Chapter 6
Why Is More Different?

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6.1 Introduction

An emergent property or phenomenon is usually defined as one that arises out of lower level constituents but is neither reducible, explainable nor predictable from them. Emergence is sometimes associated with non-reductive physicalism, a view that advocates the physical nature of all concrete entities while acknowledging that some entities/properties that arise from this physical base cannot be reduced to it. The philosophical challenge is how to understand the relation between these various ontological and explanatory levels, especially since emergentists claim a distinct status for emergent phenomena/properties, distinguishing them from straightforward aggregates of constituents. To use Anderson's words, the whole is not only greater than but very *different* from the sum of the parts (1972, p. 395).

When dealing with emergence in physics, physicalism is not an issue. No one denies that emergent phenomena in condensed matter physics (e.g. superconductivity) are comprised of elementary particles or are physical in nature. Rather, the concern is whether some variant of reduction is really at work in contexts typically associated with emergence. An advocate of reduction could easily claim that because the macro level is composed of micro constituents there is no physical difference between different levels; instead what is lacking is an appropriate type of explanatory relation. Consequently, appeals to emergence simply indicate insufficient knowledge of the relevant explanatory connections between different theoretical levels, not a physical difference.

Moreover, the definition of emergence given above, which is the one commonly used in most discussions of emergence, is fully satisfied on purely epistemological grounds; further suggesting that emergence may simply point to a gap in our

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knowledge. In keeping with this epistemic orientation we also find emergence described in terms of novelty. For example, Butterfield (2011) defines emergence in terms of "behaviour that is novel and robust relative to some comparison class". In other words, we should understand the properties or behaviour of a composite system as novel and robust compared to its components. Defining emergence in this way requires that we carefully distinguish between phenomena that are properly emergent and those that are simply aggregates. In the latter case we can reduce the composite to its constituents as in the case of a house that can be decomposed into the various materials used to build it.

However, what Anderson's characterization suggests is that emergence has a strongly ontological dimension and indeed examples of emergence in physics tend to support this way of thinking. A philosophical account of emergence that is ontologically based requires that we lay out, in an explicit way, how the micro and macro levels in emergent behaviour/phenomena are related. In other words, what causal role does the microphysics play in characterizing emergent phenomena and does this relation presuppose some implicit type of reductionism?

Perhaps the most important feature in characterizing the micro/macro relation in emergence is the notion of autonomy and the supposed independence of these two levels in explaining emergent behaviour. The relation between ontological and epistemic independence is especially important since the latter is a necessary but not a sufficient condition for emergence; the fact that we *need not* appeal to micro phenomena to explain macro processes is a common feature of physical explanation across many systems and levels. Instead, what is truly significant about emergent phenomena is that we supposedly *cannot* appeal to microstructures in explaining or predicting these phenomena even though they are constituted by them.

I begin by reviewing some arguments that address the issue of autonomy and ontological aspects of emergence (e.g. Howard 2007; Humphreys 1997a, b) and discuss why they fail to capture the features necessary for emergent phenomena in physics. From there I go on to discuss the relation between emergence and phase transitions and why we need an account of emergence at all. As an illustration of the micro/macro relation I focus on superconductivity and how it is possible to derive its characteristic features, those that define a superconductor (infinite conductivity, flux quantization and the Meissner effect), simply from the assumption of broken electromagnetic gauge invariance. I end with a brief discussion of the relation between physics and mathematics and its relevance for emergence. Emphasising the importance of emergence in physics is not to deny that reductionism has been successful in producing knowledge of physical systems. Rather, my claim is that as a global strategy it is not always capable of delivering the information necessary for understanding the relation between different levels and kinds of physical phenomena. As such, emergence becomes an important part of how we come to understand fundamental features of the physical world.

6.2 Autonomy and the Micro/Macro Relation: The Problem

In discussions of emergence in physics it is important to keep in mind that the problem is articulating the relationship between different levels of phenomena, not their ontological status. Emergent physical phenomena are typically thought to exhibit new causal "powers", meaning that new physics emerges at different energy or length scales. Philosophical debates about emergence have often appealed to non-reductive physicalism as a way of capturing the autonomy of emergent phenomena, with supervenience being the preferred way of describing the micro/macro relation. The appeal of supervenience is that it allows one to retain the beneficial features of reduction without embracing its difficulties, that is, without having to say, exactly, what the relation between x and y is, over and above the fact that the latter supervenes on the former.[1]

There are several accounts of supervenience but most involve a type of dependency relation where the lower-level properties of a system determine its higher level properties. The relation is often characterized in the following way: *A* supervenes upon another set *B* just in case no two things can differ with respect to *A*-properties without also differing with respect to their *B*-properties. In slogan form, "there cannot be an *A*-difference without a *B*-difference". Since we can assume, for the context of this discussion, that physicalism is unproblematic, an extended discussion of the pros and cons of supervenience needn't concern us at this point.[2] Instead I want to briefly look at some ontological accounts of emergence, each of which respects the "autonomy" of emergent phenomena by showing why the identity claims characteristic of reduction fail. The question that concerns me is whether these accounts of autonomy can successfully capture features associated with emergent phenomena in condensed matter physics, the field where emergent phenomena are perhaps most evident.

Humphreys (1997a, b) defines emergence in terms of a fusion relation operating between different levels of entities and properties.[3] He characterises the fusion relation (1997, p. 8) by defining a class of *i*-level properties and entities, $\mathrm{P}^{\mathrm{i}}_{\mathrm{m}}$and $\mathrm{x}^{\mathrm{i}}_{\mathrm{r}}$ respectively, as the first level at which instances of $\mathrm{P}^{\mathrm{i}}_{\mathrm{m}}\left(\mathrm{x}^{\mathrm{i}}_{\mathrm{r}}\right)$ occur. The fusion operation

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[1] Rueger's (2000) account of emergence involves a notion of supervenience defined in terms of stability or robustness. An emergent phenomenon/property is produced when a change in the subvenient base produces new behaviour that is both novel and irreducible. The causal powers that emergent phenomena have are simply those that "structural properties have in virtue of being configurations of their lower level constituents" (2000, p. 317). My difficulty with this view is that even if the emergent properties are novel and irreducible they are still the result of the system configured in a certain way. Consequently the causal powers of the whole are no different from those of the parts, making emergent properties similar to resultant properties.

[2] See Beckermann, Flohr and Kim (1992) for various discussions.

[3] If we identify emergent properties as resulting from the interaction of the constituents then it isn't immediately clear how to motivate the "more is different" claim characteristic of emergent phenomena.

