Capacity Accumulation Games with Technology Constraints

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Abstract This chapter examines the conduct and performance of large mutually dependent firms. Its objective is to study contractual relationships in a dynamic bilateral monopoly, where producers' investment choices must obey a technology constraint. This is in contrast to previous studies of accumulation games, in which technological interdependence was not explicitly allowed for. The analysis focuses on investment incentives and payoff allocation under two regimes: (1) contracting based on input quantities, and (2) contracting based on final revenues. The technologically feasible equilibrium strategies and the terms of trade that support them are characterized with intuitive necessary conditions which reflect the players' intertemporal trade-offs. To assess the factors that influence efficiency and market power, the chapter presents a linear-quadratic example. Our simulations indicate that contracts based on input quantities generate higher joint payoffs and tend to benefit the input producer.

1 Introduction

Closeness of geographic locations and development of relationship-specific assets may give rise to bilateral monopolies. That is, sometimes two firms become "locked in" to one another and can only operate in a tandem. Examples of exclusive production arrangements are numerous and include: coal mine—thermal power generator, power generator—aluminium smelter, timber mill—furniture factory. Familiarity and the desire to avoid renegotiation costs often drive bilateral monopolists into long-term contractual relationships. While the fundamentals of these contracts are rather durable, the terms of trade may undergo periodic adjustments to account for changes in the operating environment.

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The primary objective of the present chapter is to examine long-term contracting in such dynamic bilateral monopolies.[1] We study an infinite-horizon game between an input supplier and a final good producer, where investment in capacity gives rise to intertemporal spillovers. Firms are technologically interdependent: their choices are bound by a "production function." The players' strategies, as well as the terms of trade that govern surplus allocation, are assumed to have a Markovian structure. Thus, each player takes current prices as given, but also accounts for the consequences of his actions for future surplus shares.

We analyze and compare contract designs that support bilateral trade as a technologically feasible non-cooperative equilibrium. Our focus is on two payoff allocation regimes: (i) contracting based on input quantities, and (ii) contracting based on final revenue. Both arrangements are quite common in the real word. For example, Gazprom (the dominant Russian producer and exporter of natural gas) makes deliveries to European countries on the basis of contracts over input quantities. On the other hand, various software developers have an arrangement with Apple Inc. (a provider of hardware platforms) which specifies shares of final revenues.

We investigate how these regimes affect the division of surplus within the bilateral monopoly, and explore their consequences for firms' behavior. Furthermore, our model sheds light on the efficiency of such contractual arrangements. We show that strategic considerations drive investment away from the plan that maximizes the present value of the stream of joint profits. Last but not least, this study can be used by a central planner to design an allocation mechanism which ensures that neither of the bilateral monopolists will be stuck with unused production capacity in the long run.

Our methodology exhibits two desirable features. First, the Markovian assumption for contracts and investments allows us to endogenously determine the technologically feasible terms of trade.[2] Second, firms' choices are obtained as subgame-perfect equilibrium strategies in a non-cooperative game. This implies that contracts will be *self-enforcing*. Since these strategies specify optimal actions in all periods and for any history, no player will have an incentive to unilaterally deviate at any point in the game.

To quantify and compare the properties of the contractual arrangements, we analyze a numerical example based on a linear-quadratic formulation. It yields terms of trade and investment strategies that are linear in the state variables. The simulations underscore the importance of trading procedures for payoff allocation within the bilateral monopoly: for a given set of parameters, the player who has control over the value of the contractible variable is able to attain a higher surplus share. Moreover, the example shows that the choice of a contractible variable can have important consequences for economic efficiency. Specifically, joint payoffs are higher when contracts are based on input quantities rather than on final revenue.

[1] This chapter draws from and extends Petkov and Krawczyk (2004).

[2] The assumption of Markovian terms of trade relates our chapter to the literature on asset pricing originating from Lucas (1978).

We model bilateral trade using a classical capital accumulation framework that gives rise to intertemporal trade-offs. In this setting, firms are willing to incur instantaneous costs in order to gain a future strategic advantage. Thus, our analysis delivers equilibrium conditions similar to those in the literature on investment games (e.g. Hanig 1986; Reynolds 1987). More generally, our chapter contributes to the theory of dynamic oligopoly (e.g. Maskin and Tirole 1987, 1988), which studies the importance of strategic commitment in market interactions.

Previous work in this field has assumed a particular functional form for payoffs, effectively determining the strategic properties of the game (i.e., whether players' choices would be strategic complements or substitutes). In contrast, our surplus allocating procedure emerges as an equilibrating mechanism that accounts for technological interdependence. It permits the analysis and comparison of various bilateral trading relationships without making ad-hoc assumptions regarding the nature of competition.

There also exists abundant literature on bargaining and bilateral exchange with incomplete contracts which aims to explore the boundaries of the firm and asset ownership. It originates from the seminal work of Coase (1937) and is further developed by Williamson (1975, 1979, 1985), Klein et al. (1978), Grossman and Hart (1986), Hart and Moor (1990), and Whinston (2003). These papers model the allocation of final payoffs as a cooperative bargaining game, usually employing the Nash solution with exogenously fixed bargaining weights. In a more complicated dynamic setup where firms receive *streams of payoffs*, this approach may lack plausibility. It could be argued that intertemporal spillovers will cause bargaining weights to change over time. In particular, their dynamics will reflect forward-looking attempts to strategically influence the future terms of trade. While non-cooperative bargaining games can account for forward-looking behavior, they usually require the specification of restrictive bargaining procedures e.g., alternating offers as in Rubinstein (1982). Our methodology, on the other hand, is amenable to various modifications and environments.

The remainder of the chapter is organized as follows. Section 2 describes the bilateral monopoly setting and our solution concept. Section 3 provides formal analysis of the allocation arrangements and derives general equilibrium conditions. A linear-quadratic algebraic formulation of the model is motivated and solved numerically in Sect. 4. The concluding remarks are presented in Sect. 5.

2 The Setup

2.1 Key Features

The setup below is a mathematical abstraction of the following key aspects of dynamic bilateral monopolies.

- *Technological interdependence*: firms need to operate in a tandem in order to generate surplus.

- *Existence of market power*: each party has the ability to influence the future terms of trade.
- *Strategic conduct*: firms take into account the effect of their investment choices on the opponent's behavior.
- *Noncooperative decision making*: private payoff maximization ensures that contracts will be self-enforcing.

2.2 Technology and Industry Structure

Suppose that, in each period $t = 0, 1, \ldots,$ the market for a final good x is served by a single downstream producer (referred to as firm A). The production process involves the use of an intermediate good y. Input y is supplied by a single upstream firm (referred to as firm B).[3] To manufacture one unit of their goods, the two firms must use one unit of their capacities. The laws of motion of these capacities are given by the state equations

$$x_{t+1} = \mu^A x_t + u_t \tag{1}$$

$$y_{t+1} = \mu^B y_t + v_t, \tag{2}$$

where $u_t, v_t \in R$ are the (non-contractible) investment levels of players A and B, respectively, and $(1 - \mu^A)$ and $(1 - \mu^B)$ are the corresponding depreciation rates. The firms choose u_t, v_t simultaneously and non-cooperatively. They also incur convex investment costs, $C^A(u_t)$ and $C^B(v_t)$. For simplicity we assume that there are no other costs involved in the manufacturing of x and y.

The available technology implies a relationship between input and output quantities that is represented by a production function:

$$x_{t+1} = F(y_{t+1}, x_t, y_t). \tag{3}$$

This dynamic specification accounts for inherently intertemporal phenomena such as congestion, learning-by-doing, etc. The technology constraint imposed by the production function is only required to hold *in equilibrium*. Short-run violations of (3) would not cause discontinuities in the players' payoffs. In particular, we assume that:

- firm A is contractually obligated to purchase the entire production of firm B at the current terms of trade (i.e., firm B can sell y_t units at the current terms of trade even if $x_t < F(y_t, x_{t-1}, y_{t-1})$);

[3] For example, consider a thermal power station (the final good producer) which purchases coal from a nearby coal mine (the intermediate good producer). This power station produces output (i.e. electricity) that is technologically constrained by the available supply of coal, and may as well be the single most important customer of the coal mine. Other examples of such relationships were alluded to in the Introduction.

- firm A has limited reserves of the intermediate good, and thus it can operate in the event of a temporary shortage of y_t (i.e., firm A can produce and sell x_t units even if $x_t > F(y_t, x_{t-1}, y_{t-1})$).

Consequently, both producers can fully exploit their capacities although (3) may not be satisfied in the current period. It should be pointed out that it is in the players' private interest to adhere to this constraint, as any unilateral deviation from the equilibrium will be suboptimal. Even if a violation occurs, it will not persist through time: when firms implement their equilibrium strategies in the following period, (3) will hold again regardless of previous investment decisions.

