Situation Theory, Situated Information, and Situated Agents

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Abstract. Situation Theory is mathematical modelling of concepts such as information, information units, situations, states, events, context, agents, and agent perspectives. We introduce major type-theoretical objects of Situation Theory, which model situated, partial, and parametric information. The system of situated objects is defined by mutual recursion. The main contribution to Situation Theory in this article is the distinction between situated propositions, as contents of statements and intentions, and situated factuality of the verified propositions. We use this distinction to define complex, propositional types. Another contribution is that we define complex, restricted parameters by using propositional types. The article demonstrates potential applications of the introduced complex, situation-theoretical objects. Among the many applications of Situation Theory are developments of intelligent language processing and user-computer interfaces, by integrations of human and computer languages. We focus on modelling major objects that have potentials in such applications, e.g., contexts, situated agents, and usage of names to designate objects depending on agents and information available to agents.

Keywords: Situation theory · Information · Situation semantics · Parameters · Partiality · Situations · Types · Restricted parameters · Context · Agents

1 Introduction

1.1 Background

In 80's, Barwise [2] and Barwise and Perry [5] introduced Situation Theory with the ideas that partiality, factual content, and situatedness are crucial features of the meaning concepts that involve mental states, including attitudes. Situation Theory developed as a theory of the inherent relational and situational nature of information, in general, not only of linguistic meanings, by diverging from the traditional possible-world theories of semantics with type-theoretic settings, in particular from Montague's IL (see [27]). Detailed discussions and motivations

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of the situation-theoretic objects, such as situation types similar to the ones introduced in this article, are given in [5]. A more formal introduction, in the lines of our work here, is given in [17]. For an informal introduction to Situation Theory and Situation Semantics, with examples and intuitions, see [9]. One of the most distinguished applications of Situation Theory has been Situation Semantics for computational analysis of human language. Head-driven Phrase Structure Grammar (HPSG) (see [22,23]) is one of the first practical grammar frameworks, based on formal syntax of human language by using typed, linguistic feature-value structures, which was introduced by the ideas of Situation Theory for information distribution. HPSG came with ambitions to use Situation Semantics for including semantic representations in syntactic analyses. Current HPSG systems have been successfully realizing such semantic representations with a specialized language, Minimal Recursion Semantics (MRS), for handling scope ambiguities (see [8,15]). Situation Semantics has inspired other work in linguistics, e.g., it was used for semantic analysis of questions (see [10]) and for semantics of tense and aspect, in settings of logic programing, from cognitive perspective (e.g., see [28]).

1.2 Mathematics of Situation Theory

Situation Theory presented in this article is a mathematical structure consisting of primitive and complex objects defined recursively. It includes primitive and complex types that classify the system of all objects. The domain of situation-theoretical objects can be a proper class, instead of a set, depending on the needs of applications.

Situation Theory includes propositions as complex objects, which are abstract, mathematical objects, e.g., representing information asserting that some objects a is of certain type T. While some of these abstract propositions can serve in semantic representations of syntactic expressions such as sentences, the situation-theoretical propositions are not syntactic expressions per se. One of our contributions in this article is that we introduce Situation Theory that distinguishes between propositions as asserting informational units and information about verified propositions. E.g., a proposition $(a : T)$ carry asserting information that some objects a is of certain type T, while the factual information that a is of the type T is $a : T$. We also define complex, propositional types that are abstractions over propositions.

Another contribution is that we define complex, restricted parameters by using types, including propositional types. In this aspect, the parameters in this article are different from the parameters presented in [5] that are restricted with event-types, which we include as complex relations. In particular, a parameter that is restricted by a type, as a model of an underspecified object constrained to be of certain kind, can be instantiated only with objects that are of the restriction type. Such situation-theoretical parameters are especially useful for modelling context and resource situations that provide objects satisfying the information in the restricted parameters. Situation Theory with similar parametric objects has been used for semantics of attitude expressions and quantifier ambiguities (e.g., see [11–13]).

From model-theoretic point, similarly to many fields of mathematics, the meta-theory of Situation Theory is set theory. This means that it has a complex, hierarchical system of abstract objects, which are set-theoretic constructs (see [5]). Furthermore, the more powerful versions of Situation Theory are distinguished by representing circular pieces information, which are non-well-founded. The typical examples of such circularity involves situations that carry information about mutual belief and common knowledge shared by different agents. Such information units can be represented in Situation Theory by objects that do not conform with the classic axiom of foundation supporting cumulative hierarchy of sets. To accommodate such non-well-founded circularity, as discussed in [3], Situation Theory uses hypersets that are based on a version of Aczel's non-well-founded set theory (see [1]). Aczel's non-well-founded set theory replaces the foundation axiom, *FA*, of the standard axiom system *ZFC* of axiomatic set theory, with an axiom of anti-foundation, *AFA*, which was motivated by modelling non-well-founded situations in theory of processes. Applications of Situation Theory, for which non-well-founded objects and sets are not needed, use versions of Situation Theory based on standard *ZFC* set theory. A modern approach to the phenomena of circularity, in various applications, including in semantics of human languages and programming, is presented in [4].

A set-theoretic modelling of Situation Theory as an axiomatic system, which insures identification of the situation-theoretic objects as set constructions is presented in [26]. The situation-theoretic objects introduced in our paper allow variants of such axiomatic systems for modelling partial, underspecified, and parametric information, by adding restricted parameters introduced here. Of particular interest are applications to logic programming and in areas that require relational structures with partially defined and parametric objects.

1.3 Related Lines of Work: Interdisciplinary Technologies

Recent years have been characterised with technological advancements across sciences and industries, by involving hardware and software engineering. Well established, classical theories and methodologies may be fully sufficient as the foundations of some of these new technologies. But the most challenging technological advances occur concurrently with new developments of their scientific foundations, including new methodologies, and new approaches to mathematical models of the domains, for which the technologies are used and applied. From this perspective, a new interdisciplinary areas are emerging, which conjoin theoretical developments in sub-areas that are often considered to be disjoint and developed separately, but are getting co-involved in the context of new technologies. In particular, the primary sub-areas that are forming foundations of new technology advances involve (1) mathematics of the concepts of computations, e.g., mathematics of algorithms and programs (2) classic and new approaches to computational models of various domains of applications (3) hardware and software engineering (4) computational approaches in life sciences.

A representative of new interdisciplinary areas has been emerging as *Domain Science and Engineering* (DSaE) (e.g., see [7]). On its side, our article represents ongoing research on development of Situation Theory, as a computational theory of information, which contributes to domain science, by modelling domains and domain dependent entities, parts, materials, relations, situations, states, events, etc. Situation Theory is information type-theory of domains of objects, materials, their properties, and relations between them, i.e., a typed model-theory of domains. We view DSaE approach as a realisation, in its domain science, of versions of Situation Theory specialised for applications in computer software engineering. The versions of Situation Theory vary depending on areas of applications. In its current stage, DSaE encompasses series of versions of Situation Theory that are software implementable. We consider that a new line of research is in due in DSaE, on inclusion of models of states, events, actions, processes, relations, and Situation Theory is a theory of such modelling, with versions depending on areas of applications.

1.4 The Main Goals of the Article

Situation Theory as a model of information with complex relations between compound objects, partiality, underspecification, and restricted parameters. Situation Theory is a powerful mathematical model of information, with expressiveness that is broad and ranges across many contemporary applications. Computerized information systems call for reliable, faithful representation of information. This requires theory of information, which does not distort information, especially when it is partial, or underspecified, and can be dynamically specified. Partiality can appear in various ways. For example, some objects have components that are partially defined functions or relations. Some of these partial functions and relations may or may not be extended over some of the objects that are not in their domains, regardless of circumstances. In other cases, information is underspecified by missing pieces of information and components, which can be added by dynamic updates or depending on the context of usage. Parametric information is a very important kind of information, e.g., where information structure is available, but various components participate as parameters, which can be either totally unrestricted (which is rarely the case), or vary within a broader type of objects, or are restricted to vary within a narrow domain subject by various compound conditions. Naturally, such conditions are expressed by propositional constraints. Situation Theory is an information theory that targets namely such goals: representation of information, which is relational and partial. It handles partially defined objects, which comprise parametric and otherwise underspecified information. Typically, such parametric objects are restricted to satisfy constraints and their specific instantiations vary depending on context.

Applications that need models of contexts, situated agents, resource situations. The use of computational semantics of human language, which is still an open initiative, can be resourceful and ranging across many applications, alongside the area of human language processing that includes semantic representations. Many of the contemporary systems in new technologies and information processing integrate human language processing, which can be more functional when integrated with related semantic information. Computational semantics has to meet various adequateness criteria (see [14]). A primary criterion is representation of partiality, underspecification, and context-dependency of semantic information. Typically, semantic information is essentially dependent on features such as contexts, described situations, agents, and agents' perspectives. We demonstrate application of Situation Theory to modelling such information structures, which include situations and objects that naturally occur in situations and participate in relations to other situated objects. Situations can vary across these relations and objects. Information that is presented in the situation-theoretical objects, including in situations, is partial and parametric. Parameters can be subject to restrictions consisting of partial information.