[*] results in the following: if $P^i_m\left(x^i_r\right)(t_1)$, $P^i_n\left(x^i_s\right)(t_1)$ are i-level property instances, then $\left[P^i_m\left(x^i_r\right)(t_1) * P^i_n\left(x^i_s\right)(t_1)\right]$ is an i + 1-level property instance, the result of fusing $P^i_m\left(x^i_r\right)(t_1)$ and $P^i_n\left(x^i_s\right)(t_1)$. According to Humphreys it is the physical interactions represented by the fusion operation that lead to the transition from the *i* to *i* + 1 level that is responsible for emergent features. The fused $\left[P^i_m * P^i_n\right]\left[\left(x^i_r\right) + \left(x^i_s\right)\right]\left(t'_1\right)$ is a unified whole in that its causal effects cannot be correctly represented in terms of the separate causal effects of $P^i_m\left(x^i_r\right)(t_1)$ and $P^i_n\left(x^i_s\right)(t_1)$. Moreover, within the fusion $\left[P^i_m * P^i_n\right]\left[\left(x^i_r\right) + \left(x^i_s\right)\right]\left(t'_1\right)$ the original property instances $P^i_m\left(x^i_r\right)(t_1)$, $P^i_n\left(x^i_s\right)(t_1)$ no longer exist as separate entities and do not possess all their *i*-level causal powers available for use at the (*i* + 1) level. In other words, these *i*-level property instances no longer have independent existence within the fusion; they simply go out of existence in producing the higher level emergent instances.

Here the subvenient base cannot be the reason why the emergent property is instantiated since the *i* + 1 level property instances do not supervene upon the *i*-level property instances. Humphreys (15) cites the example of quantum entanglement as a case of emergence resulting from the kind of fusion he describes. The composite system can be in a pure state when the component systems are not, and the state of one component cannot be completely specified without reference to the state of the other component. He sees the interactions that give rise to the entangled states as having the features required for fusion because the relational interactions between the constituents can no longer be separately individuated within the entangled pair.[4]

Silberstein and McGeever (1999, p. 187) claim that "QM provides the most conclusive evidence for ontological emergence" and their discussion of entanglement (189) appears to endorse the appropriateness of fusion for describing the whole-part relation in this context. In Howard (2007, p. 12) the paradigm case of emergence is also quantum entanglement and he claims that in areas of condensed matter physics where there is a reasonably successful theory (superconductivity and superfluidity) there is also a clear connection to microphysical entanglement. As an example he cites the role of Bose-Einstein condensates (BEC) in superfluidity and the way that Cooper pairs in superconductivity are, in effect, BECs.[5] Consequently the phenomena of condensed matter physics supervene on the most basic property of the micro-realm—entanglement (17). Howard states that "while condensed

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[4] Humphreys also discusses examples of emergent phenomena that aren't of this sort, namely those that occur in ideal *macroscopic* systems containing an infinite number of particles (1997b, p. 342). His point is that the emergent properties cannot be possessed by individuals at the lower level because they occur only with infinite levels of constituents. Since these are exactly the sorts of examples I will have more to say about below.

[5] A Bose-Einstein condensate is a state of mater formed by bosons confined in an external potential and cooled to 0 kelvin or −273.15 °C. This causes a large fraction of the atoms to collapse into the lowest quantum state of the external potential.

matter physics does not obviously reduce to particle physics, phenomena…such as superfluidity and superconductivity do supervene on physical properties at the particle physics level and hence are not emergent with respect to particle physics…" (6). In other words, "the physical structure that [does] the explaining in condensed matter physics….is entanglement…the micro-world upon which condensed matter physics is said not to supervene" (22).

While these various claims about entanglement as an example of emergence are certainly plausible, the converse is less convincing; in other words, phenomena such as superconductivity, crystallization, magnetization, superfluidity, are neither explained by nor ontologically identified with quantum entanglement.[6] Nor can the latter account for the stability associated with these phenomena and the ability to make very accurate predictions about their behaviour. Humphreys is explicit that emergence does not require supervenience insofar as the fused properties cease to exist once the emergent phenomenon is present. But, how can this enable us to retain the ontological independence of the micro level in contexts like CMT and particle physics? Howard's solution is to understand these relations as supervenient, but this is of little help if we understand supervenience in the typical way, where the connection between the two levels requires a covariance relation to be maintained.[7] While Howard acknowledges that supervenience does not imply reduction, we shall see below that the kind of phenomena considered emergent in CMT, specifically universal behaviour, is not actually explained in terms of microphysical properties in the way he suggests, nor does it exemplify a supervenience relation. The characteristic behaviour(s) that identify phenomena as emergent (e.g. infinite conductivity) are neither explained nor identified with microphysical constituents.[8] Moreoever, one of the hallmarks of emergent phenomena is that they are insensitive to their microphysical base which challenges the dependency relation present in supervenience.

Because emergent phenomena 'arise out' of their microphysical base we need some account of the ontological connection between the levels to fully explain the exact nature of the 'emergence' relation. In the case of ontological reduction there exists a type of *identity* that cannot be upheld in cases of emergence. Reductionism assumes, among other things, that because a particular macro phenomenon is a

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[6] Although entanglement is undoubtedly operating here my use of the term 'identified' is meant to indicate that I don't subscribe to the view that emergent phenomena are *explained* via an ontolotical identification with entangled states, nor does the association with entanglement serve as an example of the supervenience relation where the basal property is associated with the higher level property.

[7] Howard cites Davidson's (1970) definition where supervenience is described as an ontic relationship between structures.construed as a set of entities. The higher level (B) entities supervene on the lower level (A) ones iff the former are wholly determined by the latter such that any change in (B) requires a corresponding change in (A).

[8] Infinite conductivity is one of the properties, along with flux quantization and the Meissner effect, that are exact regardless of the type of metal that comprises the superconductor.

collection of micro entities/properties the latter not only explains the behaviour of the former, giving us some insight as to why it behaves as it does, but it also constitutes it. Emergence shows us that the opposite is true! Initially this appears somewhat confusing because, for example, we typically understand the causal foundation of superconductivity in terms of Cooper pairing; so to claim that there is no reduction to or identification with this microphysical base requires a clarification of the exact nature of these ontological relations.

In what follows I show how the nature of universality as well as the role played by it clarifies both how and why emergent phenomena are independent of any specific configuration of their microphysical base. An important advantage of this strategy is that the microphysical entities and properties remain intact and autonomous, unlike Humphreys' fusion relation or accounts that appeal to quantum entanglement. As we shall see below, this relative independence from the underlying microphysics is crucial for understanding the difference between emergent and resultant properties and for highlighting the similarities and differences between emergence and multiple realizability.

6.3 Emergence and Reduction

In physics it has been common to think of explanation in reductivist terms, involving the elementary constituents of matter and the laws that govern them. Indeed this is the motivation behind a good deal of contemporary physics and is a strategy that has not been without success, as in the case of Maxwell's electrodynamics and Newtonian mechanics. Although the limits and difficulties associated with various forms of reductionism (ontological and inter-theoretic) have been well documented, it is still thought of as the ultimate form of explanation, as something to aspire to despite the difficulties attaining it.[9]

When evaluating the merits of reductionist explanation it is also important to inquire about its limits and how far this kind of explanation extends, specifically, what actually counts as "reduction" and at what point does the addition of free parameters undermine reductionist claims? The non-relativistic Schrodinger equation presents a nice illustration of the kind of reduction we typically associate with explanation in physics.

$$i\hbar\frac{\partial}{\partial t}|\Psi> \; = \mathscr{H}|\Psi> \tag{6.1}$$

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[9] This is especially true in the philosophy of science literature. Sklar has written extensively on the problems of reduction and the relation between thermodynamics and statistical mechanics. See his (1999) for a pointed discussion of these issues.