2.3 Revenue Sharing

For reasons explained in the introduction we study long-term contractual relationships with a time-invariant structure. In our model, this structure is characterized by two elements: (i) an observable and contractible variable z which is agreed on by the firms in a pre-play period, and (ii) a differentiable allocation function g which represents endogenously determined terms of trade.

Our analysis focuses on arrangements where the contractible variable z_t is a function of current input and output levels (capacities) i.e., $z_t \equiv z(x_t, y_t)$. Thus, we assume away trading in futures. Furthermore, we restrict attention to allocation functions that depend only on the current industry state i.e., $g \equiv g(x_t, y_t)$. This assumption implies that firms take the current terms of trade as given, but their investment choices will reflect the desire to affect future surplus allocations. Note that we do not impose restrictions on the functional form of g (other than differentiability): the terms of trade are pinned down by our solution concept.

Let $R(x_t)$ denote the period-t bilateral monopoly revenue generated from the sale of the final good, and let $S^A(z(x_t, y_t), g(x_t, y_t))$ and $S^B(z(x_t, y_t), g(x_t, y_t))$ be the revenue shares of firm A and firm B. The instantaneous period-t payoffs of the two players are defined as

$$\pi_t^A \equiv S^A\big(z(x_t, y_t), g(x_t, y_t)\big) - C^A(u_t) \tag{4}$$

and

$$\pi_t^B \equiv S^B\big(z(x_t, y_t), g(x_t, y_t)\big) - C^B(v_t). \tag{5}$$

Contracts are *allocatively feasible* if

$$S^A\big(z(x_t, y_t), g(x_t, y_t)\big) + S^B\big(z(x_t, y_t), g(x_t, y_t)\big) = R(x_t), \quad \forall x_t, y_t. \tag{6}$$

In order to account for technological interdependence, we also require that surplus allocation induces forward-looking firms to behave in a manner consistent with the production function. Thus, contracts are *technologically feasible* if the terms of trade function $g(x_t, y_t)$ gives rise to equilibrium investment strategies that satisfy (3).

As discussed earlier, we examine two types of revenue-sharing arrangements, distinguished by the specification of the contractible variable.

- Contracting over input quantities: $z_t = y_t$. Under this regime, g can be interpreted as the input price. The instantaneous revenue shares are thus defined as

$$S^A = R(x) - g(x, y)y, \qquad S^B(x, y) = g(x, y)y, \tag{7}$$

where $g(x_t, y_t) : \mathcal{R}_+^2 \to (0, R(x)/y)$.

- Contracting over the realized revenue: $z_t = R(x_t)$. In this case, g can be interpreted as firm B's share of the final revenue. The players' revenue shares are now given by

$$S^A = \big(1 - g(x, y)\big)R(x), \qquad S^B = g(x, y)R(x), \tag{8}$$

where $g(x_t, y_t) : \mathcal{R}_+^2 \to (0, 1)$.

2.4 A Solution Concept

A plausible solution concept for the bilateral monopoly problem at hand needs to allow for strategic behavior of forward-looking players, while also accounting for technological interdependence that requires coordination of investment in order to generate surplus. Given an arbitrary allocation function $g(x, y)$, we have a well-defined two-player dynamic game. We will refer to this game as Γ^g. By assumption, (i) firm A is obligated to purchase the entire production of firm B; and (ii) the final good producer has sufficient reserves to cover temporary input shortages. Therefore, both players can choose any positive or negative investment levels while maintaining full capacity utilization in the short run. Even when the technological constraint is violated in the current period, payoffs would still be given by (4) and (5).

We focus on the Markov perfect equilibrium (MPE) of Γ^g, where the players' strategies are time-invariant functions of the current industry state.[4] Let

$$u_t = f^A(x_t, y_t), \qquad v_t = f^B(x_t, y_t)$$

be the MPE strategies of firm A and firm B when the allocation function is $g(x, y)$. Payoff maximization requires that the players' choices satisfy the Bellman equations for the final good producer,

$$\begin{aligned} V^A(x_t, y_t) = \max_{u_t} \Big\{ & R(x_t) - S^B\big(z(x_t, y_t), g(x_t, y_t)\big) - C^A(u_t) \\ & + \delta V^A\big(\mu^A x_t + u_t, \mu^B y_t + f^B(x_t, y_t)\big)\Big\}, \end{aligned} \tag{9}$$

[4]This solution concept is also known as feedback-Nash equilibrium.

and for the intermediate good producer,

$$V^B(x_t, y_t) = \max_{v_t} \{ S^B\big(z(x_t, y_t), g(x_t, y_t)\big) - C^B(u_t) \\ + \delta V^B\big(\mu^A x_t + f^A(x_t, y_t), \mu^B y_t + v_t\big)\}. \tag{10}$$

Stationarity of Markovian strategies implies that

$$f^A(x_t, y_t) = \arg\max_{u_t} \{ S^B\big(z(x_t, y_t), g(x_t, y_t)\big) - C^B(u_t) \\ + \delta V^A\big(\mu^A x_t + u_t, \mu^B y_t + f^B(x_t, y_t)\big)\}, \tag{11}$$

and

$$f^B(x_t, y_t) = \arg\max_{v_t} \{ S^B\big(z(x_t, y_t), g(x_t, y_t)\big) - C^B(u_t) \\ + \delta V^B\big(\mu^A x_t + f^A(x_t, y_t), \mu^B y_t + v_t\big)\}. \tag{12}$$

As usual, the MPE strategy functions $f^A(x, y)$, $f^B(x, y)$ of the game Γ^g are a fixed point of the mapping defined by (11), (12).

Remark 1 The Bellman equations (9), (10) will have a well-defined interior solution only if the instantaneous payoffs π_t^A, π_t^B can ensure the concavity of their right-hand sides.[5]

The MPE of the game Γ^g yields investment choices that maximize the players' private payoffs for an arbitrary allocation function g. However, the equilibrium strategies are also bound by the constraint of the existing production technology. Thus, we need to focus on contracts that are technologically feasible. In other words, the terms of trade should give rise to MPE investment levels which are consistent with the production function (3) for *all* possible states (x_t, y_t):

$$\mu^A x_t + f^A(x_t, y_t) = F\big(\mu^B y_t + f^B(x_t, y_t), x_t, y_t\big), \quad \forall x_t, y_t. \tag{13}$$

Definition 1 For a pre-agreed contractible variable $z(x_t, y_t)$, a Markovian allocation equilibrium of the bilateral exchange game described above is characterized by an investment strategy $f^A(x, y)$ of the final good producer, an investment strategy $f^B(x, y)$ of the input producer, and an allocation function $g(x, y)$ with the following properties:

1. contingent on $g(x, y)$, the functions $f^A(x, y)$, $f^B(x, y)$ are the MPE strategies of the game Γ^g, i.e. they are obtained as a fixed point of (11), (12);
2. the allocation function $g(x, y)$ is such that $f^A(x, y)$, $f^B(x, y)$ satisfy the technology constraint (13).

[5]For some results on solutions to concave dynamic games see Krawczyk and Tidball (2006).

The solution concept described above has two desirable features.

- The specification of the terms of trade is quite general. The only requirements for the allocation function are differentiability and Markovian structure. Yet, these mild restrictions enable us to pin down the functional form of g. As a result, the number of possible equilibria is reduced, which boosts the predictive power of our model.
- The investment choices $f^A(x, y)$, $f^B(x, y)$ are the Markov perfect (and therefore subgame-perfect) equilibrium strategies of a non-cooperative dynamic game. Firms will choose to follow these strategies in all periods and for all states. This implies that the agreement between the input supplier and the final good producer is self-enforcing: no player will have an incentive to unilaterally breach the contract at any point in the game regardless of past history.

The above features of the proposed solution concept may lead to inefficient outcomes. In particular, the equilibrium plans may fail to maximize the joint surplus available for allocation between the firms. We will discuss how strategic considerations will distort the players' incentives in Sect. 3.2.

3 The Analysis

In this section, we analyze the bilateral monopoly game defined above. To simplify the notation, we will suppress the arguments of payoffs and strategies. In addition, we will use subscripts to denote partial derivatives. For example, φ_i would signify the derivative of a function $\varphi(r_1, \ldots, r_n)$ with respect to its i-th argument: $\varphi_i = \partial\varphi(r_1, \ldots, r_n)/\partial r_i$.[6] Finally, let φ' and φ'' be the values of φ one and two periods ahead, respectively.

3.1 Characterization of the Equilibrium

When firms choose investment levels, they take into account the direct and strategic effects of their decisions for current and future costs and revenues. The considerations that influence these choices are spelled out by Proposition 1. In this proposition, we use Bellman equations (9) and (10) to derive conditions for the players' equilibrium strategies. The derivations are provided in Appendix A.

