Research of the Optimal Algorithm in the Intelligent Materials

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Abstract. Adopting piezoelectric material as sensors and actuators, active vibration control of a cantilever beam is studied in this paper based on Linear Quadratic Regulator (LQR). Firstly, the actuator equation, sensor equation, and the vibration equation is constructed, and then the vibration equation is converted to modal state equation using modal analysis method. Secondly, the optimal control law is given by LQR method, with the detailed control flow. Finally, the active vibration control simulation is done for the vibration suppression of a piezoelectric beam. The results show that the control performance for the step response of the first and second vibration modal is good, as well as the coupled modal of the first two modal. And the effectiveness of the proposed LQR method is verified.

Keywords: piezoelectric material, smart structure, LQR control, active vibration control.

1 Introduction

Piezoelectric materials have been widely used as sensors and actuators for the smart structures [1-2]. Meanwhile active vibration control for flexible structures with attached piezoelectric materials becomes a hot spot [3-5]. And many control strategies have been proposed, Glugla and co-workers[6] presents a modification of the LMS algorithm by adjusting the underlying gradient descent alogrithm in active vibration control; Jae Hanghan and Keun Horew[7] utilizing a linear quadratic Gaussian control algorithm using the laminated composite beam with piezoelectric sensors and actuators reduce the beam vibration; Yan Ruhu and Alfred Ng[8] developed active robust vibration control method for active vibration control of flexible structures; Gustavo et al[9] proposes an on-line self-organizing fuzzy logic controller design applied to the control of vibration in flexible structures containing distributed piezoelectric actuator patches.

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With optimal configuration of the piezo patches according to the same location principle, the controlled system is guaranteed to be minimum phase system, and the observation and control spillover can be prevented from modal truncation[10]. Using modal analysis method, the vibration equation of cantilever beam is converted to modal state space equation. With LQR based independent modal control method adopted, this paper takes a cantilever piezo beam as an example, and gives simulation results of the step response and control output voltage for the first two modal frequency. And the effectiveness of the method has been approved by the simulation results.

2 Kinetic Equation for Smart Piezoelectric Structures

2.1 Character of the Piezoelectric

The piezoelectric characteristics can be described as a constitutive relation which characterizes the coupling effect between mechanical and electrical properties as follows:

$$\varepsilon_p = S_{pq}^E \sigma_q + d_{ip} E_i \,,\ \ p,q = 1,\cdots,6 \tag{1}$$

$$D_i = d_{ip} \sigma_q + \varepsilon_{ik}^{\sigma} E_k \,,\ \ i,k = 1,2,3 \tag{2}$$

Where σ_q and ε_p represent the stress and strain, respectively, E_k and D_i represent the electric field and electric displacement, respectively. Also S_{pq}^E , d_{ip} and $\varepsilon_{ik}^{\sigma}$ represent the elastic compliance, piezoelectric strain/charge coefficient, and electric permitivity, respectively. In (1) and (2) a lamina can be either a piezoelectric material or a conventional composite lamina. Take the relations (1) and (2) transformed into the relations in the geometric axes, and, recalling that the stress component except σ_x are negligible, the induced strain of an unconstrained piezoelectric actuator can be written as

$$\varepsilon_x = d_{31} E_z = d_{31} u / t_{pe} \tag{3}$$

Where d_{31} is the transformed piezoelectric constant, u is the input voltage of the piezoelectric voltage, and t_{pe} is the thickness of piezoelectric actuator.

2.2 Piezoelectric Actuator Equation

The moment of actuator force for the cantilever beam when control voltage is input by the control circuit is got from integrating (3)

$$m(x,t) = K_a u \left[h(x - x_2) - h(x - x_1) \right] \tag{4}$$

$m(x,t)$ is the moment force, and $h(x)$ is the Heaviside step function. x_1 and x_2 is the distance between the edges of the piezoelectric patch and the cantilever fixed end. K_a is the proportional constant and it can be got by (5).

$$K_a = \frac{1}{2} b d_{31} E_{pe} \left(t + t_{pe}\right) \tag{5}$$

Where b is the width of the cantilever beam and the piezoelectric patch. E_{pe} is the elastic modulus of the piezoelectric actuator. And t is the thickness of the beam.

