Teachers' Beliefs Systems Referring to the Teaching and Learning of Arithmetic

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Content

Abstract

In this paper, we focus on belief systems of six teachers of primary and secondary schools who just started their teacher trainees referring to the teaching and learning of arithmetic. Firstly, we discuss the theoretical framework of our research and outline the method. Afterwards we discuss findings of our research in three separate sections. We discuss the identification of central beliefs referring to one teacher. Further we derive findings referring to peripheral beliefs to the same teacher. Finally, we discuss types of belief systems towards teaching arithmetic. We conclude the paper with a brief summary and suggestions for further research.

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1 Introduction

Teachers' decide, *what* mathematical content they bring to the classroom; they have reasons, *why* they select specific content and – except for ad-hoc decision when interacting with students – they decide *how* they teach specific content, i.e. they individually define their way of teaching (cf. Calderhead, 1996). Although a teacher's responses to the what, why and how are dependent of his professional knowledge, his responses are strongly impacted by his beliefs about mathematics or teaching and learning mathematics that are a part of the teachers' mathematical related affect (Hannula, 2012). For example, a teacher's beliefs are crucial for a teacher's decision to what extent he will enact his knowledge about mathematics and mathematics teaching and learning referring his instructional planning and, thus, for his classroom practice (e.g. Felbrich et al., 2012).

Accepting the impact of teachers' beliefs on both the instructional planning and the classroom practice, the further impact of teachers' beliefs, i.e. on the students' learning, seems obvious. However, although research in mathematics education yielded results referring to the relationships among teachers' beliefs on the one side, and the teachers' classroom practice and the students' learning on the other side (Artzt & Armour-Thomas, 1999; Staub & Stern, 2002; Dubberke et al., 2008), this relationships are not completely investigated (e.g. Hiebert & Grouws, 2007; Skott, 2009). In this report we concern two characteristics of teachers' beliefs that potentially could yield a consistency between teachers' espoused beliefs referring to their instructional planning and those beliefs that could be derived from classroom observations, i.e. the *centrality* of the expressed beliefs concerning the internal organisation of beliefs called belief system (Green, 1971; Putnam & Borko, 2000; Wilson & Cooney, 2002; Eichler, 2011; Schoenfeld, 2011), and the specificity of these beliefs referring to a mathematical subdomain (cf. Franke et al., 2007). The centrality of teachers' beliefs seems further to be crucial when professional development or, respectively, a change of teachers' beliefs is regarded (Borko & Putnam, 1996; Wilson & Cooney, 2002).

Our research concerns both, teachers' beliefs that are relevant for their classroom practice and the development of beliefs of teachers that we have followed from their final exams at university through a phase as teacher trainees up to their starting point as a qualified teacher. For this reason, a specific interest of our research and the focus in this report is to identify teachers' central beliefs restricted to the teaching and learning of arithmetic. According to this focus, we outline the theoretical framework and describe the method of our research. In addition to findings referring to individual arithmetic teachers' central beliefs, we discuss three types of arithmetic teachers (cf. Thompson, 1984). We conclude this report summarising our findings and suggesting further research.

2 Theoretical framework

Stein, Remillard and Smith (2007) provide a curriculum model including four phases of which the latter three phases are potentially influenced by teachers' beliefs (see fig. 1).

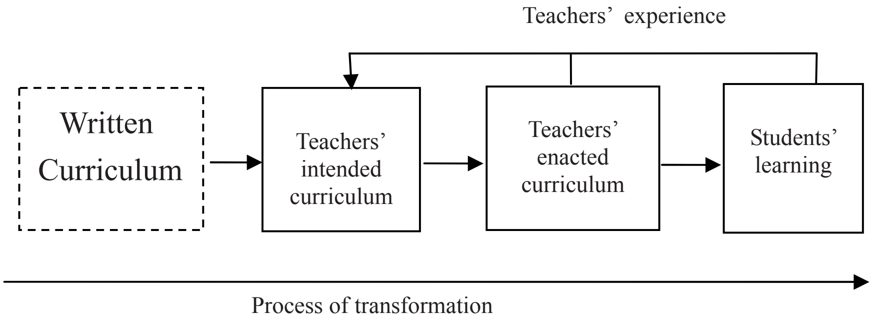


Figure 1 Four phases of the curriculum according to Stein at al. (2007)

The *written curriculum* involves instructional content, and teaching goals prescribed by national governments. The way the teachers interpret a written curriculum concerning content and goals referring to his instructional planning is called the *intended curriculum*. In this report, we mainly focus on teachers' intended curricula. However, indirectly we also regard the classroom practice involving interactions of a teacher with his or her students (*enacted curriculum*) and *students' learning*, since both have an impact on a teacher's intended curriculum through reflection on his or her experiences in classrooms (Wilson & Cooney, 2002).

