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Gabriele Kaiser · Rita Borromeo Ferri · Werner Blum *Hrsg.*

RESEARCH

Xinrong Yang

Conception and Characteristics of Expert Mathematics Teachers in China

Springer Spektrum

Perspektiven der Mathematikdidaktik

Herausgegeben von
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Foreword

The superior performance of East Asian students in recent international studies of mathematics achievement has attracted the attention of educators and policy makers worldwide. One interesting phenomenon that has been observed is that these high performing countries share a similar culture, sometimes named the Confucian Heritage Culture or CHC. Because of this phenomenon, educators and researchers have been interested in gaining a better understanding of mathematics education in China, and substantial research has been conducted on various aspects of mathematics education in China, ranging from studies on the educational policies to the official curriculum and to classroom teaching. However, very few studies have focused on one of the most fundamental issues in mathematics education — the quality of mathematics teachers. In this regard, the exploratory study conducted by Dr. Xinrong Yang based on his PhD work and as reported in this book provides important information and insight for understanding this research gap.

As Xinrong's PhD supervisor, I am very glad to witness his book seeing the light of day. I can still remember seven years ago when Xinrong started his PhD study with me, he told me that he was interested in exploring how expert mathematics teachers in China develop their expertise. However, to achieve this goal, one fundamental issue to figure out is how an expert mathematics teacher is defined, as there is no clear consensus in the literature on the definition of an expert teacher. Subsequently, Xinrong modified his research focus to exploring how an expert mathematics teacher is conceptualized by educators in China and the characteristics that expert mathematics teachers in China share. I am glad that the final thesis is a very fine piece of scholarly work, and I am sure the work will make a valuable contribution to the literature in the field of teacher education and development.

As Xinrong reviewed and argued in his work, teacher expertise is a culture-bound notion. China, as a country with a rich culture through more than five thousand years of history, has a lot of unique characteristics regarding education in general and mathematics teaching in particular. The traditional Confucian culture, or CHC, is still asserting significant influence on education in China and in many East Asian countries today. However, as widely reported in the literature, the superior mathematics performance of students in China and the rest of East Asia has been achieved despite rather unfavorable conditions such as large class size, and teacher-centered and examination-driven teaching. It is intriguing to learn how teachers develop their expertise and how the notion of expert teacher is conceived in such an unfavorable environment, as it can be expected that the conception and development of teacher expertise in this special context would be very different from other contexts. An appreciation of how teacher expertise is conceived in such a social and cultural context may provide the key for understanding other aspects of mathematics education in China, and may throw light on how teacher expertise and conception of expertise are influenced by the social and cultural context more generally.

Xinrong´s own learning experience as well as his experience in pre-service teacher education in China had enabled him to conduct his study with much insight. From his rich knowledge of the relevant literature, he adopted a sociocultural theory and a prototypical view of conception in this study of teacher expertise. He found that some of the roles expected to be played by expert mathematics teachers in China, such as being at the same time a researcher, a mentor, and an expert in examination, are quite different from the roles expected of an expert teacher in the Western culture. In addition, some characteristics of an expert teacher identified in his study are also different from the features reported in previous studies. Examples include the expert mathematics teachers´ beliefs about mathematics and its learning and teaching, and their ability to balance direct teaching and exploratory teaching.

Findings such as these should be of interest to those who are interested in mathematics education and teacher education in China, as well as those who are interested in the field of teacher expertise. Readers will no doubt gain other insights from this resourceful and inspiring book, and I am sure this book will be making an impact in the field in the years to come.

Frederick K. S. Leung

The University of Hong Kong

Abstract

This study explores: 1) how mathematics educators in mainland China conceptualize expert mathematics teachers; 2) characteristics of expert mathematics teachers; and 3) how the Chinese social and cultural context influences both. Taking a sociocultural perspective and adopting a prototype view of teaching expertise as its theoretical foundation, this study examines, through semi-structured interviews, the conception of expert mathematics teachers from the perspectives of eleven mathematics teachers, six (vice) school principals, two mathematics teacher educators, and two mathematics teaching research officers. Based on the 21 interviewees' recommendations, three expert mathematics teachers' beliefs, knowledge and teaching practices were investigated further for common characteristics. Five to six consecutive lessons in a particular class were observed and videotaped, with each of the teachers being interviewed before and after every lesson.

The constant comparative method (Glaser & Strauss, 1968) was adopted for data analysis. Characteristics mentioned by more than 50% of the 21 mathematics educators were considered as components of the conception of expert mathematics teachers, and features found in at least two of the three teachers were treated as prototypical features of expert mathematics teachers in mainland China.

It was found that expert mathematics teachers were conceptualized as teachers playing multiple roles, including demonstrating expertise in teaching, conducting research and publishing papers, and mentoring teachers. They should not only be knowledgeable in mathematics, theory, characteristics of learners, curriculum, and many other fields, but also be exemplary models for students and colleagues. Most of the characteristics described by the 21 interviewees were identified in the three expert mathematics teachers, except for some discrepancies in opinions about knowledge related to advanced mathematics and research ability. The three expert mathematics teachers were found to hold contemporary-constructivist oriented beliefs, and to possess a wide and profound knowledge base. They could teach with flexibility, balance, and coherence. They could promote students' higher order thinking and their teaching practice was consistent with the beliefs they held. They could systematically reflect on their teaching and propose modifications and improvements.

Results indicate that the concept of expert mathematics teachers is culturally bounded and that their teaching is influenced by the social and cultural context; however, they also demonstrate the ability to work against social and cultural constraints. This study's findings contribute to: 1) understanding of the conception of expert teachers in a particular subject and within a specific sociocultural context; 2) how a specific social and cultural contexts influence expert mathematics teachers' beliefs, knowledge, and practice; 3) a new perspective on mathematics education in China; 4) a new perspective on differences between the teaching of novice and expert teachers; and 5) curriculum development in pre-service and in-service teacher education. Further research is needed to explore the concept and characteristics of expert mathematics teachers at other grade levels and in other social and cultural contexts, to provide a deeper and fuller understanding of expert mathematics teachers.

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Xinrong YANG
杨新荣

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Chapter One

Introduction

1.1 Background to the Problem

Since the 1980s, Chinese students, including those from mainland China, Hong Kong, and Taiwan, have consistently outperformed their Western counterparts in large-scale international studies in mathematics, such as IAEP, TIMSS, and PISA (Fan & Zhu, 2004; OECD, 2010). Students from mainland China once ranked the first in IAEP2 (Fan & Zhu, 2004) and in PISA 2009 (OECD, 2010), and recorded excellent performance in some small-scale comparison studies in mathematics achievement as well (*e.g.*, Lee, 1998; Stevenson *et al.*, 1993). In addition, students from mainland China have been champion in the International Mathematical Olympiads (IMO) many times (IMO, 2013). While these achievements are impressive, some Western researchers have found that the Chinese learning environment, with its large class size, expository teaching methods and focus on preparation for external examinations, does not appear to be conducive to effective learning (Biggs, 1996).

Chinese students' outstanding mathematics performance, despite their unfavourable educational environment, has been identified as the so-called "the Chinese Learner Paradox" (Biggs & Watkins, 1996; Marton *et al.*, 1993), and the paradox has drawn the attention of numerous researchers, including many from the West who were disappointed with their own students' mathematics achievement (Stevenson & Stigler, 1992). The researchers explored the paradox from various perspectives and hypothesized that the differences in number systems (Fuson & Kwon, 1991; Miller *et al.*, 1995), cultural contexts (*e.g.*, parental expectations and beliefs in ability), school organizations, and mathematics curricula might contribute to Chinese students' excellent achievement (Lee, 1998; Stevenson *et al.*, 1990; Stigler & Perry, 1988).

An additional important factor attributed to Chinese students' outstanding mathematics achievement might be "schooling, more specifically, the educational practices of teaching—learning mathematics at school" (Hatano & Inagaki, 1998, p. 82). This suggests that teacher quality may play a significant role in student learning. In fact, Moir *et al.* (2009) suggested teachers' quality is the most important school-related

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factor in student learning outcomes, and “dwarfs every other school-related variable … including class size, school size, and even the heterogeneity of prior achievement within a classroom” (p. 11). Many other researchers (*e.g.*, An, 2004; Blömeke & Kaiser, 2012; Even *et al.*, 2003; Hargeaves, 1994; Leung & Park, 2002; Ma, 1999; Schmidt, Cogan, & Houang, 2011) have also described teacher quality is a major school-related factor influencing the quality of education in general and students’ mathematics achievement in particular. Thus, although the relationship is complicated, it is reasonable to conjecture that mathematics teacher expertise is a major factor affecting student achievement, as teacher expertise in mathematics instruction will affect teachers’ teaching performance (Kaiser & Li, 2011).

As such, the questions of what it means to be an expert mathematics teacher and what characteristics an expert mathematics teacher possesses are central. However, teacher expertise takes different forms in different cultures and teachers’ working conditions exert a powerful influence on the development of their expertise (Berliner, 2004). Therefore, teaching expertise and the conception of expert teacher are not universal, but culturally and contextually dependent (Berliner, 2001). As mentioned earlier, the working conditions and culture of mainland China are often described as unfavorable. In addition, as a country with more than five thousand years of history, education in China has its own characteristics and traditions (Gu, 2001, 2006). Therefore, it would be reasonable to conjecture that expert teachers in China may have some unique characteristics not shared by teachers from other cultures, which may also apply for normal mathematics teachers.

However, there is a lack of research on teacher expertise in mathematics instruction in general (Li & Kaiser, 2011) and in exploring the conception and characteristics of expert mathematics teachers in the Chinese context in particular, even though decades have passed since Cooney *et al.* (1988) intimated that “it would be interesting to learn how mathematics educators from other cultures define ‘expert teachers’” (p. 255). In view of this, the main aim of the present study is to explore how “expert mathematics teacher” is conceptualized in the mainland Chinese context, and what sorts of characteristics such expert mathematics teachers would have. Results of the study would be important for understanding what aspects count as important parts of mathematics teacher quality in this specific high-achieving education system.

1.2 Rationale of the Study

There have been many studies focusing on expert mathematics teachers and their teaching practices, and many characteristics of expert teachers have been identified in Western countries (*e.g.*, Berliner, 1995, 2004; Berliner *et al.*, 1988; Borko & Livingston, 1989; Livingston & Borko, 1990). Recently, there has been an increasing interest in studying expert mathematics teachers within mainland China (*e.g.*, Li & Huang, 2008; Li, Huang, Bao, & Fan, 2008; Li, Huang & Yang, 2011; Li & Ni, 2007; Zhu *et al.*, 2007). Many previous studies on expert teachers compared their behaviors and performances to those of novice or non-expert teachers; however, teaching expertise is not a dichotomous variable (Smith, 1999). Therefore, it might be problematic, or at least unreasonable, to compare teachers at the opposite ends of the continuum of teaching expertise.

To date, there is very little understanding of the nature of teacher expertise in mathematics education (Kaiser & Li, 2011). In particular, there is a lack of knowledge on the conception of expert mathematics teacher since very few previous studies have focused on this. Among the existing studies, those taking social-cultural contexts into account were also limited. However, essentially speaking, mathematics teaching is a cultural activity (Stigler & Hiebert, 1999). That is, the behaviors of mathematics teachers in classrooms are fundamentally influenced by cultural values existing in a specific context (Li & Kaiser, 2011). Therefore, while investigating the conception and characteristics of expert mathematics teachers, cultural values and social influences should be taken into account. Furthermore, some researchers have based their investigations on experimental or simulated tasks, rather than studying expert teachers in natural teaching contexts; as such, more investigations in natural settings are needed. Studies to date in mainland China have mainly focused on elementary school mathematics teachers and/or on some specific teacher attributes, such as mathematics knowledge (Li *et al.*, 2005) or pedagogical content knowledge (Zhu *et al.*, 2007); a systematic investigation of expert mathematics teachers has yet to be conducted. Such a segmented inquiry compromises the nature of expertise in teaching (Smith, 1999); therefore, there is a need to systematically explore the characteristics of expert mathematics teachers working at a certain grade level to provide a more comprehensive picture of expert mathematics teachers in mainland China.

However, what is the conception of expert mathematics teachers in mainland China? It is difficult to give a general definition of expert mathematics teachers (Berliner, 2004). This study has adopted Sternberg and Horvath's (1995) prototype view of teaching expertise to investigate the conception of expert mathematics teachers. In addition, characteristics shared by expert mathematics teachers in this context will also be identified and explored. Sociocultural theory is adopted to establish a link between influences of social and cultural contexts on the conception and characteristics of expert mathematics teachers. According to sociocultural theory, self-organized (voluntary) attention, categories perception, conceptual thinking and logical memory vary historically and across different cultures (Gredler & Shields, 2008). In this sense, the conception and characteristics of expert mathematics teachers are context dependent.

1.3 Research Questions of the Study

The major objective of this study is to explore how "expert mathematics teacher" is conceptualized and what characteristics are shared by expert mathematics teachers in Mainland China, which has been regarded as a high-achieving education system in international comparative studies. In particular, three research questions are investigated in this study:

1) How is "expert mathematics teacher" conceptualized by mathematics educators in mainland China?
2) What are the characteristics of expert mathematics teachers in mainland China?
3) How do Chinese social and cultural contexts influence the conception and characteristics of expert mathematics teacher?

1.4 Significance of the Study

This study makes, at a minimum, four significant contributions to the research field of teacher expertise. First, its findings allow those who interested in teacher expertise to develop a deeper understanding of the conception and characteristics of expert teachers in relation to a specific cultural background and a specific subject and from a prototype perspective. Moreover, characteristics shared by expert mathematics teachers in natural teaching contexts are richly described, and the findings are interpreted with reference to the specific cultural values found where this study was conducted. In this sense, this study helps to clarify what kind of teachers should be regarded as expert and what characteristics they might share in mainland China. More important, the findings provide information about how the cultural and social contexts influence the conception and characteristics of expert mathematics teachers.

Second, the study offers those interested in Chinese mathematics education an opportunity to understand better Chinese mathematics education. This study focuses on mathematics expert teachers in China, a country with students attaining high achievement in IMO, IAEP2, and PISA under unfavorable conditions. It is believed that this study will help readers to understand teacher quality and expertise in mainland China – not only in mathematics, but also in other subjects. Moreover, this study could also provide meaningful information to interpret the excellent achievements of Chinese students in mathematics from the perspective of teacher quality.

Third, the study's findings will be useful for the design of future mathematics teacher education programs. The study offers teacher training program designers a depiction of expert mathematics teachers in real classroom situations, as opposed to the hypothetical or theoretical situations presented in some other studies. The main aim of teacher education programs is to facilitate teachers' professional development; that is, to help teachers, especially pre-service, novice and non-expert teachers, develop their expertise and become, eventually, true experts in their fields (Kaiser & Li, 2011; Leinhardt, 1989; Li & Kaiser, 2011). As such, this study could provide rich information about what constitutes a highly qualified teacher and how such a teacher should be prepared, which has become a hot research topic in the field of mathematics teacher education (*e.g.*, Teacher Education and Development Study in Mathematics (TEDS-M study), Cognitively Activating Instruction (COACTIV study), Mathematics Teaching in the 21st Century (MT21 study)) (Blömeke & Kaiser, 2012; Kunter *et al.*, 2013; Schmidt, Cogan, & Houang, 2011), and help program designers develop more effective mathematics teacher education programs, both in mainland China and elsewhere.

Fourth, the findings offer in-service mathematics teachers a benchmark for their own further improvement. The prototypical conception of expert mathematics teacher identified in the study and rich descriptions of the characteristics of mathematics expert teachers provide a model for teachers to develop their professional skills. In addition, understanding how expert mathematics teachers construct lessons, manage teaching content, and interact with students may assist prospective and beginning mathematics teachers to develop and to overcome the difficulties they might encounter (Livingston & Borko, 1990). Therefore, the findings may facilitate the professsional growth of mathematics teachers at different development stages.

1.5 Outline of the Study

This study consists of nine chapters. The second chapter starts with discussion of three views of concepts, and then discusses the prototype view of teaching expertise, which is the theoretical perspective adopted in this study. The chapter's second section introduces and justifies the adoption of the sociocultural theory in general, and mediation theory in particular, as the theoretical underpinning of this study. The final section of the chapter reviews relevant literature directly related to this study.

Chapter 3 describes the overall social and cultural background of education in mainland China, including teachers' role in Chinese culture, the history of Chinese teacher education system, the teacher qualification and promotion system, the system of basic education and assessment, and the history of mathematics education and curriculum system.

Chapter 4 introduces the research methodology of the study. It first justifies the qualitative nature of this study, and then briefly describes the research design, research site, and participants' information. After this, the data collection and analysis methods are introduced. Strategies to enhance the trustworthiness of the findings are further described at the end of this chapter.

Chapter 5 recounts how the study's 21 interviewees conceptualize "expert mathematics teacher" in terms of her/his knowledge, ability, and traits. Chapter 6 discusses the common characteristics (beliefs, knowledge, and teaching strategies) shared by three expert mathematics teachers. Chapter 7 focuses on similar characteristics found in the three expert mathematics teachers' teaching practices, such as how they plan their teaching, deal with teaching materials, carry out lesson plans, and organize and reflect upon their teaching.

Chapter 8 discusses the conception and characteristics of expert mathematics teachers in the Chinese social and cultural context, with particular attention to sociocultural factors at four levels: classroom, school, social and cultural. Chapter 9 presents the study's major findings and insights, and makes suggestions for further research.

Chapter Two

Theoretical Orientations and Literature Review

2.1 Introduction

This chapter discusses the theoretical underpinnings and research framework of this study, and reviews the relevant literature. The first section includes information about a prototype view of teaching expertise, which serves as the theoretical perspective for this study to construct the conception of expert mathematics teachers, and to categorize common characteristics of expert mathematics teachers. The second section discusses sociocultural theory, which serves as a theoretical basis to allow the researcher to make research assumptions, develop a conceptual framework, and discuss findings from a sociocultural perspective. The third section reviews literature on expert teachers related to this study.

2.2 Theoretical Perspective of the Study

2.2.1 Views of concepts

The term “concept” has many common and technical meanings that may differ due to people’s different “knowledge representation systems, theories of natural language understanding, perceptual processors, theories of logic and semantics, and psychological accounts of semantic memory” (Cohen & Murphy, 1984, pp. 27-28). As such, a unified definition of the notion is not easily arrived at. Smith (1989) stated that a concept “is a mental representtation of a class or individual and deals with what is being represented and how that information is typically used during the categorization” (p. 502). Similarly, Howard (1987) pointed out that “a concept is a mental representation of a category” (p. 2) and people could place stimuli in this category based on similarities between them. Thus, a concept is normally seen as a mental representation. There are three general views of the notion of concept – the classical, probabilistic, and exemplar views (Medin & Smith, 1984).

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The classical view assumes that “all instances of a concept share common properties that are necessary and sufficient for defining the concept” (Medin & Smith, 1984, p. 115). In other words, as Cohen and Murphy (1984) stated:

> *…each concept corresponds to a set or collection of entities, in which membership is all-or-none. This tradition may be traced to the Aristotelian view that each concept has a definition characterizing its "essence" and providing necessary and sufficient conditions for concept membership. Membership in a concept is considered to be all-or-none: either an object fulfills all of the conditions in the definition, in which case it is a member, or else it fails some condition(s), in which case it is a nonmember*. (p. 29)

According to the classical view, an instance must have all of a concept’s defining properties to be considered an instance of that concept. That is, instances can be represented by logical conjunctive definitions (Michalski, 1993). The classical view has been criticized for failing to specify defining properties, using unclear cases, unnecessary properties and nested concepts, and for taking family resemblance as a determinant of typicality (Medin & Smith, 1984).

In the probabilistic view, “concepts are represented in terms of properties that are only characteristic or probable of class members” (Medin & Smith, 1984, p. 115). The probabilistic view rejects the notion of defining features; instead, it argues that concepts may be represented in terms of features that are typical or characteristic (Murphy & Medin, 1985). The view was developed mainly by Eleanor Rosch and Carolyn Mervis. They developed the prototype theory of concepts, which has been said to “mark a major shift in psychology away from classical theories of concepts and toward probabilistic ones” (Adajian, 2005, p. 231). Some researchers, such as Michalski (1993), treated the prototype view the same as the probabilistic view.

The core of the prototype theory is that concepts are organized around family resemblances, rather than around features that are individually necessary and jointly sufficient for categorization (Mervis & Rosch, 1981; Rosch, 1975; Rosch & Mervis, 1975). Rosch and Mervis (1975) stated that:

> *…members of a category come to be viewed as prototypical of the category as a whole in proportion to the extent to which they bear a family resemblance to (have attributes which overlap those of) other members of the category. Conversely, items viewed as most prototypical of one category will be those with least family resemblance to or membership in other categories.* (p. 575)

From this perspective, the prototype for a category is composed of the most common attribute values relevant to other members of the category. In other words, concepts are organized around a best example (Rosch, 1978; Rosch & Mervis, 1975). Every category is represented by a single prototype or best example, which is "not necessarily one that was specifically learned, but perhaps an average or ideal example that people extract from seeing real examples" (Murphy, 2002, p. 30). Therefore, the prototype is a collection of characteristic features of a certain category that tend to but need not be shared by other instances of that category (Howard, 1987). A prototype can be represented by a list of attributes generated from several members of a category (Goldstone & Kersten, 2003). In other words, prototype representtations are essentially lists of features (Barsalou, 1992) that are usually found in members in the category (Murphy, 2002).

Once the prototype for a category has been determined, category-zation can be predicated by determining how similar an object is to the prototype (Goldstone & Kersten, 2003); prototype theorists "often speak of the prototype as the 'best example' of the category and discuss the process of making category judgments in terms of having the prototype in mind or using the prototype in making comparisons" (Grandy, 1992, p. 118). According to prototype theory, similarity-based categories exhibit a graded structure wherein some category members are better exemplars of the category than are others (Rosch, 1973, 1978). Objects in the same category still probably vary in their typicality and differ in their similarity to the prototype.

Compared with classical concepts, the prototype concepts have the following characteristics:

> *prototype categories lack necessary and sufficient conditions; their members need not be absolutely "in" or "out of" the category*

> *but can be members to greater or lesser degrees; their members display family resemblances in a number of characteristic properties rather than uniformly sharing a few defining properties; and they are organized around "prototypical" exemplars.* (Pinker & Prince, 1999, p. 8)

Although the prototype view overcomes some limitations of the classical view, it has its own problems; specifically, it 1) may not adequately capture all of people's knowledge about concepts, and 2) may be too unconstrained (Medin & Smith, 1984).

The exemplar view proposes that concepts are represented by their exemplars, at least in part, instead of by an abstract summary (Smith & Medin, 1999). In other words, categories of concepts may be represented by individual exemplars rather than by a "unitary description of the class as a whole" (Murphy & Medin, 2000, p. 432). Similar to the prototype view, exemplar concepts also categorize an object by comparing it to known exemplars of the category (Medin & Smith, 1984).

There are some differences between the prototype and exemplar views. Firstly, their approaches to conception representations differ – the former involves listing essential features from a single prototype, while the latter represents concepts by a more than one exemplar. Secondly, the categorization process in the two views is different. In the prototype view, it "involves comparing an item to the prototype representation" (Murphy, 2002, p. 95), while the exemplar view "involves comparing an item to all (or many) such exemplars" (Murphy, 2002, p. 95).

Although the exemplar view also overcomes some limitations of the classical view, it has been criticized for lacking "constraints on what properties enter into concepts or even what constitutes a concept" (Medin & Smith, 1984, p. 119); the view limits neither the properties associated with any exemplar, nor the relations between exemplars included in the same representation (Smith & Medin, 2002).

Each of the three views has advantages and disadvantages. The classical view is relatively fixed because necessary and sufficient conditions are needed to define concepts; therefore, it can best be applied to represent well-defined concepts, such as in the law-like nature of the human physical, biological, or social environment (Loocke, 1999). However, the prototype and exemplar views are relatively loosely structured. As the former organizes concepts around prototypes, "only characteristic (not necessary or sufficient) features are expected" (Goldstone & Kersten, 2003, p. 606), while in the exemplar view, "a conceptual representation consists of only those actual, individual cases that one has observed" (Goldstone & Kersten, 2003, p. 606). Therefore, the prototype and exemplar views can be best applied at the beginning of concept formation, when specific instances have to be discovered first and will be further generalized.

2.2.2 A prototype approach to teaching expertise

Sternberg and Horvath (1995) proposed using the prototype view to reveal the nature of teaching expertise, as "expertise is best thought of as a prototypical concept, bound together by the family resemblance that experts bear to one another" (p. 16) because "there exists no well-defined standard that all experts meet and that no non experts meet" (p. 9). In addition, as they contended, "it is this resemblance to one another that structures the category 'expert'" (p. 9). According to Sternberg and Horvath, a prototype can represent the central tendency of all the exemplars in its category and can serve as a basis for judgments about category membership. Sternberg and Horvath proposed that "teaching expertise be viewed as a similarity-based category with something like a prototype as its summary representation" (p. 9), and as "a category that is structured by the similarity of expert teachers to one another rather than by a set of necessary and sufficient features" (p. 9). Therefore, a prototype of teacher expertise can serve as the summary representation of a similarity-based category of expertise, since it can represent the central tendency of teachers in the category.

According to Sternberg and Horvath (1995), a prototype view can contribute to the dialogue on expert teaching in the following ways:

> *1) Prototype view allows us to adopt a fuller, more inclusive understanding of teaching expertise without falling into the trap of making everyone a presumptive expert;*
> *2) A prototype view provides a basis for understanding apparent "general factors" in teaching expertise;*
> *3) The prototype view provides a basis for understanding and anticipating social judgments about teaching expertise.* (p. 9)

In this way, teaching expertise can be viewed as "a natural category, structured by the similarity of expert teachers to one another and represented by a central exemplar or prototype" (Sternberg & Horvath, 1995, p. 14) and the picture of expert teaching is broadened, and it becomes possible for researchers to use a smaller number of factors or components to describe expert, and even similarity-based categories are considered inherently fuzzy. In addition, by viewing teaching expertise as a prototype, it is possible to "distinguish experts from experienced non experts in a way that acknowledges (a) diversity in the population of expert teachers, and (b) the absence of a set of individually necessary and jointly sufficient features of an expert teacher" (p. 14).

The prototype view has been increasingly adopted by other researchers (*e.g.*, Li, Huang, & Yang , 2011; Lin, 1999; Smith, 1999; Smith & Strahan, 2004) to explore teaching expertise and is adopted as the theoretical perspective of this study, in particular the feature-based model of similarity-based categorization proposed by Sternberg and Horvath (1995). The intention of this study is to explore the conception of and common characteristics found in expert mathematics teachers in mainland China. A list of features related to the conception of expert mathematics teachers and expert mathematics teachers' teaching practices that will emerge from the collected data. This mandates the use of the prototype view. A simple list of expert mathematics teacher features cannot be deemed necessary and sufficient conditions, as claimed in the classical view, and it is paradoxical to identify an expert mathematics teacher at the very beginning of this study as a *known exemplar*, as would be required were the exemplar view employed.

2.3 Theoretical Underpinnings of the Study

2.3.1 Sociocultural theory

Sociocultural theory, as defined by Ratner (2002), is the field that "studies the content, mode of operation, and interrelationships of psychological phenolmena that are socially constructed and shared, and are rooted in other social artifacts" (p. 9). One of its fundamental claims is that its proper focus is human action (Wertsch *et al.*, 1995). Action here may be both external and internal and the action may be carried out by groups with various sizes or by individuals (*ibid*). The goal of sociocultural research is to explicate the relationship between human action and the cultural, institutional, and historical contexts in which the action occurs (*ibid*). In other words, as Lantolf (2004) explained, “despite the label ‘sociocultural’ theory is not a theory of the social or of the cultural aspects of human existence. … it is, rather, … a theory of mind … that recognizes the central role that social relationships and culturally constructed artifacts play in organizing uniquely human forms of thinking” (pp. 30-31).

Sociocultural theory has its roots in the writings of L. S. Vygotsky, in which some primary concepts are established, such as mediation, internalization and the zone of proximal development. This study adopts “mediation” in sociocultural theory as the theoretical underpinning, namely “human mind is *mediated*” (Lantolf, 2004, p. 15, emphasis in original) for an in-depth investigation of the conception of expert mathematics teachers held by mathematics educators, and of the common characteristics shared by expert mathematics teachers in mainland China.

Mediation is the central concept of sociocultural theory, and refers to “the process through which humans deploy culturally constructed artifacts, concepts, and activities to regulate (*i.e.*, gain voluntary control over and transform) the material world or their own and each other’s social and mental activities” (Lantolf & Thorne, 2006, p. 79). In other words, the term *mediated* means that individuals master a higher level of behavior through their control of cultural symbols (Gredler & Shields, 2008), including numbers, graphs and, above all, speech and writing, all of which are culturally constructed and are passed on and appropriated (often in modified form) from one generation to another (Lantolf, 1994). They are “simultaneously material and conceptual (or ideal) aspects of human goal-directed activity that are not only incorporated into this activity, but are constitutive of it” (Lantolf & Thorne, 2006, p. 62), and play an “essential role in *shaping* action” (Werstch *et al.*, 1995, p. 22, emphasis in original). In other words, “cultural activity systems and the complexes of symbolic mediation they incorporate are simultaneously the effect and the cause of the design and construction of the architecture of the mind” (Rȉo & Alvarez, 1995, p. 217).

According to Vygotsky, there are two levels of mental functions. One comprises primitive or elementary functions, such as involuntary attention, simple perception and natural memory, which are primarily controlled by one’s environment and are biologically determined and universal across historical periods and culture (Gredler & Shields, 2008). The other consists of psychological or mental functions, including self-organized (voluntary) attention, categorical perception, conceptual thinking and logical memory. These higher psychological or mental functions are "the results of learning to use the symbols of the culture to develop complex forms of thinking" (p. 81), and their development is linked to cultural symbol systems. Higher forms of thinking necessarily incorporate external symbolic forms "that are usually considered as something peripheral and accessory with respect to internal mental processes" (Vygotsky, 1999, p. 40) and are, always and everywhere, mediated by these symbol systems (Lantolf, 2004). Therefore, they are not universal; rather, they vary historically and across different cultures (Gredler & Shields, 2008).

As stated above, there exists an agreement "revolving around a view of processes of individual development as they constitute and are constituted by interpersonal and cultural/historical activities and practices" (Rogoff *et al.* 1995, p. 125). From sociocultural perspectives, the mind formation process is essentially and inescapably a sociocultural process that can only be grasped by situating individual development in its sociocultural context (Nicolopoulou & Cole, 1993). In other words, higher human mental functions are developed in and influenced by a certain cultural context. Similarly, Ratner (1997) argued that,

> *People collectively construct concepts that objectify their understanding of things (objects, animals, and humans). These cultural concepts enable people to communicate about things. Cultural concepts also organize the manner in which people perceive, imagine, think about, remember, and feel about things. In other words, collectively constructed concepts compose culture, and cultural symbols organize psychological phenomena.* (p. 93)

According to Vygotsky, humans are not autonomous and always constrained by outside context; on the contrary, human mental functioning, even when carried out by individuals, is "inherently social, or sociocultural, in that it incorporates socially evolved and socially organized cultural tools" (Wertsch & Tulviste, 1992, p. 551). Not only is the content of human thinking culturally developed and socially and historically determined, but the forms of thinking are also cultural accomplishments, developed as part of thinking with particular contents (Hedegaard & Chaiklin, 2005). In other words, “the structure of human activity and cognition is the cultural mediation between organism and environment” (Hedegaard, 1995, p. 296). Therefore, to understand or explain people’s activities or mental functioning, an exploration of the surrounding situations in which those activities are situated is required. Actually, “the fundamental tenet of sociocultural theory holds that socio-cultural and mental activity are bound together in a dependent, symbolically mediated, relationship” (Lantolf & Pavlenko, 1995, p. 109).

2.3.2 Sociocultural theory and mathematics education

In recent decades, sociocultural theory has influenced or been adopted by many mathematics education researchers (*e.g.*, Abreu, 2000; Khan, 1999; Lerman, 1998, 2000; Yang & Cobb, 1995). As Lerman (2000) argued, “mathematics education can look different in different social, economic, and cultural situations” (p. 212), while Bishop noted that “mathematics education is culturally shaped” (Bishop, 2002, p. 120). Ernest (1989) indicated that cultural and social contexts affect teachers’ beliefs about mathematics, mathematics teaching and learning, which in turn were further found to affect their teaching approaches (An, 2004). As Stigler and Hiebert (1999) argued, teaching is inherently a cultural activity, and so is mathematics learning (Forman, 1996). Other researchers have asserted that teachers’ learning to teach mathematics is better understood in terms of sociocultural practices (Goos, 2005; Lerman, 2001), and some mathematics education researchers have even argued that mathematics itself should be understood “as a kind of cultural knowledge” (Bishop, 1988, p. 180) or “cultural artifact” (Lerman, 1998, p. 303), and that or “mathematical meaning is both subjective and sociocultural rather than objective” (Even, 2003, p. 39). These authors contended that mathematics is culture-bound to the extent that it cannot be said to be culture-free (Presmeg, 1988).

Historical, cultural, and social contexts influence not only mathematics teaching and learning, but also other broader educational issues related to mathematics education, including the education system, the structure and aims of education, curriculum goals, textbooks and teacher education, as well as mathematics teaching/learning and assessment (Wong *et al.*, 2001). From a sociocultural theoretical perspective, these sociocultural and contextual influences presuppose that people's conception of expert mathematics teacher in a given society is mediated by its sociocultural context, and that social and cultural influences manifest in individual teacher's or school's practices. The manifestation of these influences at the school level were clearly discussed by Hedegaard and Chaiklin (2005), who argued that

> *the individual aspect of practice for each school within a given nation has typically some characteristics found in all schools, reflecting their interpretation of general practices that have developed within the school system, often developed and reinforced through teacher training institutions, further education, professional magazines, and so forth.* (p. 39)

Thus, there is a great possibility that characteristics found in one expert mathematics teacher will be shared by other expert mathematics teachers in the same context or society.

2.3.3 A framework for this study

Based on what has been discussed above, it is reasonable to assume that: (a) people's minds are mediated by their social and cultural contexts, and that this mediation will be apparent in their conceptualization of expert mathematics teachers; and (b) teachers' teaching practices are influenced by the social and cultural contexts in which they work. To investigate these conjectures, it is necessary for the researcher to gather, analyze and interpret data focusing on social and cultural contextual influences. As to the interpretation of social-cultural context influences on mathematics teaching and learning, numerous researchers have advanced various frameworks and methods. For example, Abreu (2000) discussed macro and micro sociocultural contexts of the mathematics classroom, while Wong *et al.* (2001) proposed a sociocultural situated model of mathematics education and Cobb and Yackel (1996) used an interpretive framework from a sociocultural perspective. Bishop (1988) suggested a comprehensive model when discussing the social dimension of mathematics education. Based on these, a framework for this study was developed, as shown in Figure 2.1.

The macro-context in Figure 2.1 refers to non-immediate interactional settings (Abreu, 2000), particularly the broad social and cultural context factors present in mainland China, and is divided into two levels – macro-context one and macro-context two. Macro-context one refers to the cultural background in mainland China. In this study, culture mainly refers to ideologies or values held by a group in a certain society (White, 1959). From this point of view, traditional beliefs about education in general and mathematics education in particular are considered as cultural factors. Macro-context two refers to the social context – factors such as China's education structure and aims, teacher education system, curriculum system, and national teacher promotion system. Micro-context one refers to the school context, such as school-based induction teacher education and in-service teacher education, teacher appraisal policy, teaching and learning assessment, workload, and textbooks. Micro-context two refers to the classroom context. Factors such as class size and pupils' characteristics are considered at this level. Even though the contexts are divided into different levels in this study, this does not mean that they influence people's conception or teacher's practices separately. On the contrary, they interact dynamically and act spontaneously.

As shown in Figure 2.1, another research focus of this study is the characteristics (mostly instructional characteristics) of expert mathematics teachers. Exploring mathematics teacher's instruction practice within one culture or across different cultures has been of increasing interest recently in the field of mathematics education research (*e.g.*, TIMSS-R, LPS). In order to understand mathematics teachers' practice as deeply and fully as possible, many researchers have developed frameworks to observe and analyze teachers' practice. The Survey of Mathematics and Science Opportunities (SMSO), a project responsible for developing instruments for the Third International Mathematics and Science Study (TIMSS), discussed a series of frameworks about curriculum and curriculum implementation (Schmidt *et al.*, 1996) and developed a model of how instruction is organized was developed (see Figure 2.2).

The model in Figure 2.2 shows the relationship between teachers' beliefs, goals, and instructional practices. Other than teachers' knowledge of the curriculum guide, teacher knowledge, such as knowledge of mathematics, pedagogy, and learner, is not included in this model. However, according to Schoenfeld (1998), what a teacher might do in any situation is "fundamentally shaped by" (p. 15) the teacher's knowledge base, including their knowledge of students, context, content,

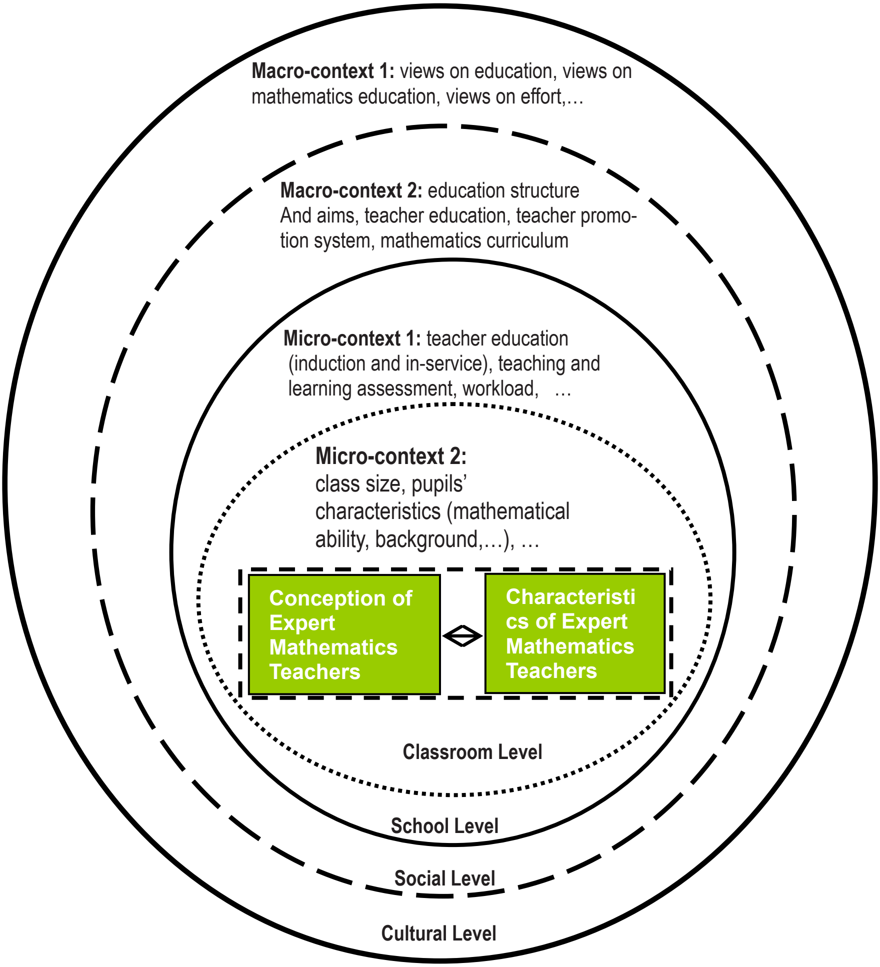


Figure 2. 1. The framework of the study

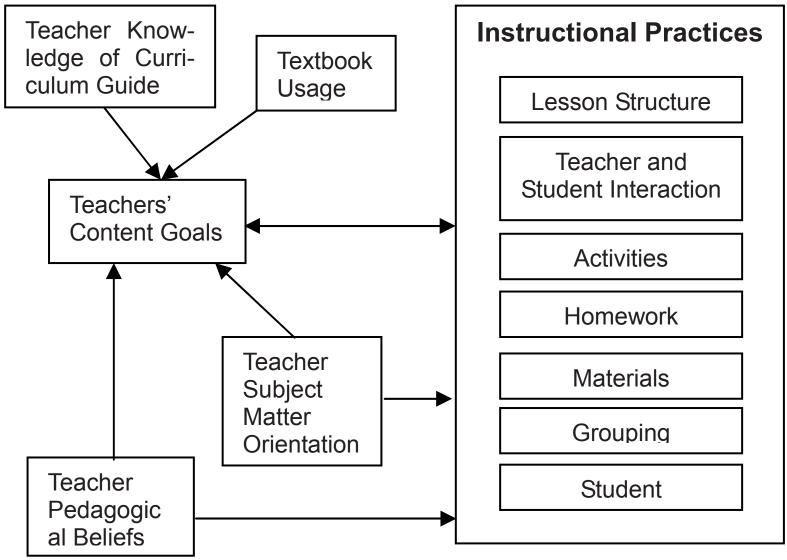


Figure 2. 2. Conceptual framework: Implemented curriculum (adopted from Schmidt *et al.*, 1996, p. 22)

and other factors; thus, to fully understand a teacher's teaching behavior, it is necessary to investigate other kinds of teacher knowledge. In addition to knowledge, Schoenfeld (2000) stated teachers' beliefs and goals, whether consciously held or not, were key factors influencing teachers' decision-making and actions. Researchers can use observation, pre- and post-observation interviews, and review artifacts, such as lesson plans and journals, to attribute a set of beliefs, goals and knowledge to a given teacher.

Moreover, as stated by Clark and Peterson (1986), teaching behavior is "substantially influenced and even determined by teachers' thought processes" (p. 255). When exploring teachers' teaching practice, it is necessary to consider teachers' thoughts behind certain behaviors. Recently, much research (Artzt & Armour-Thomas, 1998, 1999; Brown & Baird, 1993; Shulman, 1986) has moved from simply identifying what teachers do in the classroom, to investigating why and how teachers do what they do. That is, researchers have broadened their inquiries beyond merely examining teacher behavior to study teacher cognition as well.

Researchers have also tried to develop frameworks from a cognitive perspective. Artzt and Armour-Thomas (1998) examined the metacognition underlying instructional practices in mathematics using a “teaching as problem solving” perspective. They thought that teachers’ knowledge, beliefs, and goals as overarching metacognitive components directly influence teacher thinking across three stages of teaching (preactive, interactive and postactive) indentified by Jackson (1968). They developed a “Teacher Metacognitive Framework” (see Figure 2.3) to examine teachers’ mental activities associated with instructional practice.

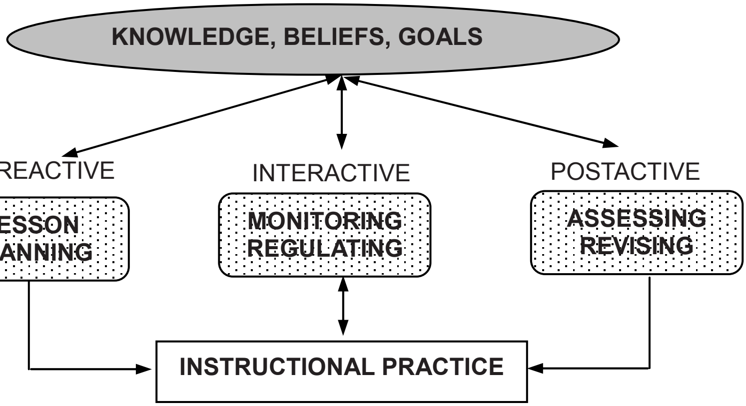


Figure 2.3. Components of metacognition (adopted from Artzt & Armour-Thomas, 1998, p. 8)

As discussed above, examining teachers’ thinking has become a main part of studying teachers’ teaching practice. This study, when exploring the characteristics expert mathematics teachers’ instructional practice, will accordingly investigate the thoughts underling the teachers’ actions to fully understand their instructional practice. To that end, this study adopts the definition of teachers’ practice proposed by Simon and Tzur (1999) to explore the characteristics of expert mathematics teachers’ teaching practice:

> *Teachers’ practice indicates not only everything teachers do that contributes to their teaching (planning, assessing, interacting with*

> *students) but also everything teachers think about, know, and believe what they do. In addition, teachers' intuitions, skills, values, and feelings about what they do are part of their practice.* (pp. 253-254)

Based on this definition and the models reviewed above, the following framework (Figure 2.4) has been developed to guide this study's literature review, research instruments development, data analysis, and interpretation.

According to Figure 2.4, teachers' knowledge, beliefs and goals influence each other and are main factors influencing the ways in which teachers deal with textbooks. These four aspects further influence teachers' instructional practice, which is divided into pre-active, inter-active and post-active stages, based on Jackson's (1968) conceptual distinctions of stages of teaching; teaching practice in turn influences teachers' knowledge, beliefs, goals, and textbook use.

2.4 Literature Review

In the previous two sections, the theoretical underpinning and theoretical perspective of this study were discussed, and a research framework for this study was developed. This section reviews literature directly relevant to this study from two dimensions: 1) studies on expert teachers; and 2) studies on the characteristics of expert mathematics teachers.

2.4.1 Studies on expert teacher

This sub-section focuses on four main topics: 1) a developmental perspective on pedagogical expertise; 2) the notion of expert teachers; 3) selecting criteria for expert teachers; and 4) characteristics of expert teachers.

2.4.1.1 A developmental perspective of pedagogical expertise

Over the past years, several models or theories of teacher development have been posited such as the development of teacher concerns (Fuller, 1969; Fuller & Brown, 1975), the developmental stages of preschool teachers (Katz,

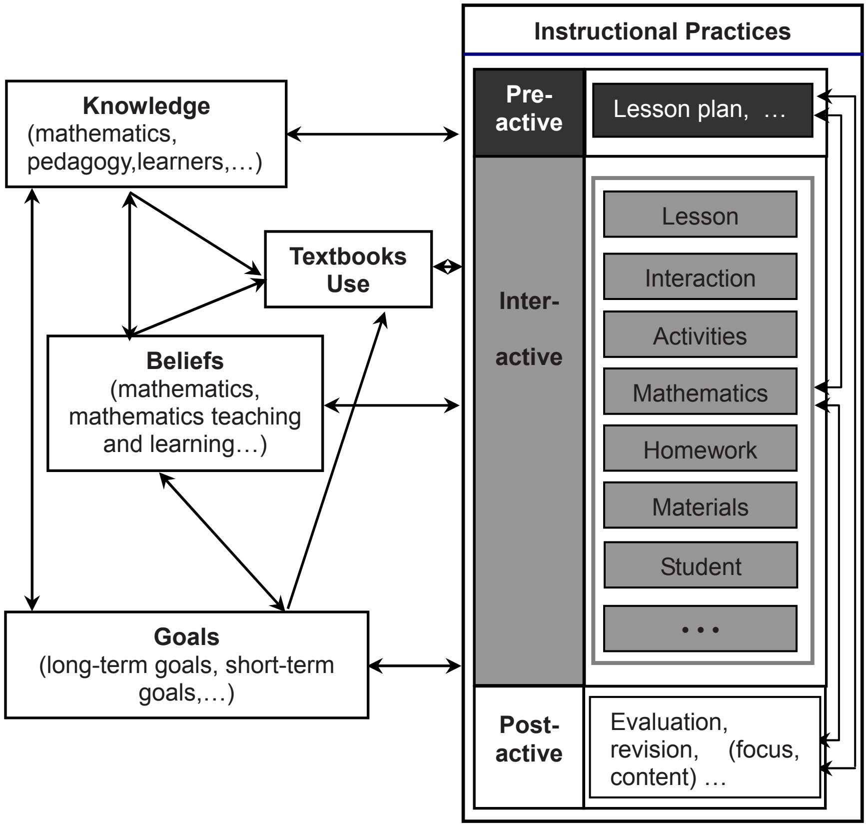


Figure 2.4. The research framework for teaching practice of the study

1972), and teachers' professional life cycle (Huberman, 1993). Borrowing from Dreyfus and Dreyfus's (1986) skill development model, Berliner (1988) developed a five stage model of pedagogical expertise development, which includes novice, advanced beginner, competent, proficient and expert.

Stage 1 Novice. In Berliner's theory, all teachers start their careers as novices, and student teachers and many first-year teachers fall into this category. Novice teachers are usually considered to be more rational and less flexible in their behavior. They are also more likely to follow existing rules and procedures. Only minimal skill at teaching tasks should be expected of a novice. At this stage, the novice teachers learn the objective facts and features of situations and begin to gain experience through real-world experience, which is, as attested to by generations of student teachers, more important than verbal information.

Stage 2 Advanced beginner. Teachers with two to three years of teaching experience belong to the advanced beginner stage. Advanced beginner teachers' experience is melded with verbal knowledge, where episodic and case knowledge is built up. It is at this stage that practical knowledge starts to accumulate and is gradually integrated with book knowledge, and conditional and strategic knowledge is built up. However, because of a lack of case knowledge, sometimes teachers at the advanced beginner level may have difficulty knowing what to do when they encounter some student challenges, and still have no sense of what is important.

Stage 3 Competent. Without more experience and motivation to succeed, not every advanced beginner teacher will become a competent teacher. Some teachers remain "fixed" at a less-than-competent teacher performance level. Many third-, fourth- and fifth-year, as well as more experienced teachers, may belong to this stage. Teachers at this stage share two characteristics. Firstly, they consciously choose what they are going to do, can make rational goals for their teaching and choose sensible means to reach their ends. Secondly, they can determine what is and is not important, make decisions about what to attend to, and stop making timing and targeting errors. They learn to make sensible curriculum and instruction decisions based on particular teaching contexts and student characteristics. Comparatively speaking, they are more responsible for what happens than beginner and novice teachers, more personally in control of events around them and more likely to follow plans. However, they are still not very fast, fluid, or flexible in their behaviors.

Stage 4 Proficient. Generally, after approximately five years of teaching, a small number of competent teachers can move into the proficient stage. At this stage, teacher intuition or know-how becomes prominent. They have accumulated rich teaching experience after long teaching practice, can recognize similarities among teaching events they encounter, and compared with novice teachers, can predict classroom events more precisely. However, proficient teachers still tend to make decisions on what to do analytically and deliberatively, despite having intuitive knowledge of pattern recognition and ways of knowing.

Stage 5 Expert. Only a very small number of proficient teachers move on to this stage. Expert teachers are categorized as arational, in that they grasp situations intuitively and respond in seemly non-analytic and non-deliberative ways, without consciously choosing what to attend to or what to do. They deal with teaching problems fluidly and effortlessly. Generally speaking, when things are going smoothly, expert teachers rarely appear reflective about their performance; that is, they are not solving problems or making decisions in the usual sense of those terms. However, if anomalies arise, deliberate analytic processes come to the foreground.

Berliner’s (1988) model describes in detail the teaching expertise development process and characteristics of each of the above stages. This model provides an initial step towards understanding the process of pedagogical expertise development and offers a new perspective on the development of teachers’ pedagogical thoughts and actions across their careers.

After surveying some 3,000 novice, proficient and expert teachers’ cognition, personality, motivation and professional psychology in mainland China, Lian (2008) developed a model of teaching expertise development from a psychological perspective. In this model, a teacher was described as going though the following stages during her/his teaching career:

Stage 1 Novice teacher. This stage is further divided into two sub-stages. Teachers at sub-stage one have one or two years of teaching experience, strong external motivation for success and are still in the process of learning basic knowledge of teaching; teachers at sub-stage two have three to five years of teaching experience, grasp basic teaching skills and realize the complexity of teaching and the difficulty of becoming an excellent teacher.

Stage 2 Proficient teacher. After three to five years of teaching experience, most novice teachers will smoothly move into this stage, which includes three sub-stages. The first of these is “competent” proficient teacher, which a small number of teachers become after three years of teaching; most need around five years of teaching practice. Teachers at this sub-stage have basic teaching ability, are able to arrange their own teaching activities reasonably, and are confident in their own teaching ability. The second sub-stage is that of the "problematic" proficient teacher. Teachers at this stage generally have seven to ten years of teaching experience and have some negative attitude to their teaching; they are not satisfied in their teaching, and are confused about their workload, salary, and responsibilities. However, most can successfully pass through this stage on their own effort or with external help. The final sub-stage is the "stable" proficient teacher. After ten years of teaching, most teachers will reach a relatively stable stage, and exhibit strong teaching ability. Some will continue their development and become excellent teachers, while others will end their teaching career at this stage.

Stage 3 Expert teacher. This stage includes two sub-stages. The first, "creative" expert teacher, is generally reached by a small number of teachers after ten to fifteen years of teaching. Teachers at this stage possess rich and well organized knowledge, and are efficient and insightful when solving teaching problems. Teachers at the second sub-stage, "leader" expert teacher, generally have at least fifteen to twenty years of teaching experience and are able to influence teaching reform and development in their school or district; they become "leaders" in their teaching subject and in the district where s/he is working.

The two models were developed in different cultures, and show obvious differences, including the number of stages, the years of teaching experience needed to achieve a certain stage, and the characteristics of each stage. However, both view teaching expertise as a developmental process, which has been supported and accepted by many other researchers (*e.g.*, Borko & Livingston, 1989; Leinhardt & Greeno, 1986; Bullough & Baughman, 1995, 1997; Tsui, 2003, 2009). However, development from novice teacher to expert teacher is not necessarily a linear process (Berliner, 1994). Both models suggest that teachers generally develop at different rates and do not necessarily become a proficient, let alone expert, teachers.

In addition, as Bereiter and Scardamalia (1993) argued, "expertise is more a process than a state" (p. 461). Teaching expertise also cannot be viewed as a stable state. Bullough and Baughman (1995) stated, "having once shown expertise in teaching does not mean that one will continue to demonstrate expertise, especially in a new setting" (p. 474). Some researchers did find that it is difficult for an expert teacher to demonstrate expertise after moving to a new setting. Bullough and

Baughman (1995) found that a highly accomplished teacher was much less adept after switching from one school to another with remarkably different contexts. Similarly, Cowley (1996) also found that, after an expert mathematics teacher moved to a new school, his expertise was "mitigated by the context of the new school, particularly the culture of the students" (p. 15), and curriculum and practical knowledge and expertise in pedagogy and management were lessened. These examples indicate that, even when a teacher is viewed as expert, s/he has to continually work hard or extend their boundaries to maintain their expertise (Bereiter & Scardamalia, 1993; Bullough & Baughman, 1995).

2.4.1.2 Notion of expert teacher

In Berliner's (1988) and Lian's (2008) model, teachers at the final stage were deemed expert teachers. The research on expert teachers or teaching expertise has its root in research on expertise in other domains, such as chess (de Groot, 1965), physics (Chi *et al.*, 1981) and computer programming (Adelson, 1981). However, teaching is more complex than the activities in those other domains, whose problems are specific and isolated from the social and cultural context, and "expert teachers have to be performers in problems situated in socially and culturally complex contexts" (Ropo, 2004, p. 162).

Due to the complexity of the working context, and to factors contributing to classroom instruction that go beyond cognition, it is more difficult to identify expert teachers than expert chess players or physicists (Berliner, 2007; Li & Kaiser, 2011). According to Bond *et al.* (2000), teaching expertise is difficult to define operationally and assessed. Thus, not only the kind of teachers who can be called expert teachers needs investigation, but also the nature of teacher expertise in different cultural contexts and education systems (Li & Kaiser, 2011).

Even so, some researchers still tried to define this in previous studies. For example, Weinert *et al.* (1992) stated that "pedagogic expertise is not a uniform, homogeneous, and coherent class of knowledge" (p. 251), but comprises four expertise sub-domains that are independent of each other and acquired independently: subject matter expertise; classroom management expertise; instructional expertise; and, diagnostic expertise. In this model, teacher competence is determined by their combined use of knowledge within these four sub-domains, and the term “expert teacher” should refer “only to those teachers who can draw on a rich knowledge base in all four sub-domains” (p. 252).

Other researchers have argued that expert teachers are similar in many aspects (*e.g.*, Ropo, 2004), what Sternberg and Horvath (1995) characterized as “family resemblance” (p. 9) in their prototypical model for diversity among expert teachers. In their opinion, the prototype view serves as middle ground between a definitional and *ad hoc* description of teacher expertise. In their prototypical model, expert teachers differ from novices in three basic ways: domain knowledge; efficiency of problem solving; and, insight. As to domain knowledge, the prototype expert teacher should have extensive and accessible knowledge of both the subject matter and of teaching, as well as knowledge of the political and social context in which the teaching occurs. Domain knowledge is split into three categories, namely content knowledge, pedagogical knowledge, and practical knowledge. Pedagogical knowledge is further divided into content-specific and content-non-specific knowledge. Practical knowledge is further divided into explicit and tacit knowledge.

As to the efficiency of problem solving, Sternberg and Horvath (1995) stated:

> *…the prototype expert teacher is efficient in solving problems within the domain of teaching. By virtue of his or her extensive experience, the prototype expert is able to perform many of the constituent activities of teaching rapidly and with little or no cognitive effort. This routinized skill enables the prototype expert to devote attention to high-level reasoning and problem solving in the domain of teaching. In particular, the prototype expert is planful and self-aware in approaching problems-he or she does not jump into solution attempts prematurely.* (p. 13)

Efficiency of problem solving is classified into three categories: automatization, executive control, and reinvestment of cognitive resources. Executive control is further split into planning, monitoring and evaluating. As to insight, they stated:

> *…the prototype expert teacher is insightful in solving problems within the domain of teaching. He or she is able to identify information that is promising with respect to a problem solution*

> *and is able to combine that information effectively. The prototype expert is able to reformulate his or her representation of the problem at hand through a process of noticing, mapping, and applying analogies. Through processes such as these, the expert teacher is able to arrive at solutions to problems in teaching that are both novel and appropriate.* (p. 14)

Insight is divided into three categories, namely selective encoding, selective combination, and selective comparison. The prototypical view of expert teacher offers a new approach to understanding the nature of expert teacher, and many researchers have tried to define expert teachers from this perspective. After reviewing much research on expert teachers, Cowley (1996) developed an extended prototypical model of expert teachers (see Table 2.1).

Table 2. 1 An extended prototype model of expert teacher *(adopted from Cowley, 1996, p. 12)*

Dimension	Sub-dimension	Features
Knowledge	Content Knowledge (pure and applied)	Domain specific/specialized; extensive; accessible; meaningful patterns perceived
	Curriculum Knowledge	Domain and context specific; extensive
	Pedagogical Knowledge (content specific & content non-specific)	Extensive; accessible; up-to-date; applied/realistic; problematic concepts understood; appropriate demonstrations
	Practical Knowledge (explicit & tacit)	Administrative; political; social
Skills/ Abilities	Pedagogy	Work focuses; routines established; individual differences considered; flexible; fluid; improvisational performance; learning supported; clear and coherent lesson structures; learning time maximized; mind-map of lesson (mental schemata); connections/links made; confident; opportunistic

	Management (administrative and behavioral)	Routines established; practical constraints accommodated; expectations /reputation established; task demand sensitivity; fluid
	Reflection/ Problem Solving	Informed by experience; intuitive; fast and accurate pattern recognition capabilities; bring rich and personal information to bear; efficient; insightful; novel and appropriate solutions to problem found
Personal Attributes	Attitude/Disposition	Challenges sought; confident; satisfied with career; work at boundaries; positive self-image; high standards set; atypical attended to
	Relationships (with students, colleagues, parents, administrators)	Respectful; empathic; social situations sensitivity; fair; unprejudiced; personal responsibility emphasized

Cowley's (1996) extended model provides a relatively more detailed structure of what an expert teacher should be; however, as Cowley (1996) argued, it "needs to be validated and possibly modified" (p. 6).

In order to understand the conception of "expert teacher" more deeply, Zhang (2005) separated the term into two parts: "expert" and "teacher". According to Zhang (2005), several things should be noted in defining "expert". Firstly, an expert only has expertise in a particular subject or field. Secondly, an expert is so called because s/he is compared with non-experts in the same field. Thirdly, an expert is a person with some special expertise. Ropo (2004) also pointed out a similar opinion — "a person is an expert because he or she seems to understand the requirements of the situation better and is able to fit his/her own decision, actions and interaction into the context" (p.163). However, as Bucci noted, "expert is a term with varied meanings, dependent upon the social and experiential travels of the rhetorician" (Bucci, 2003, p. 82); thus, the social and cultural context should be considered when describing its meaning.

Further, adapting information from Sternberg and Horvath's (1995) prototype model of expert teacher, Zhang (2005) asserted that:

> *expert teachers could be described as the teachers with comprehendsive and systematic professional knowledge, sensitivity to students' needs, and ability to efficiently solve problems in a certain teaching domain.* (p. 482)

Zhang further explained that expert teachers, in practice, do not share an absolute unified model; expert teachers of different ages, characters, interests and capabilities might well have very different teaching styles, methods and attitudes. Therefore, "expert teacher" is a categorical concept, and it is possible that expert teachers will demonstrate different characteristics at different times and in different places. In addition, as Zhang (2005) pointed out, teaching expertise is relative to a certain teaching domain; the concept of expert teachers is different in different subjects and even in different grades (Berliner, 2004; Sternberg & Horvath, 1995). Borko *et al.* (1992), for example, found that, compared with expert mathematics teachers in their study, "expert science teachers employed a *greater variety* of instructional strategies" (p. 67, emphasis in original).

2.4.1.3 Criteria for selecting expert teachers

In previous studies on expert teachers, different researchers adopted different ways to choose participants. Carter *et al.* (1988) chose teachers nominated by school superintendents and/or principals and who had more than five years of teaching experience as expert teachers. In Allen *et al.*'s (1997) study, expert teachers were those teachers who cooperated with a local university, had been recommended by their principals as effective teachers with excellent teaching skills, and who had a minimum of 10 years teaching experience. Smith and Strahan (2004) chose those who had achieved National Board certification as expert teachers.

