ST5: A 5-Valued Logic for Truth-Value Judgments Involving Vagueness and Presuppositions*

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Abstract. Both presuppositional and vague expressions may yield non-classical truth-value judgments. Given that expressions of these kinds may combine together, I propose a single logical system intended to deal with them, which would account for our truth-value judgments. The system I propose is based on Cobreros&al's [4] 3-valued system for vagueness, ST, which comes with a notion of assertoric ambiguity that I claim naturally deals with our non-classical judgments for vagueness. I show that the specificities of presuppositions with respect to truth-value judgments can be accounted for within this system if we add two logical values to it. I discuss a specific 5-valued system that I call ST5.

Keywords: vagueness, presuppositions, 5-valued logic, truth judgments.

1 Introduction

In this paper, I will focus on *truth-value judgments* concerning vagueness and presuppositions. I start from the position that we observe *conflicting* judgments for vague sentences as well as for presuppositional sentences in specific situations. For instance, consider the presuppositional sentence (1):[1]

(1) The amplifiers have stopped buzzing.

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[1] Aspectual verbs such as *stop* are well-known to trigger a presupposition. See for instance the article "Presupposition" in the Stanford Encyclopedia of Philosophy [3].

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If I'm told (1) and I know that, in fact, the amplifiers have never buzzed, I can say that (1) is *both false and not false*: it is *false* because the amplifiers were *not* buzzing before, and it is *not false* because if (1) were false, it would mean that the amplifiers *were* buzzing before. Similarly, consider the vague sentence (2), involving the vague adjective *loud*:

(2) The amplifiers are loud.

If I'm told (2) and I find the volume of the amplifiers to be neither clearly loud nor clearly not loud, I can say that (2) is *both true and false*: it is true to some extent, because the amplifiers are not clearly not loud, but it is false to some extent too, because they're not clearly loud either.[2]

My aim here will be to offer a semantics that assigns *logical truth values* to propositions involving vague and presuppositional expressions on the basis of which one could correctly predict the *truth-value judgments* of speakers in regular *and* conflicting-judgment contexts. In Sect. 2, I begin by reviewing truth-value judgments that we find for positive and negative counterparts of sentences involving vague expressions and sentences involving presuppositional expressions. Section 3 presents the 3-valued ST system [5], which has been developed for vagueness and which offers a natural way of accounting for the conflicting truth-value judgments to which vagueness gives rise. I then consider a 5-valued extension of this system, which I call ST5, in order to incorporate presuppositional expressions. In Sect. 4, I consider the interactions between vagueness and presuppositions, by looking at sentences that involve both vague and presuppositional expressions (hybrid sentences). I propose a semantics for presuppositional sentences in ST5 that makes predictions for hybrid sentences and for sentences with iteratively embedded presuppositional expressions. Finally I briefly consider alternative multi-valued systems in Sect. 5 and show why one should prefer ST5 to deal with vagueness and presuppositions.

2 Truth-Value Judgments

By a *truth-value judgment* I here mean any position that a speaker can have toward the *truth* or the *falsity* of a sentence. My use of this notion then refers to the set of combinations of *true* and *false* closed under *not*, *and*, *(n)or*, *both* and *(n)either*.[3]

Each element of this set is a truth-value judgment. It is clear that, as truth-value judgments, some of the elements in the set are so-to-speak "regular": speakers often judge sentences *true*, *false*, *not true* or *not false*. But other elements are far less "regular" (*neither true nor false*) and some even sound contradictory:

[2] Serchuk & al. [23] conducted several experiments revealing this apparent contradictory characteristic of truth-value judgments for vagueness.

[3] Importantly, the set of *truth-value judgments* is to be distinguished from the set of *logical values* that a system assigns to propositions. There is no necessary one-to-one correspondence between their elements; and the system I will eventually propose exhibits no such correspondence.

both true and false, *both true and not true*, *both false and not false* for instance.[4] Yet, I claim that speakers can use these elements to qualify some sentences. That is to say, I claim that speakers can exhibit apparently *conflicting* truth-value judgments. Even though some dialetheists, such as Priest [19], endorse the view that there are true contradictions, Lewis [17] for instance proposed to see underlying ambiguity in judgments of this kind.[5]

In the next two subsections, I present some evidence that speakers have access to these kinds of judgments concerning vagueness and presuppositions. The account I will eventually give for this relies on a notion of assertoric *ambiguity* developed in the 3-valued logic ST [5]. So far, there have been few experiments exploring the *truth-value judgments* of speakers concerning vagueness or presuppositions, I will therefore rely on indirect evidence that speakers have access to conflicting truth-value judgments in the cases of vagueness and of presuppositions.

2.1 Vagueness

In an experiment conducted by Alxatib & Pelletier [2], participants were presented with a series of five men of different heights. For each of these men, participants were shown a particular description that they could choose to label as *true*, *false* or *can't say*. In particular, they were asked to judge whether conflicting descriptions such as (3-a) and (3-b) were true or false.[6]

(3) a. This man is both tall and not tall
 b. This man is neither tall nor not tall

While participants almost unanimously judged these descriptions false when considering clearly not tall and clearly tall men, about half of them judged the conflicting descriptions true when considering the man whose height was average. Other experiments showed similar results (see [20] and [6] for instance).

All these experiments consider the use of a particular vague predicate and all show that for borderline cases of this vague predicate, people can use conflicting descriptions to qualify it. I assume that a speaker can regard (4-a) as respectively *true* or *false* when she *accepts* to qualify the man as respectively *tall* or as *not tall*; and that a speaker can regard (4-a) as respectively *not true* or *not false* when she *refuses* to qualify the man as respectively *tall* or *not tall.* Therefore, on the basis of the results of these experiments, I take it to be plausible that speakers, when asked to evaluate a vague sentence such as (4-a) regarding a borderline-tall man, can judge it *both true and false* or *neither true nor false*; and such judgments are *conflicting truth-value judgments.*

(4) a. This man is tall
 b. This man is not tall

[4] Note the italics that distinguish between judging a sentence both *false* and *not false* and judging a sentence *both false and not false.*

[5] See Kooi & Tamminga[13] for support for Lewis' view contra Priest.

