The Scaling Properties of the Turbulent Wind Using Empirical Mode Decomposition and Arbitrary Order Hilbert Spectral Analysis

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Abstract. In this work, we present an analysis of one-year period measured wind speed in the atmospheric boundary layer from a wind energy production site. We employ a Hilbert-based methodology, namely arbitrary-order Hilbert spectral analysis to characterize the intermittent property of the wind speed in a joint amplitude-frequency space. The measured scaling exponents implies intermittent nature of the wind on mesoscales.

1 Introduction

Turbulent atmospheric wind speed is a complex process with a very large Reynolds number *Re* [1]. This Reynolds number leads to huge intermittency of wind speed fluctuations involving a wide range of temporal and spatial scales (the planet scale to the dissipative scale) [2]. Knowledge of the dynamics of this process is crucial for wind energy applications. Several works have highlighted the universality of the scaling and intermittent properties of turbulent wind speed in the inertial range in

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the atmospheric boundary layer [3, 4, 5, 6]. However the knowledge of variations ranging from minutes to few days corresponding to 1 to 1000 km, i.e. the mesoscale range, is necessary to provide efficient information for management and control of the wind power generation. The studies concerning this scale range are less numerous than that for the small-scale range, due to the possible non-universality of the power law slope in the mesoscale range [7, 8]. Recent works [7, 8] have been dedicated to scaling and multiscaling properties of the atmospheric wind speed in the mesoscale range. They highlighted the multiscaling and intermittency properties of atmospheric surface layer-winds in the mesoscale range. However, as pointed by Huang et al. [18, 20] that the traditional methodologies might be strongly influenced by large-scale energetic structures, e.g, very-large-scale-motion in atmospheric boundary layer [21]. In this paper, the scaling properties of the atmosphere are investigated using a new Hilbert-based approach, namely arbitrary-order Hilbert spectral analysis, in which the large-scale influence can be constrained [9, 18, 20].

2 Arbitrary-Order Hilbert Spectral Analysis

The arbitrary-order Hilbert spectral analysis [9, 10] is an extended version of Hilbert-Huang Transform (HHT) [11, 12], which is designed to characterize the scale invariance of a given time series in a joint amplitude-frequency space. The same as the HHT, it proceeds in two steps: i) empirical mode decomposition, ii) Hilbert spectral analysis. Empirical mode decomposition is an efficient tool to separate a nonlinear and nonstationary time series into a sum of Intrinsic Mode Functions without *a priori* basis as required by traditional Fourier-based method [11, 12, 13]. An IMF must respect two conditions: i) the difference between the number of local extrema and the number of zero crossings must be zero or at most one, ii) the running mean value of two envelopes estimated by the local maxima and local minima is zero [11, 12]. Thus the original signal $u(t)$ is decomposed in a sum of $n-1$ IMF modes with the residual $r_n(t)$

$$u(t) = \sum_{m=1}^{n-1} C_m(t) + r_n(t) \tag{1}$$

To obtain a physical meaningful IMF, this sifting process must be stopped by a certain criterion [11, 12]. More details of the EMD decomposition are given in Refs. [11, 12, 14, 13, 17].

In order to determine the time-frequency energy distribution from the original signal $P(t)$, HSA is performed to each obtained IMF component $C_m(t)$ to extract the instantaneous amplitude $\mathscr{A}_m(t)$ and frequency $\omega_m(t)$ [11, 15, 16]. The Hilbert transform is written as

$$\tilde{C}_m(t) = \frac{1}{\pi} U \int_{-\infty}^{+\infty} \frac{C_m(s)}{t-s} ds \tag{2}$$

with U the Cauchy principle value [15, 16]. One can define an analytical signal $z_m(t)$ for each IMF mode $C_m(t)$, i.e.,

$$z_m(t) = C_m(t) + j\tilde{C}_m(t) = \mathscr{A}_m(t)e^{j\varphi_m(t)} \quad (3)$$

where $\mathscr{A}_m(t) = |z_m(t)| = \sqrt{C_m(t)^2 + \tilde{C}_m(t)^2}$ represents an amplitude and $\varphi_m(t) = \arg(z) = \arctan\left[\frac{\tilde{C}_m(t)}{C_m(t)}\right]$ represents the phase function of IMF modes. Hence, the instantaneous frequency $\omega_m(t)$ is defined from the phase $\varphi_m(t)$