In addition to these selection criteria, other relatively complicated criteria were adopted in several other studies. Leinhardt (1986) chose mathematics expert teachers by: 1) tracing students' "growth" scores on standardized tests over a five-year period and picking those teachers whose students' had been in top the 15 percent over the previous three years; then 2) asking principals and supervisors to review that list and suggest outstanding teachers. In Livingston and Borko's (1990) study, expert teachers were deemed to be: experienced cooperating teachers who had been identified by their school principal as experts on the basis of teaching performance and student achievement, and who had been likewise recommended as such by the county teacher center coordinator (also a university faculty member). The expert teachers in Moallem's (1998) study were experienced master teachers who had: 1) an undergraduate degree in their subject, and a graduate degree in either education or their subject; 2) no record of serious classroom management or discipline problems; 3) at least seven years of classroom experience, including three years or more in the subject s/he now taught; 4) a good reputation among colleagues and students; 5) knowledge about curriculum and organization; 6) excellent regard from her/his principal; and 7) great competency during classroom observations.

Similar selection criteria were used by researchers outside of Western countries to identify putative expert teachers. For example, in Lin's (1999) study, conducted in Taiwan, the three expert teachers selected had an average of 12.6 years teaching experience and had been recommended by teacher education program professors who had observed their teaching. Furthermore, these teachers served either as senior teaching consultants in their school district or chairs of mathematics teacher committee in local schools, and all had, at some point, mentored student teachers. In mainland China, researchers also adapted similar criteria to choose expert teachers (*e.g.*, Li *et al.*, 2005). However, due to mainland China's different working culture, teachers have opportunities to observe other teachers' classroom teaching; as such, colleagues' recommendations were also taken into account in some research (*e.g.*, Han, 2005; Li *et al.*, 2005). Some researchers in mainland China chose expert teachers based mainly on the teacher's rank and teaching experience. For example, Lian (2004, 2008) chose teachers at senior or special level with more than fifteen years of teaching experience as expert teachers. Some other researchers (*e.g.*, Gu & Wang, 2006; Li, Huang & Yang, 2011; Li & Huang, 2008) mainly chose senior level or special rank teachers with at least ten years of teaching experience.

Even though specific participant selection criteria may vary between studies, the following similarities can be identified: 1) teaching experience; 2) student achievement; 3) social recognition and reputation; 4) principal's nomination; and 5) professional or social group membership. Among them, principals' opinions were particularly stressed. However, it is also obvious that "little attention has been given to the consistency of the selection criteria used to identify 'expert teachers' across studies" (Palmer *et al.*, 2005, p. 13). Some criteria are not reasonable to a certain degree. Firstly, teaching experience and teaching expertise are not synonymous, even though it is difficult to distinguish between the two (Berliner *et al.*, 1988). It is clearly pointed out in the models proposed by Berliner (1988) and Lian (2008) (reviewed above) that some teachers will retire as competent or proficient teachers, the growth of their teaching expertise stalled despite their continuing to accumulate teaching experience. Secondly, student achievement is influenced by many factors other than teacher expertise (Berliner, 2004), and the degree a teacher, even an expert teacher, influences student achievement is unknown (Ropo, 2004).

2.4.1.4 Characteristics of expert teacher

Although it has been noted that it is difficult to define and assess teaching expertise (Bond *et al.*, 2000), since 1980s there has been, without much theoretical guidance (Berliner, 2004), increasing research interest in exploring expert teachers' characteristics, many of which have been identified based on comparing expert and novice teachers, and may have been mentioned in the pedagogy expertise development model or the prototypical models. This section systematically reviews characteristics attributed to expert teachers, both in general and across different subjects.

Rollett (1992) once summarized the characteristics of expert teachers by stating that they: 1) have a large repertoire of strategies and skills which they can call on automatically; 2) possess a distinct and original way of approaching teaching tasks and get to the core of problems rather than pay attention to peripheral features; 3) analyze situations for longer and in greater depth before planning actions than do novice teachers; 4) automatically employ knowledge and routines without realizing why they prefer a certain plan of action, can reconstruct their reasons when asked to do so; and 5) care about their students. The summary emphasizes some characteristics of expertise that exist outside of the field of teaching, such as automation, effortlessness, wealth of knowledge, and insight (Bereiter & Scardamalia, 1993), as well as the teachers' attitude towards their work. Similar characteristics have also been identified by other researchers. Based on his own work and that of his colleagues, Berliner (2001) outlined several propositions about expert teachers:

> *expert teachers excel mainly in their own domain and in a particular context;*
> *expert teachers often develop automaticity and reutilization for the repetitive operations that are needed to accomplish their goals;*
> *expert teachers are more sensitive to the task demands and social situation when solving pedagogical problems;*
> *expert teachers are more opportunistic and flexible in their teaching than are novices;*
> *expert teachers represent problems in qualitatively different ways than do novices;*
> *expert teachers have faster and more accurate pattern-recognition capabilities;*
> *expert teachers perceive more meaningful patterns in the domain in which they are experienced; and*
> *expert teacher may begin to solve problems slower, they bring richer and more personal sources of information to bear on the problem that they are trying to solve*. (p. 472)

Berliner (2001) stated that all of these propositions are supported by one or more research program; that is, all of these characteristics have been identified in expert teachers in practice.

In an effort to investigate the effectiveness of a teacher certification system, Bond *et al.* (2000) specified expert teachers' classroom teaching as comprising 13 prototypical characteristics, and invented unique measures to assess each. The prototypical features are (summarized by Berliner, 2001):

> *better use of knowledge; extensive pedagogical content knowledge, including deep representations of subject matter knowledge; better problem solving strategies; better adaptation and modification of goals for diverse learners, better skills for*

> *improvisation; better decision making; more challenging objectives; better classroom climate; better perception of classroom events; better ability to read the cues from students; greater sensitivity to context; better monitoring of learning and providing feedback to students; more frequent testing of hypotheses, greater respect for students and display of more passion for teaching.* (pp. 469-470)

At the same time, the following characteristics of student outcomes under instruction by expert teachers were also hypothesized (summarized by Berliner, 2001):

> *higher motivation to learn and higher feelings of self-efficacy; deeper, rather than surface understanding of the subject matter; and higher levels of achievement.* (p. 470)

Their investigation found that expert teachers, referred to in the study as board certified teachers, excelled at each of the prototypical characteristics, with statistically significant differences in 11 of the 13 features compared to non-certified teachers. In terms of student outcomes, 74% of the students of expert teachers were found to demonstrate higher understanding through more relational and more abstract student work, compared to only 29% of students of non-expert teachers.

Similarly, based on findings in previous studies, Ropo (2004) listed six propositions regarding expert teachers: 1) expertise develops in a narrow filed of knowledge and the knowledge base is bound to a specific context; 2) experts react to frequently recurring situations automatically; 3) compared to novice teachers, experts are more sensitive to individual students in both class situations and task situations; 4) compared to novice teachers, expert teachers are faster and more accurate in their observations; 5) expert teachers take longer to represent a problem, but can end up with a better representation of the problem; and 6) compared with novice teachers, expert teachers' knowledge is wider concerning levels of abstraction, and more hierarchical.

The characteristics summarized by these researchers suggest some common features of expert teachers, such as automaticity, flexibility, insight, sensitivity, wide knowledge base, and deep interpretation of problems. Some of these characteristics were discovered under experimental conditions, while some others were observed in expert teachers in natural teaching settings. For example, adopting a prototype view of teaching expertise, Smith and Strahan (2004) recently found that the three expert teachers in their study: 1) have confidence in themselves and in their profession; 2) mention their classrooms as learners' communities; 3) emphasize the importance of developing relationships with students; 4) demonstrate a student-centered teaching approach; 5) contribute to the teaching profession by leadership and service; and 6) have mastered the content in their teaching field. However, it must be pointed out that the participants in this study taught different subjects, and that the researchers only observed one lesson from each teacher; as such, the data and findings may not be comprehensive enough to explain teaching expertise. What is more, limited attention was paid to sociocultural influences, even though teaching expertise has been described as highly contextualized (Berliner, 2001; Berliner & Carter, 1989). Particularly, as Berliner (2001) argued, "one might be considered an expert teacher in one culture, say one like the United States that values student participation in the teaching-learning process. But the teacher would be considered terrible in another culture, one that purposely limits student participation, like India" (p. 467). This suggests that being an expert teacher or having teaching expertise might be culturally dependent, and that, to study expert teacher or teaching expertise, the social and cultural context within which the expertise is developed should be highly stressed.

In practice, researchers have found that the social and cultural context exerts an important influence on expert teachers' performance and teaching expertise. For instance, although automaticity is identified above as a very important characteristic of expert teachers, Tsui (2009) found that describing experts' work as automatic and effortless does not tally with the way the expert English teacher planned lessons in her study, conducted in Hong Kong. Tsui's expert teacher would not "treat lesson planning as something which was routinized and unproblematic" (p. 432), and would spend long hours planning and rehearsing lessons before teaching. Similarly, differences found between expert, beginning and novice teacher in their thinking about planning and in their curricular decision-making in Western cultures (for example, in Livingston and Borko's study (1989) and in Leinhardt and Greeno's (1986) study) were not found in Lin's (1999) study, conducted in Taiwan. The main reason for this is that Taiwan has adopted a unified national curriculum, teacher guides, and textbooks and teachers have no need to plan lessons on their own.

2.4.1.5 A brief summary and implications for the current study

This sub-section reviewed models of the development process for pedagogical expertise, conceptualizations of expert teachers, criteria for selecting expert teachers, and characteristics of expert teachers. Previous studies have discovered some characteristics about expert teachers or teaching expertise and generated a basic construct for expert teachers. However, as mentioned above, in studies on expert teachers “little attention has been given to the consistency of the selection criteria used to identify ‘expert teachers’ across studies” (Palmer *et al.*, 2005, p. 13). This indicates that the concept of expert teacher is still unknown. In some studies, participants were chosen based on the recommendation of principals and colleagues (an approach typically used by Chinese researchers), but the criteria the recommending parties used were not examined.

Moreover, as found in other studies, the social and cultural context and working conditions “exert a powerful influence on the development of teaching expertise” (Berliner, 2001, p. 463). In addition, teaching expertise takes different forms in different subjects and at different grade levels (Berliner, 2004; Sternberg & Horvath, 1995). Therefore, it is meaningful to explore what it means to be an expert teacher in a particular subject, at a particular grade level, and in a specific cultural context.

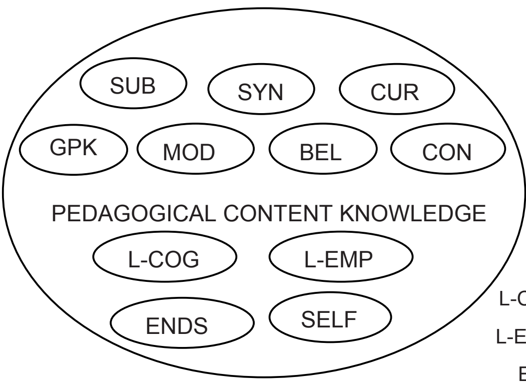
2.4.2 Studies on expert mathematics teacher

This sub-section reviews studies on expert mathematics teachers; however, whenever appropriate, information related to expert teachers or teaching expertise in other subjects is reviewed to allow for a more comprehensive picture of expert (mathematics) teachers.

2.4.2.1 Knowledge of expert mathematics teacher

From studies on expert performance and problem solving in various domains outside of education, psychologists have already learned that knowledge plays a central role in expert performance. In addition, differences have been identified in the organization of knowledge between novices and experts, with the former tending to be “organized around the literal objects explicitly given in a problem statement” (Glaser, 1984, pp. 98) and the latter tending to be “organized around principles and abstractions that subsume these objects” (Glaser, 1984, pp. 99). The richly structured and accessible bodies of knowledge allow individuals to engage in expert thinking and action.

This understanding of expertise also leads education researchers to devote increased attention to teachers’ knowledge and its organization; as can be seen above, in every prototypical model of expert teachers, knowledge is always a major component. Sternberg and Horvath (1995) once claimed that the fundamental difference between expert and novice teachers is that the former use more knowledge in solving problems in their own professional field. Teaching, as a highly complex activity, draws on many kinds of knowledge. A number of models of teacher knowledge have been proposed by researchers in this field (*e.g.*, Elbaz, 1983; Fennema & Franke, 1992; Grossman, 1990; Shulman, 1986), and Turner-Bisset (1999) even developed a model of expert teacher knowledge bases (see Figure 2.5). In this sub-section, characteristics of expert teachers’ knowledge will be reviewed, with a particular focus on expert mathematics teacher knowledge.



KEY TO CODES

SUB-Substantive Knowledge

SYN-Syntactic Knowledge

BEL-Beliefs about the Subject

CUR-Curriculum Knowledge

CON- Knowledge of Contexts

SELF- Knowledge of Self

MOD- Knowledge/Models of Teaching

L-COG- Knowledge of learners: Cognitive

L-EMP- Knowledge of Learners: Empirical

ENDS- Knowledge of Educational Ends

GPK-General Pedagogical Knowledge

Figure 2.5. Knowledge bases for teaching (adopted from Turner-Bisset, 1999, p. 47)

General pedagogical knowledge. General pedagogical knowledge, as a focus of most research on teaching, includes various knowledge, beliefs and skills needed for teaching (Grossman, 1990). Leinhardt and Greeno (1986), comparing elementary school mathematics lessons taught by expert and novice teachers, found that the expert teacher had efficient routines for checking homework and used hand-raising to gain attention and signal the beginning and end of various lesson segments. Leinhardt *et al.* (1987) found that, in expert teachers' teaching, around one-third of routines would be developed during the first two days in a new semester, and that later use would be modeled, such as raising hands, cycling through students until the right answer was found, and choral responses. Tsui (2003) also found that the expert teacher (English as a second language) in her study established routines and norms for teaching effectively and efficiently; moreover, even though the experienced and novice teachers studied adopted similar routines, there were qualitative differences in how they used them. This indicates that, even though non-expert teachers may have relevant knowledge of routines, they may not be able to use them as properly and effectively in practice as expert teachers do. This further suggests that the expert teacher's knowledge is accessible (Berliner, 2004).

The proficiency and automaticity found in experts in other fields was also found in experts in the field of mathematics teaching. Expert mathematics teachers were found to control their teaching rarely or not at all if everything went smoothly (Leinhardt, 1986), and were further found to be very skillful at keeping their teaching on track and accomplishing their objectives (*ibid*). Moreover, during their teaching, their students were allowed to raise questions and make comments, which the expert teachers were then able to use to further the discussion (Borko & Livingston, 1989). Ropo (1987, as cited in Ropo, 2004) also found that, compared to novice teachers, expert teachers were better able to take students' answers into account and adjust their teaching accordingly. Similarly, Berliner and Carter (1989) found that expert teachers tend to emphasize information that has instructional significance. This might suggest, as Ropo (2004) hypothesized, that expert teachers have a more-developed pedagogical knowledge structure and, more important, can access it without deliberate control.

Expert teachers' beliefs about teaching or about their role as a teacher were also found to be different from those of non-expert teachers.

For example, Li *et al.* (2005) found that expert mathematics teachers in mainland China tend to hold a problem-solving view about mathematics and mathematics learning, whereas non-expert mathematics teachers hold a knowledge-mastering view. In Tsui's (2003) study, it was found that, compared with non-expert teachers, the expert teacher studied held different perceptions about teaching and the roles of a teacher; the expert teacher approached teaching with the belief that a teacher should exercise authority, yet should be kind and caring towards students. However, experienced teachers who were not considered expert teachers approached teaching with the belief that teacher should be "knowledgeable, 'qualified', academically competent, and able to help students academically" (p. 116) and effectively impart knowledge. The novice teacher surveyed thought that a teacher should be very strict and stern, and knowledgeable in her/his subject.

As reviewed above, compared with non-expert teachers, expert teachers were found to have richer pedagogical knowledge and hold different beliefs about mathematics and teaching. However, several other studies reported some contradictory findings. For example, through a questionnaire survey, Yu (1999) found that the general pedagogical knowledge of novice teachers was higher than that of the expert teachers surveyed. In Shen and Li's (2001) study, pedagogical and psychological knowledge were also found to decline as teachers gained more teaching experience. According to these authors, this is because teachers with more than twenty years of teaching experience in mainland China have received little education on pedagogical and psychological theories in the decades and after their graduation, they have limited opportunities to receive in-service education. Moreover, experienced teachers have developed their own teaching routines and would not like to change them. Similarly, Borko and Putnam (1996) also claimed that what experienced teachers already know and believe about teaching, learning, and learners highly influences their willingness to learn new ways. However, the degree to which the questionnaires are valid in Yu's (1999) study and Shen and Li's (2001) study is open to question. Due to the complexity of teaching, teachers' general pedagogical knowledge cannot be fully examined through questionnaire survey or examined by a limited number of questions.

Pedagogical content knowledge. Pedagogical content knowledge in mathematics instruction is domain-specific teaching knowledge that integrates mathematics content and pedagogy (Shulman, 1986). Previous studies found that there exist differences between expert and novice teachers' pedagogical content knowledge. Gudmundsdottir and Shulman (1989, p. 33) once summarized that "the most dramatic differences between the novice and the expert are that expert has pedagogical content knowledge that enables him to see the larger picture in several ways and he has the flexibility to select a teaching method that does justice to the topic".

Gudmundsdottir and Shulman (1989) found that the expert teacher in their study has multiple, broad ideas on ways in which the subject matter can be organized and segmented, and could visualize larger and larger curriculum units due to his developed pedagogical content knowledge, unlike the novice teacher, who was only able to formulate short-term plans due to limited pedagogical content knowledge. The expert teacher was found able to select the best teaching strategy for a topic, while the novice had to expend a great deal of effort to find and use an appropriate teaching strategy.

Adopting the concept of "schema", Livingston and Borko (1990) compared review lessons taught by expert and novice mathematics teachers at the senior secondary school level. They found that expert teachers 1) were able to take cues from student questions to carry on their teaching and simultaneously cover the essential concepts and relationships of the lesson; 2) pointed out common errors and potential pitfalls to students; 3) stressed concepts and procedures with proper responses, questions and diagrams; and 4) organized reviews that generalized across separate problems to stress common concepts, strategies and relationships. Their study illustrates that expert teachers know well the teaching content, like its essential parts, and understand their students' difficulties. Moreover, they know how to promote properly students' conceptual and procedural understanding and connect it to their teaching content. The findings further indicate that expert mathematics teachers' pedagogical content knowledge is elaborated and accessible. Ropo (1987, as cited in Ropo, 2004) also investigated three expert mathematics teachers and four novice teachers at the secondary school level and found that expert mathematics teachers had the ability to make correct interpretations about different students and act accordingly in instructional interventions.

Zhu *et al.* (2007) compared the pedagogical content knowledge of one expert mathematics teacher to that of one novice teacher at the elementary school level in mainland China. They found that the expert teacher knew students' prior learning experience, knew topics related to the teaching topic, and could flexibly use both in practice. However, the novice teacher did not possess similar qualities. It was also found that the expert teacher knew students' problems and difficulties well and could make appropriate preparations, while the novice teacher could not. During teaching practice, the expert teacher was found to conduct his teaching based on students' prior experience and knowledge base and emphasize students' exploration and understanding, while the novice teacher's teaching was seen as mechanical. In sum, the expert mathematics teacher was found to have more pedagogical content knowledge than the novice teacher. More recently, Li, Huang and Yang (2011) identified a central tendency among five expert mathematics teachers— appropriately identifying and dealing with difficult content points in students' learning. That is, expert mathematics teachers were able to identify important content knowledge and design instructional strategies to help students understand and master it. In the mean time, they could also anticipate students' difficult content points and design possible ways to help students to overcome these difficulties.

Subject matter knowledge. In the field of mathematics education at the primary or secondary school level, subject matter knowledge refers to "the knowledge that a teacher needs to have or use in the course of teaching a particular school-level curriculum in mathematics" (Leinhardt *et al.*, 1991, p. 88). That is, subject matter knowledge of mathematics at this level does not imply knowledge of advanced mathematical concepts; thus, taking advanced courses in mathematics will not, by itself, make a teacher a better mathematics teacher; s/he will improve, however, if s/he can deepen her/his understanding of the knowledge about particular school topics (Leinhardt *et al.*, 1991).

In the field of teaching expertise, previous studies have generally shown that, compared with novice teachers, expert teachers know the subject they are teaching more deeply and intensively (*e.g.*, Smith & Strahan, 2004; Tsui, 2003, 2009). The situation is similar in mathematics education in mainland China. Some researchers (*e.g.*, Shen & Li, 2001) found that teachers in mainland China tend to demonstrate a deeper understanding of mathematics as they accumulate teaching experience.

Ma (1999) also found that teachers from mainland China who have most profound understanding of fundamental mathematics are those who have the most teaching experience; novice teachers were not found to understand mathematics as thoroughly. Li *et al.* (2005) investigated 16 expert and 16 non-expert elementary school mathematics teachers' understanding of mathematical knowledge and identified significant differences between the two groups. Compared with non-experts, the expert mathematics teachers had a profound understanding of mathematics knowledge, were able to understand multiple representtations of a certain topic, and had well-structured knowledge. More recently, Li, Huang and Yang (2011) found that the five expert mathematics teachers in their study have a sound subject content knowledge.

However, the consistent findings regarding expert mathematics teachers' or experienced teachers' profound understanding of mathematics in mainland China were not found among expert mathematics teachers in Western cultures. For example, Leinhardt and Smith (1985) explored the nature, level, and use of teachers' knowledge of fractions by four expert teachers, and found wide variations in their knowledge. Two expert mathematics teachers were found to have relatively high mathematics knowledge, one had relatively middle-level knowledge, and one had barely sufficient mathematics knowledge for classroom instructtion; all novice teachers in the study were found to have low knowledge. The findings suggest, on the one hand, that expert mathematics teachers tend to understand mathematics more deeply than novice teachers do and, on the other hand, that not all expert mathematics teachers demonstrate a deep understanding of mathematics.

The differences between studies conducted in mainland China and in other cultures might be attributed to Chinese mathematics teachers' pre-service teacher training experience (Li, Huang, & Yang, 2011) and the fact that mathematics teachers in mainland China continuously develop their understanding through the intensive study of textbooks (Ma, 1999; Yang, 2009). For example, Leung and Park (2002) repeated Ma's (1999) study and found that, even though elementary mathematics teachers in Hong Kong and Korea understand mathematics as well as their Shanghai counterparts, they lack a profound understanding of mathematics and do not organize their understanding into an explicit knowledge package.

Knowledge of learners. Knowledge of learners is a very important kind of knowledge in teaching practice. Some researchers have even asserted that, of the various types of knowledge possessed by expert teachers, knowledge of students is the most developed part of practical knowledge (Moallem, 1998). Compared with novice or non-expert teachers, expert teachers were found to have more knowledge of their students (*e.g.*, Carter *et al.*, 1987; Lin, 1999; Zhu *et al.*, 2007). Ropo (1990, as cited in Ropo, 2004) found that, compared with novice teachers, experienced teachers (those with at least 10 years of experience) seemed to know more about their students' past or current family events and made more connections between the student's family background and her/his school behavior or problems. Experienced teachers were better able to explain their students' performance level in different subjects, which further suggests that experienced teachers can effectively make practical use of their knowledge of learners. Berliner (2004) also pointed out that expert teachers: 1) know well the cognitive abilities of the students they teach regularly, and thus can determine the appropriate level of difficulty for the teaching content; and 2) know their regular students personally, so that there is no need to rely on bureaucratic and formal mechanisms of control in their own classrooms.

Previous studies also found that expert teachers tended to construct their own knowledge of students rather than believing what other teachers told them (Berliner & Carter, 1989). Possibly for this reason, findings in previous studies show that expert teachers' rich knowledge of learners is mostly limited to the students they are teaching or have taught for many years; that is, their knowledge is "often circumscribed" (Berliner, 2004, p. 16). For example, Stader *et al.* (1990) found that, when watching videotapes of instruction, expert teachers could not decide whether students they did not know were comprehending lesson materials or not. In another experimental study, expert teachers claimed to need more time to prepare their teaching, because they do not know the students they were going to teach (Carter *et al.*, 1987).

Development and scope of knowledge. The literature reviewed above suggests that, compared with novice teachers, expert teachers tend to have more elaborated, interconnected and easily accessible knowledge, and possess a wider and more comprehensive knowledge base. Lin (1999) found that expert teachers possess more knowledge than beginning and novice teachers in most domains, such as lesson content and structure, classroom context, and education system. More important, expert teachers' knowledge was further found to be broadly distributed along the 10 domains identified by Lin, whereas novice teachers' knowledge fell into only a few domains. This indicates that expert teachers' knowledge base is relatively complete and homogenously developed. Gu (2003) has developed a model (see Figure 2.6) to summarize the differences that exist in knowledge among expert, experienced and novice teachers at different development stages.

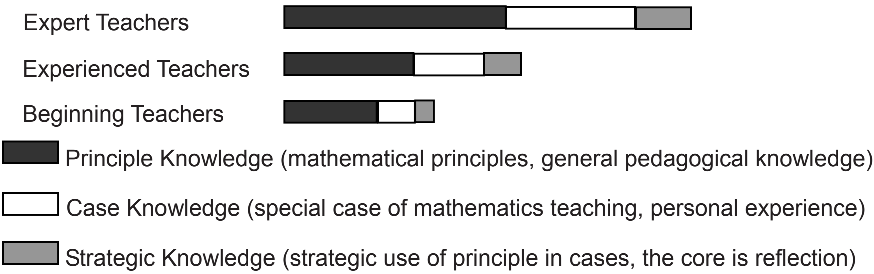


Figure 2.6. The teachers' professional growth and the changing of knowledge structure (adopted from Gu, 2003, p. 8).

Previous studies indicate that expert and novice mathematics teachers differ not only in the knowledge they possess, but also in how that knowledge is organized. Along with growth in teaching expertise, novice teachers will develop various teaching styles and knowledge. In practice, expert teachers can integrate various aspects of knowledge in relation to the teaching act (Tsui, 2009). This suggests that, as found in some previous studies (*e.g.*, Tsui, 2003, 2009), the way in which expert and novice teachers use their knowledge is different, and that the teaching practice of expert and novice teacher might also be different. In the sub-section below, characteristics of expert teachers' teaching practice will be reviewed, with a particular focus on expert mathematics teachers' teaching practice.

2.4.2.2 Characteristics of expert mathematics teachers' teaching practice

This sub-section reviews characteristics of expert mathematics teachers' teaching practice in the following three phases: pre-active, inter-active, and post-active (Jakson, 1968).

Pre-active phase. Planning is the most important part for a teacher in the pre-active phase, because s/he needs to make necessary planning to achieve lesson goals more effectively. Undoubtedly, decisions made by teachers while planning lessons have a profound influence on their classroom behavior. At the planning stage, teachers translate syllabus guidelines, institutional expectations, and their own educational beliefs and ideologies into guides for in-class action; planning also defines the structure for and the purpose of what teachers and pupils do in classroom (Calderhead, 1984). Generally, there are several kinds of planning –specifically, yearly, chapter and daily lesson planning (Yinger, 1979). In previous studies, expert teachers demonstrated the ability to plan their teaching at different levels reasonably.

Firstly, Ropo (1991, as cited in Ropo, 2004) found that experienced teachers categorize instructional goals differently than do novice teachers. Experienced teachers were able to group teaching objectives hierarchically by differentiating between school levels (*e.g.*, elementary and secondary school), grade levels (*e.g.*, grade levels 7, 8, and 9), and the generality of the goals and objectives. For example, one expert teacher in the study clearly stated that the overall goal for mathematics education is to show students the beauty of mathematics; this expert teacher could divide this overarching objective into more specific, more limited goals suitable for each grade level. Further, expert teachers were found to have individual goals and teaching objectives for particular students, suggesting that expert teachers are capable of considering students' individuality and differences. However, novice teachers typically described their instructional goals at the level of individual lessons, without having the same kind of hierarchies of objectives.

The expert (science) teacher in Moallem's (1998) study was found to use at least three types of planning: course yearly planning, unit planning, and daily planning. A variety of factors (*e.g.*, previous experience, available resources, context of school and classroom) rather than just course objective and/or leaning outcomes, affected this expert teacher's thinking during the planning process. This expert teacher was found to have well-developed mental plans in practice, including detailed images of different activities, the sequence of these activities, instructional strategies, students' probable reactions and responses, the arrangement of the classroom, materials, and possible problems.

Viewing teaching as a complex cognitive process, Leinhardt (1989) compared expert mathematics teachers' lessons with those of novices with respect to three key lesson features: agenda, role of lesson segments and nature of explanation. Agenda refers to an operational plan that includes "both the objectives or goals for segments and the actions that can be used to achieve them" (p. 55). The study found that expert teachers' agendas were far richer and more detailed than those of novice teachers. Specifically, novice teachers' agendas had no unifying logic guiding their planned instructional action, while there was at least one systematic logical statement indicating the flow of the lesson in almost all of the expert teachers' agendas. In the meantime, even though expert teachers generally did not use written lesson plans, they nonetheless had a specific overarching goal that ordered the planned actions so that the lessons would move from the broad and general procedures to the focused and narrow algorithm.

In Tsui (2009)'s study, an expert ESL teacher was further found to problematize her previous lesson plans and their enactment by considering the actual situations of her current students rather than treating lesson planning as routine and unproblematic. This teacher would spend more time planning the details of her lesson – the questions she would ask, students' possible response, examples she will give, etc. – and would then rehearse her lesson in her mind. This indicates that expert teachers treat lesson plans very seriously and that, the author argued, their work was never as automatic and effortless as that found in experts in other fields.

The studies reviewed above indicate a common characteristic of expert teachers across different subjects — the tendency to develop or plan lessons well in their mind. Borko and Livingston (1989) compared three expert mathematics teachers' planning with that of three novice mathematics teachers and found that the three expert teachers perform: 1) yearly planning, to establish the general content and curriculum sequence for the course and constructing a timeline for content coverage; and 2) chapter planning, to determine a timeline for specific topics and make relevant changes based on their previous year's plans. However, unlike the expert teacher in Tsui's (2003) study, all the expert teachers in this study made decisions about instruction details shortly before the actual instructional event. None used written lesson plans, just laid out a general sequence of lesson components and content in their mind, without including exact time planning, pacing or the number of examples or problems they would present to their students; these aspects were all determined extemporaneously, on the basis of student questions and responses.

Another similar characteristic of expert teachers' planning found by Borko and Livingston (1989) is that expert mathematics teachers seem to plan their teaching based on their previous teaching experience and their reflections thereon. In contrast, the novice mathematics teachers in their study mostly focus on short-term, individual lesson planning and rehearse instructional strategies before teaching. Another study by Livingston and Borko (1990), this one comparing two review lessons from two secondary school mathematics student teachers and their cooperating, expert teachers, found that these expert teachers also used planning materials from previous years as cues for their metal planning and had detailed mental plans although their planning.

In previous studies, expert teachers were further found to be able to make their plans richly connected. For example, Leinhardt *et al.* (1991) found that expert mathematics teachers always start their planning statements by recounting what they had done the day before, as they see individual lessons as part of connected whole. Expert mathematics teachers' agendas are "richer in detail, in connectedness, and in constraints (tests for continuing, logic for flow, and student actions)" than those of novice teachers (p. 64). Even *et al.* (1993) also examined the differences in connectedness in instruction and found that, in expert mathematics teacher's planning, each lesson segment was connected to both the previous and following lesson. Similarly, in Zhu *et al.*'s (2007) study, the expert mathematics teacher considered students' prior experience and future learning when planning his teaching.

In addition to establishing teaching goals at different levels reasonably, and planning lessons rich in details and connectedness in their minds, expert teachers were further found to plan lessons from a student perspective, rather than from their own; in other words, they think about student learning instead of their own teaching. Tsui (2009) found that when the expert ESL teacher selected materials and designed activities, she would consider what her students like to do, not what she would like them to do. To a certain degree, this might explain why, sometimes, novice teachers' plans were not accessible, while expert teachers' lesson plans were explicit and available (Leinhardt *et al.*, 1991).

Moreover, Leinhardt (1986) found that expert mathematics teachers are "good at constructing series of lessons that successfully transmit the content that needs to be learned" (p. 29). In other words, expert mathematics teachers do not include irrelevant information in their lesson plan.

Inter-active phase. As reviewed above, expert teachers could determine long term and short term goals for their teaching and maintain a largely unscripted mental model for teaching. However, in practice, planning is one thing, carrying it out is another thing. Classroom teaching is complex and relatively unpredictable, and many things might happen quickly at the same time. Researchers also explored how expert mathematics teachers implement their plans and relevant characteristics were discovered in these studies.

Leinhardt (1986) compared experts to novices in terms of what makes an expert elementary mathematics teacher expert. This study found that expert mathematics teachers' lessons are clear, accurate and rich in examples and demonstration of a particular piece of math; also, expert teachers present new material within a coherent but flexible lesson structure; their lessons take place in an academic environment that focuses on the contents that students are expected to learn.

Similar conclusions were reached by Leinhardt (1989), who compared expert and novice mathematics teachers' lessons by focusing on transition, presentation, guided practice and monitored practice. The expert mathematics teachers tended to "use well-known representations and also to use the same presentation for multiple explanations" and "use something familiar to teach something new, whereas novices often use something new to teach something new" (Leinhardt, 1989, p. 66). Compared with the novices, the expert mathematics teachers gave better explanations and were more likely to complete an explanation; moreover, their explanations contained more critical features and were less likely to contain errors. These findings suggest that expert mathematics teachers have strong ability to explain accurately and clearly, implement their teaching flexibly, connect various topics together to enrich connectedness in their teaching, and demonstrate critical features of relevant content to their students.

The ability to point out critical features of knowledge and connect similar topics together was also noted in other studies. For example, Livingston and Borko (1989) found that expert teachers' explanations highlight the characteristics of problems and core concepts, and make explicit connections across the problems employed, therefore connecting the activities to the broader scheme. In contrast, explanations by novices were largely procedural and not linked conceptually. This further suggests that expert mathematics teachers emphasize essential meaning rather than teaching relevant knowledge superficially. In Even *et al.*'s (1993) study, during expert teachers' teaching practice, it was found that "connections were found not only in the expert's development of the lesson towards the main objective (lesson connections) but also across various topics (content connections)" (p. 53).

Similarly, the ability to implement teaching flexibly was also found in other studies. Adopting an improvisational performance framework, Borko and Livingston (1989) explored the difference between expert and novice mathematics teachers' ability to improvise in practice. In their study, expert mathematics teachers were found to be very skillful at keeping their lessons on track and accomplishing their objectives, while still allowing for student questions and comments, which were used as springboards for follow-up discussions. This suggests that expert mathematics teachers were able to flexibly use student input to further their teaching, rather than following a detailed script for action.

At the same time, expert teachers could successfully generate those problems needed to illustrate or those concepts and skills needed to reinforce lessons, or quickly locate them in the text or notebooks. In Even *et al.*'s (1993) study, the expert teacher made relevant changes to her plans once she realized that her students were having difficulty. In these studies, expert mathematics teachers were found to be able to make flexible and instant decisions or changes to their lesson plans during the teaching process. In addition, they also demonstrated the ability to quickly provide examples and to connect students' comments or questions to the lesson objectives. This further suggests that expert mathematics teachers have an extensive, interconnected and easily accessible knowledge base.

Borko and Livingston (1989) further found that expert teachers balance content-centered and student-centered instruction, with minimal use of written plans or textbooks. More recently, Li, Huang and Yang (2011) found that expert mathematics teachers in mainland China appreciated and implemented student-centered instruction through various strategies, and encouraged students to participate in problem-solving activities through creating learning situations familiar to the students so as to motivate them better.

Expert mathematics teachers' lesson structures were also different from those of novice teachers. Gu and Wang (2006) found expert teachers used more time to review relevant knowledge and spent around 21 minutes creating a complicated situation within which the main teaching topic is embodied. Expert mathematics teachers were good at creating mathematical problem situations and using them to encourage students to explore mathematical formula, rather than directly imparting knowledge to students. Moreover, in comparison with novice teachers, the expert teachers spent more lesson time on deducing formulae and questions, and used fewer exercises during the teaching process. Similarly, Zhu *et al.* (2007) found that, compared with novice teachers, expert mathematics teachers tend to spend more time exploring new topics and less time on practice. In Leinhardt's (1989) study, novice teachers were generally found to spend more time on "transition" and "guided practice"; expert teachers tended to spend more time on "monitored practice".

Discourse interaction in expert mathematics teachers' classes was also different from that in non-expert mathematics teachers' classes. Li and Ni (2007) found that, compared with non-expert teachers, expert teachers asked more questions that required students to explain underlying principles and analyze the relationships among and differences between various solutions. Expert teachers also tended to emphasize students' ideas and answers in their teaching, and would explore the process of how students arrived at their answers. The classroom discourse in expert mathematics teacher's classroom generally followed the pattern of "student statement – teacher questioning – student explaining". In contrast, non-expert teachers more often asked students questions that required them to recall relevant facts or describe the process of relevant solutions, and placed less emphasize on students' ideas and answers; they would directly evaluate students' answers and then continue their teaching. The discourse in their teaching was summarized by these authors as "teacher initiation – student response – teacher evaluation".

Previous studies also identified differences between the types and number of questions employed by expert and non-expert mathematics teachers. However, the findings are not always consistent. For example, Zhang (2000) found that expert mathematics teachers asked fewer questions than novice teachers did, and would wait longer before giving correct answers. By contrast, in Guo and Song's (2008) study, the expert mathematics teacher asked 62 questions, the proficient teacher 50 questions, and the novice teacher 28 questions. Moreover, the expert teacher asked meta-cognition and open-ended questions that could facilitate students' deeper understanding of the teaching topic, while the other two teachers did not. During teaching practice, the expert teacher was better able to use problems to stimulate students' thinking and maintain their thinking activity. Zhu *et al.* (2007) found that the expert mathematics teacher in their study asked more questions that could facilitate students' understanding and encourage students' exploration. They also found that the expert teacher could organize problems into multiple levels to meet individual students' needs. Differences identified between the questions and discourse interaction in expert and non-expert mathematics teachers' teaching generally suggest that in practice, experts emphasize the development of students' mathematical thinking and ability (Li, Huang, & Yang, 2011).

Post-active phase. In some literature, expert teachers were found to have a tendency not to use the same techniques repeatedly; rather, they would reflect on their actions and try different methods (Cushing *et al.*, 1992). In Tsui's (2009) study, after her reflecting on her previous lesson, the expert teacher reframed her understanding of classroom discipline from one of maintaining order, to one of managing the classroom for teaching.

However, not much emphasis was placed on how expert teachers reflect on their teaching in these studies, probably because the distinction between the pre- and post-active phases in teachers' thinking is not as marked as that between the pre-active and interactive phases (Clarke & Peterson, 1986; Tsui, 2003). The few studies which have explored expert teachers' performance in the post-active phase have mainly focused on teachers' reflections on their teaching just finished, or reflection-on-action, as Schön (1983) proposed. Moallem (1998) found that expert teacher reflection seemed to occur in two phases: 1) reflection shortly after classroom action, during which the expert teacher considered the teaching and learning activities just completed and made immediate adjustments for future lessons; and, 2) reflection at the end of the day, week, unit or semester, in which the expert teacher more systematically examine their lessons, learnt from their experience, and redefined practical theories for future use. The content of reflection was technical and evaluative in nature. Although reflection in the second phase was not explicitly mentioned in other studies on expert teachers, as reviewed above, most expert teachers make relevant adjustments and modifications to their lessons based on their systematic reflections on their experience. To a certain degree, this also requires expert teachers to systematically reflect on their previous experience to make relevant adjustments and modifications.

Concerning reflection in the first phase, which was proposed by Moallem (1998), Borko and Livingston (1989) found that expert mathematics teachers' reflections were fairly concise and focused. Expert teachers mainly focused on students' understandings of the material or students' active role when generating problems and problems solutions. Little attention was paid to student behavior or affect, and selective attention was paid to specific classroom events; only those events the expert teachers believed affected an instructional goal were mentioned. Expert teachers seldom mentioned classroom management, and offered very little assessment of the effectiveness of their own teaching. In another study, Borko *et al.* (1992) explored the differences between an expert and a novice science teacher's teaching, and found that the expert teacher reflected more on students' understanding. In Even *et al.*'s (1993) study, compared with the novice teacher, the expert mathematics teacher considered making connections between different stages of the lessons and between different topics very important, and deliberately planned to make their teaching connected when asked to reflect thereon.

2.4.2.3 A brief summary and implications for the current study

This section has reviewed the characteristics of expert teachers' knowledge and teaching practice, with a focus on expert mathematics teachers. In the above-discussed studies, expert (mathematics) teachers were found to have rich, connected, well structured and accessible knowledge. They were also found to be able to plan their lessons in detail with rich connectedness, implement their lesson plans flexibly, point out critical features of knowledge to students, and promote students' mathematical thinking, although contradictory findings emerged from some studies. Moreover, the studies suggest that teaching is a cultural activity (*e.g.*, Stigler & Hiebert, 1999) and that teaching expertise is highly contextualized (Berliner, 2001, 2004). In addition, expertise in different subjects and at different grade levels takes different forms (Berliner, 2004; Sternberg & Horvath, 1995).

2.4.3 Summary of literature review

The literature review conducted in this section indicates that studies on expert teacher have the following main gaps:

Firstly, although there have been many studies on the characteristics of expert teachers, few have clearly conceptualized what constitutes an expert teacher (Li & Kaiser, 2011). Moreover, not many studies paid attention to the particularity of a single subject, even though it has been argued that teaching expertise "may differ as a function of subject taught" (Sternberg & Horvath, 1995, p. 15). Therefore, studies on the conception of expert teachers in a particular subject and in a particular cultural context are needed.

Secondly, many of the studies that systematically explored expert teachers' teaching were conducted in Western countries. Although there have been increased research interest in expert teachers in mainland China, previous studies have mostly focused on specific aspects, like pedagogical content knowledge, problems used in teaching, and classroom discourse. Very few studies have systematically explored the characteristics of expert teachers' beliefs, knowledge, and teaching practice as a whole. Moreover, most previous studies focused on expert teachers at the primary school level, and many were conducted with limited attention to the social and cultural background. However, teachers' working conditions exert a powerful influence on the development of their expertise (Berliner, 2004) and teaching expertise may take different forms in different grades (Sternberg & Horvath, 1995). Therefore, it is reasonable to assume that expert teachers in different cultures and at different grade levels will demonstrate different characteristics. As such, studies on expert teachers within a clearly identified cultural context and at particular grade level are needed (Kaiser & Li, 2011).

Thirdly, most of the studies compared expert and novice teachers; very few explored similarities among expert teachers, especially those working in the same subject within the same social and cultural context. Therefore, it is meaningful to explore the common characteristics of expert mathematics teachers working in the same cultural context.

In view of these gaps, this study explores 1) how “expert mathematics teacher” at the junior secondary school level are conceptualized by mathematics educators; 2) the common characteristics of expert mathematics teachers in mainland China; and 3) how the Chinese social and cultural context influences the concept and characteristics of expert mathematics teachers.

2.5 Summary of the Chapter

This chapter started with a discussion of views of concept and, particularly the prototype view of teaching expertise, which serves as the study’s theoretical perspective. Then, sociocultural theory, which serves as the theoretical underpinning of the study, was discussed. Literature on expert teacher was reviewed and relevant research gaps were pointed out. In the next chapter, educational background of mainland China will be introduced.

Chapter Three

Research Background

3.1 Introduction

This chapter provides an overview of the social and cultural background related to education in mainland China, to help readers understand the findings of this study as fully and clearly as possible, and to illuminate some unique phenomena in mainland China which might mediate educators' perception of expert teachers and teachers' practice. It starts with a brief description of the role of teachers in Chinese culture. After that, the social and educational situation of mainland China will be briefly introduced in four stages: 1) teacher education history and its system; 2) regulation of teacher qualifications and promotion policy; 3) basic education and assessment system; and 4) history of development of mathematics education and characteristics of mathematics curriculum in mainland China.

3.2 The Role of Teachers in Chinese Culture

3.2.1 The role of teachers under traditional Chinese culture

Education is an integral part of culture; as such, traditional Chinese education is also a component of traditional Chinese culture and has been shaped by this culture (Gu, 2006). China, a country of more than five thousand years history, has heavily stressed education and afforded a special respect to teachers. Historically, teachers in China have been given special importance by society and have played different roles at different times in Chinese history (Jin, 2008).

Confucianism has been considered the orthodox tradition of Chinese culture (Gu, 2006; Zhang *et al.*, 2004). Confucius, recognized as China's "foremost teacher" by Chinese and foreign scholars alike (*e.g.*, Mao *et al.*, 2001; Sprenger, 1991), thought that education was one of three fundamental factors (along with people and wealth) promoting national progress (Sun & Du, 2009). He saw education as one of the key factors influencing people's growth; without education, people would be ignorant. Confucius spent most of his life in education, and many of his

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words and experience have deeply influenced Chinese teaching, both in ancient and modern times (Sun & Du, 2009; Xiao, 2001), including:

1) *Studying diligently and inquiring widely* (勤于学习，广于见闻). Confucius emphasized acquiring knowledge and improving one's ability through exercise and broad inquiry. For Confucius, education did not just mean schooling in a narrow sense; anything that trains a person's behavior and character or increases one's knowledge and skills is a form of education (Sprenger, 1991). For example, he once stated that "whenever walking in a company of several persons, there among them must be someone worth my learning from" (三人行，必有我师) (*Analects*, translated by Lao, 1992, p. 119).

2) *Learning and reviewing what has been learned over time to gain new insights through reviewing old material* (学而时习，温故知新). Confucius said that "is it not a pleasure to learn and then constantly carry into practice what has been learned?" (学而时习之，不亦说乎？)(*Analects*, translated by Lao, 1992, p. 27) According to this view, teachers should guide students to exercise, practice and review regularly what they have learned. "Exercise" and "practice" are presented by one character in Chinese, *Xi* (习), which means to try things out and experience (Gu, 2003). The experience of exercising and reviewing not only helps students to consolidate what they have newly learned, but also helps them to get new insights from prior knowledge. In addition, it also makes students realize that the new knowledge is based on prior knowledge, and this helps them build a connected knowledge structure.

3) *Studying as well as reflecting* (学思并重，学思结合). Confucius not only stressed studying, but also reflecting. He claimed that "mere reading without thinking causes credulity; mere thinking without reading results in perplexities" (学而不思则罔，思而不学则殆) (*Analects*, translated by Lao, 1992, p. 43). According to Confucius, the point of learning is to integrate "studying" and "reflecting" (Mao *et al.*, 2001), which benefit each other (Xiao, 2001). During the process of learning, students need to inquire widely. More important, they should not blindly believe and accept whatever they learn but should doubt and question it. They should not only know "what it is", but also think about "why it is like this" (Sun & Du, 2009). Thinking and reflecting is based on studying, rather than merely thinking; therefore, teachers should urge students to study and reflect simultaneously while acquiring knowledge.

4) *Teaching heuristically and gradually* (启发教学，循序渐进). Confucius emphasized the active participation of students, and advocated heuristic teaching. His famous saying is that “I do not open up the truth to one who is not eager to get knowledge, nor help anyone who is not anxious to explain himself. When I have presented one corner of a subject to anyone, and he cannot from it learn the other three, I do not repeat my lesson” (不愤不启，不排不发，举一隅，不以三隅反，则不复也) (*Analects*, translated by Legge, 1960, as cited in Sprenger, 1991, P. 460). In other words, it is useless to teach students who are not yet ready or willing to learn. Teachers need to teach heuristically and, before providing any guidance or hints, they should encourage students to explore by themselves and involve them deeply and intellectually in learning. For new knowledge, students should have the chance to experience necessary extensions to deepen their understanding. In addition, Confucius also emphasized that teachers should teach gradually; that is, move from superficial meaning to deep meaning, from easy content to difficult, complex content (Xiao, 2001). When students asked Confucius a question, he gradually told them the answer or guided them to relevant content step by step, rather than directly telling them everything at the very beginning.

5) *Knowing students and teaching them accordingly* (了解学生，因材施教). Confucius was the first educationalist in Chinese history who proposed that teachers should teach according to students’ characteristics (Sun & Du, 2009), and should teach everyone regardless of social status or race. In Confucian culture, recruiting students regardless of family background and teaching without discrimination are basic principles for running a school (Mao *et al.*, 2001; Sun & Du, 2009). Because students’ abilities and personalities are diverse, in practice, Confucius emphasized that a prerequisite of teaching is coming to know students’ shortcomings and strong points, and teaching them accordingly. In Confucian culture, how teachers teach their students differs according to the situation and the students’ individual abilities.

These teaching principles indicate that teachers play a guiding role during teaching, and that students’ independent thought and initiative during teaching are strongly advocated. In addition to these teaching principles, Confucius also made some statements about teachers’ virtue (Xiao, 2001; Sun & Du, 2009), the two most important of which are:

1) *Teaching by setting an example* (以身作则). Confucius thought that a teacher should teach not only with words, but also with his own behavior and personality. Confucius strongly emphasized the important exemplary effects teachers have on students. He thought that, to a certain degree, teaching by example was more important than teaching by words (Sun & Du, 2009). He stated that "if a leader himself does what is right, the common people will follow him without being ordered. But if the leader himself does not do what is right, even though he gives orders, the common people will not be convinced and obey". (其身正，不令而行；其身不正，虽令不从) (*Analects*, translated by Lao, 1992, p. 217). While this statement was made about leaders, it has also been applied to Chinese teachers (Mao *et al.*, 2001), who are "viewed as models of good conduct and learning for students in the Chinese tradition" (Gao, 1998, p. 4).

2) *Learning painstakingly and insatiably* (学而不厌) *and teaching with tireless zeal* (诲人不倦). A teacher constantly ought to study, according to Confucius, in order to build a wide and broad knowledge base. Confucius stated that people will not know things naturally and that, as teachers, they should accordingly pursue knowledge constantly and continuously. If one stops one's own learning, one will lose one's qualification as a teacher one day (Sun & Du, 2009). Confucius said "I am not a man born well versed in everything; I am merely one who loves ancient studies and tries to gain his knowledge through diligent and earnest work" (我非生而知之者，好古，敏以求之者也) (*Analects*, translated by Lao, 1992, p. 119). In Confucius' opinion, only when one struggles to study to the extent that he forgets his meal, and enjoys studying to the extent that he forgets his personal worries and his own age, can one pursue the lifelong career of a teacher ("发愤忘食，乐以忘犹，不知老之将至云尔"). In addition, Confucius felt teaching is a noble profession, and that a teacher should be responsible for his students and society, should feel happy for his work (Sun & Du, 2009) and, more important, should dedicate his life to teaching and his work (Xiao, 2001). Confucius thought that a teacher should possess tireless zeal for his profession, no matter how demanding his students might be, and should persistently try to teach everything he knows to his students (Sun & Du, 2009; Xiao, 2001).

In Confucius' opinion, teaching not only benefits students, but also teachers. Teachers can learn how to teach and improve their own knowledge through teaching practice (Sun & Du, 2009). In general, in Confucian culture, teachers in China are shouldered with heavy responsibilities and high expectations. They are expected not only to transmit knowledge, wisdom and virtue, but also to serve as moral models for their students; an ideal teacher is not only knowledgeable, but also has a noble personality. These expectations influenced teachers in ancient China, and are still advocated in China today (Lo, 1984).

Moreover, Confucius did not want students to be subservient to whatever he said but encouraged them to express their opinions. He advanced the idea that, before attaining "virtue", students have no need to defer to their teachers (Gu, 2001; Mao *et al.*, 2001), which suggests that the relationship between teachers and students is one between equals. Confucius' ideas have heavily influenced Chinese education, both in history and in modern times (Mao *et al.*, 2001; Sun & Du, 2009; Zhang *et al.*, 2004), and some have influenced the thinking of other famous Chinese educationalists. For example, Xunzi, who lived in the Warring States Period (475 B.C.-221 B.C.), also believed that a country should be ruled by the combination of etiquette (*Li*, 礼) and law, and that the teacher is the key to proper etiquette (Gu, 2001). Hence, in Xunzi's opinion, the role of teachers is related to the rise and fall of a country; he stated that "if a country is to prosper, it must have respected and venerated teachers（国将兴，必贵师而重傅）" (Sun & Du, 2009, p. 81). In addition, he specified that "Heaven, Earth, Emperor, Parents, Teacher" (天，地，君，亲，师) are the five objects of worship for the Chinese people. This indicates that teachers in Chinese history were given a solemn, divine status, which is equal to heaven, earth, the monarch and the forefathers (Yang *et al.*, 1989). Xunzi also declared that "heaven and earth are the origin of all living things; ancestors are the origin of the entire human race; the Emperor and teachers are the cornerstone of an orderly society" (天地者，生之本也；先祖者，类之本也；君师者，治之本也)" (Gu, 2001, p. 191). This suggests that teachers and the emperor have the same unparalleled authority and that students therefore have to obey and respect teachers under all circumstances (Gu, 2001; Jin, 2008).

Another old Chinese saying, which echoes Xunzi's statement to a degree, says that "a teacher for one day is a father for a lifetime" (一日为师，终生为父). Johnston (1934) further explained this as follows:

> *the Chinese code requires a student to pay the same or very similar respect to his teacher that he would to his father, on the principal that just as a child owes his body to his parents, so he owes his intellectual equipment to his teacher. Physically, he is the offspring of his father and mother; intellectually, and to a large extent morally, he is the offspring of his teacher.* (p. 47)

All these statements indicate that teachers were well-respected sources of knowledge and enjoyed authority and high social status in ancient China. This tradition was transferred from one dynasty to another throughout Chinese history. In the Tang Dynasty, Han Yu (768-824), a renowned writer and educator, put forward the famous and very influential doctrine that teachers were those who transmit the way (principles or ideals) of life, instruct knowledge and skills, and solve puzzling problems（师者，传道，授业，解惑也）(Yang *et al.*, 1989). This doctrine not only deeply affected the role of teachers in ancient China, but also is "still frequently referred to in teacher-training" (Hui, 2005, p. 20) nowadays in mainland China. Under this tradition, the role of teacher was once described as 'teaching books and cultivating people' (*jiaoshuyuren*, 教书育人), which modern Chinese teachers still see as their main duties (Hui, 2005). Han Yu also argued that "it is not necessary for students to be inferior to teachers, and teachers may not necessarily be better than students. Teachers can be teachers because they know the Tao first and are more specialized in their studies" (弟子不必不如师，师不必贤于弟子，闻道有先后，术业有专攻, translated by Gu, 2001, p. 193). This suggests that a teacher might only be knowledgeable in a specific field. However, the tradition that "teachers must be respected" was still emphasized, and teachers were still in a position of authority. Furthermore, as the civil service examination was very popular, the authority of teachers was unchallenged under this examination system (Gu, 2001).

3.2.2 The role of teachers under contemporary Chinese culture

After the establishment of the People's Republic of China (P. R. China), the Soviet Union's model of education was, for political reasons, imported, and some Soviet educational literature was adopted as standard teaching references for Chinese teachers (Swetz, 1974). In particular, ideas advocated by Kairova, a Russian educationalist, such as that the aim of education is “to transmit the most stable knowledge accumulated over thousands of years to students” (Kailov, 1951, as cited in Li *et al.*, 2008, p. 67), were widely accepted by teachers in mainland China. To a certain degree, these ideas well match the Chinese notion of teaching (Li *et al.*, 2008). In particular, Kairov’s (1951) five-stage teaching model, consisting of “organizing teaching—reviewing prior knowledge—introducing new topics—consolidating new knowledge—assigning homework”, was widely accepted by teachers in Mainland China (Shao & Gu, 2006; Shao *et al.*, 2012). Under this influence, the teacher-centered teaching approach and whole class teaching have dominated classroom teaching in mainland China since then (Li *et al.*, 2008; Shao *et al.*, 2012; Zhang *et al.*, 2004; Zheng, 2006), which indicates that teachers still exercise a great deal of authority during teaching in mainland China.

In the 1950s, teachers were honored as “glorious gardeners” or “engineers of the human soul”, as they not only conveyed knowledge, but also served as moral models for students, as advocated by Confucius. This suggests that teaching was still a very noble profession (Leung & Xu, 2000); however, teachers’ socio-political status declined greatly during the Cultural Revolution (1966-1976). Teachers were downgraded as “poisonous weeds” and even described as belonging to the “stinking ninth class” (Leung & Xu, 2000). After the Cultural Revolution, the Chinese government attached greater importance to the function of teachers in education and the reputation of teachers was gradually restored. In 1987, a national Teachers’ Day on September 10 was established to honor the teaching profession. In a policy document, *Outline for Education Reform and Development in China*, teachers were placed at the center of national development efforts. The document clearly stated that “a strong nation lies in its education; and a strong education system lies in its teachers” (Ministry of Education [MOE], 1993). Many other government documents and laws since have also clearly stated that teachers should be respected. For instance, the Law of Compulsory Education, released in 2006, clearly stipulates that “teachers should be respected by all people” (MOE, 2006). This indicates that the status of teachers has been gradually enhanced and the tradition of “respecting teachers” has been revived in mainland China.

3.3 Mathematics Teacher Education in Mainland China

3.3.1 A brief history of teacher education in mainland China

As described above, teachers have been highly respected and education has been valued in traditional Chinese society for more than two thousand years. However, a formal training system for the teaching profession was not established in China until the 1890s (Cui, 2006; Jin, 2008), when one modeled on Japan's was implemented (Cui, 2006; Jin, 2008). Later, teacher education systems modeled after those in Germany, America and France were successively introduced, and influenced the teacher education system in mainland China from 1912 to 1949 (Jin, 2008). After the establishment of the People's Republic of China, China's education system began to emulate the Soviet model. The Soviet teacher education model was also adopted and remained in existence in mainland China for more than two decades (Jin, 2008; Pepper, 1996). Under its influence, China relied heavily on an independent teacher training system in which teachers were exclusively trained by normal schools, provincial/regional colleges of education, and normal colleges/universities. Due to recent developments in technology, China's economy, and education itself, however, the Soviet model is no longer able to meet the demand for teachers, and some comprehensive universities have begun to train teachers. Nevertheless, the independent system currently still dominates teacher education in mainland China (Li, 2006).

3.3.2 Pre-service mathematics teacher education

Under the independent teacher training system, in the early 1950s, specialized institutions at three levels were established to prepare teachers for different levels of schooling in mainland China (Paine & Fang, 2007). Grade 9 graduates could apply to a three-year full-time teachers' training course to be trained as primary school or kindergarten teachers in normal schools. Grade 12 graduates could apply to a three-year (at a teacher college) or four-year (at a normal university) full-time teacher training course, based on their results of College Entrance Examination. Most of the three-year trained graduates taught in junior secondary schools, whilst the four-year trainees taught in senior secondary schools. However, as a result of economic and technological developments, a new three-stage teacher education system has gradually emerged: 1) primary school teachers are trained at three-year teacher colleges or four-year colleges; 2) junior and senior high school teachers are trained at four-year colleges and normal universities; and 3) some senior high school teachers are required to attain postgraduate level studies (Huang *et al.*, 2010; Li *et al.*, 2008).

Teacher education at the teacher college and normal university levels is subject-oriented. Under the Soviet model, the curriculum during pre-service training stage heavily stressed mastering subject matter, with little time devoted to teaching practicum and professionally oriented studies (Huang *et al.*, 2010; Leung & Xu, 2000; Yang *et al.*, 2009). This reflects a long-standing belief in mainland China that teachers cannot be prepared by university programs alone (Paine *et al.*, 2003), and that student teachers can learn teaching skills from their practical teaching after they become teachers (Li *et al.*, 2008). Thus, student teachers graduate from normal universities are deemed a “semi-product” (Paine *et al.*, 2003). The pre-service teacher education curriculum includes: 1) foundation courses, such as politics, moral education, second language and physical education, which together take around 20% of total teaching hours; 2) professional education courses, including pedagogy and psychology; 3) courses related to subject matter; 4) optional courses; and 5) teaching practicum in schools, which normally take around 8 weeks (Leung & Xu, 2000).

For pre-service mathematics teachers, advanced mathematics courses (such as advanced algebra, analytical geometry, mathematical analysis, function analysis, abstract algebra, topology, etc.) are taken as a priority. In the 1960s, around 70% of the total pre-service teacher curriculum hours were devoted to mathematics subject matter; even now, the proportion is around 50% (Paine at al., 2003; Yang *et al.*, 2009), with the main aim being to “foster prospective teachers with a bird’s eye view of understanding elementary mathematics deeply rather than immediately connecting to what they will teach in schools” (Li *et al.*, 2008, p. 70).

Another characteristic of curricula at the pre-service stage in mainland China is the emphasis on reviewing and studying basic mathematics (Li, 2008; Li *et al.*, 2008). Some courses, such as basic mathematics in depth, problem solving and mathematical competition, can promote prospective teachers’ profound understanding of elementary mathematics and improve their problem solving ability. In addition, pre-service mathematics teachers are also required to take three education courses (educational psychology, general pedagogy, and mathematical pedagogy) to help them learn mathematics and pedagogy as they directly relate to mathematics teaching in practice (Li *et al.*, 2008). Secondary school mathematics textbooks are used as a regular part of the mathematical pedagogy course, which emphasizes instruction for the national mathematics syllabus, deeper understanding of teaching content, general pedagogy and specific methods for organizing mathematics content for teaching (Li, 2008). In this course, pre-service teachers learn to analyze the "important points", "difficult points", and "key points" in secondary school textbooks and how to teach them. This indicates that even though pre-service teachers gain limited teaching experience during their training, during their pre-service training stage they start to study the curriculum they might teach in future (Paine *et al.*, 2003). In general, unlike pre-service teachers in the United States, who are trained to be specialists in curriculum and instruction, pre-service teachers in China are trained to be specialists in content, curriculum, and instruction (Li, 2008).

