Chapter 3 Study on Onboard Engine Thrust Estimation Based on GNSS Precision Orbit Determination Technology

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Abstract Electric propulsion technology which has the advantages of high specific impulse, small thrust becomes the preferred way of space exploration in future. To ensure the safety and effectiveness of electric propulsion technology applications, the assessment of its performance is needed. The GNSS technology is now used for orbit determination, time service and so on, and it can be used for onboard thrust estimation. The paper gives the thrust model first. Then it gives the method to estimate the thrust for low and circle satellite orbit and the workflow using GNSS precision orbit determination Technology. At last, the simulation is made to analyse the semi-major axis variation under the thrust in the velocity direction. The result of Grace real time orbit and precision orbit is also give to prove the validity of the method in paper. The paper puts forward a suggestion that the real time orbit determination technology can be used for onboard engine thrust rapid assessment.

Keywords Precision orbit determination · GNSS · Electric thruster · Real time orbit determination

3.1 Introduction

In order to shorten the time to reach the planet or star and carry more scientific observation instruments with less propellant in space exploration, we need to find a more efficient propulsion technology than rocket engine. Electric propulsion

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technology research and development and its application has attracted the attention of space powers. Electric thruster which can repeat the start and has high specific impulse, small thrust, light weight and long life characteristics, has been used on many satellites. For example, Boeing Company has delivered eighteen satellites for military and commercial satellite applications [\[1](#page-9-0)] used the electric thruster. Currently electric propulsion technology is mainly used to keep track position, and when the satellite main propulsion system is invalid, electric propulsion will play an important role in the transfer orbit, such as the U.S. Air Force ''Advanced Extremely High Frequency satellite"-1 (AEHF-1) and the Japanese "Hayabusa" for asteroid detection and so on.

Electric propulsion technology is the preferred way forward for the future of space exploration. In order to ensure the effectiveness of its use, pre-test is needed to be carried out to evaluate its performance. The paper use the precision orbit determination technology with spaceborne GPS dual frequency data to complete the engine thrust Estimation. The paper gives the Engine thrust estimation principle first, and the method to estimate the thrust for low and circle satellite orbit. Then the simulation data and GRACE-A [[2\]](#page-9-0) data is used to verify the validity of the method. Engine thrust Estimation based on GNSS precision orbit determination can get high accuracy, but it do not have real-time characteristic. So the paper puts forward a suggestion that the real time orbit determination technology can be used for onboard engine thrust rapid assessment at last.

3.2 Engine Thrust Estimation Principle

3.2.1 Mathematical Model of Electric Thruster

In general, the equation of satellite motion [[3\]](#page-9-0) in the inertial coordinate system can be described by the following equation:

$$
\ddot{\vec{r}} = \vec{a}_g + \vec{a}_{ng} + \vec{a}_{emp} \tag{3.1}
$$

where, \vec{r} represents the position vector of the satellite center of mass, \vec{a}_g represents the sum of the acceleration caused by conservative forces on satellites, \vec{a}_{n} represents the sum of the acceleration caused by non-conservative forces on satellites, \vec{a}_{emp} represents empirical acceleration. Aside from the natural forces discussed above, the motion of a spacecraft may also be affected by the action of onboard thruster system. Despite the variety of satellite thruster system, the thruster model can be described using impulsive thrust model and constant thrust model [\[4](#page-9-0)].

1. Impulsive thrust model: the thrust duration is small as compared to the orbit period, so the thrust change with time can be approximated as a function of the pulse function (equal to impulse multiplied the Dirac- δ function). Satellite motion may conveniently be treated as instantaneous velocity increments.

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2. Constant thrust model: the thrust duration is long and even comparable to the orbit period. The equation of satellite motion can be described by the following equation:

$$
\ddot{\vec{r}} = \vec{a}_g + \vec{a}_{ng} + \vec{a}_{emp} + \vec{a}_E \tag{3.2}
$$

where, \vec{a}_E represents the acceleration caused by thrust forces.