It describes, in fairly accurate terms, a large number of physical phenomena and can be completely specified by a few quantities such as the charges and masses of the electron and atomic nuclei, as well as Planck's constant. It can be solved accurately for small numbers of particles (isolated atoms and small molecules) and agrees in minute detail with experiment. However, as Laughlin and Pines (2000) point out, when the number of particles exceeds around ten this is no longer the case. It is possible to perform approximate calculations for larger systems which yield knowledge of atomic sizes and the elastic properties of solid matter, etc. but the use of approximation techniques means that these are no longer deductions from first principles or fundamental theory—instead they require experimental input and specific, local details. What this indicates is a breakdown of the reductionist ideal of deriving explanations of a large number of phenomena from a few simple equations or laws.

But does this *really* undermine reduction as an explanatory strategy? The answer depends, in part, on how many free parameters one is willing to accept into the explanation; in other words, at what point does it no longer make sense to call an explanation reductive when the explanatory information comes via the free parameters rather than fundamental features of theories/laws. Of course one might also argue that calling a phenomenon "emergent" is simply a stop gap measure indicating we haven't yet hit on the right theoretical principles. The difficulty with this type of response is that it offers only a promissory note and fails to help us understand the phenomena/system under investigation. Put slightly differently: Our lack of understanding results, in the first instance, from a failure in the reductive strategy; hence the need for an alternative framework. Whether we might someday be able to perform the right sort of derivations or calculations from first principles is irrelevant for evaluating the merits of reduction in the cases where it currently fails to provide the relevant information.

But, when it comes to articulating the important features of emergence we need to move beyond the failure of reduction or limiting inter-theoretic relations since this too can be indicative of an epistemic problem. Moreover, if emergence simply means that a phenomenon at one level, characterized by a particular theory, fails to be fully explainable by the theory at the next lower level then it becomes much too pervasive. Instead the focus should be on what is ontologically distinct about emergent phenomena such that they are immune from the contingencies of reduction.

Laughlin and Pines (2000) point out that the parameters $e, \hbar$, and m appearing in the Hamiltonian for the Schrodinger equation

$$\mathcal{H} = -\sum_{j}^{N_e} \frac{\hbar^2}{2m} \nabla_j^2 - \sum_{\alpha}^{N_i} \frac{\hbar^2}{2M_\alpha} \nabla_\alpha^2 - \sum_{j}^{N_e} \sum_{\alpha}^{N_e} \frac{Z_\alpha e^2}{|\vec{r}_j - \vec{R}_\alpha|} + \sum_{j \not< k}^{N_e} \frac{e^2}{|\overline{r}_j - \vec{r}_k|} + \sum_{j \not< \beta}^{N_j} \frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha - \vec{r}_\beta|}. \tag{6.2}$$

can be accurately measured in laboratory experiments involving large numbers of particles but can't be derived or predicted by direct calculation.[10] For example, electrical measurements performed on superconducting rings can determine, to a very high level of accuracy, the quantity of the quantum of magnetic flux $hc/2e$ and four point conductance measurements on semiconductors in the quantum Hall regime accurately determine the quantity e^2/h. Because it is impossible to derive these exact results using either first principles, or approximation techniques, the natural question that arises is what explains the stable behaviour in these cases?

Although no reductive explanation is possible the examples indicate, in a more pointed way, the need for 'emergence' in order to account for the stability. Laughlin and Pines claim that these type of experiments work because "there are higher organizing principles in nature that make them work" (2000, p. 28). Examples of such principles are continuous symmetry breaking which renders exact the Josephson quantum and localization which is responsible for the quantum Hall effect.[11] They claim that both effects are "transcendent" in that neither can be deduced from the microphysics and would continue to be true even if the theory of everything were changed. These are classified as emergent phenomena or 'protectorates'.

When Laughlin speaks of organizational principles he seems to have in mind the kind of order that is produced as a result of some type of collective action that is essentially independent of the details of the underlying microphysics. For example, he mentions principles governing atomic spectra that can be understood without any reference to the quark structure of nucleons and the laws of hydrodynamics which would be roughly the same regardless of variations in detailed intermolecular interactions. However, in both of these cases we need to differentiate explanatory from ontological claims since emergence isn't simply about different organizational principles being important at different scales or laws not requiring specific micro-details. More is required.

In Laughlin's and Pines' discussion of continuous symmetry breaking they don't elaborate on the notion of 'transcendence' or the status of organizing principles, but in the latter case independence from specific theoretical content is going to be necessary if emergent phenomena are to be properly autonomous from the microphysical domain. While many physical theories/phenomena incorporate or involve various types of symmetry breaking, the notion itself is not linked to any *specific* theoretical framework. Rather, it functions as a structural constraint on many different kinds of systems in both high energy physics as well as condensed matter physics.

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[10] The symbols $Z\alpha$ and $M\alpha$ are the atomic number and mass of the αth nucleus, $R\alpha$ is the location of this nucleus, e and m are the electron charge and mass, r_j is the location of the jth electron, and h is Planck's constant.

[11] Localization involves the absence of diffusion of waves in a random medium caused by a high concentration of defects or disorder in crystals or solids. In the case of electric properties in disordered solids we get electron localization which turns good conductors into insulators.

I refer to symmetry breaking as a “structural/dynamical feature of physical systems” because of the way order and structure emerge as a result of the phase transitions associated with symmetry breaking. In fluid dynamics the emergence of new order and structure occurs when a dynamical system is driven further and further away from thermal equilibrium. By increasing control parameters like temperature and fluid velocity old equilibria become unstable at critical points, break down, and new branches of local equilibria with new order emerge. Spontaneous symmetry breaking (SSB) is manifest in, among other things, the acquisition of rigidity and the existence of low energy excitations in condensed matter physics; superconductivity incorporates symmetry breaking via Cooper pairing as a basic feature in the BCS theory. Particle masses in high energy physics are also thought to be generated by SSB. In each of these cases we have dynamical processes that produce specific effects. Because these processes involve a connection with microphysics, the challenge for the emergentist is to explain how and why we should think of symmetry breaking as distinct from the type of fundamental physics associated with reduction. We’ll see why this is the case below.

Before discussing that point it is important to mention that the status of SSB in the case of local gauge symmetries (of the kind relevant for superconductivity) is not entirely clear. Elitzur’s theorem (1975) states that local gauge symmetries cannot be spontaneously broken. Although the theorem was proved for Abelian gauge fields on a lattice it is suggested that it doesn’t rule out spontaneously broken global symmetries within a theory that has a local gauge symmetry, as in the case of the Higgs mechanism. There is a good deal of controversy regarding the interpretation of SSB as a “physical” phenomenon with the main arguments enumerated and addressed by Friedrich (2013) who also argues against the realistic interpretation. Although I certainly cannot provide a proper discussion of the issue here, let me mention a few points worth keeping in mind regarding the role SSB plays in the theoretical context of phase transitions.