In the first part of this article, we concentrate on mathematics of its objects and concepts. In Sects. 2–5, we introduce Situation Theory as a type based information theory. It takes some set-theoretic objects as its primitive, basic objects and uses them in construction of more complex situation-theoretic objects, including situated types. We give examples from human language. They provide a clear grasp of the abstract mathematical objects, which can be used in other areas of application. In Sect. 6, we give a brief motivation of situation-theoretical objects as biologically realistic.

In the second part of the article, we demonstrate the potentials of Situation Theory for applications related to human language processing. E.g., we demonstrate that phenomena such as linguistic contexts and agents, which are traditionally considered as pragmatic and external to computational models, are subject to precise mathematical modelling in Situation Theory. This provides the foundation of integrating such objects in computational systems. In Sect. 7, we focus on modelling contexts and situated agents. In Sect. 8, we give situational models of objects designated by names and other definite descriptions, which depend on information available to agents and agents' references in contexts. This section provides a strong motivation of restricted parameters.

2 Situation Theory — Typed Information Theory

In this section, we introduce situation theoretical notions and objects that are fundamental for fine-grained modelling of information and information components. Situation Theory takes some set-theoretic objects as its basic objects. These basic objects then are used in the recursive construction of more complex situation theoretic objects. Informally, the basic informational pieces, called *infons*, are composite objects carrying information about relations and objects filling the arguments of the relations, at certain time and space locations. Infons can be basic or complex, by recursively defined system of objects. Infons are the ground, informational content of basic and complex informational objects, the informational content of situated propositions (introduced in Sect. 3), and other objects that carry information about situations. Infons are facts when supported by actual situations, e.g., in real or virtual worlds, theoretical models, or computerized models.

Primitive Individuals. A collection (typically, a set) $\mathcal{A}_{IND}$ is designated as the set of primitive *individuals* of the Situation Theory:

$$\mathcal{A}_{IND} = \{a, b, c, \ldots\} \tag{1}$$

The objects in $\mathcal{A}_{IND}$ are set-theoretic objects, but they are considered as primitives, not as complex situation-theoretic constructions. In various versions of Situation Theory, designated for specific applications, some of the individuals in $\mathcal{A}_{IND}$ may be parts of other individuals in $\mathcal{A}_{IND}$, and as such can be in respective *part-of* relations.

Space-Time Locations. Simplified versions of Situation Theory use a collection (typically, a set) $\mathcal{A}_{LOC}$ of space-time points and regions units:

$$\mathcal{A}_{LOC} = \{l, l_0, l_1, \ldots\} \tag{2}$$

The collection $\mathcal{A}_{LOC}$ is endorsed with relations of time precedence $\prec$, time overlapping $\circ$, space overlapping $\circ$, and inclusion $\subseteq_t$, $\subseteq_s$, $\subseteq$, between locations. In some versions of Situation Theory, the space-tile locations can be given by complex objects. E.g., a simple option (equivalent to the above) is that space-time locations are pairs of two components, one for space locations, and one for time points or periods.

Primitive Relations. Significantly, Situation Theory has a collection (typically, a set) $\mathcal{A}_{REL}$ of abstract, primitive objects that are relations:

$$\mathcal{A}_{REL} = \{r_0, r_1, \ldots\} \tag{3}$$

The elements of $\mathcal{A}_{REL}$ are abstract representatives of real or virtual relations. For example, if Situation Theory is used to model real world situations, these are abstract representatives of properties of objects and relations between objects. E.g., humans (as well as other living species) are attuned to distinguish properties of and relations between objects, perceptually in the reality, or cognitively, i.e., conceptually. We normally can recognise the property of an object to be a book, while the specifics of that property may be context dependent, a hardback book, a paperback, or e-book.

Note 1. In set theory, set-theoretic relations are defined as sets of tuples of set-theoretic elements that are being in those relations. On the contrary, the primitive relations of Situation Theory, i.e., the objects in $\mathcal{A}_{REL}$, are conceived as primitive entities: they are not sets of tuples of individuals being in those relations. E.g., to model this, the primitive relations in $\mathcal{A}_{REL}$, as well as the other primitive objects in Situation Theory, such as individuals and types, can be taken as urelements of the modelling set theory.

We maintain the notion of *extension*, by introducing more complex situation-theoretic objects: for a given relation $r \in \mathcal{A}_{REL}$ and a situation s, the extension of r in s is the set of all tuples of objects that are in the relation r in s. For example, we can distinguish when a primitive relation of reading holds between two objects: a reader and an object that is read.

The set $\mathcal{A}_{REL}$ depends on the actual application of Situation Theory[1]. For example,

$$\mathcal{A}_{REL} = \{\, man, woman, dog, run, smile, like, \dots \} \tag{4}$$

Primitive Types. A collection (typically, a relatively small set) of objects, which are called *primitive* or *basic* types:

$$B_{TYPE} = \{\, IND, LOC, REL, POL, ARG, \tag{5a}$$

$$INFON, SIT, PROP, PAR, TYPE, \models \} \tag{5b}$$

where the listed basic types are used in the following way: *IND* is the type for individuals; *LOC*: for space-time locations; *REL* for relations, primitive and complex (see (3) and Definition 10); *TYPE*: for primitive and complex types (see (5a)–(5b) and Definition 9); *PAR*: for basic and complex parameters (see (19a)–(19e) and Definition 12); *POL*: for two polarity objects, e.g., presented by the natural numbers 0 and 1; *ARG*: for abstract argument roles, basic and complex (see Definitions 1, 2, 9, 10); *INFON*: for situation-theoretical objects that are basic or complex information units (see Definition 4); *PROP*: for abstract objects that are propositions (see Definition 7); *SIT*: for situations (see Definitions 6, 5); $\models$ is a designated type called "supports".

We assume that Situation Theory has a set of basic $\mathcal{BA}_{ARG}$ argument roles, which are associated with primitive relations and properties, by respecting the following Definition (1).

Definition 1 (Assignment of basic argument roles). *A set of argument roles is assigned to each of the primitive relations and each of the primitive types, by a function Args having domain and range such that $Dom(Args) = \mathcal{A}_{REL} \cup B_{TYPE}$ and $Range(Args) \subseteq \mathcal{A}_{ARG}$, where $\mathcal{A}_{ARG}$ is the set of basic and complex (see Definition 2, 9, 10) argument roles, for a given set of basic roles $\mathcal{BA}_{ARG} \subset \mathcal{A}_{ARG}$.*

For example, we can associate relations, such as *smile*, *read*, *give*, respectively denoted by the lexemes smile, read, give, etc., with arguments roles:[2].

$$Args(smile) = \{smiler\} \tag{6a}$$

$$Args(read) = \{reader, read\text{-}ed\} \tag{6b}$$

$$Args(give) = \{giver, receiver, given\} \tag{6c}$$

Another option is to use a common set of shared primitive objects for argument roles: $\mathcal{BA}_{ARG} = \{arg_1, \dots, arg_n\}$, for a specific, sufficiently large natural number $n \geq 0$. Depending on applications of Situation Theory, the set $\mathcal{BA}_{ARG}$ of the

[1] The set-theoretical meta-theory of Situation theory, including representation of $\mathcal{A}_{REL}$, is not the subject of this article.

[2] In what follows, we shall follow a practice of naming the argument role of the object that is read, by the "misspelled" notations *read-ed* and *readed*.

available, basic argument roles can be chosen to be infinite. We can use as many argument roles as needed, e.g.:

$$Args(smile) = \{arg_1\} \tag{7a}$$
$$Args(read) = \{arg_1, arg_2\} \tag{7b}$$
$$Args(give) = \{arg_1, arg_2, arg_3\} \tag{7c}$$
$$Args(\gamma) = \{arg_1, \dots, arg_n\}, \text{for any relation } \gamma \text{ with } n\text{-arguments} \tag{7d}$$

Note that there is no implicit order over the argument roles in (7a)–(7d), where the indexing with numbers has the sole purpose of distinguishing the argument roles. Which role is for what in a relation depends on the actual modelling[3] of the relations and their arguments in the abstract theoretic constructions. For example, one can fix that: in (7b), arg_1 is for the *reader* and arg_2 — for what is *readed*; and in (7c), arg_1 is for the *giver*, arg_2 — for the *recipient*, and arg_3 — for the object *given*. After such setting, it has to be used consistently throughout in the constructions and in the modeled situations.

$$Args(read) = \{reader, readed\} \tag{8a}$$
$$Args(give) = \{giver, recipient, given\} \tag{8b}$$

Each relation that has a single argument role is called a *unary relation*, or more commonly a *property*.