[6] The percentage of "can't say" answers proved to be insignificant.

Furthermore, participants gave similar judgments for positive ((4-a)) and negative ((4-b)) counterparts of vague sentences for borderline cases across these experiments. For this reason, I assume that we can judge *negative* vague sentences about borderline cases in the same way as their positive counterparts (ie. we can also say that (4-b) is *both true and false/neither true nor false* when the man is borderline-tall).[7]

2.2 Presuppositions

To my knowledge, there have been very few experiments on *truth-value* judgments concerning presuppositions.[8] Nonetheless, if we focus on what has been said about truth-value judgments concerning presuppositional sentences when the presupposition is not fulfilled, we find some clues suggesting that conflicting truth-value judgments might be accessible. Notably, Strawson [24] argued that a sentence such as (5) is *neither true nor false* when there is no king of France, contra Russell [21] according to whom such a sentence is simply *false* in these circumstances. Von Fintel [8] endorses the former approach, but also admits that speakers might judge a presuppositional sentence either *true* or *false* when its presupposition is not fulfilled depending on the meaning of the sentence.[9]

(5) The king of France is bald

Things get even more intricate when one considers the following pair of presuppositional sentences, when it is known that the amplifiers have never buzzed:[10]

(6) a. The amplifiers have stopped buzzing
    b. The amplifiers have not stopped buzzing

As noted earlier, my intuitions, shared with several speakers I have consulted, are the following: I can judge (6-a) *both false and not false*. Of course, if I were talking to someone, I would no doubt add something like "On the one hand, it is not *false* that the amplifiers have stopped buzzing because for the amplifiers to have failed to stop buzzing, the amplifiers would have to *have been buzzing* before; but on the other hand it *is* false to the extent that it can't be true that the amplifiers have *stopped* buzzing: the amplifiers have *never* buzzed!".

[7] These assumptions reflect my intuitions and those of people I've informally surveyed.

[8] Though Abrusán & Szendrői [1] recently explored the judgments of speakers for some positive and negative counterparts of presuppositional sentences.

[9] In this respect, my distinguishing between truth-value *judgments* and formal *logical* values is reminiscent of his approach where (5) is semantically *neither true nor false* but would be judged *false* by speakers.

[10] In Abrusán & Szendrői's experiment, almost no participant gave a *true* judgment for "the king of France is not bald", but they did for other negative presuppositional sentences. They explain this contrast by positing that certain linguistic factors affect speakers' judgments. Taking those factors into account goes beyond the scope of this paper. All the linguistic pairs of positive and negative counterparts given here will be reduced to mere logical counterparts in the ST5 system: ϕ and $\neg\phi$.

However, I would never judge this sentence *true* given that the amplifiers were not buzzing before.[11]

By contrast, I can judge the negative counterpart (6-b) *both true and not true*, for the very same reasons. It is not *true* to the extent that the amplifiers have never buzzed; but it *is* true to the extent that the amplifiers have not *stopped* buzzing: the amplifiers were never buzzing in the first place.[12]

2.3 Summary

The important point, ultimately, is that some speakers (such as myself) seem to have access to *conflicting* truth-value judgments both concerning presuppositional sentences (when the presupposition is not fulfilled) and concerning vague sentences (describing borderline cases). Moreover, we see that their judgments are the same concerning the positive and the negative counterparts of vague sentences (describing borderline cases); whereas they differ concerning the positive and the negative counterparts of presuppositional sentences (when the presupposition is not fulfilled). When one tries to sketch a system that would account for the truth-judgments associated with vague sentences as well as the truth-judgments associated with presuppositional sentences, one should ensure that one's system accounts for both this common point and this difference.

The intuitions concerning hybrid sentences, that is to say sentences such as (7-a) or (7-b) that involve both vague and presuppositional expressions, are more complex and, to my knowledge, have never been dealt with.

(7) a. The amplifiers have stopped being loud
b. The amplifiers are loud and they have stopped buzzing

[11] Note that putting stress on the emphasized words can help to bring out these judgments.

[12] An anonymous reviewer has noted that, in justifying the conflicting judgments, I make use of statements like the following, which by all appearances threaten the transitivity of the consequence relation. If you endorse transitivity, it seems that by accepting (i-a) and (i-b), you should conclude that "if the amplifiers have never buzzed, then the speakers used to buzz", which is a contradiction:

(i) a. If the amplifiers have never buzzed, then (6-a) is false.
b. If (6-a) is false, then the amplifiers used to buzz.

I take the simultaneous acceptance of these sentences to reveal an important fact, namely the ambiguous use of the expression "false". The system I propose offers a natural way to loosen (as in (i-a)) and/or strengthen (as in (i-b)) the meaning of "false".**[R1.2]** This ambiguity might in fact explain the variation found among speakers for truth-judgments about presuppositional sentences evaluated in situations of presupposition failure: maybe not all speakers have equal access to the loose and to the strong senses of "false".**[R1.1]**

Not surprisingly, but still interestingly, this approach is reminiscent of the analysis of the sorites paradox and of the Liar paradox advanced by Cobreros&al. [5], who developed the three-valued system that I extend to a five-valued system: in critical cases, one might have to abandon the transitivity of the consequence relation.

To my knowledge, no theory considers such sentences and therefore no theory makes any prediction regarding the semantic status of (7-a) or (7-b): Section 4 tries to sketch an account of such sentences.[13]

3 ST5

3.1 The Original 3-Valued ST System

ST is a trivalent logical system developed to deal with vague predicates [5], and more specifically to account for conflicting judgments such as those diagnosed by responses to "X is tall and not tall".[14] There are two reasons for which I base my 5-valued system on ST: first, ST already comes with an account for vagueness. Hence only half of the work remains to be done. Second, ST comes with a notion of *assertoric ambiguity* that leads to a nice explanation for our conflicting judgments.

