Logistics Service Supply Chain Network Equilibrium Model

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Abstract In this paper, a logistics service supply chain network model is developed which consists of four tiers of decision-makers: the logistics service subcontractors, the logistics service providers, the logistics services integrators, the logistics service demand sides. The model optimizes the behavior of the decision-makers, with the integrators' multi-criteria, concerned with both profit maximization and risk minimization, and under the equilibrium conditions derived. Finally, an example is used to illustrate the application and solution of this equilibrium problem.

Keywords Logistics service supply chain · Network equilibrium · Nonlinear complementarity problems · L-M algorithm · Logistics service integrator

1 Introduction

The structure of Logistics Service Supply Chain (or LSSC) is "logistics service subcontractors (LSS) ←logistics service providers (LSP) ← logistics services integrators (LSI) ← logistics service demand sides (LSD)" (Cui 2008). In recent years, more and more scholars begin to pay attention to LSSC(Liu and Xie 2013).

The equilibrium model (EM) has been widely used to express the complicated interactions between members of the supply chain. Nagurney (1999) first studied the equilibrium problem between economic networks and markets by using variational inequalities (VI). Dong et al. (2004) researched the supply chain EM under the stochastic market demand based on Nagurney. Li et al. (2011) discussed the closed-loop supply chain EM of random demand multi-commodity flow. Hu et al.

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(2012) studied the online supply chain EM, and gave the equilibrium condition s based on VI.

LSSC EM study is rarely seen. Kong (2012) studied it, but they didn't consider risk factors. In this paper, we study the overall EM of LSSC. Different from previous literature, *LSSC model consists of four tiers of decision makers: LSS, LSP, LSI and LSD, who make decision upon their own interests and members' interaction effect*. Here we assume:

(1) Transition cost in LSSC borne by downstream members, the cost includes information cost, negotiation cost and regular contracting cost;
(2) Decision makers in all tiers don't need to consider storage cost;
(3) LSI's goals are not only profit maximization, but also risk minimization.

2 LSSC Model and Optimal Decisions in Each Layer

This section designs the LSSC model, and establishes the optimal decision models of each layer in LSSC which consists of K LSSs, N LSPs, M LSIs, and I LSDs, with *k*, *n*, *m* and *i* as the respective serial numbers. When LSPs receive LSIs' orders, they make all or part orders to LSSs as a lack of logistics capabilities. Supposing each member competes in non-cooperative condition, the transaction costs borne by the downstream members besides the demanders.

Parameters

$q_{xy}a$	*Orders assigned from x to y, q^*_{xy} is the optimal value, x, y=k, n, m, i;*
$p_{xy}(q_{xy})$	*Unit prices of providing logistics capabilities from x to y, x, y=k, n, m, i;*
$c_{xy}(q_{xy})$	*Transaction cost between x and y, continuously differentiable convex functions (CDCF for short), $c^*_{xy} = c^*_{xy}(q^*_{xy})$, x, y=k, n, m, i;*
$s_{xy}(q_{xy})$	*Operating costs between x and y, CDCF, $s^*_{xy} = s^*_{xy}(q^*_{xy})$, x, y=k, n, m, i;*
q_k	*Total volume of logistics capabilities of the subcontractor k;*
$f_k = f_k(q_k)$	*Cost function of LSS k, CDCF.*

2.1 The Decisions and the Optimal Conditions of LSS

q_k must satisfy equation:

$$q_k = \sum_{n=1}^{N} q_{kn} + \sum_{m=1}^{M} q_{km} \tag{1}$$

The model of LSS k pursuing profit maximization is as follow:

$$\max \sum_{n=1}^{N}(p_{kn}q_{kn}-s_{kn}(q_{kn}))+\sum_{m=1}^{M}(p_{km}q_{km}-s_{km}(q_{km}))-f_k(q_k)$$
$$\text{s.t}\, q_k=\sum_{n=1}^{N}q_{kn}+\sum_{m=1}^{M}q_{km},\quad q_{kn}\ge 0,\ q_{km}\ge 0,\ \forall k=1\cdots K \tag{2}$$