3.3.3 In-service mathematics teacher education

The Chinese government heavily stresses in-service teacher education. According to a document published by the Ministry of Education (MOE, 1999), the main content of in-service teacher education includes politics and moral education, subject matter knowledge, educational theories, educational research, teaching skills and educational technology. Generally speaking, there exist two different types of in-service teacher education – degree and non-degree education (MOE, 1999). The first helps teachers attain a higher level degree, such as a bachelor or master's degree. Since 1996, the Educational Masters Degree in Subject Teaching (Ed.M.), which is different from the academically oriented Masters Degree in Education, has been established in mainland China. In-service elementary and secondary mathematics teachers with at least three years of teaching experience are qualified to register for the course, but need to pass an entrance examination, which mainly includes foreign language, political theory, education, psychology and mathematics subject courses. Non-degree education, by comparison, includes induction education, on-the-job-training, and *gugan* teachers training (骨干,

backbone teachers). *Gugan* teachers are those teachers who "were identified as the most experienced and effective teachers in their schools and regarded as leaders of the teaching force" (Zhou & Reed, 2005, p. 208). For example, a government project (MOE, 1998), *Gardener Project Crossing the New Century*, was established to provide in-service training for all teachers, with the goal of improving in-service teachers' quality. The project trained 10,000 *gugan* teachers from 2000 to 2003 through teacher colleges, normal universities, or comprehensive universities. Other institutions, such as provincial, regional or county schools of education and some TV colleges, can also provide professional training or education (Leung & Xu, 2000; Sun, 2000).

As mentioned above, the system and structure of teacher education in mainland China were modeled on those of the former Soviet Union, which was, in turn, based on the commune model, with an emphasis on enhancing school-based teachers' professional development through collective effort (Lin, 2008). Teacher's professional development in China is also taken as activities that are practical in nature (Li & Huang, 2008) and can be a part of a teacher's daily life and is pertinent to the teacher's needs, through a school-based teaching research mechanism (Huang, 2006; Yang & Ricks, 2012) rather than providing lengthy recommendations and workshops (Li & Huang, 2008).

Under the influence of the former Soviet teacher education model, a very special and unique teaching research system was developed in mainland China (Cong, 2009; Yang & Ricks, 2012). In 1956, Teaching Research Offices were established at the district/county, city, and provincial/municipal levels under the corresponding government education departments (Lin, 2008; Yang *et al.*, 2009; Yang & Ricks, 2012) (see Figure 3.1). The main functions of these offices was, and still is, to help education departments at the various levels enact relevant policy documents, organize seminars for teachers from the district to learn the curriculum framework and teaching syllabus, study teaching materials and teaching methods, and communicate teaching experience. The Teaching Research Officer (*Jiaoyanyuan*) in a particular subject would also instruct teachers on how to improve their teaching ability (Paine *et al.*, 2003; Zhong, 2003, as cited in Cong, 2009). Teaching Research Offices at different levels also organize regular subject–based teaching contests, which are well-organized formal professional activities (Huang *et al.*, 2010; Li & Li, 2009; Li & Li, 2012); for in-service teachers, and especially novice teachers, attending teaching contests provides a "concentrated opportunity to learn" and an opportunity to work on "basic teaching skills" (Paine *et al.*, 2003, p. 43). Teaching competition champions enjoy good reputations and play key roles in future professsional development activities. Sometimes, Teaching Research Officers will ask some very experienced teachers to deliver demonstration lessons that can be observed by teachers from various subjects and schools, who can obtain useful information from the demonstration lessons and post-lesson conferences.

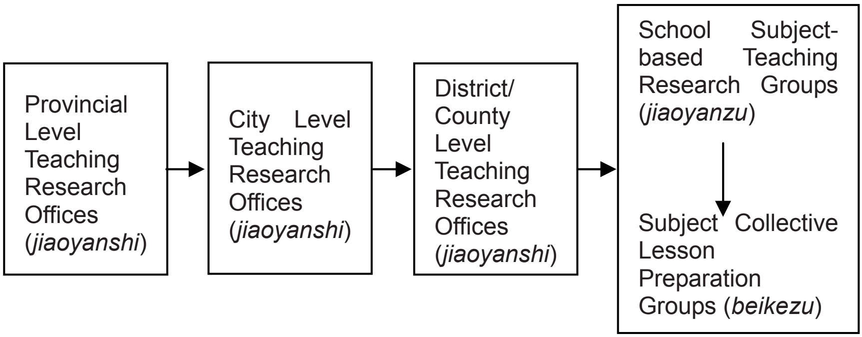

Figure 3.1. Structure of teaching research system in mainland China

As shown in Figure 3.1, in most middle schools, especially large-size ones, there are two different subject-based teaching research organizations to facilitate and organize teaching activities (Ma, 1999; Paine & Ma, 1993; Tsui & Wong, 2009; Yang, 2009; Yang & Ricks, 2012). The teaching research group for all mathematics teachers in a given school is organized to enable teachers to make and implement a detailed schedule of mathematics teaching and research based on the academic year schedule, organize teachers to learn national curriculum standards, observe and reflect on each other's lessons, discuss matters related to teaching and examination, work together to help teachers to prepare for school-based teaching competitions or opening lessons, and so on (Han, 2012; Huang *et al.*, 2010; Yang & Ricks, 2012). The other research organization is the lesson preparation group, a sub-organization of the teaching research group, which includes all mathematics teachers at the same grade level. The teachers in this group study the curriculum materials, plan lessons together, and share teaching experiences on a regular basis (Wang & Paine, 2001, 2003; Yang, 2009; Yang & Ricks, 2012). The activities organized by these two groups provide a platform and opportunity for teachers, especially novice teachers, to discuss and reflect on their teaching and to learn from excellent practice (Tsui & Wong, 2009). In this way, teachers are able to improve their teaching skills and mathematics knowledge at the same time.

Another popular professional development model adopted in mainland China is apprenticeship practice (*shituzhi*, 师徒制) (Huang *et al.*, 2010; Han, 2012), or "the old guiding the young" (*laodaiqing*, 老带青) (Tsui & Wong, 2009). Normally, an experienced teacher will be assigned to mentor a newly graduated or novice teacher on a one-to-one basis in all aspects of their work as a teacher. The mentor's teaching will be observed by the mentee and, in turn, the mentor will observe the mentee's teaching; after that, the mentor will provide critical feedback and constructive suggestions for further improvement. As found in some studies (*e.g.*, Wang *et al.*, 2004), compared to their counterparts in the United States, novice teachers in mainland China receive more specific, critical, and subject-focused feedback from their mentors. This school-based apprenticeship model helps novice teachers to resolve the problem of insufficient teaching-related training during their pre-service training stage.

3.4 Regulation of Teacher Qualifications and Promotion Policy

In mainland China, being a teacher involves meeting some basic qualification standards. The *Teachers Law of People's Republic of China* (MOE, 1993) recognized teaching, for the first time, as a profession. According to this law, teachers are required to comply with the Chinese constitution and law, demonstrate excellent moral character, and love teaching. Two years later, the teacher professional qualification system in mainland China was further shaped by the release of the *Regulation of Teacher Qualification* (MOE, 1995), which normally required would-be teachers to obtain the appropriate teacher certificate by graduating from a minimum three-year program offered by normal schools (for elementary school teachers), a three-year program offered by normal colleges (for junior secondary school teachers) or a four-year bachelor degree program offered by normal colleges or universities (for senior secondary school teachers).

In addition to these qualifications, there exists a rank system for elementary and secondary school teachers in mainland China. According to the *Regulation of Secondary Teachers' Position Promotion* (MOE, 1986), the ranks for junior secondary school teachers include intermediate levels 3, 2 and 1, and senior level (see Figure 3.2), each of which has specific political, moral and academic requirements, in addition to additional requirements published by the Ministry of Education. For example, the basic requirements for intermediate level 3 mathematics teachers are: a diploma from a three-year program at a teacher college; one year of probation; a passing score on an examination demonstrating their basic knowledge of education, psychology and pedagogy; mastering mathematics teaching skills; and being capable of teaching mathematics at the junior school level independently.

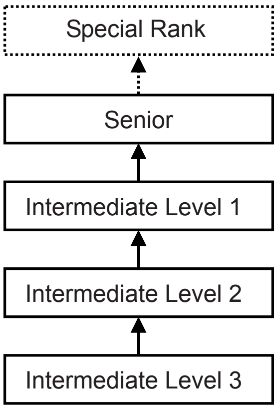

Figure 3. 2. Teacher rank system in mainland China

Basic requirements for senior rank mathematics teachers include: 1) five or more years of teaching experience as a secondary school mathematics teacher at intermediate level 1, or a PhD; 2) a systematic and solid mathematics subject knowledge base, rich teaching experience, outstanding teaching performance, and rich experience in educating students in political and moral aspects or special expertise as class teacher; and 3) engaging in research activities related to secondary school education and teaching situations, being able to write teaching reflections, reports and research papers that integrate theory and practice at academic level, and making contribution to improving other teachers' academic quality and teaching ability.

Senior teachers have extra requirements in addition to those for intermediate level 3 teachers, including the demonstrated ability to conduct research and mentor other teachers. Teachers are required to attend educational research activities to be promoted. Once being promoted to the rank of intermediate level 1, teachers are expected to mentor those at a lower rank. Moreover, additional requirements for promotion might be added or considered at the provincial level. For example, in Chongqing, in addition to the basic requirements mentioned above, other requirements, such as computer skills, English language ability, and the level of Mandarin (*Putonghua*), have been added by the Education Department, and a senior candidate teacher is expected to have published at least one research paper.

In addition to the four official rankings, a special rank (not a formal component in the rank system) has existed in mainland China since 1973 (MOE, 1973). It is an honorary rank afforded those senior teachers who have profound theoretical knowledge, have extensive teaching experience in the subject taught, demonstrate extraordinary teaching ability, obtain outstanding achievement in education reform and teaching research, make significant contribution to education, and make outstanding contribution in mentoring non-experienced teachers. Special rank teachers should also be devoted to teaching.

3.5 Basic Education and Assessment System

The Chinese education system's 12-year school education includes six years of elementary education, three years of junior secondary school, and another three years of senior secondary school. The Nine-Year Compulsory Education Law, passed in 1986, requires parents to send their children to schools until they have completed a junior secondary school education. Junior secondary school graduates can either advance to senior secondary schools (if they pass the required tests) or seek other opportunities for their future life.

There are two important examinations for students in their school education in China: the *Zhongkao* (High School Entrance Examination) and the *Gaokao* (College Entrance Examination). Grade 9 is a critical point for most students, because, at its end, students are required to take the *Zhongkao* (High School Entrance Examination). Not every junior secondary school graduate is admitted to senior secondary school. Normally, basing on their *Zhongkao* results, students will be offered enrolment in either a general senior secondary school, a vocational school, or neither; about half of all junior secondary school graduates who pass the *Zhongkao* go on to general senior secondary schools, and the other half to vocational schools. After three years of senior secondary schooling, students will take the *Gaokao* to determine whether they will be allowed to go on to university. Only a portion of senior secondary graduates will pass the *Gaokao,* which has been described as "thousands of cavalrymen and infantrymen crossing a single-plank bridge" (千军万马过独木桥，translated by Hui, 2005, p. 30). In the past ten years, due to the rapid expansion of higher education in mainland China, more and more senior secondary school graduates can enter to university. However, the expansion of higher education has also made it quite difficult for university graduates to find a job (Postiglione, 2005).

The actual situation in Mainland China is that the better a student performs on the *Zhongkao*, the more likely s/he will be admitted to a higher-quality senior secondary schools with a good reputation; in addition, there is a greater chance for her/him to enter prestigious universities three years later, which will enhance her/his competitiveness or improve her/his chances in a tough job market and further secure a better future (Gao, 2009). Therefore, like the *Gaokao*, the *Zhongkao* is also competitive in nature; indeed, academic competition in these two examinations has become more intense than ever.

The competition of these two examinations has its roots in traditional Chinese culture. In Chinese history, the civil service examination (*keju*, 科举) was used as a means for searching out and selecting capable persons for taking up official positions based on their academic merits rather than their social background or hereditary aristocracy (Siu, 2004; Zhang *et al.*, 2004). Passing the civil service examination was an opportunity to change one's life and future (and possibly, the future of one's whole family). As such, passing the civil service examination was viewed as a chance to glorify one's ancestors and to gain dignity for one's family and ancestors (Hui, 2005). Although the civil service examination no longer exists in mainland China, its importance is never forgotten (Li, 2006; Zhang, 2010); nowadays, scoring well on the *Gaokao* is viewed in much the same way as passing the civil service examination was in days gone by – as a crucial chance to change people's life and future (Zhang

& Ren, 1998; Hui, 2005). This tradition contributes to the intensity of the competition and motivates students to perform better on the *Gaokao* (and the *Zhongkao* as well). Moreover, students' exam performance reflects on the reputation of their schools, because society judges a school's quality by the number of its students that are admitted to colleges and universities (Zhang & Ren, 1998).

The examination culture dominates teaching practice in mainland China (Tu, 2009; Zhang *et al.*, 2004); there are mid-term and final-term tests and, sometimes, assessment tests at the end of each unit. Mathematics is very important in these tests, with students required to complete a certain number of questions within a given amount of time. The average *Zhongkao* mathematics examination paper contains around 24 questions (some of which may have two to three sub-questions), and the average time allotted for each question or sub-question is around three minutes (Wu, 2012). Moreover, students' test scores are not only used to assess their learning, but also used as a very important criterion by which to judge teacher effectiveness (Liu & Teddlie, 2003; Zhang & Ren, 1998). As such, teachers stress the basic knowledge and skills students will need in their examination in order to earn higher scores. For example, teachers will emphasize problem-solving skills, offer tips for solving the types of questions that will be found on the examination, and increase the quality and difficulty of examination practice questions to make sure students can solve them correctly and quickly (Li, 2006; Zhang & Ren, 1998; Zheng, 2006).

3.6 Mathematics Curriculum and Textbooks in Mainland China

3.6.1 A brief history of mathematics curriculum development

Mathematics education in mainland China has a history dating back more than three thousand years (Li & Dai, 2009; Siu, 2004). As early as in the *Xizhou* dynasty (more than 2000 years ago), mathematics, then referred to as "arithmetic", was already one of the "six arts" (rites, music, archery, chariot-riding, calligraphy, and arithmetic) for scholars in China. For many years, a classic work, *The Nine Chapters on the Mathematical Art*, exerted great influence on mathematics and mathematics education in China. This work established some of the basic characteristics of traditional mathematics in Chinese history, such as the understanding that mathematics is a subject related to calculation and that can be applied in real life (Li & Dai, 2009), and was one of the most popular mathematics textbooks in Chinese history until European mathematics was introduced around 1600 (Li, 2008). At the beginning of 20th century, some textbooks then used in Japan were translated and introduced to China. During the 1930s and 1940s, textbooks written by European and American scholars (*e.g.*, *College Algebra* written by Fine, *Plane Geometry* and *Solid Geometry* written by Schultze, Sevenoak and Schuyler) were adopted in China (Wang & Zhang, 2009).

From the establishment of the People's Republic of China to 1956, the Soviet model of education was followed. In 1952, the Ministry of Education released a teaching guideline for secondary school mathematics, mostly based on the Soviet model (this guideline was revised in 1954 and 1956 respectively), that emphasized "two basics" (basic knowledge and basic skills); this tradition has influenced mathematics teaching in China ever since (Li, 2006; Shao *et al.*, 2012; Zhang, 2006; Zhang *et al.*, 2004). Mathematics textbooks used at that time were mostly recomposed Soviet mathematics textbooks, and characteristics of the Soviet mathematics curriculum framework, such as integrity, coherence, focus, and rigor, were "widely accepted, insisted, and even developed by Chinese educators" (Li, 2008, p. 129). Under the influence of the Soviet philosophy of mathematics, mathematics curriculum, and pedagogy, mathematics is regarded as an abstract, rigorous and widely applied subject in Mainland China. Logical deduction and formal mathematical operations were emphasized, and were manifested in textbooks and mathematics teaching philosophy in mainland China, until today(Li *et al.*, 2008; Zhang *et al.*, 2004).

From 1956 to 1966, the Ministry of Education adjusted curriculum content in elementary and secondary schools to correct the dogmatism made in the process of learning from the Soviet model. By systematically investigating and comparing Chinese curricula, syllabi and teaching guidelines published between 1912 and 1956, as well as those from other countries (such as those in Europe), China started to explore its own way of developing mathematics curricula (Lv, 2007). During this period, most students in China used the same textbooks, but a minority used experimental versions (Lv, 2007). In 1963, a mathematics teaching syllabus was published that established the aim of secondary school mathematics teaching to be, in addition to helping students grasp basic knowledge, to develop students' ability in calculation, logical reasoning, and spatial visualization. This statement has influenced mathematics teachers' teaching practice heavily (Wang & Zhang, 2009).

From the end of the Cultural Revolution (1976) to 1988, several teaching guidelines and textbooks were published successively in mainland China, with content and its presentation being modified several times. Over the same period, however, mathematics teaching guidelines and textbooks in mainland China were unified. Until the late 1980s, all students in China used the same set of textbooks, all published by the People's Education Press. In the 1990s, the Chinese Government readjusted this system by allowing Zhejiang province and Shanghai to develop their own curricula, and other publishers to produce textbooks (Ma *et al.*, 2002). However, mathematics textbooks continued to be developed according to the same teaching syllabus.

Since 2001, there has been a new round of curriculum reform in mainland China, from the elementary school level to senior secondary school level (Liu & Li, 2010). In this reform, mathematics teaching ideas, the role of the teacher, mathematics teaching methods, and mathematics teaching content and its arrangement have been innovated and changed. One characteristic of this reform is a shift from knowledge transmission to developing students' learning ability and preparing students for lifelong learning. Since then, mathematics teaching in China has changed its emphasis from teacher-directed learning to teacher-guided learning (Rao *et al.*, 2009), with a strong focus on students' learning experience during teaching. To help students attain necessary mathematics experience, teachers strongly encourage them to participate in various activities and to observe, explore, and communicate during teaching. The new curriculum reforms also stress connecting mathematics teaching and learning to real life situations. The use of calculators for complex problems is also encouraged so that students can spend more time on exploratory and creative mathematical activities (Zhang, 2005). Nevertheless, the teaching of basic mathematics knowledge and skills remains the most important aim of mathematics teaching. As to mathematics textbooks, there are now nine sets of junior secondary school mathematics textbooks available, and every region can choose the textbooks most suitable to its own situation; normally, there are two or three choices of textbook sets in a region.

3.6.2 Characteristics of curriculum system and mathematics textbooks

Curriculum and assessment at the national level in China is currently centralized, as shown in Figure 3.3 (Eckstein & Harold, 1993; Liu & Li, 2010), but there is a trend towards more decentralization (Li, 2008; Wang & Paine, 2001). As to the mathematics curriculum in mainland China, it is much more linearly organized with almost no repetition (Li, 2008) and "seems to cover more advanced topics and has a quicker rate of introducing topics than its U.S. counterpart" (Jiang & Eggleton, 1995, p. 190). Although there are nine set of textbooks available at the junior secondary school level, all are written under the guidance of the newly published mathematics curriculum standard and must be approved by the government before publication and use. Hence, as shown in Figure 3.3, textbooks basically dominate the influence on the implemented curriculum in mainland China (Li, 2008).

Textbooks play a very important role in teaching and learning in mainland China (Fan *et al.*, 2004); as in other Asian countries and regions, they are regarded as "a body of the minimum and essential knowledge that everyone must learn and understand" (Park & Leung, 2006, p. 229). Therefore, teachers cannot, in practice, implement their teaching completely free of the control of the textbook. Comparatively speaking, Chinese textbooks include fewer but more mathematically challenging content topics, and fewer mathematical connections to other scientific disciplines (Cai *et al.*, 2004; Li, 2008; Park & Leung, 2006). The content in mathematics textbooks is very concise and coherent (Fang & Gopinathan, 2009; Li, 2008) with "few illustrations" (Stevenson & Stigler, 1992, p. 139). This characteristic, on the one hand, provides teachers room to expand and reorganize content for classroom instruction; on the other hand, it also challenge teachers to interpret content, develop and structure lessons, and expand content and/or select other problems to enrich students' experience (Li, 2008).

To help teachers interpret and use textbooks more effectively, a teaching reference book accompanies each textbook, providing in-depth introductions to the whole book, each chapter, and even each lesson. The reference books also provide teaching suggestions related to key part of a given lesson or chapter. However, there are no instructional recipes that teachers can directly adopt, nor any time-saving lesson preparation tools (Li, 2004). Chinese mathematics teachers need to make their own lesson plans based on the suggestions contained in the teaching reference books and textbooks, and taking into account their students' abilities and the actual classroom situation.

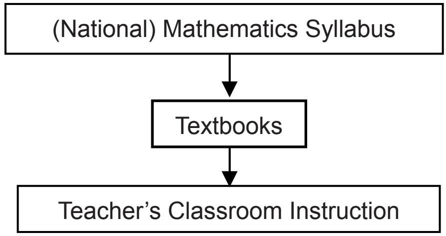

Figure 3.3. System context of implemented curriculum in mainland China

3.7 Summary of the Chapter

This chapter introduced background information for this study, including the role of the teacher in Chinese culture, teacher education, teacher promotion system, education and assessment system, and mathematics curriculum system and characteristics of textbooks in mainland China. In the next chapter, the research methodology for the study will be discussed.

Chapter Four

Research Methodology and Design of the Study

4.1 Introduction

With a particular focus on mathematics educators' conceptualization of expert mathematics teachers and expert mathematics teachers' characteristics in mainland China, this study addresses the following questions:

1) How is "expert mathematics teacher" conceptualized by mathematics educators in mainland China?
2) What are the characteristics of expert mathematics teachers in mainland China?
3) How do the Chinese social and cultural contexts influence the conception and characteristics of expert mathematics teachers?

To answer these questions, this study adopts a variety of data collection methods, including in-depth interviews, classroom observations, and document analysis. This chapter focuses on the study's methodological approach and research design. It begins by outlining the qualitative nature of the study. Other sub-sections include: research design; participants; data collection methods; data analysis procedures; the validity of results; and ethical considerations.

4.2 Justification for Choosing Qualitative Research

4.2.1 The features of qualitative research

Broadly speaking, there are two different research methodologies in social research: qualitative and quantitative research. Each methodology's philosophical assumptions about the nature of reality, epistemology, values, the rhetoric of research, and methodology are different (Creswell, 1994). Quantitative and qualitative research traditions have philosophical roots in positivistic and naturalistic philosophies, respectively (Newman & Benz, 1998); that is, the most basic assumptions contained in the two approaches are different.

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Quantitative research is based on "the argument that both the natural and social sciences strive for testable and confirmable theories that explain phenomena by showing how they are derived from theoretical assumptions" (Ary *et al.*, 2001, p. 422). It involves hypothesis-testing (Auerbach & Silverstein, 2003), emphasizes measuring and analyzing casual relationships between variables (Denzin & Lincoln, 1994) and is associated with deductive reasoning – moving from general principles to specific situations (Wiersma, 1995).

On the other hand, the key philosophical assumption of qualitative research is that "reality is constructed by individuals interacting with their social worlds" (Merrian, 2002, p. 6). Moreover, "social reality (for example, cultures, cultural objects, institutions and the like) cannot be reduced to variables in the same manner as physical reality" (Ary *et al.*, 2001, p. 422). Therefore, qualitative research, according to Denzin and Lincoln (1994):

> *is multimethod in focus, involving an interpretative, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them. Qualitative research involves the studied use and collection of a variety of empirical materials—case study, personal experience, introspective, life story, interview, observational, historical, interactional, and visual texts—that describe routine and problematic moments and meanings in individuals' lives.* (p. 2)

As indicted above, a key characteristic of qualitative research is that it studies real-world behavior occurring in natural settings, and describes phenomena in words instead of numbers or measures, in order to understand human and social behavior in a particular setting (Krathwohl, 1993). In qualitative research, "theory developed this way emerges from the bottom up (rather than from the top down), from many disparate pieces of collected evidence that are interconnected" (Bogdan & Biklen, 1998, p. 6). In other words, qualitative research is essentially an inductive process – moving from specific situations to general principles (Wiersma, 1995).

4.2.2 Why choose qualitative research as methodology

The philosophical and methodological differences between quantitative and qualitative research described above give them distinct applicability. Qualitative methodology has moved towards center stage in educational research since the mid-1980s (LeCompte *et al.*, 1992) and its use in studying educational issues has recently deepened (Bogdan & Biklen, 2003). Current trends in mathematics education research reflect a paradigm shift from "an emphasis on scientific or quantitative studies to the use of qualitative, interpretative methodologies" (Teppo, 1998, p. 10). In addition, this paradigm "has begun to dominate research in mathematics education" (Ernest, 1998, p. 22) because some notions "of represent-tativity, replicability, and generalizability are fundamental to quantitative research but not necessarily to work in all areas of mathematics education" (Pirie, 1998, p. 18). Moreover, the incorporation of qualitative research into mathematics education has made it possible to investigate mathematics teaching and learning, not only at new and different levels of complexity, but also from multiple perspectives (Teppo, 1998).

Indeed, some phenomena in the field of mathematics education, such as mathematics teaching and learning, are very complicated. Moreover, "we cannot ignore the affective and socially influential domains surrounding the teachers and students we study" (Pirie, 1998, p. 18). Therefore, phenomena such as mathematics learning and teaching could best be studied in their natural settings and, more important, with awareness and consideration of their contextual influences. Implicit in the purpose of this study is an exploration of the prototypical conception of expert mathematics teachers held by mathematics educators, and the characteristics of expert mathematics teachers within the social and cultural context of mainland China. To that end, there is a need to carry out investigations without being constrained by too many assumptions. As such, the interpretative, naturalistic approach of qualitative research is more suitable for this study. Qualitative methodology will yield more detailed and rich data, which could lead to a more in-depth understanding of the conception and characteristics of expert mathematics teachers in mainland China. In addition, sociocultural influences on the conception and characterization of expert mathematics teachers can also be examined.

4.3 Research Design

The study consists of two parts. The first investigates the conception of expert mathematics teachers through semi-structured interviews with mathematics educators. It might be possible that there exist differences in how mathematics teacher educators, mathematics teaching researching officers, school principals, and mathematics teachers conceptualize expert mathematics teachers; the aim of this study is to explore the conceptions held by different stake-holders rather than those of any one group, hence commonalities rather than differences are its focus. Mathematics educators chosen for the first part include:

1) mathematics teachers with different teaching experience and working in schools with different background (*e.g.*, key middle school and non-key middle school);
2) mathematics teacher educators who are in charge of in-service teacher education or know junior secondary mathematics education well (as opposed to those who merely concentrate on academic research and do not understand the real mathematics teaching situation);
3) school principals with mathematics education background, in the hopes they could provide information on teacher and teaching evaluation in the field of mathematics education; and,
4) mathematics teaching research officers who work with junior middle school mathematics teachers and know the situation well.

The interviewees were also asked to recommend some expert mathematics teachers for the second part of the study, which explores common characteristics of expert mathematics teachers through observation, interviews, and document analysis. The detailed research design is shown in Table 4.1.

4.4 Research Site and Participants

Qualitative research aims to study cases instead of variables; thus, fewer participants are involved than in quantitative research. Qualitative research looks for particular participants who can offer rich and specific information.

Table 4.1. Research design of the study

	Aims	Data Collection Methods	Participants
Part I	Conception of expert mathematics teachers	Semi-structured interview	Mathematics teachers; Mathematics teacher educators; (Vice) School principals (with mathematics education background) Mathematics teaching research officers
Part II	Characteristics of expert mathematics teachers	Semi-structured interview Observation Documents analysis	3 expert mathematics teachers

Guba and Lincoln (1981) argued that "sampling is almost never representative or random but purposive, intended to exploit competing views and fresh perspective as fully as possible" (p. 276). The participants in this study were purposely chosen. Before introducing the participant selection process, basic information about the research site will be introduced in this section.

4.4.1 Introduction and rationale to research site

This study was conducted in Chongqing. Chongqing is situated in southwestern China. It has a history of more than 3000 years. In 1997, Chongqing became China's fourth municipality (the other three are Beijing, Shanghai and Tianjin), the only one in the western part of China; as such, its municipal government ranks directly under the central government of the People's Republic of China. The municipality covers a total area of 82,400 square kilometers and contains around 33 million people, the majority of whom are ethnic Han; although their numbers are small, 49 ethnic minority groups also inhabit Chongqing. In 2012, its *per capita* GDP (for the permanent population) was 39,256 RMB, which ranked twelfth among the 31 provinces, municipalities, and autonomous regions of mainland China (see http://www.cq.gov.cn/). In terms of its economic situation, Chongqing is at the middle level in mainland China.

This suggests that, from an economic perspective, Chongqing might be more representative of China than either more economically developed regions (*e.g.*, Guangdong Province and Zhejiang Province) or undeveloped ones (*e.g.*, Guizhou Province and Yunan Province). In addition, unlike China's other three municipalities, Chongqing has a comparatively low urbanization level with a ratio of 50%.

Furthermore, like most places in mainland China, Chongqing has adopted the 6+3+3 schooling system. According to the China Education Yearbook 2008, in 2007, there were 7990 primary schools (with around 2.4 million students), 1099 junior secondary schools (around 1.3 million students), and 262 general senior secondary schools (around 0.5 million students) in Chongqing (see http://www.edu.cn/). Furthermore, Chongqing has adopted the national curriculum and uses textbooks developed following national curriculum standards. All these factors serve to make Chongqing more representative of China as a whole.

4.4.2 Process of choosing participants

Two different participant categories were included in this study. Participants in the first category were purposely chosen to maximize information gathered about the conception of expert mathematics teachers. In addition, since that conception is unclear, and since it would be contradictory to identify expert mathematics teachers at the beginning of the study, a snowball sampling technique (Hoyle *et al.*, 2002) was used to choose participants, which is as followed:

1) Two mathematics teaching research officers, one at the Chongqing city level and the other at the district level, were purposely chosen for their familiarity with the actual situation of mathematics teaching in Chongqing. Two teacher educators, one from Southwest University (SWU) and the other from Chongqing Normal University (CQNU), were also chosen, as they were in charge of in-service mathematics teacher training at that time and had a great deal of experience working with secondary school teachers. All four interviewees had rich knowledge of mathematics education in Chongqing. At the end of their interviews, each was asked to recommend some junior secondary mathematics teachers who could be referred to as expert mathematics teachers. In total, they recommended seven teachers, five of whom (from different schools) were recommended by at least two interviewees, and were chosen as tentative

participants for classroom observation in this study.

2) The researcher invited (through phone calls and with the help of government officials) the principals and vice-principals of these five schools to participate in this study; all accepted the invitations. Subsequently, the researcher interviewed those school principals and vice-principals with mathematics education background; in one school, all the principals and vice-principals had majored in other subjects, and therefore were not interviewed, but only asked to nominate potential expert mathematics teachers in their school. One vice-principal was nominated by the two mathematics teacher educators as an expert mathematics teacher; as a result, only two mathematics teachers were interviewed in that vice-principal's school. In addition to the nominated teachers, one or two mathematics teachers in each school were interviewed, then asked to recommend teachers in their school who were, in their view, expert mathematics teachers. All five tentative expert mathematics teachers mentioned above were also recommended by their school principals and/or colleagues (of course, some other teachers were also recommended).

3) Next, the researcher met with each of the five teachers to explain the aim of the study. Four initially agreed to participate, although one subsequently changed his mind for fear that the research results would be reported to his principal, even though he had been assured that all information collected would remain confidential and be used only for academic purposes. As a result, a total of three mathematics teachers identified as expert mathematics teachers participated in the present study, as noted in the initial research plan.

4) As all of the five nominated teachers were working in key secondary schools, it was necessary to seek more diverse information, so as to gain a comprehensive understanding of the conception of expert mathematics teachers. Hence, three principals and three mathematics teachers from three non-key secondary schools were also invited to participate in this study (with the help of the district-level mathematics teaching research officer). A total of 21 participants (referred to as the first category of participants) were chosen for the investigation of the conception of expert mathematics teachers. Three teachers (referred to as the second category of participants) were recommended by the interviewees for exploring the characteristics of expert mathematics teachers.

4.4.3 Basic information of participants

4.4.3.1 First category of participants

Detailed information of the 21 participants in the first category (for more information about the teachers and school principals, see Appendix 1) is listed in Table 4.2:

Table 4.2. Information of participations in the first category

Teacher Educators	Mathematics Teaching Research officers	(Vice) School Principals	Mathematics Teachers
1 (SWU): PhD in mathematics education, 10 years of working experience;	1 (City level): 15 years of teaching experience in secondary school, 10 years of working experience as research officers;	3 from key secondary school, average teaching experience: 24.33 years; average principal experience: 4.67 years	8 from key secondary schools and (five of them have teaching experience in non-key secondary school);
1 (CQNU): MEd in mathematics education, 18 years of working experience	1 (District level): 14 years of teaching experience in secondary school, 15 years of working experience as research officers;	3 from non-key secondary school, average teaching experience: 19 years; average principal experience: 7.33 years	3 from non-key secondary schools; Teaching experience varies from 4 years to 27 years. Average teaching experience: 13 years.

4.4.3.2 Second category of participants

Based on the first category of participants' recommendations, three expert mathematics teachers, Mr. Zhao, Ms. Qian, and Ms. Sun (all names are pseudonyms), were chosen for this study. Their general background information is listed in Table 4.3. More detailed information is introduced below:

Mr. Zhao

Mr. Zhao is a senior mathematics teacher and *gugan* mathematics teacher in Chongqing. He was about 40 years old at the time of data collection. In 1993, he obtained his first degree in mathematics education from a teachers' college and started working as a junior secondary school mathematics teacher in a village school in Chongqing. The condition in the school was not very favorable; however, he worked very hard to improve his teaching ability through observing other teachers' teaching, studying on his own, reflecting, and seeking help from other teachers. He adapted to the context very soon and started to attend teaching competitions. In 1997, he won second place at a mathematics teaching competition at the Chongqing City level, which was very rare for a village school teacher at that time (according to Mr. Zhao).

In 1998, his performance in the village school provided him a chance to transfer to another secondary school in the same county, and he worked very hard to adapt to the new context. Soon, he began to participate in teaching research activities and published papers. He also trained students to participate in different levels of mathematics competitions, some of whom turned in excellent performances. During his teaching at this school, he won such awards as "Expert Teacher in Chaoyang City" (pseudonym) (教学能手) and "The Top 10 Best Young Teachers of Chaoyang City" (十佳青年教师).

In September, 2002, he moved to a famous key secondary school in Chongqing and, again, worked to adapt himself to the new context. He also took on new challenges, such as tutoring mathematically talented students, mentoring newly graduated teachers, and helping young teachers prepare for teaching competitions, and continued to participate in teaching research. In 2004, he was chosen to attend a "*Gugan* Mathematics Teacher Program" in Chongqing; the following year, he was named *gugan* mathematics teacher in Chongqing. Meanwhile, he played a key role in developing examination papers for the *Zhongkao* in Chongqing. His abilities resulted, in 2004, in his being named Dean of Teaching Affairs at his school, and being promoted to senior mathematics teacher in Chongqing.

Table 4.3. Information of participants in the second category

	Professional Rank	Teaching Experience	Degree	Administrative Duty	Main Awards	Teaching Load Lessons/ Week	Working School	Students Background in Observation Class
Mr. Zhao	Senior *gugan* teacher	15 years -- 5 years of teaching experience in a village middle school; -- 4 years of teaching experience in a junior secondary school in Chaoyang City; -- 6 years of teaching experience in a key secondary school.	bachelor	dean of teaching affairs	The second class award "Young Teacher Teaching Competition in Chongqing" (1997); "Teaching Expert" of Chaoyang City; "The Top 10 Best Young Teachers of Chaoyang City"; "Excellent Class Teacher" of Chongqing	14	key secondary school	47 students; around half of students with poor background compared to average students within his school

Ms. Qian	Senior *gugan* teacher	15 years -- 6 years of teaching experience in an average secondary school in a district in Chongqing ; -- 9 years of teaching experience in a key secondary school.	master	vice-principal	The first class award teaching competition in Chongqing; The first class award teaching competition in China; “Top 100 best teacher” in a district in Chongqing;	7	key secondary school	60 students; around half students with strong background compared to average students within her school
Ms. Sun	Senior *gugan* teacher	15 years -- 8 years of experience in an average secondary school in a district; --4 years of teaching experience in a key secondary school in Chongqing; --3 years of teaching experience in another key secondary school.	bachelor	head of the seventh -grade	The first class award teaching competition in Chongqing; The second class award teaching competition in China; “Excellent Teacher” in Chongqing;	14	key secondary school	58 students; most students with poor background compared to average students within her school

Ms. Qian

Ms. Qian is a senior mathematics teacher and *gugan* mathematics teacher who had 15 years of teaching experience when the data was collected. She began her teaching life in 1993, as a junior secondary school mathematics teacher at an average secondary school in a district in Chongqing City. The working conditions in the school were not very supportive for her growth. Although many students had poor academic backgrounds, she worked very hard to adapt herself to the context, and improved her teaching ability with help from her colleagues and her mentor. As she became a more capable teacher, she started to participate in teaching competitions. In 1998, she won the first class award at a teaching competition in Chongqing and, as a representative of Chongqing, participated in the national teaching competition held that same yearShe won the first class award as well.

Her great teaching performance won her many awards, including "Top 100 Best Teacher" in the district she was working in at that time, and led to her transfer to a famous key secondary school in Chongqing. During her time at this school, she continued to win awards in teaching competitions at the Chongqing City and national levels. After adapting to the context of her new school, she started to take on new challenges, such as training students to participate in mathematics competition at various levels, teaching bilingually (English and Chinese), conducting research, and mentoring young teachers. During her work in this middle school, she participated in a "*Gugan* Mathematics Teacher Program" in Chongqing and completed a mathematics education master course program. Her outstanding performance also won her many awards. She was promoted to senior mathematics teacher in Chongqing in 2003, and appraised as a *gugan* mathematics teacher in 2005. In 2004, she was named a vice-principal of the school.

Ms. Sun

Ms. Sun, in her 40s, was a senior and *gugan* mathematics teacher in Chongqing with 15 years of teaching experience when the data were collected. After obtaining her first degree in 1993, she worked as a junior secondary school mathematics teacher at an average secondary school in a district in Chongqing. The teaching conditions and students' academic backgrounds of this school were comparatively poor at that time. She worked very hard to help her students achieve excellence, and to improve her teaching ability as well. In her second year of teaching, she was fortunate to attend a district teaching competition. She won the first class award and was chosen to attend another teaching competition at the Chongqing City level. She won the first class award again, followed by a second class award at the national level. These awards not only brought her recognition, but also improved her teaching ability remarkably; as a result, she was promoted to junior 1 teacher ahead of schedule and won many other awards, including "Excellent Teacher in Chongqing".

Her outstanding teaching performance also enabled her to transfer to a famous key secondary school in Chongqing, in 2000. After she adapted to the new context, she started to take on new challenges, such as conducting research, mentoring young teachers' and helping them to prepare for teaching competitions. Due to her outstanding teaching ability, she was chosen as a model teacher by Beijing Normal University Press to deliver a lesson using the latest mathematics curriculum reform ideas, which was video-taped and further used as model lesson to instruct other teachers throughout mainland China how to implement reform ideas in their teaching. In 2004, she was promoted to the position of senior mathematics teacher in Chongqing and, the following year, was named a *gugan* mathematics teacher after finishing a "*Guguan* Mathematics Teacher Training Program" in Chongqing. In the meantime, she began work as a key member developing test papers for the *Zhongkao* in Chongqing.

In 2005, she moved to another famous key secondary school in Chongqing, where she continued to perform very well. In addition to continuing her work developing *Zhongkao* examination papers, she also started to take on administrative tasks in her school. In 2007, she started to work as panel head of a certain grade, and worked as chief editor of several student exercise books published by various publishing presses.

4.5 Data Collection Methods

To meet the research objective of the study, semi-structured interview, classroom observation and documentary analysis were used for data collection.

4.5.1 Semi-structured interview

Interview is one of the most widely used methods for gathering qualitative data. It enables participants to "discuss their interpretations of the world in which they live, and to express how they regard situations from their own point of view" (Cohen *et al.*, 2000, p. 267). In other words, interview can be used to "gather data in the subject's own words so that the researcher can develop insights on how subjects interpret some piece of the world" (Bogdan & Biklen, 2007, p. 95). Interviews can gather information that cannot be obtained through observation, and can also be used to verify earlier observations (Ary *et al.*, 2002). There are different qualitative interviews according to the degree to which they are structured - structured interview, semi-structured interview, and unstructured interview (Fontana & Frey, 1994). The choice of interview type depends on the purpose for which it intends to serve in the study. When the purpose is to gather comparable data, the more standardized, structured interview is often more suitable; in contrast, when the purpose is to gather unique, non-standardized, personalized information about individuals' views of the world, the unstructured interview is more suitable (Cohen *et al.*, 2000).

In this study, semi-structured interview was adopted as a major method, as it can be used to "gain a detailed picture of a respondent's beliefs about, or perceptions or accounts of, a particular" (Smith, 1995, p. 9). In this type of interview, a number of pre-determined questions and/or special topics are used. During the interview, questions are typically asked by the interviewer purposely; however, the interviewees have the freedom to digress when they so desire. This allows the interviewer to probe far beyond the answers to the predetermined questions (Berg, 1998). Semi-structured interview also, gives the researcher and participants much more flexibility of coverage, enables the interview to enter new areas, and produces richer data than structured interview or questionnaire survey.

Semi-structured interview was used in this study for gathering data about: 1) how mathematics teacher educators, school principals, mathematics teachers, and mathematics teaching researching officers conceptualize expert mathematics teachers; and 2) expert mathematics teachers' thoughts on their own teaching (characteristics). As mentioned above, in semi-structured interviews, some predetermined questions are constructed in advance. Berliner's (1988) teaching expertise development model, which shows interviewees information about teaching expertise development, was employed in the interview scheduled to explore mathematics educators' description of expert mathematics teachers. In addition, a deeper understanding of interviewees might emerge from asking them to state and justify the differences among teachers at different development stages, which would also be valuable information that reflects how they conceptualized "expert mathematics teacher" (the interview outline was given to every interviewee one week before the formal interview). The major questions included:

1) What percentage of junior secondary school mathematics teachers can be regarded as expert mathematics teachers? Why?
2) Describe an expert mathematics teacher at the junior secondary school level.
3) State the differences and similarities between expert mathematics teachers and teachers at other development stages. (see Appendix 2, 3)
4) How would you define "expert mathematics teacher"?

Semi-structured interview was also used with the three expert mathematics teachers in this study to collect the following information:

1) Basic information about the teacher's work and the observed class, such as the number of students, students' background, and so on (see Appendix 4);
2) The expert mathematics teacher's (brief) personal history, including secondary school, pre-service and in-service experience (see Appendix 4);
3) Their beliefs about mathematics, mathematics learning, and mathematics teaching (see Appendix 5);
4) The teachers' thoughts about their own teaching.

To deeply understand their thoughts on their teaching, semi-structured interviews were conducted before and after every observed lesson. The pre-lesson observation interview questions included: ① what prior knowledge should students have to learn the topic; ② which parts of the content in the lesson are important and/or difficult, how does s/he identify them, and in what ways s/he will deal with them; and, ③ how does s/he plans the lesson (see Appendix 6). The post-lesson observation interview mainly focused on teacher's reflections on her/his teaching and events that happened during the observed lesson (see Appendix 7).

4.5.2 Classroom observation

Observation gives the researcher a chance to examine what is taking place *in situ* instead of using a secondhand source. In addition, observation data can help the researcher enter and understand the research situation (Patton, 1990). Observation can be highly structured, semi-structured or unstructured; the last two demand that the researcher review observation data before attempting any explanations of the phenomenon being observed (Cohen *et al.*, 2000). As to the observer's role is concerned, it may involve: 1) participant observation, in which the observer actively participates and becomes an insider in the event; and 2) nonparticipant observation, in which the observer does not participate or take any active part in the situation (Ary *et al.*, 2002). In this study, the researcher used unstructured observation to let the characteristics of expert mathematics teachers emerge from their teaching and because classroom teaching is too complex for pre-determined instruments. In addition, nonparticipant observation was adopted because the researcher did not want to disturb the participants' classroom teaching. In order to gather more complete data, video-recordings and field notes were taken during the observation process.

4.5.2.1 Video-recording

Video-recording is an important, flexible instrument for collecting audio and visual data, and has been widely used by researchers in education and from other fields. In Powell *et al.*'s (2003) opinion, in methodological terms, video technology lends itself to "a mixture of qualitative and quantitative approaches in both data collection and analyses" (p. 407). The benefits of using video are that video: 1) can preserve more aspects of interaction; 2) allows repeated observation and supports microanalysis and multidisciplinary analysis; 3) enables researchers to leave controlled laboratory settings and enter naturalistic fieldwork; and 4) provides analytical benefits, in that new categories will emerge from the video information (Roschelle, 2000). In addition, video data offer many advantages for data analysis. A main advantage of video data is its permanence; unlike the ephemeral nature of live observations, researchers can view videotaped events as frequently as they wish and in many flexible ways, including through the use of "real time, slow motion, frame by frame, forward, backward," (Bottorff, 1994, p. 246) to attend to their different features.

In the meantime, video makes it possible for researchers to examine the recording events repeatedly and from multiple points of views. As Lesh and Lehrer (2000) pointed out, the analysis of videotapes involves viewing through multiple aspects, including theoretical (*e.g.*, mathematical, psychological and teaching), physical (*e.g.*, observers' notes, transcripts, and videotapes from different cameras), and temporal aspects (*e.g.*, analyses of isolated sessions, analyses of group sessions, and analyses of similar sessions across several groups). Moreover, repeated viewing has the potential to enhance triangulation in data analysis, and to "make possible a cyclical analytical process that takes advantage of the fact that they can be used as both quantitative and qualitative research tools" (Jacobs *et al.*, 1999, p. 718).

Given its advantages, video-recording was used to capture the teaching behaviors in this study. Each teacher was video-taped for one week. The reason for video-taping for one week of instructional practices is straightforward. As found in some research, teachers' teaching may vary depending on the phase of the unit or on students' understanding level of the topic taught (Clarke, 2003; Jablonka, 2003; Mesiti *et al.*, 2003; Shimizu, 2003). Therefore, if just one or two lessons are video-taped, these are unlikely to be an accurate representation of an individual teacher's instructional characteristics. In addition, a one-week period of observation allows students and teachers to become accustomed to the video cameras, which allows more natural information to be obtained.

Since the focus at this stage was to explore characteristics of expert mathematics teachers, only one digital camera, which focused on the teacher, was used. Since this research was to explore expert mathematics teachers' practice in natural classroom settings, the teachers were told not to make any special preparations or instructional changes for the video-taping. Detailed information about the teaching content of the three teachers is listed in Table 4.4.

Table 4.4. Information of observed lessons

Teacher	Grade	Total Number of Observed Lessons	Teaching Content
Mr.Zhao	8	6	Definition, graph and properties of inverse proportion function
Ms.Qian	8	6	Proportional segments, golden section, similar figures and similar polygons
Ms. Sun	7	5	Definition of triangle, angle bisector of triangle, height of triangle, median of triangle

4.5.2.2 Field notes

Video-recording, however, has some disadvantages as it: 1) is incapable of selectivity because of its mechanical limitations; 2) is incapable of discerning the subjective content of behavior being recorded; and 3) is usually unable to convey the historical context of observed objects (Bottorff, 1994). To counter this, field notes were taken in this study. Taking field notes is another common method to record data during classroom observation. Field notes include two main components: 1) a descriptive part, in which a complete description of the setting, the people and their reactions and interpersonal relationships, and accounts of events are considered; and 2) a reflective part, in which the researcher's reflections are included (Ary *et al.*, 2002).

In this study, field notes were taken to capture information on the research settings, including the schools, classrooms and, in particular, events that occurred in the classrooms, such as students' behaviors and their reactions to teachers' questions, which may not be captured by the video. A reflective summary was written by the researcher after every observation. Some parts of the field notes were also used for post-observation interviews.

4.5.3 Documents

Documents are another primary data source in qualitative research. To understand the history and context of a specific setting, some documents should be reviewed. Documents can offer rich information to portray "the values and beliefs of participants in the setting"(Marshall & Rossman, 2006, p. 116). There are three types of documents: 1) personal documents, which are produced by individuals for private purposes and limited use; 2) official documents, which are produced by organizational employees for record-keeping and dissemination purposes; and, 3) popular culture documents, which are produced for commercial purposes to entertain, persuade, and/or enlighten the public (Bogdan & Biklen, 2003).

In this study, personal and official documents were gathered. Personal documents include lesson plans (if any), copies of textbooks, and PowerPoint files used in the observed lessons, and were collected during the classroom observation process. In addition, to explore characteristics of their knowledge, every teacher was asked to develop a knowledge structure picture based on the topic of the observed lessons. Official documents refer herein to curriculum standards and relevant regulations published by government, and were downloaded from the relevant official web sites. Teacher assessment policy and lesson evaluation materials (if any) within the schools in which the three expert teachers were working were also collected.

4.6 Data Analysis

Data analysis is the heart of qualitative research, and the process that most distinguishes it from quantitative research (Maykut & Morehouse, 1994). In qualitative research, data is analyzed inductively (Ary *et al.*, 2002; Johnson & Christensen, 2000) and theory is developed in a "bottom up" (Bogdan & Biklen, 1998, p. 6) manner. In other words, theory emerges from the data. In this study, the *constant comparative method* (Glaser & Strauss, 1968) was adopted to analyze data. This method combines inductive category coding with simultaneous comparison of all units of meaning obtained (Glaser & Strauss, 1968). When a new unit of meaning emerges from the data, it is compared to all other units of meaning, and then is grouped with similar categories. If there are no similar units of meaning, a new category is constructed. Therefore, data analysis is a process of continuous refinement. As the analysis progresses, initial categories may be changed, merged, or omitted, new categories may be generated, and some new relationships may be discovered (Goertz &

LeCompte, 1993). As mentioned above, three different types of information were collected in this study. Detailed information about how these data were analyzed will be provided in the following sections.

As discussed in Chapter Two, a prototypical view of teaching expertise was adopted as the theoretical perspective of this study. “The prototype is a collection of characteristic features of the category: features that instances tend to have but need not have” (Howard, 1987, p. 94). According to prototype theory, “the concept is represented as features that are usually found in the category members” (Murphy, 2002, p. 42). In the prototype view of concept, the way to measure similarity and to identify prototypical features is to track the number of times that each feature occurs in each category (Barsalou & Hale, 1993). Therefore, a threshold to identify prototypical features is needed. This study adopts the threshold suggested by Barsalou and Hale (1993), that “if a feature occurs for more than 50 percent of a category’s exemplars, it might be placed in the prototype” (p. 117). Therefore, those features mentioned by more than 50 percent of the interviewees were included as components of the conception of expert mathematics teachers; features mentioned in less than 50 percent of interviews were not. In addition, those attributes found in at least two of the three expert mathematics teachers were also treated as prototypical features.

4.6.1 Interview data

The interview data here refers only to that collected from the first category of participants. All of these interviews were transcribed by the researcher immediately after the interview concluded. If the interviewee spoke the Chongqing dialect, the transcription was further checked by a native speaker who was also fluent in Putonghua. After this, all the transcriptions were e-mailed to the interviewees to allow them to check for the accuracy and to add or delete relevant information if needed. The whole analysis process is as shown in Figure 4.2 and each of the aspects in the process will be discussed below:

4.6.1.1 Organizing the data

At the beginning of data analysis, interviewees' personal data, such as teaching experience, and school background, were added to every transcript.

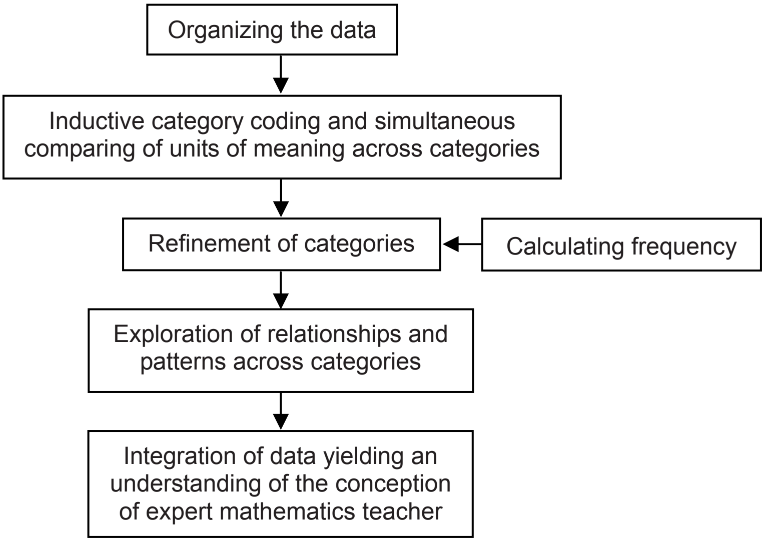

Figure 4. 1. Interview data analysis process

4.6.1.2 Inductive category coding

This stage began during the interview process. The researcher tried to understand what the interviewees said and identify key words for posing followed-up questions. Systematic analysis was then carried out after all the interviews had been completed. The researcher started the analysis by reading the data repeatedly, as inductive analysis requires a thorough sense of what is included in the overall data set (Hatch, 2002). Some key words or phrases were identified during the reading process – for example "strong research ability", "solid knowledge base", "profound understanding of teaching content", and "inquiry-oriented teaching beliefs"– and some provisional categories were formed for these key words. As the reading progressed and a new unit of meaning emerged, the researcher compared it with the provisional categories; if it were not similar to any of the provisional categories, a new category was constructed.

4.6.1.3 Refinement of categories

At this step, the researcher re-read each category in which similar units of meaning had been grouped together to ensure each unit fit the category. Next, a name was given to each category, which served as a code to mark the units of meaning that comprised the category.

4.6.1.4 Calculation of the frequency

As mentioned above, those features mentioned with a frequency of more than 50 percent were considered components of the conception of expert mathematics teachers. Therefore, the frequency of every code was calculated, and those codes whose frequency was less than 50 percent were excluded.

4.6.1.5 Exploration of relationships and patterns across categories

After the above steps, the relationships and patterns across the remaining categories were explored. Some categories were left alone, while some were combined together to form more general categories.

4.6.1.6 Integration of data and interpretation

In the above stage, a set of categories were listed and classified into different levels, and the relationships among the categories were also developed. Next, the researcher re-read the data to discover examples to support relevant codes or sub-codes. Thus, integrated and credible interpretation was subsequently established.

4.6.2 Observation data

At this stage, the main aims were, first, to identify prominent characteristics of every teacher, and then to generalize some common characteristics among the three expert mathematics teachers. Data analysis was conducted from within-case analysis to cross-case analysis, as will be discussed in the following sections.

4.6.2.1 Organizing the data

Before formal analysis, all the interviews were transcribed by the researcher himself. As Mr. Zhao spoke the Chongqing dialect in all the interviews, his transcripts were further checked by a master student who is fluent in both the Chongqing dialect and Putonghua. In addition, transcripts related to interviews of beliefs and teaching history were further checked by the three teachers respectively.

4.6.2.2 Within-case analysis

As mentioned above, different types of information were collected from individual teachers and they were analyzed separately.

Beliefs. Transcripts regarding each teacher's beliefs were examined repeatedly to identify key terms used by specific teachers to express their beliefs about mathematics, mathematics learning, and mathematics teaching.

Teaching practice. The data analyzed at this level included video data, field notes, interview transcripts (pre-observation and post-observation interviews), and copies of lesson plans, textbooks and students' exercises. For each case, the analytical process was generally divided into the following stages:

1) Watching the videos and reading the transcripts to make the researcher become more familiar with their content without intentionally imposing on them a specific analytic aim.

2) Identifying critical events and terms. After becoming familiar with the videos and transcripts, the next step was to identify critical events in the video or terms in the transcript of the teacher; that is, begin open coding (Glaser & Strauss, 1968) or generate some hypotheses.

3) Developing codes and coding. At this step, codes were developed to categorize the rest of the data according to the codes. However, a new code could be established if new events or terms were found. After viewing the videos of all three teachers, the following codes were developed (some codes developed by other researchers were also

adopted or adapted in this study to describe the teacher's teaching). To enhance the reliability of the codes, a research student majoring in mathematics education checked their meaning related to relevant critical events. The codes included:

Lesson structure. Every lesson was divided into different segments according to the purpose of the activities organized by the teacher. The following codes (see Table 4.5) were identified after the researcher had watched all the teachers' videos many times:

Table 4.5. Codes for lesson structure

Code	Descriptions
Reviewing relevant content	The main purpose of the activity is to review concepts, ideas, problem solving procedures as presented in a prior lesson, or which students had learned previously in relation to what will be taught in the current lesson.
Checking homework	The main purpose is to check the answers of homework assigned in prior lessons.
Presenting new topic	The main purpose is to introduce some new content, such as a new concept or a new theorem, which students have not worked on before.
Practice	a) *Demonstrating example exercises.* The main purpose is for the teacher to demonstrate some problem solving strategies which students could use in the following exercises or in future; b) *Practicing and consolidating.* The main purpose is to ask students to practice more exercises to consolidate the newly introduced content.
Summarizing and assigning homework.	The main purpose is to summarize or draw conclusions about the new content presented in the current lessons, or highlight some points to which students should pay attention. It also includes assigning homework related to the newly presented content.

With the help of Studiocode (a video analysis software widely used in the Learners' Perspective Study), every lesson was coded. After that, the reasons why these activities were chosen and why they were organized in a certain way within the lesson were further analyzed in relation to relevant interview transcripts. The ultimate purpose was to discover characteristics of expert mathematics teacher based on her/his choice and organization of these classroom activities.

Teaching approach. To analyze the characteristics of teaching approaches and activities employed by teachers in the observed lessons, codes developed by Mok and Lopez-Real (2006) (see Table 4.6) were adopted, as they had been developed from the analysis of an experienced mathematics teacher's teaching approach in Shanghai and were found to meet the actual situation of this study well.

Table 4.6. Codes for teaching apporach

Code	Descriptions
Exploratory	The focus is on a relatively open or difficult problem which has more than one possible answer; The teacher gave a signal for pair or group discussion; A whole class discussion with the following feature: inviting more than one student to give answers, inviting explanations, inviting peer comments.
Directive	No comment on the student's answer, no attempt to discuss the answer with the other students, simply stating what should be done (*e.g.*, the conventional notation); Emphasis is purely on following a convention; Insistence on precise language; Repetition of what had been learnt in an earlier part of the lesson at a fast pace, using this as a foundation for establishing further knowledge; Insistence on articulation of procedures; Clear and directive definition of a concept or method after an illustrative example or discussion; Teacher plays the role of directing students to work on problems; Probing for "expected" answers; Directive explanation by teacher.
Summarization	Teacher does summarization during the lesson, or to conclude the topics or problems discussed.
Exercises and Practice (sometimes includes whole class checking of exercises)	In the situation of doing textbook exercises, there can be a teacher talking about/explaining the question, and students having seatwork; Teacher checks exercises with students.
Assigning Homework	Teacher assigns homework or questions for students to do at home.

Note. Codes adopted from Mok and Lopez-Real, 2006, pp. 238-239.

With the help of StudioCode, every lesson was coded. Next, time spent on each approach within a lesson and across lessons was calculated and compared to discover characteristics related to the teacher's teaching approach.

Class organization. To analyze how the teacher organized her/his class, codes developed by Mok and Lopez-Real (2006) (see Table 4.7) were adopted.

Table 4.7. Codes for class organization

Code	Descriptions
Classwork	It refers to teacher talk only and teacher-led discussion carried out in a whole class setting. For example, the teacher explains a definition or gives instruction for a worksheet; the teacher asks a question which may be answered by one or several students and there may be some follow-up questions.
Seatwork: Individual	In such activities, students work on a task individually, without any discussions with other students.
Seatwork: Small Group	In these activities, two or more students discuss or do a task amongst themselves.

Note. Codes adopted from Mok and Lopez-Real, 2006, p. 238.

Content coherence. To analyze the coherence of teaching content, every activity (such as hands-on activities, situational problems used by the teacher to introduce relevant concepts, proof of relevant theorems, and exercises) was first treated as a problem. Problems here were defined as “events that contained a statement asking for some unknown information that could be determined by applying a mathematical operation” (Hiebert *et al.*, 2003, p. 41). The standard used in TMISS 1999 Video Study (Hiebert *et al.*, 2003) was adopted to analyze the relationship among problems (see Table 4.8).

Table 4.8. Codes for content coherence

Code	Descriptions
Repetition	The problem is the same, or mostly the same, as a preceding problem in the lesson. It required essentially the same operations to solve although the numerical or algebraic expression might be different.
Mathematically related	The problem was related to a preceding problem in the lesson in a mathematically significant way. This included using the solutions to a previous problem for solving this problem, extending a previous problem by requiring additional operations, highlighting some operations of a previous problem by considering a simpler example, or elaborating a previous problem by solving a similar problem in a different way.
Thematically related	The problem was related to a preceding problem only by virtue of it being a problem of a similar topic or a problem treated under a larger cover story or real-life scenario introduced by the teacher or the curriculum materials. If the problem was mathematically related as well, it was coded only as mathematically related.
Unrelated	The problem was none of the above. That is, the problem required a completely different set of operations to solve than previous problems and was not related thematically to any of the previous problems in the lesson.

Note. Codes adopted from Hiebert *et al.*, 2003, p. 76.

Mathematics exercises. The types of mathematics exercises used during teaching were systematically analyzed. Firstly, the exercises were classified as either: 1) from textbooks, or 2) outside of textbooks. The latter was further divided based on being either: 1) chosen from other sources; or 2) posed by the teacher. After this, every exercise was further analyzed using the following three steps (see Table 4.9).

