Interactions Between Fiscal and Monetary Authorities in a Three-Country New-Keynesian Model of a Monetary Union

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Abstract In this paper we consider the effectiveness of various coordination arrangements between monetary and fiscal authorities within a monetary union if an economic shock has occurred. We address this problem using a multi-country New-Keynesian model of a monetary union cast in the framework of linear quadratic differential games. Using this model we study various coordination arrangements between fiscal and monetary players, including partial fiscal cooperation between only a subgroup of countries, which, to the best of our knowledge, has not been considered yet in the New-Keynesian literature. Using a simulation study we show that, in many cases and from the global point of view, partial fiscal cooperation between a subgroup of fiscal players is more efficient than non-coordination and that, in general, full cooperation without an appropriate transfer system is not a stable configuration. Furthermore, in case there is no full cooperation we show that the optimal configuration of the coordination structure depends on the type of shock that has occurred. We present a detailed analysis of the relationship between coordination structures and type of shock.

1 Introduction

The creation of the (multi-country) European Monetary Union (EMU), with a common central bank, yet independent national fiscal policies, urged the ongoing discussion about the need and the feasibility of macroeconomic policy coordination within a monetary union. Since the 'one-size-fits-all' policy of the European Central Bank

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(ECB) cannot address country-specific shocks, and the other stabilisation mechanisms in the euro-zone (such as labour force mobility and financial assets mobility) are limited, the general consensus is that the main burden of stabilisation should be born by fiscal policies. However, the abuse of fiscal policies can be detrimental to both financial and economic stability and may result in undesirable suboptimal outcomes. Consequently, budgetary positions in the EMU Member States are constrained (mainly by the provisions of the Stability and Growth Pact, SGP) and are monitored by the European Commission (EC). Due to the recent experience with the current economic crises it is to be expected that this monitoring will be much more strict in the future. This situation gives rise to several relevant questions, the most important of them being, whether the coordination of monetary and fiscal policies is desirable in the aftermath of a shock.

Many studies analysing the desirability of policy coordination in a monetary union have been performed and they provide mixed results. On the one hand, many authors support the classic result of the Optimal Currency Area (OCA) that policy coordination is desirable in case of symmetric shocks. For instance, Buti and Sapir (1998) argue that coordination of fiscal policies should be implemented to tackle large symmetric shocks. However, on the other hand, there are a number of studies that demonstrate that policy coordination can provide inferior levels of welfare. Notably, in a two-country model of a monetary union, Beetsma et al. (2001) find fiscal cooperation to be counter-productive as a result of the elevated conflict with the central bank.

Against this background, in this paper we present and analyse a multi-country New-Keynesian (NK) model of a monetary union which is cast in the framework of open-loop linear quadratic differential games (LQDGs) including multi-player strategic elements. To the best of our knowledge, it is the first model in the New(-Keynesian) Open Economy Macroeconomics (NOEM) spirit to feature strategic elements between more than three players. Essentially, the starting point of the NK approach is the explicit derivation of macroeconomic relationships from underlying microeconomic foundations. This principle is largely shared with New Classical macroeconomics, although the former includes a great deal of imperfections in the goods and labour markets. Recently, NK macroeconomics has constituted the core of the macroeconomic paradigm world-wide, with a great deal of research effort directed towards the issue of optimal monetary policy. However, until now, relatively little attention has been paid to the interactions between fiscal and monetary policies when stabilising an economy after a shock; something that is especially important in the EMU context.[1] The strength of our NK model is its multi-player (monetary union countries and the central bank) strategic dimension, which allows for extensive analysis of desirability of policy coordination.

In particular, within this framework of a dynamic multi-country NK model we try to identify coordination configurations that outperform others under different types of shocks. We show that, in many cases, partial fiscal coordination of a sub-

[1] See the next section for a selection of papers that analyse this issue in the spirit of NOEMs.

group of fiscal players is more efficient, from the social point of view, than non-coordination. In other words, coordination of fiscal policies is likely to be counter-productive when the coordinating group of countries is large enough, thus increasing the conflict with the central bank. The intuition for this result is that, in case countries cooperate in smaller groups, their policies are less likely to be completely symmetric. Therefore, there are less likely to be in conflict with the central bank that targets only union averages.

Our approach adds new dimensions to the current literature. First of all, in our approach, strategic interactions between various parties become dynamic; and this within a model with rich NK specification. Secondly, a multi-country model enables us to study various intermediate coordination regimes between fiscal players (partial fiscal cooperation). Finally, we consider cooperation between the central bank and multiple fiscal players (i.e. the grand coalition) in the same manner in which other recent, but only two-country NK studies, have done.[2]

The remainder of the paper is organised as follows. The next section gives a brief overview of the literature on cyclical stabilisation in a monetary union and related issues. Section 3 outlines the model of a monetary union whereas Sect. 4 introduces an alternative concept for social loss. Section 5 presents results of numerical simulations and discussion. The last section concludes and proposes directions of future research.

2 Policy Coordination in a Monetary Union

The inability of the common monetary policy to tackle country-specific shocks is generally considered to be the single and most important macroeconomic cost of monetary unification. However, the size of this cost depends on how good other mechanisms, which may be helpful in adjusting to idiosyncratic shocks, are functioning too.[3] Unfortunately, factor markets (especially the labour market) are relatively immobile in Europe and capital flows are limited (Buti 2001). Consequently, these two basic mobility mechanisms, which play an important role in the US economy, are rather dysfunctional in the EMU case.[4] This means that, the stabilisation burden in the case of country specific shocks should be placed on other policies. One such policy is the federal tax-transfer system; another mechanism that is vital

[2]See next section for a short literature review on policy coordination in a monetary union.

[3]It should also be noted that from the macroeconomic perspective the real cost of accessing a monetary union is reflected by the shadow cost of the abandonment of an own monetary policy so that it cannot be used as an adjustment tool in the case of an idiosyncratic shock. In other words, the cost of entering the union depends, to a large extent, on the effectiveness of the national central bank to tackle idiosyncratic shocks. It is especially important in the environment of closely integrated economies, like the EU, and in the case of small countries, such as Belgium.

[4]See, for instance, Pierdzioch (2004) who presents a dynamic general equilibrium two-country optimising sticky-price model to analyse the consequences of international financial market integration for the propagation of asymmetric productivity shocks in the EMU.

to the US state-specific adjustments (see Mélitz and Zumer 1998; von Hagen 2000). Also, in the EU framework, tax-transfers already take place, however, in spite of public perception, the amounts of funds transferred are limited and are related to the long-term economic development, rather than to short and medium term conditions. A deeper tax-system integration, which may be able to address idiosyncratic shocks, seems to be politically implausible in the EMU, at least in the near future.[5] In the light of this, the national fiscal policy is considered to be the most important instrument left to policymakers.[6] In theory, this policy might be able to circumvent the problem of country specific shocks in the euro-zone. However, whether the incurred cost of stabilisation via national fiscal policies in a (relatively) integrated economic environment like the EMU is lower than the benefits, remains to be answered, especially since spillovers from national fiscal policies may be counteractive w.r.t. each other and lead to suboptimal outcomes. Another closely related problem is the response of a common monetary authority which will react to national fiscal policies. Such a reaction can be counteractive as the objectives of fiscal and monetary policies tend to be dissimilar. All of these issues warrant a discussion on the profitability and feasibility of policy coordination in a monetary union.

Aside from certain institutional issues, such as the effectiveness of enforcing the cooperation agreement, profitability of any coordination arrangement depends on the nature of the shock, i.e. whether it is symmetric or asymmetric, inflation or output gap, etc. We will focus on symmetric and asymmetric inflation shocks. In the case of symmetric shocks, policy coordination was traditionally considered to be beneficial because it internalised externalities emerging from individual policies (as argued by Uhlig 2003; Plasmans et al. 2006a). In particular, the usual argument in favour of international policy coordination is based on direct positive demand spillovers. In contrast to this, more recent micro-founded models of the EMU tend to conclude in favour of negative fiscal spillovers by emphasising the adverse terms-of-trade effects of balanced-budget foreign fiscal expansion on the domestic economy. For example, should governments perceive negative spillovers from other countries, they would reconsider a non-cooperative ("beggar-thy-neighbour") policy in response to bad economic shocks and would agree on a more restrictive stance in all countries.[7] Conversely, should governments perceive positive spillovers, coordination should eliminate free-riding behaviour of individual countries and promote

[5] The issue of fiscal transfers within the EMU has been studied by a number of authors such as Kletzer and von Hagen (2000), van Aarle et al. (2004), Evers (2006). The latter considers direct transfers among private sectors and indirect transfers among national fiscal authorities showing relative efficiency of such solutions.

[6] Already Kenen (1969) emphasised the possible role which fiscal policies might play in a monetary union as potential chock-adjustment mechanisms.

[7] One country attempts to improve its output-inflation trade-off by running a "beggar-thy-neighbour" policy. This is followed by the reaction of (an)other country(-ies) and the resulting non-cooperative outcome is a deflationary bias with all countries worse off with regard to a cooperative situation in which each country takes care of domestic inflation without attempting to affect the exchange rate (Cooper 1985).

a more expansionary policy in response to some economic shocks. Evidently, incentives for fiscal policy coordination in a monetary union are directly linked to the sizes and signs of the spillovers resulting from national fiscal policies. Fiscal spillovers are crucial, since they ultimately determine whether coordination should lead to a more expansionary or a more restrictive fiscal stance in the member states.

Other arguments in favour of coordination are that (see e.g. Hughes-Hallett and Ma 1996): (i) it restores policy effectiveness; (ii) speeds up an economy's response to policy actions; and (iii) enables to exploit comparative policy advantages.

In contrast to the above traditional arguments, more recent works question the profitability of policy coordination. Beetsma et al. (2001) show that policy coordination in a monetary union can be counter-productive in case of a symmetric shock.[8] Their model emphasises the conflict between governments and the central bank which share the stabilisation burden.[9] They argue that the coordination of budgetary stance between countries in the union makes fiscal policy more effective, thus governments are more willing to accept changes in their deficits. In other words, cooperation increases the use of fiscal policies. This, in turn, has important consequences on the behaviour of the central bank, which, generally has objectives different from those of fiscal policymakers. For instance, in case of a negative demand shock, both fiscal and monetary authorities are interested in stabilising output, which, from the central bank's point of view, is a means of stabilising inflation. Since cooperating fiscal authorities are eager to bear more stabilisation burden, the (non-cooperating) central bank free-rides on their efforts and does not loosen monetary policy as much as when governments do not collude. So, under this type of shock this cooperation structure is probably not optimal. But, at least, both authorities do not enter into a conflict situation. However, in case a supply shock occurs it is clear that this cooperation structure is far from optimal. Supply shocks make output and inflation move in opposite directions. The stronger fiscal response encouraged by cooperation exacerbates the conflict between price and output stability and, therefore, between monetary and fiscal authorities. As a result of this, the central bank is more restrictive when governments cooperate, compared to a case where governments would not act like that.

In other words, for both types of shocks, fiscal coordination may turn out to be counter-productive, albeit for different reasons.

In contrast, fiscal cooperation may be beneficial in the case of a country-specific shock. The free-riding of (or the conflict with) the central bank can in such a case largely be avoided as the monetary authority is interested in aggregate inflation (and

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[8]It should be noted that prior to Beetsma et al. (2001), it has been observed in empirical studies (see e.g. Neck et al. 1999) that coordination does not necessarily lead to superior results as a result of either time-inconsistency and/or coalition formation of the fiscal policymakers against the monetary authority (see also next footnote). Furthermore, Beetsma and Bovenberg (1998) show that fiscal coordination may have a negative influence on a tax and public spending discipline, i.e. they may reduce the positive effects of monetary unification.

[9]Rogoff (1985) already stated that there is a potential for a negative impact of coordination among a subset of actors (in this case the two fiscal authorities, leaving out the common central bank).

possibly, to some extent, aggregate output). Therefore, the reaction of the monetary authority to idiosyncratic shock is limited.

Lombardo and Sutherland (2004) confirm the above point of view by showing, in a micro-founded model, that fiscal coordination is advantageous when country-specific shocks are negatively correlated. This study also suggests that the best results are delivered by an appropriate mix of both fiscal and monetary instruments.