Assume λ_k is the Lagrange multiplier, from (1) (2), we have

$$\max \sum_{n=1}^{N}(p_{kn}q_{kn}-s_{kn}(q_{kn}))-f_k(q_k)+\sum_{m=1}^{M}(p_{km}q_{km}-s_{km}(q_{km}))$$
$$+\lambda_k\left(q_k-\sum_{n=1}^{N}q_{kn}-\sum_{m=1}^{M}q_{km}\right)\quad \text{s.t}\, q_{kn}\ge 0,\ q_{km}\ge 0,\ \forall k=1\cdots K \tag{3}$$

According to the existing literature (Nagurney 1999), the optimal conditions of LSS k is:

$$\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\lambda_k^*+\frac{\partial S_{kn}^*}{\partial q_{kn}}-p_{kn}\right)(q_{kn}-q_{kn}^*)+\sum_{k=1}^{K}\sum_{m=1}^{M}\left(\lambda_k^*+\frac{\partial S_{km}^*}{\partial q_{km}}-p_{km}\right)$$
$$(q_{km}-q_{km}^*)+\sum_{k=1}^{K}\left(\frac{\partial f_k(q_k^*)}{q_k}-\lambda_k^*\right)(q_k-q_k^*)+\sum_{k=1}^{K}\left(q_k^*-\sum_{n=1}^{N}q_{kn}^*-\sum_{m=1}^{M}q_{km}^*\right)$$
$$(\lambda_k-\lambda_k^*)\ge 0\quad \forall q_{kn}\ge 0, q_{km}\ge 0, q_k\ge 0, \lambda_k\ge 0 \tag{4}$$

The economic significance of VI (4) is obvious. The first two terms indicate if the trading volume is positive, the marginal operating cost of LSS *k* plus its willingness to pay a cost must be equal to the unit price of logistics capabilities charged by *k* to *n*. Otherwise, the trading volume is zero. The third term indicates that the marginal cost of production $\partial f_k(q_k^*)/q_k$ must be equal to its willingness to pay. The last term indicates the total amount of stock by LSS *k* must be equal to the sum of logistics capabilities that LSS *k* provides for *n* and *m*.

2.2 The Decisions and the Optimal Conditions of LSP

It assumes that logistics capabilities subcontracted from *n* to *k* are equal to the orders gotten from *m*. The decisions model of the logistics service provider is:

$$\max \sum_{m=1}^{M}p_{nm}q_{nm}-\sum_{k=1}^{K}(p_{kn}q_{kn}+c_{kn}(q_{kn}))$$
$$\text{s.t.}=\sum_{k=1}^{K}q_{kn}=\sum_{m=1}^{M}q_{nm},\quad q_{nm}\ge 0,\ q_{kn}\ge 0,\ \forall n=1,2,\cdots N \tag{5}$$

Assume λ_k is the Lagrange multiplier, the optimal conditions of LSP is:

$$\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{\partial c_{kn}(q_{kn}^*)}{\partial q_{kn}}+p_{kn}-\lambda_n^*\right)(q_{kn}-q_{kn}^*)+\sum_{n=1}^{N}\sum_{m=1}^{M}\left(\lambda_n^*-p_{nm}\right)(q_{nm}-q_{nm}^*)$$
$$+\sum_{n=1}^{N}\left(\sum_{k=1}^{K}q_{kn}^*-\sum_{m=1}^{M}q_{nm}^*\right)(\lambda_n-\lambda_n^*)\ge 0\quad \forall q_{nm}\ge 0, q_{kn}\ge 0, \lambda_n\ge 0, \forall n=1\cdots N \tag{6}$$

It shows from (6) that the unit price from *n* to m must be equal to the minimum acceptable price of n, and the marginal transaction cost must be equal to the price difference between the purchase price paying from *m* to *n* and from *n* to *k*.

2.3 The Decision and the Optimal Conditions of LSI

Logistics capabilities provided by LSI to LSD are equal to which sub-contracted to LSP and LSS by LSI. LSI both maximizes their own interests and minimizes their risk on LSSC. Let risk function $r_m = r_m(Q^1, Q^2, Q^3), \forall m = 1, 2, \cdots M, Q^{1*} \in R_+^{km}, Q^{2*} \in R_+^{nm}, Q^{3*} \in R_+^{mi}$ respectively denotes one-dimensional vector space of trading volumes.