Table 4.9. Analysis procedures and codes for mathematics exercises

Procedures	Code	Descriptions
Types	Open-ended Problems vs. Closed-ended Problems	An open-ended problem is a problem with several or many correct answers. A closed-ended problem is a problem with one answer, no matter how many different approaches there are to reach the answer. (Zhu & Fan, 2006, p. 613)
	Application Problems vs. Non-application Problems	A non-application problem is a situation that is unrelated to any practical background in everyday life or the real world. An application problem is a problem related to or arises under the context of a real-life situation. In this study, problems related to other school subjects, like physics, chemistry, geography and so on are also coded as application problems. (Zhu & Fan, 2006, p. 613)
	Combination Problems vs. Non-combination Problems	In this study, combination problem is a situation that is related to other mathematics knowledge students have already learned or will learn in the future. A non-combined problem is a situation that only relates to the mathematics knowledge of a particular topic which students are studying currently.
Complexity	I Low complexity	Solving the problem, using conventional procedures, requires four or fewer decisions by students. The problem contains no sub-problems.
	II Moderate complexity	Solving the problem, using conventional procedures, requires more than four decisions by the students and can contain one sub-problem at most.
	III High complexity	Solving the problem, using conventional procedures, requires more than four decisions by the students and contains two or more sub-problems.
Ways to work on every exercise	Characteristics of the ways to deal with an exercise were also analyzed, which include who discovers solutions, use of alternative solution and how to get different solutions, by the teacher or by students	

Note. Codes for "Complexity" were adopted from Hiebert *et al.*, 2003, p. 71.

4) Categorizing and refining categories. In this stage, every critical event or characteristic within each individual lesson was constantly compared with events and characteristics identified in other lessons. Similar events and characteristics were further categorized to form a new category. Next, each category in which similar units of meaning had been grouped together was further examined to determine whether its units fit the category, after which a list of characteristics was identified for further across-case analysis.

4.6.2.3 Across-case analysis

After identifying every teacher's critical features, further exploration of similar characteristics among the three teachers was conducted via case analysis based on the following:

Teachers' beliefs. Every critical term identified from individual teacher's transcript of beliefs was compared with those of the other two teachers. If at least two teachers were found to make a similar statement about mathematics, mathematics learning, and mathematics teaching, this statement was viewed as a common belief among the three expert mathematics teachers.

Teaching practice. Every critical characteristic of an individual teacher was compared with those of the other two teachers. If a characteristic were shared by at least two teachers, this characteristic was viewed as a common characteristic of all three teachers.

4.6.2.4 Selecting data excerpts to support relevant codes or sub-codes

After common characteristics of the three teachers were identified, the researcher watched the relevant videos or re-read the relevant transcripts to search for examples that could be used to support relevant codes or sub-codes in data reporting.

4.6.3 Documentary data

Personal documentary data – that is, lesson plans, copies of textbooks and students' exercises – were analyzed, together with the video data. Official documents were analyzed to identify social and cultural factors that could be used to explain the similarities found in the participants' conceptualizations of expert mathematics teachers and the common characteristics found among the three expert mathematics teachers.

4.7 Validity of the Study

Validity in qualitative research is defined as how accurately the account represents participants' realizations of the social phenomena and is credible to them (Schwandt, 1997). In order to avoid invalid conclusions, the following measures were taken:

1) *Triangulation* (Denzin, 1978). In this study, data triangulation and methodological triangulation were used to achieve validity. When exploring the conception of expert mathematics teachers, the researcher interviewed mathematics teachers with different teaching experiences, principals with mathematics education background, mathematics teaching research officers, and mathematics teacher educators, so as to triangulate the data sources. When exploring common characteristics of expert mathematics teachers, the study employed semi-structured interviews, observations, and documents to contribute to methods triangulation. In the meantime, to explore teachers' subject matter knowledge, in addition to the researcher's observations and interviews, teachers were all asked to draw knowledge structure pictures, which could be considered data source triangulation.

2) Collecting *rich data* to the greatest extent possible (Miles & Huberman, 1994; Patton, 1990). The semi-structured interviews employed in this study yielded very rich data related to the conception of expert mathematics teacher and the three expert teachers' thoughts on their teaching, beliefs, etc. In particular, video recordings provided rich data on the three expert mathematics teachers' teaching practices.

3) *Thick, rich description* (Denzin, 1989). When interpreting the data, the researcher cited relevant examples or evidence from the interviewees' answers to support the codes. In addition, this "enables readers to make decisions about the applicability of the findings to other settings or similar contexts" (Creswell & Miller, 2000, p. 129).

4*) Member checking* (Lincoln & Guba, 1985). As mentioned above, the interview transcripts regarding the conception of expert mathematics teacher were checked by each interviewee, as were those relating to beliefs and teaching history. The meaning of the codes developed was checked by a research student in the field of mathematics education.

5) Researcher reflexivity (Creswell & Miller, 2000). In qualitative research, the researcher is also an important research instrument, as s/he will inevitably influence the research conclusions. In this study, the researcher tried his best not to let his knowledge and experience influence the data source. When producing the interview outline, the researcher purposely did not include any questions related to the conception and characteristics of expert mathematics teachers that had been identified in the literature. When conducting the interviews, the researcher let the interviewees speak in their own dialects because it was more comfortable for them and enabled them to provide the richest possible information. In addition, the researcher purposely did not mention any new concepts or topics to the interviewees.

4.8 Research Ethics

In addition to methodology, ethics is another important concern in this study. Johnson and Christensen (2000) state five guidelines for ethical acceptance in research involving humans: participants' informed consent; justification of deception; participants' freedom during the research process; the avoidance of physical and/or mental harm resulting from the research; and anonymity and confidentiality. Based on these, the following points were given special consideration when this study was designed and conducted:

1) The purpose and use of this study were clearly explained to the participants at the outset. After the participants agreed to participate in this study, all were told they had the right to refuse to answer any question or questions at any time during the process. In addition, the participants chose the time and place for all interviews to ensure they felt comfortable. The participants also chose the date, content and specific class for classroom observation, to again ensure that they were comfortable. The students in each observed class were also assured before the video-taping began that the camera would not focus on them.

2) The interviewees and the schools from which the informants were chosen were kept anonymous and confidential. In addition, the interviewees were guaranteed that their real identity and background information would not be released to the public. In short, all possible

measures were taken to protect participants from any form of potential harm resulting from the research.

4.9 Summary of the Chapter

This chapter introduced how the present study gathered and analyzed data. The following chapters present the findings of this systematic and comprehensive process of data collection and analysis. Specifically, the conceptualizations identified from the study will be reported in the next chapter – Chapter Five. Common characteristics, such as beliefs, knowledge, teaching wisdom, and teaching practice, found among the three expert mathematics teachers will be reported in Chapter Six and Chapter Seven, respectively.

Chapter Five

Conception of Expert Mathematics Teachers

5.1 Introduction

This chapter reports the findings on the conception of expert mathematics teachers identified from interviews conducted with the 21 mathematics teachers, school (vice) principals, teacher educators, and mathematics teaching research officers. Three main themes were identified from the interviewees' descriptions – knowledge, ability, and traits. Under each theme, some sub-themes were grouped. Details of each theme and sub-theme will be discussed in the following sections.

5.2 Knowledge

The first important factor emphasized by interviewees is that an expert mathematics teacher should have a profound and broad knowledge base, including knowledge of mathematics, theory, curriculum, learners, and other subjects, as reported below.

5.2.1 Knowledge of Mathematics

According to the interviewees' descriptions, an expert mathematics teacher should have a firm mathematics base, which means 1) solid mathematics content knowledge; and 2) strong problem solving ability.

5.2.1.1 Solid mathematics content knowledge

Every interviewee mentioned that an expert mathematics teacher should have a profound mathematics content knowledge base, which mainly includes 1) comprehensive mathematics knowledge base; 2) profound understanding of mathematics knowledge; and 3) a connected knowledge structure:

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Comprehensive mathematics knowledge base. It was emphasized that, in comparison with teachers at other developmental stages, an expert mathematics teacher should have a broader and more comprehensive mathematics knowledge base. This includes:

1) Familiarity with various branches of mathematics. An expert mathematics teacher should know mathematics in a variety of areas, such as algebra, geometry, and statistics. S/he should have a full picture of the overall structure of mathematics in her/his mind. Even though s/he may not understand a particular area deeply, s/he should know basic concepts and principles, or at least have some basic ideas about this branch. As one teacher said:

> *I think the first facet is that s/he [expert mathematics teacher] has a very wide knowledge scope in mathematics itself. S/he knows more mathematics [than teachers at other stages do]. S/he may even be familiar with some fields that other teachers never touch on as secondary school mathematics teachers. (Teacher 1)*

2) Knowing the development process of mathematics well. In the interviewees' opinions, an expert mathematics teacher should know the developmental or evolutionary process of mathematics very well. S/he should not only know the historical development of particular mathematic concepts, but also current developments and trends in mathematics as a whole. In addition, s/he should be familiar with some cutting edge research on advanced mathematics topics. As one teacher educator pointed out:

> *At first, s/he [expert mathematics teacher] should have a relatively complete mathematics knowledge structure. In the meantime, s/he is familiar with the historical backgrounds or origins of some mathematical topics, including their current development situation and tendency. (Teacher educator 1)*

Profound understanding of mathematics subject knowledge. Another aspect emphasized by every interviewee is that an expert mathematics teacher should understand mathematics content deeply; this was viewed as a critical difference between expert and non-expert mathematics teachers. Some interviewees pointed out non-expert mathematics teachers may also have a relatively comprehensive knowledge base, but may not understand some topics as deeply as expert mathematics teachers do. According to some interviewees, it is only when a teacher understands mathematics profoundly that can s/he grasp the essence or inherent meaning of mathematics, which was thought to be a level that an expert mathematics teacher should reach. As one mathematics teacher mentioned:

> *An expert mathematics teacher, compared with teacher at other development stages, should have more mathematics knowledge. Moreover, s/he should understand it more thoroughly and deeply. In addition, s/he should grasp the essence of relevant knowledge. (Teacher 2)*

An expert mathematics teacher was further expected to understand mathematics from a relatively higher perspective or level, and to appreciate the thinking and methods underlying mathematical concepts and theorems. Some interviewees even stated that an expert mathematics teacher should have her/his own unique understanding of mathematical thinking and methods. As one teacher argued:

> *S/he [expert mathematics teacher] should have her/his own unique understanding of mathematics thinking and mathematics methods. How to say, if as an expert mathematics teacher, s/he only can transmit the content in textbooks to her/his students, I will not think s/he is an expert mathematics teacher. This is completely a process of transmitting knowledge. …. As an expert mathematics teacher, I think s/he should have her/his own unique understanding of mathematics thinking and mathematics methods [underlying relevant mathematics knowledge]. (Teacher 3)*

Connected knowledge structure. An expert mathematics teacher should be able to organize her/his knowledge systemically, as a net, and know every node well. Even as an expert mathematics teacher at the junior secondary school level, s/he should also understand mathematics at the primary, senior secondary and university levels as well. Moreover, s/he should be able to discover connections among relevant topics in different branches and at different grade levels. As one principal noted:

> *As to some similar topics, s/he [expert mathematics teacher] should clearly understand the connections among them. That is, s/he must know the connection of knowledge, from Grade 7 to Grade 9, or even from primary school to secondary middle school, or to some part of mathematics at university level, s/he knows the connections very well. As to every chapter, every particular topic, s/he knows [the inner connections] very well. In the meantime, s/he knows the relationship among [this topic or this chapter] and other topics very well. (Principal 1)*

This statement highlights another important characteristic of expert mathematics teacher's subject knowledge – connectedness, an understanding of the linkages both within mathematics and between mathematics and other subjects. This statement was echoed by many other interviewees. For example, one teacher said:

> *As an expert mathematics teacher, s/he should also know the connections between knowledge in different subjects, at least some basic knowledge from other subjects. As an expert mathematics teacher, s/he should not only know mathematics, s/he should also know other subjects. For example, when we teach contour line, we can use some examples from geography [to teach this topic]. (Teacher 4)*

5.2.1.2 Strong mathematics problem solving ability

Seventeen out of the 21 interviewees mentioned that an expert mathematics teacher should have strong mathematics problem-solving abilities. Eight went so far as to suggest that an expert mathematics teacher should be good at solving Mathematics Olympics Competition problems. However, some interviewees cautioned that having problem-solving skills or ability alone does not make one an expert mathematics teacher. According to their responses, problem-solving ability has the following characteristics: 1) being sensitive to solutions; 2) being able to approach problems from a higher perspective; and 3) being able to generalize ways of solving similar problems.

Being sensitive to solutions. An expert mathematics teacher must be able to identify possible directions for solutions to a problem quickly, even if s/he may not be able to solve it thoroughly. In other words, s/he should be sensitive to how to find a solution to, or to approach a problem when s/he encounters a new or difficult problem. As one school principal mentioned:

> *Solving mathematics problems is different from solving other kind of problems. Her/his [expert mathematics teacher] ability of solving mathematics problems should be very strong. S/he is very sensitive of what kind of mathematics thinking and methods used in a particular problem. For example, when s/he [expert mathematics teacher] meets a problem, no matter how difficult it is, what kind of mathematics thinking or what kind of method could be used, from which direction to start to approach it, s/he can quickly get them. Maybe s/he cannot solve this problem as quickly as some talented students do, s/he knows from which direction to start her/his approaching, no matter how difficult a problem is, s/he can find a way to solve it. (School principal 2)*

Approaching problems from a high perspective. An expert mathematics teacher should be able to approach a problem from a higher perspective, rather than only from an elementary mathematics perspective, even though s/he has no need to teach students in this way. That is, instead of relying solely on methods contained in junior secondary school textbooks, s/he should be able to use advanced mathematics thinking or methods to guide her/his approach. In addition, while non-expert or novice teachers may be easily constrained by the problem itself, an expert mathematics teacher should not only be able to solve the problem, but also to reflect on the problem-solving process, identify the essence of the problem (*e.g.,* the kinds of knowledge it examines and the mathematics thinking and methods embodied in it), and make extensions.

Generalizing a way to solve a series of similar problems. An expert mathematics teacher should be able to discover or generalize ways in which to solve a series of similar or related problems. As one teacher educator stated:

> *As to problem solving ability, s/he [expert mathematics teacher] should be able to know how to solve a series of problems. That is, s/he can find a way to solve some similar problems. S/he may know many special methods to some particular or difficult problems, and s/he may also know some ways to solve some similar problems. (Teacher educator 2)*

This educator's statement further indicates that an expert mathematics teacher should have methods of solving certain types of problems that differ from those of other teachers.

5.2.2 Knowledge of theory

Every interviewee stressed that an expert mathematics teacher should have a profound theoretical knowledge base, particularly regarding educational and psychological theories (such as theories about instruction, learning theories, cognitional theories, and theories underlying some ideas in curriculum standards). Some interviewees even mentioned that expert mathematics teachers should have theoretical knowledge equivalent to that of mathematics teacher educators at the university level. From the interviewees' perspective, it is having a profound theoretical knowledge base that makes an expert mathematics teacher a scholar. In addition, having such a base, together with strong research ability (as will be reported in 5.3), makes a mathematics teacher outstanding or prominent enough to be called an expert teacher. In other words, theoretical knowledge and research ability make a teacher an *expert;* without either of them, s/he will always be, at best, a proficient or competent teacher, no matter how well s/he teaches or how well her/his students perform on examinations. As one principal stated:

> *If you want to be an expert mathematics teacher, I think if you do not have theoretical quality, you are definitely just a craftsman, you are a Jiaoshujiang(教书匠), you are not an expert. Therefore, I think that an expert mathematics teacher should have a profound educational and pedagogical theory base. What is more, you cannot only have some theories, if you want to be an expert, you should have your own critical or unique understanding of*

> *these theories. [Basing on this], and you should be able to develop your own theories, which can be used as references or studied by other teachers. (Principal 3)*

This principal's statement further points out that an expert mathematics teacher should be able to develop her/his own unique opinions and have an understanding of education theories that is based on actual situations, rather than simply accepting them without reflection or critical judgement. According to some interviewees' descriptions, proficient and competent teachers may sometimes also have theoretical knowledge; however, it is the expert teacher's unique understanding of these theories, and her/his ability to develop her/his own theories based on teaching experience that separates her/him from proficient or competent mathematics teachers. As one teacher argued:

> *Nowadays, there are many theories. As an expert mathematics teacher, I think that s/he should refine them according to her/his own teaching experience. Right? What I said is that s/he should be able to form her/his own educational theories basing on her/his own experience. Nowadays, many teachers, including some very good teachers, use those theories developed by some educators. It seems that they do not have their own educational theories. (Teacher 2)*

Furthermore, an expert mathematics teacher should be able to interpret teaching behavior and educational phenomena at a theoretical level, rather than being constrained by the phenomena. This means that s/he should be able to make tacit knowledge or experience explicit, to a certain degree. This kind of interpretation was thought to be useful for other non-expert teachers, because they can gain insights from it.

Expert mathematics teachers should also be able to connect theories with practice. It was pointed out by many interviewees that, unlike those who teach without knowing or understanding underlying principles, an expert mathematics teacher clearly realizes the connection between her/his teaching and relevant theories. However, an expert mathematics teacher should connect theories with practice critically, rather than blindly accept them simply because educationists or psychologists have recommended them. This indicates that the theoretical knowledge possessed by expert mathematics teachers should not be detached from actual situations, but should be used to explain, structure and justify teaching practices. As one teacher stated:

> *As to those theories, like educational theories or pedagogical theories, you [expert mathematics teacher] could not have all those theories. You should have those theories that can be linked with your teaching practice. You cannot only have some pure theories, that is to say, your theoretical knowledge should be able to connect with your practice. Only those theories which can be linked with your teaching practice are actually useful theories in secondary school teaching. (Teacher 5)*

5.2.3 Knowledge of learners

Each interviewee mentioned the need for expert mathematics teachers to know her/his students' characteristics well. As a school principal pointed out:

> *An expert mathematics teacher at junior secondary school level cannot merely be an expert of solving mathematics problems. S/he must be an expert of researching students' problems. S/he should know her/his students very well. (School principal 4)*

Many interviewees pointed out that, in comparison with students in primary and senior secondary schools, junior secondary school students have their own characteristics. To teach students effectively, an expert mathematics teacher should know those characteristics well, such as their current knowledge base, family backgrounds, interests, personality, learning habits, strengths, weaknesses, and gender differences.

Some interviewees pointed out that the development of students' mathematical ability and thinking is a long-term process. At different age levels, students' characteristics also differ. An expert mathematics teacher should not only know the whole development process from primary school to senior secondary school well, but also students' situation at their current school level. In addition, some interviewees thought that students' thinking and ability are also different at different times, and that an expert mathematics teacher should be able to notice these differences, and adapt her/his teaching methods and theories accordingly.

In addition, an expert mathematics teacher should also appreciate the differences between individual students in a given class. Some interviewees pointed out that individual students' abilities, thinking patterns, interests and habits differ due to differences in cognitive development, family background and knowledge base. For example, a teacher pointed out:

> *We cannot say that there is absolutely no difference among individual students. It is impossible. Some students are very good at mathematics; however, some students cannot learn mathematics well even though they try very hard. … Some students have very strong calculation ability, some students have strong logical reasoning ability, and some students have very strong spatial visualization ability. There exist some differences, definitely. For example, two students, one can easily understand a concept, the other one may not be able to understand it no matter how many times you explain. Therefore, as an expert mathematics teacher, you should know their individual differences well and also know how to teach them effectively. (Teacher 6)*

5.2.4 Knowledge of curriculum

This theme mainly includes knowledge about curriculum standards, textbooks, and examinations, which will be reported in the sub-sections below.

5.2.4.1 Deeply understand curriculum standard and syllabus

Fifteen out of the 21 interviewees stressed that an expert mathematics teacher should be very familiar with the current direction of curriculum reform, and with the ideas and thinking contained in newly-released curriculum standards. Moreover, for every topic, an expert mathematics teacher should portray a more accurate grasp of teaching requirements and a greater ability to apply this knowledge in practice than non-expert mathematics teachers. In other words, s/he should not teach too easy nor too difficult content to his/her students. In addition, expert mathematics teachers should understand relevant ideas in curriculum standard and syllabus more deeply and essentially than non-expert teachers. As one teacher pointed out:

> *As to expert mathematics teacher, s/he should think about why this topic is stated like this in the curriculum standard. As to every statement, every statement to a particular topic, s/he will really study it, think about it and reflect on it. Why it is stated in this way. S/he has her/his own understanding and opinions. Some teachers at other development stages do not read or care about this at all. (Teacher 8)*

Some interviewees further emphasized that an expert mathematics teacher should be able to interpret relevant ideas and requirements in the curriculum standard, understand them, and express them in her/his own words. Moreover, in many interviewees' opinions, an expert mathematics teacher should systematically study both the reasonable and unreasonable ideas contained in the new curriculum reform. For unreasonable ideas and requirements, s/he could provide insightful and reasonable suggestions for further modification. An expert teacher should be able to facilitate the implementation of the current curriculum standard and offer insights on the overall development of mathematics education at the junior secondary school level.

5.2.4.2 Knowing textbooks well

Eighteen out of the 21 interviewees emphasized that expert mathematics teachers should have a greater understanding of textbooks than do regular teachers. In particular, after many years of teaching, s/he should develop a unique understanding of their content and ways in which to use them. The most frequently mentioned facets include:

① Familiarity with the structure of textbooks, which means knowing the overall arrangement of topics, their sequence, and their connections with each other. Expert mathematics teachers should have in mind a clear and complete picture of textbooks' knowledge structure, not only in junior secondary school textbooks, but also in primary, senior secondary, and even (to a degree) university textbooks. That is to say, an expert mathematics teacher should very clear about in which grade students once learned a topic or will learn a similar topic.

② Knowing students' difficulties. An expert must know which textbook topics are particularly difficult for students at a certain time or junior secondary school grade. Moreover, for a particular topic, s/he should fully understand individual students' difficulty.

③ Understanding textbook writers' intentions. Compared with non-expert mathematics teachers, an expert mathematics teacher should be more able to understand textbooks from the textbooks writers' perspective and understand why a particular content is arranged and written as it is;

④ Knowing the strengths and weaknesses of textbooks. Some interviewees pointed out that textbook writers cannot handle everything perfectly, and that it is not possible for one set of textbooks to meet every student's every need. An expert mathematics teacher should recognize the weaknesses and strengths of textbooks and demonstrate the ability to replace them with materials that are more suitable.

5.2.4.3 Acquaintance with High School Entrance Examination

Nineteen interviewees emphasized that, compared to non-expert teachers, an expert mathematics teacher has more knowledge about how to evaluate students' learning. Particularly, for every topic, s/he should know its examination requirements in future examinations, especially in the *Zhongkao*. Familiarity with the *Zhongkao* was an important factor used by interviewees to judge whether a mathematics teacher is qualified as an expert teacher. In addition, some interviewees even thought that an expert mathematics teacher should have the ability to influence the direction or tendency of the *Zhongkao* within a district. According to their descriptions, being acquainted with *Zhongkao* has the following means:

① Possessing rich information related to the *Zhongkao*. An expert mathematics teacher should know the direction and tendencies of the *Zhongkao* in her/his own city and elsewhere. Some interviewees even stated that an expert mathematics teacher can anticipate the direction of the coming *Zhongkao* and what kinds of problems are most likely to be used. As one teacher pointed out:

> *As to the Zhongkao, s/he [expert mathematics teacher] should never stop collecting and studying relevant information. The*

> *direction and types of problems used in Zhongkao keep changing every year. S/he should compare problems used in Zhongkao in the past three or four years. S/he should compare problems used in Zhongkao in Chongqing, in Chengdu, Shanghai, Beijing, and other provinces or cities as well. After her/his comparison, [s/he] knows the proportion and frequency of a certain topic [is examined]. …, In addition, s/he should study the problem styles in Zhongkao, like how many application problems, how many pure mathematics problems, how many cross-discipline problems, and so on are used in Zhongkao. (Teacher 9)*

Moreover, for a particular topic, an expert mathematics teacher should grasp its role in the Zhongkao *more accurately than non-expert teachers do. As a teacher pointed out:*

> *[for a particular topic], what is its requirement in Zhongkao, to what difficulty I should teach, what content I need to let students understand,…, an expert mathematics teacher knows better than us [non-expert mathematics teacher].(Teacher 5)*

②Integrating *Zhongkao* information in regular teaching. An expert mathematics teacher should be capable of integrating, or combining *Zhongkao* information with her/his teaching to ensure students have the necessary skills for *Zhongkao* or other examinations. As a teacher stated:

> *As to our mathematics teaching, I think if a teacher can make her/his students be familiar with problems used in Zhongkao, it can give them a big help when they actually take Zhongkao and can make students study more effective. An expert mathematics teacher should be more capable of using problems or information from Zhongkao in her/his ordinary teaching. (Teacher 10)*

③ Knowing how to develop *Zhongkao*-related problems. The ability to develop *Zhongkao*-related problems was also stressed by these interviewees. As a teacher educator pointed out:

> *S/he [expert mathematics teacher] knows how to evaluate a student's understanding of relevant topics. If students' ability is*

divided into different facets, s/he knows how to evaluate them, that is, s/he knows to use what kind of ways or problems to test students' understanding, or to test to what degree students understand this or that topic. This is not a simple job. Therefore, we always choose expert mathematics teachers to develop test problems. It is difficult for teachers at other stages. (Teacher educator 2)

5.2.5 Knowledge about other subjects

Every interviewee emphasized that, in addition to having a profound knowledge base in mathematics, an expert mathematics teacher should also have a broad and deep knowledge foundation in other fields. In these interviewees' opinions, an expert mathematics teacher should be knowledgeable about the past and the present, and be conversant with both China and the West. As a teacher educator pointed out:

I think that this teacher [expert mathematics teacher] should have very broad and rich literature knowledge. S/he cannot only know mathematics. Like some teachers, they are very good at teaching mathematics, however, they know very little about other fields, like literature, and science. I do not think they can be called expert mathematics teachers. They do not have the qualification. …. Actually, mathematics is very popularly used in different subjects, like physics, chemistry, or even aesthetics, arts, and so on. I think if you were an expert mathematics teacher, your knowledge about these subjects should not only be broad and wide, but also be deep. (Teacher educator 1)

Regarding knowledge of other subjects, an expert mathematics teacher should have a broader knowledge base than a non-expert, which can facilitate her/his professional development and make her/his teaching more vivid and closely related to real life. As a mathematics teacher mentioned:

If you [expert mathematics teacher] do not integrate other subject knowledge into your teaching, you only teach mathematics subject knowledge. After some time, your students will feel that *your teaching content is not so rich; they will think that your teaching is strictly according to textbooks. Therefore, they will lose interests to your teaching. (Teacher 10)*

5.2.6 Discussion

This section has reported how the interviewees conceptualize expert mathematics teachers' knowledge. As found in previous studies on expert teachers (*e.g.*, Berliner, 2001; Cowley, 1996; Smith & Strahan, 2004; Sternberg & Horvath, 1995), a broad and profound knowledge base is a very important factor in determining whether a teacher is an expert. The 21 interviewees emphasized that an expert mathematics teacher should be knowledgeable in mathematics, educational and psychological theory, curriculum, students, and other subjects. Theoretical knowledge and knowledge about the *Zhongkao* or similar examinations were emphasized by the interviewees, but were seldom mentioned in previous studies, especially those conducted in Western countries.

The characteristics of knowledge described by the 21 interviewees have certain social and cultural roots. Historically, teachers in mainland China were expected to be knowledgeable. Confucius emphasized that a wide and broad knowledge base is a prerequisite for being a teacher (Sun & Du, 2009) and that a teacher should painstakingly and insatiably enhance her/his knowledge base. In addition, in mainland China, the primary professional duties of a teacher have been defined as teaching students "to learn and acquire knowledge, skills, and values" (Yang *et al.*, 1989, p. 49). From this perspective, a teacher can be seen as an old master who possesses knowledge that can be transmitted to the younger generation (Paine, 1990), and a good teacher "is distinguished by possessing an exceptional amount of knowledge" (Paine, 1990, p.51). It is therefore not difficult to understand why the interviewees emphasized the importance of knowledge to expert mathematics teachers. Especially the subject knowledge of a teacher has been highly emphasized in China. As Leung (2001) stated, while "expertise in pedagogy is important, a good grasp of the subject matter is more important. The teacher should primarily be a scholar before she is able to play the role of a facilitator of learning" (p. 45). From this perspective, a mathematics teacher is an expert or a learned figure (a scholar) in mathematics. This might make the interviewees think that an expert mathematics teacher should have a profound mathematics knowledge base, even in advanced mathematics.

Moreover, in the Chinese traditional teaching culture, as emphasized by Confucius, a teacher should be able to discover individual students' characteristics after interacting with them for a period of time, and should be able teach them according to those characteristics. This belief might lead the interviewees to emphasize that an expert mathematics teacher should not only understand the class as a whole, but also each individual student – including their background, interest, habits, prior knowledge base, and cognitive development process.

China's mathematics teaching tradition might be another important influence. In China, problem solving is not only an instructional goal, it is also an important instructional approach (Cai & Nie, 2007; Shao *et al.*, 2012). Typical problem-solving activities in Chinese mathematics class include "'one problem, multiple solutions', 'multiple problems, one solution', and 'one problem, multiple changes' " (Cai & Nie, 2007, p. 459). This kind of mathematics teaching tradition might lead the interviewees to emphasize that an expert mathematics teacher should have strong problem-solving abilities, such as being able to generalize ways of solving similar problems so as to be able to achieve the goal— "multiple problems, one solution".

The emphasis on knowledge in curricula at the pre- and in-service training stages is another influence. Curriculum in China has long been described as academic-oriented (Williamson & Morris, 2000), with a great deal of time being spent on advanced mathematics courses at both the pre-service (Li, Huang, & Shin, 2008; Li, Huang, & Yang, 2011; Yang *et al.*, 2009) and in-service stages (Ma, 2000). This kind of curriculum tradition might make interviewees think that an expert mathematics teacher should understand mathematics deeply and should have a comprehensive mathematics knowledge structure. At the same time, curriculum related to theory (*e.g.*, educational and psychological theory) is a major part of the pre-service stage. Pre-service teachers are required to take three theory-related courses – pedagogy, psychology, and mathematics education. Moreover, at the in-service stage, teachers need to take many additional theoretical courses (Ma, 2000). This training experience could lead them to think that an expert mathematics teacher should have a profound theoretical knowledge base.

Examination culture, which has been said to dominate teaching practice in China (Li, 2006; Tu, 2009; Wu, 2012; Zhang & Ren, 1998), constitutes another important influence, as it burdens teachers with the task of imbuing students with skills that they can employ in future examinations (Li, 2006; Ma *et al.*, 2002; Zheng, 2006).As such, even though knowledge of examinations is seldom mentioned in previous studies conducted in Western countries, it is easy to understand why the interviewees emphasized that an expert mathematics teacher should know examination requirements well, and should be able to anticipate the *Zhongkao* and integrate examination information his/her regular teaching. Usually, outstanding teachers of good reputation are chosen to develop test papers for the *Zhongkao*, which might make some interviewees emphasize the ability to develop *Zhongkao* problems.

The practice of teaching profession development activities is another influence. As introduced in Chapter Three, teachers in mainland China tend to work together to learn to how to teach lessons well and to learn the theoretical underpinnings of good instruction (Li, Qi, and Wang, 2012; Paine, 1993; Paine *et al.*, 2003; Tsui & Wong, 2009; Wang & Paine 2003; Yang, 2009; Yang & Ricks, 2012). Since theoretical underpinnings are emphasized, this might make the interviewees stressed the importance of having a profound theoretical knowledge base for expert mathematics teachers. Similarly, the emphasis during in-service training courses on deepening teachers' understanding of mathematics content and textbook structures (Huang *et al.*, 2011; Li, Tang, and Gong, 2011) might also lead interviewees to think that expert mathematics teachers should understand mathematics deeply and know textbooks well.

Finally, the requirements outlined in school-level documents might influence how interviewees conceptualize expert mathematics teachers. Some school documents require teachers to enhance their knowledge and, in particular, to "have a good command of the subject s/he is teaching". For example, in a document in Ms. Sun's School, it is clearly stated:

> *Teachers should have broad and profound knowledge base and continue to study the subject s/he is teaching. …, Teachers in arts should have knowledge in mathematics, physics, chemistry, geography, biology and other technology knowledge; teachers in science should also be familiar with arts, history, philosophy and other knowledge in humanities. All teachers should study Chinese, English, and improve their knowledge of education and psychology.*

5.3 Ability

Another important factor in judging whether a mathematics teacher is an expert mathematics teacher is the various abilities s/he should have. According to the interviewees, an expert should have excellent research and teaching abilities, and exceptional mentoring skills.

5.3.1 Research ability

Research ability is generally among the first factors used by the interviewees to determine whether a teacher is an expert mathematics teacher or not. According their responses, research ability was a critical and fundamental factor differentiating expert and proficient mathematics teachers. "Research ability" was viewed as an important facilitative factor during a teacher's professional development; without it, a teacher would not be able to reach the final stage of Berliner's (1988) teaching expertise development model — expert teacher. Once a teacher becomes an expert, s/he should have excellent research abilities and a strong passion for research. As a teacher educator mentioned:

> *Of course, s/he [expert mathematics teacher] should be capable in teaching. In addition, s/he should have research ability. If s/he is only good at teaching, I think that s/he only has the qualifications to be a proficient teacher or a competent teacher. Only when s/he has both abilities, s/he can be a real expert mathematics teacher. (Teacher educator 1)*

Research ability here includes the ability to: 1) research teaching and educational phenomena; 2) theorize experience; and 3) conduct projects.

5.3.1.1 Ability to research teaching and educational phenomena

The first aspect related to research ability mentioned by the interviewees was the ability to conduct research into teaching and educational phenol-mena, and into (junior secondary level) mathematics itself. An expert mathematics teacher should have the ability to explore or study alternative mathematics problem solving methods or strategies, and to generalize or discover a general method to a series of similar mathematics problems at the junior secondary or other levels. In addition, s/he should be able to research teaching and learning methods, students' difficulties, students' characteristics, ways to deal with textbooks, ideas in the curriculum standard, tendencies in the *Zhongkao*, and the implementation of some educational theories. Based on these, s/he can publish journal articles, which can provide useful information for teachers at other developmental stages. In the interviewees' opinions, the ability to publish journal articles or even books was the very embodiment of research ability. Those teachers they mentioned as examples of expert mathematics teachers had all published some journal articles or books. Some interviewees even used the number of papers a teacher had published as a criterion to judge whether s/he is an expert mathematics teacher.

5.3.1.2 Ability to theorize experience

According to some interviewees' descriptions, a teacher's growth or development is a function of their experience accumulation. Normally, it takes a teacher at least ten years to develop into an expert mathematics teacher. During her/his working experience, an expert mathematics teacher should frequently reflect on her/his own teaching, other teachers' teaching s/he has observed, students' learning, the *Zhongkao*, problems in mathematics education, and tendencies in the development of mathematics education, rather than simply carrying on her/his teaching repeatedly. Basing on these reflections, s/he should be able to develop unique opinions on these matters. S/he should strive to elevate her/his experience to a theoretical level, and to publish scholarly articles or books based on the theories so developed.

In the interviewees' opinions, the ability to theorize experience was even more important to an expert mathematics teacher than the ability to research problem-solving methods, teaching and learning methods, or teaching ideas. While proficient or even some competent teachers can also write papers on problem solving, or opinions about teaching and learning, only an expert mathematics teacher has the ability to theorize experience and develop cohesive theories. This ability is a critical and fundamental difference between expert mathematics teachers and proficient mathematics teachers. As one teacher pointed out:

> *The biggest difference between a proficient teacher and an expert teacher is that the expert teacher will constantly accumulate experience during her/his teaching process and will generalize a set of mathematics education theories with her/his own characteristics. As to some experiences, s/he can develop them into some theories with her/his personal characteristics. At the same time, s/he can popularize them. This might be the main difference between those two kinds of teachers. (Teacher 1)*

Moreover, once an expert teacher has developed her/his own teaching theories, s/he should be able to use them to guide her/his teaching practice. In addition, as indicated in Teacher 1's statement above, s/he should be able to popularize the theories s/he has developed in a school, in a district, or throughout China.

5.3.1.3 Ability to conduct research projects

Another research-related ability an expert mathematics teacher should have is the ability to apply government grants to conduct projects. An expert mathematics teacher should be able to act as principal investigator and lead other teachers to conduct district-, provincial-, or even national-level education research projects. As a mathematics teaching research officer pointed out:

> *An expert mathematics teacher must do some projects, some little projects. For example, explore effective methods to teach a particular type of function, or some particular geometric concepts…., Anyway, s/he should closely combine her/his teaching with conducting projects. (Mathematics teaching research officer 2)*

This statement indicates that an expert mathematics teacher should be able to bind her/his teaching tightly to conducting projects. According to some interviewees' descriptions, the ability to conduct projects involves embarking on a systemic process with very clear research objectives, rather than simply writing a paper on teaching phenomena. An expert mathematics teacher should therefore be very familiar with current research trends and emphases, and should be willing and able to find meaningful research topics from which other teachers could gain useful information. That is, the research should make a rich practical contribution. As a teacher pointed out:

> *The projects should be researchable and meaningful. That is, they can bring us some social benefits during and after the research process. There should be some social benefits. …, I think, the research results of expert mathematics teacher should be able to promoted, no matter in good schools or some general schools, no matter in a single school or in many schools. (Teacher 3)*

5.3.2 Ability to mentor other teachers

Sixteen of the 21 interviewees mentioned that an expert mathematics teacher cannot focus on her/his own teaching or working alone, but should also mentor non-expert teachers, especially novice and advanced beginner teachers. The ability to effectively mentor other teachers is another fundamental difference between expert and proficient mathematics teachers. In comparison with proficient teachers, an expert should have greater ability to mentor other teachers and more effective strategies and methods to facilitate other teachers' growth. According to interviewees, mentoring includes offering insights to other teachers' work, delivering demonstration lessons, and organizing workshops and seminars.

5.3.2.1 Insightful opinions and comments on other teachers' work

An expert mathematics teacher should be able to make insightful comments about other teachers' teaching and work, and provide useful suggestions for their modification. It was emphasized that, when an expert mathematics teacher observes a teacher's teaching, s/he is very sensitive to the problems in this teacher's teaching. After her/his observation, instead of giving very general comments, s/he should be able to point out the critical and essential problems in their teaching. In other words, her/his comments should be insightful and to the point, and should include constructive suggestions for modification and improvement. In short, her/his comments should help the teacher overcome specific challenges and facilitate their growth and development. As one teacher pointed out:

> *When s/he [expert mathematics teacher] evaluates other colleagues' teaching, s/he can really point out some problems of these teachers. After s/he observed one teacher's teaching, s/he can pointed out some problems we never think about, some good points and shortcomings of this teacher's teaching, they are very insightful. S/he can point out some problems from a very high perspective or level. (Teacher 10)*

This teacher's statement further points out that an expert mathematics teacher should be able to evaluate other teachers' teaching comprehensively, theoretically and from a high perspective, rather than by focusing on isolated problems. This indicates that her/his observations should be guided by sound educational theories, and s/he should use her/his theoretical knowledge to interpret her/his observation and comments. As one teacher educator explained:

> *S/he [expert mathematics teacher] should know why we need to do this in this way, s/he can explain it to other teachers at a theoretical level, and therefore, s/he can give other teachers effective suggestions. (Teacher educator 2)*

5.3.2.2 Delivering demonstration lessons

An expert mathematics teacher should be able to demonstrate her/his teaching ideas, thinking, methods, and strategies to other teachers through demonstration or open lessons. It was generally thought that non-expert mathematics teachers can acquire much useful information from her/his demonstration. As one teacher pointed out:

> *S/he [expert mathematics teacher] will share her/his understanding with other teachers, for example, s/he will deliver some demonstration lessons very often to let other teachers learn teaching ideas and methods from her/him and her/his teaching. (Teacher 7)*

5.3.2.3 Organizing workshops and seminars

An expert mathematics teacher should be able to organize workshops and seminars with other teachers to share her/his rich working experience, theoretical knowledge, and profound understanding of specific phenomena. In some interviewees' opinions, an expert mathematics teacher has stronger ability to organize workshops and seminars than does a proficient teacher. As one teacher stated:

> *As an expert mathematics teacher, s/he should organize some seminars and workshops (to share her/his experience). S/he should have this ability to give other teachers some training. As to this facet, proficient teachers might not be so capable. (Teacher 5)*

5.3.3 Teaching ability

This theme covers abilities related to classroom teaching stressed by the interviewees. While not every capacity was emphasized equally, it was generally agreed that expert mathematics teachers have special strategies that make their classroom teaching unique. Even though a teacher may have excellent teaching research ability and profound knowledge base, s/he is not an expert mathematics teacher if s/he cannot demonstrate strong teaching ability. As one mathematics teaching researching officer argued:

> *Some teachers are very good at solving mathematics problems, even some extremely difficult ones, or they have published many papers. However, they are not good at classroom teaching. They do not know how to effectively impart knowledge to students. Therefore, they are not expert teachers. Because as an expert mathematics teacher, first of all, you are still a teacher, it is not to say that you are good at solving problem or you have some publications, therefore you are an expert teacher. To be an expert teacher, you should not only be good at solving problem and research, you should also have strong or excellent teaching ability. (Mathematics teaching research officer 1)*

The statement points out the importance of strong teaching ability for expert mathematics teachers. The following common aspects were identified in the interviewees' descriptions:

5.3.3.1 Planning teaching and using textbooks flexibly

Every interviewee emphasized that, in comparison with non-expert teachers, expert mathematics teachers have stronger instructional design capabilities and greater ability to deal with teaching content flexibly. An expert teacher can plan her/his teaching without being overly constrained by textbooks, and should have special ways of dealing with textbooks that make her/his teaching unique. Moreover, s/he should be able to make macro-level teaching plans (*e.g.,* year, term, and chapter plans) reasonably according to the textbooks and make relevant changes to textbook content for individual lessons in order to facilitate student understanding. As one principal pointed out:

> *A novice teacher will present content strictly according to textbooks. S/he will not creatively use the textbooks. However, an expert mathematics teacher will creatively use textbooks. S/he will integrate different new knowledge, or integrate sample problems with other content effectively…., S/he will use different ways to present teaching content to students according to the actual situation. (Principal 5)*

Typical ways of using textbooks emphasized by interviewees include: 1) adopting or adapting real life examples to introduce relevant concepts and theorems; 2) integrating similar topics into a single lesson; and 3) choosing and posing appropriate problems according to knowledge characteristics and students' background. As one principal stated:

> *There are millions of different mathematics problems. However, not all the problems are suitable for students. S/he [expert mathematics teacher] knows which problems are the most suitable ones for her/his students. S/he can quickly make a good choice and is very sensitive to this. In the meantime, s/he will not only consider the current situation of her/his students, but also*

consider the long-term development of her/his students. (School principal 4)

5.3.3.2 Implementing teaching flexibly

Twenty of the 21 interviewees mentioned that an expert mathematics teacher should be able to carry out her/his lesson plans flexibly. S/he should be sensitive to students' difficulties and confusion, and immediately adjust her/his lesson plan to reduce students' difficulties and confusion. This further suggests that s/he should be able to read students' cues and have alternative teaching methods at hand. As one teacher pointed out:

> *As to an expert mathematics teacher, s/he originally planned a lesson in a certain way, however, s/he found that students have difficulty in understanding. S/he should change [her/his plan], s/he should stop teaching new content. What s/he should do is to find out students' difficulty and employ other materials to solve their difficulty. This is to say that s/he has alternative plans or methods in her/his mind. This is to say s/he has rich "teaching thoughtfulness". (Teacher 10)*

Even though s/he should be able to make changes as necessary, many interviewees emphasized that expert mathematics teachers should still arrange her/his teaching activities reasonably and systematically; her/his lesson should still look well structured.

5.3.3.3 Explaining difficult content with simple language

An expert mathematics teacher should be able to impart profound knowledge through simple language, translate abstract or difficult concepts into examples her/his students can understand and use the most understandable language possible to explain or solve difficult problems. As one teacher pointed out:

> *No matter how difficult a problem is, or how abstract a concept is, or how difficult one topic is, it is very easy to understand after her/his [expert mathematics teacher] explanation. (Teacher 1)*

5.3.3.4 Conducting student-centered teaching

Sixteen interviewees mentioned that an expert mathematics teacher should have inquiry-oriented beliefs about mathematics teaching. S/he should be able to construct a democratic classroom teaching atmosphere and treat students with dignity and respect. In addition, s/he should be able to design activities that embody new knowledge or problems, and that encourage students to participate actively in these activities. In the interviewees' opinions, an expert mathematics teacher should have many strategies to encourage students to think, explore relevant activities and raise questions during her/his teaching. Every student should therefore have opportunities to explore, discover, reason, generalize and seek methods to solve problems on their own or, when needed, with the advice or guidance of the teacher. This indicates that an expert mathematics teacher should be able to conduct student-centered teaching, rather than simply imparting knowledge to students directly. As one principal pointed out:

> *As an expert mathematics teacher, you should, or try your best to, let students construct their own knowledge. That is, you should try your best to let your students ask questions, encourage your students to ask questions. During your teaching, your main task is to let students ask questions, encourage students to discover questions, solve questions rather than you keep talking all the time. (Principal 6)*

5.3.3.5 Stimulating students' interests

Seventeen interviewees mentioned that an expert mathematics teacher should be able to inspire students' interests. Mathematics, in their opinions, is an abstract subject that students sometimes find boring. As an expert mathematics teacher, s/he should be good at adopting or adapting daily life situations to introduce relevant knowledge, and should use body language and change her/his tone to inspire students' interest, and even sometimes make jokes to amuse her/his students, if necessary. In other words, s/he should be able to build and regulate a harmonious and relaxed classroom atmosphere. S/he should have the ability to attract her/his students so that students enjoy her/his teaching or mathematics studying and, finally, like mathematics.

In addition, an expert mathematics teacher should be able to change students' interests or attitudes toward mathematics, especially those students who do not like mathematics at the beginning. As one teacher educator pointed out:

> *… to those students, or to a whole class, maybe there are some students do not like mathematics. However, after your [expert mathematics teacher's] teaching, after a period of time, those students start to like mathematics. (Teacher educator 1)*

Moreover, once students have developed an interest in mathematics, an expert mathematics teacher should be able to maintain that interest.

5.3.3.6 Laying down a firm knowledge foundation for students

Another aspect highly stressed by every interviewee was that an expert mathematics teacher should be able to lay down a firm knowledge foundation for students during her/his teaching. Broadly speaking, this includes:

1) Stressing the knowledge development process and background. An expert mathematics teacher should be able to construct situations from which students can gain necessary background experience of relevant mathematics topics and, in particular, the knowledge development process;

2) Emphasizing the difficult and important points of the teaching content. An expert mathematics teacher should, through emphasis, be able to make her/his students (or observing teachers) clearly understand or feel which parts of her/his teaching are difficult and/or important. For the difficult parts, s/he should be able to construct a reasonable "knowledge ladder" from easier to more difficult parts, so that her/his students may gradually study them and finally understand them. For the important ones, s/he should use various exercises or methods to facilitate and consolidate students' understanding.

3) Building a connected knowledge structure for students. It was highly emphasized that the knowledge structure in an expert mathematics teacher's classroom should look very logical and rigorous. During her/his teaching, s/he should be able to encourage her/his students to explore

and discover connections among similar topics. If the connections are implicit, s/he should make them as explicit as possible. As one principal pointed out:

> *As to teaching content in an individual lesson, s/he [expert mathematics teacher] cannot only focus on the topic of this lesson. S/he will definitely make some preparation for following teaching. In addition, s/he should also refer to prior knowledge which has connection with this topic. I think s/he will let students study in a very connected knowledge environment. (Principal 5)*

In addition, s/he should be able to connect a particular topic to relevant knowledge from other fields or subjects, and even real life. By learning this topic, her/his students could gain some knowledge external to mathematics. More important, they should acquire some knowledge about how to apply mathematics in other fields. In other words, students' ability to apply mathematics is also developed.

4) Stressing the essence of mathematics. According to many teaching examples mentioned by the interviewees, an expert teacher should be able to make her/his students understand the core meaning of relevant concepts or theorems during her/his teaching. An expert teacher should be able to help students to explore mathematics deeply, and to get the essential meaning of mathematics rather than superficial knowledge.

5.3.3.7 Effectively developing students' mathematical thinking and methods

Twenty of the 21 interviewees mentioned that, in comparison with non-expert, especially novice, teachers, an expert mathematics teacher should be more capable of developing students' mathematical thinking and mathematics problem-solving ability. Knowledge, in an expert mathematics teacher's teaching, mainly functions as a vehicle through which students' mathematics thinking and methods are developed. This is to say, an expert mathematics teacher should be able to expand her/his teaching to another level — develop in her/his students the ability to think mathematically and solve problems in a mathematical way. As one teacher clearly pointed out:

> *As an expert mathematics teacher, s/he should not merely impart textbook knowledge to her/his students in a simple way. If so, every teacher can do this. I think a competent teacher has the ability to impart knowledge to her/his students. An expert mathematics teacher should provide her/his students with more chances to let them experience mathematical thinking and methods, which are embodied in relevant mathematics knowledge. During students' mathematics learning process, they can feel that mathematics is really useful, and it can actually facilitate their thinking. (Teacher 2)*

5.3.3.8 Ability to teach students with various backgrounds

Every interviewee emphasized that an expert mathematics teacher cannot be good only at teaching one type of students, *e.g.* students talented in mathematics; on the contrary, s/he should be able to teach students with various backgrounds, including students with different mathematics abilities, mathematics knowledge bases, family backgrounds, personalities, and interests. In addition, as the characteristics of students continually change, an expert mathematics teacher should be able to notice student changes and have the ability to adapt to new situations quickly. As one principal described:

> *As an expert mathematics teacher, s/he cannot only be able to teach a part of students, like those students who are good at mathematics, who can find solutions quickly. However, those students who are not very good at mathematics cannot have development [from her/his teaching]. If you are really an expert mathematics teacher, you should be able to teach every type of students. Like an expert doctor, s/he can treat unusual illness, at the same time; s/he can also treat some common illness. (Principal 1)*

5.3.3.9 Ability to teach mathematics efficiently

Nineteen interviewees mentioned that an expert mathematics teacher should be more capable of teaching mathematics than non-expert teachers, and should generate excellent achievements in less time using fewer exercises – in other words, an expert mathematics teacher should be able to teach mathematics efficiently. Moreover, it is generally believed that an expert teacher should have the ability to make every student, no matter her/his level of mathematics ability, improve their mathematical ability, knowledge, and achievements. There should be improvement in mathematics achievement both at the whole class and individual student level. As one teacher educator pointed out:

> *In a class, some students might have some difficulty in mathematics learning, or some do not like mathematics. After your [expert mathematics teacher's] teaching,…, and their mathematics achievement make a great progress. If you cannot do this, I do not think that you can be called as an expert teacher. (Teacher educator 1)*

However, this does not mean her/his students will necessarily be the best in the school or district. Many interviewees argued that, in terms of mathematics examination results, there is no significant difference between a proficient mathematics teacher's students and those of an expert mathematics teacher. In those interviewees' opinions, student achievement alone cannot determine whether a teacher is an expert mathematics teacher or not. As one teacher said:

> *If you are only good at teaching, you are not so qualified to be an expert mathematics teacher. If your students get excellent mathematics achievement, you are also not definitely an expert mathematics teacher. Sometimes, if you spend more time and ask your students to do more extra exercises, your students can have good achievement. It is very hard to say [you are an expert mathematics teacher or not] only according to your students' achievement. (Teacher 8)*

5.3.4 Discussion

This section reported some abilities highlighted by interviewees. It was generally thought that an expert mathematics teacher should be capable, not only of teaching, but also of researching and mentoring other teachers. In particular, the ability to research and the ability to teach were viewed by some interviewees as co-dependent and mutually beneficial abilities (see Figure 5.1).

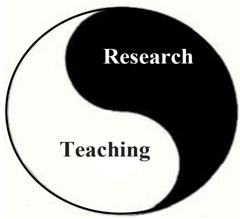

Figure 5.1 Relationship between the ability to teach and the ability to research

5.3.4.1 Research ability

In this study, the ability to conduct research was seen as the foremost important ability differentiating expert and proficient teachers in China, even though the research described is not always methodologically guided. This echoes the findings of other studies on expert teachers in China (*e.g.*, Kang, 2009; Li & Huang, 2008; Li *et al.*, 2008), although it was seldom mentioned in studies conducted in Western cultures. This suggests that, therefore, as argued below, that the conception of expert mathematics teacher held by the 21 interviewees is influenced by the Chinese social and cultural context.

Even though the ability to conduct research is seldom mentioned in Western literature on expert teachers, the concept of "teachers as researchers" was introduced in the late 1960s by Lawrence Stenhouse (as cited in Li, 2006), who states that "it is not enough that teachers' work should be studied: they need to study it themselves" (Stenhouse, 1975, p. 143). Teachers need to "treat everything they undertake as hypotheses to be tested" (Barnes, 1992, p. 9) and research activities carried out by teachers are "very different from those of academic educationists" (Barnes, 1992, p. 10). However, according to the 21 interviewees descripttions, an expert mathematics teacher in China should not only be able to carry out classroom-based research – *e.g.,* how to teach mathematics effectively and efficiently, how to implement new teaching thinking, and why it is difficult for some students to learn a particular topic – s/he should also be able to research methods of solving particular mathematics problems. More important, s/he should be able to theorize her/his teaching experience and conduct projects, and should thus be able to publish papers in professional or academic journals or books. This seems to go beyond ordinary mathematics teaching requirements and add new challenges to teachers' role, particularly from a Western perspective.

In China, research is widely accepted as an important part of a teacher's duty and is a growing trend in teacher professional development (Ning & Liu, 2000; Yang, 2001; Zhang & Ng, 2011). Teachers in Mainland China have been encouraged to conduct research. For example, a teaching research system set up at the establishment of People's Republic of China continues to exist today. The teaching research group is an organization that works to bring teachers together to conduct teaching research, rather than deal with administrative affairs (Lin, 2008; Yang & Ricks, 2012). Under this system, in addition to regularly bringing teachers together to participate in such teaching activities as how to improve teaching practices, teachers are also gathered together to conduct research (Zhong, 2003, as cited in Cong, 2009). This kind of working culture might make these interviewees emphasize the ability to research.

According to teacher qualification regulations in mainland China (MOE, 1986), teachers at all levels are required to either attend or conduct research activities (MOE, 1986), particularly teachers at senior and special rank levels. Moreover, the ability to conduct research is a very important factor to determine whether a teacher can or cannot be promoted from a lower rank to an upper rank (Zhang & Ng, 2011). As described in Chapter Three, a teacher needs to publish several papers if s/he wants to be promoted from intermediate level 1 to senior level. This kind of regulations and requirements might make the interviewees think that an expert mathematics teacher should have numerous publications, which they call the embodiment of "teaching research ability".

The curriculum at the pre- and in-service training stages is an additional influence. Pre-service teachers in mathematics education methodology courses, for example, are trained to write academic research papers and to conduct research projects (*e.g.*, Zhang & Song, 2005). At the in-service training stage, content related to conducting research is important, as in the *gugan* teacher-training program, where candidates must develop a research proposal in order to gain certification (Ma, 2000).

In some schools, the ability to conduct research and publish papers is a major component of teacher evaluations (Ying & Fan, 2001). For example, in Ms. Qian's school, teachers are required to develop their teaching research ability and must "actively participate in some experimental projects, develop the ability to choose meaningful research topic, collect data, and write academic papers on individual effort".

5.3.4.2 Teaching ability

Even though the ability to research and publish papers was emphasized by every interviewee, it must be noted that many interviewees pointed out that research ability is a necessary qualification for expert teachers, not a sufficient one; strong teaching ability is at least as important. Some abilities highlighted by the interviewees are actually found in previous studies, like flexible lesson planning and implementation and conducting student-centered teaching. (*e.g.*, Borko & Livingston, 1989; Leinhardt, 1989; Li, Huang, & Yang, 2011; Moallem, 1998; Zhu *et al.*, 2007).

Among the teaching abilities emphasized by the interviewees, one thing that must be pointed out is that, in the present study, many interviewees emphasized that a teacher cannot be called an expert mathematics teacher simply because her/his students achieve excellent results in mathematics examinations. They stated that an expert mathematics teacher should be more capable of improving students' learning attitude and interest in mathematics, rather than merely improving students' examination performance. Many interviewees mentioned that the factors contributing to students' examination achievements are complex. The findings of this study might make using students' examination results, which is the criterion adopted in some previous studies to determine whether a teacher is an expert teacher, problematic.

It may seem surprising that in Chinese teaching, which has been described as a culture dominated by examinations (Li, 2006; Tu, 2009; Wu, 2012; Zhang & Ren, 1998), student achievement is not a prerequisite for expert mathematics teachers. Considering the actual situation in China, however, this is not difficult to understand. As reported above, many interviewees noted that even a novice teacher, if s/he works hard enough and compels her/his students to do enough extra exercises, is likely to have her/his students excel on their examinations. However, her/his actions might not inspire students to learn, change their attitudes towards mathematics, or increase student enjoyment of the learning process, all of which are seen as key attributes of an expert mathematics teacher. In addition, according to some interviewees, an expert mathematics teacher should be able to get excellent levels of student achievement without resorting to extra lessons or exercises. Relatively speaking, these are more demanding for a teacher, especially considering the large class sizes in China. This illustrates that expert mathematics teachers have stronger teaching abilities than do non-expert mathematics teachers.

The abilities described by the interviewees have social and cultural roots. First, as described in Chapter Three, Confucius proposed certain teaching theories, such as reviewing recursively and gaining new insights by reviewing old materials, studying as well as reflecting, knowing students and teaching them accordingly, and teaching heuristically and gradually (Mao *et al.*, 2001; Sun & Du, 2009). These theories influenced the interviewees' descriptions of the teaching abilities that expert mathematics teachers should have, such as the ability to teach students of various backgrounds and abilities, the ability to help students to construct network-like knowledge structures, and the ability to develop students' thinking.

In addition, centre-periphery curriculum system adopted in mainland China is also an important influence. Textbooks in mainland China are concise (Fang & Gopinathan, 2009) and "have few illustrations" (Stevenson & Stigler, 1992, p. 139); teachers need to make relevant changes to relate teaching content more closely to students' real situations and to facilitate student understanding (Li, 2008). This ability varies based on teaching experience, teaching conditions and other factors; it is reasonable to assume, as the interviewees clearly do, that an expert mathematics teacher would be better able to adjust her/his teaching content to suit her/his students' actual situations.

In the meantime, the culture of teacher professional development in China might be another important influence. As described in Chapter Three, Chinese teachers tend to work collectively, and to use collaboration and sharing to improve lesson plan quality (Li , Qi, and Wang, 2012). Chinese teachers also work together to explore effective ways of teaching, such as organizing problem sequences and setting instructional objectives (Huang *et al.*, 2011). This culture might also make the interviewees think that an expert mathematics teacher should have strong teaching ability, such as planning their teaching and using textbooks flexibly. Teaching conditions in China are an additional influence. Class sizes in China are very large, with students from various backgrounds in a same classroom. This situation might also make these interviewees think that an expert mathematics teacher should be able to teach students with various backgrounds in a single classroom.

5.3.4.3 Ability to mentor other teachers

As reported above, an expert mathematics teacher was further expected to have effective ways of mentoring teachers at other developmental stages, including making insightful comments on their teaching, demonstrating ideas on or methods of teaching, and organizing workshops and seminars. Even though some participants chosen as expert teachers in other studies were in charge of mentoring student teachers, the ability to mentor others is not clearly mentioned as a prototypical characteristic of expert teacher in previous studies, especially those conducted in Western cultures. This may be the result of differences in social and cultural backgrounds.

First of all, unlike the teaching culture in the United States, in which a good teacher is thought to be born rather than made, Chinese culture holds that all teachers can teach if they are properly guided and trained (Lee, 1998). In mainland China, a long-held belief is that teachers cannot be prepared by pre-service trainings programs alone, and learn to teach after they take up teaching positions (Li, 2008; Paine *et al.*, 2003). Newly graduated teachers are regarded as a “semi-finished product” (Paine *et al.*, 2003, p. 33) and are expected to learn to teach by and from doing the job. The professional development of teachers in China is practical in nature (Li & Huang, 2008). A popular model is apprenticeship practice (Han, 2012) or “the old guiding the young” (*laodaiqing*) (老带青) (Tsui & Wong, 2009). Experienced teachers are appointed to mentor newly graduated or beginner teachers. This kind of professional development culture might have made some interviewees think that an expert mathematics teacher should be able to mentor other teachers.

The regulation of teacher qualification stresses the ability and experience to mentor novice teachers, particularly among teachers at intermediate level 1 and senior level, and a teacher’s ability to mentor novice teachers influences her/his chances for promotion. These social and cultural factors may also have made the interviewees emphasize that an expert mathematics teacher's ability to mentor novice teachers.

5.4 Other Traits

In addition to the common characteristics related to knowledge and ability identified above, there were additional traits that interviewees thought expert mathematics teachers should possess, including a noble personality, the spirit of working diligently and studying rigorously, wide horizons, and a strong social reputation.