Other works defend the view of desirability of policy coordination. von Hagen and Mundschenk (2003) argue that, in the long run, there is little need for coordination, however, in the short term, there are substantial gains from fiscal cooperation. Furthermore, if the central bank also pursues a goal of output stabilisation, the grand coalition of all the authorities together is advisable. Buti et al. (2001), Engwerda et al. (2002), Beetsma and Jensen (2004), Kirsanova et al. (2007) also support the active role of fiscal policy in stabilisation.

Cavallari and Di Gioacchino (2005), in the framework of a two-country static model, show that coordination of fiscal policies can only reduce output and inflation volatility w.r.t. the non-cooperative regime in the case of a demand shock, and that it can be potentially counter-productive otherwise. This adverse effect of union-wide coordinated fiscal measures can be circumvented by "global coordination," i.e. grand coalition. In more complex micro-founded general equilibrium models, Galí and Monacelli (2005b), Beetsma and Jensen (2004, 2005) also consider the case for fiscal and monetary policies' coordination. Specifically, Beetsma and Jensen (2005) extend the framework developed by Benigno and Lopez-Salido (2002) and develop an NK two-country monetary union model whereby national fiscal authorities pursue active stabilisation policies using public spending. Their model reveals that the relative advantage of using fiscal stabilisation policy is unchanged when the correlation of the supply shocks decreases. However, from a welfare point of view, the use of fiscal policy for the purpose of stabilisation appears to be relevant. Beetsma and Jensen (2005) argue that the governments should be active in situations in which a restriction on fiscal policy in order to equalise this policy with its natural level leads to welfare losses being equivalent to a permanent reduction in consumption of the order from 0.5 to 1 percentage point. A similar view is shared by Galí and Monacelli (2005b) who argue in favour of active fiscal policies.

In addition to cooperative scenarios, Forlati (2007) focuses on a non-cooperative regime showing that, in such a situation, the central bank does not stabilise the average monetary union inflation as it has to accommodate the distortions caused by non-cooperative national governments. At the same time, the non-existence of an agreement between countries calls for an active fiscal stance, even in case of symmetric shocks.

In conclusion, a lot of work already has been done in this area of policy coordination. Further, though some results seem to be contradictory at first sight, in most cases these differences can be attributed to (not formalised) assumptions being made. As already indicated in the previous section we will drop some of these assumptions here and study the consequences w.r.t. optimal cooperation configurations.

Most of the initial works (like the static model of Beetsma et al. 2001) were tractable enough to deliver analytical solutions. However, the much more complex dynamic general equilibrium modelling with a higher number of cooperation arrangements requires resorting to numerical methods. These were used in van Aarle et al. (2002a, 2002b), Beetsma and Jensen (2004, 2005), Plasmans et al. (2006a), among others, and will also be applied in this work. More specific, we will use the numerical toolbox developed by Michalak et al. (2011) to perform simulations for infinite-planning horizon affine linear quadratic open-loop differential games. For an introduction on dynamic games we refer to Başar and Olsder (1999) and, more specific, on LQDG, to Engwerda (2005).

3 A Multi-country NKOEM Model of a Monetary Union

During recent years the theoretical and empirical research in NK macroeconomics has been extended steadily and produced a whole new series of results and insights about the workings of the macroeconomy. Essentially, the starting point of the NK approach is the explicit derivation of macroeconomic relationships from underlying microeconomic foundations. This principle is shared with New Classical macroeconomics, although the former includes a great deal of imperfections in the goods and labour markets. The NK approach now constitutes the core of the macroeconomic paradigm world-wide.

Our modelling objective is to cast the NK model of a monetary union in the LQDG framework in order to analyse strategic interactions between a comparatively large number of players. By definition, LQDGs concern continuous-time models but, unfortunately, the vast majority of NK/NOEM models were constructed in the discrete-time framework. Notable exceptions are Benhabib et al. (2001), Linnemann and Schabert (2002), Buiter (2004), Kirsanova et al. (2006). However, with the exception of the last one, all these are single economy models, thus would obviously require extensions to allow them to be applied to a monetary union setting.[10] In line with this, our strategy will be to transform a discrete-time NK model of a monetary union into its continuous-time counterpart. This methodology is also convenient from the point of view of model parametrisation as most of the empirical studies, useful for calibration purposes, concern discrete-time models. The second important modelling issue is the computational complexity of an LQDG, which grows with the number of dynamic equations of the model and/or the number of players. Having this in mind, we aim to describe every country in a monetary union in a manner as concise as possible. In fact, as explained in Svensson (1997) and Ball (1999), short-term macrodynamics can be analysed using a relatively simple system consisting of an aggregate demand (AD) curve showing the evolution of the output gap driven by the real interest rate (see e.g. Woodford 2003), and a Phillips curve describing the dynamics of inflation. Despite its relative simplicity, such models have been widely used to understand the basic mechanisms of macroeconomic policies. Consequently,

[10]There are two countries in the model developed by Kirsanova et al. (2006) but this particular framework becomes computationally difficult when we add another, third country.

our multi-country monetary union model will consist of as many AD equations as countries, as many Phillips curves as countries, and a number of real exchange rate relationships.

Due to space constraints, we will refrain from deriving the NK model of a monetary union from micro-foundations. Instead, we will refer to results from various studies in the literature. Let fiscal and monetary players from a set N be divided in two groups: n countries i ($i \in F$) and one central bank b ($b = B$, with $N = F \cup B$). Following, among others, Lindé et al. (2004), AD equations are:[11]

$$\begin{aligned} y_{i,t} = {} & \kappa_{i,y} E_t y_{i,t+1} + (1-\kappa_{i,y}) y_{i,t-1} - \gamma_i (i_{U,t} - E\pi_{i,t+1}) + \eta_i f_{i,t} \\ & + \sum_{j \in F/i} \rho_{ij} \big[-\kappa_{i,y} E_t y_{j,t+1} + y_{j,t} - (1-\kappa_{i,y}) y_{j,t-1} \big] \\ & + \sum_{j \in F/i} \delta_{ij} \big[-\kappa_{i,y} E_t s_{ij,t+1} + s_{ij,t} - (1-\kappa_{i,y}) s_{ij,t-1} \big] + v_{i,t}^{y}, \end{aligned} \tag{1}$$

where E_t is an expectation operator at time t; $y_{i,t}$, $p_{j,t}$, $\pi_{i,t}$, $f_{i,t}$, $s_{ij,t} := p_{j,t} - p_{i,t}$ denote the (logarithmic) output gap, price level, inflation, fiscal policy in country i and competitiveness between countries i and j, respectively, whereas $i_{U,t}$ denotes the union-wide common nominal interest rate. All parameters are non-negative. The current output gap in country i depends positively on the expected output gap, the past output gap, the real interest rate, the government's fiscal deficit, the dynamics of other countries' output gaps and competitiveness, defined as the difference between respective price levels. Finally, $v_{i,t}^{y}$ is an output gap shock. This functional form of the AD equation may be obtained from a linearised model of optimisation behaviour on the part of consumers, in particular, from the resulting Euler equation, in which consumption is replaced with output gap, as shown, for instance, in Lindé et al. (2004). Inertia term $(1-\kappa_{i,y})y_{i,t-1}$ reflects so-called "habit formation" in consumption (see for example Smets and Wouters 2002; Plasmans et al. 2006b), which measures the sluggishness of households in changing their choices over time. Foreign output gap and competitiveness elements in (1) reflect the economic linkages between countries. In particular, the first one is a trade channel, where, intuitively, higher foreign output gaps contribute to higher domestic output gaps as a result of increased import. Similarly, domestic export increases when a foreign price level becomes higher than the domestic one. The forward-looking and backward-looking dynamics of foreign output gap and competitiveness spillovers result from habit formation in consumption and have a similar form as the dynamics of domestic output gaps in the AD equations.

The second set of equations in our model are NK Phillips curves, which relate inflation to cyclical activity. In the New-Keynesian model, these are derived from optimising firms' price-setting decisions subject to constraints on the frequency of

[11] By F/i we denote the set of all countries except for country i.

price adjustment. We assume Phillips curves of the form:

$$\pi_{i,t} = \beta_i \left[\kappa_{i,\pi} E_t \pi_{i,t+1} + (1 - \kappa_{i,\pi}) \pi_{i,t-1} \right] + \xi_i \left(y_{i,t} + \sum_{j \in F/i} \varsigma_{ij} s_{j,t} \right) + v^{\pi}_{i,t}, \tag{2}$$

where we follow various studies in the literature allowing for some degree of price inertia for $0 < \kappa_{i,\pi} < 1$.[12] Inflation shock $v^{\pi}_{i,t}$ in (2) independent, exogenous, stationary, zero mean AR(1) shock with damping parameter $0 < \psi_{i,\pi} < 1$, i.e. $v^{\pi}_{i,t} = \psi_{i,\pi} v^{\pi}_{i,t-1} + \varepsilon^{\pi}_{i,t}$, where $\varepsilon^{\pi}_{i,t}$ is an independently and identically distributed (*i.i.d.*) error term.

For n countries there are as many as $n(n-1)$ competitiveness relationships s_{ij}, however, as shown in the Appendix, it is possible to rewrite all of them only in terms of $s_{1j,t} := p_{j,t} - p_{1,t}$ where $j \in F/1$:

$$s_{1j,t+1} = s_{1j,t} + \pi_{j,t+1} - \pi_{1,t+1}, \tag{3}$$

which are only $n-1$ dynamic equations.

In his seminal work, Taylor (1995) demonstrated that actual US monetary policy could be described by a simple rule that relates the real interest rate to inflation and to output gap. This relationship became known as the (monetary) Taylor rule. In the monetary union case, the Taylor rule of the central bank might be written in the form:

$$i_{U,t} = \theta^U_\pi \pi_{U,t} + \theta^U_y y_{U,t}, \tag{4}$$

where $\pi_{U,t} := \sum_{i=1}^n \omega_i \pi_{i,t}$ is the average union inflation and $y_{U,t} := \sum_{i=1}^n \omega_i y_{i,t}$ is the average union output gap with parameter ω_i indicating the relative weight of country i in a monetary union ($\sum_{i=1}^n \omega_i = 1$).[13] The first term in the Taylor rule shows that the central bank responds to the rise in average inflation with a more restrictive monetary policy in order to weaken demand across the union. This, in turn, should hinder the growth in inflation. The second term shows that the real interest rate is also raised if output rises as this indicates a future inflation acceleration.

Taylor (2000) also points out that fiscal policy can be approximated by a policy rule (for further discussion see van Aarle et al. 2004). The fiscal Taylor rule can be written as:

$$f_{i,t} = \theta^i_\pi \pi_{i,t} + \theta^i_y y_t. \tag{5}$$

We extend the above definition of both rules so that:

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[12] See, among others, Fuhrer and Moore (1995), Galí and Gertler (1999), Woodford (2003), Lindé et al. (2004), Evans and McGough (2005) or Plasmans et al. (2006b).

[13] For a similar formulation of the monetary policy rule in a model of a monetary union see van Aarle et al. (2004).

$$\tilde{\imath}_{U,t} = i_{U,t} + \hat{\imath}_{U,t} = \theta_\pi^U \pi_{U,t} + \theta_y^U y_{U,t} + \hat{\imath}_{U,t}, \quad \text{and} \tag{6}$$

$$\tilde{f}_{i,t} = f_{i,t} + \hat{f}_{i,t} = \theta_\pi^i \pi_{i,t} + \theta_y^i y_t + \hat{f}_{i,t}, \tag{7}$$

where $\hat{f}_{i,t}$ and $\hat{\imath}_{U,t}$ are the players' control variables in the LQDG and denote deviations of the fiscal deficit and nominal common interest rate from (4) and (5), respectively. In particular, as Taylor (2000) argues, a simple fiscal rule can be used to explain most fluctuations in fiscal deficits. The starting point of his analysis is the division of the fiscal deficit into a cyclical component and a structural component. The first part can be interpreted as the systematic response of fiscal policy to output fluctuations (the so-called automatic stabilisers); the second part contains structural and discretionary components of fiscal policy. In our case, the standard Taylor fiscal rule $\theta_\pi^i \pi_{i,t} + \theta_y^i y_t$ is to be interpreted as an automatic stabiliser, whereas $\hat{f}_{i,t}$ is a discretionary component. For the monetary Taylor rule, $\hat{\imath}_{U,t}$ is the discretional component of the central bank's policy.

