Why Do Experts Pose Problems for Mathematics Competitions?

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Content

Abstract

Inspired by the recurrent findings on the steady decrease in students' interest in mathematics, this paper is concerned with sources of experts' motivation for posing problems for mathematics competitions. Twenty-six experts from nine countries participated in the study. The inductive analysis of the data suggests that experts utilise posing problems for mathematics competitions for fulfilment of their internal needs: an intellectual need for enriching their mathematical knowledge base and a socio-psychological need for belonging, recognition and appreciation. Educational implications of the findings for students are discussed and future research directions are presented.

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1 Introduction and Theoretical Background

Motivation has been acknowledged as one of the key factors in teaching and learning. Empirical research conducted in mathematics education has shown that motivation is closely related to students' performance, achievements and beliefs (e.g., Lewis, 2013; Middleton & Spanias, 1999). Moreover, motivation is a part of the students' scholastic mathematical experience. This experience plays a significant role in a student's decision to learn mathematics courses in the future and pursue a career in mathematics (Middleton & Spanias, 1999). In light of the above, the recurrent findings on the steady decrease in students' motivation and interest in mathematics are truly disturbing (e.g., Lewis, 2013; Middleton & Spanias, 1999).

In order to develop new insights into sources of students' motivation, this paper focuses on experts in mathematics. This focus is in line with the established practice of using research on experts as a source of ideas for fostering mathematics learning for novices (see a detailed discussion on this point in Kontorovich and Koichu, in press). Several categories of experts were considered for the purposes of this study: practicing mathematicians, lecturers, teachers, task designers and mathematics textbook writers. Eventually, the decision was made to recruit expert problem posers (EPPs) for mathematics competitions (MCs). The decision was based on the assumption that in many cases EPPs for MCs receive neither fiscal rewards nor academic accreditation for being engaged in mathematics, unlike the aforementioned categories of experts. Thus, identifying the sources of the EPPs' motivation could shed new light on intrinsic motivation in engaging in mathematics.

The preliminary study with 22 adult participants from the Competition Movement (i.e. coaches, EPPs and organizers of MCs) was presented at the 17th MAVI conference (Kontorovich, 2012). The findings of the preliminary study suggested that the participants shared a pedagogical agenda consisting of four interrelated goals: to provide students with opportunities to learn meaningful mathematics, to strengthen their positive attitudes towards mathematics, to create cognitive challenges for the students and to surprise them. Competition problems were perceived by the participants of the preliminary study as a means of achieving these goals.

In light of the above, it can be suggested that fulfilling this pedagogical agenda is a possible source of experts' motivation to pose competition problems. However, EPPs might also consider alternative ways of engaging in students' mathematical education. For instance, they could publish books or teach mathematics at school. The experts' decision to invest their time and effort in creating problems for MCs implies that they may possess additional motivational sources. Identification of these sources is the goal of the current study.

Lewis (2013) wrote that research on motivation in mathematics education is frequently concerned with attitudes and that it is undertaken from a statistical point of view. He reminds us that motivation is an emotional phenomenon, and therefore phenomenological methods propose an added value in its exploration. Lewis's approach has been adapted in the current study. Namely, the study is conducted using qualitative methods, and motivation is analysed from the participants' point of view, i.e. people who actually possess it.

2 Method

Twenty-five men and one woman participated in the study. All the participants are active problem posers for national, regional and international MCs, such as The Baltic Way, Mathematical Kangaroo and Tournament of the Towns. Twenty-three experts were above the age of 40, and the average age of the experts was 50 (SD=12). The experts' problem-posing experience ranged from seven years to more than 30. The academic background of the experts is as follows: one expert holds a master's degree in mathematics education; six received a master's degree in mathematics; two have a PhD in mathematics education; and 17 experts hold a PhD in mathematics.

