Oil Shock Transmission to Stock Market Returns: Wavelet-Multivariate Markov Switching GARCH Approach

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Abstract Our understanding of the nature of crude oil price shocks and their effects on the stock market returns has evolved noticeably in recent years. Evidence of spillover effects between several kinds of markets has been widely discussed around the globe, and yet the transmission of shocks between crude oil market and stock market returns has received little attention. Extending earlier work in the literature, we use data on monthly crude oil returns and stock market returns of five developed countries (USA, UK, Japan, Germany and Canada) to investigate two issues that have been at the centre of recent debates on the effect of crude oil shocks on the stock market returns. First, we analyse whether shocks and or volatility emanating from two major crude oil markets are transmitted to the equity markets. We do this by decomposing monthly real crude oil prices and analysing the effect of the smooth part on the degree of the stock market instability. The motivation behind the use of this method is that noises can affect the quality of the shock and thus increase erroneous results of the shock transmission to the stock market. Second, under the hypothesis of common increased volatility, we investigate whether these states happen around the identified international crises. In doing so, flexible model is implemented involving the dynamic properties of the Trivariate Markov switching GARCH model and the recent Harr A trous wavelet decomposition, in order to achieve a strong prediction of the abovementioned situations The proposed model is able to circumvent the path dependency problem that can affect the prediction's robustness and also provides useful information for investors and government agencies that have largely based their views on the notion that crude oil markets negatively affect stock market returns. Indeed, the results show that the A Haar Trous Wavelet decomposition method appears to be an important step toward improving accuracy of the smooth signal in detecting key real crude oil volatility features. Additionally, apart from UK and Japanese cases, the responses of the stock market to an oil shock depend on the geographic area for the main source of supply whether it is from the North Sea or from North America (as two oil benchmarks are used, WTI and Brent respectively).

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1 Introduction

The stock market movements as contained in the stock price (among other economic indicators) send us some obvious "signals" of a country's economic strength and development. For instance, a bull stock market, i.e. a market which goes up and maintains upward trends, is associated with increasing business investment and vice versa.

However, the majority of Organisation for Economic Co-operation and Development (OECD) countries have become increasingly dependent upon oil over the last century and this is now recognised as the most essential energy source. In 2008, the US was the largest consumer of oil, consuming around 20 million barrels per day, followed by China (7.8) and Japan (4.8) (EIA 2008). The 2007–2008 period marked the fastest price changes in the history of oil. In fact, oil prices rose dramatically to more than 140 dollars per barrel in August 2008 (the record peak), and then sharply dropped to around 30 dollars per barrel in December 2008.[1] This (and also other sequences of very large increases and decreases observed in crude oil prices over the last three decades) will obviously affect companies' earnings very significantly as oil operating costs lead to a remarkable change in stock prices.

Despite the considerable attention that has been paid to the investigation of the relationship between changes in the price of crude oil and stock prices, conclusions on these effects cannot yet be drawn. More than 20 years ago, Jones and Kaul (1996) observed that stock market returns of USA, Canada and Japan respond negatively to oil shocks. However, Huang et al. (1996) found no evidence of the relationship between US stock returns and changes in the price of oil futures. Wei (2003) argued that the decline in stock prices after the 1973/74 oil crisis seems too large to be explained by the rise in oil prices. Chen et al. (1986) in contrast, concluded that oil price changes have no impact on asset pricing. Using structural VAR, Kilian and Park (2009) demonstrate that it is useful to differentiate between three distinct sources of oil shocks in the global market for crude oil before assessing the impact of an oil price shock on aggregate US real stock returns. In particular, they report that only an oil price increase driven by a precautionary demand for oil associated with concerns about future oil supply shortfalls, namely "precautionary demand shocks", negatively affects stock prices. In contrast, shocks to the production of crude oil "oil supply shocks" have no significant impact on the US stock returns. Finally, shocks driven by strong global demand for industrial commodities including crude oil, "aggregate demand shocks", have persistent positive effects on cumulative stock returns within the first year of the expansionary shock.

[1] Source: Wikipedia, the free encyclopedia; http://en.wikipedia.org/wiki/Price_of_petroleum.

However, the impact of oil prices on other macroeconomic variables such as inflation, real Gross Domestic Product (GDP) growth rate, unemployment rate and exchange rates, is a matter of great concern for all economies. Hamilton (1983) documents that oil price increases have often been followed by economic recessions in the US since the Second World War. However, Hooker (1996) did not confirm Hamilton's results and argued that the negative relationship between oil prices and output no longer exists when the sample is extended to the 1990s. Lee et al. (1995), Ferderer (1996) and Hamilton (1996) demonstrate for sample periods that include recent years that nonlinear transformations of oil price changes restore that relationship. More recently, several studies have highlighted that economic activity is significantly affected by oil price changes (Kilian (2008) and Cologni and Manera (2008)) among others). Blanchard and Gali (2009) also found that oil price shocks have exhibited a decreased impact on GDP since 1990 for the US and other developed countries. This result can thus be explained by the fact that *"US has become less volatile and more insolent from external shocks, better economic policy, lack of large adverse shocks, or a smaller degree of energy dependence (i.e. more efficient use of energy resources and a larger share of the services sector in the economy)"* (Wu and Cavallo 2009, p. 3).

A number of studies have given special attention to the Multivariate Generalized AutoRegressive Conditional Heteroskedasticity models (*M-GARCH*) as they provide a better understanding of both volatility and co-volatility dynamics for multiple series than the nested univariate model, namely GARCH of Bollerslev (1986). The specifications include the Baba et al. 1987 (BEKK) (Engle and Kroner 1995), constant correlation model (CCC) (Bollerslev (1990), dynamic conditional correlation model (DCC) (Engle 2002) …etc.[2] The *M-GARCH* with the parameterisation BEKK (*BEKK M-GARCH*) model introduced by Engle and Kroner (1995) appears to be an appropriate methodology to reveal much more crucial information on the interaction among a given set of financial time series. Examples of recent studies on this subject include; Agren (2006) who use weekly data on the aggregate stock markets of Japan, Norway, Sweden, the UK and the US to investigate volatility spillovers from oil prices to stock markets within an asymmetric *BEKK* model. He found strong evidence of volatility spillovers for all stock markets with the exception of Sweden where evidence was weak. On the other hand, Aloui et al. (2008) find that changes in crude oil prices have a significant effect on the volatility of the stock market return of six developed countries, namely; US, UK, France, Japan, Germany and Canada using univariate (cross correlation functions) and *BEKK M-GARCH*) approaches.

Several authors have discussed in detail the inadequacy of linear models for capturing asymmetries. Therefore, regime switching models arose as an alternative to standard GARCH models allowing the behaviour of dynamic variables to depend on the state that takes place at any given point in time. The main advantage of the Markov Switching processes, often advocated in the literature, is that they can handle many

[2] For an extensive survey, see Bauwens et al. (2003).

crucial features of time series such as nonlinear phenomena, temporal asymmetries as well as persistence of the macroeconomic times series (Diebold 1986; Hamilton and Susmel 1994; Lamoureux and Lastrapes 1990). Univariate regime switching models were first proposed by Hamilton (1989, 1990) to examine the relation between turning points and changes in regimes. Markov Switching models are utilised to investigate the heteroskedastic behaviour of asset returns (Schwert 1989), the effects of oil prices on US GDP growth (Raymond and Rich 1997)…*inter alia*. Aloui and Jammazi (2009) have used univariate Markov switching *EGARCH* model with constant or time varying transition probabilities to analyse the response of the stock market returns to the oil shocks in UK, France and Japan.

Most studies to date have assumed that shock spillover intensity does not vary over time. To overcome this problem, some authors extend the standard methodology by allowing for regime switches in the volatility and spillover parameters (Beale 2002). Assuming state-dependent conditional correlations, several different Multivariate versions of Markov Switching GARCH models (*M-MSG*) have also been developed. *M-MSG* models are nested within constant conditional correlation (*CCC-GARCH*), time-varying conditional correlation (*DCC-GARCH*) of Engle and Sheppard (2001) and *BEKK-GARCH* of Gray (1996). In order to solve the path dependency problem of the Markov Switching GARCH model, i.e. the conditional variance and conditional covariance will depend on all past information, Gray (1996) suggests a tractable formulation for the conditional variance process by using the conditional expectation of the variance without giving up GARCH terms (the latter was elaborated by Hamilton and Susmel (1994) and Cai (1994) as a first solution to the path dependency problem). Haas et al. (2004), among others, modify Gray's approach to circumvent the path dependency problem. Gray's (1996) bivariate *BEKK MSG* models is perhaps the most applied model in a wide variety of applications such as estimating time-varying optimal hedge ratios (Alizadeh et al. 2008, or Lee and Yoder 2007), understanding the source and the intensity of shock spillover between stock market returns (Beale 2002). Based on Gray's approach, we propose a tractable model, namely the trivariate *BEKK MSG* model, which is more suitable for modeling the relationship between real crude oil price volatility and international real stock market returns.

In addition, using this kind of models represents another major contribution to the literature on the crude oil—stock market relationship. In fact, one limitation of existing work on the analysis of this relationship is that the price of crude oil is often treated as exogenous. However, Kilian (2008) suggests that models relying on exogenous oil price variables have been misleading in recent years. Further, Kilian (2008a, b) argue that "*direct measures of exogenous shocks to the production of crude oil have low explanatory power for the real price of crude oil*" (Kilian 2009, p. 19). Therefore, based on Kilian' arguments, our new class of model again proves to be helpful to understanding the relationship between real crude oil prices and stock market returns.

In particular, this paper analyses the shock and volatility transmission from the crude oil market to the stock market returns of US, UK, Germany, Japan and Canada under the trivariate *BEKK MSG* approach with two common states in the period January 1989 to December 2007. We combine the former with the wavelet decomposition approach, especially the Â Haar Trous Wavelet approach (*Â HTW*) in order to glean a better understanding of crude oil transmission.

Undoubtedly, *GARCH* models worked well to capture the leptokurtosis and volatility clustering generally observed in financial time series but they demonstrate some inaccuracies in terms of changes of time scales (Yalamova 2006). One major advantage afforded by wavelets analysis is its ability to perform local analysis—that is, to analyse a localised sub image area of a larger image (or signal). Therefore, wavelet analysis is capable of revealing aspects of data that other signal analysis techniques (like *GARCH* models) usually miss; aspects like trends, sharp spikes, discontinuities in higher derivatives, self-similarity…etc. Similarly, wavelet analysis can often compress or de-noise a signal without appreciable degradation (Misiti et al. 2008) because it affords a different view of data from that presented by traditional techniques. In their brief history within the signal processing field, wavelets have already proven a very useful tool for data de-noising and deconvolution (separation between two convolved signals namely smooth and detail). In this paper, we restrict our attention to "the *Â HTW* transform", introduced by Murtagh et al. (2004) and designed as well suited for outlier detection in order to decompose the real crude oil returns into six scales and a smooth part. We therefore extract the smooth series in light of the empirical evidence suggesting that the latter contains less noise than the original signal, allowing for more accurate detecting dynamic regime shifts, see Jammazi and Aloui (2009).

In summary, this paper introduces a novel insight for characterizing the relationship between crude oil market and real stock market returns. Firstly, using 6 levels *Â HTW* decomposition, we extract the main information from the real crude oil signal which is designed by the smooth low frequency part of the original series. Secondly, we examine the transmission mechanisms between the desired variables under a trivariate *BEKK MSG* model with common two states that are characterised as low mean high variance regime and high mean low variance regime. Specifically, we allow volatility in the different equity markets to depend purely on shocks and/or volatilities originated from crude oil market.

The rest of the paper is organised as follows: Sect. 2 presents the two econometric methodologies, namely *Â HTW* decomposition method and the trivariate *BEKK MSG* model. Section 3 presents the data and discusses how the smooth fluctuations of the real crude price of oil might be transmitted to the real stock market returns and Sect. 4 concludes.

2 Econometric Methodology

In this section we give a detailed description of the wavelet transform used for the crude oil data decomposition together with the multivariate *BEKK MSG* applied in our analysis.

2.1 *Signal Decomposition Using the Wavelet Method: Â Haar Trous Wavelet (Â* HTW*)*

The *Â HTW* approach was performed according to Murtagh et al. (2004). Below, we briefly recall the basic notions of the discrete wavelet theory; we present the main characteristics of the "â trous" algorithm as an alternative to the Discrete Wavelet Transform *DWT* and finally we discuss the properties of the "Â Haar Trous" wavelet decomposition approach.