First, it is important to note that Elitzur’s theorem is specific to the lattice because on the lattice it isn’t necessary to fix a gauge. Moreover, many claim that the lattice description is the appropriate one because it eliminates any reliance on perturbation theory. While there are conflicting pictures presented by the continuum and lattice formulations (see Frolich et al. 1981) one further point is worth emphasising from the “realist” perspective. It is certainly possible to carry out perturbative calculations with a Lagrangian having a local symmetry in which scalar fields that are not invariant under the symmetry have non vanishing vacuum expectation values (VEVs). This, it would seem, deserves to be called a spontaneously broken local symmetry. Perhaps the difficulty and confusion surrounding this issue arises as a result of perturbation theory; nevertheless, let me assume for the sake of argument that SSB does in fact occur in phase transitions. What are the interesting implications for emergence?

When symmetries are spontaneously broken the result is the occurrence of ordered states of the sort Laughlin refers to. For example, magnetisation results from broken spin rotation symmetry and massive particles break a phase rotation symmetry. These symmetries impose structural constraints on the physical world in that they give rise to and explain certain forms of dynamical interactions. As we shall see below these constraints are *general* structural features of physical systems that can apply in a variety of theoretical contexts. By contrast, fundamental theory is concerned with details, expressed via laws and models, of *specific* physical systems and how they behave. It is these general features rather than specific details of micro-processes that prove important for emergence. In order to clarify the ontological relations among emergence, symmetry breaking and microphysics let me turn to the example of superconductivity which nicely illustrates these features.

6.4 Phase Transitions, Universality and the Need for Emergence

As we saw above one of the organizing principles Laughlin and Pines mention is continuous symmetry breaking. While many physical theories/phenomena incorporate or involve various types of symmetry breaking the notion itself is not linked to any *specific* theoretical framework. Rather, it functions as a structural constraint on many different kinds of systems in both high energy physics as well as condensed matter physics. For example, the electroweak theory postulates symmetry breaking via the Higgs mechanism which allegedly explains bosonic masses; superconductivity also incorporates symmetry breaking via Cooper pairing as a basic feature in the BCS theory. Because these processes appear to involve a connection with microphysics, the challenge is to explain how and why we should think of symmetry breaking as an organizing principle and not part of "fundamental" theory.

Other types of organizing principles like kinship and valency function as either a principle for organizing individuals into groups or in the latter case as a measure of the number of chemical bonds formed by the atoms of a given element. Valency, understood as an organizing principle, has evolved into a variety of approaches for describing the chemical bond such as valence bond theory and molecular orbitals, as well as methods of quantum chemistry. In that sense it provides a foundational framework within which different methodological approaches can be unified and also functions as a kind of heuristic principle in the elementary study of covalent bonds. But, when Laughlin speaks of organizational principles he has in mind the kind of order that is produced as a result of some type of collective action that is essentially independent of the details of the underlying microphysics. For example, he mentions principles governing atomic spectra that can be understood without any reference to the quark structure of nucleons and the laws of hydrodynamics which would be roughly the same regardless of variations in detailed intermolecular interactions.

While this seems like a claim about different organizational principles being important at different scales, emergence isn't captured simply by an appeal to different levels nor do physical explanations always require an appeal to "fundamental" theories. True independence from fundamental theory, as characterised by emergence, requires that we locate the relevant explanatory details in more general features capable of explaining how emergent phenomena arise. Crystals arise from the breaking of translation symmetry, magnetisation from broken spin rotation symmetry and massive particles break a phase rotation symmetry. These symmetries impose structural constraints on dynamical features of the physical world described by our theories. To that extent they do more than simply organize phenomena into certain types, they function as meta-laws via their role in explaining certain forms of dynamical interactions. To the extent that symmetry breaking explains certain features/behaviour of physical phenomena we can distinguish it from the role that fundamental theory plays in explanation, explanations whose focus is microphysical phenomena and the laws that govern them. Maintaining this distinction is crucial for upholding the autonomy of emergent phenomena.

On a very basic level we can think of symmetry constraints as providing us with general structural principles that apply in a variety of theoretical contexts; fundamental theory, on the other hand, is concerned with more specific types of physical systems and the details of how those systems behave. Those details take the form of theoretical laws or models that describe and explain the behaviour of particular types of phenomena. For example, the Schrodinger equation and the Pauli exclusion principle are part of the theoretical framework of quantum mechanics, as are models like the finite potential well. By constrast symmetry principles like those mentioned above are associated with a wide variety of physical theories and laws, both quantum and classical and operate at a meta-theoretical level furnishing the very general features that systems possess. It is these general features rather than specific details of micro-processes that prove important for emergence. In order to clarify the sense in which this ordering could be thought of as 'transcendent' let me turn to the example of superconductivity to illustrate the relation between emergence, symmetry breaking and microphysics.[12]

Many of the physical properties of superconductors such as heat capacity and critical temperature (where superconducting properties are no longer present) vary depending on the type of metal. However, there is a class of properties that are independent of the specific material and are exact for all superconductors, properties such as infinite conductivity (very low electrical resistance and currents that can circulate for years without perceptible decay), flux quantization and the Meissner effect.[13] These can be predicted with extraordinary accuracy; but in deriving them and other phenomena associated with superconductors one typically uses models that are just reasonably good approximations. There are macroscopic models like Ginzburg-Landau where cooperative states of electrons are represented

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[12] This is necessary especially as an answer to Howard (2007).

[13] The former is a quantum phenomenon in which the magnetic field is quantized in the unit of $h/2e$ while the latter simply refers to the explusion of a magnetic field from a superconductor.

using a complex scalar field and the microscopic model(s) of the Bardeen-Cooper-Schrieffer (BCS) theory where electrons appear explicitly and are assumed to interact only by single phonon exchange. The latter is the widely accepted account that explains the superconducting phase of a metal as involving many pairs of electrons (Cooper pairs) bound together at low temperatures. This pairing in a superconductor is a collective phenomenon analogous to magnetization in a magnet and, as with magnetism, involves symmetry breaking and a phase transition. The essence of the BCS theory is the appearance of a pair field which is the order parameter of the superconducting state, just as magnetization is the order parameter of the ferromagnet.[14] Exactly how this pairing occurs is the subject of different model explanations, one of which was provided by BCS themselves in their original paper (1957).[15]

It is tempting to see the story about Cooper pairing as a reductive, micro-causal explanation insofar as the electron pairs seem to be the defining characteristic of superconductivity. However, the story is more complicated than might first appear. Recall the discussion above of the Josephson effect and the problem of deriving exact results from approximations. The same situation arises with superconductivity where the properties (infinite conductivity, flux quantization etc.) are exact and the same for all superconductors. Since they are exact results they must follow from general principles rather than simply derived using approximations. So, while highly precise predictions about superconductors follow from the models they do so because the models embody a symmetry principle—the spontaneous breakdown of electromagnetic gauge invariance (Weinberg 1986, 1996). One needs detailed models like BCS to explain the specifics of *how* the symmetry breaking (SSB) occurs, at what temperature superconductivity is produced, and as a basis for approximate quantitative calculations, but not to derive the most important *exact* consequences of this breakdown—infinite conductivity, flux quantization and the Meissner effect—properties that define superconductors.[16]

This fact is crucial for our account of emergence because it shows that the microphysical details about *how* Cooper pairing takes place are not important in deriving and explaining fundamental features of superconductivity. Put differently, it isn't that instances of superconductivity in metals don't involve micro-processes, rather the characteristics that define the superconducting state are not explained or predicted from those processes and are independent of them in the sense that changes to the microphysical base would not affect the emergence of (universal) superconducting properties. Although the breakdown of gauge invariance involves the formation of Cooper pairs—a dynamical process—the micro story figures simply as the foundation from which superconductivity emerges.