Typically, properties of objects, like the property of smiling, and relations between objects, like the relation of reading, pertain in space-time locations. An optional choice is to consider such properties and relations as having a specialized argument role for a location:

$$Args(read) = \{reader, readed, Loc\} \tag{9a}$$
$$Args(give) = \{giver, recipient, given, Loc\} \tag{9b}$$

Another option is to take space-time locations as a special component of the basic informational units, which we shall introduce shortly. Our choice is based on our vision for future developments and applications of Situation Theory, by inclusion of complex spice-time models. For example by using time models integrated with three dimensional space models, objects, such as individuals, that are components of informational pieces can occupy specific space locations at various times. Informational pieces with relations, properties, and actions involving objects as components typically pertain to space-time locations.

Similarly to relations, each type is associated with a set of argument roles. If a type T has a single argument role, we call it a *unary type*, or a *property type*. In particular, IND, LOC, POL, PAR, $TYPE$, are unary types, each with one argument role, that can be declared as filled only by elements of the corresponding sets:

[3] Another option, "intermediate" between the above two, is to accept a relatively small set of common, abstract roles, which are similar to those used by traditional grammarians, and reintroduced in linguistics by the so-called Θ-theory of the Government and Binding Theory (GBT).

$$
\begin{aligned}
IND\colon \xi,\;& \text{for each } \xi \in \mathcal{A}_{IND} \cup \mathcal{P}_{IND} && \text{(10a)}\\
LOC\colon \xi,\;& \text{for each } \xi \in \mathcal{A}_{LOC} \cup \mathcal{P}_{LOC} && \text{(10b)}\\
REL\colon \xi,\;& \text{for each } \xi \in \mathcal{A}_{REL} \cup \mathcal{P}_{REL} && \text{(10c)}\\
& \text{and for each complex relation } \xi \text{ (introduced later)} &&\\
POL\colon \xi,\;& \text{for each } \xi \in \{0,1\} \cup \mathcal{P}_{POL} && \text{(10d)}\\
PAR\colon \xi,\;& \text{for each } \xi \in \mathcal{P}_{IND} \cup \mathcal{P}_{LOC} \cup \mathcal{P}_{REL} \cup \mathcal{P}_{POL} \cup \mathcal{P}_{SIT} && \text{(10e)}\\
& \text{and for each complex parameter } \xi \text{ (introduced later)} &&\\
TYPE\colon \xi,\;& \text{for each } \xi \in B_{TYPE} && \text{(10f)}\\
& \text{and for each complex type } \xi \text{ (introduced later)} &&
\end{aligned}
$$

Argument Roles and Appropriateness Constraints. The argument roles of both relations and types can be associated with types as constraints for their appropriate filling.

Definition 2 (Argument roles with appropriateness constraints). *A set of argument roles is assigned to each of the primitive relations, and to each of the primitive types, by a function Args, with its domain and range of values such that*

$$Dom(Args) = (\mathcal{A}_{REL} \cup B_{TYPE}), \tag{11a}$$

$$Range(Args) \subseteq (\mathcal{A}_{ARG} \times TYPE) \tag{11b}$$

so that for every n-ary primitive relation and every n-ary type γ, i.e., for every $\gamma \in \mathcal{A}_{REL} \cup B_{TYPE}$, which has n arguments:

$$Args(\gamma) = \{\langle arg_{i_1}, T_{i_1}\rangle, \ldots, \langle arg_{i_n}, T_{i_n}\rangle\}, \tag{12}$$

where $arg_{i_1}, \ldots, arg_{i_n} \in \mathcal{A}_{ARG}$ and $T_1, \ldots, T_n$ are sets of types (basic or complex).

The objects $arg_{i_1}, \ldots, arg_{i_n}$ are called the argument roles *(or* argument slots*, or simply* arguments*) of γ. The sets of types $T_1, \ldots, T_n$ are specific for the argument roles γ and are called the* basic appropriateness constraints of the argument roles *of γ.*

Notation 1. *Often, we shall use the notation (13):*

$$Args(\gamma) = \{T_{i_1} : arg_{i_1}, \ldots, T_{i_n} : arg_{i_n}\} \tag{13}$$

The most basic appropriateness constraints can be expressed by associating argument roles with primitive types, $T_{i_1}, \ldots, T_{i_n} \in B_{TYPE}$. For example:

$$
\begin{aligned}
Args(give) = \{\, & IND : giver, && \text{(14a)}\\
& IND : receiver,\; IND : given\,\} && \text{(14b)}
\end{aligned}
$$

For any relation or type (which can be primitive or complex), the objects that fill its argument roles are restricted to satisfy the constraints associated with the roles.

Definition 3 (Argument filling). *For any given relation $\gamma \in \mathcal{R}_{REL}$ and for any given type $\gamma \in \mathcal{T}_{type}$ associated with the set of argument roles $Args(\gamma) = \{T_{i_1} : arg_{i_1}, \ldots, T_{i_n} : arg_{i_n}\}$, an* argument filling *for γ is any total function θ with $Dom(\gamma) = \{arg_{i_1}, \ldots, arg_{i_n}\}$, which is set-theoretically defined by a set of ordered pairs $\theta = \{\langle arg_{i_1}, \xi_1\rangle \ldots, \langle arg_{i_n}, \xi_n\rangle\}$, so that its values, $\theta(arg_{i_1}) = \xi_1$, $\ldots$, $\theta(arg_{i_n}) = \xi_n$, satisfy the appropriateness constraints of the argument roles of γ: $T_{i_1} : \xi_1$, $\ldots$, $T_{i_n} : \xi_n$.*

Infons, State of Affairs (soas), Situations. Next, we shall give a mutually recursive definition of several sets of situational objects:

– the set $\mathcal{I}_{INF}$, the elements of which are called infons, and are basic or complex information units;
– the set $\mathcal{R}_{REL}$ of all primitive and complex relations (complex relations are defined later): $\mathcal{A}_{REL} \subset \mathcal{R}_{REL}$;
– the set $\mathcal{T}_{TYPE}$ of all primitive and complex types: $B_{TYPE} \subset \mathcal{T}_{TYPE}$;
– the collection $\mathcal{S}_{SIT}$ of situations.

The basic informational units are identified by a unique relation, an assignment of its argument roles and a corresponding negative or positive polarity.

Definition 4 (Infons). *The set $\mathcal{I}_{INF}$ of all infons:*

1. Basic infon *is every tuple $\langle \gamma, \theta, \tau, i\rangle$, where $\gamma \in \mathcal{R}_{REL}$ is a relation (primitive or complex), $LOC : \tau$ is a space-time location, (i.e., $\tau \in \mathcal{A}_{LOC}$), $POL : i$ is polarity (i.e., $i \in \{0, 1\}$), and θ is an argument filling for γ, i.e.:*
$$\theta = \{\langle arg_{i_1}, \xi_1\rangle, \ldots, \langle arg_{i_n}, \xi_n\rangle\} \tag{15}$$
for some situation-theoretical objects $\xi_1, \ldots, \xi_n$ satisfying the appropriateness constraints of γ.
2. *Let $\mathcal{BI}_{INF}$ be the set of all basic infons. $\mathcal{BI}_{INF} \subset \mathcal{I}_{INF}$.*
3. *For representation of conjunctive and disjunctive information,* complex infons *are formed by operators (i.e., primitive relations, for which locations are irrelevant) for conjunction and disjunction:*
For any infons $\sigma_1, \sigma_2 \in \mathcal{I}_{INF}$,
$$\langle \wedge, arg_1 : \sigma_1, arg_2 : \sigma_2\rangle \in \mathcal{I}_{INF} \tag{16a}$$
$$\langle \vee, arg_1 : \sigma_1, arg_2 : \sigma_2\rangle \in \mathcal{I}_{INF} \tag{16b}$$
Other complex infons are constructed from various situation theoretic objects, which we can add later.

Notation 2. *Often, in this article, we shall use a traditional linear notation of basic infons:*
$$\ll \gamma, arg_{i_1} : \xi_1, \ldots, arg_{i_n} : \xi_n, LOC : \tau; i \gg \tag{17a}$$
$$\ll \gamma, \xi_1, \ldots, \xi_n, \tau; i \gg \tag{17b}$$
$$\sigma_1 \wedge \sigma_2 \in \mathcal{I}_{INF}, \quad \sigma_1 \vee \sigma_2 \in \mathcal{I}_{INF} \tag{17c}$$

Note 2. The notation (17a) does not assume any innate order over the argument roles of γ. On the other hand, in case that γ has more than one argument roles, the notation (17b), e.g. as in (18b), (18d), makes sense only by having some agreement about a notational order over the argument roles of γ and their assignments, which does not imply that this is a 'natural order' of the argument roles of the relevant relation or type.