Nineteen experts are employed in colleges and universities; one expert teaches in a high-school and another works in the high-tech industry. One expert has an official paid position in the organizational committee of national competitions. Only one expert (Olivia) makes her living posing riddles for daily newspapers and publishing riddle books. Thus, the study's a-priory assumption that problem posing for most of the experts is a non-profitable practice proved to be a correct one.

The participants reside in Australia, Bulgaria, Israel, Latvia, Lithuania, the Russian Federation, Spain, Sweden and the USA. Eighteen experts chose to interact with me in Russian, 4 preferred English and the remaining 4 preferred Hebrew. Thus, the majority of the experts participating in this study were influenced, or nurtured, by the [former] Soviet mathematics education. In other words, the variety of the experts' countries of residence does not fully represent the diversity of mathematical views and perceptions.

The data were collected using semi-structured in-depth interviews (e.g., Evans, Patterson & O'Malley, 2001) and open questionnaires. The Israeli participants were interviewed face-to-face. Nine participants from other countries were interviewed by telephone or Skype. The interviews lasted between 60 and 90 minutes. The questions were sent to participants by e-mail a week before the interview. This was done in order to “prep” the participants for the forthcoming interview. All the interviews were audio-taped and transcribed. The remaining eight experts preferred to answer in writing by filling in a questionnaire consisting of the central questions of the interview. After a preliminary analysis of the data the experts were then asked to take part in follow-up interviews and to answer a set of question for clarification and validation. Nine experts consented to take part in these follow-up activities.

In both the interviews and the questionnaires, the experts were asked to tell about the role that the competition movement plays in their life. They were asked specifically about their problem-solving and problem-posing experience, their reasons for joining the competition movement in the first place, and their motivation for staying in the movement and becoming problem posers. The data were analysed using an inductive approach in order "[…] to allow research findings to emerge from the frequent, dominant or significant themes inherent in raw data, without the restraints imposed by structured methodologies" (Thomas, 2006, p. 238).

3 Findings

3.1 Experts' Intellectual Need

According to Wikipedia (n.d.) the *intellectual need* of a person is a special type of intrinsic motivation for gaining a new piece of knowledge. In the excerpts below, two of the study participants, Ari and Mike (pseudonyms), explicitly refer to their "built-in" curiosity and desire to learn new mathematics:

> Ari: All my life I was aware that the only things I need in my free-time are a pen and a sheet of paper. I am always preoccupied with some geometrical configurations and their properties… which is why I usually have too many questions about them yet very few answers. I think this is the fate of any researcher. […] My motivation for conducting [mathematical] research is curiosity and the desire to reveal what is true: [For instance,] Is some property generalizable from a special case to other cases as well? Does a particular configuration conceal any additional properties that we are not familiar with, or have we reached the maximum? [Translated from Hebrew.].

> Mike: When I pose a problem, I learn [i.e. acquire new mathematical knowledge]. Actually, it is one of my character traits. I don't even understand where it comes from. Every time I see a problem that interests me, I feel a powerful desire to "strengthen" it, to take it one step forward, to learn new things about it. In this way many new problems are created.

> Interviewer: What motivates you "to strengthen the problems"?

> Mike: An ambition to know how this world works. Why does a problem include a particular given, rather than another one? The discovery and understanding create an incredible feeling, which gives you wings! [Translated from Russian.].

Two phenomena can be observed in Ari's and Mike's statements. First, they both describe their desire to learn new mathematics in a highly emotional manner and report that its fulfillment creates a pleasant and rewarding feeling. Second, Ari and Mike refer to this desire as a significant characteristic of their personality rather than a situational feeling. The intensiveness and ubiquity of this desire bear resemblance to the physiological and socio-psychological needs described by Maslow (1943), which facilitates referring to this burning desire as an *intellectual need for enriching their mathematical knowledge base.*