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[14] The order parameter is a variable that describes the state of the system when a symmetry is broken; its mean value is zero in the symmetric state and non-zero in the non-symmetric state.

[15] For more on the topic of superconductivity, theories and models see Morrison (2007, 2008).

[16] See Weinberg (1986, 1996) for details.

The key to understanding this relationship involves the connection between phase transitions and symmetries. Symmetry breaking is reflected in the behaviour of an order parameter that describes both the nature and magnitude of a broken symmetry. In the ferromagnetic state the order parameter is represented by the vector describing the orientation and size of the material's magnetization and the resulting field. In the superconducting case the order parameter is the amplitude $\langle \varphi \rangle$ of the macroscopic ground state wave function of the Cooper pairs. The electromagnetic properties in a superconductor are dominated by Cooper pairs whereas electrons in a metal normally behave as free particles that are repelled from other electrons due to negative charge. Because Cooper pairs only appear at T_c (their presence indicates that the system has undergone a phase transition) they give rise to the order parameter which implies that the Cooper pairs must form a single wave function. In general the order parameter can be thought of as an extra variable required to specify the macroscopic state of a system after the occurrence of a phase transition. In non-superconducting metals gauge invariance ensures that $\langle \varphi \rangle = 0$. It should be noted here that an order parameter can have a well defined phase in addition to an amplitude and it is the phase that governs the macroscopic properties of superconductors and superfluids.

Given this picture we now need to disentangle the relation between the order parameter and the emergent nature of superconductivity. Recall that the broken symmetry associated with the order parameter in superconductivity is electromagnetic gauge invariance. The electromagnetic properties are dominated by Cooper pairs with each pair j having a wave function

$$\psi_c^j(\boldsymbol{r}) = V^{-1/2} \mathrm{a}_j(\boldsymbol{r}) \exp \mathrm{i}\phi_j(\boldsymbol{r}) \tag{6.3}$$

where $\mathrm{a}_j(\boldsymbol{r})$ and $\phi_j(\boldsymbol{r})$ represent the amplitudes and phases respectively. The mean separation at which pair correlation becomes effective is between 100 and 1,000 nm and is referred to as the coherence length, ξ, which is large compared with the mean separation between conduction electrons in a metal. In between one pair there may be up to 10^7 other electrons which are themselves bound as pairs. The coherence volume ξ^3 contains a large number of indistinguishable Cooper pairs so one must define a density of wave functions averaged over the volume. The average will only be non-zero if the phases $\phi_j(\boldsymbol{r})$ are close together; i.e. the neighbouring Cooper pairs are coherent. In the case of the groundstate wavefunction density

$$\psi(\boldsymbol{r}) = 1/\xi^3 \sum_{j \in \xi^3} \psi_j(\boldsymbol{r}_j) \propto \sqrt{} \mathrm{n_s} \exp i\phi(\boldsymbol{r}) \tag{6.4}$$

we can identify $|\psi(\boldsymbol{r})|^2$ with the density of Cooper pairs at point $\boldsymbol{r}$ and then define creation and annihilation operators for particles at $\boldsymbol{r}$. In a normal conducting metal the expectation value for these operators takes value zero but in superconductors the operator $\psi(\boldsymbol{r})$ acquires a non-zero expectation $\langle \psi(\boldsymbol{r}) \rangle$. So, at zero temperature

$$\left\langle \psi^{\dagger}(\boldsymbol{r})\psi(\boldsymbol{r})\right\rangle = \left\langle \psi^{\dagger}(\boldsymbol{r})\right\rangle \ \langle \psi(\boldsymbol{r})\rangle. \tag{6.5}$$

The order parameter is then defined as the expectation value of operator $\psi(\boldsymbol{r})$.

Above I claimed that one can derive the exact properties of superconductors from the assumption of broken electromagnetic gauge invariance. To show that this is, in fact, the symmetry that is broken we consider the following: In a superconductor it is generally possible to choose the gauge $\Lambda(\boldsymbol{r})$ of the vector potential which determines the phase of the wave function of each particle, i.e.

$$\begin{aligned} \psi'(\boldsymbol{r}) &= \psi(\boldsymbol{r}) \exp(2\pi i \Lambda(\boldsymbol{r})/\Phi_0), \\ \boldsymbol{A}'(\boldsymbol{r}) &= \boldsymbol{A}(\boldsymbol{r}) + \nabla \Lambda(\boldsymbol{r}) \end{aligned} \tag{6.6}$$

If the particles are independent it is possible in principle to choose a different gauge to describe the motion of each particle. However, phase coherence between the various Cooper pairs requires that all the particles have the same gauge. Consequently, the symmetry broken by the order parameter is local gauge invariance. The same choice of vector potential must be made for all of the particles. The system thus selects a particular phase in the same way a magnet selects a particular direction below the Curie temperature. Choosing a particular phase for the order parameter amounts to choosing a particular gauge for the vector potential $\boldsymbol{A}$—hence the physical significance of the electromagnetic gauge in this context.

We can now go on to show how to derive the exact (emergent) properties of superconductors from the assumption of broken electromagnetic gauge invariance. To demonstrate this we consider how the consequences of broken gauge invariance for superconductors can be derived from a formalism that deals solely with the general properties of the Goldstone mode which is a long-wavelength fluctuation of the corresponding order parameter.[17] The general framework is set up in the following way: The electromagnetic gauge group $U(1)$ is the group of multiplication of fields $\psi(x)$ of charge q with the phases $\psi(x) \rightarrow \exp(i\Lambda q/\hbar)\ \psi(x)$. Because the q are integer multiples of $-e$ the phases Λ and $\Lambda + 2\pi\hbar/e$ are taken to be identical. $U(1)$ is spontaneously broken to Z_2 the subgroup consisting of $U(1)$ transformations with $\Lambda = 0$ and $\Lambda = \pi\hbar/e$. According to the general understanding of SSB the system described by a Langranian with symmetry group G, when in a phase where G is broken to a subgroup H, will possess a set of Nambu-Goldstone excitations described by fields that transform under the symmetry group G like the coordinates of the coset space G/H. In this case there will be a single excitation described by a field $\varphi(x)$ that transforms under $G = U(1)$ like the phase Λ. The $U(1)$ group has the multiplication rule $g(\Lambda_1)g(\Lambda_2) = g(\Lambda_1 + \Lambda_2)$ so under a gauge transformation with parameter Λ, the field $\varphi(x)$ will undergo the transformation $\varphi(x) \rightarrow \varphi(x) + \Lambda$. Because $\varphi(x)$ parameterizes $U(1)/Z_2$ rather than $U(1)$, $\varphi(x)$ and $\varphi(x) + \pi\hbar/e$ are

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[17] My discussion follows Weinberg (1986).

regarded as equivalent field values. The characteristic property of a system with broken symmetry is that the quantity $\varphi(x)$ behaves like a propagating field.