5.4.1 Noble personality

Nineteen of the 21 interviewees mentioned that an expert mathematics teacher should have a noble personality, In addition to being outstanding and capable, s/he should be a modest, low-key and pleasant individual who has earned the respect of her/his students and colleagues, and s/he is cooperative and willing to help her/his colleagues whenever possible. One teacher educator said:

> *S/he [expert mathematics teacher] should have a very good relationship with her/his colleagues. S/he should have cooperative consciousness. …, s/he is very capable at every aspect; moreover, s/he is willing to help others. S/he will discuss teaching matters with her/his colleagues. If s/he had very good teaching experience and teaching methods, s/he will tell and share with other teachers. S/he is very unselfish. (Teacher educator 1)*

Moreover, it was thought that this noble personality could positively influence the growth and development of colleagues and students. A teacher commented:

> A*s to her/his [expert mathematics teacher] morality or personality, her/his behavior or words, can consciously or unconsciously influence her/his students or her/his colleagues. S/he can influence a person's growth. I think that s/he can use herself/himself as a model to influence her/his students and colleagues. (Teacher 10)*

5.4.2 Working diligently and studying rigorously

Every interviewee thought that an expert mathematics teacher should work diligently. According to their descriptions, an expert mathematics teacher has tremendous self-discipline and is very strict with herself/himself, both as a person and as a teacher. In addition, s/he is very responsible and will always take her/his work seriously. As a school principal pointed out:

> *S/he [expert mathematics teacher] is very serious with planning lessons, implementing teaching, tutoring students, and marking students' assignments. S/he is very responsible on the issues within her/his classroom teaching or outside her/his teaching.… Responsibility is a very important factor for becoming an expert mathematics teacher. Without it, it is impossible for a teacher to develop as an excellent teacher, no matter to say an expert mathematics teacher. (School principal 5)*

Moreover, an expert mathematics teacher should continuously study rigorously and make efforts to improve her/his abilities in mathematics, teaching, and other fields. Some interviewees pointed out that, even after a teacher becomes as an expert mathematics teacher, s/he should keep studying to maintain that expertise. As a school principal argued:

> *If s/he [expert mathematics teacher] stops her/his own effort, s/he will drop behind. You are an expert mathematics teacher at this moment, you are successful, it does not grantee that you will still be an expert mathematics teacher tomorrow. Things keep changed and developed, if you stop studying, you cannot adapt to the development of education, technology, and science. (School principal 6)*

5.4.3 Wide horizons

Twenty interviewees emphasized that an expert mathematics teacher should have broad horizons and a long-term vision. As s/he accumulates experience and knowledge, an expert mathematics teacher's horizon becomes broader and wider, which was seen as a fundamental difference between expert and non-expert mathematics teachers. The most popular statements made about broad horizons or long-term vision by the interviewees were that: 1) an expert mathematics teacher will view and think about problems or teaching phenomena, deal with teaching materials and plan and implement teaching from a higher perspective or macro-level, and will not be overly constrained by concrete problems, phenomena or activities themselves; and, 2) in addition to focusing on teaching content, an expert mathematics teacher will pay attention to many other things, such as the effect of new education theories, good experiences in other schools or cities, and trends in mathematics education development. As one school principal pointed out:

> *I think that a teacher should broaden her/his horizon, especially expert mathematics teachers, s/he should know more experience in other schools, other provinces, or other countries as well. Some good experience from others, you [expert mathematics teacher] should be able to critically borrow them and adopt or adapt them according to your own situation. If you always work behinds the doors, it is not good. If your horizon is not broad, there are many things that you never know, most of the time, you like a frog in a well, you only see those happens in your own school or class. In this case, you will never be an expert mathematics teacher. (School principal 3)*

5.4.4 Strong social reputation

Fourteen of the 21 interviewees mentioned that an expert mathematics teacher should possess a strong social reputation. In other words, an expert mathematics teacher should not only be known to colleagues within her/his own school, but should also be well-recognized in her/his district, in Chongqing, or even throughout China. As one teacher educator pointed out:

> *As to social reputation, it might be possible that a proficient mathematics teacher is only well known in her/his own school. However, as an expert mathematics teacher, s/he is well known not only in a school, s/he is known in many schools, or in a*

> *district, or even wider, like in Chongqing. It is possible. … S/he [expert mathematics teacher] has prestige or is well known in a wide area. (Teacher educator 1)*

Moreover, an expert mathematics teacher should use her/his reputation to influence other teachers' growth and to lead other teachers to implement or experiment with new teaching ideas. As one teacher pointed out:

> *I think in any curriculum reform, as an expert mathematics teacher, s/he should lead other teachers to implement the ideas of curriculum reform. S/he should act as a leader; otherwise, it is difficult to implement the thinking and ideas of a new curriculum reform. S/he has prestige in a certain area; therefore, s/he should lead her/his colleagues or teachers from other schools to try some new methods or teaching thinking. (Teacher 11)*

5.4.5 Discussion

This section has reported some highly emphasized traits of expert mathematics teachers, many of which echo findings in previous studies, such as constant self-improvement, respect for colleagues and students, and personal responsibility (*e.g.*, Bond *et al.*, 2000; Cowley, 1996). However, some traits, such as working diligently and having broad horizons, are less frequently mentioned in other studies.

These traits have their social and cultural roots. For example, in the Confucian culture, a teacher was described as a moral model for students (Sun & Du, 2009; Xiao, 2001), and was expected to teach by example of her/his own behavior and personality. This tradition may lead the interviewees to think that an expert mathematics teacher should have a noble personality. In addition, teachers were required to learn painstakingly and insatiably throughout Chinese history, which cultural tradition might influence the interviewees to highlight expert mathematics teachers' need to study rigorously. Teaching with tireless zeal and teachers' responsibility were stressed in Confucian culture as basic qualifications for a teacher (Sun & Du, 2009); this tradition might have led the interviewees to think that an expert mathematics teacher should also be very responsible and work very diligently.

In addition, in traditional Chinese culture, an individual's diligence and efforts are central to her/his success (Bond, 1996; Lee, 1998; Li & Yue, 2004). It is widely believed in Chinese society that effort is more important to success than ability, and that ability itself can be improved through hard work (Hau & Salili, 1996; Salili, 1996). There are many sayings in Chinese culture that highlight individual effort, hard work, diligence and dedication to the pursuit of success, such as "in time, a string may saw through wood and drops of water can penetrate a stone" (绳锯木断，水滴石穿), "diligence can remedy mediocrity" (勤能补拙), "one excels through diligence" (业精于勤), and "by not giving up, you can change an iron rod into a needle" (只要功夫深，铁杆磨成针). This cultural influence might influence some interviewees to think that an expert mathematics teacher should work hard to improve her/his ability and maintain her/his expertise.

Moreover, some popular metaphors in mainland China (*e.g.,* "teachers are like candles, who sacrifice themselves to light others" (教师如蜡烛，燃烧自己，照亮别人)) might make the interviewees think that an expert mathematics teacher should work hard, study rigorously, and be dedicated to teaching. Some interviewees actually used this phrase to explain why an expert mathematics teacher should love and be dedicated to teaching.

5.5 Summary of the Chapter

This chapter reported on the conception of expert mathematics teachers from the perspective of the study's 21 interviewees. Their descriptions indicate that an expert mathematics teacher in mainland China should not only be knowledgeable in mathematics, curriculum, students, educational and psychological theory and other subjects, s/he should also be capable of teaching, mentoring other teachers and conducting research. S/he should study rigorously, work diligently, and have a noble personality and good reputation. Even though there are some similarities between these findings and those in previous studies on expert teachers, some descriptions (*e.g.*, ability to research and knowledge of examination) were unique to the Chinese context. The characteristics emphasized by more than 50% of the 21 interviewees are summarized in Table 5.1 below.

Table 5.1 constructs and themes: Conception of expert mathematics teachers

Themes	Sub-themes	Categories	Percentage
Know-ledge	Mathematics base	Solid mathematics content knowledge	100%
		Strong mathematics problem solving ability	67%
	Theoretical knowledge		100%
	Knowledge of the characteristics of learners		100%
	Curriculum knowledge	Deeply understanding curriculum standard and syllabus	72%
		Knowing textbook well	86%
		Acquaintance with Zhongkao	90%
	Knowledge about other subjects		100%
Ability	Research ability		100%
	Mentoring teachers		76%
	Teaching ability	Planning teaching and dealing with textbooks flexibly	100%
		Implementing teaching flexibly	95%
		Explaining difficult content with simple language	90%
		Conducting student-centered teaching	76%
		Stimulating students' interests	81%
		Laying down firm knowledge foundation for students	100%
		Developing students' mathematical thinking and mathematical methods	95%
		Ability to teach students with various background	100%
		Ability to teach mathematics efficiently	90%
Other Traits	Noble personality		90%
	Study religiously		76%
	Wide Horizon		95%
	Wide popularity		76%

Chapter Six

Beliefs and Knowledge of Expert Mathematics Teachers

6.1 Introduction

This chapter includes three parts. The first reports common beliefs held by the three expert mathematics teachers, including their beliefs about mathematics, mathematics learning, and mathematics teaching. The second reports common characteristics of the three expert mathematics teachers' knowledge, which are categorized into mathematics subject knowledge, pedagogy content knowledge, curriculum knowledge, and knowledge of learners. The third reports teaching strategies identified among the three expert mathematics teachers.

6.2 Beliefs

6.2.1 Beliefs about mathematics

To explore the three expert mathematics teachers' beliefs about mathematics, they were asked questions related to the nature of mathematics. Table 6.1 summarizes the main aspects related to what mathematics is in their views. Although their answers vary, there exist some commonalities as follow:

6.2.1.1 A vehicle for developing students' thinking and ability

As shown in Table 6.1, the first common statement made by the three expert mathematics teachers is that mathematics is a vehicle for developing students' thinking and abilities. They all mentioned that studying mathematics can change people's thinking style; for example, Ms. Qian stated "I think the essential function of it (mathematics) is to develop students' thinking", while Ms. Sun said mathematics can "develop some methods to think about problems". Mr. Zhao commented that:

> *the main task of mathematics is to develop students' logical thinking system, and the ability to learn [by themselves], and the ability to analyze and solve problems.*

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Table 6.1 Expert mathematics meachers' beliefs about mathematics

	Mr. Zhao	Ms. Qian	Ms. Sun
A vehicle to develop students thinking and ability	+	+	+
Being from solving problems in real life and in turn, being able to be applied in real life	+	+	+
Basics for other subjects and science	+	+	+
A school learning and examination subject	+	+	+
Developing students' views of mathematics		+	+
An accomplishment of human being	+		
An instrument to explore relationships in space and quantity relationships		+	

Note. "+" indicates that the teacher made a corresponding statement.

6.2.1.2 Application in real life

The second common statement is that mathematics can be applied in real life. Ms. Sun emphasized that mathematics is tightly linked with real life situations, and the fact that some mathematics theories cannot be applied in real life at certain times does not mean that they cannot be applied at all; some seemingly inapplicable theories might one day be used to solve practical problems. Ms. Qian also mentioned that, even though not every student will become a mathematician in the future, mathematics makes students view the world around them mathematically and will influence their ways to solve problems in daily life. Mr. Zhao pointed out that "mathematics is from real life; mathematics can be applied in real life situations; mathematics can be used to serve our life".

6.2.1.3 Base for other subjects and science

According to the three expert teachers' descriptions, mathematics acts as the base for other fields or subjects, such as physics and information technology. Mr. Zhao noted that "mathematics can be infiltrated into other subjects, for example, physics". Ms. Qian saw mathematics as an instrumental subject that serves as a basis for the development of other subjects. All three teachers emphasized that mathematics is the basis for scientific development; as Ms. Sun argued, "without the development of mathematics, it is quite difficult for science to make any progress".

6.2.1.4 A school learning and examination subject

The last commonly-held view among the three expert mathematics teachers was that mathematics is not only a school subject, but also an important examination subject at different levels, such as the *Zhongkao* and *Gaokao*. According to Mr. Zhao, “mathematics is a necessary subject in examination in secondary school”. In Ms. Qian’s view, mathematics is also a subject students have to learn for their senior secondary, university and life-long learning. Ms. Sun thought that, based on the arrangement of the curriculum in mainland China, mathematics has a high status as a school subject at both the primary and secondary levels, and students have to learn it well, because it is also a very important subject in examinations at both levels.

6.2.2 Beliefs about mathematics learning

Teachers’ beliefs about how their students learn influence how they plan their teaching and how they interact with students while teaching (Calderhead, 1996). The three expert teachers’ beliefs about students’ ability to learn mathematics, the best ways to learn mathematics, and the most important parts of mathematics learning were examined to investigate their beliefs about mathematics learning.

6.2.2.1 Beliefs about the ability to learn mathematics

According to the three teachers, not every student can learn mathematics well under the same standard. In their opinions, giftedness is a very important, factor to determine whether a student can learn mathematics well or not, but it is not the only one. Ms. Sun pointed out that some students “are not sensitive to numbers”, whilst Ms. Qian added that “there exist some differences between students’ ability”. Mr. Zhao thought that learning mathematics is a process, and that a student cannot learn mathematics well at a time does not mean s/he will never learn it well; however, he admitted that “learning mathematics well depends on one’s wisdom”. All three teachers further emphasized that, even if a student is very talented in mathematics, s/he cannot learn mathematics well if s/he does not study hard.

6.2.2.2 Beliefs about the best ways to learn mathematics

Despite the commonalities expressed about ability to learn mathematics well, the best ways to learn mathematics described by the three teachers varied. Ms. Sun thought that students should preview what they will learn and review what they have learned regularly. Ms. Qian summarized the best way to learn mathematics as "more practicing, more questioning, more reflecting". In other words, students need to practice exercises at their own ability level, ask their teacher and peers questions, think and reflect deeply. Mr. Zhao thought that the best way to learn mathematics is that students should become intellectually involved in the learning process and deeply understand the knowledge development process. In addition, he thought that students need to memorize some definitions and theorems based on understanding, that is, not memorize everything blindingly.

A common characteristic implicitly embodied in the beliefs described above is that students should engage themselves in mathematics learning and become intellectually involved in the process of mathematics learning, rather than superficially receiving or memorizing by rote. This indicates that students cannot learn mathematics well simply by receiving teachers' direct instruction and rote memorization. In other words, students' deep understanding is essential to learning mathematics well. For example, Ms. Qian said that students should more often ask themselves how to reason a particular formula and question why it should be reasoned in that way when they study mathematics. Similarly, Mr. Zhao commented that students should learn to summarize and conclude based on their own discovering and understanding.

6.2.2.3 Beliefs about the most important parts of mathematics learning

According to the three teachers, the most important parts of mathematics learning at the junior secondary school level are mathematical thinking, the ability to apply mathematics, problem-type training, and knowledge; of these, all three view mathematical thinking as the most important. In their views, mathematics thinking, not knowledge, influences students' life forever. The three teachers also commented that the ability to apply mathematics to solve either mathematics problems or problems from real life is a very important part of mathematics learning. Mr. Zhao and Ms. Sun further thought it quite necessary for students to summarize or generalize problem solving strategies they can employ in the future, because it is unrealistic to expect students to finish all mathematics problems. Lastly, Mr. Zhao and Ms. Qian thought that it is quite necessary to lay a solid knowledge foundation for students, which will benefit their future development in mathematics and in other areas.

6.2.3 Beliefs about mathematics teaching

Teachers' beliefs about mathematics teaching are "the key determinant of how mathematics is taught" (Ernest, 1989, p. 22). To understand a teacher's teaching, it is necessary to investigate her/his beliefs about teaching. In this study, beliefs about the goals of mathematics teaching, what constitutes a successful mathematics lesson, and effective mathematics teaching methods were examined to investigate the three expert mathematics teachers' beliefs about mathematics teaching.

6.2.3.1 Beliefs about the goals of mathematics teaching

Common goals of mathematics teaching mentioned by the three teachers include developing students': 1) mathematical thinking, 2) ability to apply mathematics, and 3) ability to learn mathematics independently. The three teachers saw the first of these as the most important. They thought it quite normal that students will forget their knowledge one day; however, the mathematical thinking they develop will influence their life forever. As Ms. Qian said:

> *As a saying goes, mathematics is like gymnastics of thinking to develop students' thinking. After our students' graduation, they will work in different fields. However, those students who have strong mathematics ability, whatever things they work on, they will think logically and finish it logically. Therefore, I think the most important objective of mathematics teaching is to develop students' mathematics thinking.*

Ms. Sun and Mr. Zhao also mentioned that it is very important to develop students' ability to apply mathematical knowledge to solve both mathematics problems and problems in real life. Ms. Qian and Mr. Zhao commented that mathematics teaching should develop students' ability to learn mathematics by themselves to benefit their future study and life.

6.2.3.2 Beliefs about a successful mathematics lesson

To evaluate whether a mathematics lesson is successful or not, the following three aspects were frequently mentioned by the three teachers: 1) students' active participation; 2) students' deep understanding; and, 3) linking mathematics with students' real life situations. All three teachers noted that, for a successful mathematics lesson, a teacher should employ various teaching strategies to inspire students' interests and encourage students to participate in activities. Intellectual engagement was particularly emphasized. Mr. Zhao said:

> *A successful mathematics lesson cannot be judged only according to its active classroom atmosphere. In contrast, it should make students really involve in the logical system of mathematics. They completely involve in mathematics and they solve some problems by themselves. During the whole process, teacher only act as a guider.*

Mr. Zhao further pointed out that students should have the opportunity to construct mathematical knowledge through their own experience and exploration, which was echoed by Ms. Qian and Ms. Sun. In addition, all the teachers mentioned that a successful mathematics lesson should promote students' deep understanding and help them to master knowledge, skills and problem-solving strategies in an appropriate way. Ms. Sun and Ms. Qian stated that a teacher should employ examples or construct situational problems related to students' real life so as to have students experience knowledge development process.

6.2.3.3 Beliefs about effective mathematics teaching methods

Although there exist variations in the effective teaching methods mentioned by the three teachers, common facets identified from their responses include: 1) encouraging students' intellectual engagement; 2) inspiring students' interests; and, 3) emphasizing learning methods and problem-solving strategies. All three teachers mentioned that, to teach mathematics effectively, a teacher should encourage students to involve themselves intellectually in activities or problems designed by the teacher. For example, a teacher should provide students enough time to explore the relationship between prior knowledge and current knowledge, look for solutions or alternative solutions to a problem, and summarize their own discoveries. Ms. Sun and Ms. Qian further commented that, to teach mathematics effectively, a teacher should be able to build an approachable relationship with students, and inspire students' interests to learn. Ms. Qian used a traditional Chinese saying, "interest is the best teacher", to convey this. Mr. Zhao and Ms. Sun thought that, to teach mathematics effectively, a teacher should stress developing students' problem solving strategies. As Ms. Sun said:

> *Necessary methods, include learning methods, problem-solving methods and methods to apply some basic knowledge, like "two basics" we are talking about, there are "practicing basic knowledge" and "practicing basic skills" in "two basics". We should stress these in teaching.*

6.2.4 Discussion

The three expert mathematics teachers' beliefs were qualitatively investigated through semi-structured interviews. As reported above, when the three teachers answered questions related to the nature of mathematics, they talked about mathematics as a vehicle for developing students' thinking and ability and saw it as a basis for other subjects and the progress of science. In addition, mathematics was thought of as coming from real life problems and as applicable to real life. Their comments on the function of mathematics seem to close to a combination of the instrumentalist and the problem-solving view of mathematics as proposed by Ernest (1991).

The three teachers' beliefs about mathematics are different from some beliefs expressed by Chinese mathematics teachers in previous studies, such as the "more rigid view of mathematics being more a product than as a process" (Leung, 1995, p. 315), or clear statement that mathematics is a type of knowledge that students need to learn (Wong *et al.*, 2002). None of the three teachers mentioned these two factors for several possible reasons. First, previous studies were conducted before or near 2000. As mentioned in Chapter Three, since 2001, there has been a mathematics curriculum reform, which depicts mathematics as an accomplishment of human beings coming from real life (MOE, 2001), and which might have influenced the three teachers' beliefs about the nature of mathematics. Second, the teachers in the two previous studies were not expert mathematics teachers. In Li *et al.*'s (2005) study, they found that non-expert mathematics teachers at elementary school level in China tend to hold a "mastering of knowledge" view of mathematics, whereas expert mathematics teachers tend to hold a "problem solving" view of mathematics.

Viewing mathematics as coming from real life and as a vehicle for developing students' ways of thinking differs from long-held beliefs on mathematics education in China. Under the influence of the former Soviet Union, viewing mathematics as an abstract and rigorous subject with wide application (as suggested by Aleksandrov *et al.*, 1964), was widely accepted in China (Li *et al.*, 2008; Zhang *et al.*, 2004). Though none of the three expert teachers mentioned this, Wang and Cai (2007) recently found that some very experienced elementary mathematics teachers still held the belief that "mathematics itself is an abstract and coherent knowledge system" (p. 290).

The three teachers tended to believe that mathematics teaching and learning should promote the development of students' thinking and abilities. To learn mathematics effectively, students need to explore mathematics knowledge on their own and become intellectually involved in the learning process. Similarly, to teach mathematics effectively, teachers should encourage students to participate intellectually in activities or tasks. Their beliefs about mathematics teaching are similar to the "learner-focused" model described by Kuhs and Ball (1986). Moreover, according to the three teachers' descriptions, the teacher's role in the classroom is similar to the facilitator model proposed by Ernest (1989). In other words, they tended to believe that students should have opportunities to experience the process of knowledge development and to construct their own knowledge through experience and exploration. This view of developing students' mathematical thinking is also found in other studies on Chinese mathematics teachers (*e.g.*, An, 2004; Wang & Cai, 2007; Wong *et al.*, 2002).

The above ideas (such as emphasis on students' experience, participation, and exploration, emphasis on knowledge development process during teaching, and linking teaching content to real life) have been strongly advocated in China since the latest mathematics curriculum reform (MOE, 2001), and the three expert teachers' beliefs about mathematics teaching and learning might be influenced by relevant ideas in this curriculum reform. However, the teachers asserted that they tended to hold these kinds of beliefs even before the curriculum reform. They mentioned that, after several years of working, they started to demonstrate the knowledge development process and to emphasize student experience and participation. This suggests that, even though they worked in a teaching context dominated by traditional beliefs (*e.g.*, Zhang *et al.*, 2004; Zheng, 2006), they still could develop non-traditional beliefs about mathematics, mathematics learning, and mathematics teaching.

This is not to say that their beliefs are not influenced by the social and cultural context in which they are working. Their beliefs are still influenced by the social and cultural context to a certain degree. In the Confucian culture, students' thinking is stressed; for example, Confucius said that "mere reading without thinking causes credulity; mere thinking without reading results in perplexities" (学而不思则罔，思而不学则殆) (*Analects*, translated by Lao, 1992, p. 43). This culture might make them believe that mathematics is a vehicle for developing students' thinking; that mathematics teaching and learning should aim to develop students' mathematics thinking; that to learn mathematics well and teach mathematics effectively, students should be intellectually engaged, and so on. In Chinese learning culture, individual effort and diligence are highly valued; according to Li (2004), resolve, diligence, hardship, perseverance, and concentration are highly recommended as the five most important cultural and learning merits. This might make the three teachers think that students need to work hard to learn mathematics well, regardless of their talent in mathematics.

Moreover, one important characteristic of Chinese mathematics teaching tradition is its emphasis on seeking deep understanding and promoting students' mathematics thinking and mathematical ability (Kang, 2010; Shao *et al.*, 2012; Zhang, 2010; Zhang *et al.*, 2004; Zheng, 2006). For example, the idea of developing students' thinking and abilities has long been emphasized in mathematics teaching syllabi and curriculum standard (Research Institution of Curriculum and Textbooks, 2004).This tradition might have influenced the teachers' beliefs about mathematics, mathematics learning, and mathematics teaching. In addition, they all believe that mathematics is a school subject students need to learn for examination, and that mastering basic knowledge and skills is a necessary objective of mathematics teaching and learning. These suggest that China's examination culture might have influenced their beliefs. In fact, during interviews, the teachers often mentioned that, under the current examination system in China, they have to help students prepare for examinations.

6.3 Knowledge Base

Teachers' knowledge is an important factor influencing their teaching (Shulman, 1986). The characteristics of the three expert mathematics teachers' mathematics knowledge, pedagogical content knowledge (PCK), curriculum knowledge, and knowledge of the characteristics of learners were investigated through observations and interviews to gain a more comprehensive picture. The following sub-sections will report relevant characteristics identified in the three teachers:

6.3.1 Mathematics knowledge

Teachers' knowledge of mathematics is an important factor affecting their teaching practice and students' learning (Ball, 1991; Ball *et al.*, 2001; Blömeke & Kaiser, 2012; Schmidt, Cogan, & Houang, 2011; Hill *et al.*, 2005). According to observations and interviews, it could be safe to say that the three expert mathematics teachers have a solid mathematics knowledge base. Two common characteristics of the teachers' mathematics knowledge were identified: 1) a profound understanding of teaching content; and 2) a connected knowledge structure.

6.3.1.1 Profound understanding of teaching content

The first common characteristic is that all three teachers deeply understand teaching content:

Understanding the essence of concepts. Firstly, the three teachers demonstrated that they understand the essence of relevant concepts. For example, after Ms. Qian presented the definition of “Ratio of Two Line Segments”, she further made three statements to this definition: 1) the value of the ratio is independent of the unit for measuring the length of the line segments; 2) when calculating the value, the lengths should be measured under the same unit; and 3) the value of the ratio is a positive real number. These statements indicate that Ms. Qian comprehended the essence of the definition. Moreover, these statements are not clearly stated in the textbook and teacher manual; according to her responses in the post-observation interview, she attained this deep understanding by her own work.

In Mr. Zhao’s sixth lesson, he explored the geometric meaning of “k” (named by him) in an algebraic expression of y=k/x. He used several examples similar to the one shown on the left side of Figure 6.1 to explore the relationship between the value of the area of rectangle BPAO and the value of “k” so as to make students realize that the two values are equal. After this, he further explored the value of “k” with the value of the area of a triangle as shown on the right in Figure 6.1. These examples are not found either in textbooks or in teacher manual. It appears safe to judge that Mr. Zhao not only has a thorough understanding of the definition of the inverse proportion function, but also its relevant extensional meaning.

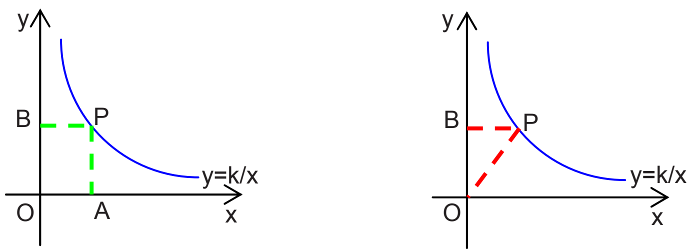

Figure 6.1 Examples used by Mr.Zhao to explore the geometric meaning of "k"

Comprehending critical differences between similar concepts. Secondly, the three teachers could differentiate critical differences between newly learned topics and similar prior knowledge. For example, Mr. Zhao once pointed out that, at the junior secondary school level, graphs of linear functions, direct proportion functions, and quadratic functions are continuous. However, graphs of inverse proportion function are not continuous. This suggests that Mr. Zhao had the ability to identify some distinguishing characteristics of various types of functions. Similarly, when Ms. Sun taught the definition of “bisector of an angle of a triangle”, she pointed out that the bisector of an angle of a triangle is a line segment; however, the bisector of an angle (as taught in primary school) is a ray. This also indicates that Ms. Sun not only understood the critical feature of angle bisector of a triangle and bisector of an angle, but also that, more important, she could discover critical differences between these two concepts.

Comprehending multiple representations. The three teachers were able to understand various representations of a concept or theorem. For example, regarding the definition of the inverse proportion function, Mr. Zhao demonstrated that he understood its algebraic expression, graph, table, and verbal description. In Ms. Sun’ teaching, she could use concrete paper triangles or figures to present a concept related to triangle. Furthermore, she could also give a verbal explanation or definition. For example, when she examined the relationship between lengths of the three sides of a triangle, she clearly stated that “the sum of the lengths of any two sides is greater than the length of the third side” and “the absolute value of the difference of the lengths of any two sides is less than the length of the third side”. After this, she used the following inequality, $|a-b|<c<a+b$, to summarize her statements.

6.3.1.2 Connected knowledge structure

Another characteristic of the teachers’ mathematics knowledge is that all three can interconnect relevant mathematics topics. During data collection, the three teachers were asked to draw a knowledge structure picture related to the teaching topic in the observed lessons. Due to the characteristics of knowledge, Mr. Zhao’s picture was relatively simple. However, as shown in Figure 6.2 and Figure 6.3, Ms. Qian and Ms. Sun developed a very comprehensive and interconnected knowledge structure on “Similar Figure” or “Triangle”. According to the two pictures, first of all, Ms. Qian and Ms. Sun well understood the interconnections among the topics within a certain chapter. For example, as shown in the shadowed frames in Figure 6.2, in the unit of “similarity”, Ms. Qian could link the ratio, golden section, similar polygons, and similar triangles together. In addition, as shown in Figure 6.2 and Figure 6.3, both the teachers clearly knew about students' prior knowledge and what students are going to learn in near future related to "similarity" or "triangle". This suggests that, even though they did not mention much knowledge at the senior secondary school level related to "Similar Figure" or "Triangle", they may still have a very comprehensive knowledge structure network in their mind at the level they are teaching.

The comprehensive knowledge structures further demonstrate that the teachers could organize and re-organize relevant topics to form a spider's web-like knowledge structure. This illustrates that they could discern relationships among similar topics and build proper connections among them. It has been well argued in the literature that "the degree of understanding is determined by the number and the strength of the connections" (Hiebert & Carpenter, 1992, p. 67). The rich connections built by the three teachers also suggest that they deeply understand the teaching content.

6.3.2 Pedagogical content knowledge

Pedagogical content knowledge (PCK), which refers to "the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9), is another important type of knowledge affecting teachers' teaching practice. From interviews and observations, it could be said that the three expert mathematics teachers have extensive PCK. In particular, the following common aspects were identified:

6.3.2.1 Knowing students' prerequisite knowledge

The first characteristic related to PCK is that the three teachers clearly know how what students learned previously related to the topic in a certain lesson, or what students should or might already grasp before they learn a certain topic, or to what degree students already understood relevant content necessary for learning the new topic. For example, in the first pre-observation interview, Ms. Qian pointed out that what students had already learned was related to what she was going to teach in those two lessons (double lessons):

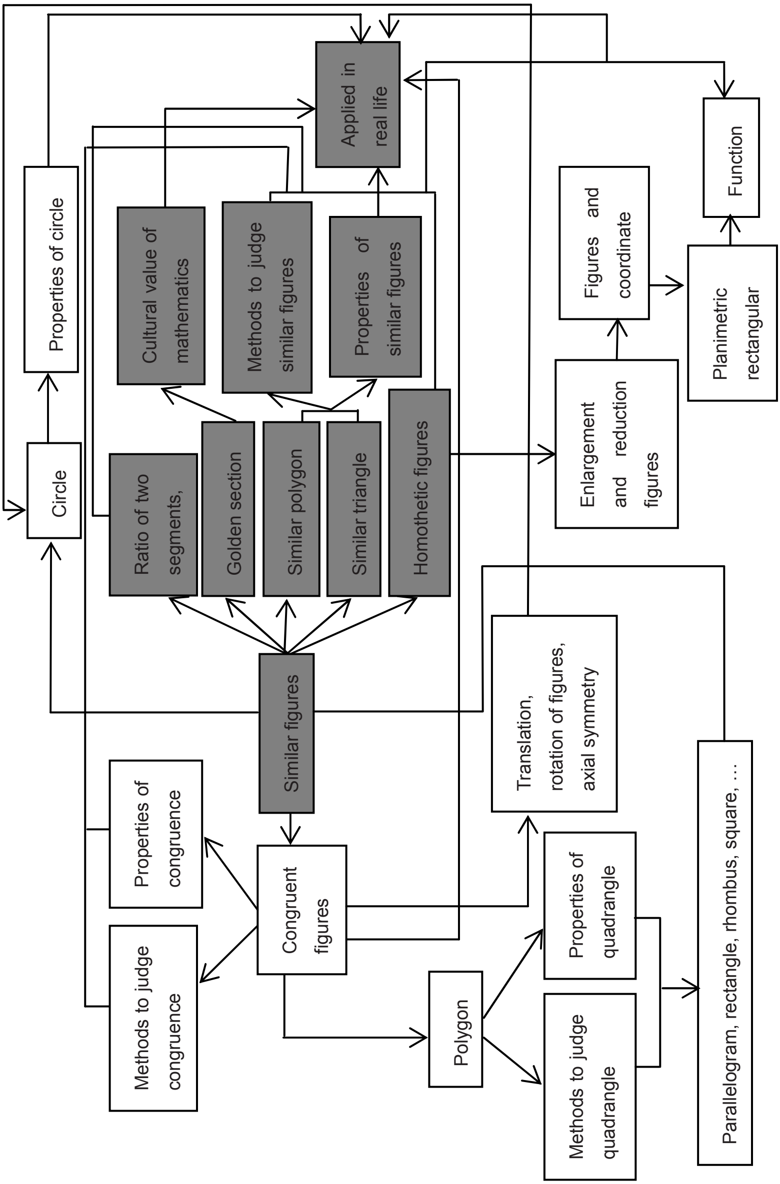

Figure 6. 2. Knowledge structure related to "Similar Figures" developed by Mr. Qian

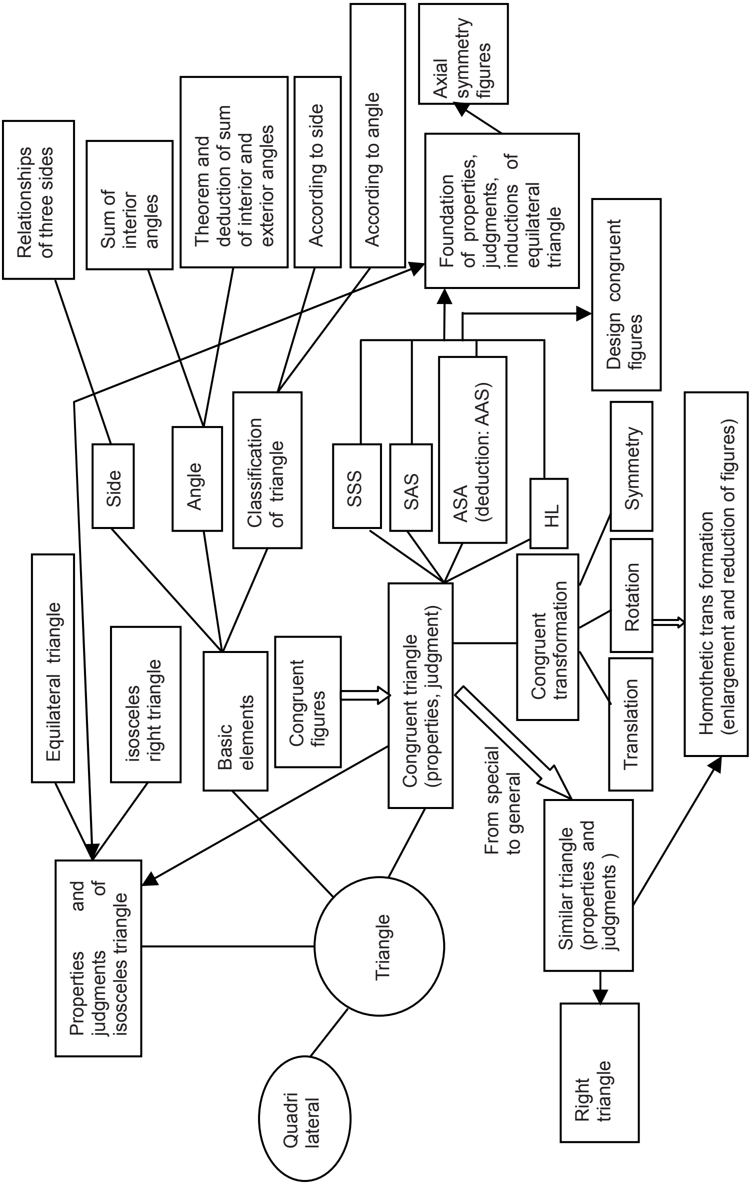

Figure 6.3. Knowledge structure related to "Triangle" developed by Ms.Sun

The first part is ratios of numbers, which they learned at Grade 6. This is the basic [for today's topic]. …, another part is what they learned at Grade 7 and in the previous chapter, the properties of equations. Meanwhile, they know well about linear function and fraction equations, they will use some knowledge of linear function and fraction equations to solve some difficult problems today.

Similarly, Mr. Zhao also stated that what students had learned before related to the definition of the inverse proportion function:

Firstly, they should know the relationship between two variables, understand the definition of direct proportion function, and linear function thoroughly …., and the inverse proportion they learned in elementary school.

6.3.2.2 Anticipating students' difficulty and various ways of working on it

For some new topics, the three teachers knew what parts would be easy and what parts would be difficult for their students. For a particular topic, they could also anticipate the kinds of mistakes students might make. For example, in Ms. Qian's second lesson, she asked students to prove the property of ratio of equality; however, most of her students could not find a way to do so at first. In the post-observation interview, she explained:

Like what I thought before the lesson, as to the property of ratio of equality, they did not know how to prove, they could not find the way right after I presented [this property].

Similarly, Ms. Sun also pointed out the difficulty her students might have in her fourth lesson (introducing the concept of the height of a triangle). She said:

The height of a triangle is a difficult part. First of all, students had difficulties when they learned vertical line. They did not know how to draw a vertical line. They might solve the difficulties at that time. However, they did not understand it thoroughly. Therefore,

> *they should have difficulties on today's topic, how to draw a height of a triangle, especially a height of an obtuse angled triangle, they will make some mistakes. …, I will review how to draw a vertical line first before I present the methods to draw a height of a triangle.*

For difficult parts, the three teachers would prepare various methods to facilitate students' understanding, such as reviewing relevant content to make sure students have necessary foundational knowledge, employing concrete examples to help students visualize abstract concepts, demonstrating some positive examples or counterexamples, discussing in small groups, and breaking the difficulties down into several lessons. For instance, Mr. Zhao thought that the graph of the inverse proportion function would be a difficult point for his students. In view of this, he divided the topic into several lessons, instead of teaching it in a single lesson. In his first lesson, even though the main aim was to teach the definition of the inverse proportion function, Mr. Zhao mentioned information such as how to list a table (which is an important step before drawing the graph of a function), possible locations of graphs of inverse proportion functions in the coordinate plane, and so on. He started to teach how to draw a graph systematically in the third lesson.

The three teachers could also anticipate which individual students might have difficulties with a given topic, and paid special attention to those students during the relevant lessons. Their most common ways of doing so included: 1) asking students with weak academic backgrounds to answer a particular question before moving to another topic, in order to make sure that they understood the current topic; 2) staying close to those students to identify what kind of difficulties they were having or what kind of mistakes they were making, tutoring them individually, and later discussing their difficulties publicly to facilitate and enhance their understanding.

6.3.2.3 Integrating various representations and selecting proper representation in teaching

The three teachers could integrate various representations into one lesson. For example, there are different representations of inverse proportion function, such as its verbal definition, graph, and table. In Mr. Zhao's teaching, he could choose an appropriate representation according to different situations and then translate one presentation to another flexibly. In the first lesson, he mainly emphasized tables, verbal representations, and analytic expressions. He used the tables to explore the relationships between two variables to facilitate students' understanding of the definition. In this lesson, he also mentioned the graph of the inverse proportion function. In his fourth and fifth lessons, he mainly focused on the analytic expression of inverse proportion function.

In Ms. Qian and Ms. Sun's teaching, due to the features of their teaching content, they initially used concrete materials or pictures to let students experience relevant concepts. Then, they used verbal representations and algebraic expressions to enhance students' understanding. For example, in Ms. Sun's third lesson, after she reviewed the concept of an angle bisector, she asked students to find the angle bisectors of three angles of an acute angled triangle by folding a paper triangle. Then, she asked them to explore their intersection and its location. After this, she asked students to fold a right angled triangle and an obtuse angled triangle to find out their angle bisectors and intersections. Through this, students discovered that three angle bisectors of a triangle join at a same point and this point locates inside the triangle. Finally, she presented the verbal definition of an angle bisector of a triangle. The reason that she chose to let students fold paper triangles is as followed:

> *I feel that asking students to fold paper triangles can impress them more deeply. Why the angle bisectors of a triangle join at a same point, is related to the properties of angle bisector and its judgment. This is very difficult to prove at this moment. However, I can let students realize that they can get this [conclusion] through their manipulation.*

Ms. Sun's explanation suggests that she can choose an appropriate representation to encourage students to discover and experience the property, which is difficult to mathematically prove at that moment. By doing so, she can help students to have a deep impression of this property and facilitate their understanding. In other words, she demonstrated the ability to make students more deeply understand relevant knowledge by changing its representation style, rather than mechanically teaching it.

In general, the three teachers all demonstrate the ability to use different representations flexibly in their teaching. More important, they can connect and integrate different representations of a topic to help students build connections among them, which plays “a role in learning mathematics with understanding” (Heibert & Carpenter, 1992, p. 66).

6.3.2.4 Designing appropriate teaching tasks according to the characteristics of the content

Another characteristic related to PCK is that the three teachers can design appropriate teaching tasks according to the characteristics of the teaching content to enrich students’ experience and facilitate students’ understanding. For example, Mr. Zhao adopted several methods to introduce “inverse proportion function”. In the first lesson, Mr. Zhao chose to let students explore several situational problems in which the idea of inverse proportion function was embodied. Based on this, he guided students to discover some common properties and further extract the definition of inverse proportion function from these situational problems. His reason for doing so, according to Mr. Zhao, was to let students experience the thinking behind inverse proportion function and the process of modeling, so that they could understand its definition more easily. In the third lesson, Mr. Zhao let students work in groups to draw graphs of some inverse proportion functions, and then made relevant modifications to those graphs. The main purpose of doing so was to let students explore how to draw the graph of inverse proportion function, based on which they could then discover some characteristics of the graphs. The fourth lesson’s main content was the properties of inverse function, and the teacher chose to explore these properties with students together.

Similarly, in Ms. Sun’s first lesson, she displayed many pictures of applications of triangles to introduce the definition of triangle and make students realize that a triangle is very useful in the real world. In the second lesson, she explored, together with students, some methods of proving that the sum of three interior angles of a triangle is 180^0, because Ms. Sun thought that this part is difficult for most of her students. In the third and fourth lessons, she asked students to find the bisectors of a triangle, median of a triangle, and height of a triangle by folding concrete paper triangles. This, in her view, gave students the opportunity and time to discover that three bisectors of a triangle join at a point, three median lines of a triangle join at a point, and three heights of triangles join at a point. In addition, it gave students opportunities to compare the locations of these three intersections, and more easily understand these characteristics.

6.3.2.5 Choosing and posing appropriate problems or exercises

For most mathematics lessons, the mathematics problem is a main tool to consolidate what students have newly learned and enhance students' understanding. The problems used by the three expert mathematics teachers suggest that they have the ability to choose or pose appropriate problems according to students' background, characteristics of teaching content, and teaching sequence to facilitate students' understanding and challenge students as well.

Various types of problems with different difficulties. Using the categories described in Chapter 4, types of problems were classified and their distribution was listed in Table 6.2. Due to differences in teaching content and students' background, the number and characteristics of problems used in different lessons varied among the teachers. Except for lessons introducing new topics, like Mr. Zhao's first and second lesson, Ms. Qian's first lesson, and Ms. Sun's second lesson, the three teachers used around 10 problems per lesson (most were taken from textbooks, and were very simple and basic for their students). In the other lessons, they used fewer problems, some of which were drawn from other materials. Although as shown in Table 6.2, many problems were routine and closed-ended, when the content was suitable for posing an application problem, the three teachers employed or posed application problems.

Moreover, based on the categories described in Chapter Four, the procedure complexity of every problem was analyzed, and the results were listed in Table 6.3. The distribution shows that the complexity of problems varies in each lesson. Normally, problems with moderate or high complexity are comparatively difficult. In addition, as shown in Table 6.2, some problems are combination problems. Prior knowledge and skills are needed to solve them. These problems are relatively difficult for students, especially those with weak mathematics backgrounds, and give students more mathematics challenges. This suggests that the three teachers could choose or pose problems with various levels of difficulty in teaching.

Table 6. 2. Distribution of different types of problems used in the three teachers' lessons

	Mr. Zhou				Ms. Qian						Ms. Sun				
	Lesson				Lesson						Lesson				
	1	2	5	6	1	2	3	4	5	6	1	2	3	4	5
Routine Problem	12	4	9	3	8	3	4	1	6	3	4	11	3	4	6
Non-routine Problem	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
Open-ended Problem	0	0	0	0	0	0	0	0	0	0	1	0	3	0	0
Closed-ended Problem	12	4	9	3	8	3	4	1	6	4	4	11	0	4	6
Application Problem	2	1	0	0	3	0	2	0	2	1	2	0	0	0	0
Non-application Problem	10	3	9	3	5	3	2	1	4	3	3	11	3	4	6
Combination Problem	2	3	5	1	3	0	3	1	3	1	0	1	1	3	3
Non-combination Problem	10	1	4	2	5	3	1	0	3	3	5	10	2	1	3

Table 6. 3. Distribution of complexity of problems used in the three teachers' lessons

	Mr. Zhou				Ms. Qian						Ms. Sun				
	Lesson				Lesson						Lesson				
	1	2	5	6	1	2	3	4	5	6	1	2	3	4	5
Low complexity	9	0	4	1	5	3	1	0	4	2	3	9	2	1	4
Moderate complexity	1	3	2	0	3	0	3	0	2	1	2	0	1	2	1
High complexity	2	1	3	2	0	0	0	1	0	1	0	2	0	1	1

The varieties of problems in individual lessons indicate that the three teachers consider individual students' differences and needs in their teaching.

Problems with increasing complexity and difficulty. As described above, the three teachers employed problems with different levels of difficulty in their teaching. They were further able to organize the sequence of the problems both within and across lessons in a reasonable manner. They could arrange problems by degree of difficult in a reasonable manner, so as to establish "scaffolding" for students, especially for those with average mathematics ability. Moreover, this further indicates that the three teachers are capable of identifying the complexity and difficulty of problems properly, based on teaching content. As shown in Tables 6.2 and 6.3, when presenting new topics, they employed more problems of low complexity and difficulty to help students consolidate the newly learned knowledge. In the following lessons, the teachers started to use problems with increasing difficulty to promote and reinforce students' understanding. This also suggests that the teachers are able to teach students with different academic backgrounds within the same class.

For particular problems of high complexity and difficulty, the three teachers demonstrated the ability to establish an appropriate ladder of difficulty among sub-problems. The first sub-problem would be relatively easy, and could be solved by most students with the use of newly learned knowledge. For example, the first problem in Mr. Zhao's fifth lesson was as follows:

> *Let* $y = \frac{-1}{x}$,
> *1) if point A (1, y_1) and point B (2, y_2) are in its graph, which one is bigger? y_1 or y_2?*
> *2) if point A (x_1, y_1) and point B (x_2, y_2) are in its graph and $x_1 < x_2 < 0$, which one is bigger? y_1 or y_2?*
> *3) if point A (x_1, y_1) and point B (x_2, y_2) are in its graph and $x_1 < x_2$, which one is bigger? y_1 or y_2?*

As the condition moves from special situation to general situation, the difficulty level of the sub-problems gradually increased. However, though the third sub-problem is difficult, with the preparation of the first two sub-problems, students with average mathematics ability might be able to solve it, or at least know how to begin to approach it. Moreover, as the level of difficulty level increases gradually and the conditions move from special to general, students can also get some experience of reasoning from special to more general situations.

Problems adhering examination requirements. The three teachers can integrate relevant examination requirements, especially *Zhongkao* requirements, into problems to teach students skills they can use in future. For example, in Ms. Sun's fourth lesson, she used the problem shown in Figure 6.4. Her reason for doing so is as followed.

In the figure, ABC is a triangle. The angle bisectors BD and CE meet at I. What is the relations between $\angle BIC$ and $\angle A$? Let α be $\angle A$. Find $\angle BIC$ in terms of α. Using the above relations, calculate:
$\angle BIC$ if $\angle A = 50^0$;
$\angle A$ if $\angle BIC = 130^0$.

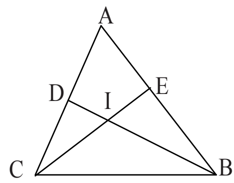

Figure 6. 4. A problem used in Ms. Sun's fourth lesson

Interviewer: When you solved this problem, you guided students to prove the relationship between $\angle A$ and $\angle CIB$. Why did you choose *to do so?*

Ms. Sun: Honestly, for examination; this one is important in different levels of examinations.

Interviewer: This is important in examinations?

Ms. Sun: Actually, this one is not only an important part in examination, we will learn it again in grade 9 because this point, point I, is the incenter of this triangle. In the second semester of grade 9, we will learn it again. At that time, it [conditions of the problem] will not tell you [students] that the concept of bisector of angle, it will only tell you that point I is the incenter of the triangle. You should know BI is the bisector of $\angle ABC$.

Interviewer: So you chose this one here.

Ms. Sun: Yes.

Interviewer: Don't you think this one is difficult?

Ms. Sun: *Not so difficult. Or, for students in our school, it is not so difficult, we will use this kind of problems in our regular examinations. Not only in regular examinations, in our unit examinations and other supplementary materials, this problem is very popular. It is a relatively classic problem, and in the second semester of Grade 8, we will use this problem again.*

Ms. Sun's explanation suggests that she knows not only what is important in examinations at different levels, but also knows how to choose important and typical problems with the integration of examination requirements whenever necessary to train students' relevant skills.

6.3.2.6 Making necessary preparation for future teaching

Making preparation for future teaching firstly means that the three teachers are able to prepare for the following teaching content within a lesson; that is, the flow of content within a lesson. In addition, they were able to make necessary preparations for the coming lesson(s). In other words, the three teachers did not consider an activity or a lesson in isolation. Instead, they could consider all the tasks within a lesson or in a unit, a chapter or relevant content at the junior secondary school level together. In the current lesson, the three teachers were laying the foundation for future lessons and connecting to previous lessons.

For example, at the beginning of Ms. Qian's third lesson, she let students view several pictures in which knowledge of golden section is embodied, and posed some problems for their consideration. After she presented the definition of golden section, she referred back to these problems. In Mr. Zhao' first lesson, he presented problem 1.8 to the students. After they finished the table, he guided them to explore the location of points in different quadrants and their relationships, Next, he asked the students to guess the possible location of its graph (graph of the inverse proportion function in this problem). His reason follows the problem:

Problem 1.8: Let y be the inverse proportion function of x. In the following table, some values of x and y are given. Answer the following questions:

x		-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1		3
y	$\frac{2}{3}$		2				-1	

1) *Find the analytical expression of this inverse proportion function;*
2) *According to the analytical expression, finish the table above.*

Mr. Zhao: …. especially when I presented the third exercise (problem 1.8), I made some variation. After I made this change, which aims to make students realize and understand that if the coordinates of two points are given, they are not always in the graph of inverse proportion function. If I have more time, I will dig it deeper. I will let them think about the situation of coordinates of three points are given.

Interviewer: Why did you choose to do so?

Mr. Zhao: It is for the coming lessons. It makes some preparation for the coming lesson (s), the combination of linear function and inverse proportion function.

6.3.3 Knowledge of the characteristics of learners

Teacher's knowledge of learners' characteristics is believed to significantly contribute to teaching practice (Even & Tirosh, 2002). As some information related to knowledge of learners has already been included in PCK reported above, this section reports on two other common characteristics related to knowledge of learners, namely knowledge related to students' personality and family background, and to students' mathematical ability and cognitive development.

6.3.3.1 Knowing learners' personality and family background well

The three teachers know students' personality well, including their interests, habits and learning attitudes. For example, Mr. Zhao once mentioned that boys in his class are very active and talkative. Ms. Sun mentioned that most students in the observed class are very shy and not brave or confident enough to express their opinions, even though, sometimes, they can solve problems successfully. Mr. Zhao mentioned that most of his students are not so self-disciplined and do not spend much time on their homework. Ms. Qian mentioned that most of her students are very self-disciplined and work very hard on mathematics after her teaching.

In addition to knowing current students' personality well, the three teachers were also aware of the differences between present students and those they had taught before. For example, Mr. Zhao and Ms. Qian once mentioned that there exist some differences in beliefs about learning, world, and self between their current and former students. These differences, in the teachers' opinions, make them update their knowledge and adjust how they deal with the teacher-student relationship.

In addition, the three teachers also know their students' family background well. Mr. Zhao mentioned that most of his students come from working-class families of average economic status. The parents do not pay much attention to their children's studies or are not knowledgeable enough to tutor their children's learning. Therefore, he has to try to help students solve their difficulties in his teaching. Ms. Sun and Ms. Qian mentioned that most of their students are from upper-class families of good economic status, and most of their parents are well educated. They heavily stress their children's learning and, sometimes, according to Ms. Sun and Ms. Qian, even like to put their own ideas into their teaching. However, Ms. Sun and Ms. Qian said that they would ignore this and continue on their own way, since they know how to teach students effectively.

6.3.3.2 Knowing learners' mathematical ability and cognitive development

The three teachers knew their students' mathematical abilities and cognitive development well, and could give detailed information about their students' academic backgrounds. For example, Mr. Zhao said there are few differences in mathematics achievement among the students in the observed class, and not many are mathematically gifted. He also mentioned that, even though boys are very talkative during lessons, they do not really think or reason mathematically. Similarly, Ms. Sun mentioned that most of her students are not very good at mathematics and that it will take her some time to improve their mathematics achievement. Ms. Qian mentioned that half of her students have strong mathematics ability and that some are good at logical reasoning.

In addition, they also know individual students' academic backgrounds well. For example, in their teaching they chose particular students to answer particular questions, either very easy or very difficult questions, because they know the student can successfully solve it or would have difficulty in solving it. In Ms. Qian's third lesson, when she presented a problem, she let students work on their own for a while. Then, she asked a boy to demonstrate his method. The reason she chose this boy was as follows:

> *I want to test their [all students] real understanding situation. This student is at the middle level [in the class]. He might have some difficulty in solving this problem. To successfully solve this problem, students need to use knowledge of square root, that is, irrational number. For most students, this is a difficult point and they learned it in last semester, most students might already forget it. Therefore, I asked this student to do some demonstrations. I knew he might make some mistakes. It does not matter. I can make some corrections or asked some students with good mathematics ability to correct it. This can also improve students' understanding.*

Ms. Qian's explanation indicates that, on certain topics or problems, she not only knew individual students' mathematics ability, but also some possible mistakes they might make. More important, it seems that she can make good use of such information in her teaching to correct her students' mistakes and enhance their understanding. Moreover, she used the information to assess her students' understanding.

6.3.4 Curriculum knowledge

Curricular knowledge, as described by Shulman (1986), is an important knowledge influencing teachers' teaching practice. Based on interviews and observations, the three teachers were found to have critical judgments about the latest curriculum reform, know the strengths and weaknesses of textbooks, have well structured vertical mathematics curricular knowledge, and know the connection between mathematics and other subjects or the real world.

6.3.4.1 Critical judgments about the curriculum reform

The first characteristic is that all three teachers have their own opinions about the latest curriculum reform, which has been implemented in mainland China since 2001. First, they seem to know the underlying ideas well. During the interviews, they could use ideas from the new curriculum standard to explain their teaching behaviors, such as "every student attains relevant development in mathematics", "different student should get different improvement", "students should have chance to experience the process of mathematics knowledge development".

However, this does not mean that they blindly, or even entirely accept the ideas in this curriculum reform. Instead, Mr. Zhao and Ms. Sun raised concerns regarding those parts they think are not so reasonable. For example, Mr. Zhao mentioned:

> *In the current mathematics curriculum at the junior secondary school level, the content related to proof has been reduced. Therefore, students' mathematics thinking cannot be effectively developed. In addition, because of this curriculum reform, students' computation ability also becomes poorer and poorer. I personally think this is not good for students' continuous development and learning in future. …., Therefore, if you want to link your teaching with the content they will learn in senior secondary school, you should add some content, not only some extra content, but also some difficult content. You should enhance the difficulty of your teaching content.*

Mr. Zhao's statement indicates that he not only knows the unreasonable parts of the new mathematics curriculum well, but also has clear ideas about how to make relevant modification to remedy these parts. Similarly, Ms. Sun commented on ideas in the curriculum reform. In the first post-observation interview, when she explained why she stressed heavily the process of exploring the definition of triangle in her teaching, she said:

> *As to the definition of triangle, in our new mathematics curriculum, it does not emphasize too much on this definition. However, I want to spend more time on exploring this concept with students. In my point of view, for those basic methods, which can be employed [as basics] to explore new knowledge in future, we should explore its fundamental elements thoroughly. Although the new mathematics curriculum does not stress heavily this definition, in my teaching, I stressed heavily it because it acts as a foundation to explore some theorems later.*

6.3.4.2 Well aware of strengths and weaknesses of textbooks

As mentioned in Chapter Four, all three teachers used textbooks published by Beijing Normal University Press. However, they also read textbooks published by other publishing houses and made comparisons among them. In the meantime, they also made comparisons between their current textbooks and the ones they used before. After this, they were able to identify the strengths and weaknesses in the current textbooks. All three teachers pointed out that their students are not familiar with some situational problems or tasks in the textbooks because they are set in the context of the northern part of China. They also mentioned that some content is not difficult or challenging enough for their students because their students' academic backgrounds are above the average (they all work in top key middle schools). They also pointed out that the textbook is not as readable for students compared with other textbooks. Therefore, it affects students' self-study. For example, Mr. Zhao mentioned:

> *As to the textbooks published by Beijing Normal University Publishing Press, my personal feeling is that they choose many excellent problems once used in Zhongkao in different cities, or some typical sample exercises. For our teachers, these provide us with some materials you have to deeply study before [you] teach them. However, my first feeling is that the textbooks are not readable for students. Students cannot get much information from it if they study by themselves. For teachers, we can analyze it gradually, but for students, it does not have too much readability. Sometimes, students cannot understand it at all, and*

> *sometimes, there is not much information that students need to read.*

In addition to identifying some of the textbooks' weaknesses, they also realized their strengths. They all felt this set of textbooks gives teachers much freedom in their teaching, even though they sometimes felt very challenging. In addition, this set of textbooks gives students opportunities and space to experience and explore mathematics because many situational problems are used to introduce new concepts. In the meantime, they pointed out that the arrangement of knowledge in this set of textbooks is relatively reasonable. The teachers mentioned that, unlike in former textbooks, in which a "linear type" of knowledge arrangement was adopted, these textbooks adopt a "spiral type" that reduces students learning pressure.

6.3.4.3 Well structured vertical mathematics curricular knowledge

Vertical curricular knowledge, according to Shulman (1986), refers to teacher's familiarity with the topics and issues that have been taught before and will be taught later. The three teachers were found to be able to clearly articulate the relationship between the newly presented knowledge, knowledge students learned before (even at the primary school level), and knowledge students will study later (including at the senior secondary school level). For example, Ms. Qian knew the knowledge structure within a certain topic very well. Before she taught "similar polygons", she clearly described the knowledge structure:

> *"Similar polygons" makes preparation for tomorrow's learning, making knowledge preparation for the learning of "similar triangle". It is a transitional part between the "similar figures" what learned yesterday and the "similar triangle" what will be learned tomorrow.*

She clearly knows the role and position of a given topic in a given unit. That is, she clearly knows the basis on which the new concept develops and what role it plays in students' future mathematics learning. In addition, she knows what knowledge students learned before was related to the topic, and how. For example, she could describe what kind of knowledge was related to the ratio of line segments and its properties:

> *In grade 6, they already learned ratio of numbers and its properties. These are knowledge foundation for this topic [ratio of line segments and its properties]. …., the second foundation is the properties of equality they learned in grade 7 and what they learned in the chapter right before this chapter is fraction and its properties. In addition, they are also very familiar with linear function. All these are the knowledge foundation for today's topic.*

6.3.4.4 Familiarity with the relationship between mathematics and other subjects/real life

The three teachers were quite familiar with the relationships between mathematics and other subjects, as well as real life. That is, they had rich lateral curricular knowledge, as defined by Shulman (1986). For example, Mr. Zhao knew which knowledge in other subjects was relevant to the inverse proportion function quite well. In his first lesson, one of the two situational problems constructed by Mr. Zhao to introduce the inverse proportion function involved the relationship between distance, speed, and time, which related to physics. After Mr. Zhao presented the two problems, he asked students to suggest another example from physics. The reason he chose to do so is that there are many topics in physics related to the inverse proportion function. In the mathematics textbook, there is an example showing the relationship among electric current (I), electric resistance (R) and electric voltage (U), which also relate to the inverse proportion function; however, Mr. Zhao did not adopt this example because students had not learnt that topic when he taught inverse proportion function. This indicates that Mr. Zhao knows not only the relationship between mathematics topics and those in other subjects, but also the teaching sequence of other subjects as well.

Similarly, Ms. Sun employed many examples from architecture and arts to introduce the definition of triangle. At the end of the fourth lesson, she mentioned the intersection of the three bisectors of a triangle could be named its "incenter" and the intersection of the three heights of a triangle could be named its "orthocenter". Her reason was:

> *I mentioned these two concepts here. According to textbook, it is not necessary. ….. However, I mentioned them here because students will start to learn physics in next term. Orthocenter is also an important concept in physics. There exist some relationships among topics in different subjects.*

6.3.5 Discussion

Investigating the characteristics of teachers' knowledge is critical to understanding the complexities of teaching (Schoenfeld, 2000). Similar to previous studies (*e.g.*, Rowland, 2008; Schoenfeld, 2000), this study used observations and interviews to explore the three expert mathematics teachers' knowledge because it is difficult to pose questions to test their knowledge before the classroom observation since the observation topic was decided by the three teachers themselves. Some knowledge, like PCK, knowledge of learners and knowledge of curriculum, is difficult to measure fully with pre-designed questions. The three teachers' knowledge was analyzed mainly based on information from observations, interviews and some other artifacts, such as lesson plans and knowledge structure pictures drawn by the three teachers.

Even though some knowledge reported above was identified mainly based on the three teachers' descriptions and the researcher's observation, as Schoenfeld (2000) proposed, observation, pre- and post-observation interviews, and artifacts such as lesson plans and journals can be used to attribute knowledge to a teacher. Evidence from these suggests that the three teachers have a wide and profound knowledge base. All of them understand mathematics deeply and can connect different topics into a network structure; they know students' knowledge backgrounds and cognitive development well; for individual topics, they know students' prerequisite knowledge and possible difficulties; they know how to design various teaching tasks and pose various problems to maximize students' engagement and reduce students' difficulties; and they not only know the structure, strengths and weaknesses of textbooks, but what and how topics from other fields relate to topics in mathematics.

