



# Movement of the Flux Quanta, Flux Flow Resistance

# 8

The question of how exactly the electrical resistance in superconductivity disappears to zero came into focus in the early 1960s, when newly discovered superconductor materials first appeared to make their technical applications at high electrical currents possible. Especially, the new superconducting niobium alloys  $\text{Nb}_3\text{Sn}$  and  $\text{NbZr}$  were very promising. Careful measurements, especially at the Bell laboratories in the USA, revealed indications of high values of critical electric current density and critical magnetic field. A novel “critical state” was also found, above which a finite, albeit relatively small, electrical resistance occurs. The American Philip W. Anderson recognized at that time that a new process must be involved, namely a movement of magnetic flux quanta caused by the flow of electric current. This process then became famous as “flux creep” and “flux flow.”

Every movement of magnetic flux quanta in a superconductor due to a force acting on them generates an electric field in the superconductor and thus an electric voltage. This “flux flow voltage” grows proportionally to the speed and number of moving flux lines. In the case of an electric current of density  $\mathbf{j}$ , the Lorentz force  $\mathbf{f}_L = \mathbf{j} \times \boldsymbol{\varphi}_0$  acts on the flux quanta. The Lorentz force is oriented perpendicular to the direction of the electric current and the magnetic field of the flux lines. The movement of the flux lines thus caused generates the electric field  $\mathbf{E}$ :

$$\mathbf{E} = -\mathbf{v}_\varphi \times \mathbf{B} \tag{8.1}$$

The magnetic flux density  $\mathbf{B}$  is given by the areal density  $n$  of the magnetic flux quanta  $\boldsymbol{\varphi}_0$ :  $\mathbf{B} = n \boldsymbol{\varphi}_0$ . The quantity  $\mathbf{v}_\varphi$  denotes the velocity of the flux lines. The electric field (Eq. (8.1)) is always oriented perpendicular to the direction of motion and to the magnetic field of the magnetic flux lines. Since the electric field

and the electric current have the same direction, energy is dissipated by the flux line motion in the superconductor and electric losses occur. This process of flux motion follows the (here simplified) equation of forces

$$\mathbf{j} \times \boldsymbol{\varphi}_0 - \eta \mathbf{v}_\varphi = 0 \quad (8.2)$$

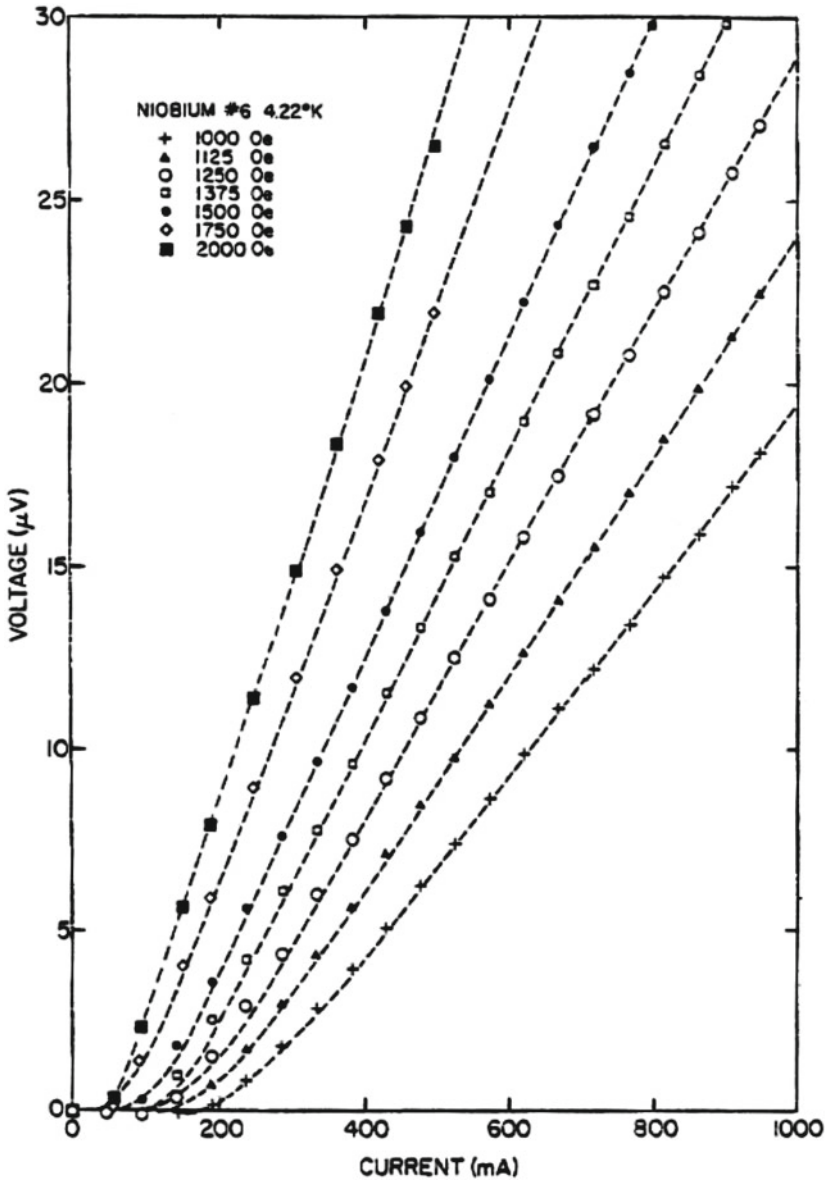
Here  $\eta \mathbf{v}_\varphi$  denotes the dissipative contribution and  $\eta$  a damping constant. In Eq. (8.2), the forces refer to a unit length of the flux line. From Eqs. (8.1) and (8.2), one finds the *specific flux flow resistance*