Two Notions of Satisfaction. Let's consider as our language $\mathcal{L}$ a non-quantified fragment of monadic first-order logic such that:

Definition 1 (Syntax)

i. *For any predicate $P \in \mathcal{L}$ and any individual name $a \in \mathcal{L}$, Pa is a well-formed formula (wff).*
ii. *For any wff ϕ, $\neg\phi$ is a wff.*
iii. *For any ϕ and ψ such that ϕ and ψ are wff, $[\phi \wedge \psi]$, $[\phi \vee \psi]$ and $[\phi \rightarrow \psi]$ are wff.*

Nothing else is a wff.

$\mathcal{M}$ consists of a non-empty domain of individuals $\mathcal{D}$ and an interpretation function $\mathcal{I}$ such that:

Definition 2 (Semantics)

i. *For any predicate $P \in \mathcal{L}$ and any individual name $a \in \mathcal{L}$, $\mathcal{I}(Pa) = \frac{1}{2}$ iff a is the name of a borderline case for P, $\mathcal{I}(Pa) \in \{0, 1\}$ otherwise.*

[13] It is worth noting that supervaluationism has been used independently for vagueness (Lewis [16], Fine [7], Kamp [15]) and for presuppositions (van Fraassen [10]). None of these supervaluationists seems to have specifically entertained a unified treatment of these two phenomena.**R2.1**

[14] ST is a built-in 3-valued version of TCS [4], which assumed bivalent extensions for vague predicates on which it built their trivalent extensions. As I present it here, ST seems to be committed to the existence of a sharp boundary between eg. clearly tall men and borderline tall men, which might sound unrealistic. This point is related to the question of higher-order vagueness, which is much discussed in the literature on vagueness. A discussion of higher-order vagueness goes far beyond the scope of this paper. I will therefore just endorse the assumption that vagueness defines a well defined trivalent extension in the rest of the paper, with no further justification.**R2.3**

ii. *For any wff* ϕ, $\mathcal{I}(\neg\phi) = 1 - \mathcal{I}(\phi)$.
iii. *for two wff* ϕ *and* ψ, $\mathcal{I}(\phi \wedge \psi) = min(\mathcal{I}(\phi), \mathcal{I}(\psi))$,
$\mathcal{I}(\phi \vee \psi) = max(\mathcal{I}(\phi), \mathcal{I}(\psi))$ *and* $\mathcal{I}(\phi \rightarrow \psi) = \mathcal{I}(\neg\phi \vee \psi)$

The system ST owes its name to the definition of two notions of satisfaction:[15]

Definition 3 (Strict and Tolerant Satisfaction)

Strict satisfaction: $\mathcal{M} \models^s \phi$ *iff* $\mathcal{I}(\phi) = 1$
Tolerant satisfaction: $\mathcal{M} \models^t \phi$ *iff* $\mathcal{I}(\phi) \geq \frac{1}{2}$

Now, imagine a is the name of a borderline case for P. We have $\mathcal{I}(Pa) = \frac{1}{2}$ and $\mathcal{I}(\neg Pa) = 1 - \frac{1}{2} = \frac{1}{2}$. Hence, we get $\mathcal{I}(Pa \wedge \neg Pa) = min(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2}$ and $\mathcal{I}(\neg(Pa \vee \neg Pa)) = 1 - max(\frac{1}{2}, \frac{1}{2}) = 1 - \frac{1}{2} = \frac{1}{2}$. This leads us to:

i. $\mathcal{M} \models^t Pa$ but $\mathcal{M} \not\models^s Pa$
ii. $\mathcal{M} \models^t \neg Pa$ but $\mathcal{M} \not\models^s \neg Pa$
iii. $\mathcal{M} \models^t Pa \wedge \neg Pa$ but $\mathcal{M} \not\models^s Pa \wedge \neg Pa$
iv. $\mathcal{M} \models^t \neg(Pa \vee \neg Pa)$ but $\mathcal{M} \not\models^s \neg(Pa \vee \neg Pa)$

With P standing for "is tall" and a standing for borderline-tall "John", what we have is that none of "John is tall", "John is not tall", "John is tall and not tall" and "John is neither tall nor not tall"[16] is *strictly* satisfied, but all of them are *tolerantly* satisfied. Cobreros & al. propose to account for the results of Alxatib & Pelletier [2] by assuming that speakers can assert vague sentences either strictly or tolerantly. To this, I add the following bridge principles:[17]

Principle 1 (Truth-Value Judgments). *One can judge a proposition* ϕ*...*

1. *"true" if* $\mathcal{M} \models^t \phi$
2. *"false" if* $\mathcal{M} \models^t \neg\phi$
3. *"not true" if* $\mathcal{M} \not\models^s \phi$
4. *"not false" if* $\mathcal{M} \not\models^s \neg\phi$
5. *"both true and false" if 1 and 2.*
6. *"neither true nor false" if 3 and 4.*
7. *"both true and not true" if 1 and 3.*
8. *"both false and not false" if 2 and 4.*

It is straightforward that, for borderline-tall John, "John is tall" as well as "John is not tall" can be judged *both true and false* and *neither true nor false.*

No Room for Presuppositions. Now, looking at the bridge principles, it would be ideal if we could add presuppositional propositions ϕ to our language in such a way that, *when the presupposition of* ϕ *is unfulfilled*:

1. $\mathcal{M} \models^t \neg\phi$ (so that a speaker can judge ϕ *false*)

[15] See [5] for a discussion of inference rules in this system.
[16] Here, I regard *neither... nor...* as the negation of a disjunction
[17] In formulating these bridge principles, I use $\mathcal{M}$ to stand for a model determined by the belief state of the speaker.**R2.4**

2. $\mathcal{M} \not\models^s \neg\phi$ (so that a speaker can judge ϕ *not false*)
3. $\mathcal{M} \not\models^t \phi$ (so that a speaker *cannot* judge ϕ *true*)

But the only way in ST to have 1. and 2. is for ϕ to get the value $\frac{1}{2}$, and then we would have $\mathcal{M} \models^t \phi$ and a speaker could judge ϕ *true* as well. More specifically, ST has the following property (see [5]):

Lemma 1 (Duality in ST). *For any wff ϕ, $\mathcal{M} \models^{s/t} \phi$ iff $\mathcal{M} \not\models^{t/s} \neg\phi$*

The solution I propose consists in breaking this duality by adding two logical values to the system: propositions that get one of these two extra values will obey the three constraints above, but propositions that get one of the three initial values will still present the equivalence noted in Lemma 1.