Example 1.

$$\ll \mathit{book}, \mathit{arg} : b, \mathit{Loc} : l; 1 \gg \tag{18a}$$

$$\ll \mathit{book}, b, l; 1 \gg \tag{18b}$$

$$\ll \mathit{read}, \mathit{reader} : a, \mathit{readed} : b, l; 1 \gg \tag{18c}$$

$$\ll \mathit{read}, a, b, l; 1 \gg \tag{18d}$$

Definition 5 (States of affairs, events, situations). *We define the following complex situational objects:*

1. State of affairs (soa) *is any set of infons that have the same location component.*
2. *An* event (course of event, coa) *is any set of infons.*
3. *A* situation *is any set of infons.*

Basic Parameters. For each of the basic types IND, LOC, REL, POL, SIT, Situation Theory that has a collection (a set) of *basic (primitive) parameters*:

$$\mathcal{P}_{IND} = \{\dot{a}, \dot{b}, \dot{c}, \ldots\}, \tag{19a}$$

$$\mathcal{P}_{REL} = \{\dot{r_0}, \dot{r_1}, \ldots\}, \tag{19b}$$

$$\mathcal{P}_{LOC} = \{\dot{l_0}, \dot{l_1}, \ldots\}, \tag{19c}$$

$$\mathcal{P}_{POL} = \{\dot{i_0}, \dot{i_1}, \ldots\}, \tag{19d}$$

$$\mathcal{P}_{SIT} = \{\dot{s_0}, \dot{s_1}, \ldots\}. \tag{19e}$$

Basic parameters are also called *indeterminates.* here we follow the original Situation Theory, by denoting specific basic parameters by dots. Often, we shall use "meta-variables" for basic parameters and the type shall be either explicitly stated or understood, e.g., typically, x is any parameter of type IND.

Definition 6 (Parametric states of affairs, events, situations). *Infons, states of affairs, and situations, in which some of the argument roles, including the space-time location and polarity components, are filled by parameters, are called, respectively,* parametric infons, parametric soas*, and* parametric situations.

Example 2

$$\ll \mathit{read}, \mathit{reader} : \dot{a}, \mathit{readed} : \dot{b}, \dot{l}; 1 \gg \tag{20a}$$

$$\ll \mathit{read}, \mathit{reader} : a, \mathit{readed} : \dot{b}, \dot{l}; 1 \gg \tag{20b}$$

$$\ll \mathit{read}, a, b, l; \dot{i} \gg \tag{20c}$$

3 Situated Propositions and Constraints

The version of Situation Theory that we introduce in this article is general, especially with respect to the nature of many of the primitive objects, and has capacities for covering a broad spectrum of applications. We use a specialized primitive type $PROP \in B_{TYPE}$, with two argument roles: a type $\mathbb{T} \in \mathcal{T}_{TYPE}$, and an appropriate argument filling θ for $\mathbb{T}$. We shall use the type $PROP$ for constructing abstract objects (set-theoretic tuples) to model the abstract notion of a proposition, which states that the objects given by θ are of the type $\mathbb{T}$, in the following way:

Definition 7 (Propositions). Proposition *is any tuple* $\langle PROP, \mathbb{T}, \theta\rangle$, *where* $\mathbb{T} \in \mathcal{T}_{TYPE}$ *is a type that is associated with a set of argument roles*

$$Args(\mathbb{T}) = \{T_{i_1} : arg_{i_1}, \ldots, T_{i_n} : arg_{i_n}\} \tag{21}$$

and θ *is an argument filling for* $\mathbb{T}$*, i.e.:*

$$\theta = \{\langle arg_{i_1}, \xi_1\rangle, \ldots, \langle arg_{i_n}, \xi_n\rangle\} \tag{22}$$

for objects $\xi_1, \ldots, \xi_n$*, such that* θ *satisfies the appropriateness constraints of* $\mathbb{T}$*:*

$$T_{i_1} : \xi_1, \ldots, T_{i_n} : \xi_n. \tag{23}$$

Notation 3. *We use the notation* $(\mathbb{T}, \theta)$ *for* $\langle PROP, \mathbb{T}, \theta\rangle$*.*

When a proposition $\langle PROP, \mathbb{T}, \theta\rangle$ is true, we say that the objects $\xi_1, \ldots, \xi_n$ are of type $\mathbb{T}$ with respect to the argument role filling θ, and we write $\mathbb{T} : \theta$, or, in case it is clear which roles are filled by which objects, $\mathbb{T} : \xi_1, \ldots, \xi_n$. I.e., propositions are the result of filling up the argument roles of a type with appropriate objects. We shall use a special kind of propositions defined by Definition 8, based on the primitive type $\models$. The type $\models$, pronounced "support", has two argument roles, one that can be filled by any object that is of the type SIT of situations, and the other can be filled by any object that is of the type INF of inforns. I.e.:

$$\begin{aligned} Args(\models) &= \{\, \langle arg_{sit}, SIT\rangle, \langle arg_{infon}, INF\rangle \,\} && \text{(24a)} \\ &\equiv \{\, SIT : arg_{sit}, INF : arg_{infon} \,\} && \text{(24b)} \end{aligned}$$

Definition 8 (Situated propositions). Situated proposition *is a situation-theoretical object*

$$\langle PROP, \models, s, \sigma\rangle \tag{25}$$

where $s \in \mathcal{P}_{SIT}$ *and* $\sigma \in \mathcal{I}_{INF}$*.*

Notation 4. *We use the notation* $(s \models \sigma)$ *and say "the proposition that* σ *holds in the situation* s*" or "the proposition that the situation* s *supports the infon* σ*".*

Example 3.

$$\begin{aligned} (s \models &\ll book, arg : b, Loc : l; 1 \gg \wedge && \text{(26a)} \\ &\ll read, reader : x, readed : b, Loc : l; 1 \gg) && \text{(26b)} \end{aligned}$$

4 Complex Types and Relations

Situation Theory uses an abstraction operator, which recalls the λ-abstraction in functional λ-calculi, but, in Situation Theory, the abstraction operator is different. It is purely semantic, i.e., informational abstraction (not for a syntactic construction of a λ-expression in a language), and defines abstract, complex types and relations, some of which can be encoded by functions, but some of them can not. In this version of Situation Theory, we introduce the abstraction operator as producing complex types, with abstract argument roles.

Definition 9 (Complex types and appropriateness constraints). *Let Θ be a given proposition, and $\{\xi_1, \ldots, \xi_n\}$ be a set of parameters that occur in Θ. Let, for each $i \in \{1, \ldots, n\}$, T_i be the union of all the appropriateness constraints of all the argument roles that occur in Θ, and which ξ_i fills up*[4]*.*

Then the object $\lambda\{\xi_1, \ldots, \xi_n\}\, \Theta \in \mathcal{T}_{TYPE}$, i.e., $\lambda\{\xi_1, \ldots, \xi_n\}\Theta$ is a complex type*, with abstract argument roles denoted by $[\xi_1], \ldots, [\xi_n]$ and corresponding* appropriateness constraints *associated in the following way:*

$$Args(\lambda\{\xi_1, \ldots, \xi_n\}\Theta) = \{T_1 : [\xi_1], \ldots, T_n : [\xi_n]\} \tag{27a}$$

The type $\lambda\{\xi_1, \ldots, \xi_n\}\Theta$, where Θ is a proposition, is alternatively denoted by

$$[\xi_1, \ldots, \xi_n \mid \Theta] \tag{28a}$$

$$[T_1 : \xi_1, \ldots, T_n : \xi_n \mid \Theta]. \tag{28b}$$

Sometimes, we shall use a mixture of λ and bracketed notation, for discriminating between the types of the abstracted away parameters.

Example 4. The situation-theoretical object (29a) is the type of situations and locations where the specific individual a walks; (29b) is the type of individuals that walk in a specific situation s and a specific location l; (30a)–(30b) is the type of individuals that read a specific book b, in a specific situation s and a specific location l; (31a)–(31b) is the type of situations, locations and individuals, where the individual reads a specific book b:

$$\lambda \dot{s}, \dot{l}\, (\dot{s} \models \ll walk, walker : a, Loc : \dot{l}; 1 \gg) \tag{29a}$$

$$\lambda x\, (s \models \ll walk, walker : x, Loc : l; 1 \gg) \tag{29b}$$

$$\lambda x\, (s \models \ll read, reader : x, readed : b, Loc : l; 1 \gg \wedge \tag{30a}$$

$$\ll book, arg : b, Loc : l; 1 \gg) \tag{30b}$$

$$\lambda \dot{s}, \dot{l}, x\, (\dot{s} \models \ll read, reader : x, readed : b, Loc : \dot{l}; 1 \gg \wedge \tag{31a}$$

$$\ll book, arg : b, Loc : \dot{l}; 1 \gg) \tag{31b}$$

[4] Note that ξ_i may fill more than one argument role in Θ.