When one turns on the interaction of the superconductor with the electromagnetic fields **B** and **E** their interaction is governed by the principle of local gauge invariance where the Nambu-Goldstone field $\varphi(x)$ transforms under $U(1)$ with a space-dependent phase $\varphi(x) \longrightarrow \varphi(x) + \Lambda\ (x)$. The potentials transform as usual and all the other field operators are gauge invariant. The Lagrangian for the superconductor plus electromagnetic field is:

$$\mathrm{L} = {}^{1}\!/_{2} \int d^3\mathrm{x}\left(\mathbf{E}^2 - \mathbf{B}^2\right) + \mathrm{L_m}[\nabla\varphi - A,\ \dot{\varphi} + \mathrm{A}^0,\ \tilde{\psi}] \qquad (6.7)$$

where the matter Lagrangian is an unknown function of the gauge invariant combinations of $\partial_\mu\ \varphi$ and A_μ as well as the unspecified gauge-invariants $\tilde{\psi}$ representing the other excitations of the system. From $\mathrm{L_m}$ one obtains the electric current and charge density as variational derivatives

$$J(\mathrm{x}) = \delta \mathrm{L_m}/\delta A(\mathrm{x}) \qquad (6.8)$$

$$\varepsilon(\mathrm{x}) = -\delta \mathrm{L_m}/\delta \mathrm{A}^0(\mathrm{x}) = -\delta \mathrm{L_m}/\delta \dot{\varphi}(\mathrm{x}). \qquad (6.9)$$

Because $\varphi(x)$ is the only non gauge-invariant matter field we can use just the Lagrangian equations of motion for $\varphi(x)$ to derive the equation for charge conservation. The structure of the functional matter Lagrangian need not be specified, instead one need only assume that in the absence of external electromagnetic fields the superconductor has a stable equilibrium configuration with vanishing fields

$$\nabla\varphi - A = \dot{\varphi} + \mathrm{A}^0 = 0. \qquad (6.10)$$

The assumption that electromagnetic gauge invariance is spontaneously broken is equivalent to the claim that the coefficients of the terms in $\mathrm{L_m}$ of second order in $\nabla_\phi - A$ and $\dot{\varphi} + \mathrm{A}^0$ have non-vanishing expectation values which makes φ behave like an ordinary physical excitation. As we shall see in deriving the consequences of these assumptions, the important point is that $\varphi(x)$ is not understood as the phase of a complex wave function used in an “approximate” model/treatment of electron pairing, but rather, a Nambu-Goldstone field that accompanies the breakdown of SSB. Put differently, we don’t need a microscopic story about electron pairing and the approximations that go with it to derive the exact consequences that define a superconductor. Planck’s constant $\hbar$ simply does not appear in the differential equations governing φ.

From this framework one can derive fundamental properties of superconductors like the Meissner effect, flux quantization and infinite conductivity. For example, in the case of flux quantization we have a current flowing through a superconducting loop in thick closed rings that is not affected by ordinary electrical resistance. It cannot decay smoothly but only in jumps. However, when dealing with infinite conductivity one needs to take account of time-dependent effects. We saw above (6.9) that charge density is given by $-\varepsilon(x) = \delta L_m/\delta\varphi(x)$ where $-\varepsilon(x)$ is the dynamical variable canonically conjugate to $\varphi(x)$. In the Hamiltonian formalism H_m is a functional of $\varphi(x)$ and $\varepsilon(x)$ with the time dependence of φ given by

$$\varphi(x) = \delta H_m/\delta(-\varepsilon(x)). \tag{6.11}$$

The voltage at any point is defined as the change in the energy density per change in the charge density at that point

$$V(x) \equiv \delta H_m/\delta\varepsilon(x). \tag{6.12}$$

Consequently the time-dependence of the Nambu-Goldstone field at any point is given by the voltage $\dot{\phi}(x) = -V(x)$. From this it follows that a piece of superconducting wire that carries a steady current with time independent fields must have zero voltage difference between its ends, which is just what is meant by infinite conductivity. Without this zero voltage the gradient $\nabla\varphi(x)$ would have to be time dependent leading to time dependent currents or fields.

A crucial part of the story, which is significant for emergence, is the relation between infinite conductivity and the presence of an energy gap in the spectrum of the Cooper pairs. Typically it is the presence of an energy gap that distinguishes superconductivity from ordinary conductivity by separating the Fermi sea of paired electrons from their excited unpaired states. The process is thought to be due, essentially, to quantum mechanics and it implies that there is a minimum amount of energy ΔE required for the electrons to be excited. As temperature increases to Tc, Δ goes to 0. Although some accounts of superconductors relate infinite conductivity directly to the existence of the gap, the treatment above shows that infinite conductivity depends only on the spontaneous breakdown of electromagnetic gauge invariance and would occur regardless of whether the particles producing the pairing were fermions instead of bosons. This is further evidenced by the fact that there are known examples of superconductors without gaps.

The advantages of thinking about emergence in this way is that it encompasses and clarifies both the ontological and epistemological aspects. Although superconductors are constituted by their microscopic properties, their defining features (infinite conductivity, flux quantization, the Meissner effect) are immune to changes in those properties (e.g. replacing fermions with bosons). This is the sense in which we can refer to the properties of a superconductor as 'model independent' and not causally linked to a specific microphysical account. In other words, symmetry breaking (here the breakdown of electromagnetic gauge invariance) provides the explanation of emergent phenomena but the specific microphysical details of *how* the symmetry is broken are not part of the account. In that sense the emergent phenomenon is not reducible to its microphysical constituents yet both retain full physical status. This also allows us to see why supervenience, understood in terms of a dependency relation, is inapplicable in explaining the part-whole aspects of emergent phenomena—there is no determining linkage between the micro and macro levels. But this is exactly as it should be. What makes an emergent phenomenon emergent is that it satisfies certain conditions, one of which is that it can't be captured using a supervenience relation.

Although we can explain emergent phenomena in terms of the symmetry breaking associated with phase transitions, the physics inherent in this explanation is not entirely unproblematic (Bangu 2009; Callender 2001; Earman 2004; Menon and Callender 2013). A well known fact about phase transitions is that even though they take place in finite systems they can only be accounted for by invoking the thermodynamic limit $N \rightarrow \infty$. The link between assumptions about infinite systems and the physics of symmetry breaking/phase transitions is provided by renormalization group (RG) methods which function as a framework for explaining *how* certain types of phenomena associated with phase transitions arise, as well as the similarity in behaviour of very different phenomena at critical point (universality) (Wilson 1983). RG provides the interconnection between mathematics and physics; fleshing out those details will further exemplify the ontological independence of the microphysics in accounting for emergent phenomena.

6.5 Renormalization Group Methods: Between Physics and Mathematics

Part of the importance of the RG is that it shows not just that we can focus on the energies or levels we are interested in, leaving out the rest, as we sometimes do in idealization and model building; it also illustrates and explains the ontological and epistemic independence between different energy levels—the defining features of emergent phenomena. One of the hallmarks of a phase transition is that it exhibits the effects of a singularity over the entire spatial extent of the system. Theory tells us that this happens only in infinite systems (particles, volume or sometimes strong interactions) so phase transitions produce a variation over a vast range of length/energy scales. As a mathematical technique RG allows one to investigate the changes to a physical system as one views it at different distance scales. This is related to a scale invariance symmetry which enables us to see how and why the system appears the same at all scales (self-similarity). As we saw above phase changes of matter are often accompanied by discontinuities such as magnetization in a ferromagnet. At critical point the discontinuity vanishes so for temperatures above Tc the magnetization is 0. We also saw that the non-zero value of the order parameter is typically associated with this symmetry breaking, so the symmetry of the phase transition is reflected in the order parameter (a vector representing rotational symmetry in the magnetic case and a complex number representing the Cooper pair wavefunction in superconductivity).

