Chilean and Finnish Teachers' Conceptions on Mathematics Teaching

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Content

Abstract

This paper analyses the cases of two Chilean and two Finnish elementary teachers' reflections on their own professional development during a research project where they learned about and used open-ended problems to teach mathematics. The data indicates that during the project the teachers increased their pedagogical content knowledge, subject matter knowledge and motivational components. Teachers claim they give more room for pupils' ideas and rely on pupils' learning in pairs or in groups. Furthermore, they allege that also the weakest pupils seem to be involved with problem solving.

C. Bernack-Schüler et al. (eds.), *Views and Beliefs in Mathematics Education,*
Freiburger Empirische Forschung in der Mathematikdidaktik,
DOI 10.1007/978-3-658-09614-4_4,

1 Background

This study is part of a larger research project in the field of mathematics education, financed by the Academy of Finland (project #135556) and Chilean CONICYT (project # AKA 09). The focal point of the project is teachers' professional development along with both teachers and pupils' development in mathematical thinking and understanding as well as pupils' performance when dealing with open-ended problems. The same problems are dealt with in both Chile and Finland. This paper is particularly focused on the teachers' own conceptions of their development and not on the results on pupils' development which are described in other papers (e.g. Laine & al. 2012, Varas & al. 2012).

The initiative for a joint research project came from Chile. The Academy of Finland and the Chilean CONICYT made an agreement for a research enterprise on education where mathematics education was a part of it.

2 Theoretical Framework

In mathematics education, problem solving is considered as a method to promote pupils' high-order thinking and understanding (e.g. Schoenfeld 1992). In the 1970's in different countries (cf. Nohda 1987), the methods of using open-ended problems were developed, in order to confront challenges of constructivism.

A problem is said to be open-ended, if it has an exactly stated starting point, but there are many possible goals where a solver might end, with equally correct methods (cf. Pehkonen 1995). Therefore, such problems do not have only one solution, but there might be many possible correct results, depending on the choices the solver has done.

When discussing the teaching of open-ended problems, the focus should be on teachers' pedagogical content knowledge and subject matter knowledge (cf. Shulman 1986). The use of open-ended problems challenges teachers to modify their roles in class. A teacher is no more a deliverer or transmitter of information, but a guide and facilitator for learning, and a planner of learning environments. Thus, teachers need to alternate and to improve their own conceptions of teaching and learning (cf. Pehkonen 2007). In order for teachers to be able to make such an adjustment, they need, for example, to learn to be sensitive to pupils' ideas and solution efforts, and to listen deeply to and try to pick up pupils' understanding (Stein & al. 2000). This all means a huge modification in teachers' pedagogical conceptions; the change might even be a radical one (cf. Merenluoto 2005).

A new idea for helping teachers to evolve is to accept that they are experts in developing new teaching solutions if only they are given the access to the newest results of theoretical studies (e.g. Wiliam 2002). This idea was empirically confirmed in the research paper by Roddick and Begthold (2004) where the authors found one of the critical change factors to be teachers' participation in the project from planning to implementation.

On the basis of previous research (e.g. Merenluoto 2001), the study at hand works with the hypothesis that when individuals encounter a phenomenon unknown to them, it leads to different cognitive effects and levels of learning. On the basis of the empirical research, another hypothesis sustains that teachers' prior understanding is quite resistant to the change (cf. Pehkonen 2007). Since the use of open-ended problems is difficult for most teachers, its learning takes time. However, there are other studies showing that such changes can also be very rapid (cf. Liljedahl 2010).

2.1 Focus of the study

The main question to be answered is the following: How do the teachers themselves perceive their conceptions about mathematics and teaching of mathematics have changed during the project?

3 Implementation of the study

The project is ongoing and will be implemented as a three-year (2010–13) follow-up study in the elementary school using quasi-experimental design. In the experimental group there are10–20 elementary classes with their teachers from Santiago and Helsinki involved and committed to the study; and there is the same amount of classes in the control group in both countries. In these experimental classes, the teachers have implemented an open-ended problem once a month, from grade 3 to grade 5, in both countries. The researchers have followed the teachers' planning, implementation and reflection (self-evaluation) of their mathematics lessons, when the same open-ended problems are used in both countries.