Proposition 1 *The Markovian equilibrium strategies* $f^A(x, y)$, $f^B(x, y)$ *and the allocation function* $g(x, y)$ *of the bilateral monopoly game satisfy the private Euler*

[6]Since our objective is to derive necessary conditions for the equilibrium strategies and allocation function, we do not need to compute second order derivatives. For a brief discussion on concavity see Remark 1 and footnote 5 on page 167.

equations,

$$-C_1^A + \mu^A \delta C_1^{A\prime} + \delta R_1^{A\prime} + \delta^2 f_1^{B\prime} R_2^{A\prime\prime} - \frac{\delta f_1^{B\prime}(\mu^B + f_2^{B\prime\prime})}{f_1^{B\prime\prime}} \left(-C_1^{A\prime} + \mu^A \delta C_1^{A\prime\prime} + \delta R_1^{A\prime\prime}\right) = 0, \tag{14}$$

$$-C_1^B + \mu^B \delta C_1^{B\prime} + \delta R_2^{B\prime} + \delta^2 f_2^{A\prime} R_1^{B\prime\prime} - \frac{\delta f_2^{A\prime}(\mu^A + f_1^{A\prime\prime})}{f_2^{A\prime\prime}} \left(-C_1^{B\prime} + \mu^B \delta C_1^{B\prime\prime} + \delta R_2^{B\prime\prime}\right) = 0, \tag{15}$$

and the technological feasibility condition,

$$\mu^A x + f^A(x, y) = F\left(\mu^B y + f^B(x, y), x, y\right), \quad \forall x, y, \tag{16}$$

where $R^A(x, y) \equiv S^A(z(x_t, y_t), g(x_t, y_t))$ *and* $R^B(x, y) \equiv S^B(z(x_t, y_t), g(x_t, y_t))$ *are the revenues of firm A and firm B*, *respectively.*

When players contract over input quantities, the firms' marginal revenues R_1^A, R_1^B, R_2^A, R_2^B take the form

$$R_1^A = R_1 - g_1 y, \qquad R_2^A = -g_2 y - g$$
$$R_1^B = g_1 y, \qquad R_2^B = g_2 y + g.$$

If, instead, the arrangement specifies shares of final revenues, R_1^A, R_1^B, R_2^A, R_2^B become

$$R_1^A = R_1 - g_1 x - g, \qquad R_2^A = -g_2 x$$
$$R_1^B = g_1 x + g, \qquad R_2^B = g_2 x.$$

The Euler equations reflect the economic factors underlying the players' decision making process. In the following description of these factors, we focus on firm A's condition (14). The intuition behind Euler equation (15) is analogous.

The left-hand side of (14) incorporates the effects of a marginal change in firm A's current investment on the costs and revenues over its planning horizon. In equilibrium, the change in costs must be equal to the change in revenues. Thus, these effects will sum up to 0. Their interpretation is provided below.

- An increase in current investment generates an additional adjustment cost of C_1^A in the current period. However, capacity carry-over will create cost savings $\mu^A \delta C_1^{A\prime}$ by reducing the need for future investment.
- The additional capacity has a delayed direct effect of $\delta R_1^{A\prime}$ on future revenues through two channels: the contractible variable, $S_1^{A\prime} z_1'$, and the terms of trade, $S_2^{A\prime} g_1'$.

- Furthermore, a marginal change in firm A's investment will invoke a reaction from its opponent in the subsequent period, which will have repercussions for firm B's future capacity, y'', and induce a strategic revenue effect, $\delta^2 f_1^{B\prime} R_2^{A\prime\prime}$.
- Finally, firm A anticipates B's reaction, and will concurrently re-adjust its capacity. This gives rise to additional strategic cost effects $\delta f_1^{B\prime}(\mu^B + f_2^{B\prime\prime})(C_1^{A\prime} - \mu^A \delta C_1^{A\prime\prime})/f_2^{B\prime\prime}$ and delayed revenue effects $-\delta f_1^{B\prime}(\mu^B + f_2^{B\prime\prime}) R_1^{A\prime\prime}/f_2^{B\prime\prime}$.

Differentiating the technology constraint with respect to x and y yields

$$\mu^A + f_1^A = F_1 f_1^B + F_2,$$
$$f_2^A = F_1\big(f_2^B + \mu^B\big) + F_3.$$

Substitution in (15) shows that technologically feasible input choices must also satisfy

$$-C_1^B + \mu^B \delta C_1^{B\prime} + \delta R_2^{B\prime} + \delta^2\big(F_1'\big(f_2^{B'} + \mu^B\big) + F_3'\big) \times \left(R_1^{B\prime\prime} - \frac{\delta(F_1'' f_1^{B\prime\prime} + F_2')}{F_1''(f_2^{B\prime\prime} + \mu^B) + F_3''}\big(-C_1^{B\prime} + \mu^B \delta C_1^{B\prime\prime} + \delta R_2^{B\prime\prime}\big)\right) = 0. \quad (17)$$

3.2 Efficiency Results

In this subsection we compare the Markovian allocation equilibrium characterized in Sect. 3.1 to an efficient outcome. For presentation purposes, we define efficiency as maximization of the net present value of the stream of joint surplus. The question of interest is whether the strategies defined by (14), (15) and (16) are consistent with this type of efficient behavior.

Let $w_t \equiv R(x_t) - C^A(u_t) - C^B(v_t)$ denote the period-t joint surplus generated by the bilateral monopoly.

Definition 2 A bilateral trade contract (z, g) is efficient if it maximizes the net present value of the stream of joint surplus $W = \sum_{t=1}^{\infty} \delta^{t-1} w_t$.

Now we show that the equilibrium input and output paths generated by (14), (15) and (16) are usually inefficient. Although firms fully utilize the available capacities in each period, their decisions are distorted by strategic considerations regarding the allocation of future revenues. In the game studied here, the Markovian structure of contracts implies that current investments have repercussions for the subsequent values of the terms of trade and the contractible variable. Forward-looking players take these repercussions into account. They choose their actions to bias the division of future surplus in their favor, thus creating inefficiencies. This phenomenon is analogous to the problem of "hold-up," which often arises in the literature on incomplete contracts (see Grossman and Hart 1986; Klein et al. 1978).

To illuminate the underlying reasons, we now derive necessary conditions for the efficient investment policies of firm A and firm B, $h^A(x, y)$ and $h^B(x, y)$. We will argue that the private Euler equations (14), (15) are generically inconsistent with these efficiency conditions.

Technological feasibility requires that

$$u_t = F\big(\mu^B y_t + v_t, x_t, y_t\big) - \mu^A x_t. \tag{18}$$

Substitution of (18) allows us to write the period-t joint surplus as

$$w(x_t, y_t) = R(x_t) - C^A\big(F\big(\mu^B y_t + v_t, x_t, y_t\big) - \mu^A x_t\big) - C^B(v_t). \tag{19}$$

Therefore, efficient investment in input capacity will solve the Bellman equation

$$\begin{aligned} W(x_t, y_t) = \max_{v_t}\big\{&R(x_t) - C^A\big(F\big(\mu^B y_t + v_t, x_t, y_t\big) - \mu^A x_t\big) \\ &- C^B(v_t) + \delta W\big(F\big(\mu^B y_t + v_t, x_t, y_t\big), \mu^B y_t + v_t\big)\big\}. \end{aligned} \tag{20}$$

In Appendix B, we use (20) to derive the following Euler equation:

$$\begin{aligned} &F_1\big(\delta R_1' - C_1^A + \delta\mu^A C_1^{A\prime}\big) - C_1^B + \delta\mu^B C_1^{B\prime} \\ &\quad + \delta\big(F_1 F_2' + F_3'\big)\big(\delta R_1'' - C_1^{A\prime} + \delta\mu^A C_1^{A\prime\prime}\big) \\ &\quad - \frac{\delta F_2''(F_1 F_2' + F_3')}{(F_1' F_2'' + F_3'')}\big(F_1'\big(\delta R_1'' - C_1^{A\prime} + \delta\mu^A C_1^{A\prime\prime}\big) - C_1^{B\prime} + \delta\mu^B C_1^{B\prime\prime}\big) \\ &= 0. \end{aligned} \tag{21}$$

Condition (18) in conjunction with (21) defines the efficient investment policies $h^A(x, y)$ and $h^B(x, y)$.

In general, the equilibrium strategy functions that solve Euler equations (14) and (15) will fail to satisfy (21). Hence, the contracts considered here are typically inefficient. As already discussed, this failure is driven by the strategic nature of interactions. More precisely, both players will attempt to influence the future contractible variable and terms of trade in order to increase their payoffs. Such behavior precludes the implementation of efficiency: firms will deviate from the investment policies that maximize joint surplus.