By taking the derivative of (4) with respect to x, the actuator force is

$$\frac{\partial m(x,t)}{\partial x} = k_a u[\varphi_i(x_2) - \varphi_i(x_1)] \tag{6}$$

Where $\varphi_i(x)$ is the mode shape of free vibration.

2.3 Piezoelectric Sensor Equation

While the beam is subjected to symmetric bending stress and small deformation and the applied electric field is zero, the output voltage of the i-th piezoelectric sensor amplified by the charge amplifier is shown in (7).

$$U_i = \frac{k d_{31} b_{pe} r_{pe} E_{pe}}{C_{pe}} \int_{x_1}^{x_2} \frac{\partial^2 w(x,t)}{\partial x^2} dx \tag{7}$$

Where $i = 1, \cdots, r$, and r is the number of the sensors, k is the amplification factor of the charge amplifier. $w(x,t)$ is deflection function. b_{pe} is the width of the sensors. r_{pe} is the distance between the sensor's neutral plane and the flexible beam's neutral plane in z direction. C_{pe} is the capacity of the piezoelectric sensors.

2.4 Vibration Equation of the Cantilever Beam

Assuming the principal inertia axis of all the cross section and the external load are in xoy plane, the transverse vibration of the beam is also in the same plane, then the major transformation of the beam is bend transformation. And if the shear deformation and the rotational inertia influence is ignored, the beam is called Bernoulli-Euler beam. And its vibration mode can be expressed by the deflection $w = w(x,t)$, a function of coordinate x and time t. $f(x,t)$ is the external force of unit length beam.

$m(x,t)$ is the applied moment, $\rho(x)$ is the linear density of the beam, A is the cross section area. E is the elastic modulus of the cantilever beam. $I(x)$ is the inertia moment of the cross section and the neutral surface.

$$EI(x)\frac{\partial^4 w(x,t)}{\partial x^4} + A\rho(x)\frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) - \frac{\partial m(x,t)}{\partial x} \tag{8}$$

Equation (8) is the transverse vibration differential equation of the Bemoulli-Eule beam. And the boundary conditions for fixed end beam is,

$$w(0,t)=0\,,\quad EI\frac{\partial w(0,t)}{\partial x}=0 \tag{9}$$

According to the related theory of the vibration mechanics and the given boundary conditions, to use the assumed modes method, the function $w(x,t)$ is expanded as an infinite series in the form:

$$w(x,t)=\sum_{i=1}^{\infty}\varphi_i(x)q_i(t) \tag{10}$$

In the above (10), $q_i(t)$ is the generalized displacement.

Substitute (10) into (8), the transverse vibration differential equation can be gained as follows:

$$\boldsymbol{EI(x)\sum_{i=1}^{n}\phi_i^{(4)}(x)q_i(t)+A\rho(x)\sum_{i=1}^{n}\phi_i(x)q_i^{(2)}(t)=f(x,t)-\frac{\partial m(x,t)}{\partial x}} \tag{11}$$

Both side of the (11) multiply the mode function $\varphi_i(x)$, and calculate the integral along with the whole length of the beam, with the perpendicular of the mode function, the decoupled mode coordinate equation can be expressed as:

$$\ddot{q}_i(t)+\omega_i^2 q_i(t)=k_a u[\varphi_i(x_2)-\varphi_i(x_1)] \tag{12}$$

Assume: $B_i=k_a[\varphi_i(x_2)-\varphi_i(x_1)]$, (12) change into:

$$\ddot{q}_i(t)+\omega_i^2 q_i(t)=B_i u \tag{13}$$

Substitute the (10) into (7), the output voltage of the ith piezoelectric sensor can be gained as follows:

$$\begin{aligned}\boldsymbol{U_i} &= \boldsymbol{\frac{kd_{31}b_{pe}r_{pe}E_{pe}}{C_{pe}}\sum_{i=1}^{\infty}q_i(t)\int_{x_{1j}}^{x_{2j}}\frac{\partial^2\phi_i(x,t)}{\partial x^2}dx}\\ &= \boldsymbol{\frac{kd_{31}b_{pe}r_{pe}E_{pe}}{C_{pe}}\sum_{i=1}^{n}\left[\phi_i'(x_2)-\phi_i'(x_1)\right]q_i(t)}\end{aligned} \tag{14}$$