In the following, there are three examples of open-ended problems used in the project:

1) Coloring a flag (grade 3): When using exactly three colors, plan as many different flags with three stripes as possible.

2) Snail-Elli (grade 4): Elli the snail climbs a wall very slowly. Some days the snail ascends 10 cm, some days she ascends 20 cm, some days she sleeps

and it does not move, and other days she is sound asleep and falls 10 cm. The wall has 100 cm of height. At the end of the tenth day Elli is at the half of the wall's height? What could have happened in the first 10 days? Show as many ways as possible.

3) Rectangles (grade 5): Find rectangles the perimeter of which is 16 cm. Find the area of each of them, and the rectangle with the biggest area.

Altogether there are seven problems per school year, i.e. about one problem lesson per month. The purpose of using such problems is to develop pupils' thinking skills and creativity. The main point is to let them see that there is not only one solution in a mathematical problem, but there might be many. Therefore, the problems dealt with are open, in the sense that they have many correct answers; very often the amount of correct answers is infinite. Additionally, the pupils are encouraged to find out the strategies of finding all solutions, if possible.

During the project, the problems were discussed with the teachers before and after the implementation in a joint meeting with the experimental teachers and the researchers' group in each country. In the meeting, they were talking about the experiences in the previous implemented problem and discussing about the next problem coming. In addition, in Finland there was a theoretical topic (around problem solving) presented to and discussed with the experimental teachers. In Chile, this was done in a two-week training seminar regarding open-ended problems previous to the beginning of the project for all the Chilean experimental teachers.

For teachers, the use of open problems means a new approach for teaching mathematics. They should modify their teaching habits, from much talking to more listening. And this might be for some teachers a huge conceptual change in their pedagogy, perhaps even a radical one (cf. Merenluoto 2005).

3.1 The interviews

For this paper, we analyze part of the research data that has been gathered through teacher interviews, classroom observations, videotaped discussions and the researchers' field notes. The teacher interviews took place during March-April 2012 in Finland and during November-December 2012 in Chile.

In this paper we analyze four of the cases, two teachers from each country, in order to reach a deeper level of analysis and more detailed reporting. The interviews of the Chilean teachers (Amelia and Nicholas) were selected from a sample of 10 interviews, and they were chosen because of their differences in a particular measurement regarding the quality of the introduction of a problem-solving task (arithmagon) that has been analyzed previously (see Varas & al.

2012). In Finland, only two teachers (Fiona and Dana) from 10 experimenting teachers volunteered to be interviewed. However, the measurement regarding the quality of the introduction of a problem solving task indicated that all four teachers were considerably different, which met our interest for answering the research question in different cases, therefore, providing a broader perspective (see Varas & al. 2012).

Interviews were conducted as semi-structured individual interviews that lasted between forty-five minutes and an hour and a half. The same questions were used in both countries, but because interviews were conducted with interviewees' native language two different researchers were used. Analysis of the interview data was carried out as a joint analysis in English.

The interviews were recorded. Each interview was divided into six different sections; in these the participants were asked about their background, the mathematical thinking of their pupils, the characteristics of their mathematics lessons, their involvement in the project, their expectations within the project and their professional development.

The observations and field notes were taken in Chile during November and December 2012, during visits to the different schools of the project and open discussions with the teachers and other school staff. In Finland the observations and field notes were from March, April and May 2012.

4 Findings

Here are the findings for the four teachers (Nicholas, Amelia, Fiona and Dana) based on the data collected through interviews, observations and field notes. The findings are gathered into the following table:

4.1 The case of Nicholas

Nicholas is a 49 year old Chilean male teacher. He claims that he gives freedom to his pupils to work, and listens and values pupils' ideas. In general, he believes that in his lessons he clearly becomes a mediator of pupils learning by being active and perceptive of the pupils' ideas and promoting the appearance of diversity in the pupils' production and responses.

Nicholas was obliged to join the project by the headmaster. At first, he had no expectations, and had a feeling there was nothing in it for him. He claims he felt tired of teaching. Nicholas considers he used to be a structured teacher in mathematics and having structured ideas about mathematics. However, he claims that during the project, he started to feel comfortable with mathematics. He feels that

mathematics no longer has the same authoritative position and thus often there are no single right answers. On the contrary, he believes that people construct mathematics in a difficult way, and they should not be afraid of it. According to him, mathematics is everywhere and mathematics teaching is to enable the child to understand how things are formed. He says: *"God might have been a great mathematician, because wherever you look around… there is maths"*.