These findings support statements made by researchers who proposed a prototype model of expert teachers (*e.g.*, Cowley, 1996; Sternberg & Horvath, 1995) and relevant statements by the 21 interviewees in the present study as well. In the prototypical models for expert teachers, having extensive, accessible knowledge of subject matter, teaching and curriculum is an important component. Consistent with previous research findings such a knowledge base was found to be possessed by the three expert teachers in this study. For example, Smith and Strahan (2004) and Tsui (2003, 2009) found that expert teachers are masters of their content areas, Li *et al.* (2005) found that expert mathematics teachers tend to understand mathematics more deeply than non-experts; Zhu *et al.* (2007) found that the expert mathematics teacher in their study knows students' prior experience and prior knowledge base much better than the novice teacher; Li, Huang and Yang (2011) found that middle school expert mathematics teachers in mainland China have sound subject content knowledge and the ability to identify and deal with students' difficulties appropriately in learning; and, many other studies have found that expert teachers, including expert mathematics teachers, know the students they are teaching well (*e.g.*, Berliner & Carter, 1989; Lin, 1999; Ropo, 1990).

However, most of these studies mainly focused on one or two aspects of expert teachers' knowledge, rather than exploring expert mathematics teachers' knowledge as a whole. Moreover, unlike the findings found in Leinhardt and Smith's (1985) study, in which not all four expert mathematics teachers were able to understand mathematics deeply, the three expert mathematics teachers in this study all demonstrated a deep understanding of the teaching content, even though the sample is very small.

Many social and cultural factors might influence the three expert mathematics teachers' knowledge base, such as their schooling experience. As Ms. Sun and Mr. Zhao recalled, they were very good at mathematics when they were in secondary school. As found in Ma's (1999) research, many secondary school graduates in mainland China already have a profound understanding of mathematics. In addition, Ms. Sun and Mr. Zhao mentioned that the methods used by their secondary school mathematics teachers impacted on their teaching. This indicates that they gained some knowledge about how to teach mathematics from their own learning.

Their pre-service training experience might also have helped them to understand mathematics deeply. As described in Chapter Three, teacher training institutions in mainland China have a "strong academic emphasis" (Williams & Morris, 2000, p. 268). In particular, under the influence of the former Soviet education model, pre-service mathematics teachers in China are required to study many courses related to both basic and advanced mathematics (Li *et al.*, 2008). Indeed, the international comparative study, *Teacher Education and Development Study: Learning to Teach Mathematics (TEDS-M)*, recently found that pre-service mathematics teachers' mathematics content knowledge is influenced by their learning experience and opportunities in training institutions (Blömeke & Kaiser, 2012; Schmidt, Blömeke, & Tatto, 2011; Schmidt, Cogan, & Houang, 2011). Even though the relationship between opportunities to learning and mathematics content knowledge is described as "complex" (Tatto & Senk, 2011, p. 133), pre-service mathematics teachers exposed to advanced or basic university mathematics generally performed significantly better than those who only learned school mathematics (Blömeke & Kaiser, 2012). More specifically, Schmidt, Cogan and Houang (2011) found that pre-service mathematics teachers at the junior secondary level in countries with the highest mathematical content knowledge scores took almost twice as many mathematics courses, and significantly more mathematics methods courses, than their counterparts in lower performing countries. Since, as Ms. Qian and Ms. Sun recalled, they had spent a lot of time learning advanced mathematics in teachers colleges, the three teachers' learning and training experience at the pre-service stage may have helped them to understand mathematics deeply, at least mathematics at the junior secondary school level.

The school-based research system might also have made some contribution to their knowledge base. According to the teachers, in-service teacher models adopted in China –like the apprenticeship model, public lessons, and teaching contests – provide teachers opportunities to develop their knowledge base and, particularly, to learn how to teach (Han, 2012; Yang & Ricks, 2012). Activities organized by lesson preparation groups, such as discussing how to teach a certain topic and planning lessons together, provide opportunities for teachers to learn how to teach from other teachers(Han, 2012; Huang, Li, & Su, 2012; Li, Qi, & Wang, 2012; Li, Tang & Gong, 2011; Yang, 2009). In the meantime, since the activities are closely connected to mathematics content, teachers would enhance their deep understanding of mathematics content gradually as well (Huang *et al.*, 2011; Li, Tang & Gong, 2011; Yang, 2009). At the very beginning of their teaching careers, the teachers said that they learned a great deal of firsthand experience from their mentors and other teachers. They also acknowledged that they benefited from participating in teaching competitions at various levels. Even now, the three teachers stated that they still learn from other teachers, sometimes even from novice teachers.

One thing that needs to be pointed out is that the three teachers mentioned knowledge at primary school level related to certain topics; however, they did not mention knowledge at the senior secondary school level, neither in their knowledge structures diagrams nor in the interviews. One reason might be that there really are not many relationships between the topic and knowledge at the senior secondary school level. Moreover, mathematics teachers in other places, such as Hong Kong, have the opportunity to teach at various grade levels at the junior and senior secondary school levels; in mainland China, junior secondary mathematics teachers only teach students from grade 7 to grade 9. Therefore, they might not be as familiar with senior secondary content. However, as all three teachers taught at the junior secondary school level for about 15 years, such an experience might have enhanced their understanding of teaching content, such as the structure of textbooks and the characteristics of students, especially students' difficulties and mistakes, at the junior secondary school level. During their fifteen years of teaching, they used several sets of textbooks; this kind of experience might have contributed to their ability to compare the strengths and weakness of various sets of textbooks and therefore, more easily discern the strengths and weakness of newly adopted textbooks.

6.4 Teaching Strategy

In addition to having various kinds of knowledge, the three expert mathematics teachers were also found to have rich teaching strategies, which include using their previous teaching experience in current teaching, showing respect to students, using class time effectively, and making effective use of the blackboard.

6.4.1 Using previous teaching experience

All three teachers were skillful at using their previous experience in their current teaching. For example, they could adjust their previous teaching design to suit current students' characteristics rather than designing a completely new lesson plan. They could also employ those effective examples they had used before and make assumptions based on their former students' difficulties. For example, in Ms. Sun's second lesson, she guided students to prove the sum of the three interior angles of a triangle equals to 180^0 using three different methods as shown in Figure 6.5, even though it was not compulsory in the textbook. Her reason is:

> *According to my previous experience, as to how to add auxiliary line, it is very difficult for most students. However, it is very important in proof, especially for students to learn geometry. At junior secondary school level, to solve a problem, we need to add one or two auxiliary lines.*

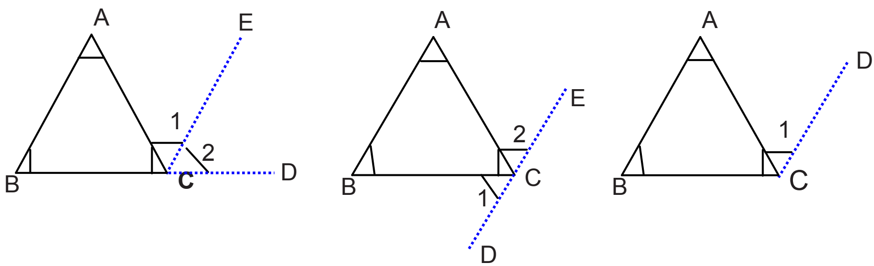

Figure 6. 5. Auxiliary line added by Ms.Sun

Similarly, in Mr. Zhao's first lesson, he used problem 1.8 (see Section 6.3.2.6) to demonstrate the combination of a linear function and an inverse proportion function, for the following reason:

> *this problem, en, actually, when I taught the last cohort of students, it was in a public lesson, I had a brainwave and decide to use this problem in that lesson. It was highly appreciated by the teachers who observed this lesson. They said that it is very good to deal with it in this way. Therefore, I used it again.*

6.4.2 Showing respect to students

The three teachers were very skillful at creating a harmonious classroom atmosphere to inspire students' interests and showed respect to their students during teaching. All were able to use just their tone of voice to attract their students' attention and to emphasize important topics or critical points during their teaching. When students were very tired or not concentrateing, instead of criticizing them, they would make little jokes to relax the students, and then continue with their teaching. According to their explanations in the post-observation interviews, they thought their approach was effective and they should respect students.

The three teachers demonstrated the wisdom of respecting students during their teaching. For example, in Ms. Qian's fifth lesson, she approached a girl with a weak academic background about a question that the girl did not know how to solve. Ms. Qian did not explicitly ask the girl if she could answer the question (even though she knew the girl could not have solved it, according to her later explanation); she merely asked whether she finish the problem. After the girl told her she had not, Ms. Qian asked her to continue to work on it. Similarly, Ms. Sun used eye contact to urge some of her students to concentrate on her teaching instead of criticizing them publicly for their inattention.

6.4.3 Effective use of lesson time

The three teachers were able to use their lesson time effectively in the observed lessons and seldom spent time on matters irrelevant to teaching. Before the lessons, they would make necessary preparations. For example, before Mr. Zhao began his first lesson, he drew three tables on the blackboard that he wanted to use in the coming teaching. While students worked on their exercises, he wrote down other exercises for followed-up teaching. Similarly, as Ms. Qian and Ms. Sun often used an overhead projector, they would turn it on before their teaching. When they planned to use hands-on activities in their teaching, they would ask students to make necessary preparation in advance, which also indicates that they planned their teaching well and had the ability to maximize lesson time.

6.4.4 Effective use of blackboard

Mr. Zhao and Ms. Qian demonstrated the ability to design their writing on the blackboard (*Banshu*) logically and clearly (Ms. Sun used the overhead projector in every lesson). As shown in Figure 6.6 and Figure

6.7, important points, such as definitions and problem solving procedures, were written down by Mr. Zhao and Ms. Qian on the blackboard and remained there as a written record for the entire lesson. During their teaching, they referred to topics written down previously to make the knowledge relationship more explicit and to help students understand more easily and construct an organized knowledge structure.

6.4.5 Discussion

In this section, some characteristics related to teaching strategies were reported. The three expert mathematics teachers used previous experience to plan their lessons, respected their students, effectively used lesson time, and logically designed *banshu* to facilitate students' understanding. These characteristics were also found in other studies. For example, respecting students was described as a prototypical proposition of expert teachers (Berliner, 2001, 2004). Borko and Livingston (1989) found that expert mathematics teachers use their previous experience to plan their lessons and Leung (1995) found that mathematics teachers in Beijing spend less time on matters un-related to teaching.

The teaching strategies identified among the three expert mathematics teachers might be influenced by social and cultural factors. For example, teaching mathematics is traditionally mainly by the use chalk and blackboard (Li *et al.*, 2008); as such, the ability to design *banshu* clearly and neatly is highly emphasized in both pre- and in-service training, and is an important factor in some teaching evaluation systems (Jiang, 2001). Furthermore, as Paine (1990) argued, the teacher in Mainland China is "virtuoso", that is, teaching is an act of performance. This tradition might have contributed to the three teachers' excellent performance as actors in their teaching, such as using their voices well and making facial expressions. However, the tradition of teachers having and exercising a high degree of authority during teaching, as described by Paine (1990), was not found to influence the three teachers' teaching; instead, they respected their students, built equal teacher-student relationships, and created a harmonious classroom atmosphere.

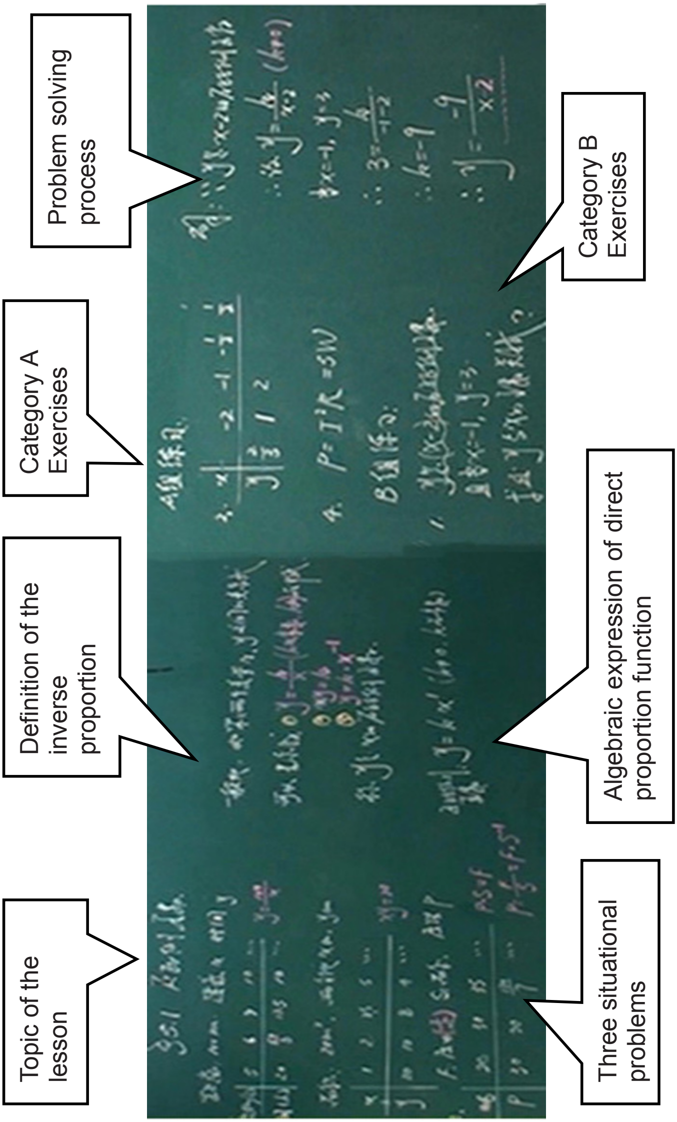

Figure 6. 6. Banshu of Mr. Zhao's first lesson

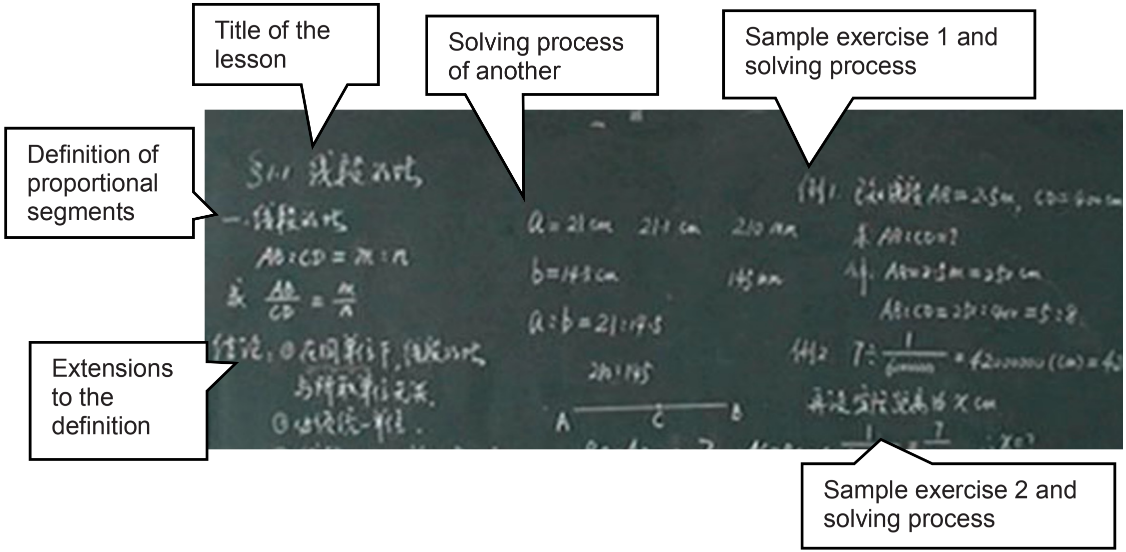

Figure 6. 7. Banshu of Ms. Qian's first lesson

6.5 Summary of the Chapter

This chapter reported the study's findings related to the characteristics of the three expert mathematics teachers' beliefs, knowledge, and teaching wisdom. Their beliefs about mathematics were found to be a combination of the instrumental view of mathematics and the problem solving view of mathematics (Ernest, 1990). They tended to believe that, to learn mathematics well, students need to become intellectually involved in the process of mathematics learning; and that, to teach mathematics effectively, teachers must also involve students intellectually. Moreover, they were found to have a wide and profound knowledge base in mathematics, pedagogy content knowledge, curriculum, and learners. They were also found to use lesson time effectively, design *banshu* clearly and logically, make use of previous teaching experience, and respect their students. The beliefs they hold and the knowledge they possess influence their teaching practice. The next chapter will report how the three expert mathematics teachers plan their lessons, use the curriculum, implement their teaching, and reflect on their teaching.

Chapter Seven

Classroom Teaching Practice of Expert Mathematics Teachers

7.1 Introduction

The previous chapter reported on characteristics common to the three expert mathematics teachers: their beliefs, knowledge and teaching wisdom. Beliefs and knowledge affect the ways in which they set teaching objectives, deal with teaching materials, design teaching tasks, and implement lesson plans. This chapter reports on characteristics of their classroom teaching practice, as identified from video-taped lessons, interviews, field notes and documentary materials. These characteristics are categorized as follows:

7.2 Teaching with Flexibility

An important characteristic of the three expert mathematics teachers' teaching is their ability to plan lessons, use teaching materials, structure and organize their lessons with flexibility.

7.2.1 Planning lessons thoughtfully and flexibly

The three teachers had well-thought-out plans for the main activities in each lesson before attempting to teach it. Evidence from pre-observation interviews shows that they had clear teaching goals in mind, and knew clearly what content they needed to cover in the lesson. They mentally prepared for each lesson, even though sometimes without a detailed written plan (*e.g.*, Mr. Zhao). For example, in Mr. Zhao's fifth pre-observation interview, he said:

> *In this lesson, I will mainly present three topics. The first one is to review the properties of inverse proportion function; the second one is the ways to find the algebraic expression of inverse proportion function; and the third one is the combination of inverse proportion function and linear function.*
> *(Note: "combination" here means some problems with the com-*

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bination of knowledge of inverse proportion function and linear function)

He delivered that observed lesson according to his planned procedure. Similar statements can also be found in Ms. Qian's and Ms. Sun's pre-observation interviews and lesson plans. This indicates that the three expert mathematics teachers are able to: 1) understand the difficulty and complexity of the teaching content well so as to choose and design tasks with appropriate difficulty to challenge students; 2) organize activities in a reasonable manner; and 3) anticipate and be prepared for students' responses. Such well-planned teaching processes made their teaching looks well-structured and in control, even when students worked on hands-on activities or in small groups.

While the three teachers planned every lesson well, they seldom planned every activity in detail or carried out their planned lesson mechanically; instead, they only outlined some main activities, which gave them the freedom to carry out these activities flexibly according to students' understanding and reactions. This might suggest that they had the ability to plan activities in detail and to carry a variety of activity plans in their mind. Sometimes, they made alternative plans in case the first option did not work out as expected. For example, in Ms. Qian's second pre-observation interview (aiming to introduce golden section), she said:

If everything goes very smoothly, if students can understand what I teach well, I will make some further application (note: add some tasks with the application of "golden section) in this lesson if I have time. This part is not a compulsory one. I will make my final decision [to add it or not] according to the actual situation.

In addition to preparing alternative plans, their lesson planning was ongoing and dynamic, occurring both before and during specific teaching activities. The teachers mentioned that, as it was difficult or even impossible to predict students' reactions accurately, they needed to make necessary change to their plans according to students' actual reactions. For example, for her second lesson, Ms. Sun had planned to add the topic "Sum of Exterior Angles of a Triangle" to the lesson if time permitted (in accordance with her lesson preparation group's requirements). However, in her actual lesson, she did not do so because:

> *I do not have that much time. I planned to teach this. However, I found that they [students] had difficulties in proving the theorem of sum of interior angles of a triangle. I spent some time on its proof. As you noticed, I used three different methods to prove this theorem. If I mention the sum of exterior angles of a triangle, I have to mention its definition, how to find an exterior angle and so on. … I do not have that much time. Therefore, I did not mention it.*

Ms. Sun's explanation indicates that she was able to flexibly adjust her lesson plan once she noticed that her students had difficulty in proving the theorem on the sum of interior angles of a triangle. Similar phenomenon happened several times during her teaching, and in Ms. Qian's and Mr. Zhao' teaching as well. This suggests that all three teachers were able to plan their teaching activities in a flexible way to account for students' reactions. However, they still demonstrated the ability to make their lessons appear well structured. This indicates that they could read cues from students (Berliner, 2001) and had the ability to make on-going adjustments; however, the changes were seldom found to be very large. This suggests that they would not plan everything according to students' actual reactions during teaching.

7.2.2 Using teaching materials flexibly

Another characteristic common to the three teachers is that all were able to use textbooks and teaching reference materials flexibly at both a macro and individual lesson level:

7.2.2.1 Using teaching materials at a macro-level flexibly

The three teachers were able to use teaching materials at a relatively macro-level, usually the chapter level, flexibly. They would read the textbook chapter intensively to understand its overall goal and its position at the grade level and junior secondary school level. Next, they would sort out the relevant topics that had been or were to be presented before and after this chapter, and how it relates to relevant topics in this chapter. They would list all the topics (in Chinese, "knowledge points") and then constructed them in a logical sequence, particularly when they found the original one in the textbook was not reasonable or suitable for their students.

Even though the three teachers did not make many changes to the knowledge point sequence in the observed lessons, they still made some in other lessons. For example, when Mr. Zhao taught methods to solve quadratic equations with a single unknown, he rearranged the textbook sequence (completing the square, using formulae and factorization) to factorization, completing the square and using formula. His reason was as follows:

> *Interviewer: Any problem with the original arrangement?*
> *Mr. Zhao: I think that there exist some gap between students' cognition and this arrangement [original arrangement].*
> *Interviewer: Why you think so?*
> *Mr. Zhao: I think that it will be much easier for students to understand the method of factorization because this method is very flexible. After students learn this method and become familiar with it, it will be much easier for them to learn method of completing the square and using formula.*

His explanation suggests that he had his own understanding of the textbook content and knew how to arrange it more reasonably, at least from his perspective. Similarly, Ms. Sun mentioned that, even though she was then teaching a Grade 7 class, she would add some Grade 9 content on the triangle to her lesson because doing so helped students build a more complete knowledge structure. To that end, she had to adjust the teaching sequence.

In addition to making changes to textbook content to meet their students' situation, the three teachers also used teaching reference materials flexibly. For example, the textbook content related to the inverse proportion function includes its definition, graph, properties and application, all of which were to be taught over a six-lesson period, according to the teaching reference material. However, Mr. Zhao felt this was too demanding for his students and that they would not understand the concepts fully after only six lessons. As such, as shown in Figure 7.1, he used two lessons to teach the definition, one lesson for graph and five lessons for its properties. The reason for his taking five lessons to teach its properties was that he added some content related to the combination of inverse proportion and linear functions, a topic he thought would be difficult for his students to understand but very important for examinations, including the *Zhongkao*.

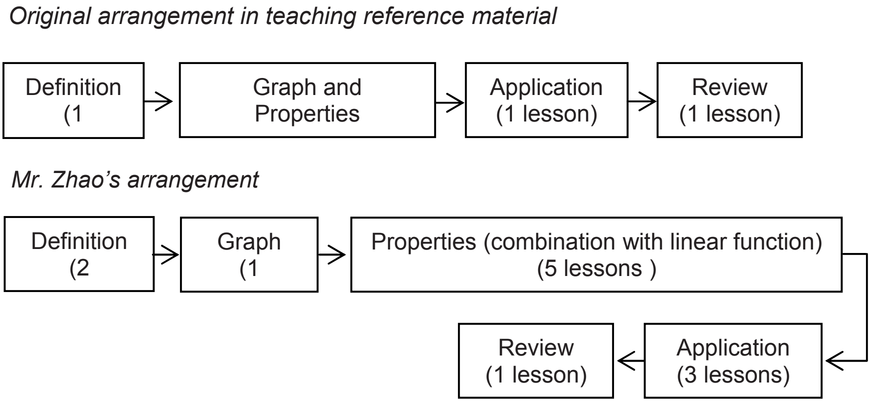

Figure 7.1. Comparison between Mr. Zhao arrangement and the one in teaching reference material

7.2.2.2 Dealing with teaching content for individual lessons flexibly

When planning individual lessons, the three teachers demonstrated the ability to deal with teaching materials flexibly. The following typical characteristics were identified:

Adopting activities from textbooks selectively. When preparing individual lessons, the teachers would make critical judgments about textbook content and selectively adopt introductory activities, sample exercises, and exercises according to their students' actual situations. For example, in Mr. Zhao's fourth lesson, he let students discover some common characteristics of three graphs of inverse proportion functions that were included in the textbooks, as he thought this would provide students with basic information about the properties of inverse proportion functions. Moreover, directly adopting the graphs would save him some instruction time. Similarly, in Ms. Sun's second lesson, she took two tasks directly from textbooks to introduce the new topic, and used a lesson structure nearly identical to that in the textbook. She explained:

> *I think this structure [in the textbook] is reasonable, if it is not reasonable, I would not teach in this way. It provides students with chance and time to experience the process of abstracting the concept from practice in real life, moreover, students have chance to explore the relationship between the lengths of three sides of a triangle from a mathematics perspective, that is, from a pure mathematics perspective, from practice to theoretical level. …, I think it fits my students' cognition development process.*

Adapting activities from textbooks flexibly. The three teachers were all able to change certain activities to suit their students' actual situation. In Ms. Qian's first lesson, she used different-sized photos of the students and of the Chinese flag to introduce the idea of similarity, rather than using the two photos of the Temple of Heaven provided in the textbook. As she said:

> *we still can use these pictures in textbook. However, I feel that they are far away from students' life even though it is a picture of a very famous place of interest in China. As to photos of students themselves, they are more closely to their life…., the second samples are five stars in Chinese flag, it is even closer to their life. We will raise the national flag on every Monday morning. Furthermore, using this example can also give them moral education.*

Similarly, in Mr. Zhao's first lesson, he constructed a situation that involved carrying goods to help survivors of the devastating *Wenchuan* earthquake (which had occurred only days before) to illustrate the relationship between distance, speed and time, rather than using the textbook example. The *Wenchuan* situation, he noted, resonated with the students and provided an opportunity for moral education. These two examples indicate the two teachers are able to adapt textbook examples to make their teaching relate to students' real life and to provide opportunities for many other possibilities, such as moral education.

The three teachers also made frequent changes to sample exercises, such as changing a problem's background or conditions, while maintaining its original aims and functions. A common explanation was given by Ms. Qian:

> *I will make changes to some sample exercises to inspire students' learning interests. If I always present the same sample exercises in textbook to students, some students will not really think about them, they will not think about these exercises on their own because there are answers in textbook. They will give you the answers in the textbook. …, Normally, if those exercises in textbooks are very good, I will choose them. Otherwise, I will adapt them to make them more closely to students' actual situation, inspire students' learning interest, and more important, make students feel novel. Of course, I will not change their original objectives and functions. I still use them to teach students the same problem-solving skills as the ones in textbook intend to.*

Extra supplementary activities. The three expert mathematics teachers added extra activities or information to nearly every lesson to achieve teaching objectives more effectively and to make their teaching relate more closely to their students' actual situations. In particular, problems with higher levels of difficulty and complexity than those in textbooks were added to: 1) challenge students, as the original ones in textbooks were not challenging enough; 2) catch the interest of students with high mathematics ability; and 3) train students' problem-solving skills and mathematics thinking.

They also added extra information during their teaching to enrich students' mathematics experience and broaden their horizons. For example, in Ms. Qian's third lesson, intended to introduce the golden section, she used many pictures of different architectural structures, including the Parthenon and Notre-Dame de Paris, to show how the golden section is widely applied in real life and can be seen in the real world. Similarly, in Ms. Sun's first lesson, aimed at defining triangles, she showed students many pictures depicting the applications of triangles, because:

> *society develops very quickly; however, our textbook writers cannot update information in time. They cannot re-write textbooks every year. Therefore, there exists an information gap between textbook content and information development in real world.*

> *There are so many things happened every day nowadays. It is impossible for information in textbooks keeps updated. Therefore, there is some outdated information in textbooks. …, however, for our teachers, we need to make our teaching catch up with the development of society. We need to add or incorporate new information in our teaching to show students that mathematics can be applied in real life.*

Ms. Sun's explanation suggests that she had not only the intention of relating her teaching to her students' real world, but also the ability to do so by integrating current information into her teaching. The ways in which Ms. Qian and Ms. Sun dealt with the textbooks further indicates that they were able to supplement their teaching with relevant information to construct a classroom environment that reflected mathematics as a part of human culture.

7.2.3 Flexible lesson structure

How a teacher organizes a lesson or series of lessons may constrain both what the content taught and the ways in which it can be taught (Hiebert, *et al.*, 2003). The kinds of lesson activities employed by the three teachers, the time spent on those activities, and the ways in which these activities were organized were analyzed using the codes described in Chapter Four. The results are listed in Figure 7.2 below. No clear teaching pattern can be identified among the three teachers' lessons or within an individual teacher's lessons. Rather, there exists much variation in the types of activities used, as well as the time spent on, and the sequence of these activities.

Comparatively speaking, Ms. Qian spent less time presenting new topics than did Mr. Zhao and Ms. Sun, who spent nearly half of their lesson time presenting new topics. This might be due to differences in their students' academic backgrounds. As mentioned in Chapter Four, half of Ms. Qian's students were strong in mathematics, while most of Mr. Zhao's and Ms. Sun's students were not. Therefore, the latter two teachers might have had to spend more time guiding students to explore the new topics to ensure they understood them deeply and thoroughly.

The ways in which the three teachers structured their lessons suggest that they never blindly and mechanically taught according to

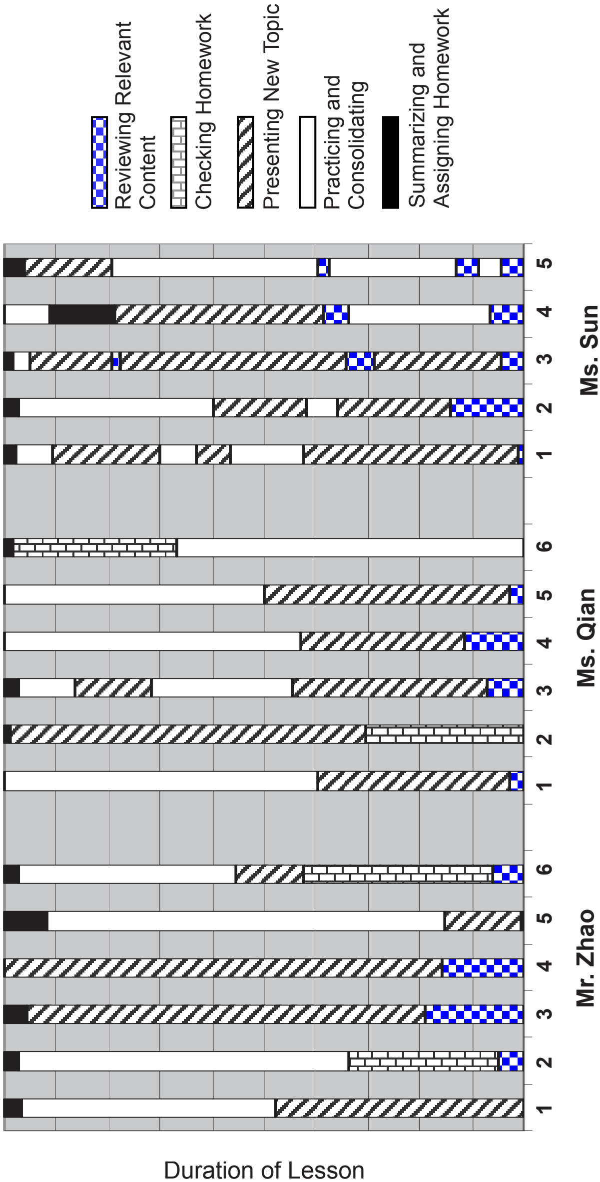

Figure 7. 2. Lesson structure of the three expert mathematics teachers' lessons

procedures suggested by other teachers, textbook writers or scholars; rather, they flexibly structured their lessons to suit the teaching content and their students' backgrounds. Even though there exist some variations in the three teachers' lesson structures, if regardless of the differences in the amount of time spent on each activity, in those lessons aimed at introducing new topics, a general and implicit process can be identified: review relevant teaching content; present the new topic; practice and consolidate the information; and summarize and highlight important aspects.

7.2.4 Flexible lesson organization

The ways in which a lesson is organized provide the context within which teachers engage their students (Stigler & Hiebert, 1999). Lesson organization for every lesson was coded using the codes described in Chapter Four. Figure 7.3 displays the distribution of lesson time spent on each activity during individual lessons, while Figure 7.4 shows the relative aggregate amount of time spent on each type of activity in each lesson. As shown in Figure 7.3 and Figure 7.4, one common characteristic is that all three teachers tended to spend at least 50% of their lesson time, and in some lessons more than 90%, on "classwork", with most of the remaining time used for "seatwork individual". When combined with the results shown in Figure 7.2, it can be seen that "classwork" occurred frequently and lasted a long time when new definitions or theorems were introduced. This indicates that the teachers did not let students explore new concepts blindly or without adequate guidance.

However, this does not mean that the three teachers never let students work in groups; if the teaching content were suitable, the three teachers would ask students to work on some tasks with their peers. As shown in Figure 7.3, in Mr. Zhao's third lesson, he spent more than 20% of lesson time on "group work" in order to let students explore how to draw graphs of the inverse proportion functions. He chose to do so because:

> *Mr. Zhao: The important points of today's topic are "the graph [of inverse proportion function]" and "table expression [of inverse proportion function]", how to draw its graph is the most important*

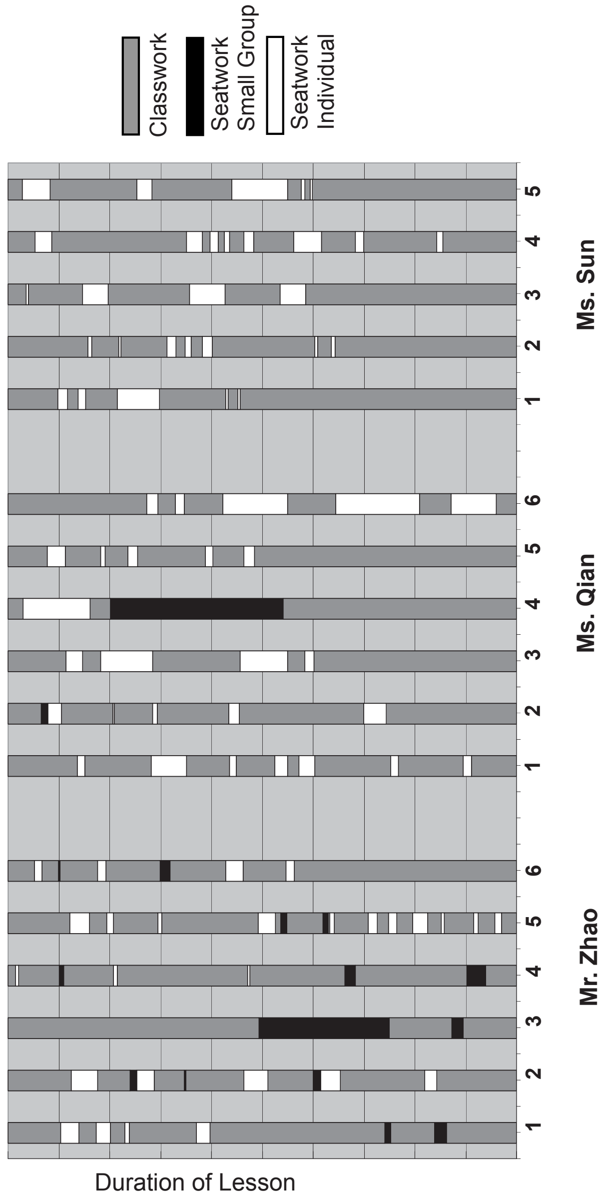

Figure 7. 3. Classroom organization of the three expert mathematics teachers

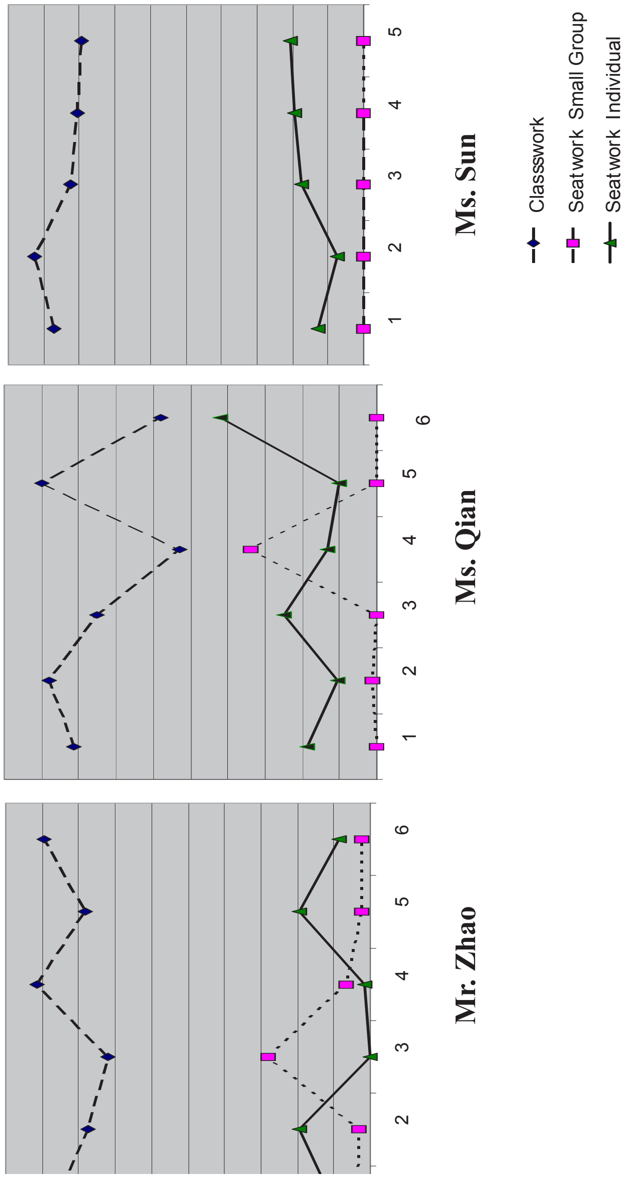

Figure 7. 4. Percentages and distributions of classroom organization over the three expert mathematics teachers' observed lessons

> *point. The procedure to draw its graph is similar to the one used to draw the graph of linear function. I will review the procedure to draw graph of linear function with students first today and after that, I willask students to work in groups to explore how to draw graph of inverse proportion function. …, if students can draw the graph of inverse proportion graph, then I have achieved the teaching goal of this lesson.*
> *Interviewer: Why will you choose to let students work together?*
> *Mr. Zhao: They have to experience it on their own. Of course, I can do some demonstration. Eventually, they have to be able to draw it by themselves.*

Mr. Zhao's explanation indicates that he knew both the characteristics of the lesson's topic and its teaching objective quite well, and chose to develop his students' abilities through their own exploration. Similarly, in Ms. Qian's fourth lesson, she spent more than 25% of lesson time on "group work" in order to let students experience how to draw similar figures. She explained:

> *The main aim of today's topic is make students be able to identify which figures in real life are similar….., In the meantime, with the consideration of [students'] feeling, attitude, and [the demonstration of knowledge] process, I will encourage students to participate [in some activities]. I designed a manipulative activity according to the one in textbook. I will ask them to draw similar figures with the use of elastic. Through their activity, make them feel that mathematics can be applied in real life, therefore, it can enhance their interest of learning mathematics and the idea of applying mathematics.*

Mr. Zhao's and Ms. Qian's explanations indicate the ability to choose lesson organization flexibly according to teaching content characteristics, teaching objectives and students' backgrounds.

7.2.5 Discussion

This section has reported one characteristic common to the three expert mathematics teachers' teaching — teaching with flexibility. Even though the teachers all had well-thought-out plans for some key teaching activities and procedures before commencing their lessons, they each made ongoing adjustments according to students' actual understanding and reactions; in other words, they did not strictly follow a detailed script. They also dealt with teaching materials flexibly at both a macro and individual lesson level, rather than being constrained by these materials. They demonstrated the ability to structure and organize lessons flexibly according to teaching content characteristics, teaching objectives, and their students' backgrounds.

This characteristic differs from findings in previous studies on non-expert mathematics teachers' teaching in mainland China. For example, Leung (1995) found that, in lessons taught by some mathematics teachers in Beijing, "each step of the lesson followed another in a structured and sometimes rigid manner" (p. 303), that "the content of most lessons followed the textbooks closely" and that "the examples were in general taken directly from the textbook" (p. 305). Fan *et al.* (2004) also found that beginning teachers "followed the textbooks very closely" (p. 258). In Li *et al.*'s (2009) study, teachers' lesson planning was found to be "clearly guided" (p. 729) by the textbooks and teaching reference materials. Differences between the present study and previous studies suggest that the three expert mathematics teachers might indeed have sufficient expertise to plan and implement their teaching flexibly.

In the literature, flexibility has been described as a proposition of expert teachers (*e.g.*, Berliner, 2001, 2004; Cowley, 1996; Rollett, 1992) and has been noted in previous studies on expert teachers' practice. For example, expert (mathematics) teachers in Western cultures demonstrated the ability to plan lesson at a macro level (*e.g.*, Borko & Livington, 1989; Livingston & Borko, 1989; Moallem, 1998). Expert mathematics teachers in Livingston and Borko's (1989) study were found to be able to make ongoing adjustments to their plans. However, the latter teachers were found to make decisions about instructional details shortly before the actual event; moreover, their "lesson plans did not include details as timing, pacing, and the exact number of examples and problems; these aspects of instruction were determined during the class session on the basis of student questions and responses" (p. 480). In the present study, the three expert mathematics teachers carefully chose some main activities and, to a large degree, organized them in a logical sequence before teaching. They only made ongoing adjustments to individual activities based to students' reactions, rather than making every decision during teaching. Similar findings were also found in other studies. For example, expert mathematics teachers in Leinhardt's (1989) study were found to have "a more complete action list, with each action supported by more complete subplans and routines" (p. 64), and the expert teacher in Tsui's (2009) study was also found to be never automatic or effortless, but spent much time planning and rehearsing details of her lesson in her mind.

Even though the three teachers were found not to follow textbook and teaching reference materials closely, this is not to say that they completely developed their own curriculum as some very experienced teachers in Western countries did, or departed from textbooks and teaching reference materials to a large degree. For example, expert mathematics teachers in Borko and Livingston's (1989) appeared to make "minimal use" (p. 481) of textbooks in teaching, while Remillard and Bryans (2004) summarized that experienced mathematics teachers used the curriculum intermittently and narrowly. Brown and Edelson (2003) also found that, for experienced teachers, curriculum materials only provide "seed" ideas. In contrast, whether before the start of a new chapter or before an individual lesson, all three expert mathematics teachers in this study would read the textbooks, teaching reference materials and examination requirements intensively. Once they found materials that were useful or suitable for their students, they would adopt or adapt them into their teaching. In other words, the textbooks provided them with "the sources and content and activities" (Mok & Morris, 2001, p. 466) for their further adaption and modification.

The differences between the present study and some previous studies conducted in Western countries indicate that, in practice, expert mathematics teachers in mainland China teach less flexibly than do their Western counterparts. This may be due to differences in the social and cultural contexts. In Chinese culture, lesson preparation is viewed as an important stage of teaching (Ma *et al.*, 2004), and one of a teacher's main duties is to "prepare a lesson (*beike*)" and "teach a class (*shangke*)" (Paine & Fang, 2007, p. 184). From this point of view, planning a lesson is an integral part of teaching a lesson. Moreover, the ability to prepare a lesson properly reflects on a teacher's professional ability (Ma *et al.*, 2004). After a student teacher enters a teaching position, s/he will be encouraged to learn basic lesson preparation skills from experienced teachers and to plan every lesson carefully. As Ma *et al.* (2004) noted, sometimes new teachers are told never to "enter the classroom without preparations and never teach without a plan" (pp. 422-423). This is to say that preparing for lessons is an important regulation for teachers in China. No matter whether they are new or experienced teachers, they are required to prepare for lessons and write lesson plans. School administrators will examine teachers' lesson plans at random to evaluate their work. For example, a document in Mr. Zhao's school states that the ability to plan lessons carefully is an important factor in evaluating a teacher's attitude towards their work. This culture may have encouraged the three teachers to prepare their lessons carefully ahead of time, rather than make teaching decisions at the last moment.

Moreover, due to social and cultural differences, the role of textbooks in Western countries and China is different. Even though textbooks in Western countries like the United States and England are important teaching resources, teachers may choose not to follow them (Li, 2008; Park & Leung, 2006). In Eastern countries, including in mainland China, the textbook is regarded by teachers and students as a "bible" (Park & Leung, 2006, p. 230) containing the minimum and essential knowledge that every student should learn and understand, and every student must master the knowledge to pass relevant examinations (Ma *et al.*, 2002). The cultural importance attached to textbooks might have led the three teachers rely on them rather than developing their own curriculum.

The difference in the role of textbooks also manifests itself in teacher education. For example, as American teachers learn to teach they are expected to develop their own curricula, and enjoy greater freedom to select teaching materials (Paine, 1990; Shimahara & Sakai, 1995). On the contrary, when the Chinese mathematics teachers are trained to plan lessons, they are expected to "digest textbooks" (吃透教材) (Luo & Li, 2002, p. 235) rather than developing their own curriculum. This cultural predisposition might make the three teachers treat textbooks seriously in teaching.

Moreover, Chinese teaching tradition may also influence the three teachers' flexibility in practice. First, one important teaching principle advocated by Confucius is that teachers should teach according to students' characteristics (Sun & Du, 2009), which requires using teaching materials flexibly to meet individual students' needs and facilitate their understanding. In particular, since most of the students in the three teachers' classes were above-average students, more challenging content was needed, and therefore, they needed to make relevant changes to the textbook content. In addition, as introduced in Chapter Three, teachers in mainland China traditionally enjoy a great deal of authority in their teaching. Especially, under the influence of the former Soviet Union, a five step teaching model suggested by Kairov (1951) – "organizing teaching— reviewing prior knowledge — introducing new topics — consolidating new knowledge —assigning homework" – has been widely accepted in mainland China (Shao & Gu, 2006; Shao *et al.*, 2012). This tradition might well influence the three teachers' ways of structuring and organizing their lessons, and restrict the flexibility they enjoy relative to Western teachers.

Teaching with flexibility is also influenced by teacher knowledge. As reported in Chapter Six, all three teachers have well-structured vertical curricular knowledge, rich lateral curricular knowledge, and are aware of their students' backgrounds and cognition development levels. This knowledge base can help them to prepare their lessons, make ongoing adjustments in teaching, use teaching materials, read cues from students and structure lessons flexibly. This further suggests that their knowledge is accessible, as found in previous studies on expert teachers (*e.g.*, Berliner, 2004; Bond *et al.*, 2000).

7.3 Teaching with Balance

Another characteristic identified among the three expert mathematics teachers is their ability to strike a balance between the mastery of knowledge and students' learning experience, exploration, and learning interests. Two common aspects were identified – 1) balanced teaching objectives; and 2) a balance between directive teaching and exploratory teaching.

7.3.1 Balanced teaching objectives

This characteristic was identified from the teaching objectives set by the teachers in their pre-observation interviews and in their lesson plans (Mr. Zhao did not agree to have his lesson plans copied). Generally, teaching objectives set by the three went beyond the mastery of mathematical knowledge and the training of problem-solving skills, and took into consideration students' experiences and the value of exploration. More important, the three teachers stressed developing students' mathematics thinking, abilities, and interests. In general, they would set multidimensional teaching objectives for their teaching. For example, in Mr. Zhao's first pre-observation interview, he described the teaching objectives for this lesson as follows:

> *Let students experience what is inverse proportion function, make them have some experience. The second one is to analyze the definition of inverse proportion function. …, and use its definition to solve some problems.*

Similar objectives were also found in Ms. Qian and Ms. Sun's descriptions and lesson plans. For example, in Ms. Qian's lesson plan for "Similar Polygons", her objectives were:

1) *Experience the formation process of the concept of similar polygons, master the meaning of "similar polygons" and "similar ratio";*
2) *During the exploring process of the essence of similar polygons, further develop students' ability to infer, analogy, reflect, and communicate, improve students' mathematics thinking, experience the function of counterexamples.*

As shown above, the teaching objectives set by the three teachers focused on: 1) knowledge; 2) problem-solving strategies; 3) students' learning experience, participation, and exploration; 4) mathematical thinking; and 5) various abilities. Objectives set by the teachers illustrate that, in their own words, "mathematics teaching could not entirely focus on knowledge, students' experience and abilities should also be one main component of the teaching objectives". Moreover, students' whole-person and continuous development is a main part of their teaching objectives. The objectives they set suggest that the three teachers can strike a balance between students' mastery of knowledge and students' exploration, experience, and mathematical thinking on one hand. On the other hand, these also suggest that the three teachers hold a broad perspective of mathematics teaching.

7.3.2 Balance between directive teaching and exploratory teaching

Another balance achieved by the three teachers was that between “directive teaching” and “exploratory teaching”. The three teachers' teaching approaches in the observed lessons were analyzed and coded using codes described in Chapter Four. Figure 7.5 and Table 7.2 report the general distribution of teaching approaches and the amount of time spent on each. As shown in Figure 7.5, the distribution of approaches and time spent on each vary considerably from lesson to lesson, and from teacher to teacher. This indicates that the three expert mathematics teachers do not employ specific teaching patterns.

The sequence of every lesson among the three teachers is dominated, in terms of time, by three elements – directive teaching, exploratory teaching and practicing – that take up more than 95% of lesson time. The average proportions of time spent on every approach over the lessons within individual teacher are summarized in Table 7.1.

Table 7.1. Average proportions of time spent on different teaching approaches

	Directive	Exploratory	Practice
Mr. Zhao	32.74%	29.27%	33.94%
Ms. Qian	28.63%	31.24%	38.55%
Ms. Sun	40.28%	33.54%	21.28%

As shown in Table 7.1, all three teachers spent a relatively equal amount of time on directive and exploratory teaching. In terms of time, this indicates that the three teachers are able to achieve a balance between the two approaches.

During the three teachers' teaching, they were generally found to control teaching direction and progress well. They could reasonably arrange the sequence of the three major teaching approaches. Usually, and particularly in lessons introducing new topics, they employed directive teaching, exploratory teaching and practicing alternately, with none them lasting for too long. This indicates that, even though they employed exploratory teaching often, the explorations were not very large or open. If students need to explore on their own, the teachers would first make a demonstration and review relevant knowledge. Some

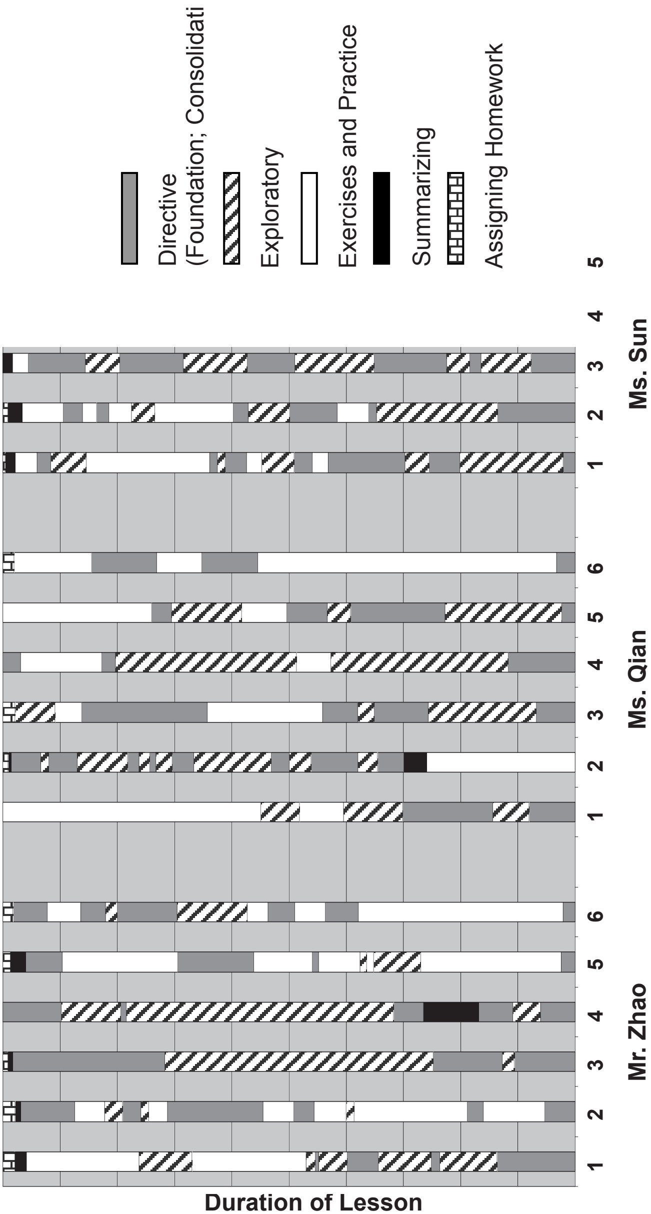

Figure 7. 5. Teaching approach of the three expert mathematics teacher

Table 7.2. Overall percentages of the teachers' teaching apporach in the observation lessons

	Mr. Zhou						Ms. Qian						Ms. Sun				
	Lesson						Lesson						Lesson				
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5
Directive	21	41	49	28	23	33	24	33	44	17	29	24	32	31	52	41	45
Exploratory	35	6	49	62	10	14	23	36	29	63	37	0	35	32	44	29	27
Exercises/ Practice	40	50	0	0	63	51	53	26	25	20	34	74	31	33	2.5	18	23
Summarizing	2	1	1	10	3	0	0	4	0	0	0	0	1.6	3	1.5	11.5	4
Homework	2	2	1	0	1	2	0	1	2	0	0	2	0.4	1	0	0.5	1

explorations were carried out under teacher supervision, such as encouraging students to find alternative ways to solve a problem, prove a theorem, or raise similar examples. For example, at the beginning of Mr. Zhao's first lesson, he guided his students to analyze two situational problems in which the idea of inverse proportion function is embodied. Then, he asked students to develop a similar example. In his view,

> *to do this [constructing situational problems] is to make preparation for the introduction of the definition [of the inverse proportion function]. [Asking students to raise an example] can let them gradually discover the essential properties of these two examples [I presented first]. After I analyzed these two with them, they can attain some [necessary] experience and knowledge and, therefore, they know how to raise examples.*

Mr. Zhao's explanation indicates that, even though he would sometimes encourage students to explore on their own, he never let them do so aimlessly. His explanation further shows that, before introducing a new concept, he would construct relevant situational problems to help students discover essential characteristics for themselves, rather than directly telling them the definition. He called these situational problems "*pudian*", which means scaffolding. This also indicates that Mr. Zhao demonstrated the ability to establish appropriate and dynamic *pudian* to set a proper "potential distance" (Gu *et al.*, 2004, p. 325) between prior knowledge or real life situations and target knowledge in a manner that would encourage students' exploration and facilitate students' understanding as well. Next, he encouraged students to discover further characteristics that were common to the three situational problems, and that acted as foundations for defining the inverse proportion function. However, as students could not discover some critical characteristics, he provided some help. This indicates that, even though his aim was to encourage students to explore mathematical concepts, he never stopped instructing them or guiding their actions. He offered further assistance to those students who could not discover critical characteristics. After presenting the definition, he introduced various algebraic expressions of the inverse proportion function, and compared its definition with that of the linear function. He explained that these parts were critical if students were to understand the definition deeply; however, it was sometimes difficult for students to discover them thoroughly on their own. In his opinion, this knowledge is the foundation for the whole unit, and he therefore made sure that students had a deep and thorough understanding of it. However, it must be pointed out that, even though he guided students to discover some characteristics, define the inverse proportion function, and compare it with the linear function, he never directly told students the relevant information. In other words, his directive teaching approach here had nothing to do with “direct telling”.

In general, to facilitate students’ understanding and to enrich their mathematics experience, the three expert mathematics teachers designed various engaging activities, such as asking them to raise examples, define new concepts, perform hands-on activities, discuss concepts with their peers, explore situational problems, and summarize their discoveries. After the students had explored the issue, the three teachers would offer further comments and corrections through their direction and instruction. They would point out critical and important aspects of new definitions and concepts, make necessary extensions, and differentiate them from relevant prior knowledge. This suggests that the three expert mathematics teachers are well able to control the progress of students’ exploration and provide proper instruction and direction at a right time; they are also capable of ending that exploration at the proper time and starting their own “directive teaching”. The hands-on activities and situational problems would provide students with direct experience of mathematics before learning pure mathematical abstraction and reasoning. This suggests that the three teachers are able to bridge students’ exploration and mathematical reasoning, and move students experience from the concrete to the abstract; that is, experience the process of mathematicization. This further illustrates that they reach a balance between direct and exploratory teaching during their teaching.

7.3.3 Discussion

This section reports a characteristic common to the three expert mathematics teachers’ teaching—teaching with balance. It was found: 1) that when the three expert mathematics teachers set teaching objectives, they balance students’ mastery of knowledge with their experience, exploration, and interests; and 2) that they can balance directive teaching and exploratory teaching. Similar abilities were found among expert mathematics teachers in Western cultures. Expert mathematics teachers in Borko and Livingston's (1989) study, for example, were able to strike "a balance between content-centered and student-centered instruction with what appeared to be minimal use of written plans or textbooks" (p. 481). Similar findings were identified in previous studies in the Chinese context as well. For example, Li, Huang and Yang (2011) found that the five expert secondary school expert mathematics teachers in their study emphasized student-centered instruction to motivate their students.

However, comparatively speaking, the three expert mathematics teachers tended to employ exploratory teaching more often than did non-expert teachers in previous studies on Chinese mathematics teaching. For example, using the same codes employed in this study, Mok and Lopez-Real (2006) found that three very experienced teachers in Shanghai spent around twenty percent of their lesson time on exploratory teaching, compared to the approximately thirty percent spent by teachers in this study.

The characteristics of the three expert mathematics teachers' teaching differ from those descriptions about Chinese mathematics teaching in the literature. Traditional Chinese mathematics teaching has been widely criticized for being heavily dominated by "mastering of knowledge" and "training examination skills", or for being "knowledge-centered" and "teacher-centered", or "learning relied on memorization" (*e.g.*, Li, 2006; Liu & Li, 2010; Zhang, 2006). Practice plays an important role in training students' relevant problem solving skills and preparing students for examinations at different levels. In particular, since the 1960s, one method – "teach only the essential and ensure plenty of practice" (精讲多练) (Zhang *et al.*, 2004, p. 195), also referred to as "concise lecture with extensive practice" (Shao *et al.*, 2012, p. 19) – has dominated Chinese mathematics teaching. Teachers tend to spend less time introducing new topics in order to allow students more time for practice exercises in class. In other words, "it is not necessary for [teachers] to spend much time to make students understand" (Zhang *et al.*, 2004, p. 195) relevant knowledge because it is believed their understanding can be improved through practice (Li., 2006; Shao *et al.*, 2012; Zhang *et al.*, 2004); as such, student exploration and experience has been largely ignored, since mathematics instruction is dominated by lecture and memorization (Liu & Li, 2010; Zhang *et al.*, 2004). Moreover, mathematics teaching in mainland China has long been criticized for offering students limited opportunities to "see the connections between mathematics and people's daily lives, other disciplines and social development" (Liu & Li, 2010, p. 11) .

The three teachers, however, spent quite a lot of time introducing knowledge and employing real-world tasks and situational problems to enrich their students' mathematics experience. The differences found between the three teachers' teaching and that of traditional Chinese mathematics indicate they have the ability to overcome social and cultural constraints. However, this is not to say that their teaching is completely free of social and cultural influence. Under the influence of Confucian culture, some teaching principles have been summarized and have affected Chinese teaching (Su & Du, 2009; Xiao, 2001), including the key principal of teaching students heuristically and gradually, which might make the three teachers encourage students to discover relevant characteristics by themselves, rather than simply and directly telling them relevant conclusions. In addition, the three teachers laid a necessary foundation for students' exploration and made necessary corrections and complements; that is, they taught students gradually.

Chinese mathematics education tradition still influences this characteristic of the three teachers' teaching. For example, the tradition of "two basics" may make the three teachers emphasize students' mathematics knowledge in their teaching objectives, and the emphasis of their instructions and direction. In addition, the tradition of developing students' abstract reasoning ability and rational thinking (Shao *et al.*, 2012; Zhang, 2006; Zhang, 2006; Zheng, 2006) might make them stress students' deeper understanding of concepts and theorems. In particular, under the influence of the former Soviet Union, the opinion of viewing mathematics as an abstract, rigorous subject (as suggested by Aleksandrov *et al.* (1964)) has widely and deeply influenced mathematics education in China for decades (Li *et al.*, 2008; Zhang *et al.*, 2004). Under this view, it has been accepted that there exists some distance between mathematics students' real lives because mathematics is essentially abstract. Students' abstract thinking cannot be fully developed if mathematics teaching only stays at the level of students' direct experience rather than extending to abstraction and reasoning (Zhang, 2006). This tradition might also lead the three expert mathematics teachers to emphasize guiding students to experience abstraction, generalization and making relevant conclusions after students' exploration, rather than letting students conduct open exploratory activities (Lopez-Real *et al.*, 2004) by themselves.

Ideas in the teaching syllabus or curriculum standard might also have influenced the three teachers' decisions. The development of students' abilities, in particular their mathematical thinking and ability to apply mathematics in real life, have been teaching objectives for many years. The mathematics teaching syllabus issued in 1986, for example, clearly states that mathematics teaching in secondary school should develop students': 1) abilities in calculation, logical thinking, and spatial visualization; 2) ability to use mathematics knowledge to analyze and solve problems in practice; and 3) interest in learning mathematics (Research Institution of Curriculum and Textbooks, 2004, p. 526). In the latest curriculum standard, ideas such as student participation and exploration are strongly advocated (MOE, 2001). These ideas might have influenced the three teachers' decisions about their objectives and teaching behavior.