Electric thruster has high specific impulse, small thrust and other characteristics. Typical ion propulsion thrust is 20–40 mN with propellant consumption of 0.8 mg/s. The thrust acceleration is expressed as follows:

$$
\vec{a}_E = \frac{\vec{F}_E}{m} = \frac{F_E \vec{e}_E}{m_0 \left(1 + \frac{\dot{m}}{m} t\right)}\tag{3.3}
$$

where, F_E represents engine thrust, m represents satellite mass, m represents the propellant consumption, m_0 represents the satellite initial mass, t represents the thrust duration. Consider electric propulsion engine propellant consumption is relatively small than the mass of the satellite, so we can ignores the satellite mass change during the maneuver. The thrust acceleration is expressed as follows:

$$
\vec{a}_E = \frac{F_E \vec{e}_E}{m} \tag{3.4}
$$

3.2.2 Electric Thrust Estimation Principle and Precision Analysis

3.2.2.1 Orbit Semi-major Axis Changes Under the Tangential Perturbation

Under the two-body problem, the trajectory of the satellite is an ellipse which can be descripted using six orbital elements. But the satellite actually is affected by the sun and moon gravity perturbation, atmospheric drag and so on which makes the orbital elements change with time. The orbit semi-major axis changes [[5\]](#page-9-0) as follows:

$$
\frac{da}{dt} = \frac{2}{n\sqrt{1 - e^2}}\sqrt{1 + 2e\cos f + e^2}U\tag{3.5}
$$

where, a, e, n, f respectively represent the satellite orbit semi-major axis, eccentricity, satellite mean motion, true anomaly, U represent the satellite perturbation in the tangential direction component. While n satisfy the following relation:

$$
n^2 a^3 = \mu \tag{3.6}
$$

where μ represent the constant of Earth gravitation. Taking the (3.6) into ([3.5](#page-2-0)), we obtain:

$$
\frac{da}{dt} = \frac{2a\sqrt{a}}{\sqrt{\mu(1 - e^2)}} \sqrt{1 + 2e\cos f + e^2} U
$$
\n(3.7)

Formula (3.7) shows that, the engine thrust along the velocity direction will cause changes of the satellite orbit semi-major axis over time and the rate of change also is the function of orbital eccentricity, semi-major axis and true anomaly.

3.2.2.2 Model Simplification of Circular LEO Orbit

First, the eccentricity of the orbit nearly circle is approximately zero, formula (3.7) can be simplified to:

$$
\frac{da}{dt} = \frac{2U}{\sqrt{\mu/a^3}}\tag{3.8}
$$

In this case, semi-major axis change is only in relation with the engine thrust and semi-major axis and its partial derivative to the semi-major axis is as follows:

$$
\frac{\partial}{\partial a} \left(\frac{da}{dt} \right) = \frac{3U}{\sqrt{\mu}} \sqrt{a}
$$
\n(3.9)

When the orbit semi-major axis changes Δa , its impact on the rate of change of the semi-major axis measured by the following formula:

$$
\frac{\frac{\partial}{\partial a} \left(\frac{da}{dt}\right) \cdot \Delta a}{da/dt} = \frac{3U\sqrt{a}\Delta a/\sqrt{\mu}}{2U/\sqrt{\mu/a^3}} = \frac{3\Delta a}{2a} \tag{3.10}
$$

When consider the semi-major axis of the LEO satellite is 500 km and Δa is 20 km, the impact above is less than 0.44 %. So we can consider the semi-major axis is a constant in the semi-major axis change computation and the constant can be substituted by the mean satellite semi-major axis a_m . The change of the orbit semi-major axis by time during Δt is as follows:

$$
\Delta a = \frac{2U}{\sqrt{\mu/a_m^3}} \Delta t \tag{3.11}
$$

Then the engine thrust can be estimated by the follows:

$$
F_E = \frac{m\sqrt{\mu/a_m^3}}{2\Delta t} \Delta a \tag{3.12}
$$

The relationship between engine thrust estimation precision and semi-major axis precision is as follows:

$$
\sigma_{F_E} = \frac{m\sqrt{\mu/a_m^3}}{2\Delta t} \sigma_{\Delta a} \tag{3.13}
$$

3.2.3 Electrical Thrust Estimation Based on GNSS Precision Orbit Determination

The problem of electrical thrust estimation has became the semi-major axis determination problem, then we can make it by orbit determination technology. With the development of GNSS technology, GPS receiver is mounted on a wide range of low-orbit satellites for real-time and precision orbit determination [[6\]](#page-9-0). The workflow to estimate the electrical thrust based on GNSS observation data is as follows:

- 1. According electric current or voltage of spaceborne electric thruster we divide the GNSS data into the data before maneuver and the data after the maneuver;
- 2. Use the data before maneuver and the data after the maneuver for precision orbit determination;
- 3. Integrate the two orbit into the whole maneuver arc;
- 4. Compare the two orbit and estimate the engine thrust.