We understand beliefs as an individual's personal conviction concerning a specific subject, which shapes an individual's way of both receiving information about a subject and acting in a specific situation (Pajares, 1992). Regarding this definition, we understand content and goals as specific forms of beliefs portraying a teacher's conviction about an appropriate way of teaching mathematics. Since an intended curriculum referring to arithmetic includes various specifications to appropriate content, goals or ways of teaching, we understand an intended curriculum as a specific form of a teacher's belief system (Green, 1971; Thompson, 1992). A belief system is characterised by a quasi-logical system of beliefs with different grades of centrality (Thompson, 1992). Although a teacher's belief system could potentially consists of clusters that need not to interact with each other, we hypothesise that a teacher's belief system is mostly consistent, if a specific mathematical domain, e.g. arithmetic, is regarded.

In this report we refer partly to overarching goals of the teachers that can be characterised by different features regarding the perception of mathematics in general (Dionne, 1984; Thompson, 1984) and to which Grigutsch et al. (1998) distinct four views:):

- A formalist view stresses that mathematics is characterised by a logical and formal approach. Accuracy and precision are most important.
- A process-oriented view is represented by statements about mathematics being experienced as a heuristic and creative activity that allows solving problems using different and individual ways.
- An instrumentalist view places emphasis on the "tool box"-aspect which means that mathematics is seen as a collection of calculation rules and procedures to be memorized and applied according to the given situation.

An application oriented view accentuates the utility of mathematics for the real world and the attempts to include real-world problems into mathematics classrooms. Further we refer to a global distinction of two different ways of teaching mathematics, i.e. a "cognitive constructivist orientation", and a "direct transmission view" (Staub & Stern, 2002, p. 344).

3 Method

The sample consists of 20 arithmetic teachers of primary and secondary school divided into two subsamples. The first subsample include 8 experienced teachers (four primary teachers, four secondary teachers) teaching arithmetic at least for five years. The second subsample consists even of 6 teachers (three primary teachers, three secondary teachers) that we have followed from their final exams at university through a phase as teacher trainees up to their starting point as a qualified teacher.

We collect data with a semi-structured interview including clusters of questions referring to arithmetic content, goals of teaching arithmetic, goals of teaching mathematics, the nature of mathematics, students' learning or materials used for the classroom practice, e.g. textbooks. In addition, the interviews incorporate prompts to evaluate given arithmetic tasks or fictitious statements of teachers or students that represent one of the views mentioned above, e.g. an application oriented view. Further, we used a questionnaire adapted form an existing scale referring to teachers' views (Grigutsch et al., 1998). We interviewed the experienced teachers once, since we assume these teachers' beliefs to be relatively stable (Calderhead, 1996). In contrast, we interviewed the teachers of the second subsample three times, i.e. at the end of their university studies, in the middle of their teacher training phase, and at the beginning of their time as a qualified teacher. The rationale for this longitudinal design is the assumption that prospective teachers' beliefs potentially change, when they get their first intense practical experience. These prospective teachers have little practical experience during their university studies including three internships that are mainly of observational nature. The teacher training between university and the beginning as a qualified teacher lasts 18 month and involves both self-dependent teaching and teaching guided by a mentor.

For analysing the data of the verbatim transcribed interviews, we used a qualitative coding method (Kuckartz, 2012) that is close to grounded theory (Glaser & Strauss, 1967). We used deductive codes derived from a theoretical perspective like "application oriented" goal and inductive codes for those goals we did not deduce from existing research concerning calculus education (Kuckartz, 2012). Further we weighted the codes with 1 or 2. If a teacher mentions a goal without a precision we weighted the code with 1. If a teacher explains a goal more deeply giving for instance a concrete example or task of his classroom practice, we weighted the code with 2. The codings were conducted by at least two persons and we proved the interrater reliability to show an appropriate value. Further, we analysed the sum of the weighted codes as triangulation to the qualitative interpretation of the interview transcripts. In a further triangulation we compared the results of the sum of weighted codes with the results referring to the questionnaire. We describe the results and the interpretation of the results of our method exemplarily in the next section referring to the structure of the belief system of one teacher.