In RG calculations the changes in length scale result from the multiplication of several small steps to produce a large change in length scale l. The physical phenomena that reflect this symmetry or scale transformation are expressed in terms of observed quantities—mathematical representations of the symmetry operation. For example, quantities that obey rotational symmetry are described by vectors, scalars etc. and in the case of scale transformations power laws reflect the symmetries in the multiplication operations. The physical quantities behave as powers l^x where x can be rational, irrational, positive etc. Behaviour near critical point is described using power laws where some critical property is written as a power of a quantity that might become very large or small. The behaviour of the order parameter, the correlation length and correlation function are all associated with power laws where the "power" refers to the critical exponent or index of the system. Diverse systems with the same critical exponents (exhibiting the same scaling behaviour as they approach critical point) can be shown via RG to share the same dynamical behaviour and hence belong to the same universality class.

The correlation function $\mathbf{\Gamma}(r)$ measures how the value of the order parameter at one point is correlated to its value at some other point. If $\mathbf{\Gamma}$ decreases very fast with distance, then far away points are relatively uncorrelated and the system is dominated by its microscopic structure and short-ranged forces. A slow decrease of Γ implies that faraway points have a large degree of correlation or influence on each other and the system thus becomes organised at a macroscopic level. Usually, near the critical point ($T \rightarrow T_c$), the correlation function can be written in the form

$$\Gamma(r) \rightarrow r^{-p} \exp(-r/\xi) \qquad (6.13)$$

where ξ is the correlation length. This is a measure of the range over which fluctuations in one region of space are correlated with or influence those in another region. Two points separated by a distance larger than the correlation length will each have fluctuations that are relatively independent. Experimentally, the correlation length is found to diverge at the critical point which means that distant points become correlated and long-wavelength fluctuations dominate. The system 'loses memory' of its microscopic structure and begins to display new long-range macroscopic correlations.

The iterative procedure associated with RG results in the system's Hamiltonian becoming more and more insensitive to what happens on smaller length scales. As the length scale changes, so do the values of the different parameters describing the system. Each transformation increases the size of the length scale so that the transformation eventually extends to information about the parts of the system that are infinitely far away. Hence, the infinite spatial extent of the system becomes part of the calculation and this behaviour at the far reaches of the system determines the thermodynamic singularities included in the calculation. The change in the parameters is implemented by a beta function

$$\{\tilde{J}_k\} = \beta(\{J_k\}) \qquad (6.14)$$

which induces what is known as an RG flow on the J-space. The values of J under the flow are called running coupling constants. The phase transition is identified as the place where the RG transformations bring the couplings to a fixed point with further iterations producing no changes in either the couplings or the correlation length. The fixed points give the possible macroscopic states of the system at a large scale. So, although the correlation length diverges at critical point, using the RG equations reduces the degrees of freedom which, in effect, reduces the correlation length.

The important point that distinguishes RG from previous renormalization methods is that the number and type of relevant parameters is determined by the *outcome* of the renormalization calculation.[18] After a sufficient number of successive renormalizations all the irrelevant combinations have effectively disappeared leaving a unique fixed point independent of the value of all of the irrelevant couplings. Assuming that a fixed point is reached one can find the value that defines the critical temperature and the series expansions near the critical point provide the values of the critical indices.[19] The fixed point is identified with the critical point of a phase transition and its properties determine the critical exponents with the same fixed point interactions describing a number of different types of systems. In that sense RG methods provide us with physical information concerning how and why different systems exhibit the same behaviour near critical point (universality).

The basis of the idea of universality is that the fixed points are a property of *transformations* that are not particularly sensitive to the original Hamiltonian. What the fixed points do is determine the kinds of cooperative behaviour that are possible, with each type defining a universality class. The important issue here isn't just the elimination of irrelevant degrees of freedom, rather it is the *existence or emergence of cooperative behaviour* as defined by the fixed points. The coincidence of the critical indices in very different phenomena was inexplicable prior to RG methods. Part of the success of RG was showing that the differences were related to irrelevant observables—those that are "forgotten" as the scaling process is iterated. Another significant feature of RG is that it showed how, in the long wave-length/large space-scale limit, that the scaling process in fact leads to a fixed point when the system is at a critical point, with very different microscopic structures giving rise to the same long-range behaviour.

What this means for our purposes is that RG equations illustrate that phenomena at critical point have an underlying order. Indeed what makes the behaviour of critical point phenomena predictable, even in a limited way, is the existence of certain scaling properties that exhibit 'universal' behaviour. The problem of

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[18] In earlier versions parameters like mass, charge etc. were specified at the beginning and changes in length scale simply changed the values from the bare values appearing in the basic Hamiltonian to renormalized values. The old renormalization theory was a mathematical technique used to rid quantum electrodynamics of divergences but involved no "physics".

[19] The equivalence of power laws with a particular scaling exponent can have a deeper origin in the dynamical processes that generate the power-law relation. Phase transitions in thermodynamic systems are associated with the emergence of power-law distributions of certain quantities, whose exponents are referred to as the critical exponents of the system. Diverse systems with the same critical exponents—those that display identical scaling behaviour as they approach criticality—can be shown, via RG, to share the same fundamental dynamics.

calculating the critical indices for these different systems was impossible prior to the use of renormalization group techniques which enable us to see that different kinds of transitions such as liquid-gas, magnetic, alloy etc. share the same critical exponents and can be understood in terms of the same fixed-point interaction.

As I noted above, epistemic independence—the fact that we *need not* appeal to micro phenomena to explain macro processes—is not sufficient for emergence since it is also a common feature of physical explanation across many systems and levels. Emergence is characterized by the fact that we *cannot* appeal to microstructures in explaining or predicting these phenomena despite their microphysical base. RG methods reveal the nature of this ontological independence by demonstrating the features of universality and how successive transformations give you a Hamiltonian for an ensemble that contains very different couplings from those that governed the initial ensemble.

Despite the explanatory power of fixed points, Butterfield (2011) has recently claimed that one needn't resort to RG in explaining phase transitions. Indeed there is a sense in which this is true if what we are trying to explain is the appearance of stable behaviour in finite systems; the sort of behaviour that we sometimes identify with phase transitions (e.g. the appearance of critical opalescence). Many (e.g. Callender 2001; Earman 2004) have argued that appeals to infinite systems required to explain phase transitions is, in fact, illegitimate since we know that the relevant behaviour occurs in finite systems. Issues related to the stability of finite system behaviour has also been pointed out by Menon and Callender (2013) and well as Huttemann, Kuhn and Terzidis (this volume). In each of these cases, however, the authors ignore a crucial feature of emergence, specifically the ability to properly explain universal behaviour and, in Butterfield's case, the role of RG in that context. The calculation of values for critical indices and the cooperative behaviour defined in terms of fixed points is the foundation of universality. RG is the only means possible for explaining that behaviour; what happens at finite N is, in many ways, irrelevant. Finite systems can be near the fixed point in the RG space and linearization around a fixed point will certainly tell you about finite systems, but the fixed point itself requires the limit.