To see this, suppose that the efficient investment policies $h^A(x, y)$ and $h^B(x, y)$ are in fact solutions to (14) and (15). Then we can rewrite the private Euler equations as

$$\delta R_1' - C_1^A + \delta\mu^A C_1^{A\prime} = D^A, \qquad -C_1^B + \delta\mu^B C_1^{B\prime} = D^B,$$

where D^A and D^B embody the strategic payoff effects:

$$\begin{aligned} D^A = {}&\delta\big(S_1^{B\prime} z_1' + S_2^{B\prime} g_1'\big) - \delta^2 h_1^{B\prime} R_2^{A\prime\prime} \\ &+ \frac{\delta h_1^{B\prime}(\mu^B + h_2^{B\prime\prime})}{h_1^{B\prime\prime}}\big(-C_1^{A\prime} + \mu^A \delta C_1^{A\prime\prime} + \delta R_1^{A\prime\prime}\big) \end{aligned} \tag{22}$$

$$D^B = -\delta\big(S_1^{B\prime} z_2' + S_2^{B\prime} g_2'\big) - \delta^2 h_2^{A\prime} R_1^{B\prime\prime} + \frac{\delta h_2^{A\prime}(\mu^A + h_1^{A\prime\prime})}{h_2^{A\prime\prime}}\big(-C_1^{B\prime} + \mu^B \delta C_1^{B\prime\prime} + \delta R_2^{B\prime\prime}\big). \tag{23}$$

Note that (22) and (23) are generically non-zero so long as the players follow state-contingent investment rules. Substitution in (21) shows that if the equilibrium was efficient, it would necessarily imply the following condition:

$$F_1 D^A + D^B + \delta\big(F_1 F_2' + F_3'\big) D^{A\prime} - \frac{\delta F_2''(F_1 F_2' + F_3')}{(F_1' F_2'' + F_3'')}\big(F_1' D^{A\prime} + D^{B\prime}\big) = 0. \tag{24}$$

However, if $D^A \neq 0$ and $D^B \neq 0$, (24) will typically fail to hold.

The features of our model suggest that these inefficiencies would arise for most allocation mechanisms. Nevertheless, some contracts might generate a smaller dead-weight loss than others. We address this issue in Sect. 4, where we compute the Markovian allocation equilibrium in a linear-quadratic example. It allows us to compare the welfare properties of the different contractual arrangements.

4 A Linear-Quadratic Formulation

In this section, we consider a linear-quadratic formulation of the bilateral monopoly game defined above. As expected, it yields a computationally tractable equilibrium with linear strategies and a linear allocation function. We use numerical simulations to explore how the contractual arrangements affect the size and the allocation of joint surplus.

4.1 Payoffs

We believe that quadratic investment costs can adequately capture the observation that marginal costs are often positively related to the magnitude of capacity adjustments. In conjunction with the assumptions of linear final revenue and production function, this specification delivers a computable equilibrium characterized by a linear allocation function and investment strategies.

Specifically, suppose that the investment costs incurred by firm A and firm B are given by

$$C^A(u) = \frac{\psi^A}{2} u^2, \qquad C^B(v) = \frac{\psi^B}{2} v^2. \tag{25}$$

Furthermore, assume a perfectly elastic demand for the final good, which translates into a linear revenue function

$$R(x) = px. \tag{26}$$

The final assumption concerns the technology constraint. Suppose that it has the form

$$x_{t+1} = a + by_{t+1} + dx_t + ey_t. \tag{27}$$

That is, output is produced according to a linear production function.

The above structure motivates the conjecture that the equilibrium strategies are linear in the state variables:

$$u_t = \alpha^A + \beta_1^A x_t + \beta_2^A y_t \tag{28}$$

$$v_t = \alpha^B + \beta_1^B x_t + \beta_2^B y_t. \tag{29}$$

Moreover, we guess a linear allocation function:

$$g(x, y) = \eta + \theta_1 x + \theta_2 y. \tag{30}$$

These conjectures, together with (25) and (26), suggest that contracting over input quantities would generate instantaneous payoffs

$$\begin{aligned} \pi_t^A &= px_t - (\eta + \theta_1 x_t + \theta_2 y_t) y_t - \frac{\psi^A}{2} u_t^2, \\ \pi_t^B &= (\eta + \theta_1 x_t + \theta_2 y_t) y_t - \frac{\psi^B}{2} v_t^2, \end{aligned} \tag{31}$$

while contracting over final revenues would yield

$$\begin{aligned} \pi_t^A &= px_t - (\eta + \theta_1 x_t + \theta_2 y_t) px_t - \frac{\psi^A}{2} u_t^2, \\ \pi_t^B &= (\eta + \theta_1 x_t + \theta_2 y_t) px_t - \frac{\psi^B}{2} v_t^2. \end{aligned} \tag{32}$$

4.2 Existence

Non-negative input and output paths and an allocation profile $\{x_t, y_t, g_t\}_{t=0}^{\infty}$ constitute an equilibrium if the investment strategies and the supporting allocation function satisfy the players' necessary conditions (14), (15), as well as the technology constraint (16). Furthermore, firms will engage in bilateral trade only if it is mutually beneficial. Hence, the equilibrium path must be such that

$$V^A(x_t, y_t) \geq 0, \qquad V^B(x_t, y_t) \geq 0, \quad t = 0, 1, \ldots$$

Finally, we would like our solutions to be dynamically stable. This requirement suggests that the eigenvalues of the capacity transition matrix

$$\begin{pmatrix} \mu^A + \beta_1^A & \beta_2^A \\ \beta_1^B & \mu^B + \beta_2^B \end{pmatrix}$$

should be inside the unit circle.

An equilibrium with these properties may not always exist in the above linear-quadratic setting. The issue of non-existence is particularly serious when contracts are based on final revenues. For some parameter values, the players' problems may not have interior solutions. For example, suppose that firm B's investment is very productive (i.e., b is high) or that it is rather durable (i.e., μ^B is high). Then the linear technology constraint (27) might never be satisfied so long as the allocation function features a positive θ_2. On the other hand, a negative θ_2 would imply that the surplus share of the intermediate good producer, $S_t^B = (\eta + \theta_1 x_t + \theta_2 y_t) p x_t$, is decreasing in his capacity y_t. But then firm B's marginal return on investment is negative, and so Euler equation (36) will not have a solution. Moreover, existence of equilibrium requires concavity of the right hand sides of the private Bellman equations, which may be lost for some parameter values (e.g., when the output price p is low).

4.3 Simulations

4.3.1 Numerical Results

To compute the Markovian allocation equilibrium, we substitute the expressions for payoffs (31), (32) and conjectures (28), (29), (30) in equations (14), (15) and (16).

- When firms contract over input quantities, the private Euler equation (14) and (15) become

$$\begin{aligned}&-\psi^A u_t + \mu^A \delta u_{t+1} + \delta(p - \theta_1 y_{t+1}) - \delta^2 \beta_1^B (\eta + \theta_1 x_{t+2} + 2\theta_2 y_{t+2})\\ &\quad - \delta\big(\mu^B + \beta_2^B\big)\big(-u_{t+1} + \mu^A \delta u_{t+2} + \delta(p - \theta_1 y_{t+2})\big) = 0 \end{aligned}\tag{33}$$

  and

$$\begin{aligned}&-\psi^B v_t + \mu^B \delta \psi^B v_{t+1} + \delta(\eta + \theta_1 x_{t+1} + 2\theta_2 y_{t+1}) + \delta^2 \beta_2^A \theta_1 y_{t+2}\\ &\quad - \delta\big(\mu^A + \beta_1^A\big)\big(-\psi^B v_{t+1} + \mu^B \delta \psi^B v_{t+2} + \delta(\eta + \theta_1 x_{t+2} + 2\theta_2 y_{t+2})\big) = 0. \end{aligned}\tag{34}$$

- If, instead, contracts are based on the final revenues, (14) and (15) are given by

$$\begin{aligned}&-\psi^A u_t + \mu^A \delta \psi^A u_{t+1} + \delta p(1 - \eta - 2\theta_1 x_{t+1} - \theta_2 y_{t+1}) - \delta^2 \beta_1^B \theta_2 p x_{t+2}\\ &\qquad - \delta\big(\mu^B + \beta_2^B\big)\\ &\qquad \times \big(-\psi^A u_{t+1} + \mu^A \delta \psi^A u_{t+2} + \delta p(1 - \eta - 2\theta_1 x_{t+2} - \theta_2 y_{t+2})\big)\\ &= 0 \end{aligned}\tag{35}$$

Table 1 The base case parameter set

Firm A	Firm B	Technology	Other
$\mu^A = 0.8$	$\mu^B = 0.5$	$a = 30, b = 0.35$	$p = 30$
$\psi^A = 0.4$	$\psi^B = 0.4$	$d = 0.2, e = -0.25$	$\delta = 0.9$

and

$$-\psi^B v_t + \mu^B \delta \psi^B v_{t+1} + \delta\theta_2 p x_{t+1} + \delta^2 \beta_2^A p(\eta + 2\theta_1 x_{t+2} + \theta_2 y_{t+2})$$
$$- \delta\big(\mu^A + \beta_1^A\big)\big(-\psi^B v_{t+1} + \mu^B \delta\psi^B v_{t+2} + \delta\theta_2 p x_{t+2}\big) = 0. \quad (36)$$

- Under both arrangements, technological feasibility requires that the equilibrium strategies satisfy the constraint (16) for all possible states:

$$\mu^A x_t + \alpha^A + \beta_1^A x_t + \beta_2^A y_t = a + b\big(\mu^B y_t + \alpha^B + \beta_1^B x_t + \beta_2^B y_t\big) + dx_t + ey_t. \quad (37)$$

Applying the method of undetermined coefficients to (33)–(37) yields nine equations that pin down the nine unknown variables needed to fully describe the equilibrium in each of the two regimes. These variables are the MPE strategy parameters α^A, β_1^A, β_2^A and α^B, β_1^B, β_2^B, as well as the parameters of the allocation function, η, θ_1, θ_2.