4 Identifying central beliefs

In this section, we restrict the focus to one teacher, Mrs. A, and her beliefs system towards the teaching and learning of arithmetic. Referring to Mrs. A, we demonstrate three steps of analysis outlined above aiming to identify central beliefs in the belief system of a teacher. In the first step of analysis, we characterise a teacher's belief system on the basis of the interview transcripts.

> Mrs. A expressed coherently a process oriented view. That means, Mrs. A repeated her process oriented view on different parts of the interview. For example, to the question of her favourite teaching style and her preferred methods she answered:

> Mrs. A: „Truly, it is important that they are able to find the solutions on their own, that they can work individually (…) that they can solve problems, that they can work on open tasks, that they can find their own strategies."

Later, nearly the same answer ensued when she was asked about pupils and their way of learning:

> Mrs. A: „It is always important for me, that it comes from the pupils themselves, that it includes a problem, I like giving pupils problem statements."

Again, being asked to the question, which goals she would like to reach with her lesson, she answered:

> Mrs. A: „And then there are strategies, i.e. to be flexible, to adapt oneself to something new. Therefore, you need the right attitude that you have the confidence to try something you don`t know and to put effort into it."

The three quoted episodes referring to different topics, i.e. the teaching style, students' learning and teaching goals give evidence that beliefs representing the process oriented view are central in the belief system of Mrs. A.

According to the process-oriented beliefs Mrs. A expressed in various episodes of the interview she responded to prompts given during the interview. For example, Mrs. A was asked to arrange eight given teaching goals into a hierarchy. Figure 2 shows her arrangement of these goals for arithmetic lessons, where Mrs. A valued problem solving and process orientation as the most important goals.

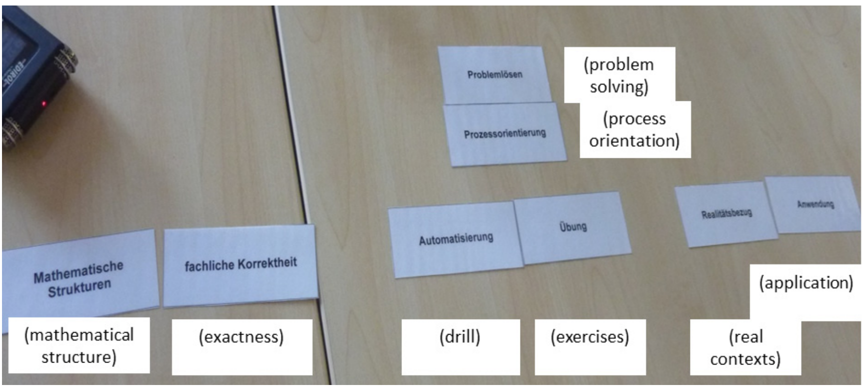


Figure 2 Mrs. A's arrangement of goals for arithmetic lessons

In figure 3 we show a further prompt consisting of students' statements representing the four views towards mathematics. The teachers were asked to arrange the statements from most desired (1) to least desired (4). Mrs. A preferred the second statement representing the process orientation.

8. statements of pupils:

[2] I like maths because there is a connection to real life problems.

[1] I like maths because hard nuts must be cracked and difficult problems can be solved.

[4] I like maths because many exercises can be solved by similar procedures/patterns.

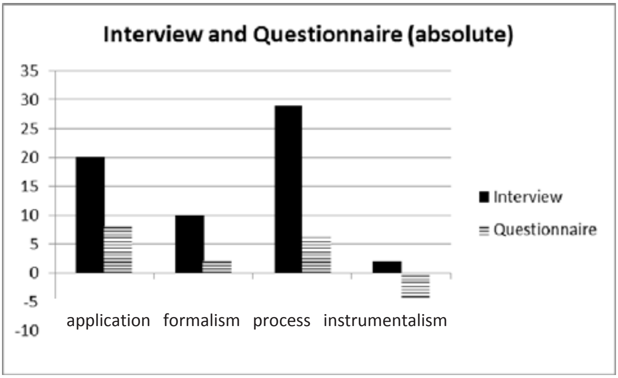
[3] I like maths because the logic is clear and it follows strict mathematical rules.