$$\rho_f = \boldsymbol{\varphi}_0 B / \eta \quad (8.3)$$

This mechanism always limits the current flow without electrical resistance and losses in superconductors. Therefore, many efforts have been made to prevent this process of flux line movement as far as possible by installing so-called pinning centers. (In Eq. (8.2) we have, for the sake of simplicity, neglected the pinning forces and a force component that causes the Hall effect when the flux lines move.)

As an example we show in Fig. 8.1 the flux-flow voltage as a function of the electric current in a niobium foil for different values of the magnetic field oriented perpendicular to the foil. (thickness of the foil = 18  $\mu\text{m}$ , width of the foil = 4 mm,  $T = 4.22$  K). The electrical resistance ratio  $R(295 \text{ K})/R(4.2 \text{ K})$  was 620, where  $R(4.2 \text{ K})$  was measured in a perpendicular magnetic field of 4,000 G. The voltage starts at a finite critical current and initially shows an upwardly curved behavior. It then grows linearly with increasing current, as can be expected from Eqs. (8.1) and (8.2). The finite critical current results from the pinning forces due to spatial inhomogeneities in the superconductor, which, as pinning centers, prevent the movement of the magnetic flux lines. The slope of the linear curve sections represents the flux flow resistance and increases with increasing external magnetic field.

Figure 8.2 shows a schematic representation of the resistivity  $\rho_f$  as a function of the external magnetic field  $H$ . Initially,  $\rho_f$  grows linearly with  $H$  and then leads into a much steeper curve branch, which reaches the normal value  $\rho_n$  of the resistivity at the upper critical magnetic field  $H_{C2}$ .



**Fig. 8.1** Flux-flow voltage as a function of the electric current in a niobium foil of  $18 \mu\text{m}$  thickness and  $4 \text{ mm}$  width for various vertically oriented magnetic fields.  $T = 4.22 \text{ K}$ ;  $T_C = 9.2 \text{ K}$

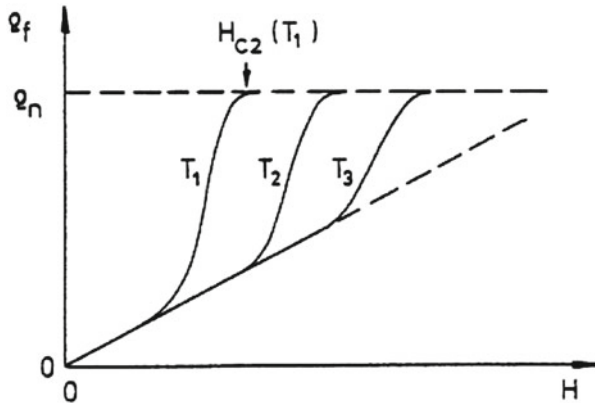
## 8.1 Thermally Activated Motion of Magnetic Flux Quanta

As already mentioned, the equation of forces (Eq. (8.2)) is an idealized approximation that neglects the effect of pinning forces on the magnetic flux lines. In the following, we want to discuss a special problem from the very complex field of the effect of pinning forces on the magnetic flux line lattice in more detail: the thermally activated movement of the magnetic flux lines.