2.1.1 Discrete Wavelet Transform

Contrary to the trigonometric functions, wavelets are defined in a finite domain and unlike the Fourier transform they are well-localised with respect to both time and scale. This behaviour ultimately makes them useful to analyse non-stationary signals. The other most important property of the wavelet method is that it can be used to recreate a series without loss of information. Indeed, the wavelet transform techniques split up a signal into a large timescale approximation (coarse approximation) and a collection of "details" at different smaller timescales (finer details). The coarse image preserves the large-scale structure and the mean of the image, whereas the "detail" or wavelet levels complement the coarse level and thus preserve the total image information. The first step of the wavelet de-noising method is the application of filters.

The dilation and the translation of the basis functions at different resolution levels are described by the scaling function φ, the so-called *father wavelet*, (Strang 1989) given by:

$$\phi_{j,k}(t) = 2^{-j/2}\phi\left(2^{-j}t - k\right) \quad \text{or} \quad \varphi(x) = \sum_k h_k \times \varphi(2x - k) \tag{1}$$

h_k denotes the low-pass filter coefficients. The low pass filter is a filter that allows only low frequency signals through its output, so it can be used to reduce the amplitude of signals with high frequencies.

Detail levels are generated from the single basic wavelet ψ, the so-called *mother wavelet*:

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(2^{-j}t - k\right) \quad \text{or} \quad \psi(x) = \sum_k g_k \times \varphi(2x - k) \tag{2}$$

where $j = 1 + \cdots + J$ in a J-level decomposition. g_k is called the high-pass (or a band-pass) filter coefficients closely related to the low-pass filter (h_k) mentioned above. The high pass filter does just the opposite, by allowing only frequency components below some threshold.

The father wavelets are used to capture the smooth, low frequency nature of the data, whereas the mother wavelets are used to capture the detailed and high frequency nature of the data. The father wavelet integrates to one, and the mother wavelet integrates to zero (Heil and Walnut 1989). Thus, an original signal *f(t)* in $L^2(R)$ may be expanded approximately using these two basic wavelet functions (φ and ψ):

$$f(t) \approx \sum_j \sum_k \alpha_{j,k} \phi_{j,k}(t) \approx \sum_k s_{J,k} \phi_{J,k}(t) + \sum_k d_{J,k} \phi_{J,k}(t) + \cdots + \sum_k d_{1,k} \phi_{1,k}(t)$$
$$\approx \sum_k s_{J,k} \phi_{J,k}(t) + \sum_j \sum_k d_{j,k} \psi_{j,k}(t) \tag{3}$$

where $s_{J,k} = \langle f(t), \phi_{j,k}(t) \rangle$ and $d_{j,k} = \langle f(t), \psi_{j,k}(t) \rangle$ are the wavelet coefficients. The coefficients $s_{J,k}$ and $d_{j,k}$ are the smooth and the detail component coefficients respectively and are given by the projections:

$$s_{J,k} = \int \phi_{J,k} f(t) dt \tag{4}$$

$$d_{J,k} = \int \psi_{J,k} f(t) dt \tag{5}$$

2.1.2 Â Trous Wavelet Transform

A potential drawback of the application of the *DWT* in time-series analysis is that it suffers from a lack of translation invariance. To overcome this problem, some authors (Coifman and Donoho 1995 among others) suggest applying a redundant or non-decimated wavelet transform.[3]

According to Zhang et al. (2001), the advantage of the redundant wavelet transform, i.e. the so-called Trous (with holes) algorithm, lies in the fact that it is shift invariant and it produces smoother approximations by filling the "gap" caused by decimation, i.e., it is non-decimated (it conserves the original dimensions of the series). A redundant algorithm is based on the so-called *autocorrelation shell representation* using dilations and translations of the autocorrelation functions of compactly supported wavelets.[4]

The scaling and the wavelet functions are chosen to satisfy the following equations respectively:

[3] A detailed description of the properties of the Â Trous and the Mallat algorithm is given in Mallat (1989) and Shensa (1992).

[4] For more details, see Saito and Beylkin (1992).

$$\frac{1}{2} \times \phi\left(\frac{x}{2}\right) = \sum_k h(k)\phi(x-k) \tag{6}$$

$$\frac{1}{2} \times \psi\left(\frac{x}{2}\right) = \sum_k g(k)\psi(x-k) \tag{7}$$

where h is a discrete scaling low-pass filter while g is a discrete high-pass filter associated with the wavelet function.

These two functions satisfy the following equation:

$$\frac{1}{2} \times \psi\left(\frac{x}{2}\right) = \phi(x) - \frac{1}{2}\phi\left(\frac{x}{2}\right) \tag{8}$$

Using the filters h and g, we obtain the pyramid algorithm for expanding into the autocorrelation shell. The smoothed and the detailed signals at a given resolution j and at a position t are obtained by these convolutions:

$$s_j(t) = \sum_{l=-\infty}^{+\infty} h(l)s_{j-1}(t + 2^{j-1} \times l) \tag{9}$$

$$d_j(t) = \sum_{l=-\infty}^{+\infty} g(l)s_{j-1}(t+2^{j-1} \times l) \tag{10}$$

where $1 < j < J$, h is a low-pass filter.

A very important property of the autocorrelation shell coefficients is that signals can be directly derived from them Zhang et al. (2001). In each step, the series is convolved with a cubic *B-spline* filter, h, with $2^{j-1} \times l$ zeros inserted between the *B-spline* filter coefficients at level j, hence the name "with holes". The convolution mask in one dimension is 1/16 [1, 4, 6, 4, 1]. Thus, we get a series of smoothed versions s_j with s_0 ($s_0(t) = x(t)$ the finest scale) as the normalized raw series. Given a smoothed signal at two consecutive resolution levels, the detailed signal $d(t)$ at level j, can be derived as:

$$d_j(t) = s_{j-1}(t) - s_j(t) \tag{11}$$

The set $d = \{d_1(t), d_2(t), \ldots d_J(t), s_J(t)\}$ represents the wavelet transform of the signal up to the scale J, and the signal can be expressed as a sum of the wavelet coefficients and the scaling coefficient:

$$x(t) = s_J(t) + \sum_{j=1}^{J} d_j(t) \tag{12}$$

2.1.3 The Â Haar Trous Wavelet Transform (*Â HTW*)

Here, we select Haar wavelet filter to implement the Â Trous wavelet transform. The asymmetry of the wavelet function used makes it a good choice for edge detection, i.e., localised jumps. However, the usual Haar wavelet transform is decimated. Consequently, Murtagh et al. (2004) develop a non-decimated or redundant version of this transform. The non-decimated or redundant algorithm is the Â Trous algorithm with a low-pass filter h = (1/2, 1/2).

The non-decimated Haar algorithm is exactly the same as the Â trous algorithm, except that the low-pass filter *h*, (1/16…etc.*)*, is replaced by the simple non-symmetric filter *h* = (1/2, 1/2). By convolving the original signal with the wavelet filter *h*, we create the wavelet coefficients.

$$s_{j+1} = \frac{1}{2}\left(s_{j,t-2^j} + s_{j,t}\right) \tag{13}$$

Thus, the scaling coefficients at a higher scale can be easily obtained from the scaling coefficients at a lower scale:

$$d_{j+1}(t) = c_j(t) - c_{j+1}(t) \tag{14}$$

2.2 *Wavelet-Multivariate Markov Switching GARCH-BEKK Model*

Several studies on the transmission volatility between different financial variables are based on the estimation of multivariate *BEKK GARCH* models (Saleem 2009; Li and Majerowska 2008; Bachmeier 2008; Malik and Hammoudeh 2007; Agren 2006 among others).

Although these models are parsimonious, they were based on constant shock and volatility transmissions. Multivariate Regime Switching models, which are both time varying and state dependent, are used henceforth to solve this problem. The main advantage of Markov-switching processes, often advocated in the literature, is their ability to take into account features such as nonlinear phenomena, temporal asymmetries as well as persistence of the macroeconomic time series: these features are crucial in the analysis of the dynamic linkage between crude oil prices and stock market returns (Aloui and Jammazi 2009). Hamilton and Susmel (1994) and Cai (1994) were the first to allow for regime-switches in the ARCH process. Gray (1996) extended their methodology to regime switching GARCH-models. In this section, we extend the standard multivariate BEKK-GARCH model of Engle and Kroner (1995) to allow for the presence of regime shifts. Finally, we discuss the trivariate wavelet *BEKK MSG* that we will use in the current analysis in order to study the transmission mechanism of shocks (volatility) originating from crude oil market to equity market returns.

2.2.1 Generalised Regime Switching GARCH Model with Path Dependent Volatility

Following Haas and Mittnik (2008), in this section we derive the multivariate *BEKK MSG* process.

Let us suppose that the joint process for a given number of series is governed by the following set of equations:

$$\begin{aligned} R_t &= \Phi + E_t \\ e_{t,s_t} &= H^{1/2}_{\Delta_t,t} E_t \qquad E_t/\Omega_{t-1} \rightarrow N(0_{M\times 1}, I_{\mathrm{M}}) \end{aligned} \tag{15}$$

Both the return R and the variance H are made regime dependent. Let R_t be the return matrix at time t, modeled as a constant plus a disturbance term. Φ constitutes the constant vector, I_M denotes the identity matrix of dimension M, The transition between the successive states is governed by a first order Markov process $\{\Delta_t\}$ with finite state space $S = \{1, 2, \ldots, k\}$ and a primitive (i.e., irreducible and aperiodic) fixed $k \times k$ transition probability matrix P,

$$P = \begin{bmatrix} p_{11} & \cdots & p_{k1} \\ \cdots & \cdots & \cdots \\ p_{1k} & \cdots & pkk \end{bmatrix} \tag{16}$$

where the transition probabilities are given by

$$p_{ij} = p(\Delta_t = j/\Delta_{t-1} = i), \quad i,j = 1, \ldots, k$$

The regime-dependent covariance matrix H is assumed to follow a *Multivariate Markov Switching GARCH (p, q, k))* in Vech form as introduced by Bollerslev et al. (1988);

$$h_{jt} = \gamma_{0j} + \sum_{i=1}^{q} \alpha_{ij}\eta_{t-i} + \sum_{i=1}^{p} \beta_{ij} h_{jt-i} j = 1, \ldots, k \tag{17}$$

where $\alpha_i = [\alpha'_{i1}, \ldots, \alpha'_{ik}]'$, $i = 1, \ldots, q$ and $\beta_i = [\beta'_{i1}, \ldots, \beta'_{ik}]'$, $i = 1, \ldots, p$ are parameter matrices of appropriate dimension. The number of the independent element of the regime-dependent conditional covariance matrices H_{jt}, is $N := M(M + 1)/2$. The "squared", (ee'_t) in $h_{jt} := vech(\mathrm{H}_{jt})$ and $\eta_t := vech(e_t e'_t)$, respectively.

A major disadvantage of using the model defined in (17) is that the positive definiteness of the estimated conditional covariance matrices is not guaranteed (Ding and Engle 2001) Every covariance matrix must be positive definite but for this model it is probably impossible to give general restrictions on parameters to insure a positive definite covariance matrix.

Parameter constraints are required to make the application trustworthy. Such a parameterisation is provided by the Baba et al. (1987) (BEKK) representation of Engle and Kroner (1995) which specifies the conditional volatility as

$$H_{jt} = \gamma_{0j}^{*}\gamma_{0j}^{*'} + \sum_{l=1}^{L}\sum_{i=1}^{q}\alpha_{ij,l}^{*}e_{t-i}e_{t-i}^{'}\alpha_{ij,l}^{*'} + \sum_{l=1}^{L}\sum_{i=1}^{p}\beta_{ij,l}^{*}H_{t-i}\beta_{ij,l}^{*'}j = \{1,\ldots,k\}$$

where γ_{0j}^{*} are k × k lower triangular matrices of state dependent coefficients, L is the lag operator. γ_{0j}^{*}, α_{ij}^{*} and β_{ij}^{*} are state dependent matrices.

By recombining the GARCH model to regime switching and given h_0^2, recursive substitution in a univariate *MS-G* (1,1) model yields Haas et al. (2004):

$$h_{t,s_t}^2 = \sum_{i=0}^{t-1}(\gamma_{s_{t-i}} + \alpha_{s_{t-i}}e_{t-1-i}^2)\prod_{j=0}^{i-1}\beta_{s_{t-j}} + h_0^2\prod_{i=0}^{t-1}\beta_{s_{t-i}} \tag{18}$$

Although the *BEKK* model involves far fewer parameters than the unrestricted *vech* form, the conditional variance as specified in Eq. (18) suffers from the path dependence problem. Indeed, in this formulation, the state dependent conditional variances are a function of the lagged values of the lagged aggregated variances and aggregated error terms (after integrated the unobserved state variable).