Figure 3 Prompt: What would you like for pupils to answer?

Just as the espoused beliefs the responds to prompts give strong evidence that process orientation is central for Mrs. A.

In the second step of analysis, we coded every episode of the interview transcript. Referring to the deductive codes, partly given by views (application (A), formalism (F), process (P) and instrumentalism (I) and weighted the codes (see above). The sum of weighted codes is shown in figure 3 on the left side.

I the third step, the teachers were asked to complete a questionnaire according to the scale of Grigutsch et al (1998) and consisting a five-point-Likert-scale including 24 items representing the four mentioned view towards arithmetic (fig, 4, left side). To compare the weighted codes and the scores gained through the questionnaire, we standardised the sums of weighted codes and the questionnaire scores, which are both shown in figure 3 on the right side. Even for the individual teacher, we preliminary proved the fit of both distributions using correlation and ICC that show a good fit.



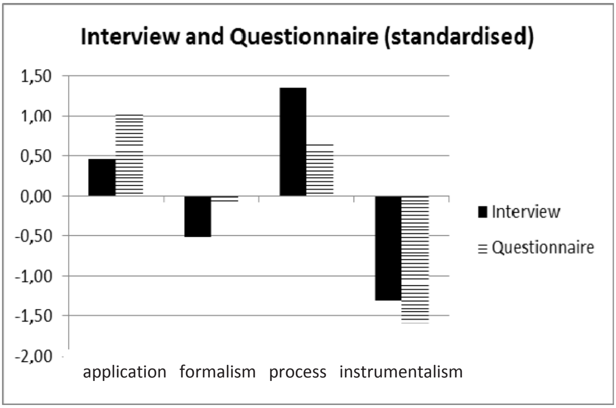


Figure 4 Weighted sum of codes and questionnaire scores

Concluding the analysis referring to the beliefs of Mrs. A concerning the teaching and learning of arithmetic, there exist several unambiguous examples for Mrs. A´s process oriented view. The high degree of coherence in different parts of the interview, the sum of weighted codes and, finally the questionnaire underline the mentioned assumption that the process oriented view is central in the belief system of Mrs. A.

5 Explaining peripheral beliefs

Since the sum of weighted codes and the results of the questionnaire facilitate the identification of central and more peripheral beliefs, the interview transcripts provide a deep insight into the relationships of central and peripheral beliefs and also into primary and derivative (subordinated) beliefs (Thompson, 1992). For example, next to the process oriented view Mrs. A emphasised the importance of application (see fig. 3). Although application oriented goals are central for Mrs. A, however, her answers concerning the application oriented view give evidence that application oriented goals are subordinated to process oriented goals. Subordination means that that application is in some sense a central teaching goal but rather a means to an end for another even central and primary goal:

> „The relation to reality is important too, as I said before referring to money and time, but it doesn´t have to be highlighted all the time. Today, for example, I just gave them a mathematical problem…"

This example shows that teachers can hold central beliefs that represent different views. In such a case we analyse relationships among the different views that were described exemplarily by regarding Mrs. A.

Concluding the analysis of the belief system of Mrs. A: On the one side, the sum of weigthed codes fit the results of the questionnaire and allows central and peripheral beliefs to be distinguished. On the other side, interpretation of the transcript allows to reconstruct the relationship of different central beliefs in terms of primary and subordinated beliefs and to explain beliefs as in detail based for example on specific tasks of a teacher's classroom practice.

6 Characterisation of teachers' belief systems

We restrict our focus to six teachers of our sample who were completely analysed yet. These six arithmetic teachers could be described by three views: Three teachers emphasise process-orientation and two emphasise application-orientation. The sixth teacher highlights partly the instrumentalism view and shows primarily a negative view towards process-orientation. Figure 5 summarises the findings for the teachers representing the three types of views in all three steps of analysis. The analysis of the interviews (column 1) shows the teachers' central beliefs, column 2 and 3 show additionally by the quotation of the interviews and questionnaires the matching of qualitative and quantitative results.