Notation 5. *For given object α and a set of appropriateness constraints T, we write $T : \alpha$ iff α satisfies all the constraints in T.*

Property 1. Let Θ be a given proposition and $\{\xi_1, \ldots, \xi_n\}$ be a set of parameters that occur in Θ. Let, for each $i \in \{1, \ldots, n\}$, T_i be the union of all the appropriateness constraints of all the argument roles that occur in Θ and ξ_i fills up. Given that $\alpha_1, \ldots, \alpha_n$ are objects that satisfy appropriateness constraints $T_1 : \alpha_1, \ldots, T_n : \alpha_n$, we have:

1. by Definition 9, $\lambda\{\xi_1, \ldots, \xi_n\}\,\Theta \in \mathcal{T}_{TYPE}$ is a complex type with argument roles such that

$$Args(\lambda\{\xi_1, \ldots, \xi_n\}\Theta) = \{T_1 : [\xi_1], \ldots, T_n : [\xi_n]\} \tag{32a}$$

2. Let θ be the total function that is set-theoretically defined by the set of ordered pairs $\theta = \{\langle[\xi_1], \alpha_1\rangle \ldots, \langle[\xi_n], \alpha_n\rangle\}$,
   (a) by Definition 3, θ is an argument filling for the type $\lambda\{\xi_1, \ldots, \xi_n\}\,\Theta$.
   (b) by Definition 7: $(\lambda\{\xi_1, \ldots, \xi_n\}\,\Theta : \theta)$ is a proposition, i.e., the proposition that the objects from the argument the filling θ are of the complex type $\lambda\{\xi_1, \ldots, \xi_n\}\,\Theta$, i.e.:

$$\langle PROP, \lambda\{\xi_1, \ldots, \xi_n\}\,\Theta, \theta\rangle \equiv (\lambda\{\xi_1, \ldots, \xi_n\}\,\Theta : \theta) \tag{33}$$

Abstractions over individuals in propositions result in *complex types of individuals*. In general, for any given proposition Θ and a parameter ξ for an individual, i.e., $IND : \xi$, which occurs in Θ, the situation-theoretical object $\lambda\{\xi_1\}\,\Theta \in \mathcal{T}_{TYPE}$ is a complex type, that is the type of the individuals for which the proposition $\Theta(\xi_1)$ is true.

In order to complete the recursive definition of the complex objects in Situation Theory, next we define complex relations, while in this article we do not use them actively.

Definition 10 (Complex relations and appropriateness constraints). *Let $\rho \in \mathcal{R}_{REL}$ be a given relation, and $\{\xi_1, \ldots, \xi_n\}$ be a set of parameters that occur in ρ. Let, for each $i \in \{1, \ldots, n\}$, T_i be the union of all the appropriateness constraints of all the argument roles that occur in ρ, and which ξ_i fills up*[5]*.*

Then the object $\lambda\{\xi_1, \ldots, \xi_n\}\,\rho \in \mathcal{R}_{REL}$, i.e., $\lambda\{\xi_1, \ldots, \xi_n\}\rho$ is a complex relation*, with abstract argument roles denoted by $[\xi_1], \ldots, [\xi_n]$, and corresponding* appropriateness constraints *associated in the following way:*

$$Args(\lambda\{\xi_1, \ldots, \xi_n\}\rho) = \{T_1 : [\xi_1], \ldots, T_n : [\xi_n]\} \tag{34a}$$

The relation $\lambda\{\xi_1, \ldots, \xi_n\}\rho$, is alternatively denoted by

$$[\xi_1, \ldots, \xi_n \mid \rho] \tag{35a}$$
$$[T_1 : \xi_1, \ldots, T_n : \xi_n \mid \rho]. \tag{35b}$$

[5] Note that ξ_i may fill more than one argument role in ρ.

5 Complex Parameters with Restrictions

Any basic parameter x of type τ (i.e., $\tau : x$) can be properly assigned only to a situation theoretic object of type τ. Complex restricted parameters can be properly assigned only to objects that satisfy the constraints associated with the restricted parameters. Associating basic parameters with types has constraining effect. Thus, parameter assignments of both basic and restricted parameters are constrained.

Definition 11 (Consistent types). *For any finite set T of types:*

1. *T is* consistent *iff there is at least one situation theoretic object that is of each of the types in T.*
2. *A type τ is* compatible *with T iff the set $\{\tau\} \cup T$ is consistent.*

Definition 12 (Parameters). *Basic* (19a)–(19e) *and restricted parameters are parameters.*

Restricted Parameters.

1. *Let T be a finite (and consistent) set of types. If x is a fresh parameter of type τ, i.e., $\tau : x$, and τ is compatible with the set T of types, then $x^{\{\tau\} \cup T}$ is a parameter of type $\{\tau\} \cup T$. We say that $x^{\{\tau\} \cup T}$ is a* parameter restricted by $\{\tau\} \cup T$.
2. *Let ξ be a parameter and $\Theta(\xi)$ a proposition, such that ξ is a constituent of $\Theta(\xi)$ (i.e., ξ fills at least one argument role in $\Theta(\xi)$). Let T be the set of all types associated with all the argument roles in $\Theta(\xi)$ that are filled by ξ*[6]*. (I.e., $\lambda\xi\, \Theta(\xi)$ is a type and T is the set of the appropriateness constraints of its argument role.) If the set T of types is consistent, and x is a fresh parameter of type τ, i.e., $\tau : x$, such that τ is compatible with T, then $x^{\lambda\xi\, \Theta(\xi)}$ is also a parameter of type τ. We say that $x^{\lambda\xi\, \Theta(\xi)}$ is a* parameter restricted by $\lambda\xi\, \Theta(\xi)$.
   With the alternative denotation of the complex type $[\xi \mid \Theta(\xi)]$, the restricted parameter $x^{\lambda\xi\, \Theta(\xi)}$ is denoted by $x^{[\xi \mid \Theta(\xi)]}$.

For any situation theoretic object $\gamma(x^r)$, in which the restricted parameter x^r is a constituent, we can "connect" some or all of the parameters in it to objects by a parameter assignment function.

A parameter assignment c is defined on x^T, where T is a set of consistent types, only if the proposition $(c(x^T) : \tau)$ is true for each type $\tau \in T$.

A parameter assignment c is defined on $x^{[\xi \mid \Theta(\xi)]}$ only if the proposition $(c(x^{[\xi \mid \Theta(\xi)]}) : [\xi \mid \Theta(\xi)])$ is true; i.e., only if there is a parameter assignment c' for $\Theta(\xi)$, such that $c'(\xi) = c(x^{[\xi \mid \Theta(\xi)]})$ and the proposition $c'(\Theta(\xi))$ is true.

Note that the restricted parameter $x^{[\xi \mid \Theta(\xi)]}$ is defined even if the proposition $\Theta(\xi)$ may not be true, but an object a can instantiate the parameter $x^{[\xi \mid \Theta(\xi)]}$ only if the proposition $(c(x^{[\xi \mid \Theta(\xi)]}) : [\xi \mid \Theta(\xi)])$ is true for $c(x^{[\xi \mid \Theta(\xi)]}) = a$, i.e., $c'(\Theta(\xi)]$ is true for some parameter assignment $c'(\xi) = a$. This has been our motivation for defining restricted parameters with types as restrictions, instead of with complex relations.

[6] Note that ξ may fill more than one argument role in $\Theta(\xi)$.

6 Biological Basis of Situation Theory

Restricted parameters represent generic patterns, "blueprints", that can be instantiated, i.e., realised, by specific objects that satisfy the corresponding restrictions and are of respective types. In nature, biological entities carry blueprints that are restricted according to shared features, e.g., of species. Parameter assignments represent specific realisations of the generic components in specific instances.

We take a stand that human cognitive abilities and faculties, that are universal for humans, are expressed by innate brain capacities for some fundamental operations:

- perception and recognition of entities, smells, sounds, etc., that are located in three-dimensional space, in time, and situated in environments
- perception and recognition of properties and relations, primitive and complex, "possessed" by entities, in space, time, and situated in environments
- human brain faculties associate properties and relations with abstract and specific objects, by argument roles and argument role assignments
- recognition of abstract patterns, i.e., of types and parametric objects
- pattern construction via primitive abstract types and abstraction over parametric objects
- pattern construction via restrictions over parameters
- pattern matching i.e., an entity $\mathcal{O}$ is of type τ, $\tau : x$.

Restricted parameters reflect innate human faculty for development and attainment of concepts of objects that have some properties and are in relations to other kinds of objects, not necessarily referring to specific objects in the reality. A youngster or an adult person can get an idea what an object with certain properties could be, without having seen any such objects, in reality or in other ways depicted. Such concepts are not necessarily expressed by or associated with language. Parameter assignments correspond to instantiations with particular objects and can represent references to particular objects, concrete and fully determined without parameters, or abstract, with parametric components.

7 Application of Situation Theory to Modelling Context Dependency

Human language is used in contexts, that can be spoken, written, pictural, virtual, in reasoning, "in the mind", or combining any of these ways of usage. Language can be used by speakers that know its abstract linguistic meanings and how the abstract linguistic meanings can be "connected", i.e., assigned to specific interpretations. Abstract linguistic meanings, taken out of any context of use, carry semantic information, which is partial, parametric and sometimes ambiguous. I.e., normally, abstract linguistic meanings, out of context, have structure with parametric constituents and abstractions over parameters. When used in specific contexts, the abstract linguistic meanings are assigned to specific interpretations, by the speakers and listeners. The interpretations in context can still be parametric and partial. Ambiguities are typically resolved by speakers' and listeners's who interprete depending on their perspectives.