To circumvent the path dependency problem, Gray (1996) introduces a recombining method that collapses the conditional variances in each regime by taking the conditional expectation of h_t^2 based on the regime probabilities.[5] As a consequence, the conditional variance and the residual depend only on the current regime, not on the entire past history of the process. Based on the Gray (1996)'s recombining method, in the following section we analyse how this path dependence problem may be resolved in our trivariate MS-G model case.

2.2.2 Circumventing the Path Dependency Problem: Case of a Trivariate Markov Switching BEKK GARCH (Trivariate *BEKK MSG*)

Since three equations complicate the estimation considerably, we have to make some choices in terms of the required number of volatility states and parameters involved in the estimation procedure. We restrict our study to the case of three equations and two states. Thus, the state-dependent crude oil and stock market returns are specified as:

[5] Gray (1996) proposes a recombining method for the univariate Markov Switching volatility model. For a detailed description of the path-dependence problem and its solution for the univariate MS GARCH process case, see Lee and Yoder (2007).

$$\begin{aligned} r_{s,t} &= \mu_{s,s_t} + e_{s,t,s_t} \\ r_{w,t} &= \mu_{w,s_t} + e_{w,t,s_t} \\ r_{b,t} &= \mu_{b,s_t} + e_{b,t,s_t} \end{aligned} \tag{19}$$

where subscribers s, w, and b denote real stock market returns, WTI and Brent real crude oil volatilities (the smooth part), see Eq. (13) respectively, μ is a constant where $\Phi = (\mu_{s,s_t}\mu_{w,s_t}\mu_{b,s_t})'$. e_{s,t,s_t}, e_{w,t,s_t} and e_{b,t,s_t} are state dependent residual terms. The unobserved state variable $s_t = \{1, 2\}$ is interpreted as the market state or regime when the process is at time t, which follows a first-order, 2-dimensional state Markov process.

The conditional variances are specified as:

$$E_{t,s_t}/\psi_{t-1} = \begin{bmatrix} e_{s,t,s_t} \\ e_{w,t,s_t} \\ e_{b,t,s_t} \end{bmatrix} /\psi_{t-1} \rightarrow TN\left(0, H_{t,s_t}\right) \tag{20}$$

TN denotes the trivariate normal. H_{t,s_t} is a state-dependent conditional variance-covariance matrix of each return.

The time-varying 3 × 3 positive definite conditional covariance matrix, H_{t,s_t}, is specified as (where $p = q = 1$):

$$\begin{aligned} H_{t,s_t} = \begin{bmatrix} h^2_{s,t,s_t} & 0 & 0 \\ 0 & h^2_{w,t,s_t} & 0 \\ 0 & 0 & h^2_{b,t,s_t} \end{bmatrix} &= \begin{bmatrix} \gamma_{ss,s_t} & 0 & 0 \\ 0 & \gamma_{ww,s_t} & 0 \\ 0 & 0 & \gamma_{bb,s_t} \end{bmatrix} \begin{bmatrix} \gamma_{ss,s_t} & 0 & 0 \\ 0 & \gamma_{ww,s_t} & 0 \\ 0 & 0 & \gamma_{bb,s_t} \end{bmatrix}' + \begin{bmatrix} \alpha_{ss,s_t} & \alpha_{sw,s_t} & \alpha_{sb,s_t} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}' \\ &\begin{bmatrix} e^2_{ss,t-1} & e_{s,t-1}e_{w,t-1} & e_{s,t-1}e_{b,t-1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_{ss,s_t} & \alpha_{sw,s_t} & \alpha_{sb,s_t} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \beta_{ss,s_t} & \beta_{sw,s_t} & \beta_{sb,s_t} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}' \begin{bmatrix} h^2_{s,t-1} & h_{sw,t-1} & h_{sb,t-1} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{ss,s_t} & \beta_{sw,s_t} & \beta_{sb,s_t} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= \Gamma_{s_t}\Gamma'_{s_t} + \mathrm{A}_{s_t}\mathrm{E}_{t-1}\mathrm{A}'_{s_t} + \mathrm{B}_{s_t}\mathrm{H}_{t-1}\mathrm{B}'_{s_t} \end{aligned} \tag{21}$$

where Γ_{st} is a *3 × 3* diagonal matrix of state dependent coefficients, A_{st} and B_{st} are 3 × 3 state dependent coefficient matrices restricted to be of 1 × 3 dimension for further simplification.

h_{sw,t,s_t} and h_{sb,t,s_t} are conditional covariance at time t given s_t, and h^2_{s,t,s_t}, h^2_{w,t,s_t} and h^2_{b,t,s_t} are conditional variances at time t given s_t. The matrices Γ_{s_t}, A_{s_t} and B_{s_t} and $E_{t\text{-}1}$ are compact representations of the state-dependent coefficients γ, α, β and e respectively.

We will refer to the model defined by Eq. (21) as a trivariate BEKK Markov-switching *GARCH (1,1;2)* process or, in short triavariate *BEKK-MSG (1,1;2)*. Since we are interested in providing the results related to the shock and volatility transmission only from the crude oil market to the stock market in presence of regime switching, we assume that only $h_{s,t}^2$ follows a *BEKK-MSG (1,1)* process under two volatility states (high volatility and low volatility) and each of $h_{w,t}^2$ and $h_{b,t}^2$ follow a constant.[6] We allow for the vectors of mean and variance parameters to switch across two regimes.

As in the univariate regime switching GARCH model, the recursive nature of the GARCH process makes the basic form of the model intractable due to the dependence of the conditional variance on the entire past history of the data. Indeed, only the first equation i.e., $h_{s,t}^2$, of the proposed trivariate GARCH model, is subject to the path-dependency problem. Hence, it depends directly on the state variable s_t and $h_{s,t-1}^2$, which itself depends on s_{t-1} and $h_{s,t-2}^2$ and so on. The computation of the likelihood function for a sample of length T requires the integration over all 2^T possible (unobserved) regime path, rendering estimation of the model infeasible in practice. This is the well-known path dependency problem in the regime switching literature (Cai 1994; Hamilton and Susmel 1994; Gray 1995, 1996). Furthermore, this problem is present not only in variances and residuals, but also in the covariance between crude oil and stock market returns $h_{sw,t}$ and $h_{sb,t}$.

Using Gray (1996)'s recombining method at time 1, the path-independent conditional variance, residual and covariance for the stock market variance-covariance equation are given, respectively, by:

$$\begin{aligned} h_{s,t}^2 &= E\left(r_{s,t}^2|\psi_{t-1}\right) - E\left(r_{s,t}|\psi_{t-1}\right)^2 \\ &= p_{1,t}\left(\mu_{s,1}^2 + h_{s,t,1}^2\right) + (1-p_{1t})\left(\mu_{s,2}^2 + h_{s,t,2}^2\right) - \left[p_{1t}\mu_{s,1} + (1-p_{1t})\mu_{s,2}\right]^2 \end{aligned} \tag{22}$$

$$\begin{aligned} e_{s,t} &= r_{s,t} - E\left[r_{s,t}|\psi_{t-1}\right] \\ &= r_{s,t} - \left[p_{1t}\mu_{s,1} + (1-p_{1t})\mu_{s,2}\right] \end{aligned} \tag{23}$$

$$\begin{aligned} h_{si,t} &= Cov(r_{s,t}, r_{i,t}|\psi_{t-1}) \\ &= E\left[r_{s,t}r_{i,t}|\psi_{t-1}\right] - E\left[r_{s,t}|\psi_{t-1}\right]E\left[r_{i,t}|\psi_{t-1}\right] \quad i = \{w, b\} \end{aligned} \tag{24}$$

where;

$$E\left[r_{s,t}r_{i,t}|\psi_{t-1}\right] = p_{1t}\left(\mu_{s,1}\mu_{i,1} + h_{si,t,1}\right) + (1-p_{1t})\left(\mu_{s,2}\mu_{i,2} + h_{si,2}\right) \tag{25}$$

$$E\left[r_{s,t}|\psi_{t-1}\right] = p_{1t}\mu_{s,1} + (1-p_{1t})\mu_{s,2} \tag{26}$$

$$E\left[r_{i,t}|\psi_{t-1}\right] = p_{1t}\mu_{i,1} + (1-p_{1t})\mu_{i,2} \tag{27}$$

[6] Henceforth, the conditional covariances $h_{ws,t-1,s_t}$ and $h_{bs,t-1,s_t}$and the variances $h_{w,t-1,s_t}^2$ and $h_{b,t-1,s_t}^2$ were fixed to be zero.

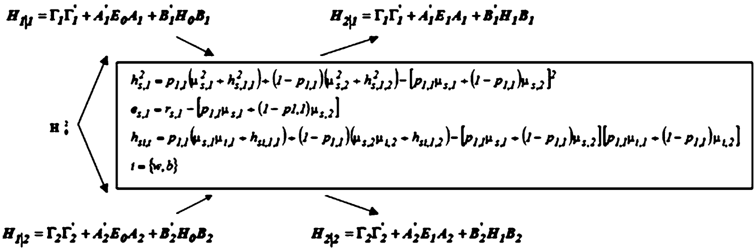


Fig. 1 Path-independent conditional variance of a trivariate BEKK-MSG model

With this definition, the conditional covariance depends only on the current regime, not on the entire past history of the process. The model is then state-independent and tractable even with large samples.

A graphical illustration for the recombining method for *BEKK* Markov Switching model is shown below (Fig. 1).

The regime probability of being in state 1 at time t is:

$$\begin{aligned} p_{1t} &= \Pr(s_t = 1|\psi_{t-1}) \\ &= P\left[\frac{f_{1t-1}p_{1t-1}}{f_{1t-1}p_{1t-1} + f_{2t-1}(1-p_{1t-1})}\right] + (1-Q)\left[\frac{f_{2t-1}(1-p_{1t-1})}{f_{1t-1}p_{1t-1} + f_{2t-1}(1-p_{1t-1})}\right] \end{aligned} \tag{28}$$

where

$$\begin{aligned} P &= \Pr[s_t = 1|s_{t-1} = 1] \\ Q &= \Pr[s_t = 2|s_{t-1} = 2] \end{aligned} \tag{29}$$

$$f_{st} = f(R_t|s_t = i, \psi_{t-1}) = (2\pi)^{-1}\left|H_{t,i}\right|^{-1/2}\exp\left\{-1/2e_{t,i}'H_{t,i}^{-1}e_{t,i}\right\}, \text{ for } i = \{1,2\} \tag{30}$$

$R_t = [r_{s,t}r_{w,t}r_{b,t}]'$ is a vector of crude oil and stock market returns at time t. H and e are defined in Eqs. (20) and (21), respectively.

The steady-state probabilities of s_t used as the initial start value for the recursive expression of the regime probability is:

$$\Pr(s_t = 1|\psi_0) = \frac{1-Q}{2-P-Q} \tag{31}$$

where P and Q are state transition probabilities assumed to follow a logistic distribution defined as in the following equations;

$$P = \Pr[s_t = 1 | s_{t-1} = 2] = \frac{\exp(p_0)}{1 + \exp(p_0)}$$
$$Q = \Pr[s_t = 2 | s_{t-1} = 2] = \frac{\exp(q_0)}{1 + \exp(q_0)} \tag{32}$$

p_0 and q_0 denote unconstrained constant terms which have to be estimated along with the regression coefficients' system.

Given the path independent *BEKK MSG* model as described by Lee and Yoder (2007), the unknown parameters that we seek to estimate for our trivariate case model are $\{p_0, q_0, \mu_{s,s_t}, \mu_{\mathrm{w,s_t}}, \mu_{\mathrm{b,s_t}}, \gamma_{ss,s_t}, \gamma_{sw,s_t}, \gamma_{sb,s_t}, \alpha_{ss,s_t}, \alpha_{sw,s_t}, \alpha_{sb,s_t}, \beta_{ss,s_t}, \beta_{sw,s_t}, \beta_{sb,s_t}\}$ for $s_t = \{1, 2\}$. We obtain the estimates parameters by maximising the following log-likelihood function.

$$LL = \sum_{t=1}^{T} \log[p_{1t}f_{1t} + (1 - p_{1t})f_{2t}] \tag{33}$$

where f_{it} for $i = \{1, 2\}$ is defined as shown in Eq. (30).