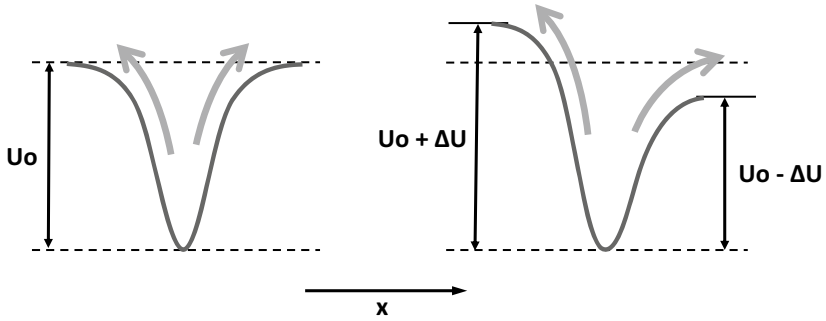
The basic ideas from the 1960s go back to the abovementioned Philip W. Anderson and Young Kim. We consider a single magnetic flux quantum that is fixed in a potential well by the pinning force. (The potential well represents a local minimum in the spatial course of the Gibbs free energy density.) The depth of the potential well is denoted by  $U_0$ . Thermal activation allows the flux quantum to jump out of the potential well, the hopping rate  $R_j$  being given by

$$R_j = v_0 \exp\left(-\frac{U_0}{k_B T}\right) \quad (8.4)$$

Here  $v_0$  denotes a characteristic attempt-frequency, and we assume  $U_0 \gg k_B T$ . In the absence of an external force acting on the flux quantum, the thermally activated hopping process of the flux quantum is the same in all directions, and the resulting flux motion vanishes. However, if an external force acts on the flux



**Fig. 8.2** Schematic representation of the specific flux flow resistance  $\rho_f$  in a type II superconductor as a function of the magnetic field for different temperatures ( $T_1 > T_2 > T_3$ ).  $\rho_n$  denotes the normal resistance



**Fig. 8.3** Thermally activated motion of magnetic flux quanta. *Left* Without an external force, the flux jumps do not show a preferred direction. *Right* In the presence of an external force, the flux jumps show a preferred direction

quantum, this spatial symmetry is broken. In the direction of this force, the wall height of the potential well is reduced by  $\Delta U$ , and in the opposite direction, it is increased by  $\Delta U$ . In Fig. 8.3, we show a schematic representation. The hopping process now has a preferred direction.

After a short calculation, two important limiting cases are found, where the critical electric current density  $j_c$  is decisive, where the energy gain by the Lorentz force exactly compensates the depth of the potential well:  $\Delta U = U_0$ . In the limit of thermally activated flux flow (TAFF limit),  $j \ll j_c$ , the following applies:

$$E = 2\rho_c \cdot \exp\left(-\frac{U_0}{k_B T}\right) \cdot \frac{U_0}{k_B T} \cdot j \quad (8.5)$$

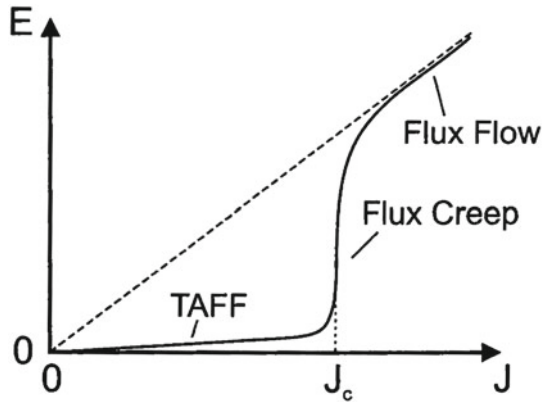
and in the limit  $j \approx j_c$ :

$$E \approx \rho_c \cdot \exp\left[-\frac{U_0}{k_B T} \left(1 - \frac{j}{j_c}\right)\right] \cdot j_c \quad (8.6)$$

In the TAFF limit, we get *Ohm's law* (Eq. (8.5)). However, the resistance  $E/j$  is strongly reduced due to the factor  $\exp(-U_0/k_B T)$ . The limit  $j \approx j_c$  of Eq. (8.6) is called *flux creep* with the current-dependent effective energy  $U_{\text{eff}} = U_0 (1 - j/j_c)$  of the barrier. This current-dependent exponent in Eq. (8.6) often causes a strong increase of  $E$  with increasing  $j$  over many orders of magnitude.

The case  $j \gg j_c$  is called *flux flow*, for which pinning effects are negligible, just like for a perfectly homogeneous sample. (In the latter case, only the edges of the

**Fig. 8.4** Electric field  $E$  as a function of the electric current density  $J$  for the different ranges of the physical behavior of the magnetic flux quanta



sample play a role as spatial inhomogeneity). Figure 8.4 shows a summary of the different ranges we have discussed.