Partiality of information about the objects designated by language parts is by introducing primitive and complex, i.e., restricted, parameters. The restriction r over a parameter x^r represents a constraint $r : a$ that is necessary for an object a to be associated with the parameter x^r in a larger piece of information $\gamma(x^r)$. The assignment of an object a to x^r in $\gamma(x^r)$ results in the instantiation $\gamma(a)$. The constraint r itself is not per-se a part of $\gamma(a)$, but is an additional, necessary-constraint information, satisfied by a, i.e., a is of type r, $r : a$. A speaker-agent uses the restriction r to designate the object a, by assigning it to x^r. The listener-agent identifies the object a filling the arguments in $\gamma(a)$, by the constraint $r : a$.

7.1 Linguistic Utterance Components

We follow a tradition of using the technical notion of an *utterance*, as a situation type representing minimal components of context, which are crucial for association of linguistic meanings with potentials for specific interpretations in specific contexts, i.e., in "utterances" of expressions, by speakers addressing listeners. In practice, the technical notion of an utterance can be realised for spoken, written, or combined language use. Depending on the areas of applications of Situation Theory, linguistic contexts can be extended. The context (discourse) components include, as a minimum, the following kinds of information:

1. *Pure linguistic information:* The expressions uttered are presented by a syntax-semantics interface structure, which determines its abstract linguistic meaning. The author of this article supports the view that the syntax-semantics interface in human language is innate faculty of brain physiology. Computational approaches to language processing would be more intelligent and adequate by taking such a perspective.
2. *Broad-linguistic information by utterance components:* Context contributes essential semantic information, which is not always explicitly present in the wording of expressions. The most prominent components of context are: the "speaker" agent that delivers the expression, for example by an utterance; the listener agent(s) that are addressed interpreters; the time and the space location of the utterance; the speaker's references that assign particular objects to language components; the knowledge and the intentions of the speaker and the listener that contribute to interpretations of abstract linguistic meanings, by assigning objects to parameters, and disambiguation. Such information can be presented by abstract utterance types, as parametric situation theoretic constructs.

3. *Extra-linguistic utterance information:* Various components of language use contribute semantic information, e.g., language specific word order and word inflection paradigms, punctuation, speech acts, intra-sentential punctuation, intonation, gesture and other means for expressing speaker's perspectives, stress, presenting "new" vs. "old" information.

7.2 Situated Linguistic Agents

Denotations of human language expressions in specific contexts may depend on reference acts. A *linguistic reference act* is an event consisting of at least the following components: a language expression, an object (real or abstract) referred to, which is called the referent of the expression, and an utterance situation (or a broader discourse). The *utterance situation* consists of subcomponents such as the *speaker*, the *speaker's reference act*, the *space-time location of the utterance*, and the *listener(s)*.

Definition 13. *1. The infon* (36a) *models the information that an individual x utters an expression α by addressing a listener y. We call any infon such as* (36a) *an* utterance infon*. By using Notation 2, this infon is represented as* (36b)*.*

2. The infon (36c) *models the information that, an individual x refers to an object z by using an expression α. We call any infon such as* (36c) *a* reference infon*. By using Notation 2, this infon is represented as* (36d)*.*

3. A situation u that supports (i.e., has as an element) an utterance infon, as in (36e)*, is called an* utterance situation *(or briefly an* utterance*). In case that u supports exactly one utterance infon, the object x filling the argument role speaker of the relation tells is called* the speaker *in u; the object y filling the argument role listener is called the* listener(s) *in u. Note that, in general, y can be a set of individuals. i.e., listeners. We allow broader utterance situations with more than one utterance infons, speakers and listeners, which may have entirely different locations that may be related by overlapping or precedence.*

4. When the expression α is an expression with which speakers can refer to objects[7]*, the utterance situation can support also a reference infon, as in* (36f)*. In such a case, the respective reference infon $\ll referes\text{-}to, x, z, \alpha, l; 1 \gg$ is called a* speaker's reference act *in u.*

$$\ll \mathit{tells}, \mathit{speaker} : x, \mathit{listener} : y, \mathit{uttered} : \alpha, \mathit{Loc} : l; 1 \gg \tag{36a}$$

$$\ll \mathit{tells}, x, y, \alpha, l; 1 \gg \tag{36b}$$

$$\ll \mathit{referes\text{-}to}, \mathit{speaker} : x, \mathit{referent} : z, \mathit{by} : \alpha, \mathit{Loc} : l; 1 \gg \tag{36c}$$

$$\ll \mathit{referes\text{-}to}, x, z, \alpha, l; 1 \gg \tag{36d}$$

$$u \models \ll \mathit{tells}, x, y, \alpha, l; 1 \gg \tag{36e}$$

$$u \models \ll \mathit{tells}, x, y, \alpha, l; 1 \gg \wedge \ll \mathit{referes\text{-}to}, x, z, \alpha, l; 1 \gg \tag{36f}$$

[7] Such reference expressions include many noun phrases (NPs) in human languages, e.g., names, pronounce, and definite descriptions.

The object z in the reference act depends on the utterance u and its components. It can be a specific, fully identified object or a parameter that may be restricted, as in the examples that follow.

By using situation theoretical objects with restricted parameters, the utterance components can be modeled by situation-theoretical objects as follows (see also [11–13]).

The proposition expressing who is the speaker x, who is the listener y, what is the space-time location, and which is the expression α uttered in an utterance situation u, i.e., a minimum of context information is expressed by the situated proposition (37):

$$pu(u, l, x, y, \alpha) \equiv (u \models \ll \mathit{tells}, x, y, \alpha, l; 1 \gg) \tag{37}$$

Then, (38) is an abstract type of an utterance situation.

$$ru(l, x, y, \alpha) \equiv [u \mid pu(u, l, x, y, \alpha)] \tag{38}$$

The type (39) is the type of a speaker agent in an utterance situation u.

$$rsp(u, l, y, \alpha) \equiv [x \mid pu(u, l, x, y, \alpha)] \tag{39}$$

The type of an individual to be a listener agent in an utterance situation u is (40):

$$rlst(u, l, x, \alpha) \equiv [y \mid pu(u, l, x, y, \alpha)] \tag{40}$$

The type of an object to be the utterance (or discourse) space-time location is given by (41):

$$rdl(u, x, y, \alpha) \equiv [l \mid pu(u, l, x, y, \alpha)] \tag{41}$$

The type (42) is a type for the *referent agent*, i.e., of the objects to be referred to by an expression α in an utterance situation.

$$r_\alpha(u, l, x, y, s_{res}) = [z \mid q(u, l, x, y, z, \alpha)] \tag{42}$$

where $q(u, l, x, y, z, \alpha)$ is a proposition such as (43a) or (44a).

$$q(u, l, x, y, z, \alpha) \equiv \tag{43a}$$
$$(u^{ru(l,x,y,\alpha)} \models \tag{43b}$$
$$\ll \mathit{referes\text{-}to}, x^{rsp(u,l,y,\alpha)}, z, \alpha, l^{rdl(u,x,y,\alpha)}; 1 \gg) \tag{43c}$$

The proposition (43a), i.e., (43b)–(43c) asserts that the speaker x^{rsp} refers to z by using the expression α, in the location $l^{rdl(u,x,y,\alpha)}$. Here

- The situation parameter $u^{ru(l,x,y,\alpha)}$ is restricted by the type $ru(l, x, y, \alpha)$ of a situation being an utterance.
- The situation parameter s_{res} is for a resource situation, which in the case of (43b)–(43c) is $s_{res} \equiv u^{ru(l,x,y,\alpha)}$.

A more elaborate representation of the names can be expressed by the following version of the proposition $q(u, l, x, z, \alpha)$:

$$q'(u, l, x, y, z, \alpha, s_{res}) \equiv \tag{44a}$$
$$(u^{ru(l,x,y,\alpha)} \models \ll \textit{referes-to}, x^{rsp(u,l,y,\alpha)}, z, \alpha, l^{rdl(u,x,y,\alpha)}; 1 \gg \wedge \tag{44b}$$
$$\ll \textit{believes}, x^{rsp(u,l,y,\alpha)}, \tag{44c}$$
$$(s_{res} \models \ll \textit{named}, \alpha, z, l_{res}; 1 \gg), \tag{44d}$$
$$l^{rdl(u,x,y,\alpha)}; 1 \gg) \tag{44e}$$

The proposition (44a), i.e., (44b)–(44e), asserts that the speaker x^{rsp} refers to z by using the name α, by believing that z is named α. Alternatively, when the speaker knows that the referent z is named α, we can use the relation *knows* instead of *believes*. In what follows, all the above restrictions shall be written without explicitly specifying the parameter arguments.

8 Named Objects and Information Dependent on Names

In this section, we turn to examples of referential expressions, such as proper names and definite descriptions, for exposition of how Situation Theory can handle such semantic phenomena. Semantics of naming expressions gives essential contributions to semantics of larger, encompassing language constructions, e.g., such as sentences and upward to larger texts. However, it is important how those contributions are handled computationally, where is their proper placement in the semantic representations, all of which should also take into account the context and agent dependency of their semantics.