In order to reduce the number of equations, it is convenient to substitute $i_{U,t}$ and $f_{i,t}$ in (1) with (4) and (5). The resulting system consists of n AD curves (1), n Philips curves (2), and $n-1$ competitiveness equations, which, together with the inflation shock AR processes, constitute a hybrid (forward- and backward-looking) stochastic NK Model (SNKM henceforth) of a closed monetary union.

This completes the description of the discrete-time NK model. Next, we used some standard transformation techniques to recast the reduced form of this model into its equivalent continuous-time counterpart. In the Appendix, Sect. 5, we explained the details of this transformation procedure.

In order to complete the construction of the LQDG, we propose the following fiscal players' objectives:

$$\min_{\hat{f}_i(t)} J_i(t_0) = \min_{\hat{f}(t)} \frac{1}{2} \int_{t_0}^{\infty} \left\{ \alpha_i \pi_i^2(t) + \beta_i y_i^2(t) + \chi_i \hat{f}_i^2(t) \right\} e^{-\theta(t-t_0)} dt, \tag{8}$$

for $i = 1, 2, \ldots, n$, where α_i, β_i, χ_i indicate fiscal players' relative preferences concerning deviations of national inflation rates, output gaps and fiscal deficits.[14] The common central bank's objective function is defined in a similar way as:

$$\min_{\hat{\imath}} J_{CB}(t_0) = \min_{\hat{\imath}} \frac{1}{2} \int_{t_0}^{\infty} \left\{ \alpha_{CB} \pi_U^2(t) + \beta_{CB} y_U^2(t) + \chi_{CB} \hat{\imath}^2(t) \right\} e^{-\theta(t-t_0)} dt, \tag{9}$$

where α_U and β_U indicate the central bank's relative preferences concerning deviations of inflation, output gap and interest rate in the MU as a whole.

[14] Since the seminal works of Kydland and Prescott (1977) and Barro and Gordon (1983), the quadratic loss functions are commonly used in the literature on strategic behaviour of fiscal and monetary authorities. See also Schellekens and Chadha (1999) for a more recent analysis supporting the quadratic form of the loss function.

4 Social Loss

Usually, it is assumed that the entire union's loss, also (often) called the social loss, is represented by the total sum of monetary and fiscal authorities' losses:

$$J_U(t_0, \Pi) := \sum_{i\in F} J_i(t_0) + J_{CB}(t_0), \tag{10}$$

where $J_i(t_0)$ and $J_{CB}(t_0)$ are defined by loss functions (8) and (9), respectively, and Π is a cooperation regime in which the combined loss is computed. Whereas the above definition seems to be plausible for a two-country model, it is not appealing in more complex settings. Since, in a general formulation of our model, there are n countries in the union and only one central bank, the relative importance of the monetary policymaker in $J_U(t_0, \Pi)$ and, hence, also of the monetary instruments gets smaller with increasing n. It is rather difficult to see the rationale behind it as the relative cost related to the interest rate volatility should be irrelevant of the size of the union.[15] To circumvent the above concerns, we propose the next MU loss function:

$$J_U^*(t_0, \Pi) := \frac{1}{2}\int_{t_0}^{\infty}\left[\sum_{i=1}^{n}\omega_i\big(\alpha_U\hat{\pi}_i^2(t) + \beta_U\hat{y}_i^2(t) + \chi_{f,U}\widehat{\hat{f}}_i^2(t)\big) + \chi_{r,U}\widehat{\iota_U}^2(t)\right] \times e^{-\theta(t-t_0)}dt. \tag{11}$$

Here α_U, β_U, $\chi_{f,U}$ and $\chi_{r,U}$ are preference parameters in the monetary union concerning deviations of inflation, output gap and both types of control instruments. Averages of variables' squares instead of squares of variables' averages guarantee that negative deviations of inflations and output gaps do not cancel out with positive ones. Furthermore, taking into account the average value of fiscal control instruments across a monetary union guarantees that volatility of interest rate is well represented in the loss (as it corresponds to its actual relative importance in a single economy).

Whether $J_U^*(t_0, \Pi)$ is smaller, equal or greater than $J_U(t_0, \Pi)$ depends largely on the preference parameters in loss functions (8)–(9) and (11). In a basic case when all these preferences coincide, i.e. $\alpha_i = \alpha_{CB} = \alpha_U$, $\beta_i = \beta_{CB} = \beta_U$ and $\chi_i = \chi_{CB} = \chi_{f,U} = \chi_{r,U}$, it is trivial to show that the (more) conventional social loss $J_U(t_0, \Pi)$ will always be higher than $J_U^*(t_0, \Pi)$ irrelevant of $\hat{\pi}_i$, $\hat{y}_i$, $\widehat{\hat{f}}_i$ and $\hat{\iota_U}$ adjustment paths. Otherwise, the result of this comparison is case dependent. In particular, it may vary with the type of shock considered.

The formula in (11) is similar to the one proposed by Beetsma et al. (2001); however, we extended the definition of cross monetary union loss with the devia-

[15] Formula (10) applied to the 50-State US and to the 13-member state EMU would show that (ceteris paribus) the relative importance of interest rate volatility for the American economy is much lower than for the Euro-zone.

tion of the interest rate, as this is an important factor influencing the welfare of the representative citizen in each country.

Furthermore, formula (11) can be interpreted in terms of NK microfoundations. Specifically, as shown by, for instance, Amato and Laubach (2003) and Woodford (2003), it is possible to derive a quadratic loss function of a form similar to (8) and (9) from a second-order Taylor series approximation to the representative household's welfare. Thus, function (11) can be interpreted as an average welfare function of all households in a monetary union. Taking this social point of view, the output gap volatility is related to the number of hours worked (i.e. employment) in the representative household's utility function whereas inflation volatility to purchasing power. As shown by Woodford (2003), a nominal interest rate in the social welfare function is related to the presence of real money balances in a representative household's utility function. In this respect, Friedman (1968) argued that high nominal interest rates result in transaction costs. Furthermore, the fiscal debt element in (11) can be attributed to the cost of excessive fiscal deficit for a society, that results in an increased price of servicing accumulated debt.

There remains an open question about the values of α_U, β_U, $\chi_{f,U}$ and $\chi_{r,U}$ that define preferences of the society. In the numerical simulations presented in the next section, it is assumed that values of these parameters correspond to preferences of fiscal and monetary authorities. The underlying assumption here is that both the governments' and the central bank's objectives are, to a large extent, representative for the public's preferences. Such an interpretation of formula (11) as a true social loss function is subject to various assumptions. In particular, we assume that both types of authorities aggregate heterogeneous preferences of a society in such a way that the aggregation is equal to the outcome of a voting mechanism or a utilitarian social welfare function determining society's weights on inflation and employment.[16]

Coalition structures for which $J_U^*(t_0, \Pi)$ is the lowest will be called social optima and will be denoted by Π^{*SOP}. Similarly, those regimes that are characterised by the conventional lowest loss $J_U(t_0, \Pi)$ will be denoted by Π^{SOP}. It is straightforward to show that, in the LQDG framework considered here, a coalition composed of all the players in the game (i.e. the grand coalition) always belongs to a set of social optima Π^{SOP} (for more details on this issue see Plasmans et al. 2006a, Chap. 2). However, as the definition of $J_U^*(t_0, \Pi)$ is not necessarily based on the same players' preferences as those used in the optimisation process, the grand coalition does not necessarily belong to Π^{*SOP}. This will be evident from various numerical simulations presented in the subsequent section.

5 Numerical Simulations

For clarity and space concerns we will focus our simulations on the three-country application of the model from Sect. 3. This number is sufficient to consider partial

[16]For further discussions see, for instance, Rogoff (1985), Persson and Tabellini (1993) and McCallum (1997).

Table 1 List of considered coalition structures

CS	Long notation	Acronym	Description
Π_1	$[C1\|C2\|C3\|CB]$ or $[1\|2\|3\|4]$	N	Non-cooperative regime
Π_2	$[C1, C2, C3, CB]$ or $[1234]$	C	The grand coalition
Π_3	$[C1, C2, C3\|CB]$ or $[123\|4]$	F	Full fiscal cooperation
Π_4	$[C1, C2\|C3\|CB]$ or $[12\|3\|4]$	P (or 4)	Partial fiscal cooperation
Π_5	$[C1, C3\|C2\|CB]$ or $[13\|2\|4]$	P (or 5)	Partial fiscal cooperation
Π_6	$[C1\|C2, C3\|CB]$ or $[1\|23\|4]$	P (or 6)	Partial fiscal cooperation

Table 2 Baseline parameters ($i, j \in \{1, 2, 3\}, i \neq j$)

Structural parameters:				
$\kappa_{i,\pi/y} = 2/3$	$\gamma_i = 0.5$	$\eta_i = 0.75$	$\rho_{ij} = 0.5$	$\delta_{ij} = 0.25$
$\beta = 0.99$	$\zeta_i = 0.06$	$\varsigma_{ij} = 0.5$		
Policy rules parameters:				
$\theta_y^i = -0.5$	$\theta_\pi^i = 0$	$\theta_y^U = 0.5$	$\theta_\pi^U = 1.5$	
Preference parameters:				
$\alpha_i = 0.02$	$\beta_i = \alpha_i/5$	$\chi_i = 0.1$	$\alpha_U = 0.02$	$\beta_U = 0.02$
$\alpha_{CB,i} = \beta_{CB,i}/5$	$\beta_{CB,i} = 0.02$	$\chi_{CB,i} = 0.1$	$\chi_{f,U} = \chi_{r,U} = 0.1$	

cooperation between fiscal authorities but still small enough to be computationally tractable. Furthermore, throughout this chapter we assume that cooperation between the central bank and only a subgroup of countries is not allowed, which yields 6 feasible coalition structures listed in Table 1. $C1$, $C2$ and $C3$ and CB denote governments within the union and the central bank, respectively.

5.1 Parametrisation

Table 2 lists all the parameters of the benchmark model. In the baseline scenario (denoted sc_1), countries are assumed to be symmetric with respect to all 7 structural parameters.

The parameters listed in Table 2 are comparable to other simulation studies, in particular, Batini and Haldane (1999), Leith and Wren-Lewis (2001) and van Aarle et al. (2004). When calibrating the IS equation with both backward- and forward-looking behaviour for the UK, they assumed $\kappa_{i,y}$ equal to 0.8 and 0.9, respectively, which are plausible values for quarterly data. For an average EMU economy we set this value to be $2/3$ in the benchmark model; however, we will pay special attention to this parameter in our sensitivity analysis. McCallum (2001), for the US case, suggests that for the interest rate elasticity of output in the IS curve (γ_i in our model), a value of 0.4 is more appropriate than Rotemberg and Woodford's (1999) value of

0.6 or McCallum and Nelson's (1999) value of 0.2. However, Cecchetti et al. (2002) estimate its average value to be 0.7 in the EU. In our case it is assumed that $\gamma_i = 0.5$ which is the value in between the above studies and, for example, corresponds to the parametrisation of Batini and Haldane (1999). In the sensitivity analysis both lower and higher values of this parameter will be considered.

The fiscal multiplier (η_i) measures the impact of changes in fiscal deficit on output gap and is estimated by the European Commission (2001, 2002) in the framework of the Commission's QUEST model and the OECD's Interlink model. The first simulations suggest an average value of 0.6 (± 0.1) in the EU countries while second ones yield values of 0.6 in France, 0.9 in Italy, 1.0 in Germany and the UK, and 1.3 in the US. The difference is to be attributed to the forward-looking nature of the first model. Having these values in mind, we assume η_i to be equal to 0.75. Parameters ρ_{ij} measure the elasticity of domestic import w.r.t. the foreign output gap and is estimated to be equal on average to 0.4 for the EU countries (Equipe MIMOSA 1996) and about 0.35 for Sweden by Lindé et al. (2004). We follow van Aarle et al. (2004) and assume the value of 0.5, which implies relative high trade integration of the economies in a monetary union. This regards also the competitiveness parameter δ_{ij} that is set to 0.25.