3.2 The ST5 System

In ST, we had three values: $\{0, \mathcal{V} = \frac{1}{2}, 1\}$, and vague predications on borderline cases got the value $\mathcal{V}$. Now, in ST5, we add two more values, $\mathcal{P}^0$ and $\mathcal{P}^1$, such that: $0 < \mathcal{P}^0 < \mathcal{V} < \mathcal{P}^1 < 1$ and such that $\mathcal{P}^0 = 1 - \mathcal{P}^1$. The syntax and the semantics of ST remain unchanged in this extended system, as well as Def. 3 of tolerant and strict satisfactions. By this simple addition, we obtain the following:

Lemma 2 (Duality lost)

- *For any proposition ϕ such that $\mathcal{I}(\phi) = \mathcal{P}^0$:*
  i. $\mathcal{M} \not\models^t \phi$ and $\mathcal{M} \not\models^s \phi$ since $\mathcal{P}^0 < \frac{1}{2} < 1$.
  ii. $\mathcal{M} \models^t \neg\phi$ but $\mathcal{M} \not\models^s \neg\phi$ since $1 - \mathcal{P}^0 = \mathcal{P}^1$ and $\mathcal{P}^1 \geq \frac{1}{2}$ but $\mathcal{P}^1 < 1$.
- *For any proposition ϕ such that $\mathcal{I}(\phi) = \mathcal{P}^1$:*
  i. $\mathcal{M} \models^t \phi$ but $\mathcal{M} \not\models^s \phi$ since $\mathcal{P}^1 \geq \frac{1}{2}$ but $\mathcal{P}^1 < 1$.
  ii. $\mathcal{M} \not\models^t \neg\phi$ and $\mathcal{M} \not\models^s \neg\phi$ since $1 - \mathcal{P}^1 = \mathcal{P}^0$ and $\mathcal{P}^0 < \frac{1}{2} < 1$.

Given that we now have propositions ϕ for which $\mathcal{M} \not\models^s \neg\phi$ but $\mathcal{M} \not\models^t \phi$ (propositions of value $\mathcal{P}^0$), Lemma 1 no longer holds in ST5. Nonetheless, the following holds in ST as well as in ST5:

Lemma 3 (Entailment). *For any wff ϕ, $\mathcal{M} \models^s \phi$ entails $\mathcal{M} \models^t \phi$.*

Now let's stipulate that any simple positive proposition ϕ whose presupposition is *unfulfilled* gets the value $\mathcal{P}^0$. It follows that its negation gets the value $\mathcal{P}^1$. Let ϕ stand for "The amplifiers have stopped buzzing", the bridge principles predict the following:[18]

i. One can judge ϕ *both false and not false* ($\mathcal{M} \models^t \neg\phi$ but $\mathcal{M} \not\models^s \neg\phi$)
ii. One can judge ϕ *neither true nor false* ($\mathcal{M} \not\models^s \phi$ and $\mathcal{M} \not\models^s \neg\phi$)
iii. One can judge $\neg\phi$ *both true and not true* ($\mathcal{M} \models^t \neg\phi$ but $\mathcal{M} \not\models^s \neg\phi$)
iv. One can judge $\neg\phi$ *neither true nor false* ($\mathcal{M} \not\models^s \neg\phi$ and $\mathcal{M} \not\models^s \neg\neg\phi$)
v. One *cannot* judge ϕ *true* ($\mathcal{M} \not\models^t \phi$)

[18] Recall that we have $\neg\neg\phi \equiv \phi$.

One should note at this point that presuppositional propositions are propositions that can get a value in $\{\mathcal{P}^0, \mathcal{P}^1\}$, in the same way that vague propositions are propositions that can get the value $\mathcal{V}$. A presuppositional proposition used when the presupposition *is* fulfilled gets a value in $\{0, 1\}$, just like a vague proposition describing a *non*-borderline case. We thus do not predict any non-classical judgment in such contexts (as desired).[19]

4 Hybrid Sentences

4.1 Presupposition Satisfaction in ST5

So far, we have considered situations where presuppositions were simply fulfilled or unfulfilled. But as it turns out, presuppositions can themselves involve vague and presuppositional expressions. Think of sentences such as (8-a) or (8-b) whose presuppositions are (8-a-i) and (8-b-i).

(8) a. The amplifiers have stopped being <u>loud</u>
  (i) The amplifiers were <u>loud</u>
 b. John knows that the amplifiers have <u>stopped</u> buzzing
  (i) The amplifiers have <u>stopped</u> buzzing

In situations where the amplifiers were borderline-loud and have never buzzed, (8-a-i) gets the value $\mathcal{V}$ and (8-b-i) gets the value $\mathcal{P}^0$ in ST5. What effect does a presupposition with value $\mathcal{V}$ or $\mathcal{P}^0$ have on the value of the proposition as a whole?