$$\omega_m(t) = \frac{1}{2\pi}\frac{d\varphi_m(t)}{dt} \quad (4)$$

The original signal $P(t)$ can be expressed as

$$P(t) = \mathscr{R}e \sum_{m=1}^{N} \mathscr{A}_m(t)e^{j\varphi_m(t)} = \mathscr{R}e \sum_{m=1}^{N} \mathscr{A}_m(t)e^{j\int_{-\infty}^{t}\omega_m(t)dt} \quad (5)$$

in which $\mathscr{R}e$ is part real [11, 12, 9].

In order to characterize the scale invariant property of a considered signal in the Hilbert frame, Huang et al. [10] proposed an extension of HHT, arbitrary-order Hilbert spectral analysis. This approach has been applied successfully on turbulence data [10, 18], river discharge data [19]. For that, a joint pdf $p(\mathscr{A},\omega)$ of the instantaneous frequency ω and amplitude $\mathscr{A}$ from all these IMF components is extracted [10, 9]. In this frame, the Hilbert marginal spectrum is rewritten as

$$h(\omega) = \int_0^{+\infty} p(\omega,\mathscr{A})\mathscr{A}^2 d\mathscr{A} \quad (6)$$

This expression concerns only the second-order statistical moment. A generalization of this definition is considered to arbitrary-order statistical moment $q \geq 0$ [10, 9, 20]

$$\Psi_q(\omega) = \int_0^{+\infty} p(\omega,\mathscr{A})\mathscr{A}^q d\mathscr{A} \quad (7)$$

Hence, in the Hilbert space, the scale invariance is written as $\Psi_q(\omega) \sim \omega^{-\xi(q)}$, where $\xi(q)$ is the corresponding scaling exponent in the Hilbert space. This scaling exponent function can be linked to scaling exponent function $\zeta(q)$ of structure functions by $\zeta(q) = \xi(q) - 1$ [10, 9, 20]. Here the Hurst exponent could be defined as $H = \xi(1) - 1$ [20].

3 Results

We present the analysis of the measured atmospheric wind speed wtih a sampling rate of 1 Hz, obtained from a wind energy production site of Petit Canal in Guadeloupe, an island in the West Indies, located at 16°15N latitude 60°30 W longitude. Figure 1 illustrates the power spectral density provided by the Fourier transform and the Hilbert transform. The corresponding scaling exponents are $\beta = 1.27$ and $\xi(2) = 1.28$ respectively in Fourier and Hilbert spaces, over a frequency range

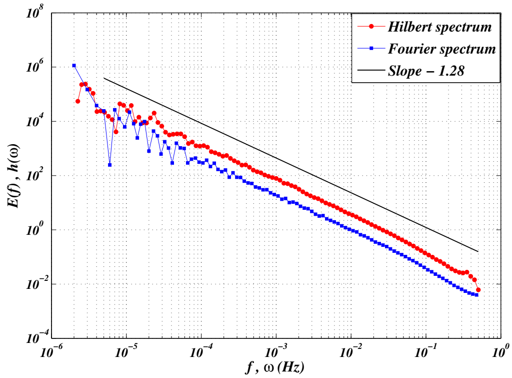

Fig. 1 Power spectral densities for wind speed $u(t)$ provided by the Fourier transform, $E(f)$ (□) and the Hilbert approach, $h(\omega)$ (○). Power law behavior is observed on a large range of scales with a scaling exponent 1.28.