The goal weight of profit maximization is 1; a non-negative weight β_m is given to a risk minimization target. The optimal conditions of LSI are:

$$\begin{aligned} &\max \sum_{i=1}^{I} (p_{mi} q_{mi} - c_{mi}(q_{mi})) - \sum_{k=1}^{K} (p_{km} q_{km} + c_{km}(q_{km})) - \\ &\sum_{n=1}^{N} (p_{nm} q_{nm} + c_{nm}(q_{nm})) - \beta_m r_m(Q^1, Q^2, Q^3) \\ &\text{s.t} \sum_{i=1}^{I} q_{mi} = \sum_{k=1}^{K} q_{km} + \sum_{n=1}^{N} q_{nm}, q_{mi} \geq 0, q_{km} \geq 0, q_{nm} \geq 0, \forall m = 1 \cdots M \end{aligned} \tag{7}$$

Assume λ_m is the Lagrange multiplier Q^{1*}, Q^{2*}, Q^{3*} are the optimal solutions meet (7), $r_m^* = r_m(Q^{1*}, Q^{2*}, Q^{3*})$. The optimal conditions of LSIs are:

$$\begin{aligned} &\sum_{m=1}^{M} \sum_{i=1}^{I} \left(\frac{\partial c_{mi}^*}{\partial q_{mi}} - p_{mi} + \lambda_m^* + \beta_m \frac{\partial r_m^*}{\partial q_{mi}} \right) (q_{mi} - q_{mi}^*) + \\ &+ \sum_{m=1}^{M} \sum_{k=1}^{K} \left(\frac{\partial c_{km}^*}{\partial q_{km}} + p_{km} - \lambda_m^* + \beta_m \frac{\partial r_m^*}{\partial q_{km}} \right) (q_{km} - q_{km}^*) \\ &+ \sum_{m=1}^{M} \sum_{n=1}^{N} \left(\frac{\partial c_{nm}^*}{\partial q_{nm}} + p_{nm} - \lambda_m^* + \beta_m \frac{\partial r_m^*}{\partial q_{mn}} \right) (q_{mn} - q_{mn}^*) \\ &+ \sum_{m=1}^{M} \left(\sum_{i=1}^{I} q_{mi}^* - \sum_{k=1}^{K} q_{km}^* - \sum_{n=1}^{N} q_{nm}^* \right) (\lambda_m - \lambda_m^*) \geq 0, \\ &\forall q_{mi} \geq 0,\ q_{km} \geq 0,\ q_{mn} \geq 0,\ \lambda_m \geq 0, s\, \forall m = 1 \cdots M \end{aligned} \tag{8}$$

The first term of (8) indicates the unit price by *m* to *i* is equal to the minimum acceptable price of *i* plus the marginal transaction costs and risks costs through weight conversion. The second and third terms indicate that the price obtained by m must be equal to a minimum price with willingness minus the marginal transaction costs and risks costs through weight conversion. The forth term indicates the orders received by LSI are equal to its subcontracted orders.

2.4 The Decision and Optimal Conditions of LSD

LSD mainly considers the minimum cost of available services. Similar to the literature (Fei et al. 2011), the optimal decision of LSD is:

$$\sum_{i=1}^{I}\sum_{m=1}^{M}(p_{mi}-p_{mi}^{*})(q_{mi}-q_{mi}^{*})\geq 0 \quad \forall q_{mi}\geq 0, p_{mi}\geq 0, \forall i=1\cdots I \tag{9}$$

Constraint (9) denotes if the price of logistics capabilities charged by LSI exceeds acceptable price by LSD, there will be no logistics transaction between them.

3 The Equilibrium Conditions of LSSC

In order to maintain the overall balance of LSSC, the optimum conditions in all layers must be satisfied, so Eqs. (4), (6), (8), and (9) must be satisfied.

Definition1 The equilibrium state of LSSC is the trading volume and price between the tiers simultaneously satisfy the sum of inequalities (4), (6), (8) and (9).

Theorem1 LSSC will reach equilibrium if it satisfies the following VI:

