Chapter 19
Decision Making via Binary Decision Diagrams: A Real Case Study

Alberto Pliego Marugán, Fausto Pedro García Márquez and Jose Lorente Lavirgen

Abstract Nowadays logistical and operational systems are being more complex. A quantitative and qualitative analysis in Decision Making (DM) is presented in this paper. Decision Trees (DT) and Binary Decision Diagrams (BDD) are used to find the best solution to the main problem. The BDD method is used to perform the quantification. A real case related to the timeliness on the deliveries is studied in this paper. Importance Measures (IMs) have been considered to rank the basic events of the DT with respect to their contribution to the top event.Thereby, an improvement of the response of a company facing certain problems and the optimization of the company resources is done. Statistical data is used to obtain an approximate measure of the occurrence probability of the events involved.

Keywords Binary decision diagrams · Decision making · Decision tree · Importance measures

19.1 Introduction

The aim in a Decision Making (DM) process is to select the most advantageous path among different situations.

According to the DM problem studied in this paper, to establish a logical structure of the problem is essential. DT has been chosen for that purpose [5]. DT leads to complete a logical relation among several single events. These events, alone or by combination of them, are the responsible of the main problem. The interrelation of mentioned events has been implemented by using logical gates.

It is possible to establish which of the events need to be set employing data analysis techniques when the logical structure of the problem, as well as the statistical data,

A. P. Marugán · F. P. G. Márquez (✉) · J. L. Lavirgen
Ingenium Research Group, University of Castilla-La Mancha, 13071 Ciudad Real, Spain
e-mail: FaustoPedro.Garcia@uclm.es

J. Xu et al. (eds.), *Proceedings of the Eighth International Conference on Management Science and Engineering Management*, Advances in Intelligent Systems and Computing 280,
DOI: 10.1007/978-3-642-55182-6_19,

is considered. Nowadays the key to optimize the resources is found on these data analysis techniques considering that a wrong approach of the problem is able to make unfruitful and costly decisions.

19.1.1 Decision Making Scenario

DT shows the main problem and it is composed by several events called basic problems. These basic problems are the responsible for the main event to occur. Not all of the basic problems have the same weight and every single event has a different occurrence frequency. The DT has different levels, from the top event to the different roots, which are called basic events. It is indeed on these basic events where it will be necessary to work in order to reduce its occurrence frequency, where a lower occurrence probability of the main problem will be achieved by reducing it.

19.1.2 Binary Decision Diagram

Binary Decision Diagrams (BDDs), as a data structure that represents the Boolean functions, were introduced by Lee [4], and further popularized by Akers [1], Moret [8], and Bryant [2]. The BDD is used in order to analyze the DT. It will allow obtaining an analytical expression depending on the occurrence probability and the logical structure of the tree of every single basic event.

A BDD is a directed acyclic graph (V, N), with vertex set V and index set N (position of v in the order of variables). Vertex set contains two types of branches. On the one hand, a terminal vertex has as attribute a value: value(v) $\{0, 1\}$, where "1" state corresponds to the occurrence of the main problem, and "0" state corresponds to the non occurrence of the main problem. All the paths that have 1 state provide the cut-sets of the DT. On the other hand, a non-terminal vertex v has as attributes an argument index index(v) $N\{0, 1, \cdots, n\}$, and two descendants, low(v) and high(v), that are connected by a branch. Each one has a vertex 0 branch that represents a non-occurrence basic event, or 1 branch that represents an occurrence basic event. For any non-terminal vertex v, if low(v) is also non-terminal, then index$(v) <$ index(low(v)), and if high(v) is non-terminal, then index$(v) <$ index(high(v)).

BDD has a root vertex v that leads to denote a function f_v defined recursively as follow: Firstly, if v is a terminal vertex and value$(v) = 1$, then $f_v = 1$. In other case, if value$(v) = 0$, then $f_v = 0$. Secondly, if v is a non-terminal vertex with index$(v) = i$, then f_v will be:

$$f_v(x_1, \cdots, x_n) = x_i \cdot f_{\text{low}(v)}(x_1, \cdots, x_n) + x_i \cdot f_{\text{high}(v)}(x_1, \cdots, x_n). \quad (19.1)$$

The cut sets of a BDD are the pathways to terminal vertices with value of 1. The occurrence probability of the top event can be calculated by the probability of the

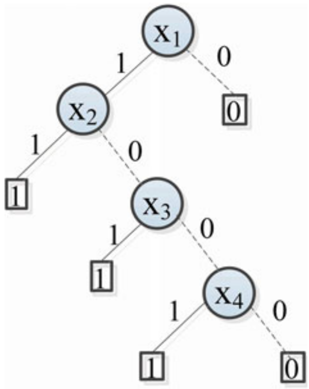

Fig. 19.1 BDD example

cut sets of the BDD. It is possible to achieve the occurrence probability of the main problem through the addition of the BDD cut sets. Thus, mentioned cut sets are able to be evaluated by changing the occurrence probability of each event [1, 2, 8].