A distinctive semantic contribution of naming expressions provides means for *potential reference to objects*, by the language users, i.e., "speaker" and "listener" agents in context, e.g., by using sentences, and so forth, up to text discourse. Typically, by utterances of affirmative sentences, speakers describe some situations (not necessarily the same as the utterances) as holding facts (i.e., infons). The objects, which are the referents of name sub-expressions, participate as fillers of arguments roles of semantic relations, in the facts that are stated to hold in the described situations, by utterance situations.

It is important not to misplace the additional, auxiliary semantic contribution of the naming sub-expressions as direct components of the facts (i.e. of the infons) that are the informational content of the proposition stated by a sentence utterance, and not directly in the facts of the utterance itself.

E.g., by an utterance u of a sentence like "Maria is reading the book", a speaker may describe a situation s_1 as holding that a specific individual $z^{r_{\text{MARIA}}}$, referred to by the name "Maria", is involved in the activity of reading a specific book $w^{d_{\text{THE BOOK}}}$, referred to by the definite description "the book", in a location $l_1^{[l|l \circ l^{rdl}]}$. This is expressed by the proposition (45):

$$(s_1 \models \ll \textit{read}, z^{r_{\text{MARIA}}}, w^{d_{\text{THE BOOK}}}, l_1^{[l|l \circ l^{rdl}]}; 1 \gg) \tag{45}$$

The described situation s_1 may be part of or the same as the utterance u, i.e., $s_1 \subseteq u$. The possibility that s_1 is fully disjoint from u, i.e., $s_1 \cap u = \emptyset$, is left open. An utterance u of this sentence and the described situation s_1 are related via the speaker's references in the utterance. The speaker uses the name "Maria" and the definite description "the book" to identify correspondingly the participants $z^{r_{\text{MARIA}}}$ and $w^{d_{\text{THE BOOK}}}$ of the fact of reading in (45). By the inflection of the verb lexeme "read" (present-time continuous), the reading is located in a space-time location l_1, which is related to the space-time location of the utterance with overlapping via the restriction over it $l_1^{[l|l \circ l^{rdl}]}$. These pieces of information, including that the reader z is named "Maria" and that the object w is having the property of being a book (the unique one to which the speaker refers) are carried by the sentence, but they are auxiliary to the major propositional content expressed by the infon in (45). Such pieces of information should not be indiscriminately conjoined into the major propositional content. The restrictions over the parameters $z^{r_{\text{MARIA}}}$ and $w^{d_{\text{THE BOOK}}}$ "distribute" such information, which is linked to the facts in the utterance and the propositional content, i.e., to facts of the described situation.

In general, for a given naming expression α, its denotation[8] $\mathsf{den}(\alpha) = z^{r_\alpha}$ is given by a restricted parameter, where r_α is like (42), i.e., $r_\alpha(u, l, x, y, s_{res}) = [z \mid q(u, l, x, y, z, \alpha)]$, which is dependent on the specific expression α and other context information expressed by the proposition $q(u, l, x, y, z, \alpha)$ as in (43a) or (44a). Depending on the expression α, context, and applications, r_α may have more or alternative constraints in it, e.g., by (44b)–(44e). Importantly, the object z^{r_α} is parametric and its instantiations are subject to the constraint r_α expressed by the semantics of a name α, e.g., as in (46a)–(46b), (47a), and a definite description α, e.g., as in (48a)–(48d).

Potentially, an utterance, the speaker's references in it, which are expressed by restricted parameters like z^{r_α}, and a broader context can provide a specific object referred to by an expression α, as an instantiation of the restricted parameter z^{r_α}. The instantiated object has to satisfy the restriction r_α. The restricted parameter z^{r_α} can get linked to a specific referent depending on the specific utterance context and the speaker agent. That specific referent, subjected to satisfaction of the constraint r_α, can fill up relation arguments in facts described by a larger expression, in which the name α occurs, e.g., as in (49). The restriction r_α, while a direct component of the restricted parameter z^{r_α} itself, provides "extra", i.e., "auxiliary", semantic information, which is linked to the direct semantic content of the larger expression.

Example 5.

$$r_{\text{MARIA}} \equiv [z \mid (u^{ru(l,x,y,\text{MARIA})} \models \tag{46a}$$

$$\ll \textit{referes-to}, x^{rsp}, z^{n_{\text{MARIA}}}, \text{MARIA}, l^{rdl}; 1 \gg)] \tag{46b}$$

[8] In order to keep the article into its major topic, we present the denotation function den without diverging to more theoretical technicalities, which are subjects to other ongoing and future work.

where the parameter z is recursively restricted by the type restriction n_{MARIA} in (47a), which expresses that the object z is named MARIA by x^{rsp} in a resource situation s_0:

$$n_{\text{MARIA}} \equiv [z \mid (s_0 \models \ll named, \text{MARIA}, x^{rsp}, z, l_0; 1 \gg) \tag{47a}$$

Example 6. The linguistic meaning of a noun phrase (NP) that is a definite description, e.g., "the book", can be expressed by $w^{d_{\text{THE BOOK}}}$, where $d_{\text{THE BOOK}}$ is the type (48a)–(48d), and s_2 and l_2 are parameters for a *resource situation* and its resource *location* for evaluation of the NP "the book". The resource situation s_2 and some of its component locations l_2 are provided by the references of the speaker agent, and while they might be the same as the utterance situation u and location, respectively, they might as well be "external" to the utterance situation and subjected to additional constraints over parameters.

$$d_{\text{THE BOOK}} \equiv [z \mid (s_2 \models \ll book, z, l_2; 1 \gg \wedge \tag{48a}$$

$$\ll unique, z, \tag{48b}$$

$$[z \mid (s_2 \models \ll book, z, l_2; 1 \gg)], \tag{48c}$$

$$l_2; 1 \gg)] \tag{48d}$$

Example 7. The abstract, linguistic meaning of a sentence like "Maria is reading the book" can be designated by the following situated propositional type:

$$\lambda s_0, s_1, s_2, l_0, l_1, l_2 (s_1 \models \ll read, z^{r_{\text{MARIA}}}, w^{d_{\text{THE BOOK}}}, l_1^{[l \mid l \circ l^{rdl}]}; 1 \gg) \tag{49}$$

where r_{MARIA} and $d_{\text{THE BOOK}}$ are, respectively, the constraints (46a)–(46b) and (48a)–(48d). The semantic λ-abstractions over the parameters $s_0, s_1, s_2, l_0, l_1, l_2$ represent the type of a relation between situations and locations, where s_0 and l_0 for the naming situation and location, s_1 and l_1 for the described situation of reading activity, s_2 and l_2 for the resource situation identifying the object that is the book.

Alternatively, the parameters for the described situation and location, any naming and resource situations and locations, in this example $s_0, s_1, s_2, l_0, l_1, l_2$, can be left as parameters that are free of λ-abstraction. In such a case, the abstract, linguistic meaning of a sentence like "Maria is reading the book" is an asserted parametric proposition, (50) for this specific sentence, where s_1 is the described situation:

$$(s_1 \models \ll read, z^{r_{\text{MARIA}}}, w^{d_{\text{THE BOOK}}}, l_1^{[l \mid l \circ l^{rdl}]}; 1 \gg) \tag{50}$$

In both cases, (49) and (50), the proposition expressed by an utterance of the sentence is $(s_1 \models \ll read, z^{r_{\text{MARIA}}}, w^{d_{\text{THE BOOK}}}, l_1^{[l \mid l \circ l^{rdl}]}; 1 \gg)$. The propositional information content is represented by the infon $\ll read, z^{r_{\text{MARIA}}}, w^{d_{\text{THE BOOK}}}, l_1^{[l \mid l \circ l^{rdl}]}; 1 \gg$, where the restricted parameters $z^{r_{\text{MARIA}}}$, $w^{d_{\text{THE BOOK}}}$, $l_1^{[l \mid l \circ l^{rdl}]}$ link the proposition with the additional information, which is carried by the sentence sub-expressions, about the objects denoted by the name "Maria" and the definite description "the book". The restrictions over the component parameters of the proposition carry information about any potential utterance u of the sentence and the utterance components, such as the speaker, the addressee, and the locations, via the situation-theoretic objects (37)–(41). These general parametric patterns for a potential utterance situation u, ($u \models \ll tells, x, y, \alpha, l; 1 \gg$), and its components are instantiated for the specific expression α, i.e., the sentence $\alpha \equiv$ MARIA IS READING THE BOOK.

9 Conclusions and Future Work

Underspecification, partiality, and context dependency present major difficulties in related theoretical developments and adequate applications, including development of dedicated software systems, decision-problem models, and solutions involving models of states, events, actions, context, and other situations, where information can be partial and parametric. Situation Theory is a finely-grained, mathematical model of information, which respects the fundamentals of information in nature. We consider versions of Situation Semantics as its prominent applications for models of informational content for semantics of both natural and artificial languages. Various domain-dependent versions of Situation Semantics are proliferating in contemporary technologies and software systems. Mathematics of Situation Theory and its applications are open for developments depending on specific areas of applications in Computer Science and new technologies, such as Computational Semantics, Natural Language Processing, Artificial Intelligence, Cognitive Science, computational approaches to neuroscience, medical sciences, health care, ontology frameworks, etc.