$10^{-5} \leqslant f \leqslant 0.5$ Hz. If one applies the Taylor's frozen hypothesis [22], this corresponds to a spatial scale from 16 m to 800 km. Figure 2 shows the measured mean frequency f_m of each IMF mode. Here we estimate the mean frequency by the following definition [11, 19]:

$$f_m = \frac{\int_0^\infty f E_m(f) df}{\int_0^\infty E_m(f) df} \tag{8}$$

where $E_m(f)$ is the Fourier spectrum of mth IMF mode C_m. It is an energy weighted mean frequency in the Fourier space [11, 19]. We obtain a relation of the form

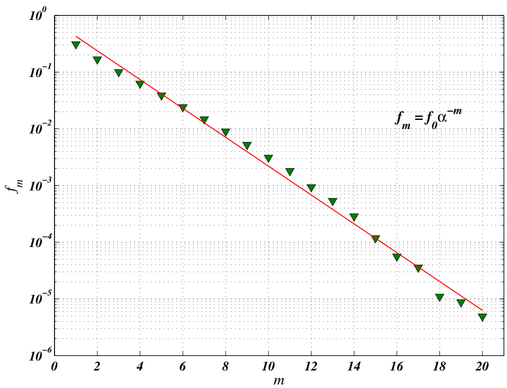

Fig. 2 Representation of the mean frequency f_m versus the mode index m, in log-linear plot. The fitting slope is 0.58, corresponding to $\alpha = 1.8$-times filter bank.

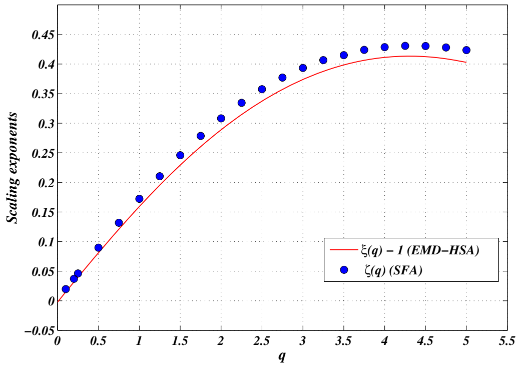

Fig. 3 Scaling exponents measured by the structure function analysis (SFA) and arbitrary-order Hilbert spectral analysis (EMD-HSA)

$f_m = f_0 \alpha^{-m}$, with $f_0 = 1.34$ and $\alpha = 1.8$, meaning that f_m of one IMF is approximately 1.8-times f_m of the next one. This characteristic corresponds to an almost dyadic filter bank in the frequency domain. This filter bank property is deduced by the data set itself, showing the fully adaptiveness of the method [13]. Figure 3 displays the corresponding scaling exponents $\xi(q) - 1$ (red line) obtained from arbitrary-order Hilbert spectral analysis and $\zeta(q)$ ($\circ$) obtained from classical structure function analysis. We can see that the scaling exponents $\xi(q) - 1$ are close to $\zeta(q)$: the obtained curves are concave and nonlinear, validating the intermittent and multiscaling properties of wind speed $u(t)$ in mesoscales.

4 Conclusion and Discussion

In this paper, we apply arbitrary-order Hilbert spectral analysis, to the measured wind speed of atmospheric boundary layer. We observe a power law behavior on a large range of time scales from $10^{-5} < f < 0.5$ Hz. This can be associated to the spatial scales from 16 m to 800 km if one applies the Taylor's frozen hypothesis. It is found that the EMD decomposition acts as a filter bank, which is close to the dyadic one reported by several authors, showing the adaptiveness of the method. Furthermore, the intermittent nature of the wind speed is characterized by the scaling exponents $\zeta(q)$ and $\xi(q)$ respectively provided the classical structure function analysis and the Hilbert-based method. Despite the use of different methodologies (resp. different analysis frameworks), the coincidence of two scaling curves confirms the intermittent nature of the flow in the atmospheric boundary layer. Therefore, this intermittency must be taken into account when a model is proposed.