Ari and Mike exposed their intellectual need more elaborately than the rest of the study participants. In the data collected on 10 additional experts, intellectual need was noted in certain sentences and phrases, in which the experts referred to their aspiration to acquire new mathematical knowledge. For instance: "From the moment I remember myself, I always wanted to understand how…", "I was always curious about …", "There are problems and questions that will keep you up all night!". Note that in the first two phrases, the idiosyncrasy of the need is emphasized (like in the case of Ari and Mike), when in the third phrase the need is described as contextual, i.e. being provoked by a particular mathematical situation. This type of situation is exemplified below in the case of Alexis and Mavlo's Theorem:

Mavlo's Theorem states that one of the arcs created by a side of a triangle in the Euler circle equals the sum of the arcs created by the other two sides. In some cases, the arcs are created by extending the sides of the triangle up to the intersection with the Euler circle. When Alexis was asked to recall his first reaction to the theorem, he said:

> "My intuition told me that it is possible to take this theorem one step forward. […] The Euler circle is the most familiar one in geometry and it leaves other circles a long way behind. I wanted to find out if some other circles also have a chance to stand out."

After that, Alexis was engaged in a search for other circles with properties related to arcs created by triangles. During his search, he acquired mathematical concepts and theorems which were new to him, and he posed problems based on his new knowledge. One of these problems appeared in 2010 in the Russian Geometry Olympiad named after Sharigin. In addition, Alexis described his mathematical journey of knowledge acquisition in a self-reflective paper (see Miakishev, 2010).

Similarly to Alexis, other experts also said that when they pose problems for competitions, they frequently get exposed to mathematics which is new to them, generate questions that they did not think about before and encounter problems they are unfamiliar with. Therefore, it can be deduced that problem posing for MCs is fertile ground for fulfilling the experts' intellectual need for new mathematical knowledge.

It is noteworthy that EPPs are not interested in just any piece of mathematical knowledge which is new and unfamiliar to them. The analysis showed that each participant possesses 1-3 fields of mathematical interests and seeks to enrich his or her knowledge base in these fields. Examples of such fields are Euclidean geometry, 3-D geometry, number theory, logic problems, linear algebra etc.

3.2 Experts' Socio-Psychological Needs

The experts tend not only to discover new [for them] mathematical facts, but turn these facts into problems and propose them to various MCs. This behavior cannot be attributed to the aforementioned intellectual need. From the perspective of experts' pedagogical agendas, it could be said that experts use these problems to achieve their goals regarding the students who participate in the competitions (see Introduction and Theoretical Background). In this sub-section I argue that the act of publishing the created problems in MCs can also be interpreted as a means for fulfilling experts' *socio-psychological needs for belonging, recognition and appreciation*. To recall, this need is included in Maslow's (1943) pyramid and its fulfillment is acknowledged by many schools as a powerful motivating factor (e.g., Maccoby, 1988; McClelland, 1971).

The importance of the sense of belonging in the case of EPPs is implied by the fact that all the experts who participated in the study were involved in MCs when they were students. Consequently, the experts have belonged to the competition movement for the greater part of their lives, when at some point they made “a switch” from solvers to posers. When asked why they decided to stay in the competition movement as adults, some of the experts explained their decision using the law of inertia: “I got used to Olympiads; it was my hobby”, “I like dealing with mathematics, so how else am I supposed to do it?!”. One of the experts, Ronny (pseudonym), gave a more detailed explanation:

> "It was when I started studying in college. It was a very stressful time for me: I finished high school, moved out of my parents' house and relocated to another city; I was away from my family. The studying [in college] was so different from what I was used to in high school. The mathematics was so different… So I looked for something to lean on, something that was familiar and natural to me." [Translated from Hebrew.].

Another expert, Olivia, explained her decision to remain in the competition movement metaphorically:

> "I participated in competitions when I was a student, but I never excelled in them. And I always had a strong desire to be noticed. This desire gave me the motivation to pose problems. I think that the desire to be better than others in something is natural. For instance, if someone likes ice skating but he isn't a good ice skater, he can

> become a good coach, or a judge or an ice show producer." [Translated from Russian].