Conclusions: Advances in Theory for Applications to New Technologies. This article is part of our broader work on theoretical development of Situation Theory for modelling complex information and development of computational syntax-semantics interfaces for natural languages. Mathematical models of the concepts of context and agents in context concern fundamentals of syntax-semantics interfaces in languages in general. Our specific goal is theoretical development of computational type-theory of information for language processing based on syntax-semantics interface. We target theory of information that is supported by the role of languages in nature, from the perspective of applications and software engineering in new technologies.

One of the primary applications of Situation Theory is to computational semantics of human languages, for modelling semantic domains and information that is designated by human language, including linguistic contexts and agents. Human language is notoriously ambiguous and context dependent. These phenomenal features present the core part of language productivity and efficiency, partly because they allow different agents, in different contexts, to express varying information, with familiar expressions. Moreover, language expressions, even when considered unambiguous, when out of context, carry partial and parametric information, which is not necessarily and fully instantiated in specific contexts when used by specific agents. In many cases, agents such as language users, speakers, listeners, and readers, appreciate parametric, partial and underspecified information expressed by language even in specific contexts. This presents needs of a theory that models partial, parametric and underspecified information, that also models the context-dependency of language and information. This means that such a theory of information has the capacities to model interrelated context components and language agents in context. Situation Theory has been under development for meeting such needs.

Future Work. Closely related line of research is development of new approach to the fundamentals of computation and the notion of intension. Moschovakis recursion (see [20,21]) models the concepts of algorithm in a novel way that covers fundamental features of computation processes. In particular, the formal language and theory of Moschovakis acyclic recursion L^{λ}_{ar} (see [21]) introduces a novel approach to modelling the logical concepts of meaning, synonymy, and referential intension, by targeting adequateness of computational semantics of human language.

The formal system L^{λ}_{ar} has been used in work (see [16,18]) on the theoretical aspects of computational syntax-semantics interface, by covering major syntactic constructions of human language, in Generalized Constraint-Based Lexicalized Grammar (CBLG). Further work is ongoing in the following directions:

- mathematical modelling of the domains of semantic structures of L^{λ}_{ar}. E.g., in this direction, we target versions of Situation Theory.
- developments of type-theory of recursion, in several directions for adequacy depending on applications (see [19]). Further work is necessary towards (1) type-theory of full recursion (2) type-theory of recursion with extended type systems, for example with dependent types (3) L^{λ}_{ar} includes states as contexts in all of its layers: a specialized type for states in its type system, variables for states in its syntax, and a specialized domain of states in its semantic structures. The concept of state, in the current stage of L^{λ}_{ar}, is rudimentary and in need of development. For this purpose, we envisage using a version of Situation Theory.

Another closely related work involves using versions of Situation Theory and type-theory of algorithms (i.e., Moschovakis recursion) in large-scale grammatical frameworks for human language. In particular, a highly expressive new grammatical framework (GF) (see [24,25]) has been under developments for multi-lingual translations, by targeting universal, typed-directed syntax that covers major semantic features of human-language. We maintain the view that GF, as a new branch of CBLG, is open and highly prospective for further work on syntax-semantics interfaces, e.g., in the lines of the new ideas and approaches presented in this article.

New foundational developments, such as Situation Theory and Typed theory of Recursion, target more adequate, reliable and intelligent foundations of technological applications. In the same time, they are part of the ever advancing, scientific understanding of the fundamentals of information and computation.

References

1. Aczel, P.: Non-well-founded Sets. CSLI Lecture Notes, vol. 14. CSLI Publications, Stanford (1988)
2. Barwise, J.: Scenes and other situations. J. Philos. **78**, 369–397 (1981)
3. Barwise, J., Etchemendy, J.: The Liar: An Essay on Truth and Circularity. Oxford University Press, New York (1995)
4. Barwise, J., Moss, L.: Vicious Circles: On the Mathematics of Non-Wellfounded Phenomena. CSLI Publications, Stanford (1996)
5. Barwise, J., Perry, J.: Situations and Attitudes. MIT Press, Cambridge (1983). republished as [6]
6. Barwise, J., Perry, J.: Situations and Attitudes. The Hume Series. CSLI Publications, Stanford (1999)
7. Bjørner, D.: Domain science & engineering: a foundation for computation for humanity. In: Computation for Humanity: Information Technology to Advance Society, pp. 157–175 (2013)
8. Copestake, A., Flickinger, D., Pollard, C., Sag, I.: Minimal recursion semantics: an introduction. Res. Lang. Comput. **3**, 281–332 (2005)
9. Devlin, K.: Situation theory and situation semantics. In: Gabbay, D., Woods, J. (eds.) Handbook of the History of Logic, vol. 7, pp. 601–664. Elsevier, Amsterdam (2008)
10. Ginzburg, J., Sag, I.A.: Interrogative Investigations: The Form, Meaning, and Use of English Interrogatives. CSLI Publications, Stanford (2000)
11. Loukanova, R.: Russellian and strawsonian definite descriptions in situation semantics. In: Gelbukh, A. (ed.) CICLing 2001. LNCS, vol. 2004, pp. 69–79. Springer, Heidelberg (2001)
12. Loukanova, R.: Generalized quantification in situation semantics. In: Gelbukh, A. (ed.) CICLing 2002. LNCS, vol. 2276, pp. 46–57. Springer, Heidelberg (2002)
13. Loukanova, R.: Quantification and intensionality in situation semantics. In: Gelbukh, A. (ed.) CICLing 2002. LNCS, vol. 2276, pp. 32–45. Springer, Heidelberg (2002)
14. Loukanova, R.: Computational syntax-semantics interface. In: Bel-Enguix, G., Jiménez-López, M.D. (eds.) Language as a Complex System: Interdisciplinary Approaches, pp. 111–150. Cambridge Scholars Publishing, Cambridge (2010)
15. Loukanova, R.: From Montague's rules of quantification to minimal recursion semantics and the language of acyclic recursion. In: Bel-Enguix, G., Dahl, V., Jiménez-López, M.D. (eds.) Biology, Computation and Linguistics – New Interdisciplinary Paradigms, Frontiers in Artificial Intelligence and Applications, pp. 200–214. IOS Press, Amsterdam (2011)
16. Loukanova, R.: Semantics with the language of acyclic recursion in constraint-based grammar. In: Bel-Enguix, G., Jiménez-López, M.D. (eds.) Bio-Inspired Models for Natural and Formal Languages, pp. 103–134. Cambridge Scholars Publishing, Cambridge (2011)
17. Loukanova, R.: Situated propositions with constraints and restricted parameters. In: Blache, P., Christiansen, H., Dahl, V., Villadsen, J. (eds.) Proceedings of the 6th International Workshop on Constraints and Language Processing. Computer Science Research Reports, vol. 134, pp. 44–55, October 2011

18. Loukanova, R.: Syntax-semantics interface for lexical inflection with the language of acyclic recursion. In: Bel-Enguix, G., Dahl, V., Jiménez-López, M.D. (eds.) Biology, Computation and Linguistics - New Interdisciplinary Paradigms, Frontiers in Artificial Intelligence and Applications, vol. 228, pp. 215–236. IOS Press, Amsterdam (2011)
19. Loukanova, R.: Semantic information with type theory of acyclic recursion. In: Huang, R., Ghorbani, A.A., Pasi, G., Yamaguchi, T., Yen, N.Y., Jin, B. (eds.) AMT 2012. LNCS, vol. 7669, pp. 387–398. Springer, Heidelberg (2012)
20. Moschovakis, Y.N.: Sense and denotation as algorithm and value. In: Oikkonen, J., Vaananen, J. (eds.) Lecture Notes in Logic, vol. 2, pp. 210–249. Springer, Heidelberg (1994)
21. Moschovakis, Y.N.: A logical calculus of meaning and synonymy. Linguist. Philos. **29**, 27–89 (2006)
22. Pollard, C., Sag, I.A.: Information-Based Syntax and Semantics, Part I. CSLI Lecture Notes, vol. 13. CSLI Publications, Stanford (1987)
23. Pollard, C., Sag, I.A.: Head-driven Phrase Structure Grammar. University of Chicago Press, Chicago (1994)
24. Ranta, A.: Grammatical framework: a type-theoretical grammar formalism. J. Funct. Program. **14**(2), 145–189 (2004)
25. Ranta, A.: Grammatical Framework: Programming with Multilingual Grammars. CSLI Publications, Stanford (2011)
26. Seligman, J., Moss, L.S.: Situation Theory. In: van Benthem, J., ter Meulen, A. (eds.) Handbook of Logic and Language, pp. 239–307. Elsevier, Amsterdam (1996)
27. Thomason, R.H. (ed.): Formal Philosophy: Selected Papers of Richard Montague. Yale University Press, New Haven (1974)
28. Van Lambalgen, M., Hamm, F.: The Proper Treatment of Events. Explorations in Semantics. Wiley-Blackwell, Oxford (2004)