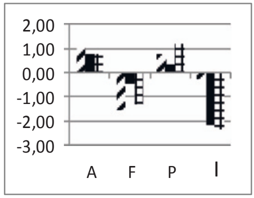
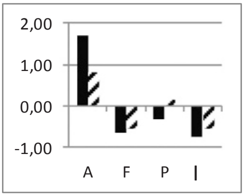
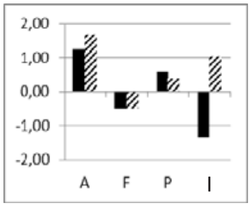
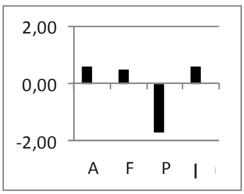
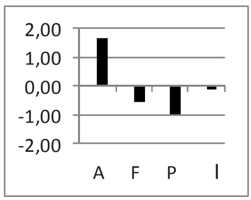
Type	standarized interview	standarized questionnaire
Process oriented view initiated by applications Mrs. A: And therefore for me good math lessons are lessons, where students work a lot on their own and/or discover a lot on their own, where you perhaps reflect collectively.	Mrs R Mrs A	 Mrs O
Application oriented view Mrs. H: On top there is the reference to reality, that's what school is there for, it should prepare for life, therefore it's important that things we calculate in school have something to do with the reality of the students.	 Mrs H	 Mr S
Instrumentalism view Mr. H: [This] should help to evoke a simple automatism with the students that they see something, and then they can immediately solve that in writing.	 Mr H	

Figure 5 preliminary typing of the six teachers

It is striking that for all teachers the application oriented view is crucial and central for teaching arithmetic. However, the status of the application oriented view varies: For type 1 (e.g. represented by Mrs. A) the application oriented view is subordinated to the process oriented view, i.e. teachers of this type tend to use applications by means of achieving a process orientation. In contrast, for type 2 real-world problems are per se a crucial part of arithmetic teaching without emphasising a process oriented view. Finally, the teacher representing type 3 highlight applications particularly as a principle of student motivation. However, this teacher tends to subordinate the application oriented view to the instrumentalism view, i.e. applications are used to initiate arithmetic procedures the students have to learn. We illustrate the goal of student motivation referring to one episode of the interview with Mr. H:

> Mr. H: „It's commonly said you should pick things from the students' everyday life. This is crucial for initiating a subject and this is totally different compared to initiating a subject without these things. For example, if I have a family with 5 people and 3 pizzas, the students know that and can empathize with that and can perhaps understand the problem more easily. That's why problems should be from the students' everyday life."

7 Discussion

In this report, we presented a method aiming to identify teachers' central beliefs, which could further be distinguished to primary and subordinated beliefs, and peripheral beliefs constituting the teachers' belief systems towards teaching arithmetic. Results show that the qualitative interpretation of the interview as well as the weighted sum of codes and, finally, the analysis of the teachers' responds to a questionnaire consisting an existing scale (Grigutsch et al., 1998) yield consistent results referring to central and peripheral beliefs and, further, the qualitative analysis yield a distinction between primary and subordinated goals. We discussed these three steps of analysis in detail referring the process orientation of Mrs. A.

Taking into account the analysis of Mrs. A and further teachers, it was possible to analyse the status of peripheral beliefs. Thus, these peripheral beliefs are subordinated to central beliefs, i.e. the teachers express peripheral goals as a means to achieve central teaching goals. For example, applications are used to initiate problem solving (process oriented view; type 1) or to motivate students to learn arithmetic procedures (schema view; type 3).

On the basis of identifying central and peripheral beliefs as well as primary and subordinated beliefs, it was further possible to match the teachers to different types of teaching arithmetic. Partly, these types agree with the findings of Thompson (1984). In contrast to Thompson (ibid.), we found an omnipresence of favouring applications for the teaching of arithmetic.

We assume the central beliefs being relevant for both the teachers' observable classroom practice and the development of teachers' beliefs. Thus, the identification of the beliefs' characteristics serves as requirement for further steps in our research programme. In these steps, we will prove the relevance of central and peripheral beliefs and primary and subordinated beliefs by an observation of the classroom practice and the longitudinal analysis of teachers' beliefs.

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