Fig. 19.1 shows a BDD example, which is composed of: A root vertex: x_1; Two non-terminal vertex: x_2 and x_3; A terminal vertex: x_4.

The BDD gives (starting at the x_1 event and finishing in every path to the ones) the cut sets needed, which indeed are:
$CS_1 = \{x_1,\ x_2\}$,
$CS_2 = \{x_1,\ \overline{x}_2,\ x_3\}$,
$CS_3 = \{x_1,\ \overline{x}_2,\ \overline{x}_3,\ x_4\}$,
where $\overline{x}_i$ is the denial of x_i, which means the non occurrence of that event.

19.1.3 Conversion from DT to BDD

The transformation from DT to BDD is achieved by applying certain mathematical algorithms [7]. Hence, it is possible to find an analytical expression of the logical structure desired.

An adequate ranking of the basic events is crucial in order to reduce the size, and thus the computational time to solve the BDD. There are different methods, and any of them will be more adequate to use according to the problem structure, number of variables, etc. In the simulations done in this paper, the AND method have been considered for listing the events [6].

Once the conversion from DT to BDD is done, it is possible to obtain an accurate expression of the main problem occurrence probability by assigning a probability value to each basic event.

$$Q_{sist} = q_{e001} \cdot q_{e002} + q_{e001} \cdot (1 - q_{e002}) \cdot q_{e003} + q_{e001} \cdot (1 - q_{e002}) \cdot (1 - q_{e003}) \cdot q_{e004}, \tag{19.2}$$

where Q_{sistis} the occurrence probability of the main problem and q_{e00i} is the probability of occurrence of the basic event 'i'. Further detailed information about the conversion and variable ordering methods can be found in [4].

19.1.4 Importance Measures

A classification and identification of the events that are influencing the most in the main problem is necessary. IMs reveal some key information such as which of the events are the ones that contribute the most to the main problem to occur.

IMs can be calculated by the Birnbaum, Criticality and Fussell-Vesely heuristic methods. The basic events with higher IM values must be the first to be considered [3]. Focusing on the events that IMs are pointing, will allow the company to reduce the probability of the main problem.

Birnbaum Importance Measure determines that the system is in a certain state that, the occurrence of a certain event causes the main problem. The mathematical expression is:

$$I_i^{\text{Birn}} = Q_{\text{sis}}(1_i, q(t)) - Q_{\text{sis}}(0_i, q(t)) = \frac{\partial Q_{\text{sis}}(t)}{\partial q_i(t)}, \tag{19.3}$$

where Q_{sis} is the unavailability of the system, $Q_{\text{sis}}(1_i, q(t))$ is the probability of occurrence of the main problem when the basic event "i" is happening, and $Q_{\text{sis}}(0_i, q(t))$ is the probability of occurrence of the main problem when the basic event "i" is not happening.

Criticality Importance Measure, unlike Birnbaum, considers the probability of the related basic event:

$$I_i^{\text{Crit}} = \left(\frac{q_i}{Q_{\text{sis}}}\right) \cdot \left(\frac{\partial Q_{\text{sis}}}{\partial q_i}\right) = \left(\frac{q_i}{Q_{\text{sis}}}\right) \cdot I_i^{\text{Birn}}, \tag{19.4}$$

where Q_{sist} is the occurrence probability of the main problem and q_i is the probability of occurrence of the basic event 'i'.

Fussell-Vesely Importance Measure is that corresponding to the union of those cut sets where such events are presented.

$$I_i^{FV} = \frac{P(E_1{}^i \bigcup E_2{}^i \bigcup E_3{}^i \cdots E_n{}^i)}{Q_{\text{sis}}(t)}, \tag{19.5}$$

where $(E_1{}^i \bigcup E_2{}^i \bigcup E_3{}^i \cdots E_n{}^i)$ is the probability of the union of those cut sets where basic event i is presented and $Q_{\text{sis}}(t)$: Main problem occurrence probability.

19.2 Case Study

This paper is focused on the reduction of the occurrence probability of an undesired main event in a decision making scenario. Mentioned event is defined by the orders delay. Firstly, detect which are the events related with the delay in the orders is compulsory (see Fig. 19.3). In order to have an efficient and real decision making, the development of the DT flowchart is crucial. The closest to real decisions the tree is, the better the results will be achieved.