In this quote, Olivia admits that she possesses the need for recognition and appreciation, and this explains her decision to remain in the competition movement. Namely, Olivia puts forward the connection between her modest achievements in solving competition problems and her decision to become a problem poser: the fact that she was a part of the competition movement in the past and that she is still a part of it today enables her to compensate for the lack of recognition and appreciation back then with the recognition and appreciation she gets today, thanks to the high-quality problems she creates.

Olivia's idea about the connection between posing problems and gaining recognition and appreciation can also be extracted from Leo's words:

> "It is important that Olympiad questions include as many ideas as possible. If all the questions are similar to each other or to the questions that appeared in the previous Olympiads, it can create an impression that we [problem posers] don't have enough ideas". [Translated from Russian.].

It can be concluded from Leo's words that he perceives the competition problems as a two-sided assessment tool: It enables one to assess the problem-solving skills of the students as well as the problem-posing skills of the experts. Therefore, when posing a problem, Leo makes an effort to create a positive impression with his problems on the intended solvers, fellow problem posers, coaches, organizers of the competitions etc.

In 25 (out of 26) cases, the needs for belonging, recognition and appreciation went hand in hand. The excerpt from the case of Mike presented below shows that these emotions can intertwine in a complex manner. Mike specializes in the field of algebraic inequalities. For more than 20 years he has been solving these types of inequalities, collecting inequalities that appeared in MCs over the world, posing his own inequality problems and even developing methods for solving and proving inequalities.

Mike said that not many people are as interested in inequalities as he is. Moreover, over the years he has reached extraordinary levels of expertise in this field and asserted that he has difficulty finding people who are able to understand and appreciate his findings. He found these people on the web-forum “Art of Problem Solving” and became an active member with more than 10,000 posts. Thus, it can be concluded that the forum serves for Mike as a platform for having meaningful interactions with like-minded others and it provides him with a sense of belonging.

However, it turned out that Mike conducts his mathematical research in writing and it is summarized in twenty-two thick notebooks. The following excerpt from the interview with Mike focused on this phenomenon:

Mike: If all my stuff was saved in the computer, someone would steal it for sure. I don't want this to happen: it is an enormous amount of work, the work of all my life actually! Why should I give it to someone else?!

Interviewer: Why you are so sure that someone would "steal" your work?

Mike: It's not paranoia, it has already occurred in the past.

Interviewer: Can you tell me about this?

Mike: Naughty XXXians [names a country] stole one of my problems [he says the words with an intonation of pride, smiles and giggles at this point]. I posted one of my problems on the forum "Art of Problem Solving" and a year and a half later, it appeared in the XXXian Olympiad.

Interviewer: And how did it make you feel then?

Mike: Well, you know… It is important to me that people see my results. If someone steals them, well… So be it! If I want, I can always prove that I was the first one to publish them.

Mike's case exemplifies the importance of the sense of belonging, recognition and appreciation in expert problem posing. Similarly to Olivia and Leo, Mike wants people to attribute their positive emotions provoked by mathematical problems to him, the person who created these problems. This idea can be extracted from Mike's usage of the word "naughty": While, on the one hand, he claims that some people stole his problem, on the other hand, they popularized it and, therefore, automatically popularized its creator. Interestingly, it is enough for Mike to know that he is the creator of the popularized problem, even if people are not aware of this fact. Indeed, he did not try to expose himself as the real creator of the problem; he is satisfied simply knowing that the exposure is achievable. Regarding the act of "stealing" his intellectual property (from his point of view), apparently, this is the "price" he is willing to pay for the sense of belonging which he is granted by the forum.

As can be seen from the excerpts above, experts' socio-psychological needs were observed in very personal and revealing materials, which they chose to share. Apparently, this special kind of material could not appear in the data on all the participants of the study. This fact partly explains why evidences of experts' socio-psychological needs were observed only in five cases. In addition, it turned out that in four of these cases, experts regularly publish their mathematical results in special journals and handbooks. This action can be interpreted as a realization of a high degree of need for recognition and appreciation, a degree which could not be completely fulfilled by publishing problems in competitions. The data collected on eight additional experts included phrases such as "I like sharing new mathematical results with people" or "I'm very proud, when people say 'wow!' about my problems". These phrases can also be seen as evidence of experts' needs for recognition and appreciation.