3 Methodology Results and Discussions

3.1 Data

Our analysis deals with two variables; (1) real stock returns of five major industrial countries, namely; US (DJIA), UK, (FTSE100), Germany (Dax30), Japan (NIKKEI225) and Canada (TSX) and (2) real prices of two major crude oil products, defined as the US price of West Texas Intermediate Cushing (WTI) and the Europe Brent which are quoted in dollars per barrel. Crude oil prices were extracted from the US Department of Energy (Energy Information Administration), while stock market prices were taken from the International Financial Statistics databases (IFS). All the data are measured on a monthly basis. The use of a monthly frequency is justified by the need to observe common high volatility phases that are expected to be coincident with the ECRI recession dating periods which are also provided in monthly frequency over the investigated period. The sample covers the period from January 1989 to December 2007, for a total of 228 observations. All the data were used in real terms. For each country, real stock returns are defined as the difference between the continuously compounded return on stock price index and the inflation rate given by the log-difference in the consumer price index. Consumer price indices are from OECD databases. On the other hand, the most accurate measure of an oil shock is the real oil price. The world oil prices were therefore deflated by the consumer price index (CPI) of each country. In other words, we take the world price of oil in US $ and divide by the CPI of each country.

This choice of variables may ultimately be crucial for comparison purposes. Indeed, many of the recent studies have shown that net oil prices have predictive content for determining stock market turning points (Aloui and Jammazi 2009). In contrast to some work, we would like to show that the real oil prices are also a useful predictor of turning points in stock markets. Figure 3 (left panel) plot the real equity returns and the smooth part of the real crude oil returns.[7] It is likely that time series include structural changes in the mean during the investigated period. For instance, real DJIA return series increases especially around 1992 and 2007. However, for the other countries, real equity returns experience several jumps throughout most of the period that roughly coincide with the major conventional crises.

The results from Fig. 3 (left panel) provide some preliminary evidence of (roughly) coincidental market volatility switches between real stock returns and the smoothed real crude oil volatility during the study period. In the following sections, we explore this issue further by applying the trivariate wavelet-BEKK MSG model. Let us start with the extraction of the smoothed series for the crude oil volatility index based on the new wavelet decomposition method described above.

3.2 Haar Trous Wavelet Decomposition: Application to the Real Crude Oil Volatility

Oil prices have traditionally been more volatile than many other commodity or asset prices (Regnier 2007). Recently, it has been claimed that "*Wavelet filtering is particularly relevant to volatile and time-varying characteristics of real world time series.*" (Chang and Fan 2008, p. 803).

To verify this, monthly real crude oil price volatilities were used to assess the performance of the $\hat{A}\ HTW$ algorithm in getting a smooth component without losing the underlying characteristics of the respective series. Indeed, the input data consists of the monthly real crude oil price volatility of the West Texas Intermediate Cushing (WTI) and the Europe Brent real oil returns (expressed in $/bbl) for the period January 1989–December 2007. The real crude oil market volatility R_{it} is taken as the log difference of real crude oil price P:

$$R_{it} = LogP_t - LogP_{t-1}$$

where P_t is the real crude oil price at date t.

The two transformed series are decomposed into their time scale components using $\hat{A}\ HTW$ which is redundant or non-decimated method. The wavelet filter used

[7] We first decompose the original signal (monthly real crude oil returns) using the THW transform. We then extract the smooth part from the signal. We will discuss this in more detail in the following section.

is the discrete low pass filter (G) of length, $L = 6$. The sifting processes produce six level details which are captured by *scale 1*, *scale 2*,…, *scale 6* plus the smoothed series (*Smooth*) each containing (the total sample size) 228 samples. At each scale, the corresponding component is reconstituted according to Eqs. (13) and (14). Figure 2 plot the original series (*signal*), the details (*scale 1 to 6*) and the smoothed series (*smooth*) for the real crude oil volatilities of US, UK, Germany, Canada and Japan. The standard deviations (*SD*) of each detail are not uniform across the series but proportional to the *SD* of the underlying signal. Since we use monthly data, the level of details represents the variations within 2^i months horizon which correspond to 4–8, 8–16, 16–32, 32–64 and 64–128 month dynamics, respectively. All the details are listed from the highest to the lowest frequency. The most short-run fluctuations are observed in the two finest components scales 1, and 2 and some in scale 3 which contain the high frequency content, so that they are extremely sensitive to non-smooth data characteristics such as noise, jumps, and spikes in the data. However, scales 4 to 6 depict medium and long-term fluctuations of the series. As the wavelet resolution level increases, the corresponding coefficients become smoother and the smooth trend (the coarsest approximation series) contains the lower frequency movements.

One of the advantages of the wavelet transform is that it can be used to analyse structural break at different time scales (Tommi 2005).

As noted in his article, Hamilton (2005) argues that nine of the last ten recessions during the post- II World War period in the US were preceded by large increases in oil prices. Suppose instead that we believe large oil shocks are followed by sharp recessions. To do so, we first look at the recession history with a particular focus on how each recession is preceded by a specific oil shock.[8] Henceforth, shaded bars in Fig. 2 indicate recessionary periods in months, as identified by Economic Cycle Research Institute (ECRI) from 1989 to 2007 (available upon request). According to ECRI dating, recession periods show some similarities and differences in the growth of business cycles. All the countries experienced six (single or double adjacent) recessions in the period studied (except for UK).[9] These recessions took place in 1990 (the mid-1990s Gulf war), 1994 (the Mexican Peso crisis), 1997 (the East Asian financial crisis), 2000 (economic recession in US), 2004 (Argentine energy crisis) and 2007 (the US mortgage subprime crisis). The 1994 recession in US and Canada lasted longer than in UK, Germany and Japan. However, the 1997 recession was longer in the US and UK. The main difference in the business cycle's growth among these countries concerns the recession in 1990. This recession started earlier in UK, US, Canada but 2 years later in Germany. On the other hand, Japan experienced double recessions during the same period. The recession in early 2000 was long for UK, lasting about 2 years, and shorter for Japan; on the other

[8] It is important to note that we do not attempt to analyse the causality between the crude oil spike volatility and recessions but are just trying to examine graphically the correlation between them at different time scale.

[9] UK experienced only five recessions compared to the other countries.

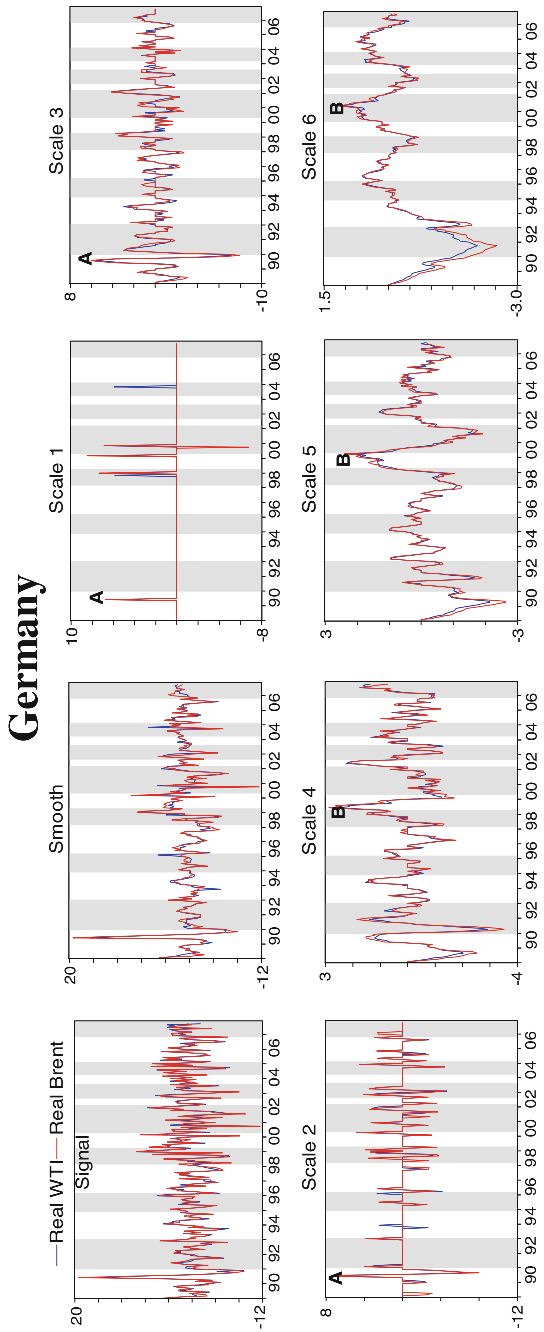


Fig. 2 Haar A Trous Wavelet decomposition of the real crude oil volatilities. The top panel: the original series (signal) and the smoothed series (smooth). The six panels namely scale 1 to scale 6: the wavelet components (vertical axis represents the amplitude of scaling coefficients (in Hertz). The shaded vertical bars indicate Growth Cycle recessions as dated by ECRI "Economic Cycle Research Institute." The sample period is January 1989 to December 2007, a total of 228 observations

Fig. 2 (continued)

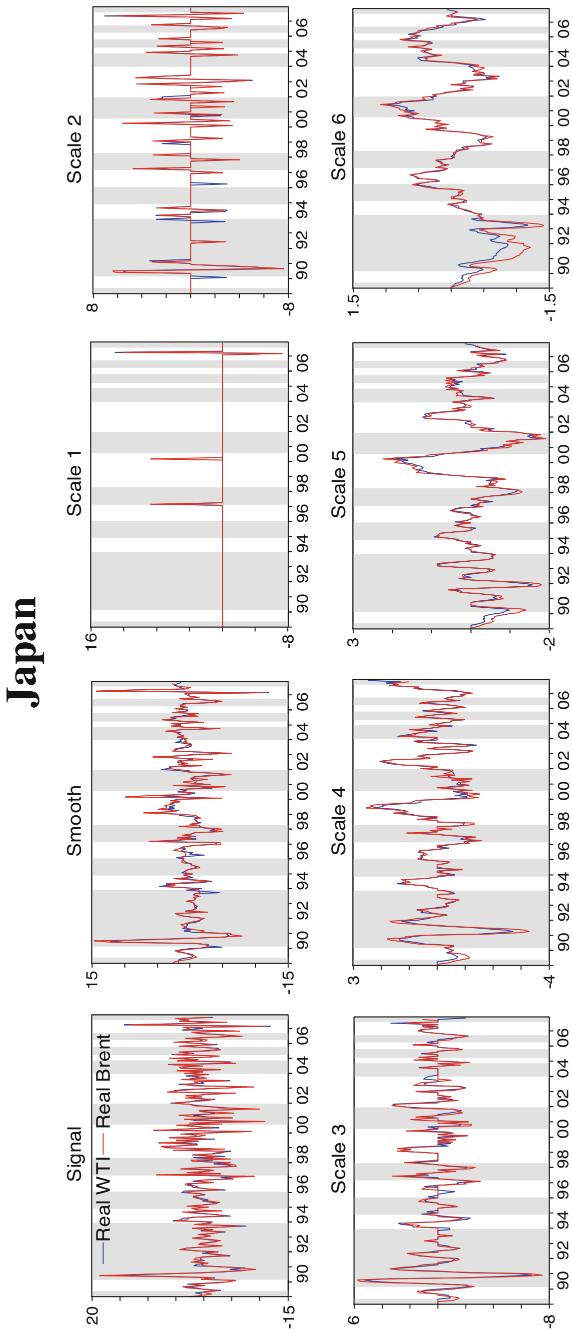
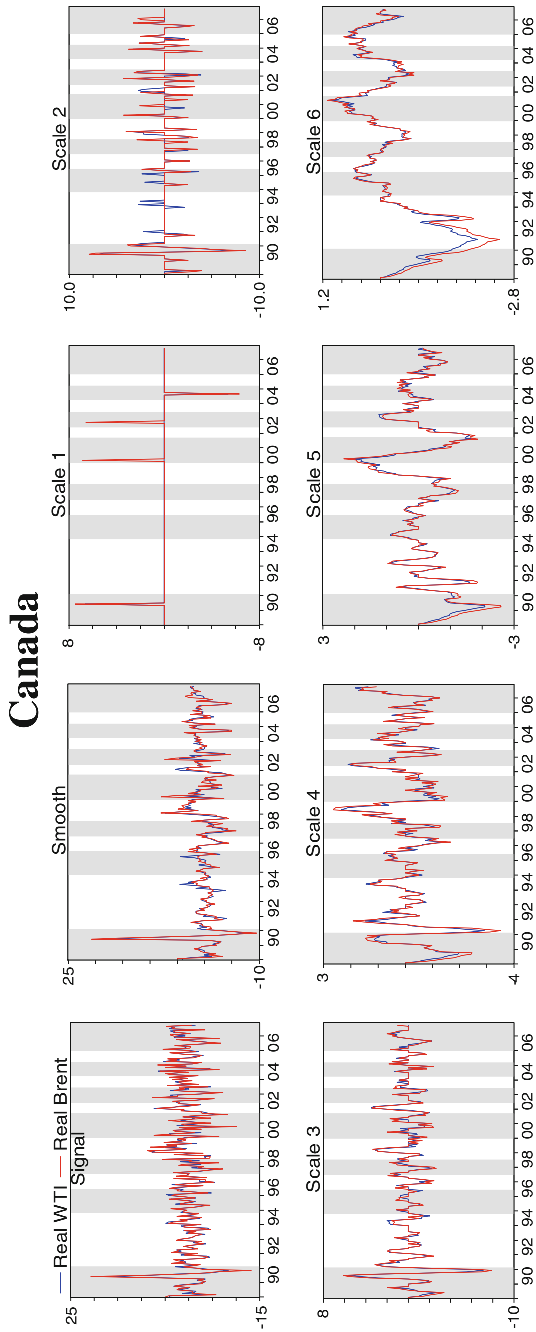