$$\begin{aligned}
&\sum_{n=1}^{N}\sum_{k=1}^{K}\left(\frac{\partial s_{kn}(q_{kn}^{*})}{\partial q_{kn}}+\frac{\partial c_{kn}(q_{kn}^{*})}{\partial q_{kn}}+\lambda_k^{*}-\lambda_n^{*}\right)(q_{kn}-q_{kn}^{*})\\
&+\sum_{m=1}^{M}\sum_{k=1}^{K}\left(\frac{\partial s_{km}(q_{km}^{*})}{\partial q_{km}}+\frac{\partial c_{km}(q_{km}^{*})}{\partial q_{km}}+\lambda_k^{*}-\lambda_m^{*}\right.\\
&\left.+\beta_m\frac{\partial r_m(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{km}}\right)(q_{km}-q_{km}^{*})\\
&+\sum_{m=1}^{M}\sum_{n=1}^{N}\left(\frac{\partial c_{mi}(q_{mi}^{*})}{\partial q_{nm}}+\beta_m\frac{\partial r_m(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{mn}}+\lambda_n^{*}-\lambda_m^{*}\right)(q_{mn}-q_{mn}^{*})\\
&+\sum_{m=1}^{M}\sum_{i=1}^{I}\left(\frac{\partial c_{mi}(q_{mi}^{*})}{\partial q_{mi}}+\beta_m\frac{\partial r_m(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{mi}}+\lambda_m^{*}-p_{mi}^{*}\right)(q_{mi}-q_{mi}^{*})\\
&+\sum_{k=1}^{K}\left(\frac{\partial f_k(q_k^{*})}{q_k}-\lambda_k^{*}\right)(q_k-q_k^{*})+\sum_{k=1}^{K}\left(q_k^{*}-\sum_{n=1}^{N}q_{kn}^{*}-\sum_{m=1}^{M}q_{km}^{*}\right)(\lambda_k-\lambda_k^{*})\\
&+\sum_{n=1}^{N}\left(\sum_{k=1}^{K}q_{kn}^{*}-\sum_{m=1}^{M}q_{nm}^{*}\right)(\lambda_n-\lambda_n^{*})\\
&+\sum_{m=1}^{M}\left(\sum_{i=1}^{I}q_{mi}^{*}-\sum_{k=1}^{K}q_{km}^{*}-\sum_{n=1}^{N}q_{nm}^{*}\right)(\lambda_m-\lambda_m^{*})\geq 0\\
&\forall (Q^1,Q^2,Q^3)^T\in R_+^{KM+NM+MI}, q_{kn}\geq 0, q_k\geq 0, \lambda_k\geq 0, \lambda_n\geq 0, \lambda_m\geq 0,
\end{aligned} \tag{10}$$

VI (10) can be proved by *Definition1* and inequalities (4), (6), (8) and (9). It can be converted into a nonlinear complementarity problem (or NCP). That is to solve the vectors $X=\left(Q^1,Q^2,Q^3,q_{kn},q_k,\lambda_k,\lambda_n,\lambda_m\right)^T\in R_+^{KM+NM+MI+KN+2K+N+M}$ to satisfy formula

$$X\geq 0, F(X)\geq 0, X^T F(X)=0, F(X): \tag{11}$$
$$R_+^{KM+MN+MI+KN+2K+N+M}\mapsto R^{KM+NM+MI+KN+2K+N+M}$$

$F(X)$ is continuous and differentiable, let $F(X)=\left(F^1(X)F^2(X),\right.$ $\left.F^3(X),F^4(X)F^5(X)\right),$

$$\mathrm{F}^1(\mathrm{X})=\left(\mathrm{F}_{11}^1(\mathrm{X}),\cdots,\mathrm{F}_{1\mathrm{N}}^1(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{K}1}^1(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{KN}}^1(\mathrm{X})\right)^{\mathrm{T}}\in\mathrm{R}^{\mathrm{KN}};$$
$$\mathrm{F}^2(\mathrm{X})=\left(\mathrm{F}_{11}^2(\mathrm{X}),\cdots,\mathrm{F}_{1\mathrm{M}}^2(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{K}1}^2(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{KM}}^2(\mathrm{X})\right)^{\mathrm{T}}\in\mathrm{R}^{\mathrm{KM}}$$
$$\mathrm{F}^3(\mathrm{X})=\left(\mathrm{F}_{11}^3(\mathrm{X}),\cdots,\mathrm{F}_{1\mathrm{N}}^3(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{M}1}^3(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{MN}}^3(\mathrm{X})\right)^{\mathrm{T}}\in\mathrm{R}^{\mathrm{MN}};$$
$$\mathrm{F}^4(\mathrm{X})=\left(\mathrm{F}_{11}^4(\mathrm{X}),\cdots,\mathrm{F}_{1\mathrm{i}}^4(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{M}1}^4(\mathrm{X}),\cdots\mathrm{F}_{\mathrm{MI}}^4(\mathrm{X})\right)^{\mathrm{T}}\in\mathrm{R}^{\mathrm{MI}}$$
$$\mathrm{F}^5(\mathrm{X})=\left(\mathrm{F}_1^5(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{k}}^5(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{K}}^5(\mathrm{X})\right)^{T}\in\mathrm{R}^{\mathrm{K}};$$
$$\mathrm{F}^6(\mathrm{X})=\left(\mathrm{F}_1^6(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{k}}^6(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{K}}^6(\mathrm{X})\right)^{\mathrm{T}}\in\mathrm{R}^{\mathrm{K}}$$
$$\mathrm{F}^7(\mathrm{X})=\left(\mathrm{F}_1^7(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{n}}^7(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{N}}^7(\mathrm{X})\right)^{T}\in\mathrm{R}^{\mathrm{N}};$$
$$\mathrm{F}^8(\mathrm{X})=\left(\mathrm{F}_1^8(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{m}}^8(\mathrm{X}),\cdots,\mathrm{F}_{\mathrm{M}}^5(\mathrm{X})\right)^{\mathrm{T}}\in\mathrm{R}^{\mathrm{M}}$$