There is no consensus in the literature regarding inflation persistence in the Phillips-curve. It is generally recognised that a backward-looking element plays an important role in this equation, but various empirical studies deliver different estimates of $\kappa_{i,\pi}$. Whereas Galí and Gertler (1999) and Benigno and Lopez-Salido (2002) find a predominantly forward-looking specification of the Phillips curve ($\kappa_{i,\pi}$ around 0.7 for Germany, 0.64 for France, 0.4 for Italy, etc.), Mehra (2004) finds a predominantly backward-looking specification ($\kappa_{i,\pi}$ around 0.1). Furthermore, Mankiw (2001) argues that stylised empirical facts are inconsistent with the predominantly forward-looking Phillips curve. In the benchmark we assume the same value of $\kappa_{i,\pi}$ as $\kappa_{i,y}$ i.e. 0.66 but we will consider different specifications later. The elasticity of inflation w.r.t. the output gap is an important parameter of the Philips curve as it ultimately determines short-run adjustment between inflation and output gap. McCallum and Nelson (1999) and Galí and Monacelli (2005a) assume this value to be 0.3, Batini and Haldane (1999), 0.2, Leith and Wren-Lewis (2001) for the UK and Rotemberg and Woodford (1999) for the US set this value to be 0.1, whereas Beetsma and Jensen (2005) choose the value of 0.04. Furthermore, Lindé et al. (2004) estimate it to be at most 0.0158 for the Swedish economy. Again, we choose the value in between the above values setting $\zeta_i = 0.06$, however, it will be one of the main parameters on which we are going to focus our sensitivity check. Gagnon and Ihrig (2002) estimate the import price pass-through parameter ($\zeta_i \times \varsigma_{ij}$ in our model) to be between 0.05 and 0.23 for most OECD countries. On the other hand, Lindé et al. (2004) estimates this value for the Swedish economy to be smaller than 0.003. We calibrate this value to be 0.03, i.e. $\varsigma_{ij} = 0.5$.

Structural model parameters are assumed to be symmetric, however, policymakers' preferences are not. The central bank's preferences differ from those of the (assumed identical) national governments. As it is common in the literature (see, for instance, Beetsma and Bovenberg 1998; Dixit and Lambertini 2001; Engwerda et al. 2002; Uhlig 2003), we assume that the central bank puts a larger weight on inflation stabilisation than on output-gap stabilisation. In contrast, fiscal players are more concerned with output-gap stabilisation than with inflation stabilisation. Moreover, as laid down in the Maastricht Treaty, the central bank's objectives concern aggregate output and inflation in the monetary union while the fiscal players are only concerned about own output and inflation. Parameter β_U is often regarded as a (counter proportional) measure of the central bank independence and it is argued, that a fully independent central bank should be concerned only about inflation, i.e. $\beta_{CB} = 0$.[17] In the benchmark we do not take such a restrictive position and we assume that the central bank is 5 times more concerned about inflation than about an output gap, however, β_{CB} is still positive. Fiscal authorities, in turn, are 5 times more concerned about output gap than about inflation. Thus, calibrated preferences appear to be the most appropriate in our model as they guarantee that no variable is overrepresented in the total loss of any player. For the social loss function $J_U^*(t_0)$, it is assumed that society should be concerned about the output gap as much as the government is, whereas it should be concerned about inflation as much as the central bank is. Hence, $\alpha_U = \alpha_{CB} = 5\alpha_i$ and $\beta_U = \beta_i = 5\beta_{CB}$. The preference parameters of control instruments are set the same as in loss functions (8)–(9), i.e. $\chi_i = \chi_{CB} = \chi_{f,U} = \chi_{r,U}$.

As far as both policy rules are concerned, for the monetary rule, we assume the parametrisation originally proposed by Taylor (1993a, 1993b) for the US, i.e.: $\theta_\pi^U = 1.5$ and $\theta_y^U = 0.5$. For the fiscal policy rule, we assume that $\theta_y^i = -0.5$, which is the value found for the sensitivity of the fiscal deficit in relation to the cyclical variation by the European Commission (2001) for the Euro-area. It is used, for instance, in the simulations of van Aarle et al. (2004). Furthermore, $\theta_\pi^i = 0$. The value of a discount factor $\theta = 0.01$ in the loss functions (8)–(9) is coherent with the assumed structural discount parameter $\beta = 0.99$, which implies a 1 % (steady-state) real rate of return on a quarterly basis. Finally, in the benchmark, we assume symmetric bargaining power in every coalition, i.e. $\tau_{C1/C2/C3} = \tau_{CB} = \frac{1}{4}$ in the grand coalition, $\tau_{C1/C2/C3} = \frac{1}{3}$ in the fiscal coalition under F, and $\tau_{C1} = \tau_{C2} = \frac{1}{2}$, $\tau_{C1} = \tau_{C3} = \frac{1}{2}$, and $\tau_{C2} = \tau_{C3} = \frac{1}{2}$ in regimes 4, 5, and 6, respectively.

5.2 Symmetric Inflation Shock

The first four rows of Table 3 contain the (optimal) welfare losses in the various coalitional arrangements for the symmetric benchmark scenario and the common inflation shock, $v_{0S}^\pi := [v_{1,0}^\pi, v_{2,0}^\pi, v_{3,0}^\pi]^T = [1, 1, 1]^T$.[18] The next two rows show social losses J_U and J_U^* whereas, in the rest of the table, a decomposition of players'

[17] This opinion was also expressed by Lars Svensson at the conference "Inflation Targeting, Central Bank Independence and Transparency," 15–16 June 2007, Trinity College, Cambridge.

[18] All (optimal) losses are multiplied by the factor 10^3.

Table 3 Optimal losses for a symmetric inflation shock, baseline parametrisation (see Table 7 in the Appendix for the number of equilibria)

(sc_1, v_{0S}^{π})	N	C	F	[12|3|4]	[13|2|4]	[1|23|4]
$C1$	2.1948	2.1211	9.3016	2.5328	2.5328	1.7296
$C2$	2.1948	2.1211	9.3016	2.5328	1.7296	2.5328
$C3$	2.1948	2.1211	9.3016	1.7296	2.5328	2.5328
CB	6.2456	5.3308	21.843	5.6049	5.6049	5.6049
$J_U(t_0)$	12.8300	11.6940	49.7480	12.4000	12.4000	12.4000
$J_U^*(t_0)$	7.1445	6.3675	29.9340	6.6853	6.6853	6.6853
$\alpha_{F,C1}\hat{\pi}_{C1}^2$	1.0782	0.8725	0.9979	0.9605	0.9605	0.9792
$\beta_{F,C1}\hat{y}_{C1}^2$	1.0880	1.0596	1.0612	1.3182	1.3182	0.7167
$\chi_{F,C1}\widehat{\hat{f}}_{C1}^2$	0.0285	0.1890	7.2424	0.2541	0.2541	0.0336
$\alpha_{F,C2}\hat{\pi}_{C2}^2$	1.0782	0.8725	0.9979	0.9605	0.9792	0.9605
$\beta_{F,C2}\hat{y}_{C2}^2$	1.0880	1.0596	1.0612	1.3182	0.7167	1.3182
$\chi_{F,C2}\widehat{\hat{f}}_{C2}^2$	0.0285	0.1890	7.2424	0.2541	0.0336	0.2541
$\alpha_{F,C3}\hat{\pi}_{C3}^2$	1.0782	0.8725	0.9979	0.9792	0.9605	0.9605
$\beta_{F,C3}\hat{y}_{C3}^2$	1.0880	1.0596	1.0612	0.7167	1.3182	1.3182
$\chi_{F,C3}\widehat{\hat{f}}_{C3}^2$	0.0285	0.1890	7.2424	0.0336	0.2541	0.2541
$\alpha_{CB}\hat{\pi}_{CB}^2$	5.3912	4.3627	4.9896	4.8337	4.8337	4.8337
$\beta_{CB}\hat{y}_{CB}^2$	0.2176	0.2119	0.2122	0.2181	0.2181	0.2181
$\chi_{CB}\widehat{\hat{i}}_U^2$	0.6367	0.7561	16.641	0.5530	0.5530	0.5530

losses into constituting elements is presented.[19] Reporting inflation, output gap and instrument shares in the total loss aims to provide additional intuition for our results.

As mentioned before, the grand coalition C is always a standard social optimum Π^{SOP} in the LQDG framework. In this particular case this regime constitutes also the social optimum as in Π^{*SOP}. For every coalition structure, $J_U^*(t_0)$ is approximately two times smaller than $J_U(t_0)$ which is caused by the following two reasons: (i) $J_U^*(t_0)$ contains only averages of inflation, output gap and fiscal debt deviations whereas $J_U(t_0)$ is composed of nominal values; (ii) $J_U(t_0)$ includes additionally the loss of the central bank from output and inflation. For some combinations of preference parameters it could be theoretically possible to obtain $J_U^*(t_0) > J_U(t_0)$ but this condition would hold only in an extreme case.

[19]For instance, $C1$'s loss $\frac{1}{2}\int_{t_0}^{\infty}\{\alpha_i\hat{\pi}_i^2(t) + \beta_i\hat{y}_i^2(t) + \chi_i\widehat{\hat{f}}_i^2(t)\}e^{-\theta(t-t_0)}dt$ reported in the top of Table 3 is decomposed into $\frac{1}{2}\int_{t_0}^{\infty}\{\alpha_i\hat{\pi}_i^2(t)\}e^{-\theta(t-t_0)}dt$, $\frac{1}{2}\int_{t_0}^{\infty}\{\beta_i\hat{y}_i^2(t)\}e^{-\theta(t-t_0)}dt$ and $\frac{1}{2}\int_{t_0}^{\infty}\{\chi_i\widehat{\hat{f}}_i^2(t)\}e^{-\theta(t-t_0)}dt$ in the lower part of the table.

In general, the structural symmetry of the model, the symmetry of fiscal players' preferences and the shocks make all the fiscal players' losses to be the same in the N, C and F regimes. Naturally, this symmetry is broken up under partial fiscal cooperation. The decomposition of losses shows that in nominal terms squared inflation deviation over time is about 5 times higher than squared output gap deviation over time. That is why inflation deviation contributes to the total fiscal loss as much as output gap deviation, even though fiscal players care 5 times less about inflation than about output gap. This observation validates our choice of benchmark weights in the loss functions.[20]

Regarding the form of the AD curve and the comment of Lambertini and Rovelli (2003), quoted in Sect. 2, that both types of authorities target the same variable, it is interesting to note that, in our dynamic setting, there is no straightforward relationship between changes in total volatility of output and changes in total volatility of inflation. Intuitively, we would expect that, since the volatility of inflation is directly linked with volatility of output gap via the Phillips Curve just as the volatility of output gap is linked to the volatility of inflation in the AD curve, the relationship between changes in the total loss of both variables should be one-directional. In other words, diminished inflation volatility would be related to either diminished or increased output gap volatility only. However, in our relatively rich dynamic setting, diminished (total) inflation volatility can be associated both with decreased (total) volatility of output gap (e.g. $\beta_{F,C1}\hat{y}_{C1}^2$ and $\alpha_{F,C1}\hat{\pi}_{C1}^2$ in F vs. N) and with increased (total) volatility of output gap (e.g. $\beta_{F,C1}\hat{y}_{C1}^2$ and $\alpha_{F,C1}\hat{\pi}_{C1}^2$ in [12|3|4] vs. N). Thus, it is clear that complex patterns of economic conditions can emerge in our model, which emphasises the need for an accurate policy regime.

The dynamics of all relevant variables in regimes N, C and F is compared in Fig. 1. Symmetric supply side (positive) inflation shocks cause output gap to decline which urges expansionary fiscal policies in all countries. In contrast, the central bank reacts to positive inflation by increasing interest rates. Thus, there is an obvious policy conflict between fiscal and monetary authorities. From decomposed losses in Table 3 we see that in the non-cooperative regime N there is a strong free-riding effect compared to the grand coalition regime C, which results from the positive fiscal spillovers characterising our setting.[21] When authorities do not cooperate, each of them tries to free-ride on the others' stabilisation efforts. The same phenomenon occurs in Beetsma et al. (2001) as their model also features positive fiscal spillovers. Consequently, all the authorities do not stabilise the economy strongly enough and the output gap and inflation deviations under regime N are comparatively high. This results, for instance, in the following total loss for fiscal players from output gap and inflation deviations: 1.0880 and 1.0782, respectively. In contrast, under regime C,

[20]More conventional preferences in which fiscal authorities care only two times as much about output gap as about inflation will be studied later on in this chapter.