Fig. 2 (continued)



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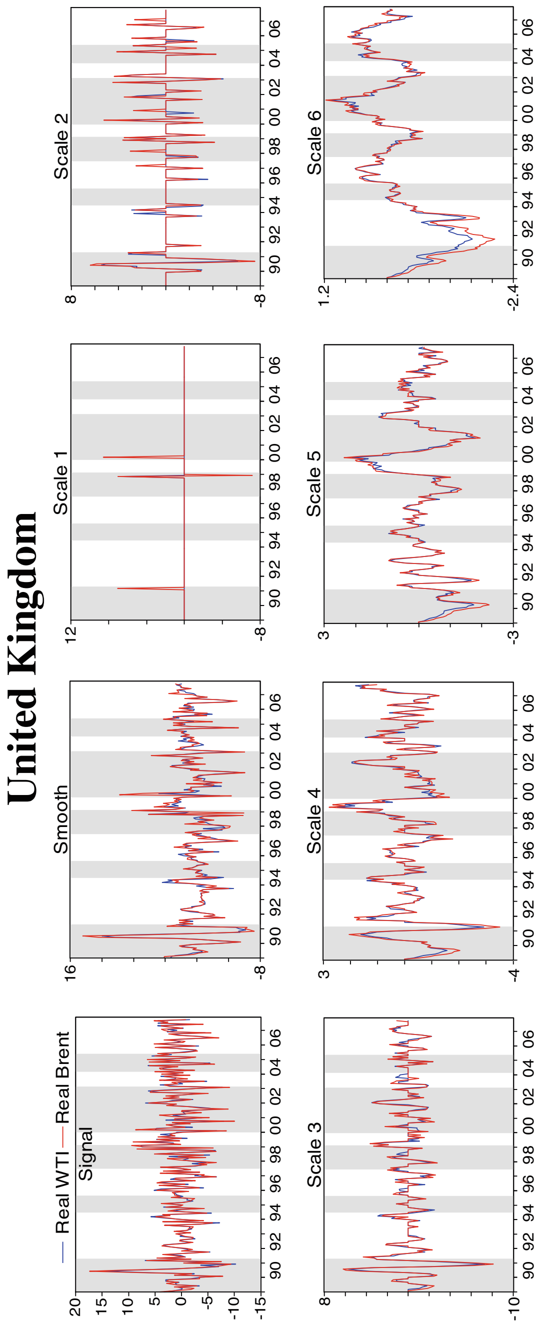

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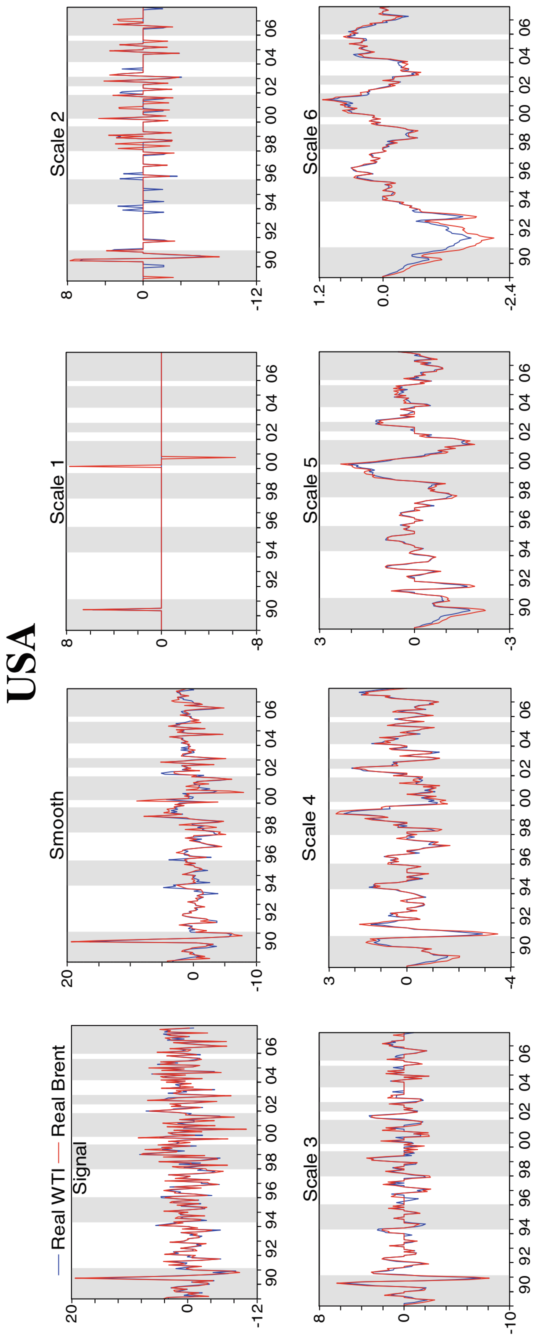

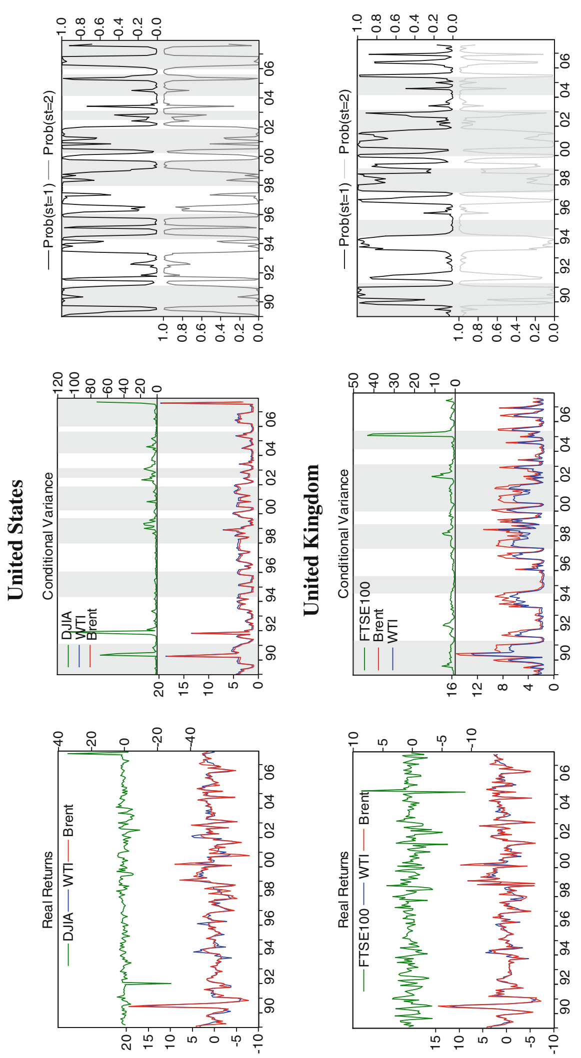

Fig. 3 The *left panel* monthly real stock market returns and the smoothed real crude oil volatilities. The *second panel* the conditional variances obtained from the trivariate RS-BEKK-GARCH model. The *right panel* smoothed probabilities of regime 1 and of regime 2 that the three markets are jointly in regime 1 (high volatility regime) at time *t* and in regime 2 (low volatility regime) at time *t* respectively. The shaded vertical bars indicate Growth Cycle recessions as dated by ECRI "Economic Cycle Research Institute." The sample period is January 1989 to December 2007, a total of 228 observations

hand, two shorter recessions occurred close to each other during the same period for US, Canada and Germany. The 2004 recession started and ended at about the same time while Japan again had two recessions during this period. In 2006, Canada,

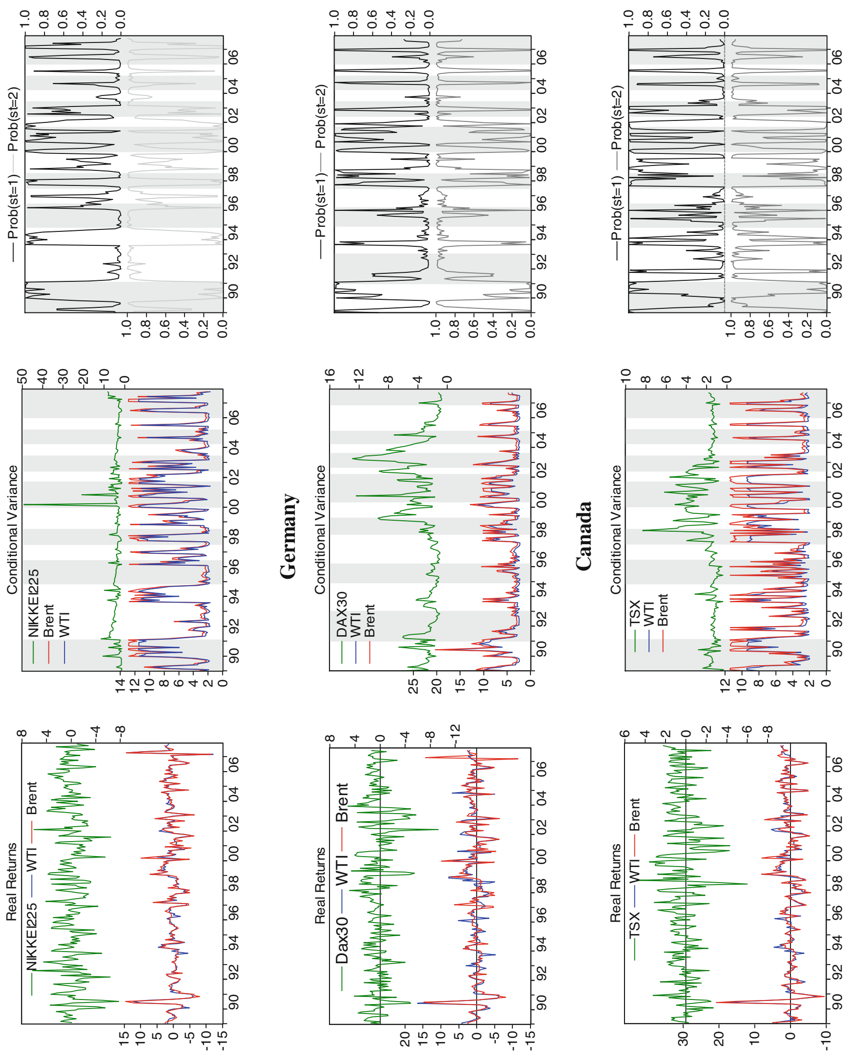

Fig. 3 continued

Germany and Japan sank into a recession at about the same time. However, this latter crisis did not hit UK.

The obtained wavelet coefficients were used to identify characteristics of the time-scale signal (smooth) that were not apparent from the original time domain signal. Therefore, Fig. 2 (scales 1–6) show that crude oil volatility peak detections are easily perceptible in the finest scales (short-term fluctuations of the series) as well as in the coarsest scales (medium and long-term fluctuations of the series). From these plots, it is easy to see which peak features are meaningful at any specific time in world history. For example,[10] in levels 1–3, the wavelets capture well the most intense volatility peak denoted by "A", which has a value of 6 or 7 and occurs in June/July 1990 for all the country cases. Essentially, this huge short-term real crude oil volatility peak leads to the 1990s recession. On the other hand, low frequency waves (scale 4–6) present fewer and thicker spikes with smaller lengths. For instance, wavelet is capable of capturing the long-term real crude oil volatility peak denoted by "B" which has a value of about 2 and occurs in 1999/2000. This followed the early 2000s recession. These plots also highlight the wavelet's strength of detecting pertinent information at varying decomposition levels. It can be seen that this evidence is also supported in the smooth series. Indeed, the studied period began with a huge oil shock in 1990 (Japan has a second largest oil shock which took place at the beginning of 2007). One can observe again that the spike of 1990 seems to be the historical spike at which the global economy can achieve a severe crisis. After this dramatic increase in real crude oil volatility, political controls try to stabilise the oil price trend. The second highest real crude oil volatility, which rises and falls in a distinct series of spikes, was at the beginning of 2000 in almost all the countries. Furthermore, it is unequivocal that there are several instances of coincidence of recessions with crude oil volatility spikes identified by the smooth series. Indeed, the initial spike volatility case was followed by a recession only for Germany and Canada[11] while the latter spike volatility case was followed by a recession for all the economies. The other ECRI recession cases were preceded by rather small oil shocks.