We can now compute a baseline scenario with parameters as listed in Table 1. Specifically, we assume a negative value of the parameter e in (27) to reflect input congestion and capacity deterioration caused by previous heavy workloads; the positive sign of d can be attributed to learning-by-doing effects.

The corresponding Markovian allocation equilibria are presented in Table 2 for contracts based on input quantities, and in Table 3—for contracts based on final revenues. The first rows in these tables show the results for our baseline scenario. Then we vary one parameter at a time (denoted by bold font in the first column of each table) and obtain the rest of the table entries.

Figure 1 and Fig. 2 depict the equilibrium transition paths for contracting over input quantities and contracting over final revenues, respectively, as well as the efficient input and output paths. Figure 3 illustrates the evolution of the terms of trade under the two regimes. The initial conditions are set at $y_0 = 10$, $x_0 = 10$. Then Fig. 4 shows a number of transition paths for various values of y_0 and x_0. The input and output paths each converge to an identical (technologically feasible) steady state, as would be expected in a Markov perfect equilibrium.

4.3.2 Equilibrium Investment Strategies

Each player's investment choice will reflect his direct and strategic considerations regarding current and future profits. These considerations are influenced by the terms of trade function, which is constructed so that the technology constraint holds

Table 2 *Contracting over input quantities*. The table shows the equilibrium strategies and allocation function, the steady-state output and input quantities, total surplus, value of the allocation function and the surplus share of firm B

	Strategy of firm A	Strategy of firm B	Allocation function	Steady state
$\mu^A = 0.8$, $\mu^B = 0.5$	$\alpha^A = 45.86$	$\alpha^B = 45.30$	$\eta = -0.775$	$\hat{x} = 48.63$
$\psi^A = 0.4$, $\psi^B = 0.4$	$\beta_1^A = -0.541$	$\beta_1^B = 0.167$	$\theta_1 = 0.008$	$\hat{y} = 89.06$
$p = 30$, $\delta = 0.95$	$\beta_2^A = -0.110$	$\beta_2^B = -0.010$	$\theta_2 = 0.009$	$\hat{w} = 1043$
$a = 30$, $b = 0.35$				$\hat{g} = 0.448$
$d = 0.2$, $e = -0.25$				$\hat{s}^B = 24.6\ \%$
$\mu^A = 0.8$, $\boldsymbol{\mu^B = 0.55}$	$\alpha^A = 39.04$	$\alpha^B = 25.83$	$\eta = -554$	$\hat{x} = 45.12$
$\psi^A = 0.4$, $\psi^B = 0.4$	$\beta_1^A = -0.565$	$\beta_1^B = 0.101$	$\theta_1 = 0.012$	$\hat{y} = 60.93$
$p = 30$, $\delta = 0.95$	$\beta_2^A = -0.074$	$\beta_2^B = -0.048$	$\theta_2 = 0.006$	$\hat{w} = 1187$
$a = 30$, $b = 0.35$				$\hat{g} = 0.364$
$d = 0.2$, $e = -0.25$				$\hat{s}^B = 28.8\ \%$
$\mu^A = 0.8$, $\mu^B = 0.5$	$\alpha^A = 42.43$	$\alpha^B = 35.40$	$\eta = -0.666$	$\hat{x} = 46.52$
$\psi^A = 0.4$, $\boldsymbol{\psi^B = 0.45}$	$\beta_1^A = -0.551$	$\beta_1^B = 0.139$	$\theta_1 = 0.010$	$\hat{y} = 72.20$
$p = 30$, $\delta = 0.95$	$\beta_2^A = -0.103$	$\beta_2^B = -0.081$	$\theta_2 = 0.009$	$\hat{w} = 1085$
$a = 30$, $b = 0.35$				$\hat{g} = 0.426$
$d = 0.2$, $e = -0.25$				$\hat{s}^B = 27.8\ \%$
$\boldsymbol{\mu^A = 0.85}$, $\mu^B = 0.5$	$\alpha^A = 48.85$	$\alpha^B = 53.85$	$\eta = -1.007$	$\hat{x} = 50.40$
$\psi^A = 0.4$, $\psi^B = 0.4$	$\beta_1^A = -0.585$	$\beta_1^B = 0.185$	$\theta_1 = 0.008$	$\hat{y} = 103.23$
$p = 30$, $\delta = 0.95$	$\beta_2^A = -0.114$	$\beta_2^B = -0.112$	$\theta_2 = 0.010$	$\hat{w} = 968$
$a = 30$, $b = 0.35$				$\hat{g} = 0.431$
$d = 0.2$, $e = -0.25$				$\hat{s}^B = 12.2\ \%$
$\mu^A = 0.8$, $\mu^B = 0.5$	$\alpha^A = 48.42$	$\alpha^B = 52.62$	$\eta = -0.969$	$\hat{x} = 50.15$
$\boldsymbol{\psi^A = 0.42}$, $\psi^B = 0.4$	$\beta_1^A = -0.536$	$\beta_1^B = 0.181$	$\theta_1 = 0.008$	$\hat{y} = 101.22$
$p = 30$, $\delta = 0.95$	$\beta_2^A = -0.113$	$\beta_2^B = -0.110$	$\theta_2 = 0.010$	$\hat{w} = 971$
$a = 30$, $b = 0.35$				$\hat{g} = 0.427$
$d = 0.2$, $e = -0.25$				$\hat{s}^B = 13.4\ \%$
$\mu^A = 0.8$, $\mu^B = 0.5$	$\alpha^A = 43.97$	$\alpha^B = 39.92$	$\eta = -0.429$	$\hat{x} = 47.47$
$\psi^A = 0.4$, $\psi^B = 0.4$	$\beta_1^A = -0.542$	$\beta_1^B = 0.167$	$\theta_1 = 0.007$	$\hat{y} = 79.77$
$\boldsymbol{p = 35}$, $\delta = 0.95$	$\beta_2^A = -0.110$	$\beta_2^B = -0.010$	$\theta_2 = 0.008$	$\hat{w} = 1325$
$a = 30$, $b = 0.35$				$\hat{g} = 0.538$
$d = 0.2$, $e = -0.25$				$\hat{s}^B = 43.4\ \%$
$\mu^A = 0.8$, $\mu^B = 0.5$	$\alpha^A = 50.53$	$\alpha^B = 64.17$	$\eta = -1.053$	$\hat{x} = 47.39$
$\psi^A = 0.4$, $\psi^B = 0.4$	$\beta_1^A = -0.530$	$\beta_1^B = 0.218$	$\theta_1 = 0.005$	$\hat{y} = 113.04$
$p = 30$, $\delta = 0.95$	$\beta_2^A = -0.141$	$\beta_2^B = -0.159$	$\theta_2 = 0.012$	$\hat{w} = 764$
$a = 30$, $\boldsymbol{b = 0.32}$				$\hat{g} = 0.540$
$d = 0.2$, $e = -0.25$				$\hat{s}^B = 16.8\ \%$
$\mu^A = 0.8$, $\mu^B = 0.5$	$\alpha^A = 36.33$	$\alpha^B = 32.37$	$\eta = -0.371$	$\hat{x} = 39.37$
$\psi^A = 0.4$, $\psi^B = 0.4$	$\beta_1^A = -0.542$	$\beta_1^B = 0.167$	$\theta_1 = 0.008$	$\hat{y} = 64.92$
$p = 30$, $\delta = 0.95$	$\beta_2^A = -0.110$	$\beta_2^B = -0.010$	$\theta_2 = 0.009$	$\hat{w} = 957$
$\boldsymbol{a = 25}$, $b = 0.35$				$\hat{g} = 0.553$
$d = 0.2$, $e = -0.25$				$\hat{s}^B = 46.2\ \%$

Table 3 *Contracting over final revenue shares*. The table shows the equilibrium strategies, allocation function and the steady-state output and input quantities, total surplus, value of the allocation function and the surplus share of firm B