[21]The term "free-riding" refers here to taking advantage of others' stabilisation policies during the stabilisation game (i.e. from time t_0, when the shock occurs, onwards). However, this term will be also used in the context of individual players breaking-up coalitional arrangements (with the same objective to take advantage of others' cooperative stabilisation policies but themselves playing non-cooperatively and constraining own costly policies).

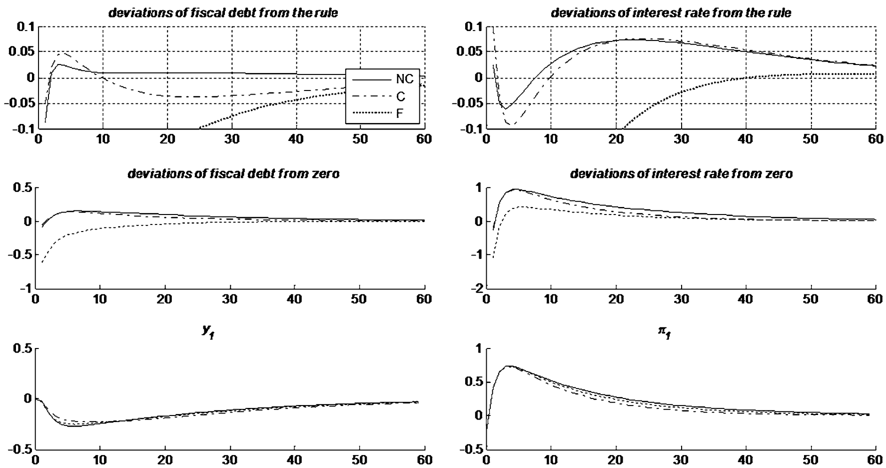

Fig. 1 Benchmark model, v_{0S}^{π}, comparison of regimes N, C and F

fiscal players: (i) do not try to free-ride on each other and (ii) take into account positive spillovers from other fiscal policies. Accordingly, they pursue a more active fiscal policy which is associated with the increased fiscal stabilisation cost from 0.0285 to 0.1890 and diminished losses from output gap and inflation deviations (1.0596 and 0.8725 respectively). The attitude of the central bank is crucial at this point. As under a symmetric inflation shock, there is clearly a policy conflict between fiscal and monetary authorities, increased fiscal activity should urge the central bank to more restrictive monetary policy than under the non-cooperative regime N. However, in the grand coalition all players, including the central bank, cooperate, and, secondly, the objective of every player is different than under non-cooperation as all of them aim to minimise the joint loss function which is a weighted sum of individual losses. Thus, in our benchmark, where all players have an equal weight in every coalition, the central bank's objective function (9) counts only for a quarter of the common loss function in regime C. Consequently, this function is "biased" towards fiscal preferences. Interestingly, under regime C these are fiscal players which find it profitable to change their policies so that there is no conflict between authorities about the direction of the stabilisation policies. Deviations of fiscal debts and the interest rate from the rules is shown in the upper part of Fig. 1. Under regime N both authorities deviate positively from the rules, which is counter-active as fiscal debt influences output gap in the opposite way to (nominal and real) interest rates. In contrast, when fiscal and monetary authorities cooperate, fiscal authorities deviate positively from the rule until period 10 and negatively since then. The sign of the central bank's deviations is exactly opposite; hence, both control instruments influence the output gap always in the same direction. In addition to the consent on policy direction, the lack of free-riding makes all the policies more active. All in all, cooperation makes the grand coalition the most attractive regime from the social point of view.

As far as now we have analysed our results mainly from the social perspective. However, even the regime which is the most desirable from the social point of view, can be very difficult to attain in the self-oriented environment. This is, for instance, the case for the grand coalition in Table 3. In a self-oriented myopic environment it could be very difficult to enforce this form of coordination, because every fiscal player has an incentive to unilaterally deviate from C.[22] For instance, $C1$ prefers [1|23|4] to C, thus, would break up the grand coalition, with hope that $C2$ and $C3$ maintain cooperation. In other words, there are strong free-riding incentives in the case of symmetric inflation shock to deviate from full cooperation. On the other hand, if the assumption of myopic behaviour is waived regime C is more likely to be stable, as it is clear that no partial fiscal cooperation is sustainable as any fiscal player involved in a coalition would prefer to break it and play in the non-cooperative regime N.

The situation is completely different under regime F, where the central bank does not cooperate with the coalition of all fiscal authorities. As before, coordination of policies among governments eliminates the free-riding and alleviates the use of fiscal instruments. However, it is clear from the decomposed losses in Table 3 that it exacerbates the conflict with the central bank as the loss from control effort is much higher in this regime both for governments and for the central bank. The policies are counter-active (see the upper part of Fig. 1) and the payoff for an increased control action is limited to only a little lower inflation and output gap deviations, which, by far, cannot make up for the increased loss. As a consequence, full fiscal coordination is worse than both cooperative and non-cooperative regimes. This latter result is exactly in line with the conclusions of Beetsma et al. (2001).[23] The difference between the results of both analyses is in the direction of the policies. In our model, when all fiscal players decide to cooperate, the central bank pursues more expansionary monetary policies than the assumed Taylor rule, whereas, the fiscal authorities pursue a more restrictive fiscal policy than the assumed fiscal rule. The direction of policies is, therefore, opposite to Beetsma et al. (2001), where supply shocks lead to more restrictive monetary policy vs. further reaching fiscal expansion.[24] However, in both studies a regime in which fiscal players cooperate against the central bank is counter-productive.

The ordering of social preferences over the regimes in Table 3 is $C \overset{J_U/J_U^*}{\succ} P \overset{J_U/J_U^*}{\succ} N \overset{J_U/J_U^*}{\succ} F$, i.e. cooperation in the grand coalition or in a partial fiscal coalition is better than playing alone. The analysis of P regimes follows similar lines as the dis-

[22]Deviations from a coalition are related to the coalition formation theory concept of internal stability (see Plasmans et al. 2006a, for further details).

[23]Note that Beetsma et al. (2001) do not consider coordination in a grand coalition.

[24]It might be argued that the above high cost of stabilisation is caused by the specific choice of policy rules which is so far away from optimum that players are forced to deviate much. In other words, it might be argued that θ_π^M should be closer to 1 and θ_π^i closer to 0. However, in other regimes, even in the fully non-cooperative regime, players are able to choose paths of stabilisation instruments close to assumed policy rules. This clearly overlures such an objection. Furthermore, the results reported in Table 3 were checked also for other parameterisations of policy rules such as: $(\theta_\pi^M = 1.5; \theta_\pi^i = 0)$; $(\theta_\pi^M = 1; \theta_\pi^i = -0.5)$; $(\theta_\pi^M = 1.25; \theta_\pi^i = 0)$; or $(\theta_\pi^M = 1; \theta_\pi^i = 0)$ and produce similar results (under all the above assumptions).

cussion of the grand coalition case. Starting from the non-cooperation, the creation of partial fiscal coalition eliminates free-riding incentives between two governments involved in cooperation which increases their activity. For example, when [12|3|4] is created, $\chi_{F,C1}\widehat{\hat{f}}^2_{C1}$ and $\chi_{F,C1}\widehat{\hat{f}}^2_{C2}$ increase symmetrically from 0.0285 to 0.2541. This increase is higher than the increase for the grand coalition, thus, it cannot be justified only by the diminished free-riding incentives. At the same time $\chi_{F,C1}\widehat{\hat{f}}^2_{C3}$ also increases (from 0.0285 to 0.0336), instead of decreasing.[25] Both the above results are to be explained by the more constrained activity of the central bank, which is caused by the asymmetry of output gap and inflation in this regime that makes union-wide averages less volatile and, therefore, not affecting the loss of a monetary authority to such an extent as before. In other words, the central bank is able to free-ride even more than in regime N and this, in turn, increases the use of control instrument in both fiscal players involved in a coalition and playing non-cooperatively.

5.3 Asymmetric Inflation Shock

Table 4 is constructed in a similar manner as Table 3 and presents (optimal) welfare losses for the asymmetric (country-specific) inflation shock, $v^{\pi}_{0S} := [v^{\pi}_{1,0}, v^{\pi}_{2,0}, v^{\pi}_{3,0}]^T = [1,0,0]^T$. Clearly, not all fiscal losses are now symmetric, as the shock directly hits the first country only and other member-states of a monetary union have to deal with its indirect consequences. However, in general, it can be said that the pattern of losses in Table 4 is quite similar to Table 3: again the ordering of social preference over regimes is $C \overset{J_U/J_U^*}{\succ} P \overset{J_U/J_U^*}{\succ} N \overset{J_U/J_U^*}{\succ} F$; full fiscal cooperation is the worst regime for everybody; the grand coalition performs reasonably well but myopic fiscal players would have an incentive to break up this regime; partial fiscal coordination is not sustainable as players involved in cooperation would prefer the non-cooperative regime. Finally, it should be noted that cooperation of a country which is not affected directly by the shock with a country which is affected (regimes [12|3|4] or [13|2|4]) is very unprofitable for both of them but especially for the former one. Furthermore, also cooperation of both countries which are not affected directly by the shock (regime [1|23|4]) is also not profitable when compared to non-cooperation. Consequently, any form of partial fiscal cooperation when the correlation of shocks gets smaller seems to be unsustainable in case of a symmetric inflation shock.

Losses in this case are smaller than for the symmetric inflation shock because there is a different reaction of the common monetary policy and national fiscal policies in countries not affected by the shock. This happens because the central bank

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[25] The decrease in $C3$ control effort would be expected as the cooperation between $C1$ and $C2$, by increasing their activism, gives even more incentives to free-ride.

Table 4 Optimal losses for an asymmetric inflation shock, baseline parametrisation

(sc_1, v_{0S}^{π})	N	C	F	[12|3|4]	[13|2|4]	[1|23|4]
$C1$	0.3409	0.3245	1.0848	0.3921	0.3921	0.2807
$C2$	0.2369	0.2329	1.0495	0.3006	0.1763	0.2896
$C3$	0.2369	0.2329	1.0495	0.1763	0.3006	0.2896
CB	0.6939	0.5923	2.4270	0.6024	0.6024	0.6084
$J_U(t_0)$	1.5092	1.3827	5.6109	1.4715	1.4715	1.4684
$J_U^*(t_0)$	0.8483	0.7618	3.3804	0.7898	0.7898	0.7932
$\alpha_{F,C1}\hat{\pi}_{C1}^2$	0.1659	0.1461	0.1584	0.1509	0.1509	0.1541
$\beta_{F,C1}\hat{y}_{C1}^2$	0.1681	0.1679	0.1607	0.2183	0.2183	0.1100
$\chi_{F,C1}\widehat{\hat{f}}_{C1}^2$	0.0069	0.0104	0.7656	0.0228	0.0228	0.0165
$\alpha_{F,C2}\hat{\pi}_{C2}^2$	0.1067	0.0823	0.0970	0.0896	0.0924	0.0899
$\beta_{F,C2}\hat{y}_{C2}^2$	0.1266	0.1215	0.1253	0.1573	0.0760	0.1512
$\chi_{F,C2}\widehat{\hat{f}}_{C2}^2$	0.0035	0.0291	0.8270	0.0536	0.0078	0.0483
$\alpha_{F,C3}\hat{\pi}_{C3}^2$	0.1067	0.0823	0.0970	0.0924	0.0896	0.0899
$\beta_{F,C3}\hat{y}_{C3}^2$	0.1266	0.1215	0.1253	0.0760	0.1573	0.1512
$\chi_{F,C3}\widehat{\hat{f}}_{C3}^2$	0.0035	0.0291	0.8270	0.0078	0.0536	0.0483
$\alpha_{CB}\hat{\pi}_{CB}^2$	0.5990	0.4847	0.5544	0.5220	0.5220	0.5230
$\beta_{CB}\hat{y}_{CB}^2$	0.0241	0.0235	0.0235	0.0242	0.0242	0.0242
$\chi_{CB}\widehat{\hat{i}}_U^2$	0.0707	0.0840	1.8490	0.0561	0.0561	0.0612

targets average inflation which in the case of an asymmetric shock is clearly much smaller than in case of a symmetric one (see Fig. 2). Also, the national fiscal policies of $C2$ and $C3$ react more moderately since these countries are only affected by cross border-spillovers and the restrained reaction of the common monetary policy.