Table 1: Findings regarding the Perceptions of four teachers: Nicolas (N), Amelia (A), Fiona (F) and Dana (the abbreviation used: OE = open-ended).

	BEFORE THE PROJECT	MIDDLE OF THE PROJECT
MATH	N: Math seemed to be an imperative A: Always liked math, but difficult and hard F: Math one subject among others, very easy D: Math structured, systematic and accurate	N: Enthusiastic about math A: Math difficult, but interesting F: Math connected to the children's world D: Perception has not changed, but deepened
OPEN-ENDED PROBLEM SOLVING	N: A problem needs to have only one solution A: Didn't know that OE problems existed F: Not familiar with the concept D: used problem-solving activities	N: OE problems are as valid as other type of math A: Sees the benefit in the use of OE problems F: OE problems are for all pupils D: Critical with some of the OE tasks
PUPILS CAPABILITIES	N: Had quite many weak pupils A: Pupils were mostly scared of math F: Wished the pupils to learn to connect problem solving with everyday's reality D: Sometimes pupils come up with unexpected solutions.	N: Some weak pupils had lost their fear to math A: Pupils seem more secure and like more math. F: Pupils think independently and not give up easily D: Pupils self- awareness has increased

	BEFORE THE PROJECT	MIDDLE OF THE PROJECT
TEACHING MATHS	N: Very structured. A: Liked to have control over the class and use concrete materials. F: Likes to lead when she teaches math, and follows the book D: Liked group work based lessons.	N: Not being an authority in the class A: Gives no room for pupils to improvise F: Has more patience to discuss and more independent from the book. D: Values more interaction with the pupils.

Nicholas declares that for him mathematics learning is based on the concrete, with games and through discovering. He insists on the importance of letting the pupils discover. After two years in the project he thinks his view of mathematics has changed. He claims that *"now math is crazy… and I owe it to the project"*. He complains that he does not like to plan the lessons and thus, the lessons result in what he calls a "salad of diverse activities". He describes himself as a mediator, and claims that the project has helped him understand how important it is for the teacher to allow pupils to express themselves.

For Nicholas a mathematical problem is a difficulty that a child has, and that the child can solve with mathematical attributes. With open-ended problems, Nicholas was surprised that the weakest pupils seemed to be motivated and involved. He considers it a challenge for the teacher to sustain the children's interest. According to him, by incorporating open-ended problems to his lessons, the result is a clear increase of participation and, pupils like mathematics more and develop their potentials better. He thinks that, in all, the project has affected his class positively and the pupils seem more open to mathematics. However, despite all the impact he claims that the use of open-ended problems has had in him, not much has changed in his lessons. The main change is that the pupils realize that he is fascinated with mathematics.

4.2 The case of Amelia

Amelia is a Chilean 47 year old female teacher. Amelia has low expectations of her pupils. Even though she is pessimistic, she is always ready to acknowledge the work of her pupils. Most of the time, the pupils' work is better than what she expects.

She got involved in the project because she thought it was interesting, and it was going to help the children. And as a result of it, she feels that there are children that are more secure and like mathematics more.

Amelia claims that mathematics is something that she has always liked, but it has been hard. She says that to teach mathematics is to give tools to the children to show the world of numbers, which is the practical and logical world where children live. Amelia remarks that mathematics is the base for everything to function well.

The use of open-ended problems has made Amelia realize that she speaks too much in her lessons, and she feels that she is mostly the main actor and thus, as a result, the pupils follow her way. She recognizes that she does not let them speak or think. She justifies that her intentions are to guide them through her experience.

Amelia believes that open-ended problems develop pupils' own abilities. She clarifies that with the traditional problems that are closed and structured, some pupils are not capable of solving them and getting engaged. Amelia claims that with open-ended problems, some of the weakest pupils are motivated and participate during the lesson.

In addition Amelia feels that, as a negative part, the open-ended problems do not connect with the contents of the curriculum, she is expected to teach. In her normal lessons she says she feels appalled to go back to fixed answers and structured ways to solve things.