What RG does is show us how to pass through the various scales to reach the point where phase transitions are not breakdowns in approximation techniques, but true physical effects. We know that if you try to approximate a sum by an integral you quickly find that exact summation can't admit a phase transition. And, although we witness stable and universal behaviour experimentally in finite N, we aren't able to understand its fundamental features without RG. The formal (mathematical) features function as indicators of the kind of phenomena we identify with phase transitions and in that sense the mathematics provides a representation and precise meaning for the relation between phase transitions and universal behaviour.

Many of the worries surrounding emergence are related to the issue of reduction and whether the former presents a telling case against the latter. Why, for example, should universality be considered more effective against reduction than multiple realizability arguments? Moreover, one could also claim that universality and symmetry breaking are part of fundamental physics and hence the emergentist story actually encorporates elements of reduction. The objection concerning symmetry breaking and fundamental physics can be answered as follows: Although what defines fundamental physics is not rigidly designated it unequivocally includes explanations that invoke microphysical entities and theories/laws that govern then. When symmetry breaking features in microphysical theories its role requires specific details of the 'breaking', i.e. an account that appeals to microstructures as in the case of the Higgs mechanism. The point of the superconductivity example was to illustrate that details of symmetry breaking were not necessary for the derivation of infinite conductivity; all that was required was an assumption that electromagnetic gauge invariance was spontaneously broken. So, while SSB bears some relation to microphysical explanation, as a general process it doesn't qualify as "fundamental" in the way the term is typically understood. The existence of universal phenomena further bears this out. Because we witness identical behaviour at critical point from phenomena that have completely different microstructures, and the explanation ignores those microstructures, the notion of fundamental physics is rendered inapplicable.

Here the reductionist might respond that surely it is possible *in principle* to derive macro phenomena from micro properties given the Schrodinger equation and the appropriate initial conditions (i.e. god could do it). But again, universality speaks against this possibility. If we suppose that micro properties could determine macro properties in cases of emergence then we have no explanation of how universal phenomena are even possible. Because the latter originate from vastly different micro properties there is no obvious ontological or explanatory link between the micro-structure and macro behaviour. More specifically, while fluids and magnets both arise out of microphysical constituents their behavioural similarity at criticality is independent of and immune from changes in those micro constituents. This is what separates emergent phenomena from resultant properties and aggregates. In the latter cases there is a direct physical link between the micro and macro that is absent in cases of emergence.

A relatively similar point can be made for cases of multiple realizability. Although macroregularities can be realized by radically heterogeneous lower level mechanisms the problem here is one of underdetermination; we simply don't know which of the micro arrangements is responsible for the macro state and hence the causal, explanatory link is unknown with respect to the competing alternatives. However, universality presents a rather different picture in that the micro-macro link is simply broken rather than being underdetermined. In other words, we know what the initial macro states are in each of the separate instances of critical behaviour, but because those are "washed out" after several iterations of RG equations they no longer play a role in the macro behaviour. Moreover, the mystery to be explained is how several different systems with different micro structures behave in exactly the same way; hence, because the micro structures are different in each case the explanation *cannot* be given in those terms. In that sense the analogy with multiple realizability breaks down.

A more direct challenge to the claim about the incompatibility of emergence and reduction comes from Huttemann et.al. (this volume). They claim that "the fact that certain features of the constituents are irrelevant in the technical RG sense does not imply that the properties and states of the constituents fail to influence macro-behaviour. Rather, it is only a small number of features of these that does the work for asymptotic critical exponents." But these features are simply the symmetry and dimensionality of the system and have nothing to do with claims about micro-reduction.

Finally one might want to claim that universality is simply another form of multiple realizability (MR) and to that extent is provides no added reason to deny reduction and embrace emergence. The possibility that macro-level regularities are heterogeneously multiply-realised is evidenced by the fact that liquids and gases exhibit the same type of behaviour at critical point while having radically different microstructures. So, the issue is whether examples of universal behaviour fall prey to some form of reduction in virtue of a supervenient relation to their microphysical base.[20]

Here again the answer is 'no'. The dependence relation required for supervenience is clearly lacking in cases of 'universal' behaviour since fixing the subvenient properties in no way fixes the supervenient ones and vice versa—the whole is substantially different from the sum of its parts. In cases of supervenience any change in higher level properties requires a difference in lower level properties, something that fails to occur in cases of emergence. For example, superconducting metals that constitute different "natural kinds" will have different transition temperatures but they exhibit the same properties as a consequence of broken electromagnetic gauge invariance. The claim so often associated with supervenience—there can be no A difference without a B difference (where A properties supervene on B properties)—is irrelevant here since once the system reaches critical point and universal behaviour (A properties) is dominant, information about micro-level structure (B properties) is simply lost.

But as I have stressed many times, the issue is not simply a matter of ignoring irrelevant details as one does in the formulation of laws or levels of explanation. In those cases changes in macro structure *are* determined by changes in micro structure and vice versa. In emergence the important *physical* relationships involve long wavelengths and cooperative behaviour defined in terms of fixed points. The systematic treatment provided by RG enables us to see behind the abstract mathematics of the thermodynamic limit and divergence of the correlation length to fully illustrate the physical processes involved in emergent 'universal' phenomena.

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[20] Although there are arguments for the claim that supervenience needn't entail reduction my argument rests on the fact that even the requirements of supervenience fails in the case of universal phenomena.

6.6 Conclusions

One of the fundamental issues in debates about emergence involves the difference between epistemic and ontological claims about what constitutes emergent phenomena. The temptation to classify everything as epistemically emergent is overwhelming, especially due to uncertainties about what future physics will reveal. For example, it seems reasonable to suggest that our inability to explain or predict phenomena we now classify as emergent will or can be resolved once a more comprehensive theory is in place. However, once we focus on the notion of universality the appeal of epistemic emergence quickly fades. For instance, the fact that phenomena as different as liquids and magnets exhibit the same critical behaviour and share the same values for critical exponents is not going to be explained by a more comprehensive micro theory. In fact, the difference in the micro structure of phenomena that share the same universality class indicates that the explanation of their stable, emergent behaviour cannot not arise from the microphysical base. In that sense universality undermines any appeals to reduction as an explanatory strategy for understanding this behaviour.

While emergent phenomena may be novel and surprising, these are not the characteristics by which they should be defined. Instead we need to focus on the ontological aspects of these phenomena to understand not only the basis for their similarity but also the stability of their behaviour patterns. The success of renormalization group methods in calculating the values of critical indices as well as exposing the reasons behind the failure of mean field theory in explaining universality further indicates the irrelevance of micro level, reductive explanations. However, it isn't simply the irrelevance of micro structures that it important here but also the way in which fixed points account for the cooperative behaviour present in cases of emergence. Without the explanation of these physical features via RG methods, emergent phenomena would remain theoretical novelties awaiting explanation in terms of some future theory.

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