Assume: $C_i=\dfrac{kd_{31}b_{pe}r_{pe}E_{pe}}{C_{pe}}[\varphi_i'(x_2)-\varphi_i'(x_1)]$

So (14) can be written as:

$$U_i(t)=\sum_{i=1}^{n}C_i q_i(t) \tag{15}$$

3 State Space Equation for Cantilever Piezoelectric Beam

Where $x(t)=\{q_1(t),q_2(t),\cdots,q_n(t),\dot{q}_1(t),\dot{q}_2(t),\cdots,\dot{q}_n(t)\}^T$ is the state space vector, then, (12) and (14) can be converted to state-space form.

$$\begin{cases} \dot{x}(t)=Ax(t)+Bu(t) \\ y(t)=Cx(t) \end{cases} \tag{16}$$

$y(t)$ is output of the sensor and $u(t)$ is the input of the actuator. A is state matrix, B is control matrix, D is output matrix.

$$A=\begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -\Omega^2 & 0 \end{bmatrix},\quad B=\begin{bmatrix} 0_{n\times 1} \\ B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix},\quad C=\begin{bmatrix} C_1 & C_2 & \cdots & C_n & 0_{1\times n} \end{bmatrix} \tag{17}$$

where

$$\Omega=\begin{bmatrix} \omega_1 & & & \\ & \omega_2 & & \\ & & \vdots & \\ & & & \omega_n \end{bmatrix} \tag{18}$$

As the low frequency response is the main concern, only the first two modal is considered in this paper. The natural frequency of the flexible beam can be obtained by (19).

$$\omega_i=\lambda_i^2\sqrt{\frac{EI}{m}} \tag{19}$$

$I=\dfrac{bt^3}{12}$ is the cross-section inertia moment about y axis, m is the mass per unit length of the cantilever beam.

$$\varphi_i(x)=\sin(\lambda_i x)+D_i\cos(\lambda_i x)+E_i\sinh(\lambda_i x)+F_i\cosh(\lambda_i x) \tag{20}$$

$$\varphi_i'(x)=\lambda_i[\cos(\lambda_i x)-D_i\sin(\lambda_i x)+E_i\cosh(\lambda_i x)+F_i\sinh(\lambda_i x)] \tag{21}$$

For the first vibration modal, $\lambda_1=1.875/l$, $D_1=-1.3622$, $E_1=-1$, $F_1=1.3622$; For the second vibration modal, $\lambda_1=4.694/l$, $D_2=-0.9819$, $E_2=-1$, $F_2=0.9819$. l is the length of the cantilever beam.

4 Linear Quadratic Regulator

4.1 A Linear Quadratic Regulator Algorithm

While the control system is linear, the performance function of the state variables and control variables is quadratic function, and this optimal control problem is called linear quadratic optimal control problem. As the solution of linear quadratic problem is linear function of state variables, and closed loop optimal control can be achieved by state variable feedback, so the practical value of this method is rather high. The quadratic performance index of the state variables and control variables is defined as,

$$J_s = \frac{1}{2}\int_0^{\infty}[x^T Qx + u^T Ru]dt \tag{22}$$

Q is the weighting matrix of state variables and R is the weighting matrix of input variables. And Q is a positive definite matrix, R is a positive definite matrix.

As $u(t)$ is not restricted, the optimal control should satisfy

$$\frac{\partial H}{\partial u} = R^T u(t) + B^T \lambda \tag{23}$$

As R is positive definite matrix and its inverse exists, thus

$$u^*(t) = -R^{-1}B^T\lambda \tag{24}$$

As $\frac{\partial^2 H}{\partial u^2} = R > 0$, $u^*(t)$ makes the minimum value of H exist.