Chinese teaching conditions are another important influence. As described in Chapter Four, there are around sixty students with various mathematics backgrounds in the three teachers' classes, and lesson content has to be conveyed within 40 or 45 minutes. It is therefore very difficult for teachers to let students carry out large exploratory activities on their own. The three teachers explained that, even though they sometimes asked students to discuss or carry out activities in small groups, students with poor backgrounds did not know how to work with others and could lose interest or just passively listen to others. In view of this, the teachers have to guide students through or demonstrate some activities before asking students to start their own exploration. This indicates that they have the expertise to maximize students' engagement in their teaching rather than simply and superficially carrying out ideas advocated in the curriculum reform.

7.4 Teaching with Coherence

A third characteristic common to the three expert mathematics teachers' teaching is their ability to make their instruction coherent. Coherence in this study was analyzed from two aspects: 1) discourse transitions between activities within individual lessons and across lessons; and 2) coherence of mathematics content within individual lessons and across lessons (Fernandez *et al.*, 1992; Stevenson & Stigler, 1992).

7.4.1 Pedagogically coherent lessons

As shown in Figure 7.1, the three expert mathematics teachers designed different activities to achieve specific teaching objectives within individual lessons. However, as Wang and Murphy (2005) pointed out, “sometimes even the well-organized nature of activities does not insure that students will grasp the relationship between activities, especially when the relatedness of activities is not explicit enough for young learners” (p. 100). To minimize students’ difficulties, teachers need to employ appropriate discourse to connect different activities explicitly. In this study, the three teachers all demonstrated the ability to use effective discourse to enhance the coherence of their teaching, and to make their teaching pedagogically coherent, rather than abruptly moving between activities.

For example, after presenting situational problems, exploring concrete examples or common characteristics related to a new topic, or reviewing prior knowledge, the three teachers would make a short summary and highlighted those critical areas related to the lesson topic as a segue into the new topic. During their teaching, the teachers would explore any connections between the new knowledge and earlier activities, sometimes making such announcements as, “now, let us compare the definition of inverse proportion function with the definition of linear function” (from Mr. Zhao), or “let us compare the relationship between angle bisector of a triangle and bisector of an angle” (from Ms. Sun). They would also make statements such as “as shown in these pictures at the beginning of the lesson, ….” (from Ms. Qian), or “do you remember the picture of the cat showed just now” (from Ms. Sun) to connect the new and the prior knowledge with reference to situational problems and examples.

After introducing the new topics, they would highlight important parts or summarize what was newly presented, then say, “OK, now, let us do some exercises”, or “please open your textbooks and turn to page xx; let us complete some exercises now” to move to another stage of their teaching. At the end of the lesson, the three teachers would recap the lesson, using such phrases as “in this lesson, we learned …..” or “we should pay attention to xx” to summarize the main content and highlight important points. In short, within individual lessons, the three teachers were able to link segments systematically as “story telling”. Moreover, reviewing at the beginning and summarizing at the end of the lesson further built connections across lessons. They were thus able to make their teaching pedagogically coherent within lessons and across a series of lessons.

7.4.2 Mathematically coherent lessons

Coherence of content is important to instructional coherence (Wang & Murphy, 2004) and is a very important factor influencing students’ learning as well. The three expert mathematics teachers were able to 1) review prior knowledge comprehensively; 2) differentiate similar topics; and 3) connect activities within a lesson mathematically or thematically.

7.4.2.1 Reviewing prior knowledge comprehensively

Before introducing a new topic, the three teachers would review relevant prior knowledge as comprehensively as possible in order to build a knowledge foundation for students. For example, in Mr. Zhao’s third lesson, he spent around 20% of lesson time reviewing the definition of the inverse proportion function, the relationship between two variables, the representations of function, and the procedures for graphing the linear function to provide students with a foundation from which to explore how to graph an inverse proportion function. As Mr. Zhou pointed out, since the students might have forgotten some of what they learned in Grade 7, the review was both necessary and useful.

Similarly, in Ms. Sun’s second lesson, she spent around 15% of lesson time reviewing what had been learned in the first lesson, knowledge related to triangles learned in primary school, and the properties of parallel lines to provide the proper foundation for the current lesson. She also highlighted that it is possible that students with weak academic background did not, at the time, thoroughly understand some of the knowledge taught them in primary school; it is necessary and useful, therefore, to review that knowledge and ensure the students fully understand it before letting them try to absorb new knowledge.

Generally, according to the teachers’ explanations, reviewing prior knowledge has multiple functions: 1) consolidating students’ understanding of relevant knowledge; 2) providing a knowledge foundation for new topics; and 3) helping students to construct connected knowledge structures. This indicates that the three expert mathematics teachers would make knowledge scaffolds before starting a new topic. Moreover, as shown in Figure 7.2, the amount of time spent on reviewing differed among the three teachers; comparatively speaking, Mr. Zhao and Ms. Sun tended to spend more time in review than did Ms. Qian (average percentage of time spent on reviewing is as followed: Mr. Zhao: 7.81%; Ms. Sun: 9.60%; Ms. Qian: 4.00%). Disregarding the differences in teaching contents, this might be related to differences of their students' academic background. This further suggests that the three teachers have the ability to adjust their teaching strategies to suit their students' academic background.

7.4.2.2 Differentiating similar topics

During their teaching, the three teachers were further found to compare new concepts or theorems with similar prior concepts and theorems. For example, in Mr. Zhao's third lesson, he compared the characteristics of the graph of the inverse proportion function to the graph of the linear proportion, and pointed out the critical differences between the two. In her fifth lesson, after presenting the definition of similar polygons, Ms. Qian guided students to explore the differences and similarities between similar and congruent polygons. She explained that differentiating between the two concepts can help students to understand both more deeply.

According to the three teachers, reviewing prior knowledge and differentiating similar topics helps students understand relevant topics more deeply, assimilate newly-learned knowledge into their prior knowledge, and build a connected knowledge structure. In their teaching, they sometimes explicitly told students how knowledge they will learn in future (including at the senior secondary level) relates to the topic they are learning at the moment. In general, as summarized in Figure 7.6, the three teachers were able to connect newly-presented teaching content to what students had previously learned (including in primary school) and would learn in the future (including in senior secondary school), to help them construct a systematical and connected knowledge structure. This made their teaching content tightly connected.

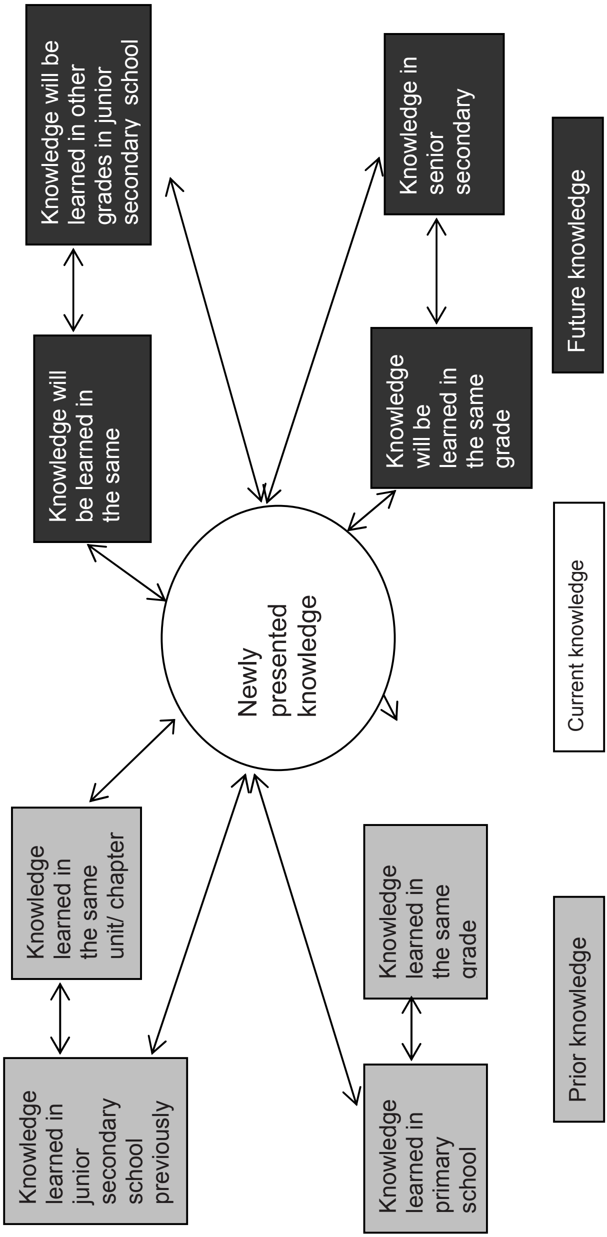

Figure 7. 6. Connected knowledge structure in the three expert mathematics teachers' lessons

7.4.2.3 Connecting activities within a lesson mathematically or thematically

Another characteristic of the three teachers' teaching is that activities within a given lesson were mathematically or thematically connected; in other words, they would not employ activities that were irrelevant to the lesson's topic. For example, with the use of criteria introduced in Chapter Four, every activity in Ms. Sun's second lesson was analyzed; the connections among activities are shown in Figure 7.7. The result suggests that the activities used in this lesson are mathematically and thematically connected. When Ms. Sun reviewed prior knowledge, she heavily stressed the definition and properties of a triangle and the properties of parallel lines, which are directly related to the proof of $\angle A + \angle B + \angle C = 180^0$. Knowing there are other means of proving this theorem, she deliberately chose three methods that were mathematically connected (see Section 6.4.1). Next, she asked her students to practice two exercises related to the theorem: $\angle A + \angle B + \angle C = 180^0$. She then used paper triangles to present a new topic –types of triangles and the properties of a right-angled triangle. The theorem, $\angle A + \angle B + \angle C = 180^0$, acts as a part of knowledge foundation for these two topics. Even though tasks three to seven were related to types of triangles and the properties of right-angled triangle, as shown in Figure 7.7, they were also mathematically related to tasks one and two, because the same theorem, $\angle A + \angle B + \angle C = 180^0$, was needed to solve them. This illustrates her ability to choose or pose mathematically related problems in teaching. At the end of this lesson, she systematically went over newly-learned knowledge with her students and clearly pointed out the connections among them, which made the knowledge relationships among the activities more explicit.

Similar knowledge relationships can also be found in her other lessons, and in Mr. Zhao's and Ms. Qian's lessons as well. Unlike the situation in Ms. Sun's second lesson, which focued on three topics, the three teachers usually focused on one particular mathematical topic and designed activities that were related to that topic mathematically and thematically. In short, during the three teachers' teaching, they reviewed relevant prior knowledge comprehensively, systematically compared newly-learned knowledge with prior relevant knowledge, and previewed what students would likely learn in the future. All of these factors make

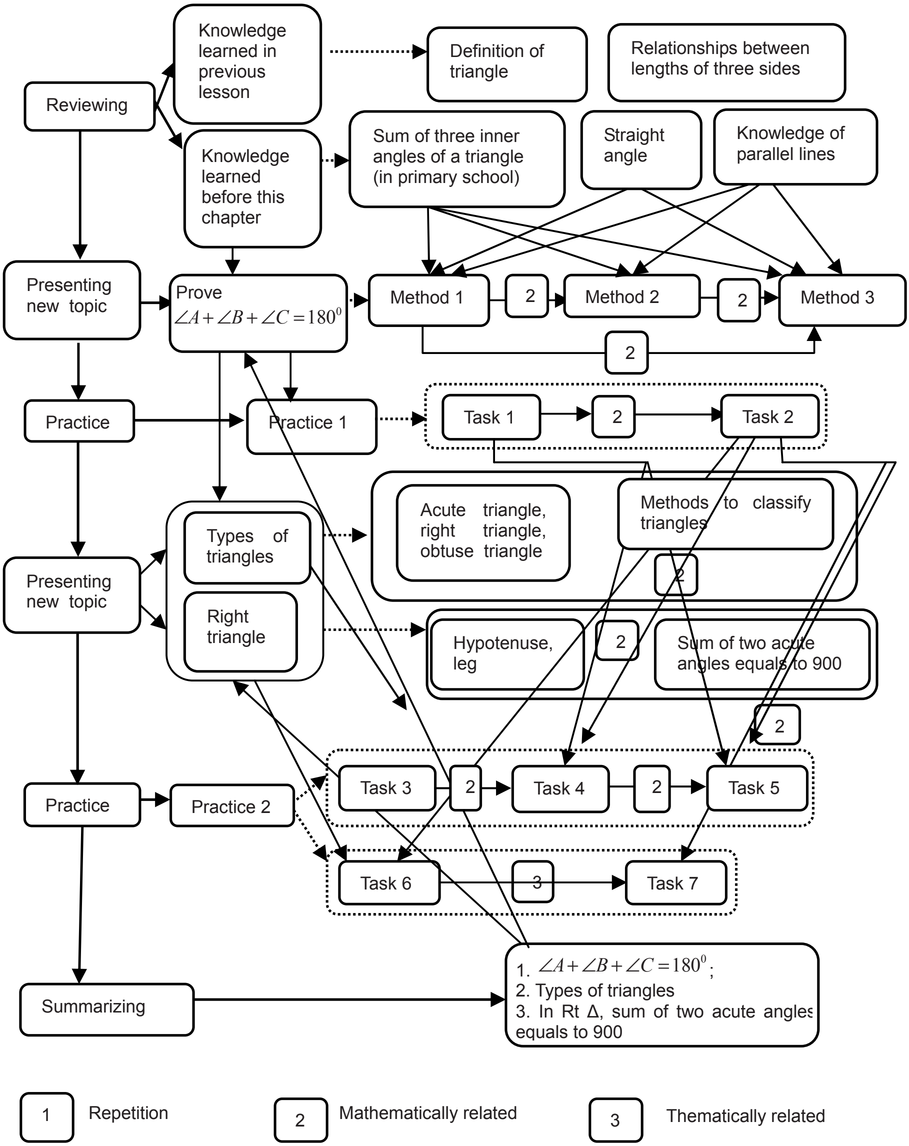

Figure 7. 7. Knowledge relationships among Ms. Sun's second lesson

their teaching appear more mathematically related, and make a series of lessons within a unit mathematically and thematically coherent.

7.4.3 Discussion

This section reported an important characteristic of the three teachers' teaching — teaching with coherence. The three teachers were found to be able to make a lesson or series of lessons pedagogically and mathematically coherent. In their teaching, they effectively employed discourse to move from one activity to another smoothly and reduce unnecessary confusion and difficulties. They systematically reviewed relevant prior knowledge to introduce new topics and compared newly presented knowledge with similar prior knowledge to help students discern critical properties of the new knowledge. Moreover, they designed activities within a given lesson that were mathematically or thematically related.

The ability to connect different topics has also been found in other expert mathematics teachers' teaching. For example, Leinhardt (1986) found that expert mathematics teachers would not include irrelevant information when planning their teaching. In another study conducted by Leinhardt (1989), it was found that expert mathematics teachers saw "lessons as connected and tied together" (p. 64), and could "construct lessons that display a highly efficient internal structure, one that is characterized by fluid movement from one type of activity to another, by minimal student confusion during instruction" (p. 73). This indicates that their teaching was pedagogically coherent. Similarly, in Livingston and Borko's study (1990), expert mathematics teachers were also found to make "explicit the relationships across problems, thus connecting the activity to the broader scheme" (pp. 383-384). Consistently, Even *et al.* (1993) found that the expert mathematics teacher in their study could connect new and previous lessons by employing contexts with which students were familiar to introduce new topics.

The three teachers' ability to make their teaching mathematically and thematically coherent may be influenced by their knowledge structure. As reported in Chapter Six, all three have a connected mathematics knowledge structure and vertical curriculum knowledge that could help them refer to other topics freely while they plan and implement their lessons. This further suggests that their knowledge is accessible, as found in other studies on expert teachers (Berliner, 2001, 2004).

Traditional Chinese educational principles might have also influenced their teaching behavior and decisions. For example, Confucius once said that "a man who can gain new insights through re-studying what has been learned may serve as a teacher" (温故而知新，可以为师矣) (*Analects*, translated by Lao, 1992, p.41). As reported above, all three teachers systematically reviewed relevant prior knowledge in order to help students discern essential points that could act as a foundation for the introduction of new topics. This, to a certain degree, also consolidated students' understanding of prior knowledge and newly-learned topics.

Chinese mathematics education tradition might have also influenced the three teachers' teaching, such as its emphasis on: 1) reviewing prior knowledge to make a knowledge foundation for new knowledge; 2) differentiating similar topics; and 3) making the teaching process logical and coherent (Shao *et al.*, 2012; Tu & Song, 2006; Xu *et al.*, 2009). The three teachers all stressed building knowledge foundations for students through the review of prior knowledge, comparing similar topics to help students discern critical properties of new knowledge, consolidating new and prior knowledge, deepening students' understanding and, more important, helping students assimilate new knowledge into their existing knowledge structure.

Teaching conditions are another influence. As discussed in Chapter Four, there were around 60 students of varying academic backgrounds in the observed classes. In Ms. Sun's and Mr. Zhao's classes in particular, around half of the students were comparatively poor in mathematics. To involve most, if not all students, in the teaching process, the teachers needed to ensure that their students had the necessary fundamental knowledge and skills. As reported above, the three teachers acknowledged that their reasons for reviewing prior knowledge included helping those students who had not understood the relevant knowledge and, therefore, allowing those students who did not understand to explore on their own. Moreover, this also suggests that the three teachers demonstrated the ability to reduce the cognitive pressure on students with relatively poor mathematics abilities.

7.5 Promoting Students' High-order Thinking Skills

A general impression of the three expert mathematics teachers' teaching is that they emphasized the promotion of students' high-order thinking skills. They were found to: 1) promote students' mathematical communication; and 2) employ an open-ended approach in working on problems.

7.5.1 Promoting students' mathematical communication

The transcripts of classroom teaching discourse show that the three expert mathematics teachers could encourage students to communicate mathematically. After students correctly answered a question, the teachers would further encourage them to explain how they came to their result. For example, in Ms. Qian's second lesson (introducing the properties of ratios), she asked students to think about how to prove: if $\frac{a}{b}=\frac{c}{d}$, then ad=bc. A part of interaction process is as follows:

Ms. Qian: Anyone can prove it?…, anyone can explain why ad=bc?..., (some students raised their hands) some students have found a way. If you found a way, please keep looking for a second way.(after a while) …, Peter, please tell us why ad=bc?
Peter: I think we can multiply bd at the both sides of the given equality $\frac{a}{b}=\frac{c}{d}$.
Ms. Qian: His method is to multiply what at the both sides of the given equality.
Students: bd
Ms. Qian: Multiply bd at the both sides, then, ***why ad equals to bc****?*
Peter: According to the basic property of equality?
Ms. Qian: ***Please specify which basic property****.*
Peter: If we multiply a non-zero number or expression at the both sides of an equality at the same time, the right side still equals to the left side.
Ms. Qian: very good, ***is bd non-zero****?*
Peter: yes, bd is non-zero.
Ms. Qian: ***Why****?*
Peter: Because according to the given condition, $\frac{a}{b}=\frac{c}{d}$, *if b is zero or d is zero, the fractions are meaningless.*
Ms. Qian: Yes, because b and d are non-zero, therefore bd is non-zero.

OK, if we multiply bd at the both sides of the equation ($\frac{a}{b}=\frac{c}{d}$), the left side equals to?

Students: ad

Ms. Qian: the right side equals to?

Students: bc

Ms. Qian: en, therefore, we get ad=bc, we can prove it successfully with the use of basic property of equality. ***Are there any other methods?***

In the above dialogue, particularly those portions shown in bold, Ms. Qian did not end the communication process once Peter gave her the correct answer; instead, she encouraged him to analyze his answers further or to explain relevant underlying principles. During the interaction process, Ms. Qian did not control Peter's thinking process, but encouraged him to explain, justify, and clarify his thoughts. As shown at the end of this quotation, she then encouraged students to find other alternative methods in an effort to broaden and deepen their thinking.

When they worked on problems, the three teachers would also encourage students to articulate their solutions. For example, in Mr. Zhao's first lesson, in which he presented problem 1.8 (see Section 6.3.2.6), after letting students think for around 75 seconds, he chose to fill this table first. A part of the teaching interaction is as follows:

Mr. Zhao: Do you finish it?

Students: Yes.

Mr. Zhao: OK! Peter, tell us how you filled this table.

Peter: The first blank should be -3.

Mr. Zhao: The first one is -3, his answer is -3, right?

Students: Yes.

Mr. Zhao: ***How did you get -3?***

John: Because $y=\frac{k}{x}$, and k=-2, therefore the first x is (-3).

Mr. Zhao: ***How did you get k= -2?***

John: Because there is a given x=-1 and y=2, therefore, k= (-1) × 2=-2

Even though Peter gave Mr. Zhao the right answers, Mr. Zhao asked other students to assess John's answer and John to explain the process by which he came to his answer. According to Mr. Zhao's comments in the post-observation interview, he wanted to ensure that this student really understood the definition of the inverse proportion function and knew how to use it to solve relevant problems. Similarly, Ms. Sun said that some students guessed the right answers without really understanding the relevant knowledge or methods, and did not know how to use the knowledge in practice. Ms. Sun further said that if students really knew how to solve a problem, asking them to articulate their process could enhance their understanding and make their thinking more organized and clear. This indicates her awareness that asking students to articulate what they know would promote students' reflection and develop understanding (Carpenter & Lehrer, 1999).

The interaction shown above indicates that, despite the constraint of large class size, the teaching process remains under the teachers' control. This is not to say that the interaction process is as simple as traditional "initiation-response-evaluation" style; on the contrary, the three teachers asked relatively open-ended questions that could not readily be answered "yes" or "no". During their interactions, they encouraged students to reflect on, and provide rational analysis for their answers; in short, they stressed "reflective communication" (Brendefur & Frykholm, 2000, p. 127). Thus, students and teachers used mathematics as a communication medium. The interaction process not only provided the three teachers a chance to test whether students really understand relevant knowledge, but also helped students to connect new knowledge to what they had previously assimilated and to construct their own understanding. More important, this deepened students' understanding and provided opportunities for mathematically reasoning.

7.5.2 Open-ended approaches in working on problems

As reported in Chapter Six, the three teachers employed many routine problems in their teaching practice. However, they encouraged students to look for alternative solutions to these problems rather than simply asking them to find correct answers. According to Shimada (1998), "students are, in a sense, facing and dealing with an open-ended problem, since what is asked for is not the answer to the problem but rather the methods for arriving at an answer" (p. 1). The "open-ended approach" proposed by Shimada (1998, p. 1) is borrowed here to capture the characteristics identified in the three teachers' ways of working with problems since some characteristics meet the above description.

7.5.2.1 Providing students time to look for solutions

The three teachers gave students time to look for possible solutions. Especially, for those difficult problems, they would give students relatively more time to think about how to solve them before beginning a public discussion so that students of average mathematics ability had sufficient time to think about how to solve a problem. Therefore, students would not only passively accept the methods provided by the students who could solve them quickly or simply copy the solutions demonstrated by the teacher. They said that if students solve a problem simply by copying the way demonstrated by other students or the teacher, they will very soon forget it. Moreover, those students do not really involve themselves in the process of looking for solutions and do not fully and deeply understand the relevant methods. This indicates that the three teachers value students' own efforts and the problem solving process above directly telling students how to solve it.

7.5.2.2 Stressing the process of analyzing/approaching problems

The three teachers emphasized the process of guiding students to approach problems, especially the complex and difficult ones. That is, instead of directly giving students answers or solutions, they emphasized the process of looking for answers and solutions. They guided students to analyze and made extensions to the given conditions to make them closer to possible solutions. During the process, they also helped students to discover critical steps or points in solving the problem, and encouraged students to link the given conditions with newly learned or prior knowledge. For example, in Ms. Sun's fourth lesson, when she presented the second problem (see Section 6.3.2.5, Figure 6.4), a part of interaction process is as followed:

Ms. Sun: What are the given conditions of this problem?
Students: CE and BD are bisectors,

Ms. Sun: OK, then, we need to find out the relationship between $\angle BIC$ and $\angle A$, Right?
Students: Yes.
Ms. Sun: Now, $\angle A$ could be represented as?
Students : α .
*Ms. Sun: OK, now, **how can we connect $\angle BIC$ with $\angle A$,** which knowledge can help us? If we need to prove the quantitative relationship between angles, we should use which knowledge point?*
Students: Vertical angle
Ms. Sun: Yes, right, they are vertical angles, can we use them?
Students (after a while, some): Not necessary.
*Ms. Sun: Then, **how can we connect $\angle BIC$ with $\angle A$?***
Students (after a while, some): Sum of three angles of a triangle.

The above dialogue shows how Ms. Sun encouraged her students to link given conditions with the problem that needed to be solved. However, she did not directly provide any new information until students mentioned some. In the rest of the lesson, she also guided students to justify why the knowledge point, “vertical angles”, is not workable in this problem.

The process of guiding students to approach a problem demonstrates the teachers' own thinking processes. This further indicates that they not only focus on correct answers, but also highly value the process finding those answers. In other words, they emphasized teaching students how to think, rather than giving them tips on looking for right answers.

7.5.2.3 Stressing alternative methods

Another characteristic common to the three teachers is that they encouraged students to look for various methods of solving a given problem. For example, in Ms. Qian's first lesson, there was a problem related to how to use scale (map) to discover the actual distance between two cities. One girl successfully solved it using a method she had learned in primary school. Ms. Qian first gave her some positive feedback; then, she asked the students to look for other solutions. After a while, a boy suggested the use of equation. Ms. Qian explained that her approach *can help them develop the idea of looking for different methods to solve the same problem. This can make them experience divergent thinking. Under this kind of situation, students will try to look for methods from different directions, therefore, those who are good at mathematics can find a new way.*

Her explanation suggests that she not only stressed encouraging looking for alternative problem solving methods, she also understood the relevant educational psychology theory (*e.g.*, divergent thinking) behind her behavior.

In Mr. Zhao's teaching, due to the characteristics of knowledge of function (it has multiple presentations, including analytical expressions, graphs and tables), he would encourage students to approach a certain problem from different perspectives in order to find alternative solutions. For example, he presented the following problem in his sixth lesson:

1) *Let $y=\frac{2}{x}$ and y=x-1, what is the value of the coordinator of their intersections?*
2) *Let $y_1=\frac{2}{x}$ and y_2=x-1, Under what situation $y_1 = y_2$? Under what situation $y_1 > y_2$? And under what situation $y_1 < y_2$?*

He spent considerable time guiding students to explore the possible locations for the points' intersections on their graphs. Then, he guided his students to compare the values of y_1 and y_2 with the help of their graphs. He believed this would make the problem more understandable and help students to figure out under what conditions $y_1 = y_2$, $y_1 > y_2$, and $y_1 < y_2$. After this, he encouraged his students to consider the problem from an algebraic perspective through the use of inequation. This indicates that Mr. Zhao was able to develop students' mathematics ability and extend their thinking by integrating different presentations of function in his teaching. The integration of multiple representations further promotes, broadens, and deepens students' conceptual understanding, and strengthens their ability to solve problems flexibly (Even, 1998). Even though inequation with two degrees is not necessary for junior secondary school students, Mr. Zhao explained that the graph of function and the ability to use graphs to solve or analyze problems are important for learning functions at the secondary school level; as such, he felt he should develop students ability to approach this kind of function problems with the use of graph and analytical expression.

7.5.2.4 Making variations

All three teachers made variations to some problems after students had successfully solved them. For example, in Ms. Sun' first lesson, after she finished the second sub-problem of problem two, she asked students to work on problem three.

> *2. In triangle ABC as shown in Figure (1):*
>
> *1) Find the corresponding side of triangle B and its adjacent side.*
>
> *2) How many triangles are there in Figure (1)? Can you list them out?*
>
> *3. How many triangles are there in Figure (2).*

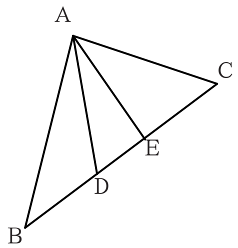

Fig. 1

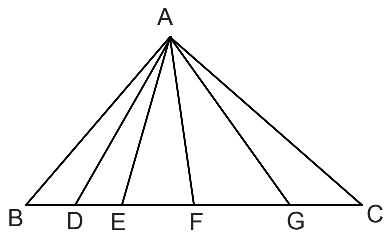

Fig. 2

In these two problems, Ms. Sun encouraged students to count triangles using two different methods with a main aim to ensure they understood the elements of a triangle. Moreover, she pointed out that problems like this will be important in future examinations and that she should make sure students understand how to solve this kind of problem.

In Mr. Zhao's first lesson, after he presented problem 1.8 (see Section 6.3.2.6) as described before, he made the following variation:

Now, if I say that y is function of x and the table is given like this:

x		*-2*	*-1*	$-\frac{1}{2}$	$\frac{1}{2}$	*1*		*3*
y	$\frac{2}{3}$	*1*	*2*				*-1*	

1) *find the analytical expression of this function;*
2) *according to the analytical expression, finish the table above;*

To successfully solve this problem, students needed to consider at least two different situations: 1) y is inverse proportion function of x; and 2) y is linear function of x. This variation, in Mr. Zhao's opinion, would help students differentiate between some critical properties of the two kinds of functions and understand more deeply the critical properties of inverse proportion function and linear function both.

Obviously, making variations to a problem increases its complexity and difficulty. This illustrates that although sometimes the three teachers chose easy tasks to start students' practice, they could still challenge students by making further variations to the problems. For those students who were not good at mathematics, such problems could further provide them with the necessary "scaffolding" to move from easy to increasingly difficult situations. Even though some students might not successfully be able to solve the most difficult problem, they at least were given some idea about how to approach this kind of problem in future. In the three teachers' opinions, this could help every student attain relevant development in their teaching.

7.5.2.5 Making further extensions

The final characteristic shared among the three teachers is that, after successfully solving a problem, they would quickly rehearse the entire process with their students, extend the methods to similar situations, and then draw conclusions before starting a new topic. During the process, they pointed out those parts to which students should pay special attention for their future learning, told students from which direction they might start to approach similar problems, and on what parts they might easily make mistakes. For some very important parts or methods, they asked students to take notes. A main reason shared by the three teachers is that, if they did not point out essential and critical parts in their teaching, some students might not be able to discover them, and would therefore continue to make similar or even identical mistakes in the future. Moreover, going over the problem-solving process, in the teachers' opinions, not only made sure students understood relevant methods. More important, it made sure students understood why the problem should be approached and solved in a certain way.

In addition, they differentiated between the methods used to solve the current problem and similar prior methods, if any. They also extended the knowledge scope of a particular problem, particularly to foreshadow future learning. For example, after Mr. Zhao guided students to finish the table in problem 1.8 (see Section 6.3.2.6), he guided them to explore the location of some points in different quadrants, the relationships among them and to guess the possible quadrants in which its graph will locate. This extended the knowledge scope to the graph and property of the inverse proportion function.

Similarly, in Ms. Sun's fifth lesson, she used the following problem to enhance students' understanding of right triangle:

If triangle ABC satisfies _________, then it is a right-angled triangle.
$\angle A + \angle B + \angle C = 180^0$

After asking a student to present his approach, she made some extensions to the second condition and third condition with the exploration of how to judge a triangle is right angled. For the third condition, she changed it to $\angle A:\angle B:\angle C = 1:4:5$ and $\angle A:\angle B:\angle C = 1:2:3$ to guide students to discover the rules behind them. Some students successfully discovered that if $\angle C = \angle A + \angle B$, then the triangle was right angled. She extended the second condition to $\angle A = \angle B - \angle C$ to let students judge whether triangle ABC was a right triangle. Basing on this, she guided students to reach the conclusion that if triangle ABC satisfies this condition ($\angle A = \angle B - \angle C$), it is also a right triangle. She explained that making relevant extensions to some conditions or problems can deepen and expand students' understanding. Moreover, it can help students construct a connected knowledge structure and understand mathematics more deeply and fully.

7.5.3 Discussion

This section reported another characteristic common to the three expert mathematics teachers' teaching — promoting students' high-order thinking skills. In their teaching, the three teachers encouraged students to communicate relevant ideas mathematically and employed an open-ended approach when working on problems. The strategy illustrates that they valued students' thinking and the process of getting relevant solutions more than correct answers. The ways in which the three teachers communicated with students and worked on problems illustrate that, for the three teachers, knowing the rules or steps by which one can solve a problem is not enough; students should also be able to discover the deeper logical and mathematical principles underlying the method. What they sought was not superficial understanding or a fortuitous right answer, but a deeper understanding of the process of finding the correct answer and its rational. In other words, what they most valued was the development of their students' thinking.

Similar findings were also found among other expert mathematics teachers. For example, in Guo and Song's (2008) study, compared to novice and proficient teachers, expert mathematics teachers were better at using "problems" to carry on their teaching, and were more able to choose appropriate problems to promote students' mathematics thinking. Expert mathematics teachers in Livingston and Borko's (1990) study were also able to connect problems together and highlight problem characteristics. However, in their study, no detailed information was given on how expert mathematics teachers worked on problems. Li and Ni (2007) found that expert teachers required students to explain the principles underlying relevant questions and analyze the relationships and differences among various solutions. More recently, Li, Huang, and Yang (2011) identified a central tendency among five expert mathematics teachers in their study —emphasizing the development of students' mathematical thinking and ability.

This characteristic differs from other descriptions of Chinese teaching. The stereotype of Chinese classroom has long been described as "cramming the duck", or teacher-talk throughout the lesson (Li, 2006; Mok *et al.*, 2001); traditional Chinese teaching is often seen as "an act of transmission, its movement unilateral", in which "the teacher plays the leading role", and "students are expected to receive the teacher's knowledge as it is presented" (Paine, 1990, p. 50). The three teachers encouraged students to communicate mathematically, look for solutions and articulate relevant answers and solutions. This indicates that the three teachers neither directly provided solutions to their students nor taught mathematics superficially.

It has been said that "mathematics educators in China have always attached great importance to the practice of solving problems for the learning of basic mathematical knowledge and skills" (Zheng, 2006, p. 383). In particular, teachers tend to employ many exercises before students really understand relevant concepts, in the belief that practice can facilitate the formation of mathematics concept and eventually enhance students' conceptual understanding (Li, 2006; Shao *et al.*, 2012). However, the actual situation is that teachers insist "more strongly on 'over-loaded exercise', instead of encouraging students to use their own ways to solve problems" (Zheng, 2006, p. 383). This makes practice in mathematics teaching in mainland China a form of "drill practice" or "repetitive practice" (Li, 2006; Zhang *et al.*, 2004; Zheng, 2006).

However, in the three expert teachers' teaching, similar to findings in other studies on expert mathematics teachers in mainland China (*e.g.*, Li, Huang, & Yang, 2011; Zhang, 2000; Zhu *et al.*, 2007), they tended to employ fewer problems and made fuller use of them through making further variations, encouraging students to look for alternative methods, and making extensions. This illustrates that, as discussed above, they highly valued students' deep understanding and mathematics thinking. The number of problems the three teachers employed in teaching and the time they spent on practice differs from those reported in other studies; a recent prize-winning lesson, for example, which was praised as exemplary and was delivered by a beginning teacher, featured 25 problems (Li & Li, 2009). Relatively speaking, the three expert mathematics teachers tended to employ a small number of problems in their teaching. Compared with the findings in Mok and Lopez-Real's (2006), the three teachers spent less time on practice. One of the three teachers in Mok and Lopez-Real's study from Shanghai spent around 70% time on exercises and practice.

However, their teaching is also influenced by social and cultural factors. According to Confucius, students' thinking and reflection should be highly stressed in teaching. He stated that , that "mere reading without thinking causes credulity; mere thinking without reading results in perplexities”（学而不思则罔，思而不学则殆）(*Analects*, translated by Lao, 1992, p. 43) and that “not until he is eager to know but feels difficulty do I instruct; not until he wants to speak out but fails to express himself do I enlighten. If I present him one corner and he cannot from it infer the other three, I do not continue the lesson.”（不愤不启，不排不发，举一隅，不以三隅反，则不复也）(p. 142). These principles suggest students' exploration and thinking should be valued, and their deep intellectual involvement should be especially stressed. These principles might move the three teachers to provide students with time to think about how to solve a problem rather than directly telling them the relevant solutions, to encourage students to look for alternative methods and approach problems from multiple perspectives, and to make variations to a problem if possible. At the end of every problem, they would go over the problem solving process with students and, in particular, make extensions to the problems.

The Chinese mathematics teaching tradition also influences the three teachers' teaching, including its pedagogical belief in seeking deeper mathematical understanding (Shao *et al.*, 2012; Zhang *et al.*, 2004; Zheng, 2006), the principle that “teachers should help students not only to know what and how, but also why” (Zheng, 2006, p. 387) and its focus on mathematics thinking and rational thinking – all of which have been traditionally emphasized in mainland China (Shao *et al.*, 2012; Zhang, 2006; Zhang *et al.*, 2004; Zheng, 2006). For example, the mathematics teaching syllabus for junior secondary schools issued in 1992 clearly stated that a teacher should guide students to make necessary generalizations in problem-solving thinking and methods (Research Institution of Curriculum and Textbooks, 2004). This tradition might make the three teachers encourage students to reflect on their solutions and make extensions to relevant conditions to ensure students clearly understand the underlying rationale.

Moreover, an emphasis on problem solving, teaching with variation (*bianshi* mathematics teaching), the sophistication of problem solving methods and skills in teaching is a tradition of Chinese mathematics teaching (Cai & Nie, 2007; Gu *et al.*, 2004; Tu & Song, 2006; Wong *et al.*, 2012; Xi, 2008; Zhang, 2008). Problem solving in China is viewed as both an instructional goal and an instructional approach (Cai & Nie, 2007). There are several popular types of problem-solving activities in Chinese mathematics classroom, such as “‘one problem, multiple solutions’, 'multiple problems, one solution', and 'one problem, multiple changes' " (Cai & Nie, 2007, p. 459). In particular, solving a problem in multiple ways is a typical teaching method for Chinese mathematics teachers (Cai & Nie, 2007; Li, 2006; Zhang *et al.*, 2004). This tradition might make the three expert mathematics teachers encourage students to look for alternative methods, make variations to problems, and make extensions to relevant conclusions during their teaching.

Encouraging students to communicate their ideas is an important idea in the latest mathematics curriculum standard (MOE, 2001). This might also make the three teachers encourage students to mathematically communicate during teaching. Moreover, teaching evaluation systems in the three schools might also influence the three teachers' teaching behavior to a certain degree. For example, a document on how to evaluate lessons in Ms. Sun' school requires teachers to encourage students to reflect and inquire, rather than passively accept what teachers say.

7.6 Consistency between Beliefs and Practice

According to the characteristics reported in the sections above, the three expert teachers generally used a process-oriented style of teaching (Cooney, 2001) and employed various activities to engage students in experiencing and understanding the process of developing definitions and theorems. For example, Mr. Zhao constructed some situational problems to let students extract the definition of the inverse proportion function, while Ms. Qian and Ms. Sun employed pictures or hands-on activities to let students experience the application of relevant concepts. They also asked students to raise their own examples, discuss in groups, publicly present and articulate solutions.

Generally speaking, students were given the chance to experience discovery, conjecture, reasoning and communication in the three teachers' teaching. In other words, they had the opportunity to construct their own knowledge, sometimes under the guidance of the three teachers, rather than directly receiving rules, formulas or problem-solving methods, or mechanically memorizing and repeatedly practicing a problem-solving strategy. This means that, even though the teachers stressed the importance of mastering knowledge and skills students can employ in future examinations, they were never satisfied; rather, they stressed students' experience and exploration, fostered a deeper understanding of mathematics and promoted mathematical thinking and mathematical reasoning ability.

These teaching characteristics are consistent with the teachers' beliefs. As reported in Chapter Six, the three teachers tended to emphasize that mathematics is rooted in real life and is a vehicle for developing students' mathematical thinking and abilities. In addition, they believed that, to learn mathematics well, students need to be intellectually engaged in the learning process. As regards mathematics teaching, they felt that students should have opportunities to experience the knowledge development process and to become intellectually involved in mathematics teaching. These kinds of beliefs might lead them to stress students' experience, deep understanding, and the development of mathematical thinking and reasoning.

In addition, they all mentioned that mathematics is an examination subject for students, and that, as such, basic knowledge and problems solving skills are a part of the goals of mathematics learning and teaching. This kind of belief might lead them to construct solid knowledge foundations for students, integrate *Zhongkao* information into regular teaching, and add extra content that will be important for future examinations in their teaching. Figure 7.8 summarizes the consistent relationship between their beliefs and practice.

Based on their comments in some interviews, it can further be said that the three expert mathematics teachers' teaching was influenced, to a certain degree, by their beliefs. For example, when Mr. Zhao explained why he constructed situational problems to encourage students' participation, he said:

> *Actually, from the very beginning [of my teaching career], I have held the view that students should experience the process of knowledge development. At the very beginning of my teaching career, I noticed that many experienced teachers would directly tell students some conclusions. I think that I should let her/him [student] understand the development process of knowledge. So, I was very proud of myself when I learned that this idea was advocated in the new curriculum standard in 2001. I think that I already have this opinion for many years.*

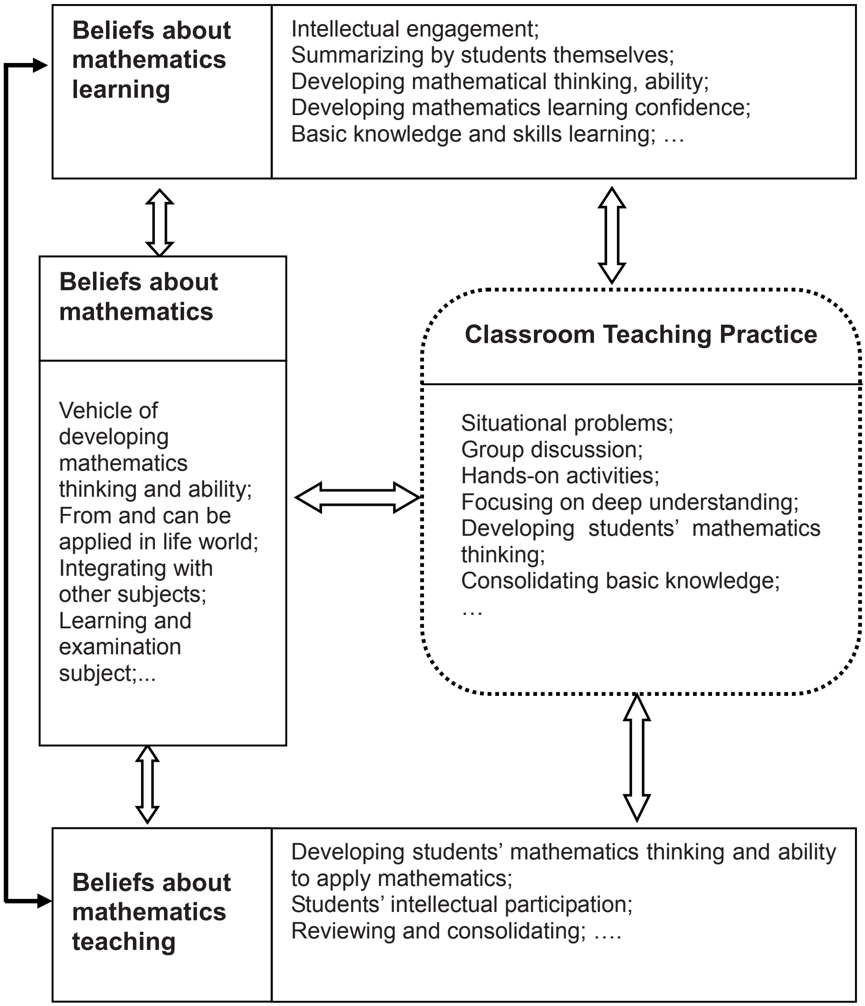

Figure 7. 8. Consistent relationship between the three expert mathematics teachers' beliefs and practice

Similarly, Ms. Qian also said that, during her teaching history, her beliefs about mathematics teaching had gradually changed and that her teaching was guided by this belief. She said:

> *many years ago, mathematics teaching in mainland China has been very traditional. Even now, some teachers still have these traditional beliefs, I can say. …, As a novice teacher, I also thought that students could attain more from my talking than their own exploration. Therefore, in the first years, I talked a lot during my teaching. …, after two or three years of teaching, after I observed some other teachers' teaching in Chongqing or in other cities in mainland China, I gradually realized that I should let students participate in some activities. I think it will be more effective.*

This section reported the consistent relationship between the three expert mathematics teachers' teaching practice and their beliefs. Generally, their beliefs were found to be consistent with their teaching practice, which is not the situation for some beginning mathematics teachers (Raymond,1997). Therefore, to a certain degree, the findings in this study support the statement that teachers' beliefs are a major factor influencing their teaching practice (Calderhead, 1996; Ernest, 1989; Kagan, 1992; Thompson, 1984, 1992). In the meantime, this further indicates that the differences between expert and novice or non-expert mathematics teachers' teaching (as reported in previous studies) might be, in part, caused by differences in their beliefs rather than being solely the result of differences in qualification and abilities. Some pre-service teachers in mainland China tend to hold traditional views of mathematics, mathematics learning, and mathematics teaching (Yang & Li, 2009a, 2009b). Novice teachers in the same two studies were found to believe that memorization and practice are effective ways to learn mathematics, and that the aim of mathematics teaching is to make students skillful in problem solving and attain excellent achievement in mathematics examinations. However, the three expert mathematics teachers in this study tended to believe that, to teach mathematics effectively, a teacher should involve students intellectually and give them the opportunity to experience the knowledge development process directly.

7.7 Reflection on Teaching

Reflection is seen as central to the improvement of teaching ability and teachers' professional development (Artzt & Armour-Thomas, 1998, 1999; Schön, 1987). In this study, the three expert mathematics teachers were found to be able to systematically reflect on their teaching. For example, as described above, they could flexibly adjust their teaching plans once they encountered unexpected student reactions. This illustrates that they could reflect on their teaching in the midst of teaching. Moreover, they demonstrated the ability to reframe their teaching plans to reduce students' difficulty; that is, practiced reflection-in-action (Schön, 1983). In addition, Schön also proposes reflection-on-action; that is, reflection before or after a given situation. This section reports some characteristics of "refection-on-action" (Schön, 1983, 1987) revealed in the post-observation interviews. When the three teachers reflected on their lessons, they mainly focused on the following two aspects: 1) students' understanding; and 2) ways to deal with teaching content.

7.7.1 Reflection on students' understanding

The first aspect the three expert mathematics teachers frequently mentioned was students' performance and understanding. They all stressed whether and to what degree their students truly understand their teaching content. For example, in Ms. Qian's second post-observation interview, she mentioned:

> *As to the concept in this lesson, golden section, I think that students understand it well. They can combine the ratio of line segments they learned in previous lessons with this concept. They know how to use ratio of line segments to define the concept of "golden section". I thought that they well understand this concept [golden section].*

Her statement indicates that she can judge students' understanding of a concept based on their performance in the lesson. Similarly, in her third post-observation interview, Ms. Sun mentioned that what she really satisfied was:

> *as to the important parts of these two lessons (double lessons), students really understand them [teaching content]. They really realized that three angle bisectors, median lines, and heights of a triangle join at a same point through folding paper triangles and their own drawing. They realized this.*

In addition to stressing students' understanding, they also reflected on students' performance, such as how deeply they were involved in mathematics thinking and how actively they participated in discussions. For example, in Mr. Zhao's first post-observation interview, when asked to evaluate his teaching, he said:

> *Not so good. There exist two problems. The first one is that students did not show their analytical ability. They were a little bit nervous. Normally, they are very active, if I ask them to discuss, they will discuss very actively. Today, they did not. Sometimes, they talked about something else.*

As indicated in the above statement, Mr. Zhao not only noticed students' performance, he could also identify some factors that had influenced their performance.

7.7.2 Reflection on teaching methods

The ways in which the three expert mathematics teachers dealt with teaching content and relevant teaching methods were another commonality.

7.7.2.1 Ways to deal with teaching content

Ways of dealing with teaching content were the first facet mentioned by the three teachers in almost every post-observation interview. In Mr. Zhao's first post-observation interview, he mentioned:

> *as to the introduction part, it is not so natural. I presented too much new information here. This introduction is not so effective. I would delete one or two situational problems if I teach this topic again. Just use one example, like the one of distance, speed, and time, is enough.*

Ms. Sun also expressed a similar opinion in her second post-observation interview. She mentioned:

> *next time, if I teach this topic again, I will not use three methods to prove this theorem ($\angle A+\angle B+\angle C=180^0$). Because according to textbook, it does not require us to prove it from different perspectives, it does not require us to teach its proof at all. I think you (the researcher) also read the textbook. In the textbook, it uses a hands-on activity as the way used in primary school textbook [to introduce this theorem]. If I teach this topic again, I will decide whether to add its proof or not according to students' situation.*

In practice, they demonstrated the ability to modify teaching plans on reflection. For example, in Ms. Sun second lesson, she did not, as suggested in textbook and teaching reference material, fold a paper triangle to guide students to discover that, in a triangle, $\angle A+\angle B+\angle C=180^0$; she explained:

> *I taught this topic in another class yesterday, …, when I taught this topic yesterday in that class, they all knew that, so, I think this process is unnecessary. Every student knows this process. It is not necessary for me to let them fold [a triangle] again. Therefore, in this class, I did not ask them (students in the observed class) to do so.*

7.7.2.2 Ways to organize the lesson

In addition to reflecting on ways to deal with the textbook, Mr. Zhao and Ms. Qian also reflected on whether the ways in which they organized their lessons were reasonable. Mr. Zhao mentioned this several times. In his fifth post-observation interview, he commented:

> *I am very satisfied about the way I arrange the lesson structure, the teaching sequence. It is reasonable, I think. In this lesson, I also made some pudian (scaffolding) for my future teaching. …*

Similarly, in Ms. Qian's second post-observation interview, she mentioned that the three activities designed by her for that lesson – looking at pictures, introducing the concept of the golden section, and hands-on activities to explore the golden section ratio – were both reasonable and effective, and that she would not change the overall lesson structure if she were to teach this topic again to a similar class.

7.7.2.3 Choices of exercises and the ways of working on them

Another important factor mentioned by the three teachers was the choice of exercises and ways of working on them. All three expert mathematics teachers mentioned several times that they did not use all the exercises they had planned to use in a given lesson because they had to adjust their teaching plan due to student difficulties. They mentioned that they would delete some exercises next time they taught the lesson. In addition, the difficulty of exercise was another significant concern. For example, in Ms. Qian's fourth post-observation interview, she mentioned that:

> *I will not make too many changes to the overall lesson plan, if give me the second chance to teach this lesson to a similar class, as to the exercises, I will make some changes because I feel that for a part of students, some problems are too easy for them. They can solve it right after I presented a problem. However, students' mathematics ability varies in my class, there are some students even cannot solve the first two basic problems. …, therefore, I would make some changes to the exercises, I will think more about how to choose problems with various difficulties.*

Her explanation indicates that she recognized not only the defects in the exercises, but also the factors causing them. In the meantime, the three teachers also reflected on the ways in which they worked on some problems. For example, as shown in part of Mr. Zhao's fourth post-observation interview below:

Interviewer: …, what else will you modify if you teach it again?
Mr. Zhao: From my point of view, from my personal point of view, I "dug" (worked) too little in the exercises.
Interviewer: You mean the last one?

Mr. Zhao: *No, all of the exercises, I should dig deeper, and make more variations and extensions.*
Interviewer: *Why do you think so?*
Mr. Zhao: *If I dig a little bit deeper and make more variations, it will make students understand the teaching content more thoroughly and deeply. I will use another lesson to review it.*

Similarly, in Ms. Sun's fourth post-observation interview, she reflected that the ways in which she worked on a problem were not effective in facilityateing students' understanding:

> *I have to review [this problem] again, because even I guided students to approach it, I do not think they really understand it thoroughly. Actually, not only did they feel this problem is difficult, former students also felt that it is difficult. I have to review this again.*

7.7.3 Discussion

Reflection is a major factor facilitating teachers' growth (Schön, 1983, 1987). In their reflections, the three teachers stressed students' understanding and the ways in which they dealt with teaching content and relevant teaching methods. They not only identified and explained what was effective or ineffective in their lessons, they also made appropriate adjustments to the inappropriate or ineffective approaches. According to Lee's (2005) criteria, their reflective thinking was at the highest level; that is, when they reflected, they "approach[ed] their experiences with the intentions of changing/improveing in the future, [and] analyze[d] their experiences from various perspectives" (p. 703).

Some characteristics identified in the three teachers' reflection are echoed in other studies. For example, Bond *et al.* (2000) stated that expert teachers reflect on how well ideas work in practice and refine future lessons accordingly. In Livingston and Borko's (1989) study, expert mathematics teachers' reflection was also found to focus on student understanding of the material and student performance. Like the three expert mathematics teachers in this study, expert mathematics teachers in Livingston and Borko's (1989) study rarely mentioned classroom management; however, unlike Livingston and Borko's subjects, who did little to assess the effectiveness of their own teaching, the three teachers in this study reflected on the effectiveness of the ways in which they dealt with the teaching content, organized lessons, and worked on problems.

Differences between expert teachers in Western culture and the three expert mathematics teachers in this study might in part be caused by differences in personality. Expert teachers in Western cultures were found to “possess a self-confidence that manifests itself in their belief that they can do a variety of things well” (Smith, 1999, p. 108), a characteristic not obviously shared by the three expert mathematics teachers in this study. Instead, they tended to be modest and always emphasized that they have many weaknesses on which they have to work very hard to overcome in future. In addition, as mentioned above, in Chinese educational culture, ways to deal with teaching materials and organize lessons are the main factor in evaluating a teachers’ teaching. This tradition might also make them heavily emphasize these aspects.

7.8 Summary of the Chapter

This chapter reported the common characteristics of the three expert mathematics teachers’ teaching practice within the Chinese sociocultural context. The teachers were able to teach with flexibility, coherence and balance, and could promote students’ high-order thinking skills in their teaching. Their teaching was consistent with their beliefs and they could systematically reflect on their lessons afterwards.

Some of these characteristics are similar to findings in studies conducted in Western cultures; like those expert teachers, the three Chinese expert mathematics teachers had flexible and detailed mental lesson plans, demonstrated improvisation skills and effectively used their connected knowledge structure to make their lesson mathematically and thematically coherent. They could strike a balance between directive teaching and exploratory teaching, and could choose relatively more demanding content to promote students’ high-order skills. On reflection, they also stressed students’ understanding and performance.

However, there are also some differences. Even though they could use textbooks flexibly, they seldom developed their own curriculum to the degree that very experienced teachers did in Western countries. While their lesson plans were flexible, they never made decisions about specific examples and problems while they were teaching. Finally, unlike expert mathematics teachers in the West, they reflected on the effectiveness of their own teaching. These differences might be caused by the different social and cultural contexts in which they are working. In the following chapter, more discussions will be conducted from the social-cultural perspective.

Chapter Eight

Sociocultural Influences

8.1 Introduction

The main aim of this study is to explore the conception and characteristics of expert mathematics teachers in mainland China from a sociocultural perspective. Findings have been reported and relevant sociocultural factors have been elaborated on and discussed in the previous three chapters. In this chapter, these sociocultural factors will be further discussed. As mentioned in Chapter Two, these social and cultural factors will be organized into four levels: the classroom, school, social, and cultural levels.

8.2 Sociocultural Factors at Classroom Level

The classroom setting is believed to provide the most influential environment for teachers' teaching and students' learning (Lee, 1998). As in many other countries and education systems in East Asia, class size in primary and secondary schools in mainland China is large (Biggs, 1996; Stevenson & Stigler, 1992) with the number of students in each classroom usually in excess of 45 (MOE, 2009), which makes it difficult for teachers to practice individualized teaching (Zhang *et al.*, 2004). Usually, individual tutoring was only considered a supplement to whole class instruction and a way to help students who had fallen behind to catch up (Stigler & Fernandez, 1995). In mainland China, classroom instruction is mainly conducted within a whole classroom setting (Cai *et al.*, 2004; Leung, 2001; Shao *et al.*, 2012) and lesson progress is under the control of teachers (Lopez-Real *et al.*, 2004). The central role of teachers is to make their decisions about what to teach in the classroom during a lesson and how much time should be spent on each teaching activity. Based on average learning ability, teachers also need to determine the pace of classroom teaching. All of these factors require teachers to understand the topic profoundly and to be able to find the best way to organize their teaching (Zhang *et al.*, 2004).

This tradition might make the 21 interviewees in this study – who comprise mathematics teachers, mathematics teacher educators, school

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principals and vice-principals, and mathematics teaching research officers – emphasize that an expert mathematics teacher should have strong capability to design her/his teaching and the ability to involve students in classroom activities. In this study, three expert mathematics teachers were found to be capable of doing so; they could plan their teaching thoughtfully before class and had the ability to control their lesson progress in class and make every lesson well organized.

Class size was also found to have profound influence over the teaching decision-making process of the three expert teachers. As introduced in Chapter Four, there were approximately 50 students in each of the three teachers' classes. All of the teachers admitted that the large class size made it impossible to have students work on very open exploratory activities on their own; lesson organization was thus dominated by "classwork". However, the three teachers also demonstrated the ability to counter the constraints of large class size, and to balance "students' mastering of knowledge" with "students' participation and exploration", as seen among expert mathematics teachers in Western cultures (*e.g.*, Borko & Livingston, 1989). Their teaching objectives stressed students' experience and exploration. In practice, they demonstrated the ability to encourage students to work on hands-on activities, discuss in small groups, and demonstrate their solutions publicly. In general, their teaching is like "both teacher led and student centered" (Wong, 2004, p. 526) but not as pointed out in literature as "teacher dominated" or "teacher-centered" (Li, 2006; Lv & Wang, 2002; Mok, 2006; Zhang, 2006; Zhang *et al.*, 2004).

The class size is large in Mainland China, however, unlike the situation in the United States, children mainland China are not commonly "tracked according to ability into different classrooms, even as early as kindergarten", with the assumption that "children are best served when they are placed in homogenous groups and given instruction that is tailored to their needs" (Stigler & Fernandez, 1995, p. 107). In mainland China, as in some other Asian countries, tracking does not occur until the later years in students' learning careers, usually at the senior secondary level (Stigler & Fernandez, 1995). This means that students in junior secondary school classrooms usually have different mathematics ability, interests and backgrounds. In Chinese education philosophy, it is believed that all students, except for some extreme cases, can learn the defined content well if they are given the opportunity and if they work hard to practice after school (Stevenson & Stigler, 1992). Therefore, in the Chinese mathematics pedagogy, teachers are supposed to "make teaching beneficial to all the students rather than some of them" (Zheng, 2006, p. 388).

As such, it is not difficult to understand why the 21 interviewees emphasized that an expert mathematics teacher should know students' backgrounds and have the ability to teach them accordingly. Teachers are supposed to be able to teach in mixed social class situations in classrooms and to know how to teach students with different abilities and interests, rather just than a homogenous cohort of students. This might explain why the interviewees thought that an expert mathematics teacher should be capable of planning lessons, dealing with textbooks, and implementing teaching plans flexibly to meet various students' needs. In addition, since students have various degrees of interest in mathematics, an expert mathematics teacher is also expected to be able to inspire, change, and maintain students' interest in mathematics learning.

In this study, the three expert mathematics teachers were found to be able to teach students with various backgrounds. Over the past fifteen years, they each had served in several schools, in each of which students' backgrounds, their academic backgrounds in particular, were quite different. The teachers demonstrated their ability to know and teach students from different backgrounds in each of these schools. In the schools which they now serve, they knew well the differences in students' academic abilities, cognitive development, family backgrounds, and habits. Moreover, they could deal with textbooks flexibly to make content better suit students' actual needs and demonstrated the ability to design teaching tasks with various and gradually increasing difficulty, to make involve more students and facilitate their understanding.

Furthermore, even though students' ability varies in every class, the situation in China is unlike American classrooms, where teachers tend to group students for instruction "according to their abilities or levels of preparation" (Stigler & Fernandez, 1995, p. 108). Chinese teachers rarely group students in the classroom based on ability (Stigler & Fernandez, 1995); students' seats are fixed in the classroom and are seldom changed in the process of teaching (Yang, 2009; Yang & Ricks, 2012). In addition, teachers in mainland China, as in other East Asian countries, are more likely to involve all students in their teaching process collectively, which is quite different from the situation in American classrooms, where teachers are more likely to encourage students to voluntarily offer their explanations or answers to the class on an individual basis (Lee, 1998). This kind of pedagogy encourages teachers to prepare their teaching with the intention of affording most, if not all, students the opportunity of participating in classroom activities.

This tradition might account for the interviewees thinking that expert mathematics teachers should have strong ability to prepare various tasks to meet various students' needs, and to involve most students in learning. The three expert mathematics teachers did demonstrate the ability to prepare tasks with different levels of difficulty and design various activities to encourage more students to engage in the classroom. Mr. Zhao, especially, divided his exercises into three categories: category A, the basic level, exercises were drawn mostly from textbooks and were intended for all students; category B, the middle level, featured problems chosen from other materials or posed by him, and covered 60-70% of the students; and category C, the high level, included problems from *Zhongkaos* or even more difficult problems, and were intended for those students with high mathematics ability.

8.3 Sociocultural Factors at School Level

Micro-context 1 in the framework developed in Chapter Two refers to the school context, with the aim of capturing social and cultural influences at the school level. The school culture not only influences teachers' work, including teaching, but also their professional development (Ma & Paine, 1992; Paine, 1993; Wang & Paine, 2001, 2003; Yang, 2009; Yang & Ricks, 2012).