	Strategy of firm A	Strategy of firm B	Allocation function	Steady state
$\mu^A = 0.8$, $\mu^B = 0.5$ $\psi^A = 0.4$, $\psi^B = 0.4$ $p = 30$, $\delta = 0.95$ $a = 30$, $b = 0.35$ $d = 0.2$, $e = -0.25$	$\alpha^A = 39.52$ $\beta_1^A = -0.500$ $\beta_2^A = -0.178$	$\alpha^B = 27.21$ $\beta_1^B = 0.286$ $\beta_2^B = -0.293$	$\eta = -7.41$ $\theta_1 = 0.504$ $\theta_2 = -0.032$	$\hat{x} = 43.76$ $\hat{y} = 50.04$ $\hat{w} = 1172$ $\hat{g} = 13.07$ $\hat{s}^B = 45.1$ %
$\mu^A = 0.8$, $\boldsymbol{\mu^B = 0.55}$ $\psi^A = 0.4$, $\psi^B = 0.4$ $p = 30$, $\delta = 0.95$ $a = 30$, $b = 0.35$ $d = 0.2$, $e = -0.25$	$\alpha^A = 39.04$ $\beta_1^A = -0.506$ $\beta_2^A = -0.182$	$\alpha^B = 25.82$ $\beta_1^B = 0.268$ $\beta_2^B = -0.357$	$\eta = -7.88$ $\theta_1 = 0.558$ $\theta_2 = -0.063$	$\hat{x} = 43.30$ $\hat{y} = 46.36$ $\hat{w} = 1197$ $\hat{g} = 13.35$ $\hat{s}^B = 44.4$ %
$\mu^A = 0.8$, $\mu^B = 0.5$ $\psi^A = 0.4$, $\boldsymbol{\psi^B = 0.45}$ $p = 30$, $\delta = 0.95$ $a = 30$, $b = 0.35$ $d = 0.2$, $e = -0.25$	$\alpha^A = 38.74$ $\beta_1^A = -0.508$ $\beta_2^A = -0.183$	$\alpha^B = 24.97$ $\beta_1^B = 0.262$ $\beta_2^B = -0.210$	$\eta = -8.14$ $\theta_1 = 0.570$ $\theta_2 = -0.052$	$\hat{x} = 43.10$ $\hat{y} = 44.82$ $\hat{w} = 1165$ $\hat{g} = 14.08$ $\hat{s}^B = 44.4$ %
$\boldsymbol{\mu^A = 0.85}$, $\mu^B = 0.5$ $\psi^A = 0.4$, $\psi^B = 0.4$ $p = 30$, $\delta = 0.95$ $a = 30$, $b = 0.35$ $d = 0.2$, $e = -0.25$	$\alpha^A = 39.20$ $\beta_1^A = -0.545$ $\beta_2^A = -0.176$	$\alpha^B = 26.28$ $\beta_1^B = 0.300$ $\beta_2^B = -0.288$	$\eta = -8.17$ $\theta_1 = 0.507$ $\theta_2 = -0.025$	$\hat{x} = 43.75$ $\hat{y} = 50.00$ $\hat{w} = 1179$ $\hat{g} = 12.76$ $\hat{s}^B = 43.5$ %
$\mu^A = 0.8$, $\mu^B = 0.5$ $\boldsymbol{\psi^A = 0.42}$, $\psi^B = 0.4$ $p = 30$, $\delta = 0.95$ $a = 30$, $b = 0.35$ $d = 0.2$, $e = -0.25$	$\alpha^A = 39.19$ $\beta_1^A = -0.497$ $\beta_2^A = -0.175$	$\alpha^B = 26.25$ $\beta_1^B = 0.296$ $\beta_2^B = -0.286$	$\eta = -8.04$ $\theta_1 = 0.503$ $\theta_2 = -0.026$	$\hat{x} = 43.73$ $\hat{y} = 49.82$ $\hat{w} = 1172$ $\hat{g} = 12.66$ $\hat{s}^B = 43.3$ %
$\mu^A = 0.8$, $\mu^B = 0.5$ $\psi^A = 0.4$, $\psi^B = 0.4$ $\boldsymbol{p = 35}$, $\delta = 0.95$ $a = 30$, $b = 0.35$ $d = 0.2$, $e = -0.25$	$\alpha^A = 41.79$ $\beta_1^A = -0.500$ $\beta_2^A = -0.178$	$\alpha^B = 33.69$ $\beta_1^B = 0.286$ $\beta_2^B = -0.293$	$\eta = -5.45$ $\theta_1 = 0.504$ $\theta_2 = -0.032$	$\hat{x} = 44.83$ $\hat{y} = 58.61$ $\hat{w} = 1381$ $\hat{g} = 15.30$ $\hat{s}^B = 52.5$ %
$\mu^A = 0.8$, $\mu^B = 0.5$ $\psi^A = 0.4$, $\psi^B = 0.4$ $p = 30$, $\delta = 0.95$ $a = 30$, $\boldsymbol{b = 0.32}$ $d = 0.2$, $e = -0.25$	$\alpha^A = 38.70$ $\beta_1^A = -0.507$ $\beta_2^A = -0.182$	$\alpha^B = 27.18$ $\beta_1^B = 0.292$ $\beta_2^B = -0.287$	$\eta = -7.58$ $\theta_1 = 0.502$ $\theta_2 = -0.019$	$\hat{x} = 41.88$ $\hat{y} = 50.06$ $\hat{w} = 1117$ $\hat{g} = 12.50$ $\hat{s}^B = 44.8$ %
$\mu^A = 0.8$, $\mu^B = 0.5$ $\psi^A = 0.4$, $\psi^B = 0.4$ $p = 30$, $\delta = 0.95$ $\boldsymbol{a = 25}$, $b = 0.35$ $d = 0.2$, $e = -0.25$	$\alpha^A = 35.20$ $\beta_1^A = -0.500$ $\beta_2^A = -0.178$	$\alpha^B = 29.15$ $\beta_1^B = 0.286$ $\beta_2^B = -0.293$	$\eta = -4.21$ $\theta_1 = 0.504$ $\theta_2 = -0.032$	$\hat{x} = 37.53$ $\hat{y} = 50.26$ $\hat{w} = 988$ $\hat{g} = 13.13$ $\hat{s}^B = 54.0$ %

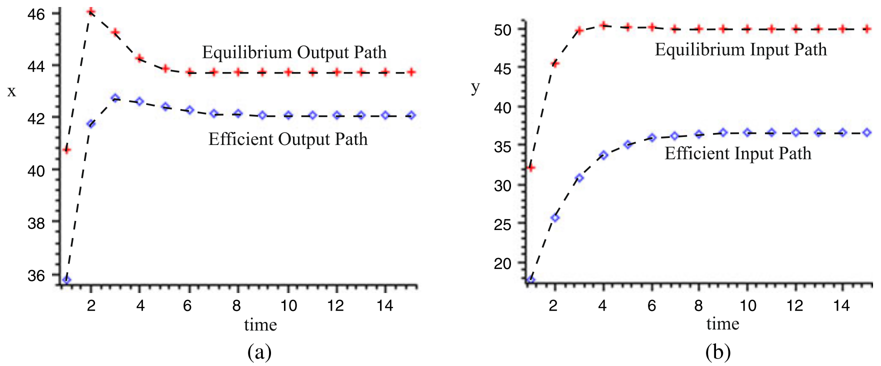

Fig. 1 *Contracting over input quantities* (**a** output paths, **b** input paths). Panel **a** illustrates the equilibrium and efficient output paths. Panel **b** illustrates the equilibrium and efficient input paths. The calculations are based on parameters $p = 30$, $\delta = 0.95$, $\psi^A = 0.4$, $\psi^B = 0.4$, $\mu^A = 0.8$, $\mu^B = 0.5$, $a = 20$, $b = 0.35$, $d = 0.2$, $e = -0.25$ and initial conditions $x_0 = y_0 = 10$

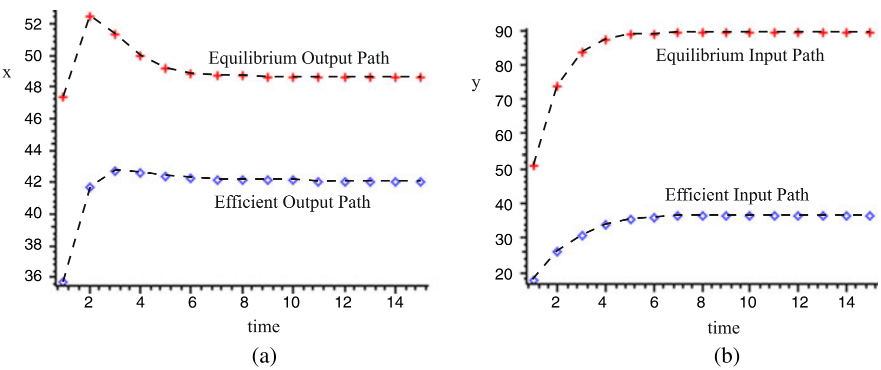

Fig. 2 *Contracting over final revenue shares* (**a** output paths, **b** input paths). Panel **a** illustrates the equilibrium and efficient output paths. Panel **b** illustrates the equilibrium and efficient input paths. The calculations are based on parameters $p = 30$, $\delta = 0.95$, $\psi^A = 0.4$, $\psi^B = 0.4$, $\mu^A = 0.8$, $\mu^B = 0.5$, $a = 20$, $b = 0.35$, $d = 0.2$, $e = -0.25$ and initial conditions $x_0 = y_0 = 10$

for any (x_t, y_t). We focus on strategies that are linear in the state variables.[7] Specifically, they are defined by (28) and (29). The strategy parameters β_1^A, β_2^B, β_2^A, β_1^B capture the effect of the observed output and input capacities x_t, y_t on the players' investment decisions. As Table 2 and Table 3 show, all numerical examples studied here yield $\beta_1^A < 0$, $\beta_2^B < 0$ and $\beta_2^A < 0$, $\beta_1^B > 0$. We contend that the signs of these parameters are in line with economic intuition. Our reasoning is explained below.