Bearing in mind that the presuppositional propositions in ST5 are the propositions that get one of the values in $\{\mathcal{P}^0, \mathcal{P}^1\}$ in at least one model, I propose that we see the values of these propositions as being determined in the following way:

Definition 4 (Factoring out Presuppositions). *Let us use the notation* ϕ_p *for a proposition whose* assertive *part can be expressed by the proposition* ϕ *and whose* presuppositional *part can be expressed by the proposition* p. *Then:*

- $\mathcal{I}(\phi_p) = \mathcal{I}(\phi)$ *if* $\mathcal{M} \models^s p$
- $\mathcal{I}(\phi_p) = \mathcal{P}^1$ *if* $\mathcal{M} \not\models^s p$ *and* $\mathcal{M} \models^t p$ *and* $\mathcal{M} \models^s \phi$
- $\mathcal{I}(\phi_p) = \mathcal{P}^0$ *if* $\mathcal{M} \not\models^t p$ *or [* $\mathcal{M} \not\models^s p$ *and* $\mathcal{M} \not\models^s \phi$ *]*

[19] A reviewer asked whether being borderline can be treated as a case of presupposition failure. ST5 allows us to adopt a liberal understanding of the notion of *presupposition*: one could suggest that any use of a proposition presupposes it to have a classical value (0 or 1). To that extent, ascribing a vague predicate to a borderline case would constitute a case of presupposition failure (for the proposition would get the value $\mathcal{V}$, which is neither 0 nor 1). Percus and I [18] argued for the usefulness of such a position, taking the TCS [4] system as background and building on the account of the sorites paradox by means of presupposition projection presented in my MA thesis [26].

In situations where the amplifiers were borderline-loud, we have $\mathcal{M} \not\models^s$ (8-a-i) but $\mathcal{M} \models^t$ (8-a-i); and in situations where the amplifiers have never buzzed we have $\mathcal{M} \not\models^s$ (8-b-i) and $\mathcal{M} \not\models^t$ (8-b-i). Additionally, let's imagine that the amplifiers are now low and that John *believes* that the amplifiers were buzzing but have stopped. We can then assume that the *assertive* parts are strictly satisfied.[20] Looking at our stipulations, we obtain: $\mathcal{I}((8\text{-a})) = \mathcal{P}^1$ (because $\mathcal{M} \models^t$ (8-a-i) and the assertive part is strictly satisfied) and $\mathcal{I}((8\text{-b})) = \mathcal{P}^0$ (because $\mathcal{M} \not\models^t$ (8-b-i)). So we predict that under such circumstances, a speaker can judge (8-a), "The amplifiers have stopped being loud", *both true and not true* and (8-b), "John knows that the amplifiers have stopped buzzing", *both false and not false.*

As noted earlier, our intuitions for sentences with iteratively embedded presuppositional expressions (henceforth *recursively presuppositional sentences*) and hybrid sentences are somewhat messy and maybe only experimental data can discriminate between theories that make different predictions regarding truth-judgments for these kinds of sentences. Nonetheless, any theory has to make *some* predictions for these sentences, and it doesn't appear to be the case for existing theories, for one simple reason: a majority of these theories only consider *bivalent* presuppositions. As long as a theory of presuppositions treats the presuppositional content as *bivalent*, it can't account for sentences where the presuppositional content is vague. This is precisely the weakness that the definitions above avoid: they let us escape the traditional duality of either "fulfilled" *or* "unfulfilled" presuppositions. Rather, all the conditions above are stated in terms of satisfaction. The first clause states that when a presupposition is strictly satisfied, the whole proposition gets the value of its assertive part: in this situation one would traditionally say that the presupposition is "fulfilled". The second clause considers the case where the presupposition is only *tolerantly* satisfied. To some extent, one could see this as a condition where the presupposition is "partly fulfilled". The whole proposition will be "partly true" if the assertive part is true itself: that's what $\mathcal{P}^1$ stands for. Finally, the third clause states that even if the presupposition is tolerantly satisfied, there is no reason for the whole proposition to be "partly true" if the assertive part is not strictly satisfied; and even less reason if the presupposition is not satisfied *at all.* But still, such a proposition is not merely false, because the presupposition is *not* "fulfilled": that's what $\mathcal{P}^0$ stands for.

Many theories do consider recursively presuppositional sentences. However, none of them deal with hybrid sentences such as (8-a) to my knowledge. Moreover, ST5 is able to make distinctions that other approaches cannot. For example, Karttunen ([12]) proposed to categorize factives (such as *know*) as what he famously called *holes*:

> "If the main verb of the sentence is a hole, then the sentence has all the presuppositions of the complement sentences embedded in it."

[20] I take "*X believes* ϕ" to be the assertive part of "*X knows* ϕ". It might well be the case that things are more complex, and that one should consider justified belief for the assertive part. But whatever we take to be the assertive part, the crucial point here is how each part contributes to the value of the whole proposition.

Regarding (8-b), this view provides no way to distinguish between a situation where the amplifiers are still buzzing and a situation where the amplifiers have never buzzed: in the first situation, the complement of the factive is false so it yields a presupposition failure; in the second situation the *inherited* presupposition is *unfulfilled* so it *also* yields a presupposition failure. By contrast, in ST5, we have the tools to make a distinction because the presuppositional part of the whole proposition would have the value 0 in the first case and the value $\mathcal{P}^0$ in the second case. It is not clear whether speakers actually would give different truth-judgments in these two situations for (8-b), and I chose here to treat them equally, as does a theory *à la* Karttunen. But I find it important that ST5 allows more easily than its competitors for the possibility of nuanced judgments for presuppositional sentences, given that it takes the relative "gradedness" of the presuppositions into account.

Because Def. 4 covers all the satisfaction possibilities, it is easy to see that the system is now completely predictive with respect to the kind of proposition (ie. regular, vague, simply presuppositional or even hybrid itself[21]) that appears as a presupposition of the whole sentence.

4.2 Conjunctions, Disjunctions and Implications in ST5

An Example. Finally, because ST5 deals with totally ordered values and defines its connectives in terms of *min* and *max*, it naturally makes predictions for conjunctions, disjunctions and implications combining vague and presuppositional propositions. Consider (7-b) repeated here that conjoins a vague sentence and a presuppositional sentence:

(9) The amplifiers are loud and they have stopped buzzing

Given that the amplifiers have *never* buzzed, if their volume is somewhere between clearly loud and clearly not loud, the first conjunct gets the value $\mathcal{V}$ and the second conjunct gets the value $\mathcal{P}^0$. Therefore in these circumstances, the whole proposition gets the value $min(\mathcal{V}, \mathcal{P}^0) = \mathcal{P}^0$: it is judged *both false and not false* (for the amplifiers were *not* buzzing before), and it's not judged *true*.