Where

$$r=1,\cdots,K;\ j=1,\cdots,N;,\cdots,K;\ d=1,\cdots,M;\ h=1,\cdots,I$$

$$\mathrm{F}_{\mathrm{rj}}^1(\mathrm{X})=\frac{\partial s_{\mathrm{rj}}(\mathrm{q}_{\mathrm{rj}})}{\partial q_{rj}}+\frac{\partial c_{\mathrm{rj}}(\mathrm{q}_{\mathrm{rj}})}{\partial q_{rj}}+\lambda_{\mathrm{r}}-\lambda_{\mathrm{j}};\quad \mathrm{F}_{\mathrm{dj}}^3(\mathrm{X})=\frac{\partial c_{\mathrm{dj}}(\mathrm{q}_{\mathrm{dj}})}{\partial \mathrm{q}_{\mathrm{dj}}}$$
$$+\beta_{\mathrm{d}}\frac{\partial t_{\mathrm{d}}(\mathrm{Q}^1,\mathrm{Q}^2,\mathrm{Q}^3)}{\partial \mathrm{q}_{\mathrm{dj}}}+\lambda_{\mathrm{j}}-\lambda_{\mathrm{d}};\quad \mathrm{F}_{\mathrm{dh}}^4(\mathrm{X})=\frac{\partial c_{\mathrm{dh}}(\mathrm{q}_{\mathrm{dh}})}{\partial \mathrm{q}_{\mathrm{dh}}}+\beta_{\mathrm{d}}\frac{\partial t_{\mathrm{d}}(\mathrm{Q}^1,\mathrm{Q}^2,\mathrm{Q}^3)}{\partial \mathrm{q}_{\mathrm{dh}}}$$
$$+\lambda_{\mathrm{d}}-\mathrm{p}_{\mathrm{dh}}^*,\qquad \mathrm{F}_{\mathrm{r}}^5(\mathrm{X})=\frac{\partial f_{\mathrm{r}}(\mathrm{q}_{\mathrm{r}})}{\mathrm{q}_{\mathrm{r}}}-\lambda_{\mathrm{r}},\mathrm{r}=1,\cdots,\mathrm{K};$$
$$\mathrm{F}_{\mathrm{r}}^6(\mathrm{X})=\mathrm{q}_{\mathrm{r}}-\sum\nolimits_{\mathrm{n}=1}^{\mathrm{N}}\mathrm{q}_{\mathrm{rn}}-\sum\nolimits_{\mathrm{m}=1}^{\mathrm{M}}\mathrm{q}_{\mathrm{rm}},\mathrm{r}=1,\cdots,\mathrm{K};$$
$$\mathrm{F}_{\mathrm{j}}^7(\mathrm{X})=\sum\nolimits_{\mathrm{k}=1}^{\mathrm{K}}\mathrm{q}_{\mathrm{kj}}-\sum\nolimits_{\mathrm{m}=1}^{\mathrm{M}}\mathrm{q}_{\mathrm{jm}},\mathrm{j}=1,\cdots,\mathrm{N};$$
$$\mathrm{F}_{\mathrm{d}}^8(\mathrm{X})=\sum\nolimits_{\mathrm{i}=1}^{\mathrm{I}}\mathrm{q}_{\mathrm{dj}}-\sum\nolimits_{\mathrm{k}=1}^{\mathrm{K}}\mathrm{q}_{\mathrm{kd}}-\sum\nolimits_{\mathrm{n}=1}^{\mathrm{N}}\mathrm{q}_{\mathrm{nd}},\mathrm{d}=1,\cdots,\mathrm{M};$$