After verifying Hamilton's assumption, we proceed with our analysis by improving further Â THW effectiveness; that is the possibility of noise level reduction while preserving the significant feature of the original signal. Indeed, although the original signal (Fig. 2 (top left panel)) presents several peaks that precede each identified international crisis, unfortunately they are noise contaminated.

It is apparent from the plot of the smooth series (Fig. 2 (top right panel)) that the noise is reduced but the peak height is also reduced slightly. Indeed, the smoothed peaks and original unsmoothed peaks are not perfectly coincident. This is not always the case as the presence of noise can shift the peak by 1–3 sample locations. After undergoing the smoothing algorithm, the peak values are higher in amplitude than the noisy peak, and this agreement is typical of the better quality data. Finally, we could easily argue that the reconstructed signal has a simple and very smooth fluctuation that allows for easy interpretation.

Further probing led to the discovery that each spike in the oil volatility series was matched by transient instabilities in another economic indicator, including stock market returns (Cologni and Manera 2009). Our interest lies in whether oil

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[10] This example is only illustrated in the case of Germany. The remaining figures generally report the same behaviour.

[11] A potential explanation of this result is that a prolonged recession occurred at the beginning of 1988 (not included in our dataset) was preceded by successive oil shocks and that conducted to the recession of 1990 for US, UK and Canada.

price changes affect the stock market returns. Figure 3 (left panels) plots real stock returns and the smooth real crude oil returns for each country. The relationships shown in this graph were correlative. Care has thus to be taken since correlation in time does not imply causation. Bearing this in mind, the hypothesis posed was that these recurring spikes of volatility in oil price destabilised the stock market returns.

3.3 Estimation Results of the Multivariate Markov Switching Model

Having the true real crude oil volatility signal in hand, the analysis that follows endeavours to investigate whether switches in this signal have a trend towards higher stock market volatility in the five developed countries. In particular, we assume that high volatility states coincide across the two markets and we use our data set to inquire whether these states coincide with the main international crises.

The estimation of our trivariate *BEKK* MSG (1,1;2) as specified in Eq. (21) already gives us five three-market combinations where each one contains three variables: WTI real returns, Brent real returns and the respective individual real developed-country stock market returns (i.e., US, U.K., Germany, Japan, and Canada). We refer to the crude oil markets as "potential originators" and the stock markets as "potential recipient markets" because we want to explore whether shocks and volatilities originating from these markets are related with shocks and volatilities of the stock markets as in the following pairs of markets[12]:

In order to reduce the computational burden, we allow the triple markets, i.e. the recipient market (the stock market) and the two originator markets (WTI and Brent crude oil markets) to share the same volatility state. In this trivariate formulation, the number of states is six. For instance, for USA, we have the following six primitive states (as for each country case):

$s_t = 1$: DJIA real stock return—low volatility, WTI—low volatility, Brent—low volatility

$s_t = 2$: DJIA real stock return—high volatility, WTI—high volatility, Brent—high volatility

The conditional variance H is specified as a *BEKK* representation where the first element (h^2_{s,s_t}) of the diagonal matrix follows a *BEKK* MSG (1,1;2) process and the two other elements (h^2_{w,s_t} and h^2_{b,s_t}) follow a constant. Regime switching is allowed through the conditional mean intercepts and all the conditional variance parameters.

These choices allows us to refine our aim which consists essentially of finding out whether shocks and/or volatilities originating from crude oil markets are transmitted to stock markets under a jointly "high-high" volatility state or "low-low" volatility

[12] This idea was inspired by that of Edwards and Susmel (2001), who analyse the behaviour of the stock market volatilities for a group of Latin America countries using both univariate and bivariate switching models.

state. Edward and Susmel (2001) call the behaviour under this hypothesis "high volatility synchronisation" which signifies that when the "originator market" is in a high or low volatility state, the "recipient market" is always in the high or low volatility state. Furthermore, we are interested in determining whether these identified transmissions happen around the time of the conventional international crises.

Therefore, it is important to use the best possible model specification. Accordingly, assuming a *BEKK* structure, we consider two different models: (1) a standard Trivariate GARCH model with $p = q = 1$ which we denote MG(1,1) and, (2) our trivariate MSG (1,1;2).

In order to pick the most likely model, Table 1 summarises the critical values of Likelihood Ratio (LR) test, suggested by Garcia and Perron (1996). The log maximum likelihood values for the MMSG (1,1,2) models are higher than for the case where no regime switching is allowed. Notice that the former performs much better than the single regime model. Additionally, one can immediately see that the MMSG ranks better than the MS model according to the SIC, HQC and AIC criteria (not reported here).[13]

The results of estimating the multivariate Markov Switching GARCH model with BEKK parameterisation for each conditional mean and conditional volatility equation are reported in Table 2. Five triple-wise models are estimated and several interesting findings merit attention. It can be seen from the results that the three markets can be separated into two regimes. It is easy to interpret these two regimes. The first regime (labeled $s_t = 1$) indicates that all the real returns are at the same time in a "crash" state with low mean (a_S, a_W, a_B) and high variance (c_{11}, c_{22}, c_{33}). Conversely, regime 2 (labeled $s_t = 2$) captures the behaviour of the real returns in the recovery state with high mean and low variance. These states can differ substantially in durations.

We derived the transition probability matrix for the "originator" and "recipient" markets. It was assumed that the probability law that causes the market to switch among states is given by a K = 2 states Markov chain, P, with a typical element given by $\mathrm{Prob}(s_t = j/s_{t-1} = i) = p_{ij}$. From the estimated transition probabilities P_{11} and P_{22}, we can calculate the duration of being in each regime.[14] In the case of USA, the average expected durations of being in regime 1 and 2 are roughly equal (6.5 months). The expected durations of being in regime 2 for the rest of country cases are about two times higher than those of being in regime 1. Thus, high variance states are less stable for UK, Germany, Japan and Canada. It is expected to persist for as long as the low volatility state in the case of the USA.

One of the study's key objectives is to find out whether the originator and the recipient market states, assumed to be in a joint high-high volatility states, occur around the identified international crises episodes. In other words, we verify

[13] Diagnostic tests for the MG model are available on request.

[14] the average duration of being in state 1 as suggested by Hamilton (1989) can be calculated as: $D_i = (1 - P_{ii})^{-1}$.

Table 1 The likelihood ratio test

	Ln_{MMSG}	Ln_{MS}	LR statistic
USA	−833.7	−898.9	130.4[a]
UK	−750.1	−782.7	65.2[a]
Germany	−842.9	−889.4	93[a]
Japan	−867.8	−886.7	37.8[a]
Canada	−785.7	−833.4	95.4[a]

Note the LR test statistic approximately follows a $\chi 2$ distribution with three degree of freedom. Ln_{MMSG} denotes the log maximum Likelihood value of the Trivariate Markov Switching GARCH-BEKK model and Ln_{MG} designates the log maximum likelihood value of the Multivariate GARCH-BEKK model. [a] denotes significance at the 1 percent level

whether the "volatility synchronisation" between the cycles of stock market and the crude oil market happens around the conventional economic recessions.

To verify this hypothesis graphically, we plot the smoothed probability for the two states $s_t = j$ ($j = 1,2$) in the right panels of Fig. 3. These figures display both the probability that crude oil market and stock markets are jointly in a high-volatility state or state 1 (black line) and the probability that the two markets are jointly in a low-volatility state or state 2 (grey line). The observations are classified following Hamilton's (1989) proposed method for dating regime switches. According to this procedure, an observation belongs to state i if the smoothed probability $\Pr(s_t = i|\psi_t)$ is higher than 0.5.

These figures show that regimes are seen to change frequently although the states are quite persistent. Table 3 compares the ECRI turning points for the five developed countries and the joint high—high volatility periods obtained from our regime switching models. In order to concentrate on the transmission of high volatility from the crude oil market to stock market, in the discussion that follows we focus mostly on the upper line of the bottom panel. As regards the dating results of the joint high—volatility regime, the model is able to delineate all the identified international crises. Additionally, Figures show that around each of the identified ECRI crises, crude oil and stock market jointly experience high volatility states. The common contraction periods differ in length and severity. The duration of the 5 or 6 contractions range from 6 to 27 months for USA, from 5 to 24 months for UK, from 2 to 33 months for Germany, from 3 to 44 months for Canada and from 2 to 23 months for Japan (see Table 3). The longest joint recession probability (a range of two or more successive recessions occurring close to each other) is associated with the 1996 East Asian crisis for USA and UK, the economic recession of 2000 for Canada and Japan and the 1990s Gulf war for Germany. Furthermore, it is obvious that the oil shock of 1990 induces the longest joint recovery period lasting about 3 years for Canada and Japan. In contrast, the oil shock of 2000 triggers the longest common recovery period for USA, UK and Germany.

The estimations of the econometric models are reported in Table 2. we first consider matrix Φ in the mean equation (Eq. 19), captured by the parameters μ_{ij} in Table 2, to see the link in terms of returns across the markets in each triple case.

Table 2 Estimates of the trivariate BEKK-MSG model

Param.	USA	UK	Japan	Germany	Canada
Mean equation					
$\mu_{ss,s_t=1}$	0.43723[c] (2.533)	0.0667 (0.157)	−0.05047 (−0.063)	0.41293 (1.094)	0.37441[a] (1.378)
$\mu_{ww,s_t=1}$	−0.86701[c] (−2.336)	−1.16371 (−0.69)	−0.57928 (−0.123)	−0.47747 (−0.656)	−0.23688 (−0.462)
$\mu_{bb,s_t=1}$	−0.81018[c] (−4.234)	−1.10085 (−0.598)	−0.64003 (−0.127)	−0.7421 (−0.971)	−0.38566 (−0.557)
$\mu_{ss,s_t=2}$	0.8655[c] (4.122)	0.53434[c] (2.786)	0.06613 (0.151)	0.69504[c] (4.57)	0.41688[a] (1.791)
$\mu_{ww,s_t=2}$	0.753669[c] (4.14)	0.71245[a] (1.558)	0.60213 (0.66)	0.64799[c] (2.697)	0.48967[a] (1.747)
$\mu_{bb,s_t=2}$	0.81946[c] (4.234)	0.76233[a] (1.56)	0.67792 (0.683)	0.71545[c] (2.86)	0.54541[a] (1.604)
Variance equation					
$\gamma_{ss,s_t=1}$	1.01169[c] (9.3243)	1.21839[c] (9,802)	1.08123[c] (2.761)	1.09181[c] (7.126)	0.72372[c] (5,453)
$\gamma_{ww,s_t=1}$	1.38159[c] (14.069)	1.58813[c] (3,002)	1.84192[c] (8.559)	1.70612[c] (7.316)	1.75198[c] (13.123)
$\gamma_{bb,s_t=1}$	1.41483[c] (12.920)	1.71815[c] (3,153)	1.89504[c] (11.273)	1.7876[c] (7.357)	1.83818[c] (12.053)
$\gamma_{ss,s_t=2}$	0.82333[c] (5.2642)	0.79844[c] (3,956)	0.64524[c] (3.164)	0.24226 (0.7963)	0.65601 (1.226)
$\gamma_{ww,s_t=2}$	0.96959[c] (17.407)	1.10553[c] (11,223)	1.14041[c] (3.760)	1.20845[c] (16.091)	1.18165[c] (12.373)
$\gamma_{bb,s_t=2}$	1.0292[c] (15.981)	1.13841[c] (10,396)	1.19702[c] (3.110)	1.27353[c] (15.761)	1.23767[c] (11.523)
$\alpha_{ss,s_t=1}$	−0.08768 (−1.287)	0.04765 (0.053)	0.35916 (1.032)	−0.16367[a] (−1.472)	−0.4012[c] (−3.828)
$\alpha_{sw,s_t=1}$	0.10004 (0.638)	−0.05232 (−0.804)	−1.1248 (−1.151)	0.56081[a] (1.776)	−0.50581[c] (−2.642)
$\alpha_{sb,s_t=1}$	−0.08387 (−0.603)	0.01302 (0.074)	1.05857 (1.113)	−0.57145[c] (−2.108)	0.32392[a] (1.894)
$\beta_{ss,s_t=1}$	−0.09402 (−0.093)	−0.39355 (−0.132)	0.0000 (0.000)	0.60485[c] (3.616)	0.67594[c] (5.661)
$\beta_{sw,s_t=1}$	−4.84532 (−1.104)	−1.15252 (−0.166)	−0.61242 (−0.001)	−0.03723 (−0.045)	1.12084 (0.07)
$\beta_{sb,s_t=1}$	5.96062 (1.198)	1.01243 (0.163)	−0.02393 (0.000)	0.21019 (0.298)	−0.81552 (−0.064)
$\alpha_{ss,s_t=2}$	0.8655[c] (4.122)	−0.67316[c] (−2.327)	0.05621 (0.647)	0.21307[c] (2.617)	0.23979[c] (3.5347)
$\alpha_{sw,s_t=2}$	0.89084[a] (1.795)	0.02,779 (0.048)	−0.34,526 (−1.185)	−0.11,484 (−1.296)	0.16,314 (1.116)
$\alpha_{sb,s_t=2}$	−0.27,778 (−0.739)	−0.06,521 (−0.303)	0.22,148 (0.981)	0.15,617[a] (1.81)	−0.07,771 (−0.58)