4.3 The case of Fiona

Fiona is a Finnish 41 year old and claims that she uses partly teacher-led practices. She plans her lessons well and in her partly pupil-centered practices, her pupils work in pairs or groups. She explained that she has high expectations of her pupils and her classes are filled with work. She said that mathematics was an easy subject at school and she felt she did not need to put extra effort into learning it.

Fiona got the information of the project from her headmaster. She was inspired to get new ideas about teaching mathematics, professional development and using problem solving in her classes. Fiona claims that she used problem-solving tasks earlier in her mathematics teaching only now and then, and mostly as some extra material for her gifted pupils. However, within the project, she says she has learned a lot of how to use problem-solving tasks in her teaching and how to instruct her pupils.

According to Fiona, her perceptions of mathematics have changed during her working years. She claims that concreteness is of increased importance, and that she wants to tie mathematics into the lives of children. She insists that within the project she has learned to understand the value of guiding and asking relevant questions. She feels confident in herself: "I know I am on the right track!"

Fiona tells that her teaching of mathematics has also changed. She explains she has learned to separate the essential from irrelevant, and has now courage to vary her teaching methods. Fiona believes now that she can weed out material and achieve the same objectives with different methods also. She affirms to be more relaxed and she is confident on her professional skills.

There have been changes in Fiona's classroom practices, too. She mentions that now she understands that problem-solving tasks can promote mathematical thinking, and she can provide them to everyone, not just for the talented pupils. Fiona reveals that in her lessons, when pupils work in pairs, they help each other and their self-efficacy seems to have increased.

4.4 The case of Dana

Dana is a 49 year old Finnish teacher. She is an affective person, who appreciates creativity and spontaneity. Dana says that she does not plan her teaching that much beforehand but relies on pupils' ideas. She got the information of the project from her headmaster. Dana explained that she had used problem-solving tasks in her mathematics teaching now and then. But she adds that her own perceptions of mathematics have not changed much though during the project. She claims that she develops her pupils' mathematical thinking by questioning, and not giving ready-made answers. She tells that she gives nice feedback often and accepts many of her pupils' answers, even though they are perhaps not quite right. According to her, during her years of work, her mathematics teaching has changed. She tells about some kind of relaxation in her teaching and claims to live in the moment.

Dana explains that there have been changes in her classroom practices, too. Nowadays Dana reveals that she allows her pupils to work in pairs for the pupils to negotiate and consult each other. In her opinion some of the tasks used in this project were inspiring and during the project Dana insists she has learned to understand different areas of her pupils' interests.

5 Conclusions

As an answer to the research question “How do the teachers themselves perceive that their conceptions about mathematics and teaching of mathematics have changed during the project?” we have the following findings:

Firstly, the teachers claimed that the use of open-ended problems had an effect on their’ conceptions about mathematics and teaching of mathematics since, for example, in the case of Dana, the subject knowledge increased and, in the case of Nicholas, the motivational component and his enthusiasm towards mathematics clearly increased.

Secondly, regarding the teacher’s perceptions about pupils as mathematics learners, their allegations of transformation were evident as well. This is in line with the findings about pupils (Laine & al. 2012, Varas & al. 2012). For instance, Amelia noticed the growing interest in pupils and Fiona declared she was able to notice pupils’ mathematical thinking. In addition, Nicholas and Amelia claimed to be surprised to see how also the weakest pupils can be involved with problem solving.

Lastly, all teachers declared that the use of open-ended problems seemed to have affected teachers’ classroom practices up to certain extend. Nicholas, Fiona and Dana revealed the fact that they think they give more room for pupils' ideas, comments and arguments, and Fiona and Dana claimed that they rely on pupils' learning in pairs or in groups. Furthermore, Nicholas and Dana noticed how different tasks awaken different learning and interest. However, Amelia and Nicholas confessed that not much has changed in their daily lessons, and they blame this on the strictness of the system in Chile.

There seem to be no differences in the development of teachers’ conceptions on mathematics and it’s teaching in Chile and Finland. In both countries one may notice the teachers evolve in similar ways that seem to be due to the use of open-ended problems: teachers give more room for their pupils' ideas, comments and arguments, notice how different tasks awake different learning and interest, and that the weakest pupils can be involved with problem solving.

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