + \mathrm{t^k} \tag{1}$$

$$\mathrm{R} = \mathrm{R^k} \cdot \mathrm{R} \tag{2}$$

$$\mathrm{t} = \mathrm{R^k} \cdot \mathrm{t} + \mathrm{t^k} \tag{3}$$

2.2 Basic Concepts About Matrix Theory

Definition 2.1 (vector inner product and vector norm): Given two n-dimensional vectors $\mathrm{a} = (\mathrm{a_1},\ \mathrm{a_2}, \cdots,\ \mathrm{a_n})$ and $\mathrm{b} = (\mathrm{b_1},\ \mathrm{b_2}, \cdots, \mathrm{b_n})$, then the squared Euclidean distance between a and b is rewritten using the inner product and norm of vector as

$$||a-b||^2 = ||a||^2 + ||b||^2 - 2(a,b) \tag{4}$$

It is easy to know (here, a, b are row vectors)

① $||a||^2 = (a,a) = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} = aa^T$

② $(a,b) = (b,a)$

③ $(ka,b) = (a,kb) = k(a,b) = k \cdot ab^T$

Definition 2.2 (orthogonal matrix): Square matrix A is called as an orthogonal matrix when $\mathrm{AA^T} = \mathrm{A^TA} = \mathrm{I}$ is satisfied, where I is an identity matrix.

Definition 2.3 (matrix trace): Let $\mathrm{A} = \left(\mathrm{a_{i,j}}\right)_{\mathrm{n\times n}}$ is a square matrix, the trace of A is defined as

$$\mathrm{tr(A)} = \sum_{i=1}^{n} a_{ii} \tag{5}$$

The properties of trace are included:

① $tr(A) = tr(A^T)$

② $tr(AB) = tr(BA)$

3 Subset-ICP Registration Method

In this section, a proof of the unknown variables of rigid transformation is provided and then our subset-ICP method is introduced.

3.1 Parameters of the Rigid Transformation

The point set registration is to find an optimal mapping F to make $\sum ||\mathrm{x} - \mathrm{F(y)}||^2$ as smaller as possible. When the rigid transformation is considered, the point set registration problem is transferred as a minimum optimization with constraints

$$\min Q(s,R,t) = \sum ||x-(sRy+t)||^2 \quad \text{Subject to} \quad R^TR = I, \ \det(R) = 1 \tag{6}$$

In order to obtain the closed form solution of (6), a lemma 1 [11] is described as follows:

Lemma 1: Let $R_{D\times D}$ be an unknown rotation matrix and $A_{D\times D}$ be a known real square matrix. Let USV^T be a Singular Value Decomposition of A, where $UU^T = VV^T = I$ and $S = d(s_i)$, with $s_1 \geq s_2 \geq \cdots \geq s_D \geq 0$. Then, the optimal rotation matrix R that maximizes $tr(A^TR)$ is $R = UCV^T$, where $C = d(1,1,\cdots,1,\det(UV^T))$.

For simplicity, we assume $X = \{x_1, x_2, \cdots, x_M\}$ and $Y = \{y_1, y_2, \cdots, y_N\}$, $M = N$ that denotes no outliers existing and the correspondences are established well. So the objective function is $Q(s,R,t) = \sum_{i=1}^{N} ||x_i - (sRy_i + t)||^2$.

Firstly, a partial derivative of Q with respect to t is computed and makes it equal to zero

$$\frac{\partial Q(s,R,\ t)}{\partial t} = -2 * \sum_{i=1}^{N}(x_i - sRy_i - t) = 0$$

We can obtain

$$t = \bar{x} - sR\bar{y} \tag{7}$$

where, the mean vector $\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$, $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$.

Based on formulae (7) and (4)

$\sum_{i=1}^{N} ||x_i - (sRy_i + t)||^2 = \sum_{i=1}^{N} ||(x_i - \bar{x}) - sR(y_i - \bar{y})||^2 = \sum_{i=1}^{N} ||\hat{x}_i||^2 + s^2\sum_{i=1}^{N} ||R\hat{y}_i||^2 - 2s\sum_{i=1}^{N}(\hat{x}_i, R\hat{y}_i) = \sum_{i=1}^{N}(\hat{x}_i{}^T\hat{x}_i) + s^2\sum_{i=1}^{N}(\hat{y}_i{}^TR^TR\hat{y}_i) - 2s\sum_{i=1}^{N}(\hat{x}_i{}^TR\hat{y}_i)$