8.3.1 Collective working culture

As introduced in Chapter Three, the school level working culture in mainland China has been described as collective (Paine & Ma, 1993; Paine *et al.*, 2003; Wang & Paine, 2001, 2003; Wang *et al.*, 2004). Teachers work collectively to learn how to teach good lessons, including lesson preparation, public teaching with observation, and post-lesson discussions, and to understand relevant educational theories that underpin their teaching (Yang & Ricks, 2012; Tsui & Wong, 2009). This indicates that teachers' professional development is "presented as activities that are practical in nature" (Li & Huang, 2008, p. 69). Moreover, it is believed in the East Asian region (includeing in mainland China) that all teachers can teach if they are properly trained and guided; by comparison, in the United States, it is commonly believed that a good teacher is born rather than made (Lee, 1998). In mainland China, teachers newly-graduated from teacher-training universities are required to learn to teach after entering teaching positions (Li *et al.*, 2008; Paine *et al.*, 2003). A popular teacher development practice in mainland China is one-on-one mentoring, also called "the old guiding the young" (*Laodaiqing*) or apprenticeship practice (Han, 2012; Paine *et al.*, 2003; Tsui & Wong, 2009; Wang & Paine, 2001, 2003).

The conception of expert mathematics teachers held by the interviewees is influenced by this kind of working culture and pathway of professional development. The ability to mentor other teachers was emphasized by different interviewees as a fundamental sign distinguishing expert mathematics teachers from proficient mathematics teachers. An expert mathematics teacher is expected to be able to evaluate insightfully other teachers' teaching and working and help them to improve. In particular, s/he is supposed to have many strategies to facilitate a novice teacher's professional development and the ability to organize workshops or seminars to share her/his experience with other teachers.

The three expert mathematics teachers did demonstrate the ability to mentor novice teachers. They all acted as mentors to novice teachers in their schools and tutored them to prepare for teaching competitions at different levels. Several teachers under their mentoring won first prize honors at the national level. Moreover, this culture also influences the three expert mathematics teachers' own professional development. They all admitted that they had learned a lot about how to teach from observing other teachers' teaching and attending relevant teaching research activities, and that through this process they gradually developed their knowledge of mathematics, pedagogy, learners, and curriculum. This was also a finding in other studies (*e.g.*, Ma, 1999; Paine & Wang, 2001, 2003).

8.3.2 Evaluation policy on teachers and teaching

Relevant requirements and the appraisal policy at school level also influenced the interviewees' conception of expert mathematics teachers and their teaching. In most schools in mainland China, conducting research and publishing paper are important criteria for evaluating teachers' performance (Ying & Fan, 2001; Zhang & Ng, 2011). For example, in Ms. Qian's school, as stated in one school document teachers are required to "actively participate in some research projects, develop the ability to choose a meaningful research topic, collect data, and write academic papers on individual effort". This sort of requirement might also make interviewees emphasize the need for expert mathematics teachers to have strong research abilities.

Currently, teaching effectiveness in mainland China is also evaluated based on their students' performance on various examinations (Ying & Fan, 2001). This could cause many interviewees to emphasize that an expert mathematics teacher should be able to teach mathematics efficiently and improve students' mathematics achievements significantly, even though s/he has no need to make her/his students perform the best within a school or district. This can also explain why the three expert mathematics teachers stressed improving students' problem solving skills, adding extra content and problems, and integrating examination information into their teaching.

Standards established at the school level to evaluate teachers' teaching might also influence the interviewees' conception of expert mathematics teachers and the three teachers' teaching practice. As mentioned in Chapter Three, in school-level teaching competitions, teachers will be evaluated by the standards established at one school, which mainly focus on teaching methods, teaching organization, teaching effectiveness, language skills, and teaching mien (Zhang & Ng, 2011). According to the standards established at Ms. Qian's and Mr. Zhao's schools, teachers are required to deal with teaching material flexibly according to students' actual situation, actively encourage students to participate in activities, stress important and difficult points of lesson content, and promote students' deep understanding. This kind of evaluation standard might contribute to the interviewees' emphasis on expert mathematics teachers having relevant abilities. It might also contribute to the three expert mathematics teachers' tendency to emphasize students' participation and conceptual understanding and their encouragement of students' hands-on activities, group discuss, and students' self-reflection.

8.4 Sociocultural Factors at the Societal Level

Mathematics education looks different in different social and economic situations (Lerman, 2000). The social context in which teachers grow up and work influences their conception of expert mathematics teachers as well their practice. As discussed in Chapter Two, factors at this level mainly include: 1) teacher education; 2) teacher promotion policy; and 3) the mathematics curriculum system.

8.4.1 Teacher education

The characteristics of the teacher education curriculum in mainland China are an important sociocultural factor that influences the conception and characteristics of expert mathematics teachers. As introduced in Chapter Three, at both the pre- and in-service training stages, teachers' ability to conduct research is a main focus of teachers' training, especially for in-service teacher training. This tradition might make the interviewees think that an expert mathematics teacher should have strong research ability. In addition, in some national in-service teacher training programs (*e.g.*, exemplary lesson development, master teacher work stations), training courses focus on teachers' understanding of mathematics content and ways of teaching mathematics effectively, such as improving lesson plan quality and setting teaching objectives (Huang *et al.*, 2011; Li, Qi, & Wang, 2012; Li, Tang, & Gong, 2011). This tradition might lead the interviewees to think that an expert mathematics teacher should have a profound understanding of mathematics and strong teaching ability.

Secondly, for a long time, the curriculum at the pre-service stage in mainland China has been described as academically-oriented (Williamson & Morris, 2000). Under the influence of the former Soviet Union, pre-service teachers are required to take many advanced mathematics courses (Li *et al.*, 2008; Yang *et al.*, 2009). At the in-service stage, advanced mathematics courses are also a major component of training programs (Ma, 2000). Reviewing and studying basic mathematics is another curriculum focus at the pre-service stage, the aim of which is to enhance pre-service teachers' deep understanding of basic mathematics at the secondary school level and improve their problem solving ability (Li *et al.*, 2008). This kind of academically-oriented curriculum might make the interviewees think that an expert mathematics teacher should have a solid and comprehensive mathematical knowledge base and strong problem solving ability. This tradition also contributes to the three expert teachers' understanding of teaching content as reported before. Even though this study did not directly test their problem solving ability, the three teachers mentioned that they had sufficient experience to train students to participate in mathematics competitions at different levels. This may, to a certain degree, indicate that they have strong problem solving abilities.

Although the three expert mathematics teachers demonstrated a deep understanding of the subject knowledge they are now teaching, in the knowledge structure pictures drawn by them and in the interviews, they seldom mentioned knowledge at senior secondary school and university levels. That is, the comprehensive knowledge structure, which included knowledge at the senior secondary and university levels as described by the 21 interviewees, is not observed in the three expert mathematics teachers at the junior secondary school level. This might be explained by the possibility that there are not many connections between the topics and knowledge they teach beyond the junior secondary level. However, another explanation could be the school system in mainland China. As discussed before, mathematics teachers at the junior secondary level in mainland China are only required to teach two classes at the same grade from grade 7 to grade 9 and thus make teachers not be as familiar with knowledge at the senior secondary level. Indeed, Mr. Zhao did mention several times that he is not so familiar with the knowledge structure at the senior secondary school level.

Moreover, the emphasis on learning to use textbooks, such as analyzing "important points", "difficult points", and "key points" in mathematics education methodology courses at the pre-service training stage (Li, 2008; Paine *et al.*, 2003), might also contribute to the interviewees thinking that an expert mathematics teacher should know the structure of textbooks and how to use them flexibly. This pre-service training experience improves the three expert mathematics teachers' knowledge of curriculum and teaching skills, on one hand, but influences the ways in which they use textbooks, on the other. As reported above, even though they never followed textbooks strictly, neither did they develop their own curriculum based on their experience as experienced teachers in Western countries often do (Brown & Edelson, 2003; Remillard & Bryans, 2004). At pre-service and in-service stages, much emphasis has been placed on learning educational and psychological theory (Li *et al.*, 2008; Ma, 2000), which accounts for the interviewees emphasis on expert mathematics teachers having rich theoretical knowledge.

8.4.2 Teacher qualification and promotion policy

Relevant requirements in teacher qualification regulations and the teacher promotion system adopted in mainland China were also important factors influencing the interviewees' conception of expert mathematics teachers. In mainland China, the ability to conduct research is viewed as a very important qualification and a key factor in teacher promotion. In some provinces, like in Chongqing, publications are also viewed as a necessary factor for promotion. To a large extent, this kind of policy contributes to the 21 interviewees highly emphasizing that an expert mathematics teacher should have strong ability to conduct research and have many publications.

Even though the three expert mathematics teachers had conducted research projects and published papers, there still exists a discrepancy between their practice and that described by the 21 interviewees, as the number of papers they had published was lower than what the interviewees suggested. The discrepancy might be the result of several reasons. One is that, when the interviewees talked about what an expert mathematics teacher should be, they had an idealized model in mind. Another reason could be that the 21 interviewees' conception of expert mathematics teachers is influenced by the expert mathematics teacher model at the senior secondary school level. In fact, the exemplar expert mathematics teachers mentioned by some interviewees were senior secondary school teachers. In addition, another reason might be, as the district-level mathematics teaching research officer mentioned, too many excellent junior secondary school teachers were reappointed to positions in senior secondary schools, which has led to a shortage of outstanding mathematics teachers at the junior secondary school level. The three teachers' quality might be another factor. All three teachers have only 15 years of teaching experience. According to Lian (2008), teachers with 15 years of teaching experience in mainland China can be categorized as "creative" expert teachers. Teachers of this category need several years of efforts to become "leader" expert teachers, the final stage in Lian's teaching expertise development model. "Leader" expert teachers may have a higher researching ability and a better publication record. The three expert mathematics teachers in this study may be somewhere between these two stages mentioned above and their ability of researching and output in terms of publication may still need to improve.

Mentoring is a very important responsibility for teachers at higher levels, and the ability to mentor novice teachers is seen as another important factor influencing teacher promotion (Zhang & Ng, 2011). This might make some interviewees think that an expert mathematics teacher should have strong ability to mentor novice teachers. As discussed above, the three teachers were found to have the ability to mentor other teachers' work and help them to prepare for teaching competitions.

8.4.3 Mathematics curriculum system

As Ma *et al.* (2002) pointed out, "China has one of the most centralized curriculum systems in the world" (p. 198). The nation-wide unified curriculum standard provides a "guideline for all teaching and learning activities at different grade levels" (Liu & Li, 2010, p. 9). Therefore, unlike teachers in decentralized curriculum systems, where teachers have "greater control over the curriculum" (Pong & Pallas, 2001, p. 257), Chinese teachers rarely, if ever, "adapt the central curriculum to meet the context of their schools and the characteristics of their students" (Ma *et al.*, 2002, p. 200). Curriculum in mainland China is not only highly centralized but also "scientific-discipline-centered" (Zhong, 2006, p. 374). As pointed out in the literature, "Chinese mathematics curricula offer a relatively narrow scope of content, but the coverage is often deeper" (Cai *et al.*, 2004, p. 546). Under this curriculum paradigm and tradition, mathematics teachers in mainland China are expected to be "content specialists" (Li, 2008, p. 192) since they need to teach students more mathematically-challenging content.

This curriculum paradigm might account for the 21 interviewees emphasizing that an expert mathematics teacher should have a deep understanding of the mathematics content they teach. Moreover, this curriculum system might also contribute to the interviewees thinking that an expert mathematics teacher should know the curriculum structure well since every teacher has to implement the curriculum established by the official government. This centralized curriculum system, on the other hand, might ensure the three teachers cannot develop their own curriculum as discussed below.

The three expert mathematics teachers' beliefs and practice are influenced by teaching syllabus requirements and mathematics curriculum standard. For example, all three teachers thought that mathematics teaching and learning should develop students' mathematics thinking and ability, which is in accordance with the national mathematics teaching syllabus and curriculum standard (Research Institution of Curriculum and Textbooks, 2004). Ideas – such as the emphasis on students' experience, participation, and exploration, knowledge development process during teaching, and linking teaching content to real life – are emphasized in the latest curriculum standard and might make the three teachers think that students in mathematics classrooms should have the chance to experience and explore mathematics by themselves (MOE, 2001). As a result, they all demonstrated the ability to construct some situational problems to let students explore, asked students to discuss in groups, and worked on hands-on activities.

Under mainland China's centralized curriculum system, textbooks are developed "in alignment with the unified curriculum standard" (Li *et al.*, 2009, p. 734). Moreover, until the late 1980s, all students used the same set of textbooks (Ma *et al.*, 2002); even now that there are several sets of mathematics textbooks from which to choose, all are compiled under the same curriculum standard and have to be government officially approved (Liu & Li, 2010). In other words, textbooks still influence the implemented curriculum in mainland China (Li, 2008). This influence gives textbooks an important role to play in teachers' teaching and students' learning. In mainland China, as in other Asian countries and regions, textbooks are regarded as a "bible" that contains "a body of the minimum and essential knowledge" (Park & Leung, 2006, p. 229) that students must learn and understand for examinations (Ma *et al.*, 2002). Since textbooks play a very important role in practice, it is reasonable for the interviewees to think that an expert mathematics teacher, or even a non-expert teacher, should know the content, the structure, the strengths, and weaknesses of the textbook very well.

The characteristic of knowing the structure, strengths and weaknesses of textbooks emphasized by the interviewees was found in the three expert mathematics teachers. The three expert mathematics teachers were well aware of the strengths and weaknesses of the textbooks, and had well-structured vertical mathematics curricular knowledge. However, the role played by textbooks in mainland China also influences the three teachers' teaching practice. Since students must understand the textbook content for future examinations, the three teachers cannot develop their own curriculum without considering teaching content unlike very experienced mathematics teachers in Western countries (*e.g.*, Brown & Edelson, 2003; Remillard & Bryans, 2004). This tradition might also make the three expert mathematics teachers' teaching appear less flexible than that of expert teachers in Western countries even though they did not follow the textbooks "from cover to cover" (Stevenson &Stigler, 1992, p. 141).

Moreover, mathematics textbooks in mainland China, as in many other East Asian countries and regions, tend to emphasize the noble logical system of mathematics "by presenting a combination of concepts, symbols, and algorithms in a decontextualised way" (Park & Leung, 2006, p. 235). This results in textbooks containing very little real world information, instead stresssing mathematical content in a deductive manner (Park & Leung, 2006). Furthermore, textbooks in China are traditionally concise (Fang & Gopinathan, 2009) with "few illustrations" (Stevenson & Stigler, 1992, p. 139); teachers need to interpret and expand textbook content and structure lessons on their own according to their students' actual characteristics (Li, 2008). This tradition also demands that teachers be content specialists. It is thus understandable that the interviewees emphasized that an expert mathematics teacher should have her/his own understanding of the teaching content and be able to deal with that content flexibly to meet students' needs. The three teachers did demonstrate the ability to deal with textbooks with both macro- and micro- level flexibility. They could employ many real life examples to enrich students' experience, replace relevant content with more appropriate and localized information, and re-organize the sequence of the teaching content. The logical system emphasized by textbooks might also make the three teachers stress students' thinking in practice and make their teaching look coherent.

In addition, as found in many comparative studies on textbooks, Chinese textbooks stress applying basic knowledge and routine procedures, solving non-contextualized and conventional problems (Bao, 2002; Fan, 1999). Activities in Chinese textbooks usually support individual rather than cooperative learning (Cai *et al.*, 2004). Even though these characteristics are not clearly found to influence the interviewees' conception of expert mathematics teachers, they do influence the three teachers' teaching. As reported before, most of problems used in the three teachers' lessons were routine problems and non-application problems. Moreover, they tended to employ more "classwork" and "individual seatwork" in their teaching. However, the three teachers all designed some application problems and employed group work when the teaching content was appropriate. To a certain degree, this suggests that they also demonstrated the ability to work against this constraint, even though they cannot completely extricate themselves from this tradition.

8.5 Factors at the Cultural Level

Traditional cultural beliefs shared by a particular group in a society influence teachers' perceptions of their role and their performance of day-to-day activities (Ratner, 2002; Rogoff, 2003; Wong *et al.*, 2001). As discussed in Chapter Two, cultural factors in this study refer to beliefs about education in general and mathematics education in particular. Relevant beliefs, such as beliefs about the role of teachers, teaching, ability, mathematics tradition, and China's examination culture influence the conception and characteristics of expert mathematics teachers in this study.

8.5.1 Beliefs about the role of teacher

The Confucian culture has been described as the orthodox tradition of Chinese culture (Gu, 2006; Zhang *et al.*, 2004). Under the influence of Confucius, teachers are expected not only to transmit knowledge, wisdom, and virtue, but also to act as moral models for their students (Sun & Du, 2009; Xiao, 2001). Given this, it is not difficult to understand why many interviewees emphasized that an expert mathematics teacher should have a noble personality and that her/his personality can influence students and other teachers' development. Great personality, emphasized by the interviewees, was also found in the three expert teachers. According to their colleagues and principals' descriptions, the three teachers are very modest, care for their students, and are willing to help other teachers.

Under the influence of Confucian culture, teachers are required to dedicate themselves to teaching and should teach with tireless zeal (Sun & Du, 2009; Xiao, 2001). This tradition might make many interviewees emphasize that an expert mathematics teacher should study diligently, dedicate themselves to teaching, treat their work seriously, and work very hard. In practice, the three expert mathematics teachers were found to work very hard, and all said that they will continue to work as a teacher for the rest of their lives. They said that, were they given a second chance at life, they would still choose to be a teacher. This indicates that they are indeed dedicated to teaching.

Moreover, under the influence of Confucius, a teacher in China is required to be knowledgeable. Confucius emphasized that possessing wide and broad knowledge is a prerequisite of being a teacher (Sun & Du, 2009); he thought that a teacher should learn painstakingly and insatiably to enhance her/his knowledge. In mainland China, teachers' professional duties have been described as "teach[ing] students to learn and acquire knowledge, skills, and values" (Yang *et al.*, 1989, p. 49). Under this image, the teacher has been described as an old master who possesses knowledge that can be transmitted to the younger generation (Paine, 1990). Furthermore, it is widely accepted that a teacher should be an expert or scholar in the subject s/he is teaching (Leung, 1995). Specific to mathematics education, this social and cultural influence might make interviewees think that an expert mathematics teacher should have a profound mathematics knowledge base, even including advanced mathematics. In practice, the three teachers were found to understand mathematics profoundly at the level they are teaching.

8.5.2 Beliefs about teaching

Chinese teaching has also been described as heavily influenced by the ideas proposed by Confucius (Li, 2006; Sun & Du, 2009; Xiao, 2001; Zhang, 2010; Zhang *et al.*, 2004). In Chapter Three, some of the teaching principles advocated by Confucius were introduced, such as knowing students and teaching them accordingly (了解学生，因材施教), reviewing what has been learned from time to time and being able to gain new insights through reviewing old material (学而时习，温故知新), teaching heuristically and gradually (启发教学，循序渐进), and studying as well as reflecting (学思并重，学思结合). All these were found to influence the interviewees' conception of expert mathematics teachers. For example, they emphasized that an expert mathematics teacher should know her/his students well and be able to teach students from various backgrounds. They also emphasized that an expert mathematics teacher should have the ability to develop students' mathematical thinking and conduct student-centered teaching.

The abilities highlighted by the interviewees were found in the three expert mathematics teachers. They were found to know their students' backgrounds and individual students' differences well. During their teaching, they spent a lot of time reviewing what students had already learned in order to facilitate their understanding of and help them to build connections between new and existing knowledge. During the teaching process, the three teachers valued students' thinking and demonstrated the ability to develop it, such as by encouraging students to analyze and explain the rationale informing their answers and solutions, pushing students to look for alternative solutions and making extensions to solutions and conclusions.

8.5.3 Beliefs about effort and enduring hardship

In traditional Chinese culture, individual diligence and personal effort are highly valued as essential factors to pursue success (Bond, 1996; Lee, 1998; Li & Yue, 2004). It is widely believed in Chinese society that effort is more important for success than ability, which can be improved by working hard (Hau & Salili, 1996; Salili, 1996). For example, as a famous Chinese saying goes, "diligence can remedy mediocrity" (勤能补拙). Enduring hardship has long been emphasized as another important factor for success in Chinese culture (Li, 2002; Li & Yue, 2004). Chinese people believe that single-minded concentration on one thing and "studying hard, regardless of favorable or difficult learning conditions" (Li, 2002, p. 263) can enable one to realize her/his dream eventually. Various Chinese sayings perpetuate this, such as "in time, a string may saw through wood and drops of water can penetrate a stone" (绳锯木断，水滴石穿), "by not giving up, you can change an iron rod into a needle" (只要功夫深，铁杆磨成针). Given these kinds of cultural beliefs, it is not difficult to understand why the interviewees mentioned that an expert teacher should study hard and continuously develop her/his expertise.

These cultural beliefs influenced the three expert mathematics teachers' professional growth and the development of their teaching expertise. As mentioned in Chapter Four, all three expert mathematics teachers had worked hard to pursue success, and were still working hard to pursue even greater success. At the very beginning of their teaching careers, the contexts in which they worked were not favorable to their growth; however, they did not cease their efforts to develop their teaching ability and expertise. Rather, they developed both through their own efforts and hard work despite their environments. From a Western perspective, teaching expertise has been seen as being developed in the context that supports it (Berliner, 2004). Some of the differences between the three expert teachers in this study and those statements made in Western studies might be explained by this cultural belief. In the meantime, this kind of cultural belief was found to influence the three teachers' beliefs. They thought that, even if a student is talented in mathematics, if s/he does not work hard, s/he could not learn mathematics well.

8.5.4 Mathematics teaching tradition

During its long history, mathematics education in mainland China has gradually gained its own characteristics, which have been summarized by a variety of researchers (*e.g.*, Kang, 2010; Shao *et al.*, 2012; Tu & Song, 2006; Xu *et al.*, 2009; Zhang, 2006, 2010). Some of these characteristics are unique to mainland China, and influence the conception and characteristics of expert mathematics teachers in this study.

8.5.4.1 Emphasis on the "two basics"

The Chinese mathematics teaching tradition places much emphasis on the "two basics" – basic knowledge and basic skills (Li, 2006; Shao et al, 2012; Zhang, 2006, 2010; Zhang *et al.*, 2004) – which have been called the foundation of students' development and creativity (Li, 2006; Zhang *et al.*, 2004) and are highly valued by teachers in daily teaching practice. In order to enhance students' "two basics", certain teaching approaches, such as "integration of teaching and practice" (讲练结合), "teach the essential parts and ensure plenty of exercises" (精讲多练), or "practice makes perfect", have been widely used to the extent that they have achieved a dominant position in mathematics teaching over the past several decades (Li, 2006; Shao et al, 2012; Zhang, 2006, 2010; Zhang *et al.*, 2004; Zheng, 2006). In particular, it is accepted by many Chinese mathematics educators that practice can enhance familiarization and proficiency, which many call the “aim of learning” (Zhang *et al.*, 2004, p. 192). Traditionally, teachers have tended to use a great many exercises in mathematics teaching to enhance students’ proficiency in problem solving skills (Li, 2006; Shao et al, 2012; Zhang, 2006; Zheng, 2006). This tradition might also make the 21 interviewees think that an expert mathematics teacher should have strong problem solving abilities. On the other hand, s/he should also be able to lay down a firm knowledge foundation for her/his students and train them in necessary skills that they can employ in the future.

The three expert mathematics teachers also emphasized that students should master the “two basics”. In setting up their teaching objectives, they stressed students’ acquiring knowledge and mastering necessary problems-solving skills. However, they also demonstrated the ability to work against this tradition to a certain degree. Their teaching objectives balanced the mastering of knowledge with students’ experience, exploration, interests, and the development of students’ thinking. Moreover, in their teaching they reached a balance between directive teaching and exploratory teaching, and did not assign their students as many practice problems as did other teachers (*e.g.*, Gu, 1999; Li & Li, 2009). Instead, they employed a relatively small number of problems and adopted open-ended approaches to working on them, such as encouraging students to look for alternative solutions, making variations to problems, and making extensions to conclusions.

8.5.4.2 Emphasis on variation

To enhance students’ understanding and consolidate what students have learned, a popular method employed by mathematics teachers in mainland China is to make variations to the representations of new topics, tasks and problems (Gu, 1992; Gu *et al.*, 2004; Li, 2006; Wong *et al.*, 2012). In particular, teachers will make many variations to mathematics problems and their solutions to help students master knowledge and enhance their necessary problems solving skills (Li, 2006; Tu & Song, 2006; Zhang, 2006, 2010; Zheng, 2006). This tradition might have contributed to the interviewees thinking that expert mathematics teachers should have relatively strong problem solving abilities so that they can freely make necessary variation to a problem and demonstrate various solutions during their teaching. This tradition also explains why an expert mathematics teacher should be expected to have the ability to pose problems and plan their teaching flexibly since they need to be able to vary the tasks they design.

The ability to make variations teaching tasks or exercises was found in the three expert mathematics teachers. In the observed lessons, they made various changes to the ways in which they presented new topics. After they introduced a new concept, they would make variations to it to help students discern its critical properties; and they made variations to the problems they worked on to consolidate students' understanding and train their problem-solving skills. This suggests that this tradition influenced the three teachers' teaching practice.

8.5.4.3 Emphasis on deep understanding, mathematics thinking, and mathematics ability

Another tradition of mathematics teaching in mainland China is its emphasis on fostering students' deep understanding and promoting their mathematics thinking and mathematics ability (Kang, 2010; Li, 2006; Shao et al, 2012; Zhang 2006; Zheng, 2006). Tu and Song (2006) pointed out that a tradition in mathematics teaching is to "deeply dig" new knowledge, such as by analyzing critical terms in the definition and exploring connections with prior knowledge. In addition, under the influence of the Soviet, viewing mathematics as an abstract, rigorous, and wide application subject as suggested by Aleksandrov *et al.* (1964) has been widely accepted and has deeply influenced mathematics education in mainland China since then(Li et al., 2008). Under this tradition, the development of students' abstract and rational thinking has been considered as one of the key and long-term teaching objective (Zhang, 2006; Zhang *et al.*, 2004). This tradition might make the interviewees emphasize an expert mathematics teacher's ability to understand mathematics profoundly, view mathematics problems at the junior secondary level from a high perspective, and command the mathematical thinking underlying a given topic. This tradition might also make the interviewees think that an expert mathematics teacher should be able to develop students' mathematical thinking and mathematics abilities through their teaching.

The ability to promote students' deep understanding was found in all three expert mathematics teachers. In teaching, they emphasized developing students' mathematical thinking. Even though they sometimes employed real life situations to introduce new topics, asked students to participate in group discussions and work on hands-on activities, after these activities, they encouraged students to use their prior knowledge to explain their solutions and answers, make clarifications to their solutions, and analyze the rationale behind their answers. This tradition might have influenced the three expert mathematics teachers' beliefs as well. They all believe that mathematics is a vehicle to develop students' mathematical thinking and that doing so is the main goal of mathematics learning and teaching.

8.5.5 Examination culture

Examination became a cultural tradition in Chinese history under the influence of the civil examination system (*keju*, 科举), the history of which can be dated back over 1400 years (Li, 2006; Zhang *et al.*, 2004), and high-stakes public examinations still affect teachers' teaching and students' learning in mainland China today (Wong *et al.*, 2004; Wu, 2012; Zhang & Ren, 1998). As Li (2006) pointed out, "'education for exam' has been almost a directional convention" (p. 132) in mainland China. The strict and unified examination systems in mainland China have made teachers "see themselves as having the responsibility of helping students get over these hurdles" (Ma *et al.*, 2002, p. 199) and have driven students "to only learn the content that will examined in future" (Zhang *et al.*, 2004, p. 191).

As such, it is easy to understand why the interviewees in this study highlighted that an expert mathematics teacher should be knowledgeable about examinations; in Western studies, by contrast, knowledge of examinations is seldom mentioned. In particular, an expert mathematics teacher is expected to know the examination requirements, how to integrate *Zhongkao* information into their teaching and how to develop *Zhongkao* problems. However, this is not to say that students' performance in examinations is an important criterion for judging whether a teacher is an expert mathematics teacher. To the contrary, the interviewees pointed out that the factors influencing students' mathematics achievements are complicated. Students taught by an expert mathematics teacher do not necessarily to perform best on mathematics examinations.

The ability to integrate examination information into their teaching was found in the three teachers in this study. Ms. Sun and Mr. Zhao acted as members of teams to develop examination papers for the *Zhongkao* in Chongqing several times. This suggests that they knew the examination's requirements and how to develop *Zhongkao*-related problems. The examination culture can be seen to have influenced their beliefs, knowledge, and teaching, inasmuch as they believed that mathematics is a school subject students need to learn for future examinations, and that basic knowledge and skills are a necessary objective of mathematics teaching and learning. In teaching, they all introduced some typical *Zhongkao* problems or other difficult problems to equip students with problem-solving skills they could use in examinations. The variations and extensions they made to a problem enhanced students' deep understanding of the topics, and also prepared them to solve similar problems quickly in the future examinations.

8.6 Summary of the Chapter

This chapter discussed the findings reported in the previous three chapters from a sociocultural perspective. The characteristics of an expert mathematics teacher, as described by the 21 interviewees, were compared with those identified among the three expert mathematics teachers. In general, most of the described characteristics were observed in the three expert teachers, although there were some discrepancies regarding their capacity for research, the number of publications an expert teacher should have, and the mathematics knowledge structure an expert teacher should possess. Moreover, it was found that the conception held by mathematics educators was influenced by social and cultural contextual factors. The three expert mathematics teachers' beliefs, knowledge, and practices were also found to be influenced by the Chinese social and cultural context, despite the fact that they sometimes were able, to a degree, to work against it.

Chapter Nine

Conclusions and Recommendations

9.1 Introduction

The findings have been reported, and the relevant sociocultural factors have been discussed in the four previous chapters. This final chapter synthesizes the main findings of the study and then presents the contributions and implications of the study. Finally, its limitations will be pointed out and some recommendations for further research will be suggested.

9.2 The Main Findings of the Study

This study aimed to explore: 1) how mathematics educators in mainland China conceptualize “expert mathematics teacher”; 2) the characteristics of expert mathematics teachers; and 3) how the Chinese social and cultural context influences the conception and characteristics of expert mathematics teachers. Adopting sociocultural theory and prototype view of teaching expertise as its theoretical foundations, this study explored 21 mathematics educators’ perceptions of expert mathematics teachers. Based on their recommendations, this study further explored the common characteristics of three expert mathematics teachers. In this section, the findings of the study will be summarized to answer the research questions.

9.2.1 Conception of expert mathematics teachers

According to the 21 interviewees’ descriptions, an expert mathematics teacher in mainland China should play multiple roles, rather than just one demonstrating her/his expertise in teaching. Other roles highlighted by the interviewees include:

1) ***Researcher***. First and foremost, an expert mathematics teacher should be able to conduct research and publish papers in professional and academic journals;

2) ***Teacher educator.*** An expert mathematics teacher should be able to mentor non-expert teachers and should have many strategies to

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effectively facilitate non-expert teachers' professional growth;

3) ***Scholar.*** An expert mathematics teacher should have a broad and profound knowledge base in mathematics and many other fields. Mastery of knowledge in pedagogical, psychological, curriculum theories is also highly emphasized;

4) ***Expert in examinations***. An expert mathematics teacher should be knowledgeable in examinations and have the ability to develop problems used in the *Zhongkao*;

5) ***An exemplary model for students and colleagues***. An expert mathematics teacher should have a noble personality, and should act as an exemplary model for her/his students and colleagues.

9.2.2 Characteristics of expert mathematics teachers

The three teachers who were recommended as expert mathematics teachers were found to have the following common characteristics:

9.2.2.1 Contemporary-constructivist oriented beliefs

The three expert mathematics teachers hold contemporary-constructivist oriented beliefs. Some traditional views about mathematics, mathematics learning, and mathematics teaching held by Chinese mathematics teachers in other studies – such as a rigid view of mathematics or viewing mathematics as some knowledge students need to learn for school and examination purposes – were not found in the three expert mathematics teachers. Their beliefs about mathematics seem to be close to a combination of the instrumentalist view and the problem-solving view of mathematics (Ernest, 1991). The three teachers were found to emphasize students' intellectual involvement in mathematics learning and teaching, and thought that enhancing students' mathematical thinking and abilities were the most important goals of mathematics teaching and learning.

9.2.2.2 Broad and profound knowledge base

The three expert mathematics teachers possess a broad and profound knowledge base. They know mathematics deeply at the level they are teaching and have a web-like knowledge structure. They have extensive pedagogical content knowledge, including knowing students' prerequisite knowledge, being able to anticipate students' difficulties, designing appropriate teaching tasks, and being able to teach students from various backgrounds. They make critical judgments about the latest mathematics curriculum and know the structure, strengths and weaknesses of textbooks well. They also know the students they are teaching well, including their family and academic backgrounds, and their interests.

9.2.2.3 Teaching with flexibility

The three expert mathematics teachers are able to teach flexibly. They can flexibly deal with teaching materials, rather than strictly following textbooks or teaching manuals. They can make ongoing and dynamic changes to their teaching plans based on their students' reactions, and can structure and organize their lesson flexibly according to the characteristics of the teaching content and their students' background.

9.2.2.4 Teaching with coherence

The three expert mathematics teachers can make their teaching coherent. First, they employ appropriate discourse to move from one activity to another smoothly, so as to reduce student confusion. Second, they systematically review relevant knowledge before introducing a new topic, differentiate similar topics, and design various activities within a lesson that tightly relate to each other and make their teaching mathematically and thematically coherent.

9.2.2.5 Teaching with balance

The three teachers achieve a balance in their teaching objectives between students' mastery of knowledge and students' experience and exploration. In their teaching, they strike a balance between directive teaching and exploratory teaching.

9.2.2.6 Teaching with the aim to promote students' higher order thinking skills

The three teachers can encourage students to analyze their answers and solutions. They employ an open-ended approach to work on problems, such as providing students time to look for solutions, encouraging students to look for alternative methods, and making variations and extensions to problems.

9.2.2.7 Consistent relationship between beliefs and practice

The three teachers' teaching practice and beliefs are found to be consistent. To a certain degree, their beliefs influence their teaching practice.

9.2.2.8 Systematic reflection on teaching

The three expert mathematics teachers can reflect on their teaching and explain the rationale behind their teaching. They also have the ability to make adjustment and improvements to those parts they feel are not appropriate or effective.

9.2.3 Chinese social and cultural influences

Factors at the cultural, social, school and classroom levels were found to influence the conception and characteristics of expert mathematics teachers. However, findings on their teaching practice suggest that the three expert mathematics teachers can sometimes work against contextual and cultural constraints to a certain degree. They developed contemporary-constructivist orientated beliefs at a time when Chinese mathematics teaching was described as teacher-centered and knowledge-centered (*e.g.*, Li, 2006; Zhang *et al.*, 2004; Zheng, 2006). Despite the burden of large class sizes, they provide students opportunities to discuss in groups, carry out hands-on activities, and explore on their own. They link teaching content with real life situations rather than teaching mathematics only "within mathematics itself" (*e.g.*, Li & Liu, 2010; Zhang, 2008; Zheng, 2006). They employ a relatively small number of problems and make full use of them to develop students' high-order thinking skills, rather than asking students to practice many exercises repeatedly as other teachers do (*e.g.*, Gu, 1999; Li & Li, 2009).

9.3 Major Contributions of the Study

Although the present study focuses on the conception and characteristics of expert mathematics teachers in the context of Chongqing, its significance and contributions are far from being merely regional. This study makes significant theoretical and practical contributions to the field of mathematics education research and teacher education research.

9.3.1 Theoretical contributions of the study

In terms of theoretical contributions, first of all, this study has identified a conception of expert mathematics teachers from the perspective of 21 mathematics teachers, mathematics teacher educators, school principals and vice-principals, and mathematics teaching research officers; relevant social and cultural factors, which were found to influence the interviewees' conception, have also been examined. Even though some facets of expert teachers mentioned by the 21 interviewees are also found in other studies, others, such as expertise in research, theoretical knowledge, and knowledge of examinations, have seldom been mentioned previous studies, and especially those studies conducted in Western cultures. These findings indicate that the conception of expert mathematics teachers is culturally bounded. The findings also suggest that the qualities of expert mathematics teachers expected in mainland China go beyond teaching and are bounded by the Chinese social and cultural context. This illustrates that teachers in mainland China take on more responsibilities than do their counterparts in other countries and regions. Hence, findings of this study further suggest that the expected qualities of a teacher are essentially a sociocultural product.

Secondly, characteristics found in the three expert mathematics teachers have contributed to the current literature on expert teachers. This study found that, at the beginning of the three expert mathematics teachers' teaching careers, they worked in contexts that did not support the development of their teaching expertise based on the opinions of researchers from Western cultures (*e.g.*, Berliner, 2001; Bullough & Baughman, 1995). Context has been described as a very important factor influencing the development of expert teachers. Berliner (2001) pointed out that "it is probably the power of context followed by deliberate practice, more than talent, which influences a teacher's level of competency" (p. 466). However, the three teachers developed their teaching ability through their own efforts despite working in an unfavorable context. The three expert mathematics teachers' teaching experience suggests that, under the Chinese social and cultural context, teachers who work hard enough can develop their teaching expertise. Moreover, even though the three expert mathematics teachers were able to plan and implement their teaching flexibly, the degree of flexibility was not as great as that found in expert teachers in Western cultures. This indicates that characteristics found in expert teachers in Western cultures, such as developing "automaticity and reutilization" or having "automatic ways", are not the same as those found in expert teachers in mainland China. Similar to the findings in Tsui's (2009) study, the description of experts' work as automatic and effortless does not tally with the ways in which the three expert mathematics teachers design their teaching and implement their lesson plans. This further indicates that teaching contexts do exert influence on the performance of expert teachers' teaching (Berliner, 2004). However, the three expert mathematics teachers sometimes demonstrated the ability to work against the constraints of the teaching context to a certain degree.

Thirdly, this study provides a new perspective for interpreting Chinese mathematics teachers' professional development, mathematics teaching, and Chinese students' mathematics achievements. Many mathematics teachers, school principals and vice-principals, mathematics teacher educators, and mathematics teaching research officers strongly emphasized that the ability to conduct research is a critical factor in a teachers' professional development and potential for promotion or professsional advancement. This finding explains how Chinese mathematics teachers learn to teach, given that they receive comparatively less education than their counterparts in Western countries (Ma, 1999) and insufficient professional training in the pre-service stage (Li *et al.*, 2008; Paine *et al.*, 2003). The ability to promote students' higher order thinking skills suggests that not all students in mainland China study mathematics superficially and procedurally as suggested by some Chinese mathematics education researchers (*e.g.*, Li, 2006; Lv & Wang, 2005; Zhang, 2006). Moreover, the balance between directive teaching and exploratory teaching illustrates that, even though lesson progress is under the control of the three teachers, it is not “necessarily detrimental to learning” (Mok, 2006, p. 140). This suggests that, while mathematics teaching in mainland China might superficially look as being content-oriented or teacher-dominated, students still have the chance to engage and become intellectually involved in mathematics learning (Huang & Leung, 2004; Lopez-Real *et al.*, 2004; Mok, 2003, 2006). The focus on high-order thinking skills and deep understanding lays down a firm knowledge foundation for students that might help them perform well in comparative studies of mathematical achievements.

Fourthly, this study provides a new explanation for differences between expert teachers’ teaching and novice teachers’ teaching. The study found that the three expert mathematics teachers tend to hold some non-traditional beliefs about mathematics, mathematics learning, and mathematics teaching, and that those beliefs were found to be consistent with their teaching practice. This characteristic indicates that differences identified between expert and novice mathematics teachers’ teaching in previous studies (*e.g.*, Borko & Livingston, 1989; Leinhardt, 1989; Zhu *et al.*, 2007) might be caused by differences in their beliefs although the relationship is complicated. This finding provides another perspective for interpreting the differences between expert and novice mathematics teachers’ teaching other than merely focusing on differences in teacher quality and experience.

Lastly, this study employed a prototype view of teaching expertise to explore the conception and characteristics of expert mathematics teachers in mainland China. Until now, very few studies have employed this view to research expert teachers in Mainland China. This study expands the applicability of the prototype view to another social and cultural context. In addition, very few studies have explored expert teachers from a specific subject when taking this view. Unlike many other previous studies on expert teachers in which the researchers explore expert teacher from an experimental psychological perspective (*e.g.*, Stader *et al.*, 1990), this study has used data collected from teachers in the natural context of their classrooms. This suggests that this study also makes contributions to the methodology for researching expert teachers.

9.3.2 Practical contributions of the study

First of all, this study has implications for both pre- and in-service mathematics teacher education programs. According to the findings of this study, an expert mathematics teacher in mainland China should play multiple roles that are emphasized by educators in practice rather than by policy makers. This indicates that these qualities are those which teachers need to possess and enhance to cope with their work. Therefore, to make every teacher perform close to expert teacher level, it is necessary to enhance these abilities at both the pre-service and in-service stages since "teachers' role have become more complex as a result of repeated efforts of reform" (Al-hinai, 2007, p. 154). Moreover, it seems difficult, if not impossible, to successfully improve students' opportunities to learn mathematics "without parallel attention to their *teachers' opportunities for learning*" (Even & Ball, 2009, p. 2, emphasize in original).

However, the fact is that either "teacher education programs are not clearly connected to the educational needs" (Murray, 1996, p. 9), or approaches to teacher education "often do not help them develop the skills and insights needed for practice" (Ball & Even, 2009, p. 255). Part of the reason may be that teacher educators do not know what teachers need in practice. Since there is an increasing research interest in how mathematics teachers should be prepared (*e.g.*, Blömeke & Kaiser, 2012; Schmidt, Blömeke, & Tatto, 2011; Schmidt, Cogan, & Houang, 2011), the findings of this study provide valuable information for the design of teacher education curricula at both the pre-service and in-service stages in China and in other countries and regions. Especially for mathematics teacher educators in mainland China, as the ability to mentor teachers is emphasized as a necessary facet for an expert mathematics teacher, this ability should be enhanced in training programs therefore s/he can more effectively mentor novice teachers. However, as in other cultures, there has been surprisingly little preparation for this (Ball & Even, 2009; Even, 2008; Even *et al.*, 2003).

Secondly, this study's findings provide practising mathematics teachers with information for their ongoing professional development. In particular, common characteristics found in the three expert mathematics teachers' beliefs, knowledge, and teaching practice provide a model of what an expert mathematics teacher looks like. Non-expert mathematics teachers can compare their own behaviours and characteristics with the findings to give direction to their further development. Moreover, the experience of the three teachers' development might also provide practicing teachers with confidence and useful information for their growth.

Thirdly, this study's findings were obtained from an "insider" perspective. Thus, this study serves as a window through which readers outside of mainland China can observe mathematics education, mathematics teaching, and teacher education in mainland China. This study's findings provide readers with more information with which to make comparisons between descriptions reported by outsiders and those of insiders related to mathematics education and teacher education in mainland China. Therefore, they can understand mathematics education and teacher education in mainland China more deeply.

9.4 Limitations and Recommendations for Further Research

Because of resource and time constraints, this study has some limitations, and leaves a range of issues for further studies.

First of all, this study focused on expert mathematics teachers at only the junior secondary school level with the intention of understanding mathematics education at this stage more deeply since it is a part of compulsory education in mainland China. However, as "grade levels present very different contexts" (Fullan & Hargreaves, 1992, p. 6) for teachers' work, the findings and conclusions in the present study should not be generalized to expert mathematics teachers at the primary or senior secondary school levels. There is a need to explore the conception and characteristics of expert mathematics teachers at those two levels to provide more information about what constitutes an expert mathematics teacher at those levels, so that a deep understanding can be developed of expert mathematics teachers in mainland China at each school level.

Secondly, this study was conducted in the specific context of Chongqing, a city located in the Western part of mainland China. China is a very large and diverse country, and small-scale studies such as this are inevitably not representative of the whole situation in mainland China. In particular, due to unbalanced economic development, there exist differences in teaching conditions between various places, and especially between cities and rural areas. Considering that contextual factors exert influence on teaching expertise (Berliner, 2004), it is likely that mathematics educators' conception of expert teachers and expert teachers' practice in other areas, such as well-developed cities or less-developed rural areas, would be different. Therefore, future studies exploring the conception and characteristics of expert mathematics teachers in those areas would help to construct a more complete picture of expert teacher in mainland China.

Thirdly, due to research time constraints and conditions, this study was conducted in a specific social and cultural context, one in which students have excellent performance in mathematics achievements. Comparative studies between different contexts or countries would provide meaningful information to understand similarities and differences in conception of expert mathematics teachers and would provide meaningful information to understand teachers' expected qualities in a specific context and their relationship with its social and cultural context as well. Moreover, similarities and differences found between expert mathematics teachers' teaching practice in different cultural contexts would provide information for understanding how social and cultural factors influence the performance of expert mathematics teachers.

Fourthly, this study only observed the three teachers for a week due to practical constraints. The teaching content in each teacher's lessons was drawn from a single field, such as algebra or geometry. It is possible that their teaching decisions and behaviours would be different when they taught different topics. Therefore, it could provide a deeper, fuller and richer understanding of the characteristics of expert mathematics teachers if a researcher was to observe a teacher for a longer period of time and, in particular, when s/he teaches different topics.

Fifthly, as the limitation of research conditions, this study was conducted by only one researcher. For the data on classroom observation, this eliminated the problem of inter-observer inconsistency and enhanced internal reliability. However, it also meant that the researcher's biases were difficult to detect and, therefore, the external reliability was reduced, even though the researcher asked relevant interviewees to check their interview transcripts and a postgraduate student majoring in mathematics education to check the meaning of the relevant codes. Moreover, due the researcher's lack of personal teaching experience, findings might not have been interpreted deeply. In the meantime, since the researcher comes from mainland China, what might be seen as interesting findings to outsiders might have been taken for granted by the researcher. Therefore, since many factors need to be taken into consideration when analyzing and interpreting teachers' behavior, future studies should be conducted by a research team rather than a sole researcher.

Lastly, due to the main focus of the present study is to explore the characteristics of expert mathematics teachers' teaching, no detailed information related to the subject teachers' teaching history was collected, even though this study conducted a brief life history interview to collect basic information about their professional development. This is far from sufficient to explore and explain how the three expert teachers develop their teaching expertise. Especially as, in this study, it was found that at the very beginning of the three expert mathematics teachers' teaching history, they all worked in secondary schools with poor conditions, which would seem to be unfavourable for the development of teaching expertise according to Western researchers (Berliner, 2001). Therefore, future studies examining expert mathematics teachers' professional development history in the context of mainland China or in other contexts could provide more information on how teaching expertise is developed.

9.5 Concluding Remark

This study is an exploratory study of the conception and characteristics of expert mathematics teachers in mainland China from a sociocultural perspective. As pointed out above, it has several limitations. Conclusions made herein are based on the current teaching situation in mainland China, information drawn from limited number of sampled cases, and the researcher's interpretation of the findings. For these reasons, care should be taken when drawing inferences from the results of this study. In spite of its limitations, this study has expanded the prototype theory of teaching expertise to the Chinese context and a specific subject. Findings from this study illustrate that the conception and characteristics of expert mathematics teachers are not only influenced by factors at the classroom, school and societal levels, but also by factors at the cultural level. This indicates that the conception of expert mathematics teachers is culturally bounded, and that the characteristics of expert mathematics teachers are social and culturally dependent as well, even though the three expert mathematics teachers sometimes demonstrated the ability to work against their social and cultural constraints to a certain degree.

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Appendices

Appendix 1:

Detailed information of the teachers and principals in the first category

Teacher	Working School	Working Experience	Gender
1	Key secondary school	3 years of teaching experience in a general secondary school, 4 years of teaching experience this key secondary school	Female
2	Key secondary school	12 years of teaching experience in a general secondary school, 2 of years teaching experience this key secondary school	Female
3	Key secondary school	21 years of teaching experience in several general secondary schools, 6 years of teaching experience in this key secondary school	Male
4	Key secondary school	16 years of teaching experience in several general secondary schools; 7 years of teaching experience in this key secondary school	Male
5	General secondary school	17 years of teaching experience in several general secondary school	Female
6	Key secondary school	10 years of teaching experience in several general secondary schools, 4 years teaching experience in this key secondary school	Male
7	Key secondary school	5 years of teaching experience in this key secondary school	Male
8	Key secondary school	10 years of teaching experience in this key secondary school	Female

X. Yang, *Conception and Characteristics of Expert Mathematics Teachers in China,*
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9	General secondary school	9 years teaching experience in this general secondary school	Female
10	General secondary school	12 years of teaching experience in several general secondary school	Female
11	Key secondary school	5 years of teaching experience in this key secondary school	Male

(vice) Principal	Working School	Working Experience	Gender
1	General secondary school	18 years of teaching experience 5 years of experience as principal	Male
2	General secondary school	21 years of teaching experience 10 years of experience as vice principal	Male
3	Key secondary school	29 years of teaching experience 10 years of experience as vice principal	Male
4	Key secondary school	19 years of teaching experience 1 years of experience as vice principal	Female
5	Key secondary school	25 years of teaching experience 3 years of experience as vice principal	Male
6	General secondary school	18 years of teaching experience 7 years of experience as principal	Male

Appendix 2

Interview outline for the conception of expert mathematics teacher

访谈提纲

尊敬的老师：

您好！

首先非常感谢您能接受我的访谈！下面是我访谈将要问到的几个主要问题，请您先考虑一下！

"专家型教师"是近年来心理学和教育学研究领域中的一个热点问题。而每位教师对什么是"专家型教师"都有自己独特的理解。本研究重点想探讨在初中阶段什么样的数学教师是"专家型数学教师"。在现阶段的研究中，我非常希望能得到您的帮助，对什么是"专家型数学教师"如实表达您的看法。您的意见非常宝贵，它将成为本研究及相关后继研究的基础。我保证您所提供的意见及个人资料仅供学术研究之用。

再次感谢您的支持与配合！

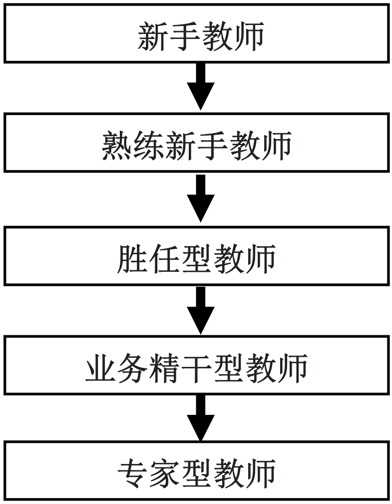
此致

敬礼

杨新荣

2008 年 4 月

附：
下图是伯林纳（1986）研究得出的一个关于教师成长的模型，根据该模型，请您思考下列问题：



1） 您认为大概多大比率的初中数学教师能称为专家型数学教师? 您为什么这样认为?

2) 在您的学习或者教学生涯中，您是否遇到过可以称之为“专家型数学教师”的初中数学教师？您为什么认为他是专家型数学教师？相比您遇到的其他数学教师，他/她有何特别或者突出之处？ 您能对他/她的个人特点做出详细的描述吗？

3) 假如您认为您没有碰到过可以称之为“专家型数学教师”的初中数学教师，您认为什么样的老师才能称之为“专家型数学教师”？

4) 您觉得“专家型数学教师”跟其他阶段的数学教师有哪些相似和相异之处？
5) 您觉得怎样才能成为一名专家型数学教师？

4） 假如现在要您给“专家型数学教师”下个定义，您会怎样定义？

Interview Outline

Dear All:

I am most grateful that you can participate in the present study of "what is an expert mathematics teacher at junior high school level"! The questions followed are the main questions what I would like to ask! Please think about them at your convenience!

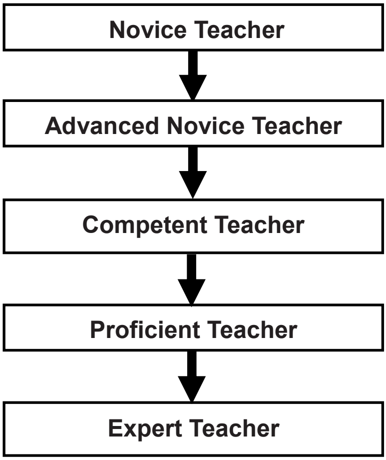
"Expert teacher" is a hot research topic in the psychological and educational research field. It is believed that every teacher will has her/his own understanding of what an expert teacher is. The main aim of the present study is to explore what kind of teacher can be defined as "expert mathematics teacher" at junior high school level. In the present phase of my study, you are invited to express your opinions about what expert mathematics teacher is. Your opinions are precious, as they will constitute the foundation of the present study and the studies follows. I promise that your opinions and personal information will only be used for the purpose of academic research.

Thank you again for you kind support and corporation!

Best Regards,

Yang Xinrong

The following model is developed by Berliner (1988) on the development of teaching expertise. Please consider the following questions according to this model.



1) How much is the percentage of “expert mathematics teacher” in junior high school in Chongqing? Why do you think so?

2) In your learning or working life, have you ever encountered a teacher that you think could be called as an expert mathematics teacher in junior high school? Why do you think her/himas an expert mathematics teacher? In comparison with the other mathematics teachers, does s/he possess any unique characteristics? Would you please describe her/himin details?

3）If you have not encountered such a teacher, what kind of teacher do you think can be called as “expert mathematics teacher”?

5） What are the differences and similarities between the “expert mathematics teacher” and the teachers at other development stages?

6） Now, if you were asked to give a definition to “expert mathematics teacher”, what kind of definition will you give?

Appendix 3

Interview schedule for the conception of expert mathematics teacher

A: 背景信息 (Background Information)

1) 姓名（Name, optional)

2) 职务（profession）

3) 教育及工作经历(Education experience and working experience)

B: Questions:

1) 您认为大概多大比率的初中数学教师能称为专家型数学教师?

(How much is the percentage of “expert mathematics teacher” in junior high school in Chongqing?)

2) 您为什么这样认为？(Why do you think so?)

3) 在您的学习或者教学生涯中，您是否遇到过可以称之为“专家型数学教师”的初中数学教师？(In your learning or working life, have you ever met a teacher that you think could be called as an expert mathematics teacher in junior high school?)

4) 您为什么认为他/她是专家型数学教师？(Why do you think s/he is an expert mathematics teacher?)

5) 相比您遇到的其他数学教师，他/她有何特别或者突出之处？(In comparison with the other mathematics teachers, does s/he possess any unique characteristics?)

6) 您能对他/她做出详细的描述吗？ (Could you please describe her/himin details?)

7) 假如您没有碰到过可以称之为“专家型数学教师”的初中数学教师，您认为什么样的老师才能称之为“专家型数学教师”？

(If you have not encountered such a teacher, what kind of teacher do you think can be called as “expert mathematics teacher”?)

8) 根据以前给您的模型，您能谈谈“专家型数学教师”跟其他阶段的数学教师的相似和相异之处？

(What are the differences and similarities between the “expert mathematics teacher” and the teachers at other development stages according to Berliner’s (1988) model?)

9) 假如现在要您给“专家型数学教师”下个定义，您会怎样定义？

(Now, if you were asked to give a definition to “expert mathematics teacher”, what kind of definition will you give?)

10) 您能给我推荐几个您认为的专家型数学教师吗？

(Could you please recommend several expert mathematics teachers to me?)

Appendix 4

Interview schedule for three expert mathematics teachers (Teaching experience)

A. 背景信息(Background information)

1. 观察班级的背景信息 (background information of the observed class)
2. 教学任务 (working load)

B. 个人经历访谈 (Personal experience)

1. 您能谈谈您的（数学）学习经历吗? (Could you please talk about your learning experience? Mathematics learning experience?)
2. 您为什么选择教师作为职业？ (Could you tell me why you choose teaching as a profession?)
3. 您能谈谈您在师范培训时的经历吗？有什么对您工作影响较大的吗？(Could you please talk about your training experience at pre-service stage? Any special events influence your teaching?)
4. 您能谈谈您的教学工作经历吗？（Could you tell me your teaching history?）
5. 您曾经或者现在仍在为成为一名好教师做过一些什么努力？(Could you tell me what particular things you did or are still doing or will do to be a good teacher?)

Appendix 5

Interview schedule for three expert mathematics teachers (Beliefs)

Beliefs about Mathematics

1、总的来说，您认为数学是什么？您为什么这样认为？

(What do you think mathematics is? Why you think so?)

2、您一般做些什么样的数学？(Normally, what kind of mathematics do you do)

3、您的学生一般做些什么样的数学？（课堂内/课堂外）(What kind of mathematics your students do? Inside/outside classroom)

Beliefs about Mathematics Learning

4、您觉得所有学生都能学好数学吗？为什么是/不是？

(Do you think all students are able to learn mathematics well? Why yes or not?)

5、对于学生数学学习困难上的差异，您怎样解释？(How you will explain students' difficulty in learning mathematics?)

6、您认为学生最佳学习数学的方式是什么?

(What are the best ways to learn mathematics for students?)

7、您自己最佳学习数学的方式是什么？(What's your best ways to learn mathematics?)

8、在初中阶段，学生数学学习最重要的部分是什么？您期望您的学生在数学上达到什么水平？为什么？(At junior secondary school level, what are the most important parts for students' mathematics learning? What kind of level you expect your students to achieve in mathematics? Why you think so?)

9、当某个学生在学习某个概念、解决某个问题遇到困难时，您会怎么做？

(If one student has some difficulties in learning a certain conceptualization or solving a problem, what you will do?)

Beliefs about Mathematics Teaching

10、您认为初中阶段，数学教学的主要目标是什么？为什么？（What are the objectives of the mathematics teaching at junior secondary school level? Why?）

11、您怎么知道您已经上了一堂成功的数学课？

(How do you know you teach a successful mathematics lesson?)

12、能否请您描述一堂您认为最成功的数学课？

(Could you please describe a successful lesson you once taught?)

13、您认为数学教学最有效的方式是什么？（这个是否与其他学科有所区别？）

(What are the most effective ways to teach mathematics? Are these ways different from those ways to teach other subjects?)

Appendix 6

Pre-observation interview schedule for three expert mathematics teachers

录像前访谈提纲

1）您能告诉我这节课您打算教什么？

(Could you describe what are you going to teach in the lesson?)

2) 这节课的教学目标是什么？您是怎样决定这些教学目标的？

(What are the objectives of this lesson? How do you determine them?)

3) 学习这些知识点前，学生需要掌握那些知识？

(Could you tell me what prior knowledge students need to understand for learning this topic?)

4) 您能告诉我这节课的重点是什么吗？您是怎样知道这些重点的？

(Could you tell me what are the most important parts of this lesson? How do you know these parts are the most important?)

5）您打算用哪些教学策略使学生更好地掌握或者理解这些重点？您为什么会认为这些方法是有效的？ 您认为还有其它有效的方法吗？您能告诉我您在何时、怎样知道这些方法的？

(What kind of teaching strategies will you use to facilitate students' understanding or enhance students' understanding of those important parts? Why do you think these methods are effective? Are there any other methods you also think are effective? Could you tell me when and how you know these ways?)

6）您能告诉我这节课的难点是什么吗？您怎样知道这些难点的？

(Could you tell me what are the most difficult parts of this lesson? How do you know these parts are the most difficult?)

7) 您打算用哪些方法使学生更好的理解或者掌握这些难点？您为什么认为这些方法有效？还有其它的有效的方法吗（那您为什么不用这些方法）？您能告诉您在什么时候，怎样得知这些方法的吗？

(What kind of teaching strategies will you use to make students easily understand these difficult parts? Why do you think these methods are effective? Are there any other better methods? Could you tell me when you know these methods? How?)

8）您有教案吗？您通常都会写教案吗？您通常大概花多长时间写教案？您写教案时，会参考其它资料吗？通常是哪些资料？您为什么要参考这些资料？

(Do you have a lesson plan for this lesson? Do you usually write lesson plan? How much time do you spend on this lesson plan? Do you refer to any other materials when you plan your lesson? What kind of materials? Why do you refer to these materials?)

Appendix 7

Post-observation interview schedule for three expert mathematics teachers

录像后访谈提纲 （基本问题包含, general questions including）

1) 您觉得您这节课上得怎样？有哪些地方您非常满意的？有哪些地方您觉得还要改进的？您为什么会这样认为？您平常上完课，也都会反思吗？一般反思些什么？

(How do you feel about this lesson? What parts you feel very successful? What parts you think that you should make modifications? Why you think so? Do you usually reflect on your teaching? Could you please tell me your foci?)

2) 对于这节课的教学内容，您觉得哪些地方大部分学生都已经掌握或者掌握得比较好了？哪些地方学生还没有很好地掌握？您为什么会这样认?

(For the content in this lesson, could you please tell me what parts that most students have already understood? Which parts most students might still have difficulties in? Why you think so?)

3)下次再教这个内容，您会有相应地改动吗？您为什么会有这些改动？您为什么认为这些改动是有效的？您在这节课为什么没这么处理？

（If you will teach this content to a similar class again, what kind of changes you will make to your plan? Why you will make these changes? Why you think these changes will make your teaching more effectively? Why you did not use these in this lesson?）

4) 您认为本节课中，哪些是学生容易混淆或出错的？您是怎样知道这些的？您还记得您用了哪些方法来解决这些混淆或者出错的地方吗？

(Could you tell me the errors and confusions that the students tend to have when learning these topics? How do you know these? Do you remember the methods you used to deal with students’ errors or confusions?

5) 我注意到您上课时没有/有按照教材顺序？您为什么要/不 进行这些变动？

(I noticed you did/ did not follow the textbook content? Why you did not / did you make such changes?)

6) 我注意到您只选用了教材上的问题，您为什么不选用一些外面的问题了？

（我注意到您选用了一些非教材上的问题，您为什么不用教材上的问题了？）

(I noticed that you only employed these exercises in the textbook, why you did not choose exercises from other materials? Or I noticed that you chose some exercises from other materials, why you chose to do so?)

7) 我注意到您用了这些方法来处理这个知识点，您为什么要这样处理这些知识点了？您认为还有其它的方法吗？您在何时，怎样知道这些方法的？

(I noticed that you teach this topic in this way, could you tell me why you deal with this topic in this way other than some others? When and where and how did you get to know this way?

8) 您能讲下今天这堂课的内容与它前后内容之间的关系吗？（在初中阶段？在整个中学阶段？）

(Could you the connections between this topic and the topics before and after it in this chapter? at this grade level? at junior secondary school level? at secondary school level?)