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References

1. Burton, T., Sharpe, D., Jenkins, N., Bossanyi, E.: Wind energy: handbook. Wiley Online Library (2001)
2. Lovejoy, S., Schertzer, D.: Towards a new synthesis for atmospheric dynamics: Space-time cascades. Atmos. Res. 96, 1–52 (2010)
3. Schmitt, F., Schertzer, D., Lovejoy, S., Brunet, Y.: Estimation of universal multifractal indices for atmospheric turbulent velocity fields. Fractals 1(3), 568–575 (1993)
4. Katul, G.G., Chu, C.R., Parlange, M.B., Albertson, J.D., Ortenburger, T.A.: Low-wavenumber spectral characteristics of speed and temperature in the atmospheric surface layer. J. Geoph. Res. 100(D7), 14,243-14,255 (1995)
5. Schmitt, F.G.: Gusts in intermittent wind turbulence and the dynamics of their return times. In: Peinke, J., Schaumann, P., Barth, S. (eds.) Proceedings of the Euromech Colloquium Wind Energy, pp. 73–79. Springer (2007)
6. Böttcher, F., Barth, S., Peinke, J.: Small and large fluctuations in atmospheric wind speeds. Stoch. Res. Ris. Asses. 21, 299–308 (2007)
7. Lauren, M.K., Menabe, M., Seed, A.W., Austin, G.L.: Characterisation and simulation of the multiscaling properties of the energy-containing scales of horizontal surface layer winds. Boundary Layer Meteorology 90, 21–46 (1999)
8. Muzy, J.F., Baile, R., Poggi, P.: Intermittency of surface-layer wind velocity series in the mesoscale range. Phys. Rev. E 81, 056308 (2010)
9. Huang, Y.: Arbitrary-order hilbert spectral analysis: Definition and application to fully developed turbulence and environmental time series. Ph.D. thesis, Université des Sciences et Technologies de Lille - Lille 1, France & Shanghai University, China (2009)
10. Huang, Y., Schmitt, F., Lu, Z., Liu, Y.: An amplitude-frequency study of turbulent scaling intermittency using hilbert spectral analysis. Europhys. Lett. 84, 40010 (2008)
11. Huang, N.E., Shen, Z., Long, S.R., Wu, M.C., Shih, H.H., Zheng, Q., Yen, N., Tung, C.C., Liu, H.H.: The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proc. R. Soc. London, Ser. A 454(1971), 903–995 (1998)
12. Huang, N.E., Shen, Z., Long, S.R.: A new view of nonlinear water waves: The Hilbert Spectrum. Annu. Rev. Fluid Mech. 31(1), 417–457 (1999)
13. Flandrin, P., Gonçalvès, P.: Empirical Mode Decompositions as Data-Driven Wavelet-Like Expansions. Int. J. Wavelets, Multires. Info. Proc. 2(4), 477–496 (2004)
14. Huang, N.E., Wu, M.L., Long, S.R., Shen, S.S.P., Qu, W., Gloersen, P., Fan, K.L.: A confidence limit for the empirical mode decomposition and Hilbert spectral analysis. Proc. R. Soc. London, Ser. A 459(2003), 2317–2345 (2037)
15. Cohen, L.: Time-frequency analysis. Prentice Hall PTR, Englewood Cliffs (1995)
16. Long, S.R., Huang, N.E., Tung, C.C., Wu, M.L., Lin, R.Q., Mollo-Christensen, E., Yuan, Y.: The Hilbert techniques: an alternate approach for non-steady time series analysis. IEEE Geoscience and Remote Sensing Soc. Lett. 3, 6–11 (1995)
17. Rilling, G., Flandrin, P.: One or two frequencies? The empirical mode decomposition answers. IEEE Trans. Signal Process, 85–95 (2008)
18. Huang, Y., Schmitt, F., Lu, Z., Fougairolles, P., Gagne, Y., Liu, Y.: Second-order structure function in fully developed turbulence. Phys. Rev. E 82, 26319 (2010)

19. Huang, Y., Schmitt, F.G., Lu, Z., Liu, Y.: Analysis of daily river flow fluctuations using empirical mode decomposition and arbitrary order hilbert spectral analysis. J. Hydrol. 373, 103–111 (2009)
20. Huang, Y., Schmitt, F.G., Hermand, J.-P., Gagne, Y., Lu, Z., Liu, Y.: Arbitrary-order Hilbert spectral analysis for time series possessing scaling statistics: comparison study with detrended fluctuation analysis and wavelet leaders. Phys. Rev. E 84, 016208 (2011)
21. Marusic, I., Mathis, R., Hutchins, N.: Predictive model for wall-bounded turbulent flow. Science, 193–196 (2010)
22. Frisch, U.: Turbulence: the legacy of AN Kolmogorov. Cambridge University Press (1995)