The objective function is written with matrix trace form as follows:

$$Q(s,R,t) = tr(\hat{X}^T\hat{X}) + s^2tr(\hat{Y}^T\hat{Y}) - 2str(\hat{X}^T\hat{Y}R^T) \tag{8}$$

where, the following notations are used $\hat{x}_i = x_i - \bar{x}$, $\hat{y}_i = y_i - \bar{y}$, $\hat{X} = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N]$, $\hat{X}^T = \begin{bmatrix} \hat{x}_1{}^T \\ \hat{x}_2{}^T \\ \vdots \\ \hat{x}_N{}^T \end{bmatrix}_{N\times n}$, $\hat{Y} = [\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_N]$, $\hat{Y}^T = \begin{bmatrix} \hat{y}_1{}^T \\ \hat{y}_2{}^T \\ \vdots \\ \hat{y}_N{}^T \end{bmatrix}_{N\times n}$, n is the dimensional number of point.

The first two terms of (8) are independent of rotation matrix R, we can denote them as a constant c_2. In addition, according to the property of trace, the last term of (8) is calculated as

$$\mathrm{tr}(\hat{X}^T\hat{Y}R^T) = \mathrm{tr}(((\hat{X}^T\hat{Y})R^T)^T) = \mathrm{tr}(R(\hat{X}^T\hat{Y})^T) = \mathrm{tr}((\hat{X}^T\hat{Y})^TR) \tag{9}$$

Thus, the objective function can be converted as

$$Q(s, R, t) = -c_1 tr((\hat{X}^T \hat{Y})^T R) + c_2 \tag{10}$$

Based on Lemma1, let $A = \hat{X}^T\hat{Y}$, $USV^T = svd(A)$, the optimal rotation R is

$$R = UCV^T \tag{11}$$

where $C = d(1,1,\cdots,1,\det(UV^T))$.

In order to solve scaling factor s, taking a partial derivative of (8) with respect to s and make it equal to zero, then we have