Here is a table summarizing the predictions of ST5 for hybrid conjunctions and disjunctions when the amplifiers (abbreviated as A) are borderline-loud and have never buzzed:

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[21] As an example of how ST5 deals with hybrid *presuppositions*, consider (i-a), its presupposition being (i-b):

(i) a. John knows that the amplifiers have stopped being loud.
b. The amplifiers have stopped being loud.

We saw earlier that in cases were the amplifiers were borderline-loud before decreasing in volume, the hybrid proposition expressed by (i-b) gets the value $\mathcal{P}^0$, which prevents it from being even tolerantly satisfied; therefore (i-a) will also get the value $\mathcal{P}^0$ by Def. 4.

Proposition	**Value**	**Judgment**
A are loud	$\mathcal{V}$	Both true and false
A are not loud	$\mathcal{V}$	Both true and false
A have stopped buzzing	$\mathcal{P}^0$	Both false and not false
A have not stopped buzzing	$\mathcal{P}^1$	Both true and not true
A are loud & have stopped buzzing	$\mathcal{P}^0$	Both false and not false
A are not loud & have stopped buzzing	$\mathcal{P}^0$	Both false and not false
A are loud & have not stopped buzzing	$\mathcal{V}$	Both true and false
A are not loud & have not stopped buzzing	$\mathcal{V}$	Both true and false
A are loud or have stopped buzzing	$\mathcal{V}$	Both true and false
A are not loud or have stopped buzzing	$\mathcal{V}$	Both true and false
A are loud or have not stopped buzzing	$\mathcal{P}^1$	Both true and not true
A are not loud or have not stopped buzzing	$\mathcal{P}^1$	Both true and not true

Left-Right Asymmetries. In view of these predictions, a word is in order about the alleged left-right asymmetry of presuppositions. Consider the pair of sentences in (10), for which the ST5 truth judgment predictions are clear. In ST5, conjunctions are totally symmetric and (10-a) and (10-b) will get the same value when the amplifiers never buzzed: $min(\mathcal{P}^0, 0) = min(0, \mathcal{P}^0) = 0$. Therefore we predict that both (10-a) and (10-b) will be judged merely *false* when we know that amplifiers have never buzzed.

(10) a. The amplifiers have stopped buzzing and they were buzzing before.
b. The amplifiers were buzzing before and they have stopped buzzing.

It's been observed since at least Stalnaker [25] and Heim [14] that sentences such as (10-a) have a status that the corresponding reversed sentence (10-b) doesn't. And the standard way of viewing this difference is in terms of presuppositions: (10-a) gives rise to a presupposition that (10-b) doesn't. The point I wish to make is the following. As far as the facts are concerned, it's unclear what *truth-value judgments* speakers would actually give for (10-a) and (10-b). We should, though, distinguish between the question whether (10-a) and (10-b) can give rise to different *truth-value judgments*, and the rather clear intuition that (10-b) is *utterable* in a broader range of conditions that (10-a).

Schlenker ([22]) pointed out that the asymmetry in conditions of use in cases like (10) could be related to a more general property of conjunctions. Indeed, the contrast we observed between (10-a) (which "sounds weird") and (10-b) is in a certain way similar to the one we observe between (11-a) (which "sounds weird" too) and (11-b):[22]

(11) a. John lives in Paris and he resides in France.
b. John resides in France and he lives in Paris.

[22] To insist on the need of distinguishing between giving a *non-classical truth-value* judgment for a sentence and feeling this sentence is "weird", note that you will judge *both* (11-a) and (11-b) completely false if you know John lives in London, but still regard (11-a) as weird.

Schlenker proposes a general constraint that has the effect of ruling out conjunctions where the first conjunct entails the second one. Note that, given the way we proposed to view presuppositional sentences in the previous section, the right conjunct in (10-a) can be regarded as expressing the *presuppositional part* of the left conjunct. Since for a presuppositional proposition to be true in ST5 its presuppositional part has to be true, whenever the left conjunct in (10-a) is true, the right conjunct is too: (10-a) would thus be ruled out by a principle *à la* Schlenker.

One should note moreover that if the only constraint on the use of (10-a) were for the presupposition of its left conjunct to be fulfilled, then (10-a) should sound totally fine in cases where (10-b) is known to be true, but this is not the case: if we know that the amplifiers used to buzz, (10-a) "sounds weird" in a way in which (10-b) does not. To this extent, the strength of the contrast between (10-a) and (10-b) should not be raised in favor of the view that (10-a) is presuppositional while (10-b) is not: as a matter of fact, we can't use our judgments on (10-a) to clearly distinguish between cases where the presupposition of its left conjunct is fulfilled from cases where it is not.

If one thinks that, nonetheless, these sentences should receive different *truth-value judgments*, a possibility is to revise the semantics of the conjunction operator so that it gives the value $\mathcal{P}^0$ to a conjunction whenever it has a proposition of value $\mathcal{P}^0$ on its left: with such a semantics, and contrary to the option above, (10-a) would come out as presuppositional in ST5 since it would get the value $\mathcal{P}^0$ in at least one model. As Fox [9] and George [11] point out, one can extend this kind of considerations to all the connectives in the system by resorting to a unifying principle in the spirit of the one proposed by Schlenker. However it is not clear whether disjunctions and implications show the same asymmetry (see (12)), and so whether one should or not revise the semantics of the connectives in the system.

(12) a. The amplifiers have stopped buzzing or they were not buzzing before.
b. The amplifiers were not buzzing before or they have stopped buzzing.
c. The amplifiers have stopped buzzing, if they were buzzing before.
d. If the amplifiers were buzzing before, they have stopped buzzing.