(continued)

Table 2 (continued)

Param.	USA	UK	Japan	Germany	Canada
$\beta_{ss,s_t=2}$	0.19,619[c] (3.125)	0.55,125[c] (2.855)	0.98,334[c] (64.439)	0.76,459[c] (10.22)	0.95,601[c] (74.63)
$\beta_{sw,s_t=2}$	4.99,591[a] (1.703)	−1.02,577 (−0.155)	3.86,618 (0.026)	0.68,817 (0.186)	−1.28,347 (−0.005)
$\beta_{sb,s_t=2}$	−7.89,867[c] (−2.389)	0.71,428 (0.14)	−2.97,633 (−0.032)	0.1982 (0.055)	0.84,935 (0.004)
Transition probabilities					
P_{11}	0.84,493	0.78,607	0.74,283	0.66,801	0.71,520
P_{22}	0.84,671	0.87,895	0.85,757	0.85,954	0.81,371
Residuals diagnostics					
Log-L	−833.757	−750.196	−867.844	−842.985	−785.767
SIC	−923.123	−839.488	−957.283	−932.351	−875.132
HQC	−889.507	−805.918	−923.62	−898.735	−841.516
AIC	−866.757	−783.196	−900.844	−875.985	−818.767

Notes The regime dependent covariance matrices H evolves according to a trivariate RS-GARCH (1,1) equation with a BEKK representation. The diagonal elements "μ" in matrix Φ represent the constant mean coefficients. While the diagonal elements "γ" in matrix Γ represent the constant variance coefficients. Elements "α" in matrix A captures own and cross-market ARCH effects. Elements "β" in matrix B measure own and cross-market GARCH effects. Subscribers: s, w, and b denote real stock market returns, WTI and Brent real crude oil returns. Student-*t* statistics of parameters are reported in parentheses. [a, b, c] denote statistical significance at 10, 5 and 1 %

The diagonal parameters $\mu_{11,st=2}$, $\mu_{22,st=2}$ and $\mu_{33,st=2}$ for all the modeled triples equations are positively significant (except for Japan) and approximately equal during expansion phases, suggesting that financial markets and crude oil markets tend to become more stable and predictable during an expansion regime. For instance, the average mean of the real DJIA return is 0.69 % while for the real crude oil returns are 0.64 and 0.71 % (respectively for the WTI and the Brent). In contrast, during high volatility states, these diagonal parameters are significant only for USA and Canada (for Canada, only one of the three parameters is significant; $\mu_{11,st=1}$). However, it is shown that while stock market returns appear to be positive, crude oil markets are characterised by negative returns during recession states. This can demonstrate that high volatility regime in crude oil markets are on average more severe, whereas American and Canadian stock markets seem to be more resistant to an economic slowdown. The Japanese case clearly distinguishes itself from the remaining countries. It shows no significant effects on the means of any of the parameters studied either during recessions or during expansions phases.

Results from the constant parameters of the variance equations show that all the intercept terms except $\gamma_{11,st=2}$ for Germany and Canada, are positively significant. However, the amplitude of these parameters is reduced slightly when volatilities switch simultaneously from state 1 to state 2. Interestingly, we observe again that

Table 3 Reference and estimated recession periods extracted from the trivariate MS-GARCH model

USA	1.1989M01–1991M02 (26 months)	1.1989M04–1989M09 (5 months)	a.1989M10–1989M12 (3 months)
	2.1994M05–1996M01 (21 months)	1990M01–1991M05 (17 months)	1991M06–1991M09 (4 months)
	3.1998M01–1999M09 (21 months)	1991M10–1991M12 (3 months)	1992M01–1993M08 (20 months)
	4.2000M04–2001M11 (20 months)	2.1993M09–1994M10 (14 months)	b.1994M11–1995M04 (6 months)
	2002M07–2003M02 (8 months)	1995M05–1995M06 (2 months)	1995M07–1996M01 (7 months)
	5.2004M03–2005M08 (18 months)	1996M02–1996M06 (5 months)	1996M07–1996M11 (3 months)
	6.2006M01–2007M12 (24 months)	3.1996M12–1997M07 (8 months)	c. 1997M08 (1 month)
		1997M09–1999M03 (19 months)	1999M04–1999M12 (9 months)
		4.2000M01–2000M05 (5 months)	d.2000M06–2000M09 (4 months)
		2000M10–2001M01 (4 months)	2001M02 (1 month)
		2001M03–2002M04 (14 months)	2002M05–2003M08 (16 months)
		2003M09 (1 month)	2003M10–2005M06 (21 months)
		5.2005M05–2005M10 (6 months)	e.2005M 11–2006M06 (8 months)
		6.2006M07–2007M02 (8 months)	f.2007M03–2007M10
		2007M11–2007M12 (2 months)	
UK	1.1989M01–1991M04 (28 months)	1.1990M03–1990M05 (3 months)	a.1990M06–1990M07 (2 months)
	2.1994M07–1995M08 (14 months)	1990M08–1992M03 (20 months)	1992M04–1993M11 (20 months)
	3.1997M07–1999M02 (20 months)	2.1993M12–1994M11 (12 months)	b. 1994M12–1997M02 (28 months)
	4.2000M01–2003M02 (38 months)	3.1997M03–1997M07 (5 months)	c. 1997M08–1997M11 (4 months)
	5.2004M03–2005M05 (15 months)	1997M12–1999M02 (15 months)	1999M03–1999M06 (4 months)
		1999M07–1999M10 (4 months)	1999M11–2000M03 (5 months)
		4.2000M04–2000M06 (3 months)	d.2000M07–2000M11 (5 months)

(continued)

Table 3 (continued)

		2000M12–2002M01 (14 months)	2002M02–2003M02 (13 months)
		2003M03–2003M04 (2 months)	2003M05–2005M09 (29 months)
		5.2005M10–2005M11 (2 months)	e.2005M12–2006M08 (9 months)
		2006M09–2006M10 (2 months)	2006M11–2007M03 (5 months)
		2007M04 (1 month)	2007M05–2007M12 (8 months)
Germany	1.1991M01–1993M01 (25 months)	1.1989M03–1989M07 (5 months)	a.1989M08–1990M03 (8 months)
	2.1994M12–1996M03 (16 months)	1990M04–1990M06 (3 months)	1990M07 (1 month)
	3.1998M03–1999M04 (14 months)	1990M08–1991M05 (10 months)	1991M06–1991M09 (4 months)
	4.2000M05–2002M03 (23 months)	1991M10–1992M12 15 months)	1993M01–1993M11 (11 months)
	2002M09–2003M08 (12 months)	2.1994M12–1995M02 (3 months)	b.1995M03–1995M11 (9 months)
	5.2004M04–2005M02 (11 months)	1995M12 (1 month)	1996M01–1996M03 (3 months)
	6.2006M 11–2007M12 (14 months)	1996M04 (1 month)	1996M05–1997M11 (19 months)
		1997M12–1998M03 (4 months)	1998M04–1998M06 (3 months)
		3.1998M07–1998M09 (3 months)	c.1989M10 (1 month)
		1998M11–1999M01 (3 months)	1998M02–1999M03 (15 months)
		4.2000M04–2000M07 (4 months)	d.2000M08–2000M11 (4 months)
		2000M12–2001M01 (2 months)	2001M02 (1 month)
		2001M03–2001M07 (5 months)	2001M08–2001M09 (2 months)
		2001M10–2001M12 (2 months)	2002M01 (1 month)
		2002M02–2002M03 (2 months)	2002M04–2003M01 (10 months)
		2003M02–2003M04 (3 months)	2003M05–2004M11 (19 months)
		5.2004M12–2005M01 (2 months)	e.2005M02–2005M09 (8 months)
		2005M10–2005M11 (2 months)	2005M12–2006M02 (3 months)
		6.2007M03–2007M04 (2 months)	f.2007M05–2007M12 (8 months)

(continued)

Table 3 (continued)

Canada	1.1989M01–1991M02 (26 months)	1.1989M04–1989M08 (5 months)	a.1989M09–1990M07 (11 months)
	2.1994M11–1996M06 (20 months)	1990M08–1991M06 (11 months)	1991M07–1991M12 (6 months)
	3.1997M07–1998M07 (13 months)	1992M01–1992M03 (3 months)	1992M04–1993M11 (20 months)
	4.2000M01–2001M09 (21 months)	1993M12–1994M02 (3 months)	1994M03–1994M04 (2 months)
	2002M06–2003M06 (13 months)	2.1994M05–1994M06 (2 months)	b.1994M07–1995M08 (14 months)
	5.2004M04–2005M03 (12 months)	1995M09 (1 month)	1995M10–1996M02 (5 months)
	6.2006M01–2007M12 (24 months)	1996M03–1996M04 (2 months)	1996M05–1996M07 (3 months)
		1996M08 (1 month)	1996M09–1997M11 (3 months)
		3.1997M12–1998M07 (8 months)	c. 1998M08/M10 (2 months)
		1998M09 (1 month)	1999M06 (1 month)
		1998M11–1999M05 (7 months)	1999M11–2000M02 (4 months)
Japan	1.1989M01–1989M05 (5 months)	1.1989M05–1989M06 (2 months)	a.1989M07–1990M01 (7 months)
	1990M03–1993M12 (46 months)	1990M02–1991M04 (15 months)	1991M05–1993M10 (30 months)
	2.1994M12–1996M01 (14 months)	2.1993M11–1994M10 (12 months)	b.1994M11–1996M04 (18 months)
	3.1997M03–1998M04 (14 months)	1996M05 (1 month)	1996M06–1997M01 (8 months)
	4.2000M08–2001M12 (17 months)	3.1997M02–1997M03 (2 months)	c.1997M04–1997M11 (8 months)
	5.2004M01–2004M11 (11 months)	1997M12–1998M04 (5 months)	1998M05–1998M12 (8 months)
	2005M04–2005M10 (7 months)	1999M01/09 (2 months)	1999M02–1999M08 (7 months)
	6.2006M04–2006M09 (6 months)	2000M02–2000M05 (4 months)	1999M10–2000M01 (4 months)
	2007M08–2007M12 (5 months)	4.2000M11–2001M06 (8 months)	2000M06–2000M10 (5 months)
		2001M09–2002M05 (9 months)	d.2001M07–2001M08 (2 months)
		2002M10 (1 month)	2002M06–2002M09 (4 months)
		2002M12–2003M04 (5 months)	2002M11 (1 month)

(continued)

Table 3 (continued)

		5.2004M11 (1 month)	2003M05–2004M10 (18 months)
		2005M09 (1 month)	e.2004M12–2005M08 (9 months)
		6.2006M08–2006M10 (3 months)	2005M10–2006M07 (10 months)
		2007M04–2007M06 (3 months)	f.2006M11–2007M03 (5 months)
		2007M08 (1 month)	2007M07 (1 month)
			2007M09–2007M12 (4 months)

Note: *Growth rate cycle peak and trough dates from 1989 to 2007 (source: Economic Cycle Research Institute (ECRI)). Figures in parentheses indicate the average length of the period in month

the volatility of crude oil returns is lengthened more than the volatility of the stock market returns in both states. Thus, high crude oil market volatilities have the potential to damage the conditions of economic growth much more and so these volatilities might be the primary cause of financial market turbulence.