The above findings, in general, correspond to the main arguments of Beetsma et al. (2001). Indeed, the clash between authorities is diminished under an asymmetric inflation shock because many effects cancel each other and policymakers have less incentives to excaberate the dispute. However, in our model, the extent of the conflict is still so substantial, that the excessive use of control instruments make the regime of fiscal cooperation, by far, unprofitable.

5.4 Sensitivity Analysis

Thus far, we have studied a number of cases characterised by different forms of coordination. It is interesting to perform sensitivity analyses of the model in order to identify which results (possibly) prevail and which parameters contribute mostly

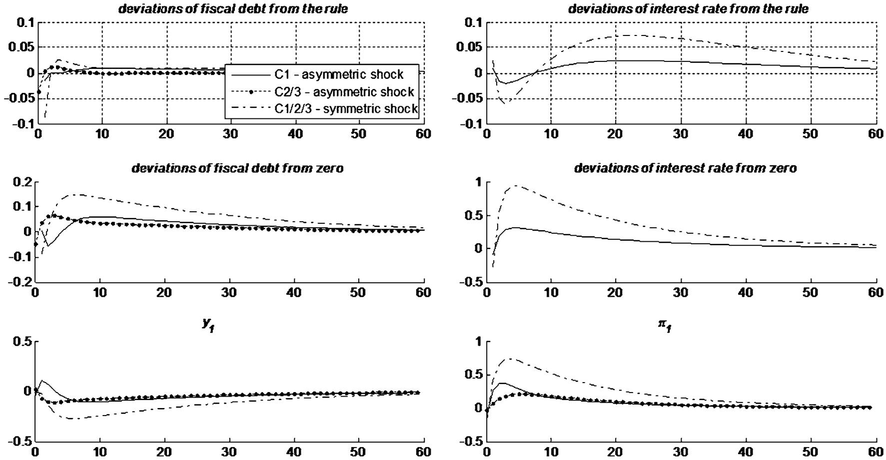

Fig. 2 Benchmark model, v^{π}_{0S} vs. v^{π}_{0A}, regime N

to particular outcomes. Some elements of a sensitivity analysis have been already present in the discussion above. In particular, we have inquired what happens to the social loss when alternative weights of the central bank in the grand coalitions are assumed. Additionally, we performed sensitivity analysis in three other dimensions:

1. Similarly to Beetsma et al. (2001) or Beetsma and Jensen (2005) we vary one structural parameter of the model at a time, assuming that it has either a *high* or a *low* value. Additionally, we perform not only one but a number (usually 4 or 5) of simulations in the neighbourhood of each high and low value;
2. Furthermore, we study various combinations of preference parameters in the loss functions of players. In particular, different ratios of preferences towards output gap and inflation are considered;
3. In the next step, the sensitivity analysis of governments' preferences is performed. A particular attention will be paid to the government preference parameter χ_i which different values can be interpreted as levels of the SGP stringency.

A detailed description of the obtained results is available in the Appendix (Sections A.1–A.4). The structural parameter sensitivity check reveals that the degree of output gap backward-lookingness is the key parameter, which can either magnify or diminish the conflict between fiscal and monetary authorities. Counter-intuitively, it turns out that cooperation can be more effective when the economies are relatively more backward-looking. Backward-lookingness makes economies more rigid and therefore more difficult to control which, in turn, should lead to increased control effort. This is the main factor inducing conflict with the central bank in the benchmark and resulting in higher losses in the full fiscal cooperation regime. However, when a certain threshold is triggered, the players realise that there is no point in setting off each other's policies as their influence on the economy becomes too limited. Consequently, they all decide to refrain from excessive actions and instead of colliding their policies, governments try to free-ride on the central bank's control effort and vice versa. Interestingly, this is not the case under the non-cooperative regime. Instead, in this type of regime, conflict between authorities increases together with the backward-lookingness of the output gap. Consequently, when the rigidity of the output gap becomes high enough, non-cooperation starts to become inferior w.r.t. fiscal cooperation. In other words, the situation is the exact reversal of that of the baseline analysis. In the benchmark case, characterised by high forward-lookingness, free-riding diminishes the conflict between fiscal and monetary authorities under non-coordination. However, under fiscal coordination, the struggle between governments and the central bank increases everybody's loss. When high backward-lookingness is assumed, the conflict is exaggerated under non-cooperation, whereas under fiscal cooperation, authorities try to free-ride on each others' policies.

The second factor which heavily influences the results of our benchmark analysis is the relative conflict between preferences of fiscal and monetary authorities. The more governments exclusively focus on output gap stabilisation and the more central banks focus on inflation stabilisation, the more pronounced the conflict between them becomes. Consequently, the counter-profitable effects of full fiscal cooperation are even greater. On the other hand, when the preferences of both types of players are more alike, there is less reason for conflict and fiscal cooperation becomes, from the social point of view, beneficial. More specifically, full fiscal cooperation initially becomes beneficial w.r.t. the non-cooperative regime, before subsequently becoming more beneficial w.r.t. partial fiscal cooperation regime and finally, more beneficial w.r.t. the grand coalition. This last effect is counter-intuitive as, by definition, the grand coalition always minimises the sum of the losses of all the players in the LQDGs. However, in our analysis we mainly refer to our own definition of the social loss in which, as has been mentioned previously, the cost of interest rate deviation from the rule is properly weighted to correspond to the one-country case. Consequently, our social optimum does not necessarily agree with the grand coalition. In fact, when the loss functions of governments and central banks become very alike, social loss obtained under fiscal cooperation tends to be smaller than under the grand coalition. This is caused by the fact that, in the simple sum of players' losses, the weight of the control instrument of a monetary authority is relatively small w.r.t. fiscal debts of individual governments. This creates an incentive to use it more extensively. If the importance of the interest rate is appropriately rescaled in the social loss, the grand coalition is no longer the most profitable regime.

Using numerical simulations, we study the way in which various combinations of policy rules' parameters influence output gap and inflation volatility. We also determine which of such combinations are likely to result in a(n) (near) optimal outcome from the social point of view. In the non-cooperative regime, and under a symmetric inflation shock, the proximity of the social optimum is reached for the combination of rules in which the central bank follows the standard Taylor rule, yet there is no automatic fiscal stabiliser to output. However, we show that if players in a monetary union were able to unilaterally choose their rules, the social optimum combination would not be sustainable. This is because the monetary authority has incentives to increase its automatic reaction to inflation and, at the same time, the government has incentives to increase its reaction to output gap. When these things occur simultaneously, the economies end up in a position that is suboptimal, not only from the social point of view, but also from the perspective of the individual.

Finally, we study various scenarios characterised by different levels of the SGP stringency and show that it is the third factor that is pivotal to the benchmark results. The increased SGP stringency reduces the incentives for fiscal players to use control instruments. Therefore, in situations where high social losses were driven by the conflict between authorities (notably full fiscal cooperation regime), this firmer stance is beneficial to the union-wide economic interest. However, in situations where free-riding is present (notably non-cooperative regime under benchmark parametrisation), increased SGP stringency may lead to more extensive free-riding of governments, since controlling the economy becomes much more costly. This, in turn, forces the central bank to intervene and increases social loss of the union. In other words, the stringent SGP has both positive and negative effects in our context and is able to render unprofitable full fiscal cooperation regime profitable w.r.t. non-cooperation.

6 Conclusions

In this work we considered a number of important issues concerning the policy coordination in the monetary union which have been discussed in the literature. In relation to this, we proposed a (stylised) Multi-Country New-Keynesian Monetary Union Model cast in the framework of linear quadratic differential games which can be used to simulate strategic interactions between an arbitrary number of fiscal authorities who interact in coalitions either in cooperation with or against the common central bank. In the above setting, we studied various coordination arrangements, including partial fiscal cooperation between only a subgroup of countries, which, to the best of our knowledge, had not previously been considered in the literature.

Our results are comparable to those of Beetsma et al. (2001) but in a much richer dynamic setting. Whereas free-riding in economics and social sciences is usually associated with inefficiency and losses, in our model, for many parameter combinations, fiscal cooperation between all countries in a monetary union turns out to be counter-productive as a result of exacerbated conflict with the central bank. Thus, the non-cooperative regime, in which policy-makers free-ride on each others' stabilisation efforts, is more profitable. The relative performance of fiscal cooperation is worst in the case of a symmetric inflation shock. When the shock is asymmetric, the response of the central bank to an average inflation in the union is more moderate, as are the losses of all the authorities.

In addition to the above results, in our multi-country framework, we were able to study intermediate regimes, in which only a subgroup of the fiscal players cooperate. Such a solution turns out to be interesting, especially from the common perspective. When a unity of governments is broken and they no longer optimise the common objective function, then the free-riding element is back into play and the ultimate choice of optimal policies is much less intense. Nevertheless, such regimes can be difficult to sustain, as they lead to a deteriorated position of some individual countries.

Furthermore, we discussed the effectiveness of the grand coalition, i.e. cooperation between all fiscal authorities and the monetary authority. Although this regime is profitable from the social point of view, it seems unlikely that it could be sustained without the creation of a central control or an effective transfer system. Since the situation is caused by the fact that individual countries are often worse off in the grand coalition than if they were in the non-cooperative regime, these countries are not willing to accept such a regime without compensation.

Finally, we performed an extensive sensitivity check of our results and determined three variables that are pivotal to the results we obtained for the benchmark of our model: (i) the degree of forward-lookingness in the union's economies; (ii) the preference conflict between fiscal and monetary authorities; and (iii) the SGP stringency. The last analysis reveals the most interesting finding as long as the issue of fiscal cooperation is concerned. Since the main source of the relative inefficiency of full fiscal cooperation is an increased conflict with the central bank due to a more intensive use of the relevant control instrument. Higher levels of SGP stringency mean that the use of fiscal debt becomes less and less attractive. This, in turn, removes the reason for conflict between two types of authorities.

The following policy conclusion can be derived from our analysis. The grand coalition is, in general, the most effective regime; however, only if the design of a cooperative arrangement takes into account the specific nature of a central bank and its policy instruments. However, this regime is difficult to implement due to various problems. In particular, the very nature of such a coalition jeopardises the idea of the independent central bank. On the other hand, under special circumstances, like present financial crises that spreads to other sectors of the global economy, we already witness various forms of cooperation between the ECB and the EMU member states as well as EU member states. However, in general, both de jure and de facto state of the affairs in the EMU can be described best by the non-cooperative regime. Since the grand coalition is rather out of question in the long term, intergovernmental coordination appears to be another interesting alternative. With respect to this, our results show that obligatory fiscal coordination between all the countries within the union can be counter-productive and that smaller coalitions of countries should be considered. This corresponds to the findings of the literature on voluntary environmental agreements that suggest that local solutions are more stable than a centralised approach as for instance the Kyoto Protocol. Similarly, partial fiscal cooperation can be an interesting option as far as macroeconomic policy coordination is concerned. However, if (possibly multiple) partial agreements are not feasible for political reasons, then full fiscal coordination can be considered again in conjunction with the increased stringency of the SGP. The SGP should prevent excessive increases in fiscal activity that might be induced by cooperation.

Our analysis can be extended in several important directions. Although we considered both symmetric and asymmetric shocks in our analysis, we assumed full symmetry of monetary union member states. Following on from this, it would be beneficial to carry out an analysis of an asymmetric model, in terms of both the economies structures and the players preferences. Furthermore, since our results suggest that more attention should be devoted to partial forms of cooperation between the governments of a monetary union, we would like to extend our analysis to (at least) a four-country case, where two non-trivial fiscal coalitions may coexist in one single coalition structure. Finally, it could be interesting to consider how the results obtained from our model are altered by the introduction of a federal fiscal transfer system in a monetary union.[26]

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Appendix

A.1 Sensitivity Analysis with Respect to Structural Parameters

The assumed changes of one structural parameter at a time are presented in the first column of Table 5. The next columns show the social preference ordering over different regimes for four shocks.[27] The following notation is used: P—stands for regimes 4, 5 and 6; regime in bold means that players in a coalition are better off than in N; $\hat{C}/\hat{F}$—means that all fiscal players in grand/full fiscal coalition are better off than in N but the CB is not in this regime; $\check{C}/\check{F}$—vice versa; **F**—all players are better off than in N. In most of the cases the ordering based on J_U^* was the same as based on J_U, with only few minor exception, therefore, only the preference ordering based on the former social loss is reported in Table 5.