An inner procedure must be in charge of compiling all the information of the basic events. The connections between the events and the logical structure will be obtained by analyzing this information. Mentioned inner procedure need to be done carefully with surveys, questionnaires, meetings and anything needed to create a feedback between employees and the company. The probability associated to each basic event is taken from mentioned feedback and questionnaires.

A repeated event in the DT is possible to be found. This is due to there are basic events which are able to cause the main problem in numerous company areas. For instance in this real case study "Sampling mistakes" ($e006$) is repeated in first and third branches.

Minimal cut sets are obtained by using a mathematical algorithm which converts the DT to BDD as aforementioned in previous sections. Figure 19.2 shows the basic events interrelation and the probability of occurrence of each basic event. The following calculations have been obtained starting from mentioned probability of occurrence.

19.2.1 System Probability Variation Over the Years

A simulation of the system through ten years has been done. The simulation consists on decreasing the probability of each single event in isolation. With this simulation, a more restrictive control of the influence of each basic event over the system is achieved.

According to the results presented in Fig. 19.2, basic events number ten, from eighteen to twenty-three, twenty-six and twenty-seven, are the ones which affect the most to the system. That means that those events must be modified if the purpose of the company is to reduce efficiently the main event probability. These results will be verified straightaway with the IMs analysis.

Figure 19.4 shows the importance of each event with the three IM methods. These events can be grouped in a ranking of importance. In this particular case, the event twenty-six has a greater importance than the rest of the events. That means it should be the first event to be considered. However, there is a group of events which has a similar IM value among them (events ten, eighteen to twenty-three and twenty-seven). It is useful to observe the three IMs values to decide how events will be placed in the importance ranking.

Delay in the orders

Projects interaction	**g001**	OR	**Projects interaction**	g001	AND	**Human Resources**	g007	OR	**Lack of notification**	g013	OR	**Detailed needs in organization chart**	g017	OR	**Delay in repair orders**	g019	OR
Human Resources	**g007**		Wrong stock project	g002		Renege on inner procedures	g008		Unreal warehouse data	g014		Mismatch between needs	g018		Tools shortage	g020	
Lack of advice	**g013**		Wrong information exchange. P=0.04	e001		Bad diet	g009		Wrong simulations	g016		Unreal capacity work P=0.1	e020		Errors in paperwork generation	g022	
Detailed needs in organization chart	**g017**		Wrong stock project	g002	AND	Renege on inner procedures	g008	OR	Unreal warehouse data	g014	AND	Mismatch between needs	g018	AND	Manufacturing closed late	g023	
Delay in repair orders	**g019**		Poor production management	g003		Lack of employees formation	g005		Theft employees. P=0.12	e014		Overcharge of capacity. P=0.05	e018		Tools shortage	g020	OR
			Lack of a global planning. P=0.03	e002		Lack of motivation	g010		No coordination between workers	g015		Wrong work planning. P=0.07	e019		Wrong hierarchy of employees. P=0.02	e021	
			Poor production management	g003	OR	Lack of employees formation	g005	AND	No coordination between workers	g015	OR				Wrong tools stock	g021	
			Forecast mistakes	g004		Employees with low qualification	e004		Poor communication with work orders. P=0.13	e015					Wrong tools stock	g021	AND
			Lack of personal formation	g005		Lack of inner formation courses	e005		Lack of daily notification. P=0.2	e016					Limited tools. P=0.03	e022	
			Forecast mistakes	g004	AND	Lack of motivation	g010	AND	Wrong simulations	g016	AND				Low tools reliability. P=0.1	e023	
			Shortage of statistic resources	g006		Limited salary. P=0.07	e009		Lack of simulations. P=0.03	e017					Paperwork generation errors	g022	AND
			Out-dated analysis techniques. P=0.05	e003		Bad work atmosphere. P=0.08	e010		Forecast mistakes	g004					Employees with limited qualification. P=0.1	e024	
			Shortage of statistic resources	g006	OR	Bad diet	g009	OR	Forecast mistakes	g004	AND				Lack of resources in some departments. P=0.2	e025	
			Sampling mistakes. P=0.08	e006		Personal reasons. P=0.06	e008		Out-dated analysis techniques. P=0.05	e003					Manufacturing closed late	g023	OR
			Parameters selection mistakes. P=0.08	e007		Poor canteen	g011		Shortage of statistic resources	g006					Limited connection between departments. P=0.3	e026	
			Lack of employees training	g005	AND	Poor canteen	g011	AND	Shortage of statistic resources	g006	OR				Manufacturing order not scheduled. P=0.06	e027	
			Employees with low qualification. P=0.06	e004		Menu variety. P=0.06	e011		Sampling mistakes P=0.08	e006							
			Lack of inner formation courses. P=0.02	e005		Food quality	g012		Parameters selection mistakes P=0.08	e007							
						Food quality	g012	OR									
						Low kitchen personnel qualification. P=0.05	e012										
						Low ingredients quality. P=0.04	e013										