Substitute $u^*(t)$ into the Hamilton canonical equation

$$\dot{x}(t) = Ax(t) - BR^{-1}B^T\lambda(t), x(t_0) = x_0 \tag{25}$$

$$\dot{\lambda}(t) = -A^T\lambda(t) - Qx(t) \tag{26}$$

It is the linear homogeneous equation of $x(t)$ and $\lambda(t)$, and the relation of $x(t)$ and $\lambda(t)$ is linear at any moment.

Take derivations of (26) about t, and put (25) together

$$\left(-Q - A^T P(t)\right)x(t) = \left(\dot{P}(t) + P(t)A - P(t)BR^{-1}B^T P(t)\right)x(t) \tag{27}$$

As equation (27) works for any value of $x(t)$

$$\dot{P}(t) + P(t)A + A^T P(t) - P(t)BR^{-1}B^T P(t) + Q = 0 \tag{28}$$

Equation (28) is the first order differential equation about $P(t)$, and it is called differential Riccati equations. And it can be approved that while the elements of A, B, Q, R is sectional-continuous function about t on $[t_0, \infty]$, equation (28) has the unique solution by discovering convergence condition of the boundary condition.

$$u^*(t) = -R^{-1}B^T P(t)x \tag{29}$$

Let $K(t) = R^{-1}B^T P(t)$, thus

$$u^* = -K(t)x \tag{30}$$

And closed linear feedback control can be realized by using state variable linear feedback.

Substituting (29) into (15), and the state equation of the closed loop system is

$$\begin{cases} \dot{x} = Ax - BR^{-1}B^{T}Px \\ y = Cx \end{cases} \tag{31}$$

The closed loop control principle diagram of LQR control with state feedback is shown in Figure 1.

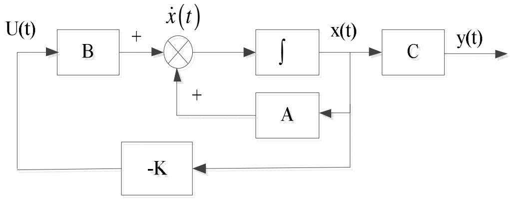


Fig. 1. LQR closed loop control principle diagram

As the low frequency vibration of the cantilever beam plays a major role, suppressing the first few order vibration energy is enough. Taking $x(t)$ as the modal control variable and using LQR method, state feedback control is designed to control $u(t)$ to drive actuators' control moment $m(x,t)$. During $[t_0,\infty]$, optimal control is used to find the optimal control force and the input voltage $u^{*} = -Kx$ to let the system performance function achieve the minimum value and transfer the system from initial state to zero state. And the output voltage of the piezo sensors reflects the vibration state of the piezo cantilever beam.

4.2 The Controllability of Control System

For linear time-invariant systems are infinite state regulator, request system can fully controllable. Because the control region in the infinite state regulator, when the time extend to infinity, so whatever control vector will be extend to infinity if the system can't be controlled. While for a limited time state regulator, Because the system performance in the upper limit of the integral term is finite, even if the system was not entirely controllable, but in the limited integration time, the integral value is limited, so for a limited time state regulators, can control the system from time to emphasize of the requirements.

The size of the aluminium base plate and piezoelectric patches and the physical parameters is listed in Table 1. According to the closed loop state (33), and the derived system state matrix A and system input B, the system matrix can be obtained,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 273.3 & 0 & 0 & 0 \\ 0 & 1712.8 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0.1565 & 0.1381 \end{bmatrix}^{T}$$

Thus $rank([B, A*B, A\wedge 2*B, A\wedge 3*B]) = 4$

Therefore, the system is controllable, as the rank of the system controllability is 4, and it is equal to the system dimension. And considering that the Q and R is symmetric positive definite matrix, optimal control exists and is unique.

Table 1. Parameters of the beam and piezo patch

	E (Gpa)	Density (kg/m^3)	Length (mm)	Width (mm)	Thickness (mm)	Piezo Constant (m/v)	Capacity (μf)
Beam	59.7	2500	400	20	2.5		
Piezo Patch	18.95	1800	100	20	0.3	4.47*1010	0.06

5 Simulation Example

The smart cantilever beam illustrated in Fig. 2 is taken as a simulation example with the sensors and actuators in the same direction. Step signal is imposed on the system. While the system is undamped, the initial conditions of the modal variables are $q_1(0)=q_2(0)$, $\dot{q}_1(0)=\dot{q}_2(0)$, Linear quadratic regulator control is used to suppress vibrations of the first two modal frequencies, while $Q=\begin{bmatrix} 10e7 & 0 \\ 0 & 10e7 \end{bmatrix}$, $R=[1]$.