$$s = \frac{tr(\hat{X}^T \hat{Y} R^T)}{tr(\hat{Y}^T \hat{Y})} \tag{12}$$

3.2 The Subset-ICP Algorithm

A simulated annealing combined with ICP scheme is used to improve the registration accuracy, at the same time, reduce the computation cost. The number of subsets is defined as a special temperature parameter T, where the increment is one for each iteration. Subset-ICP is implemented on two subsets to search for the closest points and to build the transformation mapping. The obtained mapping is employed to update the source data set Y. The pseudo-code of subset-ICP is described as follows in Figure 2:

```
Inputs: Point sets X and Y
Initialization: The scaling factor s_0 = 1, the rotation matrix R_0 = I
                and the translation vector t_0 = 0
For iterative registration
For simulated annealing (T = 1: max _T)

To update correspondences on subsets X_T, Y_T;
To update transformations, R and t;
To apply the transformation to Y;

End simulated annealing
End iterative registration

Outputs: The transformations, R and t; The matched point set Y
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Fig. 2. The pseudo-code of subset-ICP algorithm

4 Experiments

In this section, the experiments were conducted to verify the effectiveness of the proposed method in different kinds data including 2D fish clean points, 2D fish points with Gaussian noise, 3D bunny clean points, 3D bunny points with outliers points which can deteriorate the shape and 3D bunny data with missing data that means the incomplete data.

There are two point sets extracted from an object image, one is called the target set and its shape is represented by red circle, another is called source set and its shape is described by blue pentagrams. The task is to find an optimal spatial transformation (rotation matrix R and translation vector t in our experiments) according to the current locations of the target set and the source set. If these two sets can be aligned very well (overlapping totally), then the registration (matching) is successful. The performance measures are Mean Squared Error (MSE) and visual results of registered images.

4.1 The Efficiency of Subset-ICP

2D fish data [12]: Four source point sets are synthetically formed by different linear transformations, blurring and deformation of the target point set. The registration results are shown in Figure 3. We can know that the proposed method is efficient for different spatial positions of rigid source point set, involving the larger rotation data. It is also robust to the noise data but not very effective to the deformable shape.

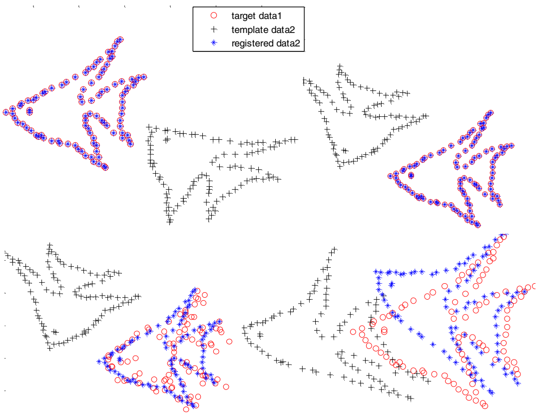

Fig. 3. The registration results of subset-ICP on 2D fish data. Red circle points form the target data1, black cross points are source data2 and the blue pentagram marks are the registered source data2

3D bunny data: we test our method on the stanford bunny data set [13]. 305 points are located manually to get the profile of 3D frontal bunny to form the target data1. Figure 4 demonstrates the qualitative registration results of subset-ICP on 3D bunny data, which validate the subset-ICP is an efficient rigid registration method.

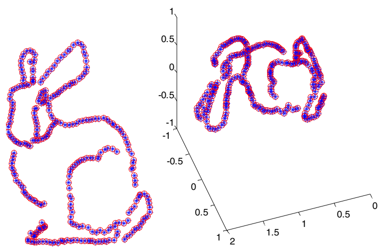

Fig. 4. The bunny data is registered by the subset-ICP. Left figure is a 2D result and the right figure is a 3D result

Missing data: The target bunny (red circle) and the source bunny (blue pentagram marks) are or/both with missing data in different parts respectively. The registration results are shown in Figure 5. We can see that the subset-ICP is robust to missing data.

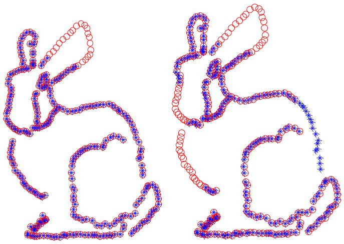

Fig. 5. The registration results of the subset-ICP when the missing data arising. The source data with missing ear data is in the left figure. The target data and the source data are both with missing data in different parts in the right figure