4 Discussion

The main contribution of this paper is in its empirical identification of two motivational sources among experts in mathematics: an intellectual need for enriching their mathematical knowledge base and a socio-psychological need for belonging, recognition and appreciation. The identified sources serve as an explanatory framework for understanding the practices of 26 Expert Problem Posers (EPPs) who create problems for Mathematics Competitions (MCs).

The analysed case of EPPs and MCs is an example of a "good match" between individuals who possess particular needs and a framework for participation that addresses these needs and enables its participants to create products of a high quality, develop their knowledge and skills, and actualize their capabilities and talents. Therefore, creating similar frameworks for learning purposes can be considered one of the central aims of education. Moreover, I argue that intellectual and socio-psychological needs are typical to all people, and therefore, they could be and should be utilized for designing these frameworks.

The educational community has been extensively utilizing students' intellectual needs for learning mathematics. For instance, Harel and his colleagues introduced and empirically based a comprehensive framework for the instruction of mathematics (e.g., Harel, 2008). According to it, learning of a new construct of knowledge occurs as a result of students being engaged in carefully designed tasks, the solution of which requires the missing knowledge. However, Harel's approach to the concept is purely epistemological. In his words:

> "There is often confusion between *intellectual need* and *motivation*. The two are related but are fundamentally different. While intellectual need belongs to epistemology, motivation belongs to psychology. Intellectual need has to do with disciplinary knowledge being born out of people's current knowledge through engagement in problematic situations conceived as such by them. Motivation, on the other hand, has to do with people's desire, volition, interest, self determination, and the like." (Harel, 2008, p. 897-898).

The data presented in the current paper show that in the case of EPPs, intellectual need is one of the most powerful motivational sources deriving from their personalities. For them it is a state of mind that enables them to sense the need for a nonspecific piece of mathematical knowledge without being involved in problematic situations. This finding is in line with Ericsson (2006), who argued that experts tend to engage in deliberate practices (problem posing in this case) in order to extend their already well-developed knowledge bases and to sharpen their professional skills.

In light of the above, there is room for rigorous empirical investigation of students' personal intellectual needs, preferences and interests, for structuring them in clusters and for designing learning settings that will match the identified clusters. Note that nowadays, the idea of a learning setting which is matched to the intellectual cluster of a student is realized in terms of a course in some mathematical area with a particular level of complexity (e.g., Algebra 3). In other cases the course is merely adjusted to the age of the students (e.g., Mathematics for the 9th grade). The aim of the proposed investigations is to address the intellectual needs of particular students with narrower aspects of the learning setting, such as mathematical topics that will interest them, problems that will challenge and engage them, exercises that will promote competencies without boring them etc. Matching students to a carefully designed learning setting can be a powerful tool aimed at unlocking their mathematical potential.

Regarding the socio-psychological need, recognition and appreciation are typically achieved through high grades, which are assigned by figures of authority. The grades are transformed into a goal to achieve, while learning is solely the means to an end. The situation is significantly different in the case of EPPs: *First*, they are rewarded by being accepted into the elite society of problem posers for prestigious MCs. *Second*, rewarding is based on high-quality products (excellent mathematical problems in this case) produced by the experts. Since these products are inseparable from learning mathematics, a direct connection is established between learning and reward. *Third*, the experts are rewarded by colleagues and consumers of their products – principally the students who participate in MCs. In this way, the rewarding communities encourage the posing of high-quality problems, and thus they enhance learning.

Research has been relatively silent about empirical attempts to model these communities in a classroom setting (see Brawn & Walter, 1996 for a rare example). Therefore, there is room for additional inquiry of students' socio-psychological needs and for the ways these needs can be addressed when learning mathematics. I believe that rigorous study of the movement of Mathematics Competitions is an insightful step for addressing these issues.

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