To demonstrate the stock market's response to crude oil market movement, Table 2 shows the estimated interaction parameters between the degrees of turbulence or stability emanating from real crude oil volatility series to real stock market returns.

As a result, we find that almost two stock markets utilised in our analysis are affected by news (i.e. shocks) and volatility generated from their own markets, namely Dax30 and TSX during joint recession state. However, almost all the markets are affected by news (except for Japan) and volatility generated from their own markets during the joint expansion state.

Table 2 provides results from estimating the model using equity markets and WTI, Brent crude oil markets subscribed by the letters s, w and b respectively.

The results apparently indicate that FTSE 100 and NIKKEI 225 stock market returns do not receive significant shocks/volatility originating from crude oil markets during either joint high volatility state or joint low volatility state.

Therefore the biggest danger to financial stability does not seem to have come from high increases in crude oil market volatility.

As shown in the second panel of Fig. 3, excepting the abnormal increase (during early 2000 and 2005 for Japan and UK respectively),[15] UK and Japanese stock market volatilities remain static over all the period despite the presence of large spikes in the volatility of crude oil markets. Henceforth, UK and Japanese equity

[15] Britain and Japanese stock market volatility saw an unprecedented rise of about 50 % (in 2005 and 2000 respectively) followed by rapid reversals. These meteoric rises may not be explained by any change in oil (or fundamentals), which barely changed during this period but may be indicative of explosive bubbles (e.g. the UK housing market bubble of 2004–2005).

market returns are not interrelated during the last 20 years in spite of the heavy dependency on oil.[16] This may indicate the important role that improvement in energy efficiency plays in reducing oil shock transmission to the volatility of the stock market. Indeed, according to the data of IEA (2009), UK and Japan have had the lowest primary energy intensities of any countries since the 1970s oil shock, indicating a higher efficiency than the other developed countries. Together, high volatility states in stock markets may be affected by diverse factors other than oil shocks such as interest rates or exchange rates (Apergis and Miller 2009).

The recessionary WTI (Brent) oil price shocks are positively (negatively) and significantly transmitted to the high volatility state of the German Dax 30 stock market. Then this transmission intensity switches to the joint recovery state and becomes negative (positive) and insignificant (significant) with 5 times lower amplitude. The finding for Canada can be interpreted in a similar way as for Germany with a difference in the amplitude and the sign of the coefficients $\alpha_{sw,st=1}$ and $\alpha_{sb,st=2}$ where the oil shock transmission switches from negative (positive) and significant during simultaneous high volatility state to positive (negative) and insignificant with a 3 times lower amplitude during simultaneous low volatility state. However, there is no evidence of volatility transmission running from the crude oil market to stock market.

This finding suggests that recessionary "external oil shocks"[17] (WTI) affect the German and Canadian (Brent) stock markets by increasing their volatilities. On the other hand, reaching the expansion regime, the underlying shocks negatively affect the stock market volatility and their transmission intensities become much less pronounced or even insignificant. In contrast, the opposite happens for "domestic oil shocks". Indeed, they stabilise the underlying stock markets by decreasing their volatilities during the joint recessionary state. This may highlight the decreased role that hedging policy efficiency plays in order to neutralise any potential oil price impact (particularly "external oil shocks") on the volatility of the stock market.

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[16] Japan imports all of its oil. It is considered the third largest oil consumer in the world (behind US and China) and the second largest net importer of oil (behind US) in spite of its limited domestic oil reserves and production. UK is largest producer of oil and natural gas in the European Union but it cannot produce enough oil to meet its domestic demand (EIA 2008).

[17] Brent oil is, by definition, produced from Europe (UK), Africa and the Middle East (Brent North Sea crude). However, WTI oil is produced from North America (North America crude such as Canada). In what follows, we denote WTI oil shock as "External oil shock", i.e. extra-North sea oil shock, for European countries like Germany and as "Domestic oil shock" for American countries like Canada. In the same way, we denote Brent oil shock by "External oil shock", i.e. extra-American oil shock (North America as well as South America), for American countries and as "Domestic oil shock" for European countries.

In 2006, Germany is the fourth largest net-oil importing country (it imported 2.483 million barrels of crude oil per day to meet most of its oil needs). It was dependent on external oil sources even in peacetime. The top three sources of German crude oil imports were Russia (34 %), Norway (16 %) and UK (12 %) (Hsing 2007). Furthermore, Canada is both an exporter and importer of crude oil. From Stats Canada for 2005, domestic crude accounts for only about 45 % of Canada's oil consumption. Imports represent the remaining 55 %, mostly coming from North Sea Countries (UK and Norway) or the Middle East (Iraq, Saudi Arabia…etc.).

Decision makers are advised to drive domestic oil production and seek renewable energy technologies in order to reduce its reliance on foreign oil.

It should be emphasised, as shown in Fig. 3 (second panels) for Canadian and German cases, that these transmissions were concentrated during the 1999–2004 period of severe worldwide economic contractions (the bursting of the equity bubble of 1990, the US terrorist attack and the Enron scandals in 2001, the Argentine energy crisis, the Iraq disarmament crisis). They were opposite and weaker than those observed before and after these crises periods. Indeed, as clearly illustrated in these figures, the conditional variances of TSX and Dax30 varied dramatically over the 2000–2003 period which coincides with the sharp increases in oil volatility. Together, as previously demonstrated in Sect. 3.2, these respective low frequency components of crude oil volatility shock take longer period to stabilise. Moreover, especially in the case of Canada, real Brent is more volatile and therefore far more vulnerable to the real TSX than do the real WTI.

The US stock market response differs systematically from that of other oil-importing countries. Table 3 shows that the crude oil market does not transmit any signals (shock or volatility) to the DJIA stock market return during a common recession state. The significant coefficient on $\alpha_{12,st=1}$ shows that shocks of WTI arising during simultaneous low volatility states are transmitted positively and significantly to the DJIA stock market. There is also evidence of positive (negative) volatility transmission from WTI (Brent) oil market ($\beta_{12,st=2}$ and $\beta_{13,st=2}$) to the US stock market during those same periods. In addition, the DJIA stock market volatility is very sensitive to volatility coming from crude oil returns (4.9 and 7.8), underlying the major role that crude oil plays in this country as the largest oil importer.[18]

The positive transmission of the WTI's shock/volatility to the expansion phase of the USA stock market may underline the latter's greater vulnerability to shocks/volatilities from American sources of crude oil prices[19] than from the North Sea crude oil prices, but not to the point of leading to a stock market crash. In fact, with the declining production volumes of the Brent fields, more of the North Sea crude oil supply is being absorbed locally and less is available for sale to the USA. US dependence on the Brent crude fell sharply; this sudden change can be explained mainly by the rapid increase in oil demand by high growth countries particularly China and India,[20] the so-called "US Middle East

[18] According to US energy information Administration 2008, USA is the world's largest net importer of crude oil. It imported 10,984 thousand barrels per day, followed by Japan (4652) and China (3858).

[19] In 2000, North and South American countries particularly Canada (17.8 %), Mexico (14.2 %), Venezuela (14 %) supplied much more crude oil to the USA. However, Middle East countries (Saudi Arabia and Iraq) provide less than 23 % of USA oil imports, 25 % comes from African countries (Nigeria, Angola and Algeria), and less than 3 % from European countries (UK, Norway). (http://import-export.suite101.com/article.cfm/usa_oil_imports_by_country_2007).

[20] In 2008, Chinese crude oil imports, largely concentrated in the volatile Middle East, was roughly 4 times higher than in 1978 (Leung 2010).

oil independence"[21] (Kraemer 2006), as well as by the improvements made in energy efficiency by the US policy to reduce the inflationary effects of oil shocks. As a result, the decreasing US dependence on Brent crude may help make the stock market more resilient to the disruption of Brent supplies.

It can be concluded from these findings that the increased dependence on American crude oil supplies and the decreased dependence on North Sea crude oil supplies (the most unstable countries in the world) may be welcomed in the stock market.

The economic intuition for our main findings is most easily explained with reference to the second panel of Fig. 3. In this panel, the crude oil variances vary considerably over time and low spikes (state 2) are associated with very moderate investments in stocks (see the period 1999–2004). In contrast sharp spikes (state 1) are associated especially with small stock and reduced allocations (the two subsequent high volatility periods occurred in 1990 and the other one in 2007). Because regimes are persistent, short-horizon investors clearly attempt to time the market by reducing (increasing) the allocation to the riskiest assets when investment opportunities are poor (good) based on the information offered by the crude oil market volatility.

As there is no spillover effect between the stock market and crude oil market for USA during the joint high volatility state, there is limited potential for making riskless excess profit on the US stock market in much less time based on information from WTI, for example. Except for these periods, volatility in US equity markets remained generally low.

4 Summary and Concluding Remarks

In this paper, we use monthly stock market prices and two crude oil data (WTI and Brent) for a group of five developed countries (USA, UK, Germany, Japan and Canada) to quantify the magnitude and time-varying nature of volatility spillovers running from the crude oil market to the equity markets (DJIA, FTSE100, Dax30, NIKKEI225 and TSX).

With the objective of finding the most efficient way to model the behaviour of crude oil price volatilities, we use wavelet filtering, particularly Trous Haar wavelet decomposition method, as it has already proved it can provide a better insight into the dynamics of financial time series.

Moreover, most studies assume that the relationship between variables (especially asset returns) is generated by a linear process with stable coefficients so the predictive power of state variables does not vary over time. However, there is

[21] Indeed, many American politicians (President George W. Bush, among others) had worked toward US energy independence in order to reduce US imports of oil and other foreign sources of energy (see also "US energy independence" article from Wikipedia, http://en.wikipedia.org/wiki/United_States_energy_independence).

mounting empirical evidence that spillover parameters follow a more complicated process with multiple "regimes", each of which is associated with a very different distribution of asset returns. The restricted trivariate *BEKK MSG* model used in our analysis is quite general and allows means, variances and parameters of shock/ volatility transmission to vary across states. Hence assuming that the two variables are in common states, the stock market return can vary across states in response to a shock or volatility originating from the crude oil market.

The results show that the *Â HTW* decomposition method appears to be an important step towards obtaining more accurate results. Indeed, we find that it seems to be very useful in detecting break-points, which implies that crude oil shock intensity varies significantly through time. Further, the resulting signals are smooth and give us a better approximation or reconstruction of the original signal. We also improve accuracy of this variable in detecting key real crude oil volatility features.

On the other hand, the trivariate *BEKK-MSG* estimations suggest that there are quite close connections between the joint equity and crude oil high volatility state and international recessions. Additionally, apart from UK and Japanese cases, the responses of the stock market to an oil shock depend on the geographic area for the main source of supply, be it from the North Sea or from North America (as we take two oil benchmarks, WTI and Brent respectively). Then, for Germany and Canada, external oil sources contribute more to causing a stock market crash even though these countries import less oil from abroad (Western America for Germany and Europe for Canada. However, oil shocks originating from Eurasian or European countries (North America) appear to be far less vulnerable.

The results for the US stock market volatility response to the crude oil shock and to volatility are different. Indeed, WTI crude oil volatility (American sources of oil) increases the DJIA stock market volatility, whereas the latters exhibit the inverse reaction to Brent crude oil. The US stock appears to be more resilient to crude oil shocks since even they exist they do not lead to a potential stock market crash.

However, Japanese and Britain equity markets do not show any reaction to shocks and/or volatilities coming from crude oil market.

Our results might be of interest to:

(1) investors; results show that the current crude oil market state is a persistent bear state with more attractive assets than in a bull crude oil market state.
(2) Monetary policy makers; the results obtained suggest that there are divergences between the hedging performance of WTI and Brent. For example, the presence of a positive transmission of the temporary WTI oil price shocks to the recessionary stock market phase highlights that the hedging policy in Germany is less efficient to neutralise the WTI oil price effect on the volatility of the German Dax30 stock market. Reaching the expansion phase, the opposite occurs but shocks take longer to stabilise. Here, monetary policy may play a more active role as a

(3) Energy policy makers; since German stock market may be more vulnerable to a WTI shock than a Brent shock (the inverse case for Canada), the government should import little to no oil from the main production countries of WTI crude but diversify sources and promote incentives for developing alternative energy sources (both in industrial and household sectors); this would reduce dependence on any one area outside the Brent crude main source countries. Conversely, the results for the US case can be attributed to the successful efforts of American policy makers to promote efficient energy since it depends mostly on WTI.

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