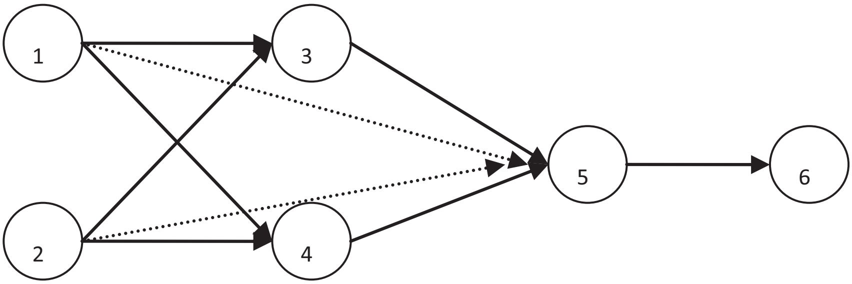

Fig. 1 Example LSSC model

Then solving the equilibrium problem (10) is transformed into solving the optimal solution of NCP (11).

There exist some iteration methods for solving NCP (11), smoothing-type algorithm, Newton-type methods, Levenberg–Marquardt method, and so on. Due to space limitations, this paper does not elaborate on solving problem, just uses an example to illustrate the application of the equilibrium problem.

4 Example Analysis

Consider a simple LSSC which composed by 2 subcontractors, 2 providers, 1 integrators and 1 market demand side, as shown in Fig. 1.

(1) The decision variables: $q_{13}, q_{14}, q_{15}, q_{23}, q_{24}, q_{25}, q_{35}, q_{45}, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ (the subscript represents each decision-maker's label).

(2) Parameters:

$$c_{rj} = 0.8q_{rj}^2 + q_{rj}, r = 1,2; j = 3,4; \ \ c_{rd} = q_{rd}^2 + 2q_{rd}, r = 1,2; d = 5;$$

$$c_{dj} = q_{dj}^2 + q_{dj}, j = 3,4; d = 5; \ \ c_{dh} = 2q_{dh}^2 + q_{dh}, d = 5; h = 6;$$

$$s_{rj} = 0.5\left(\sum\nolimits_{j-3}^{4} q_{rj}\right)^2 + q_{rj} + 5, r = 1,2; j = 3,4; \ \ s_{rd} = 0.6(q_{rd})^2 + q_{rd} + 5, r = 1,2; d = 5;$$

$$p_{56} = 12; \ \ f_1 = f_1(q_1) = 1.5(q_1)^2 + q_1q_2 + 2q_1; \ \ f_2 = f_2(q_2) = 1.5(q_2)^2 + q_1q_2 + 2q_2;$$

$$r_5 = r_5(Q^1, Q^2, Q^3) = (q_{15} + q_{25} + q_{35} + q_{45} + 2q_{56} - 3)^2$$

We solve equilibrium problems (15) in Matlab. All numerical experiments were done at a PC with Celeron (R) D CPU of 2.8 GHz and RAM of 2G.

Set $x^0 = (1,1,1,1,1,1,1,1,1)$, we get the equilibrium solutions for LSSC:

$$\lambda_1 = 4.4090, \lambda_2 = 4.4090, \lambda_3 = 6.9046, \lambda_4 = 6.9046, \lambda_5 = 10.9095, q_{13} = 0.1377,$$
$$q_{14} = 0.1377, q_{14} = 0.1377, q_{23} = 0.1377, q_{24} = 0.1377, q_{15} = 0.3269,$$
$$q_{25} = 0.3269, q_{35} = 0.2753, q_{45} = 0.2753, q_{56} = 1.2045$$

5 Conclusions

Based on the existing literature, this paper puts forward a LSSC network equilibrium model to explore the effects of LSSC network on equilibrium computation with respect to entities behavior and equilibrium conditions. It considers the characteristics of logistics capability of invisible and the risk problem of logistics services integrator. This model is transformed into a nonlinear complementarity problem. An example is used to illustrate the application and solution of the equilibrium problem. This research has some guiding significance for the complex LSSC network decision, but the network here represents only a limited form of network, the network with infinite members will be studied further.

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