[7] As noted earlier, such equilibria may not always exist. Also, there could be equilibria involving non-linear strategies, see e.g. Haurie et al. (2012).

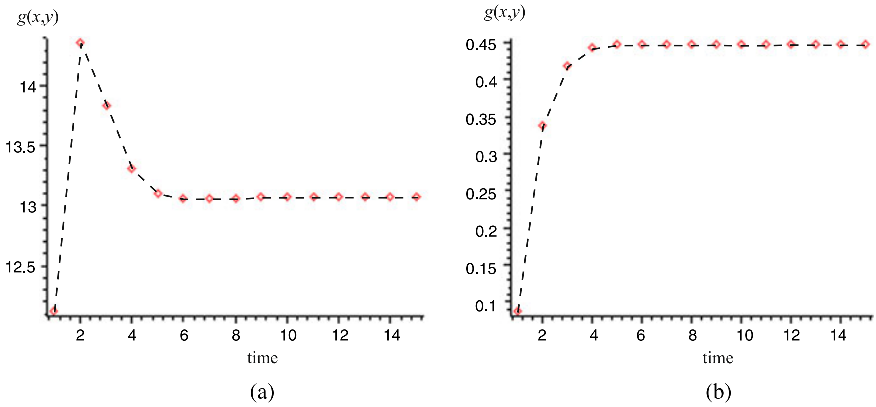

Fig. 3 *Equilibrium terms of trade* (**a** contracting over input quantities, **b** contracting over final revenue). Panel **a** shows the evolution of the equilibrium allocation function with contracting over input quantities. Panel **b** shows the evolution of the equilibrium allocation function with contracting over final revenues. The calculations are based on parameters $p = 30$, $\delta = 0.95$, $\psi^A = 0.4$, $\psi^B = 0.4$, $\mu^A = 0.8$, $\mu^B = 0.5$, $a = 30$, $b = 0.35$, $d = 0.2$, $e = -0.25$ and initial conditions $x_0 = y_0 = 10$

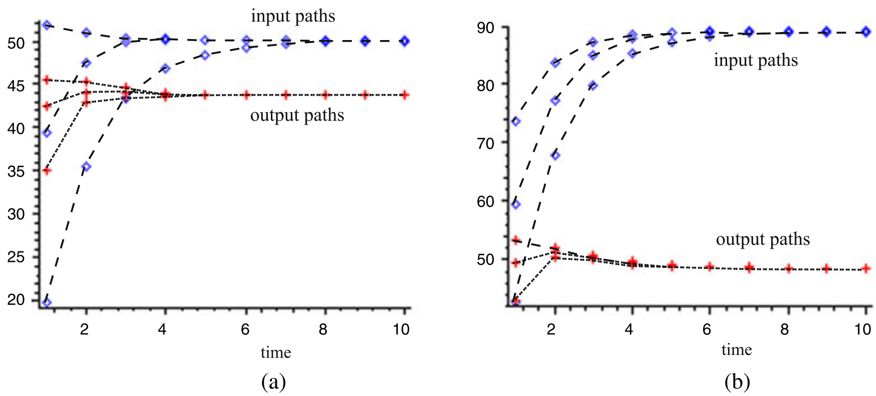

Fig. 4 *Equilibrium paths under alternative initial conditions* (**a** contracting over input quantities, **b** contracting over final revenue). Panel **a** shows the equilibrium input and output paths with contracting over input quantities. Panel **b** shows the equilibrium input and output paths with contracting over final revenues. The calculations are based on parameters $p = 30$, $\delta = 0.95$, $\psi^A = 0.4$, $\psi^B = 0.4$, $\mu^A = 0.8$, $\mu^B = 0.5$, $a = 30$, $b = 0.35$, $d = 0.2$, $e = -0.25$. The respective initial conditions are $x_0 = y_0 = 0$, $x_0 = y_0 = 25$ and $x_0 = y_0 = 50$

- The simulations yield allocation function parameters θ_1 and θ_2 whose signs suggest that marginal payoffs are constant or decreasing in the players' own capacities. However, (25) gives rise to increasing marginal costs of investment. Thus, cost considerations will motivate each firm to cut down on investment if its capacity has gone up, implying that $\beta_1^A < 0$, $\beta_2^B < 0$.

- To satisfy the technology constraint (27) for the assumed values of a, b, d and e, we need to have $\beta_2^A < 0$ and $\beta_1^B > 0$. That is, the investment of firm A should be decreasing in the capacity of the intermediate good producer, while the investment of firm B should be increasing in the capacity of the final good producer. Note that the allocation function (30) must ensure that it is in the players' self interest to behave accordingly. The resulting implications for the parameters θ_1 and θ_2 are discussed in the next subsection.

To assess the efficiency of the contractual arrangements, it may be useful to compare the equilibrium input and output paths to the plans that maximize joint surplus. As shown in Fig. 1 and Fig. 2, both types of contracting will lead to overinvestment relative to the efficient levels in our baseline scenario. However, for other parameter values (e.g., a high b) the equilibrium might involve underinvestment. Whether MPE capacities will exceed or fall short of their efficient levels will depend on the incentives provided by the revenue sharing arrangements. In general, if an incremental change in investment increases a player's private continuation payoff by more than the future joint surplus, then the equilibrium behavior of this player will involve overinvestment.

4.3.3 Equilibrium Allocation Function

Given a linear allocation function (30), the firms' ability to affect the future terms of trade will depend on θ_1 and θ_2. The signs of these parameters determine whether the capacities of the intermediate good producer and the final good producer are strategic complements or substitutes. As already established, the technology constraints in our examples are consistent with linear feedback strategies (28) and (29) whose parameters satisfy $\beta_2^A < 0$ and $\beta_1^B > 0$. Therefore, we would expect to obtain an allocation function such that: (i) firm A's marginal return on investment is decreasing in firm B's output (i.e., $\partial^2\pi_t^A/\partial x_t \partial y_t < 0$), and (ii) firm B's marginal return on investment is increasing in firm A's output (i.e., $\partial^2\pi_t^B/\partial x_t \partial y_t > 0$).

- If the parties contract over input quantities, these complementarity requirements would imply that $\theta_1 > 0$ and $\theta_2 < 0$. That is, the input price should be decreasing in the capacity of the input producer and increasing in the capacity of the final good producer. This appears to be consistent with the standard laws of supply and demand. The numerical results shown in Table 2 confirm this intuition.
- If, on the other hand, contracts are based on final revenues, these complementarity requirements would amount to $\theta_1 > 0$ and $\theta_2 > 0$. In other words, firm B's revenue share should be increasing in both production capacities. This intuition is supported by the results in Table 3.

The transitional dynamics of the terms of trade are illustrated in Fig. 3.

4.3.4 Equilibrium Surplus Allocation

The numerical examples also shed light on the factors that influence the distribution of surplus in bilateral trade. Tables 2 and 3 illustrate the allocative properties of the Markov perfect equilibrium by providing information about the steady-state surplus share of the input producer, $s_t^B = \pi_t^B/(\pi_t^A + \pi_t^B)$.

The simulations suggest that the choice of a contractible variable z plays an important role in payoff allocation. A comparison between the two arrangements shows that the firm which chooses the value of the contractible variable usually attains a higher surplus share. In particular, contracting over input quantities tends to benefit the input producer (high steady-state surplus share $\hat{s}^B$), while contracting over final revenue is more favorable for the final good producer (low steady-state surplus share $\hat{s}^B$). This result is consistent with the observation that in real-world interactions control is often advantageous.

4.3.5 Contract Efficiency

The linear-quadratic formulation also enables us to compare the efficiency of the two regimes as measured by the joint surplus w_t generated by the bilateral exchange. The numerical examples underscore the importance of design and procedure for economic efficiency.