For all combinations of parameters except for $\kappa_{i,y/\pi} = 0.5$, from the social point of view, regime N is preferred over F; in other words full fiscal coordination in counter-productive. Thus, it can be said that for the large set of parameters our model confirms results of Beetsma et al. (2001).

The ordering *CPNF* prevails for symmetric inflation shock v_{0S}^{π} and is robust to changes in parametrisation (except for lower value of $\kappa_{i,y/\pi}$).[28] The same ordering is valid for country-specific inflation shock v_{0A}^{π}, however, due to asymmetry partial

[26] An example of such an analysis (albeit only in a two-country setting) can be found in Plasmans et al. (2006a, Chap. 3).

[27] For the time being we focus on structural parameters of the model excluding policy rules, which together with preference parameters will be discussed in the next section.

[28] Note that in the case of symmetric shock regimes 4, 5 and 6 denoted jointly by P are symmetric; thus, are characterised by the same social loss. However, for asymmetric shocks which always hit $C1$ only regimes 4 and 5 are symmetric to each other where as they are, in general, asymmetric to regime 6. Consequently, for asymmetric shocks we do not the joint P-notation for partial fiscal cooperation regimes.

Table 5 Sensitivity analysis of the benchmark model

	$J_U^*(t_0)$			
	v_{0S}^{π}	v_{0A}^{π}	v_{0S}^{y}	v_{0A}^{y}
Benchmark	$\mathbf{C}PNF$	$\mathbf{C}456NF$	$P\hat{C}NF$	$CN456F$
$\kappa_{i,y/\pi} \approx 0.5$	$\mathbf{C}\check{F}\mathbf{P}N$	$\mathbf{C6}\check{F}\mathbf{45}N$	$\mathbf{C}P\mathbf{F}N$	$\mathbf{C6}45\mathbf{F}N$
$\kappa_{i,y/\pi} \approx 0.8$	$\mathbf{CP}NF$	$\mathbf{C645}NF$	$\mathbf{C}PNF$	$\mathbf{C645}NF$
$\gamma_i \approx 0.45$	$\mathbf{C}PNF$	$C45\mathbf{6}NF$	$N\hat{C}PF$	$CN456F$
$\gamma_i \approx 0.55$	$\mathbf{C}PNF$	$\mathbf{C}45\mathbf{6}NF$	$N\hat{C}PF$	$CN456F$
$\eta_i \approx 0.65$	$\mathbf{CP}NF$	$\mathbf{C}456NF$	$\hat{C}PNF$	$CN45\mathbf{6}F$
$\eta_i \approx 1$	$\mathbf{C}PNF$	$\mathbf{C}456NF$	$P\hat{C}NF$	$CN456F$
$\rho_{ij} \approx 0.4$	$\mathbf{C}PNF$	$\mathbf{C}456NF$	$N\hat{C}PF$	$CN456F$
$\rho_{ij} \approx 0.5$	$CPNF$	$C456NF$	$CPNF$	$CN456F$
$\delta_{ij} \approx 0.075$	$\mathbf{C}PNF$	$\mathbf{C}45\mathbf{6}NF$	$P\hat{C}NF$	$C45N6F$
$\delta_{ij} \approx 2/3$	$\mathbf{C}PNF$	$\mathbf{C}456NF$	$N\hat{C}PF$	$CN456F$
$\xi_i \approx 0.03$	$\mathbf{CP}NF$	$\mathbf{C}645NF$	$\mathbf{C}PNF$	$\check{C}\mathbf{6}4\mathbf{5}NF$
$\xi_i \approx 0.125$	$\mathbf{CP}NF$	$\mathbf{C}45\mathbf{6}NF$	$\mathbf{C}PNF$	$\check{C}\mathbf{P}N\mathbf{F}$
$\varsigma_{ij} \approx 0.025$	$\mathbf{C}PNF$	$\mathbf{C}456NF$	$N\hat{C}PF$	$\check{\mathbf{C}}\mathbf{6}N45F$
$\varsigma_{ij} \approx 0.1$	$\mathbf{C}PNF$	$\mathbf{C}456NF$	$\mathbf{C}PNF$	$\check{\mathbf{C}}\mathbf{6}N45F$

fiscal coalition 6 is (in general) characterised by different social loss than 4 and 5; thus, preference ordering over partial fiscal coalitions might vary. This leads to the conclusion that the grand coalition or partial fiscal coalitions should be sought as the socially efficient regimes in the case of inflation shock. The question whether such forms of coordination would be sustainable remains open. In Table 5 under v_{0S}^{π} and v_{0A}^{π} the grand coalition is preferred over non-cooperation by every player from an individual point of view. However, it does not tell us whether any player would like to deviate from this arrangement with hope that remaining fiscal players maintain cooperation and a partial fiscal regime emerges. In contrast, the fact that P-regimes are usually inferior to N for (fiscal) player(s) being in coalitions allows us to draw a conclusion that, certainly, these regimes are not sustainable in self-oriented (and myopic) environment.

The picture is less clear for output gap shocks as parameter changes have more influence on regimes' social ordering here. Full fiscal cooperation is always least preferred (except for lower value of $\kappa_{i,y/\pi}$) so the results of Beetsma hold also in this case. However, in contrast to inflation shocks, the grand coalition scores often worse than non-cooperation or P-regimes in the case of symmetric output gap shock.

To summarise, sensitivity analysis of the benchmark model confirms the result of Beetsma et al. (2001) as in all the cases but one full fiscal coordination is worse than non-cooperation. Furthermore, this result can be extended further, as it comes out that F is the worst of all regimes, including partial fiscal cooperation. For inflation shocks, the grand coalition is the socially optimal outcome, and this regime is better than non-cooperation from the individual point of view. However, whether

Table 6 Optimal losses for $(v_{0S}^{\pi}, \kappa_{i,y} = 0.5)$

(sc_1, v_{0S}^{π})	N	C	F	[12|3|4]	[13|2|4]	[1|23|4]	N^F
$C1$	4.2650	3.5500	3.5033	3.7917	3.7917	3.4779	4.0750
$C2$	4.2650	3.5500	3.5033	3.7917	3.4779	3.7917	4.0750
$C3$	4.2650	3.5500	3.5033	3.4779	3.7917	3.7917	4.0750
CB	11.7566	6.9454	8.5717	10.1221	10.1221	10.1221	12.3601
$J_U(t_0)$	24.5516	17.5957	19.0817	21.1836	21.1836	21.1836	24.5859
$J_U^*(t_0)$	13.9057	8.9598	10.3530	11.8533	11.8533	11.8533	13.7293
$\beta_{F,C1}\hat{y}_{C1}^2$	1.7332	2.1976	2.0190	2.0272	2.0272	1.4935	1.4647
$\alpha_{F,C1}\hat{\pi}_{C1}^2$	1.7692	1.0961	1.3181	1.5834	1.5834	1.5980	2.4136
$\chi_{F,C1}\widehat{\hat{f}}_{C1}^2$	0.7625	0.2562	0.1661	0.1810	0.1810	0.3863	0.1966
$\beta_{F,C2}\hat{y}_{C2}^2$	1.7332	2.1976	2.0190	2.0272	1.4935	2.0272	1.4647
$\alpha_{F,C2}\hat{\pi}_{C2}^2$	1.7692	1.0961	1.3181	1.5834	1.5980	1.5834	2.4136
$\chi_{F,C2}\widehat{\hat{f}}_{C2}^2$	0.7625	0.2562	0.1661	0.1810	0.3863	0.1810	0.1966
$\beta_{F,C3}\hat{y}_{C3}^2$	1.7332	2.1976	2.0190	1.4935	2.0272	2.0272	1.4647
$\alpha_{F,C3}\hat{\pi}_{C3}^2$	1.7692	1.0961	1.3181	1.5980	1.5834	1.5834	2.4136
$\chi_{F,C3}\widehat{\hat{f}}_{C3}^2$	0.7625	0.2562	0.1661	0.3863	0.1810	0.1810	0.1966
$\beta_{CB}\hat{y}_{CB}^2$	0.3466	0.4395	0.4038	0.3677	0.3677	0.3677	0.2929
$\alpha_{CB}\hat{\pi}_{CB}^2$	8.8462	5.4808	6.5908	7.9414	7.9414	7.9414	12.0680
$\chi_{CB}\widehat{\hat{i}}_U^2$	2.5637	1.0251	1.5770	1.8128	1.8128	1.8128	0

C is sustainable remains still an open question. Partial fiscal cooperation is suboptimal w.r.t. the grand coalition but gives better results than non-cooperation from the social perspective. Unfortunately, in many cases these regimes are suboptimal from individual point of view, thus, possibly unsustainable in the non-cooperative environment, especially if players are myopic. The ineffectiveness results about full fiscal coordination hold also for output gap shocks, but it more difficult to draw some definite conclusions about the preference ordering over the other regimes as the differences in social loss between them are small and, therefore, sensitive, to changes in parametrisation.

It is apparent from Table 5 that the influence of forward-lookingness on our model calls for more attention, as lower values of this parameter may have an important impact on the result obtained above. Table 6 shows the optimal losses for symmetric inflation shock v_{0S}^{π} and benchmark parametrisation but with $\kappa_{i,y/\pi} = 0.5$. What is the reason for the improved effectiveness of F regime w.r.t. N when the economies in a monetary union are characterised by lower forward-lookingness?

Change of an important model parameter certainly influenced the reduced form matrices $\tilde{B}_4$ and $\tilde{B}_5$ which show the influence of control instruments on output gaps

Table 7 Symmetric inflation shock, the number of equilibria in LQDGs

(sc_1, v_{0S}^{π})	N	C	F	[12|3|4]	[13|2|4]	[1|23|4]
LQDGE	158	1	1	140	140	140
PUNE	1	1	1	13	13	13

and inflations, respectively.

$$\tilde{B}_4 = \begin{bmatrix} 0.7007 & 0.1468 & 0.1468 & -0.6631 \\ 0.1468 & 0.7007 & 0.1468 & -0.6631 \\ 0.1468 & 0.1468 & 0.7007 & -0.6631 \end{bmatrix}, \quad \text{and}$$

$$\tilde{B}_5 = \begin{bmatrix} 0.0134 & 0.0026 & 0.0026 & -0.0125 \\ 0.0026 & 0.0134 & 0.0026 & -0.0125 \\ 0.0026 & 0.0026 & 0.0134 & -0.0125 \end{bmatrix}.$$

The pattern is closely comparable to the benchmark (all the values are now more or less 15 % lower than before and all the signs are preserved). With a bigger backward-looking component, economies are more persistent and converge slower to the equilibrium. This is reflected by nearly two times as more losses in regimes N, C and P for $\kappa_{i,y/\pi} = 0.5$ compared to benchmark with $\kappa_{i,y/\pi} = \frac{2}{3}$ in Table 3. As far as the cost of the control instruments is concerned, we observe the following pattern. For regimes where only fiscal players fully or partially cooperate (i.e. F and P), (total) cost of control of those players involved in the coalition gets lower when backward-looking component becomes more eminent. In the case of full fiscal cooperation. This reduction is drastic, from 7.2424 to 0.1974. In contrast, the loss of the fiscal players from the control effort (in P regimes) increases more than 10 times, from 0.0336 to 0.3682. The same holds for the non-cooperative regime, where the (non-cooperative) fiscal players have their control effort increased from 0.0285 to 0.6618. In spite of the somewhat increased volatility of inflation and output gap, the above decrease in control cost in the F regime accompanied by the increase in control cost in N regime (see Fig. 3) makes full fiscal cooperation more attractive than previously.

The reason for these more moderate actions should be sought in the fact that more backward-lookingness in the model means that economies are more persistent and more slowly converging to equilibrium. This means that it is more costly to control them as the use of instruments is less efficient, which successfully diminishes the conflict between authorities, which choose more reasonable policies.