3.3 Simulation of Electric Thrust Effect and Analysis of the Orbit Determination Result

The following first simulate the semi-major axis change under the electric thrust of 40 mN in 790 kg, 500 km orbit altitude of the satellite. Then the dual-frequency GPS data of GRACE-A is used for precision orbit determination to validate the feasibility of the estimation method.

3.3.1 Impact of Electric Thrust on Satellite Orbit

First it simulated a 500 km nearly circle orbit of a satellite whose mass is 790 kg. Then the 40 mN thrust along the velocity direction is added at different time and different lasting time. The compare between the initial and changed orbit is as follows (Figs. 3.1, 3.2, [3.3](#page-6-0)).

Figure 3.2 shows that, semi-major axis change with time under the thrust is approximately linear relationship. Figure [3.3](#page-6-0) shows that the change rate of semimajor axis change becomes faster over time; the change rate of semi-major axis change is variety at different added time of thrust; the change rate of semi-major

axis is approximately a constant value in short time such as 10 min. The relationship between engine thrust estimation precision and semi-major axis precision is as follows:

$$
\sigma_{F_E} = \frac{m\sqrt{\mu/a^3}}{2\Delta t} \sigma_{\Delta a} = 0.0073 \sigma_{\Delta a} \tag{3.14}
$$

3.3.2 Estimation Accuracy of the Electrical Thrust Based on Dual-Frequency GPS Precise Orbit Determination

GRACE (Gravity Recovery and Climate Experiment) program is cooperated by NASA and the German DLR. The project has two 220 km away from the nearly circular orbit satellites. In July 2010, GRACE-A satellite orbit altitude is about 460 km. Using the GPS dual-frequency observation data for precise orbit determination, location accuracy and the corresponding semi-axis precision are shown in Figs. [3.4](#page-7-0) and [3.5.](#page-7-0)

The precise orbit accuracy of GRACE-A using dual-frequency data is 5.183 cm. Semi-axis precision is 2.839 cm and the max semi-axis error is 9.13 cm, so the corresponding thrust estimation precision is respectively 0.296 and 1.33 mN. To a 40 mN thruster the relative precision is 0.74 and 3.33 %.

The calculating results of GRACE-A data shows, when using GPS dual-frequency observation data to estimate the thrust of a 40 mN electric thruster, the calibration accuracy can reach 1 %.

semi-major axis

3.4 Feasibility Using Real-Time Orbit Determination Method for Orbit Control

Precision orbit determination method for electrical thrust estimation can get high accuracy, but do not have the real-time characteristics. The real time orbit determination technology using Kalman arithmetic can estimate satellite state in real time, and it can be used for electric engine thrust Estimation. The GRACE-A result using real time determination algorithms are shown in Figs. [3.6](#page-8-0) and [3.7](#page-8-0).

Figure 3.7 shows that, the filter converges to a stable level of precision an hour later. The orbit accuracy is 1.3 m and semi-axis accuracy is 0.64 m. To a 40 mN thruster the relative precision is approximately 2 %.

3.5 Summary

The method of engine thrust estimation based on the GPS precision orbit determination technology was proposed and proved with GRACE-A data in this paper. The result showed that the method used in 40 mN engine thruster could reach 1 % accuracy in nearly circle LEO satellite. The paper also gives the result of real time orbit determination which has the semi-axis accuracy of 1 m after 1 h filtering.

As the satellite orbit determination is a nonlinear problem, the EKF filter need a long convergence time. Then the use of more advanced filter such as UKF may shorten the time needed for thrust rapid assessment. And also we can add the engine thrust to the filter state vector in order to realize the real time estimation of the electric engine thrust.

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