Fig. 19.2 Delay in the orders DT description

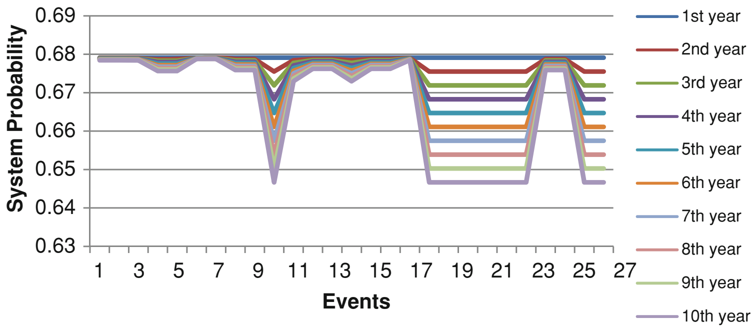

Fig. 19.3 Influence of basic events over the years

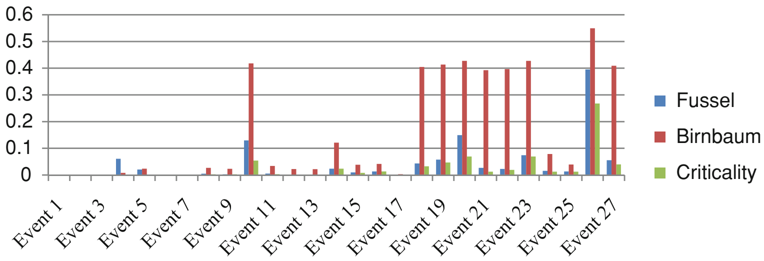

Fig. 19.4 Importance Measures

The IM value of the rest of the events is so small that they will not have to be considered until the events with the higher IM value had been solved. With this valuable information, it is possible to apply a methodology whereby to obtain a ranking showing the order of the basic events to act.

Looking for the most efficient way to act, it is suggested to start with Criticality IM. In case the basic events have the same IM value, the second step would be to obtain the Birnbaum IM and rank the events. In case the basic events still have the same importance, finally Fussell-Vesely should be obtained and thus have an importance order of the basic events.

19.3 Conclusions

A quantitative analysis in DM problems provides efficient and useful results, e.g. to determinate the basic events that have major influences. Furthermore, to establish a logical structure of the problem is necessary in order to respond as close as possible to the manner the problem is caused.

BDDs are employed to obtain the cut sets that are used to get the analytical expression of the main problem occurrence probability. Thus, the DM offers the chance to simulate dynamically the probability of the main problem when the probabilities of the basic events changeover time.

Decrease the probability of certain events simultaneously entails a lower impact over the main problem probability than to decrease only the probability of the events which the IMs are pointing as more important.

Applying aforementioned methodology provides the company with a powerful method in the DM process and also an approach to increase the reliability.

As further work the author propose to explore large DM problems, and more complex problems where consider other variables.

Acknowledgments The work reported herewith has been financially supported by the Spanish Ministerio de Economíay Competitividad, under Research Grant No. DPI2012-31579.

References

1. Akers S (1978) Binary decision diagrams. IEEE Trans Comput c-27(6):509–516
2. Bryant R (1986) Graph-based algorithms for boolean functions using a graphical representation. IEEE Trans Comput 35(8):677–691
3. Lambert H (1975) Measures of importance of events and cut sets in fault trees. Technical report, California University, Livermore (USA), Lawrence Livermore Lab
4. Lee C (1959) Representation of switching circuits by binary decision diagrams. Bell Syst Technol 38:985–999
5. Lopez D, Slyke WV (1977) Logic tree analysis for decision making. Omega 5(5):614–617
6. Márquez F, Moreno H (2012) Introducción al análisis de farboles de fallos: Empleo de bdds. Editorial Académica Española
7. Marquez FG, Pliego A et al (2011) Fault tree analysis for wind turbine. In: Proceedings of the fifth international conferenceon management science and engineering management. Editorial World Academic Union Ltd, Macao, pp 3–7
8. Moret B (1982) Decision trees and diagrams. Comput Surv 14(4):413–416