Interestingly, Table 2 and Table 3 show that, in all of the cases studied here, contracting over input quantities yields a higher steady-state joint surplus $\hat{w}$ relative to contracting over final revenue. For some parameter values those welfare differences are rather significant. A brief inspection of Fig. 1 and Fig. 2 shows that contracting over input quantities generates transition paths that are much closer to their efficiency counterparts. On the other hand, contracts based on final revenues seem to cause substantial distortions in the input supply decisions of firm B.

We can use condition (21) to compute the welfare generated by the efficient input and output paths, and compare it to the joint surplus arising in the Markovian allocation equilibrium. Our numerical example shows that contracting over input quantities gives rise to a small deadweight loss: in our baseline scenario, steady-state welfare is only 0.8 % lower than the efficient level. The corresponding number for contracting over final revenues is 11.7 %.

A comparison between Figs. 1 and 2 suggests a possible explanation for this result. While both arrangements cause the intermediate good producer to overinvest in our numerical examples, this problem is exacerbated when contracts are based on final revenues. The smaller deviations from surplus maximization observed in Fig. 1 are likely due to the effect of firm B's investment on the future value of the allocation function (30). As we already established, contracts based on input quantities yield $\theta_2 < 0$. Thus, an increase in investment of the intermediate good producer will worsen his future terms of trade. As a result, his incentives to boost input capacity will be mitigated. On the other hand, when contracts are based on final revenues, we have $\theta_2 > 0$. Therefore, an increase in investment will now improve firm B's future terms of trade. This suggests that the intermediate good producer will overinvest more relative to the other regime, adversely affecting overall efficiency.

5 Concluding Remarks

This chapter offers a novel perspective on contract design and firm conduct in dynamic environments where the production process necessitates bilateral exchange. The analysis illuminates the factors that govern surplus allocation within bilateral monopolies and explores the efficiency of different contractual arrangements.

Our model incorporates dynamic capacity constraints, where technological interdependence causes firms to engage in trade. Two types of surplus allocation procedures are considered: (i) contracting based on input quantities; and (ii) contracting based on final revenues. Furthermore, we impose a Markovian restriction on contracts and strategies. The benefit of this approach is twofold:

- it explicitly accounts for the strategic motives driving the firms' investment decisions;
- it enables us to determine the prevailing terms of trade implied by profit maximization, technological constraints and the surplus allocation mechanism.

Using dynamic programming, we derive necessary conditions for the equilibrium investment strategies that are consistent with the production technology. We argue that strategic concerns will typically prevent the firms from attaining joint surplus maximization. The adoption of a linear-quadratic payoff formulation allows us to characterize numerically the equilibrium investment decisions and the terms of trade. We find that surplus allocation arrangements based on input quantities are more efficient, but tend to benefit the input producer. This helps explain why "dominant" suppliers like Gazprom may want to entice their partners to sign contracts that are tied to input quantities.

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Appendix A: Markov Allocation Equilibrium Conditions of the Bilateral Monopoly Game

This appendix derives the necessary conditions that characterize the Markov equilibrium of the bilateral monopoly game.

A.1 Euler Equation of the Final Good Producer

First consider the problem of firm A. Differentiating Bellman equation (9) yields the first-order condition:

$$V_1^{A\prime} = \frac{C_1^A}{\delta}. \tag{38}$$

By assumption the equilibrium strategies of firm A and firm B are respectively $f^A(x,y)$ and $f^B(x,y)$. Therefore, these strategy functions satisfy the recursive equation

$$\begin{aligned} V^A(x_t, y_t) = R(x_t) - S^B\big(z(x_t,y_t), g(x_t,y_t)\big) - C^A\big(f^A(x_t,y_t)\big) \\ + \delta V^A\big(\mu^A x_t + f^A(x_t,y_t), \mu^B y_t + f^B(x_t,y_t)\big). \end{aligned} \tag{39}$$

Differentiating with respect to x_t gives us

$$\begin{aligned} V_1^A = R_1 - S_1^B z_1 - S_2^B g_1 - C_1^A f_1^A \\ + \delta\big(\mu^A + f_1^A\big) V_1^{A\prime} + \delta f_1^B V_2^{A\prime}. \end{aligned} \tag{40}$$

Substituting $V_1^A(x,y)$ from the first-order condition into (40) forwarded one period yields an equation for $V_2^A(x,y)$:

$$V_2^{A\prime\prime} = -\frac{1}{\delta f_1^{B\prime}}\left\{R_1' - S_1^{B\prime} z_1' - S_2^{B\prime} g_1' - \frac{C_1^A}{\delta} + \mu^A C_1^{A\prime}\right\}. \tag{41}$$

Furthermore, differentiating (39) with respect y_{t-1} delivers

$$\begin{aligned} V_2^A = R_1 - S_1^B z_2 - S_2^B g_2 - C_1^A f_2^A \\ + \delta f_2^A V_1^{A\prime} + \delta\big(\mu^B + f_2^B\big) V_2^{A\prime}. \end{aligned} \tag{42}$$

Substituting $V_1^A(x,y)$ from (38) and $V_2^A(x,y)$ from (41) into (42) yields (14).

A.2 Euler Equation of the Intermediate Good Producer

Now consider the decision problem of firm B. Bellman equation (10) implies that the optimal strategy solves the first-order condition

$$V_2^{B\prime} = \frac{C_1^B}{\delta}. \tag{43}$$

Furthermore, by assumption the optimal strategies of firm A and firm B are respectively $f^A(x,y)$ and $f^B(x,y)$. Therefore, these strategy functions satisfy the recursive equation

$$\begin{aligned} V^B(x_t, y_t) = S^B\big(z(x_t,y_t), g(x_t,y_t)\big) - C^B\big(f^B(x_t,y_t)\big) \\ + \delta V^B\big(\mu^A x_t + f^A(x_t,y_t), \mu^B y_t + f^B(x_t,y_t)\big). \end{aligned} \tag{44}$$

Differentiating (44) with respect to y_t yields

$$V_2^B = S_1^B z_2 + S_2^B g_2 - f_2^B C_1^B + \delta f_2^A V_1^{B\prime} + \delta\big(\mu^B + f_2^B\big) V_2^{B\prime}. \tag{45}$$

Substituting V_2^B from the first-order condition and solving for V_1^B we get

$$V_1^{B\prime\prime} = -\frac{1}{\delta f_2^{A\prime}}\left\{S_1^{B\prime} z_2' + S_2^{B\prime} g_2' - \frac{C_1^B}{\delta} + \mu^B C_1^{B\prime}\right\}. \tag{46}$$

Similarly, differentiating (44) with respect to x_t gives us

$$V_1^B = S_1^B z_1 + S_2^B g_1 - f_1^B C_1^B + \delta\big(\mu^A + f_1^A\big) V_1^{B\prime} + \delta f_1^B V_2^{B\prime}. \tag{47}$$

After substitution of (43) and (46) into (47) we obtain (15).

Appendix B: Dynamically Efficient Investment

This appendix derives the necessary condition for joint surplus maximization.

Bellman equation (20) yields the first-order condition

$$-F_1 C_1^A - C_1^B + \delta F_1 W_1' + W_2' = 0. \tag{48}$$

Differentiation with respect to x gives us the envelope condition

$$W_1 = R_1 - \big(F_2 - \mu^A\big) C_1^A + \delta F_2 W_1'. \tag{49}$$

Furthermore, differentiation with respect to y gives us the envelope condition

$$W_2 = -\big(\mu^B F_1 + F_3\big) C_1^A + \delta\big(\mu^B F_1 + F_3\big) W_1' + \delta \mu^B W_2'. \tag{50}$$

Multiplying (49) by F_1 and adding it to (50) yields

$$\begin{aligned} F_1 W_1' + W_2' &= F_1 C_1^A - C_1^B \\ &= \delta F_1 R_1' - \delta F_1 \big(F_2 - \mu^A\big) C_1^{A\prime} - \big(\mu^B F_1' + F_3'\big) C_1^{A\prime} \\ &\quad + \delta^2 \mu^B \big(F_1' W_1'' + W_2''\big) + \delta^2 \big(F_1 F_2' + F_3'\big) W_1''. \end{aligned} \tag{51}$$

Substituting $F_1' W_1'' + W_2''$ from the first-order condition into (51) gives us an equation for W_1:

$$\begin{aligned} W_1'' = \frac{1}{\delta^2 (F_1 F_2' + F_3')} \big\{ & F_1 C_1^A - C_1^B - \delta F_1 R_1' \\ & + \delta F_1 \big(F_2' - \mu^A\big) C_1'^A + F_3' C_1^{A\prime} - \delta \mu^B C_1^{B\prime} \big\}. \end{aligned} \tag{52}$$

Finally, substituting (52) into (49) delivers Euler equation (21).

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