5 A Discussion of Potential Alternatives

The system that I have described adds two logical values to $\{0, \frac{1}{2}, 1\}$. Would it have been possible to add only one? Not given the semantics for $\neg$: the semantics for $\neg$ would force us to include a value corresponding to 1 minus the new additional value; and, since our initial three-valued set was $\{0, \frac{1}{2}, 1\}$, adding a fourth value would then require adding a fifth as well. One might however wonder if one could manage with a different kind of four-valued system in which the value $\frac{1}{2}$ played no role. There are two potential ways of doing this: by making the four values totally ordered, and by making them partially ordered.

Let us consider the first possibility. We would then have a set of four values $\{0, \mathcal{P}, \mathcal{V}, 1\}$, where $\mathcal{P}$ would be a value assigned to propositions describing situations of presupposition failure and $\mathcal{V}$ a value assigned to propositions describing borderline cases. In addition, we would have $\mathcal{P} = 1 - \mathcal{V}$ in order to fit the semantics for $\neg$. But there is a problem with this solution, and it is precisely related to negation. Imagine you have a proposition ϕ_P describing a case of presupposition failure and a proposition ψ_V describing a borderline case: as such, ϕ_P gets the value $\mathcal{P}$ and ψ_V gets the value $\mathcal{V}$. But now $\neg\phi_P$ gets the value $1 - \mathcal{P} = \mathcal{V}$, which is the value of ψ_V. And conversely, $\neg\psi_V$ gets the value $1 - \mathcal{V} = \mathcal{P}$, which is the value of ϕ_P. This has two unwelcome effects: first it predicts that we should observe the same truth judgments for negative counterparts of presuppositional sentences used in case of presupposition failure and for vague sentences used to describe borderline cases; second it predicts that we should observe different truth judgments for affirmative and negative counterparts of vague sentences. We have seen that these predictions are wrong; that excludes this approach.

But what about an alternative assuming a *partial* order — a set of four values $\{0, \mathcal{P}, \mathcal{V}, 1\}$ where $0 < \mathcal{P} < 1$ and $0 < \mathcal{V} < 1$? We would then need to adapt the semantics of our connectives to a partial ordered lattice: negation could semantically contribute as a symmetric operator (ie. for $\mathcal{I}(\phi) = 1, \mathcal{I}(\neg\phi) = 0$, for $\mathcal{I}(\phi) = 0$, $\mathcal{I}(\neg\phi) = 1$, for $\mathcal{I}(\phi) = \mathcal{V}$, $\mathcal{I}(\neg\phi) = \mathcal{V}$ and for $\mathcal{I}(\phi) = \mathcal{P}$, $\mathcal{I}(\neg\phi) = \mathcal{P}$), and conjunction and disjunction could respectively semantically contribute as the greatest lower bound and as the least upper bound.[23] But note that in this system, a proposition describing a case of presupposition failure would receive the same value as its negation. We would like to avoid this result given the asymmetry in our truth judgments for presuppositional sentences.

Raising the possibility of partially ordered values does suggest additional alternatives to the system developed here, so I would like to briefly address these. One possibility would be to consider a *partially ordered five*-valued set $\{0, \mathcal{P}^0, \mathcal{P}^1, \mathcal{V}, 1\}$ such that $0 < \mathcal{V} < 1$ and $0 < \mathcal{P}^0 < \mathcal{P}^1 < 1$: positive propositions describing situations of presupposition failure would have the value $\mathcal{P}^0$ and their negation would have the value $\mathcal{P}^1$. In fact, partially ordered systems of this kind give rise to an important problem irrespective of whether or not they incorporate a fifth value. Consider the conjunction and the disjunction in (13).

(13) a. The amplifiers are <u>loud</u> and they have <u>stopped</u> buzzing
b. The amplifiers are <u>loud</u> or they have <u>stopped</u> buzzing

With either the four-valued or the five-valued version of a partially ordered lattice, in situations where the amplifiers are borderline-loud and have never buzzed, (13-a) would express the conjunction of two propositions that would receive *non-ordered values* and (13-b) would express their disjunction. With

[23] A reviewer argued that there are other ways of defining the connectives that might be as legitimate as the standard Dunn-Belnap definition. This is perfectly fair and I am currently exploring a four-valued system with alternative definitions of the connectives. But since there is no place to develop it here, I will focus on standard approaches to the connectives in the rest of this paper.**R1.3**

conjunction being defined as the greatest lower bound and disjunction being defined as the least upper bound, the proposition expressed by (13-a) would get the value 0 and the proposition expressed by (13-b) would get the value 1. Such a system would therefore predict a pure *false* judgment for (13-a) and a pure *true* judgment for (13-b) in those situations, which clearly goes against our intuitions.

One might finally consider a system with still partially ordered values but such that the greatest lower bound and the least upper bound of the values for vagueness and presuppositions are not 0 and 1. With $\mathcal{E}^0$ and $\mathcal{E}^1$ the new $\mathcal{E}$xtra values, we would have a set of six values $\{0, \mathcal{E}^0, \mathcal{V}, \mathcal{P}, \mathcal{E}^1, 1\}$ such that $0 < \mathcal{E}^0 < \mathcal{V} < \mathcal{E}^1 < 1$ and $0 < \mathcal{E}^0 < \mathcal{P} < \mathcal{E}^1 < 1$. In this system, vagueness and presuppositions seem ontologically well distinguished ($\mathcal{P}$ and $\mathcal{V}$ are not ordered with each other), and in critical situations, the conjunction expressed in (13-a) would get the value $\mathcal{E}^0$ (the greatest lower bound of $\mathcal{P}$ and $\mathcal{V}$) and the disjunction expressed in (13-b) would get the value $\mathcal{E}^1$ (the least upper bound of $\mathcal{P}$ and $\mathcal{V}$). But this raises the question of what $\mathcal{E}^0$ and $\mathcal{E}^1$ actually represent. If their existence is motivated only by the existence of conjunctions and disjunctions of propositions describing borderline cases and propositions describing cases of situation failure, this seems a large price to pay. (In addition, the six-valued system I considered here is based on a partially ordered four-valued system which doesn't distinguish between affirmative and negative presuppositional sentences in cases of presupposition failure: a partially ordered *seven*-valued system might then be more adequate.)