Outliers: The target data with 40% Gaussian outliers, the matching result is displayed in the left part of Figure 6. The source data with 20% Gaussian outliers, the matching result is shown in the right part of Figure 6. It is easy to see that the target with outliers do not affect the matching result but a slight difference exists when the source shape is disturbed by outliers. According to the experimental observation, for either the target shape or the source shape, an ill-matching happens when the structure information of shape is deteriorated by the outliers.

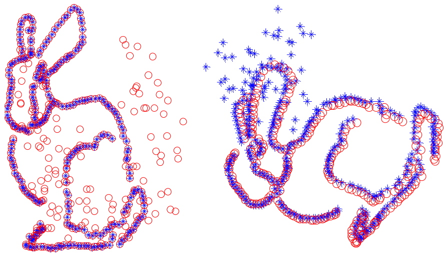

Fig. 6. The 2D matching results for the outliers. Target data with Gaussian outliers is shown in the left figure and the source data with Gaussian outliers is displayed in the right figure

4.2 The Comparison with Standard ICP

The comparisons between standard ICP algorithm and our method are displayed in Figure 7 and 8. The x-axis is the experiment times, that means ten pairs of the target sets and the source sets are generated to test the performances of subset-ICP and standard ICP algorithm. Y-axis of Figure7 is the MSE and y-axis of Figure8 is the speed (execution time (second)). Based on the experimental results, we can know that subset-ICP is faster and more efficient than standard ICP on 3D bunny data.

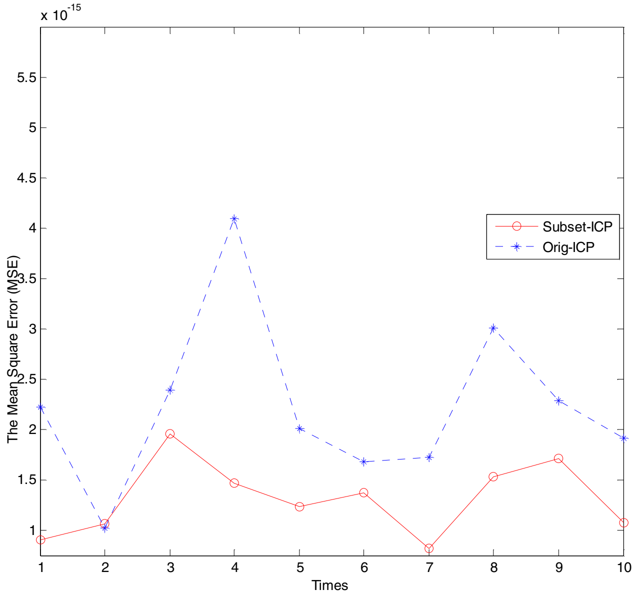

Fig. 7. The comparison of MSE on 3D bunny data

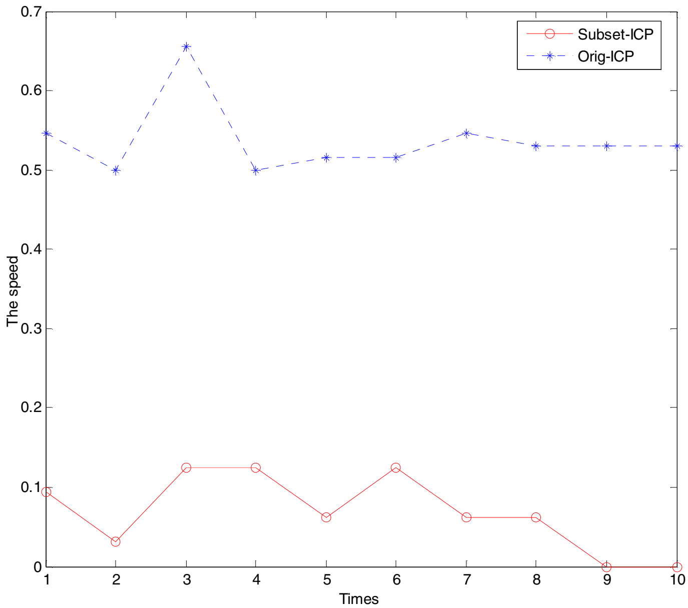

Fig. 8. The comparison of execution speed on 3D bunny data

5 Conclusions

In this paper, a subset-ICP rigid point set registration method is proposed, which is an improved version of standard ICP. Partial data instead of entire data can implicitly offer structural information that benefits the registration. Experiments are conducted on 2D and 3D synthetic data, and the matching results are analyzed and compared. All experimental results showed that the proposed method is efficient and robust. Under the viewpoint of optimization and matrix theory, the induction procedure of the rigid transformation parameters is also provided in this paper.

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References

1. Besl, P.J., McKay, N.D.: A Method for Registration of 3-D Shapes. IEEE Transactions on Pattern Analysis and Machine Intelligence **14**(2), 239–256 (1992)
2. Chen, Y., Medioni, G.: Object Modeling by Registration of Multiple Range Images. Image and Vision Computing **10**(3), 145–155 (1992)
3. McLachlan, G.J., Krishnan, T.: The EM algorithm and extensions. John Wiley & Sons, New York (1996)
4. Granger, S., Pennec, X.: Multi-scale EM-ICP: A Fast and Robust Approach for Surface Registration. In: Heyden, A., Sparr, G., Nielsen, M., Johansen, P. (eds.) ECCV 2002, Part IV. LNCS, vol. 2353, pp. 418–432. Springer, Heidelberg (2002)
5. Liu, Y.: Automatic registration of overlapping 3D point clouds using closest points. Image and Vision Computing **24**(7), 762–781 (2006)
6. Chen, J., Liao, I.Y., Belaton, B., Zaman, M.: A Neural Network-Based Registration Method for 3D Rigid Face Image. World Wide Web (2013) doi:10.1007/s11280-013-0213-9
7. Chen, J., Belaton, B., Pan, Z.: Multiple Components-Based 2D Face Images Alignment Using Neural Networks. International Journal of Computational Science and Engineering (2013) (under review)
8. Myronenko, A., Song, X.: Point Set Registration: Coherent Point Drift. IEEE Transactions on Pattern Analysis and Machine Intelligence **32**(12), 2262–2275 (2010)
9. Rasoulian, A., Rohling, R., Abolmaesumi, P.: Group-Wise Registration of Point Sets for Statistical Shape Models. IEEE Transaction on Medical Imaging **31**(11), 2025–2034 (2012)
10. Gold, S., Rangarajan, A., Lu, C.-P., Mjolsness, E.: New Algorithms for 2D and 3D Point Matching: Pose Estimation and Correspondence. Pattern Recognition **31**, 957–964 (1997)
11. Myronenko, A., Song, X.: On the Closed-Form Solution of the Rotation Matrix Arising in Computer Vision Problems. Presented at the Technical Report arXiv:0904.1613v1, Oregon Health and Science University (2009)
12. Chui, H., Rangarajan, A.: A new point matching algorithm for non-rigid registration. Computer Vision and Image Understanding **89**, 114–141 (2003)
13. The Stanford 3D Scanning Repository (2010). http://graphics.stanford.edu/data/3Dscanrep/