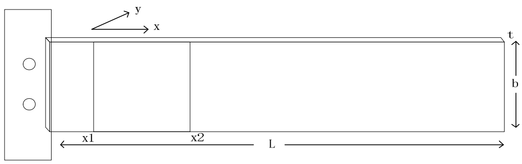
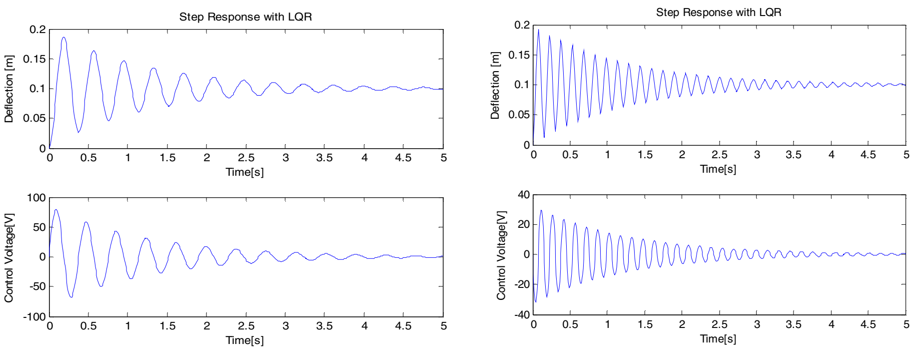


Fig. 2. Cantilever beam model with attached piezo patches



(a) the first modal frequency frequency

(b) the second modal

Fig. 3. Control response and control voltage for (a) the first modal frequency and (b) the second modal frequency

The vibration control response and the control voltage for the first modal frequency is illustrated in Fig. 3(a) while feedback control is on, and the results for the second modal frequency is illustrated in Fig. 3(b). For both of the first two modal frequencies, the mixed control response and control voltage is shown in Fig. 4(a), and the state response is shown in Fig. 4(b).

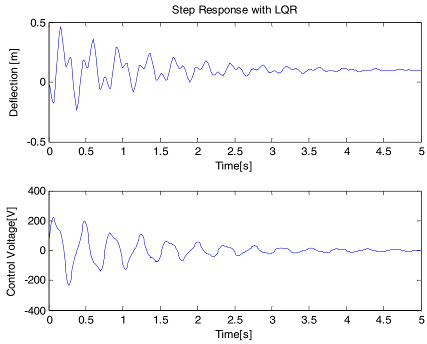


Fig. 4. Control response for the first two modal frequency

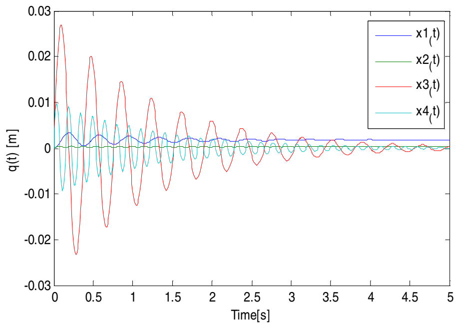


Fig. 5. State response for the first two modal frequency

As shows in Fig. 3(a) and Fig. 3(b), the LQR control is effective and can suppress the vibration response significantly with less steady-state error. As shown in Fig. 3(a) and Fig. 3(b), the control voltage for the second modal frequency is lower than that is needed for the first modal frequency. Fig. 4 and Fig. 5 shows that the first two modal vibration is coupled, though the control performance is also good, the required control voltage is much bigger.

6 Conclusion

Vibration control of flexible beam was studied in this paper. By taking piezoelectric sensors and actuators , the sensor and actuator equation was deduced, and the dynamic control equation was converted to the state space equation. LQR based independent modal space control was carried out, and the simulation was done for vibration control of smart cantilever beam. The simulation results show that the proposed method could suppress the vibration of the flexible structures effectively.

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