In the system that I have settled on, there are five totally ordered values where each value has a clear ontological status. This seems superior to all of the alternatives I considered here. [R1.4,R2.5]

6 Conclusions

ST provides us with a notion of assertoric ambiguity that, along with some bridge principles, lets us explain our conflicting truth-value judgments in case of vagueness. Adding two symmetrical values around $\frac{1}{2}$ has made it possible to capture the difference between *not true* and *false* judgments and between *not false* and *true* judgments by virtue of bridge principles based on ST notions of satisfaction. Moreover, these values lend themselves naturally to an account for the relationship between truth-value judgments for the positive and negative counterparts of presuppositional sentences. Furthermore, we now have a system that incorporates *both* vagueness and presuppositions while *also* accounting for the differences in the judgments they trigger. At the same time, there is clearly more to be said about how the presuppositions of complex sentences depend on the presuppositions of the simple sentences they embed; here we had to add some stipulations. More data would be welcome in order to test the predictions of ST5: we are currently at work on an experimental design for eliciting truth-value judgments for vagueness and presuppositions.[R2.2]

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Reviewers' Comments

Reviewer #1:

Overall, I think this is a quite nice paper. It's very clearly written. Throughout reading it I had a good sense of the goal of the project and the plan for accomplishing it. The formal material is presented clearly, without getting bogged down in unnecessary detail. It is also, as best as I can see, technically correct.

I've got three suggestions. They aren't such that they absolutely need to be addressed in the final version, but they may be food for thought (I wasn't sure whether this should be marked as a 3 or 4 on the form–it definitely can be published as is should the author prefer).

1. I think that the formal account does a really nice job of doing justice to the motivating concerns of the project. However, I have some worries about these motivating concerns. It seems as though the theory is meant to be a predictive, broadly linguistic theory–that is, it's meant to predict actual linguistic behavior. This struck me as worrisome in two ways.

Point 1: First, as you acknowledge, there's a big divide on these linguistic intuitions. As you say, you and some others have access to them, though not everyone does. Does this mean that those with different intuitions ultimately mean something different by certain terms? **(see R1.1 on p. 251)**

Second, in discussing hybrid sentences, you mention that intuitions are murky, and that perhaps we need more empirical research to settle these cases. However, I'd be worried that the empirical research wouldn't be helpful, because all the people surveyed would presumably have the same kinds of murky intuitions. It seems that a good linguistic theory would actually refrain from predictions in these kinds of cases, whereas you say that a theory must make predictions. So maybe you're not giving a linguistic theory after all, but if not it would be good to say what you are doing.

I should note that obviously this gets into very big issues very quickly, so if there's not something reasonably quick you can say here, I wouldn't worry about it.

2. **Point 2:** In motivating the initial conflicting judgments, you make two arguments. First, that (1) is false because there was no buzzing before. Second, that (1) is not false because if false that would mean there was buzzing before. That is, you seem to accept "if no buzzing, then (1) is false" and "if (1) is false, then buzzing". Given transitivity for the conditional and contraction for the conditional, these imply that there was buzzing, which you don't accept in the case where you say (1) is false and not false. So, it looks like you have to give up transitivity or contraction. That's not necessarily a problem, but it would be interesting to hear which you prefer. **(see R1.2 on p. 251)**

3. **Point 3:** When you extend your account to conjunctions in 4.2, you make verdicts based on using a definition of conjunction in terms of minimum value. However, my sense here is that you need to do something to justify this. (The justification I have in mind is the kind of intuitive justification you can give

for Strong Kleene truth tables, given their intended application) One might worry that extending the same rules from ST to ST5 is overgeneralizing in a problematic way. **(see R1.3 on p. 260)**

Point 4: Relatedly, the definition in terms of minimum value presupposes that P0 should be lower than V, for instance. I'm not totally convinced by this. I see that $0 < \mathcal{P}^0 < \mathcal{P}^1 < 1$ and $0 < \mathcal{V} < 1$, but I'm not sure I'm convinced that there's any meaning to be attached to the relative orderings of $\mathcal{P}^0$, $\mathcal{P}^1$, and $\mathcal{V}$.**(see R1.4 on p. 259)**

Reviewer #2:

I understand my charge as assessing whether the paper is fit to publish. I judge that it is: it is interesting, developed to an appropriate level of explicitness and rigor, and beautifully written.

Without requiring addressing the comments below as a condition on publication, I offer some reactions and comments that may be of some use to the author either directly or in the future.

Point 1: It might be worth mentioning that Kit Fine's 1975 supervaluation approach to handling vagueness was inspired in part by van Fraassen's theory of presupposition failure. So there are precedents for thinking that formal techniques for handling vagueness and for handling presupposition might converge. **(see R2.1 on p. 252)**

Point 2: For future work, clearly it would be relevant and interesting to get Mechanical Turk data on the judgments people actually give for the more complicated sentences, and see whether the data support the predictions of the model. **(see R2.2 on p. 261)**

Point 3: Re (2)i: Note that the theory assumes that the border between borderline and tall is crisp: the proposition that someone is tall is assigned either to 1/2 or to 1. This is a reasonable compromise, but it is unrealistic: people can be clearly borderline tall, borderline borderline tall, and so on. **(see R2.3 on p. 252)**

Point 4: fn 15: M should be a set of belief states, not a set of beliefs. **(see R2.4 on p. 253)**

Point 5: p. 7: the structure of the dialectic is a bit garbled here. It's perfectly possible to have four truth values: true, false, borderline, and presup-failure, where neither borderline nor presup-failure entails the other. At that point, the argument that if conjunction/disjunction is treated as meet/join, we get undesirable results comes into play.

But in fact, what about a whiskered diamond configuration of six truth values? $5 > 4 > 3 > 1 > 0$, $4 > 2 > 1$, but 2 (borderline) and 3 (presup-failure) do not entail each other. The join is still not full truth, and the meet is still not full false. **(see R2.5 on p. 259)**