To improve our understanding of the meaning of forward-lookingness in the model similar sensitivity check to the previous one was conducted but now assuming that $\kappa_{i,y/\pi}$ is set to 0.5 as the benchmark. The results are presented in Table 8 which was constructed in the same way as Table 5.[29]

[29]In Table 8 the orderings based on $J_U(t_0)$ are in all cases but 8 exactly the same as those based on $J_U^*(t_0)$, where the differences occurred only in the ordering of partial fiscal cooperation regimes 4, 5 and 6 under asymmetric shocks. Consequently, we report only orderings based on $J_U^*(t_0)$.

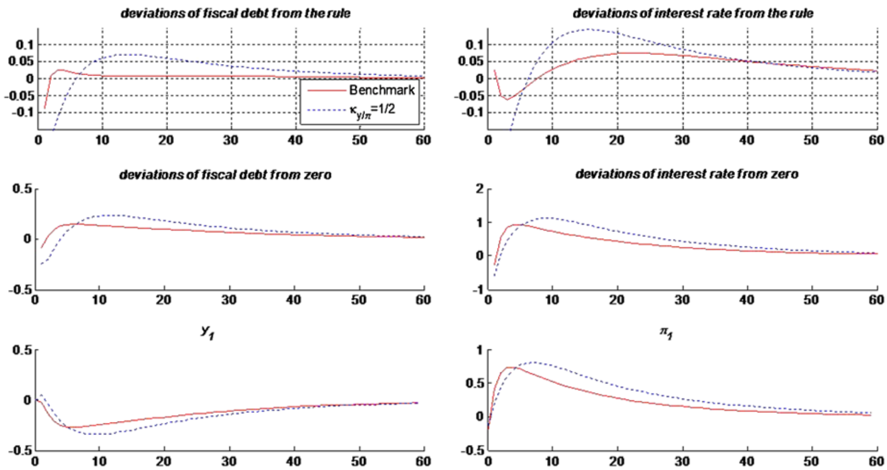

Fig. 3 Benchmark vs. model with lower forward-lookingness, v_{0S}^{π}, regime N

Table 8 Sensitivity analysis, benchmark model but with lower forward-lookingness

	$J_U^*(t_0)/J_U(t_0)$			
	v_{0S}^{π}	v_{0A}^{π}	v_{0S}^{y}	v_{0A}^{y}
Benchmark	C$\check{F}$PN	C6$\check{F}$45N	CPFN	C645FN
$\gamma_i \approx 0.45$	CF456N	CF645N	C456FN	$\check{C}$645FN
$\gamma_i \approx 0.66$	CF456N	CF645N	CNF456	$\check{C}NF$456
$\eta_i \approx 0.65$	CF456N	CF456N	C$N\hat{F}$456	$\check{C}\hat{F}N$456
$\eta_i \approx 1$	CF456N	CF645N	C456FN	$\check{C}$45FN6
$\rho_{ij} \approx 0.35$	CF456N	CF645N	C456FN	C645FN
$\rho_{ij} \approx 0.55$	CF456N	CF645N	CFN456	$\check{C}$645$\hat{F}N$
$\delta_{ij} \approx 0.075$	CF456N	CF645N	C456FN	$\check{C}$45$\hat{F}N$6
$\delta_{ij} \approx 0.5$	CF456N	CF645N	C645FN	$\check{C}$456FN
$\xi_i \approx 0.03$	CF456N	CF456N	C456$\check{F}N$	$\check{C}$6$\check{F}N$45
$\xi_i \approx 0.125$	CF456N	CF645N	CF456N	C645FN
$\varsigma_{ij} \approx 0.02$	CF456N	CF645N	C456FN	C645FN
$\varsigma_{ij} \approx 0.1$	CF456N	CF645N	C645FN	$\check{C}$456FN

In most of the cases, $J_U^*(t_0, F)$ is preferred over $J_U^*(t_0, N)$ which confirms that forward-lookingness is decisive for the social profitability of full fiscal cooperation in our model. Similarly to Table 6 the social ordering is especially robust for both symmetric and asymmetric inflation shocks. In all these cases full fiscal coordination is the second best regime just after the grand coalition. Regimes P with smaller fiscal coalitions score worse, where as the worst result is obtained for non-cooperation. Consequently, in contrast to previous findings and Beetsma et al. (2001), the above results strongly advocate the need for coordination in the case of inflation shocks.

Furthermore, any form of coordination is better than non-cooperation, not only from the social point of view, but also from the perspective of individual authorities.

The results are a little less clear for output gap shocks as, for a few parameter combinations, non-cooperation comes back to the position it took in Table 5, i.e. just after C but before N. It happens when the CB gets more powerful w.r.t. fiscal players in its influence on the output gap (either higher values of γ_i or lower value of η_i).

A.2 Sensitivity Analysis w.r.t. Preference Parameters

Thus far, our sensitivity analysis was performed only w.r.t. structural parameters of the economies (excluding policy rules). It is interesting to take a closer look into preference parameters in players' loss functions. In the benchmark, fiscal players cared five times more about development of output gaps than inflation whereas the CB cared five times more about inflation than output gap. We argued that such a preferences were in line with other parameters in the model as they guarantee that no variables are overrepresented in the total loss of players (as decompositions in previous sections confirm). In this section we vary relative preferences of fiscal and monetary authorities regarding output gap and inflation. In particular, let $r_F^{\pi/y}$ and $r_{CB}^{\pi/y}$ denote the ratio between α_i and β_i and between $\beta_{CB,i}$ and $\alpha_{CB,i}$, respectively, i.e. $r_F^{\pi/y} := \frac{\alpha_i}{\beta_i}$ and $r_{CB}^{\pi/y} := \frac{\beta_{CB,i}}{\alpha_{CB,i}}$. The sensitivity analysis will be performed for $r_F^{\pi/y}$ and $r_{CB}^{\pi/y}$ simultaneously changing from 0 to 1 with step 0.1. For $r_F^{\pi/y} = r_{CB}^{\pi/y} = 0$ we have a situation where governments are concerned only about output gaps (i.e. are very liberal in stabilisation sense) while the CB is concerned only about inflation (i.e. is very conservative). In other words, this is the situation when preferences are totally opposite. In contrast, when $r_F^{\pi/y} = r_{CB}^{\pi/y} = 1$ fiscal authorities as well as the monetary ones are equally interested in deviations of both variables, which, taking into account that the weight of the control instrument does not change and is equal between all the players, means that under symmetric shocks their objectives are the same in this extreme case. The results of the sensitivity check of the benchmark model with preferences amended in the above way are presented in Table 9. The ordering in this table, similarly to previous tables, is based on $J_U^*(t_0)$.

It is evident from Table 9 that the grand coalition is the socially optimal regime for lower values of $r_{F/CB}^{\pi/y}$, when preferences of various authorities are opposite. The second best choice are partial fiscal cooperation regimes, where as fiscal cooperation scores worst, even worse than non-cooperative regime N. This pattern is observed in the neighbourhood of the benchmark for all shocks except for v_{0S}^{y}. In contrast, when $r_{F/CB}^{\pi/y}$ becomes larger, first partial fiscal cooperation becomes more socially profitable than C, then F more profitable than N and, finally, when preferences of governments and the CB coincide, F becomes the most profitable outcome of all. This last result is interesting as previously, in the majority of situations, C was the

Table 9 Sensitivity analysis, benchmark model with altered preference

$r^{\pi/y}_{F/CB}$	$J^*_U(t_0)$							
	v^{π}_{0S}	$\overline{J^*_U}$	v^{π}_{0A}	$\overline{J^*_U}$	v^{y}_{0S}	$10^2\overline{J^*_U}$	v^{y}_{0A}	$10^2\overline{J^*_U}$
0	**C***PNF*	17.16	$\check{C}645NF$	1.93	*CNPF*	4.23	*CN*456*F*	3.17
0.1	**C***PNF*	13.14	$\check{C}645NF$	1.54	$\hat{C}NPF$	7.78	*CN*456*F*	3.38
0.2[a]	**C***PNF*	10.58	**C**456*NF*	1.23	$P\hat{C}NF$	7.37	*CN*456*F*	3.32
0.3	**C***PNF*	8.97	**C**456*NF*	1.06	**P**$\hat{C}NF$	7.13	*C*45*N*6*F*	3.31
0.4	**C***PNF*	8.05	**C**45**6***NF*	0.96	**P**$\hat{C}NF$	6.98	*CN*456*F*	3.28
0.5	*P***C***NF*	7.58	45**C6***NF*	0.91	**P**$\hat{C}NF$	6.90	$\hat{C}45NF$**6**	3.27
0.6	*P***C***NF*	7.11	45**C6***NF*	0.84	**P**$\hat{C}N\hat{F}$	6.85	$\hat{C}\hat{F}45N$**6**	3.28
0.7	*P***C***NF*	6.80	45**6C***NF*	0.81	**P**$\hat{F}N\hat{C}$	6.84	$\hat{C}\hat{F}N45$**6**	3.24
0.8	**PCF***N*	6.66	45**6CF***N*	0.79	**FP**$N\hat{C}$	6.82	**F**$\hat{C}$**45***N*6	3.28
0.9	**FPC***N*	6.58	**F6**45**C***N*	0.79	**FP**$N\hat{C}$	6.82	**F**45$\hat{C}N$**6**	3.29
1.0	**FPC***N*	6.61	**F6**45**C***N*	0.79	**FP**$N\hat{C}$	6.85	**F**45*CN***6**	3.26

[a]Benchmark

most socially desirable outcome. However, when $r^{\pi/y}_{F/CB} \approx r^{\pi/y}_{F/CB}$ this regime is not so efficient any more because under equal bargaining power assumption the loss of the CB is under-represented in the joint loss of the grand coalition. This leads to the situation in which the interest rate is less important in the joint loss than fiscal debts of individual countries and, therefore, is used more extensively than under F, where free-riding between fiscal coalition and the CB prevents both groups from an overuse of their control instruments. It is easily visible in Table 10 which shows the decomposed players losses for symmetric price shock and symmetric preferences, i.e. $r^{\pi/y}_{F} = r^{\pi/y}_{CB} = 1$. Loss from $\chi_{F,C3}\widehat{\hat{f}^2_{C3}}$ under F is bigger than under C and, at the same time, $\chi_{CB}\widehat{\hat{i}^2_U}$ is lower under F than under C as fiscal players in F cannot rely so much as in C on interest rate to stabilise economies due to free-riding of the CB. Since, in social loss based on J^*_U interest rate has relatively bigger share than in the joint loss of the grand coalition, full fiscal coordination scores better than the grand coalition where interest rate is relatively overused. Another important observation from Table 9 is that partial fiscal cooperation, as in most of the cases analysed before, is very often the second best choice.

Next to every column with social preference ordering we show average social loss obtained for different levels of $r^{\pi/y}_{F/CB}$. Obviously, the less conflicting preferences are the lower average common loss suffered by the union is. Thus, the percentage difference between the average losses for $r^{\pi/y}_{F/CB} = 0$ and $r^{\pi/y}_{F/CB} = 1$ is the highest for symmetric and asymmetric inflation shocks (61.5 % and 59 %, respectively), and much more moderate for both output shock (13.5 % and around 0), which confirms are previous results that the biggest gains from choosing an appropriate regime is to be expected in the former case.

Table 10 Optimal losses for v^{π}_{0S}, $\alpha_{F,i} = \alpha_{CB} = \frac{1}{2}\alpha_{CB} = \frac{1}{2}\alpha_{F,i}$

(sc_1, v^{π}_{0S})	NC	C	F	[12|3|4]	[13|2|4]	[1|23|4]
$C1$	6.3331	5.0871	5.8439	6.3071	6.3071	5.4947
$C2$	6.3331	5.0871	5.8439	6.3071	5.4947	6.3071
$C3$	6.3331	5.0871	5.8439	5.4947	6.3071	6.3071
CB	6.9370	6.5147	5.5070	6.2236	6.2236	6.2236
$J_U(t_0)$	25.9363	21.7763	23.0388	24.3328	24.3328	24.3328
$J^*_U(t_0)$	7.0018	6.9906	6.1134	6.5194	6.5194	6.5194
$\beta_{F,C1}\hat{y}^2_{C1}$	1.0667	1.1640	1.0812	1.3336	1.3336	0.6526
$\alpha_{F,C1}\hat{\pi}^2_{C1}$	5.2015	3.4472	4.1563	4.6365	4.6365	4.7429
$\chi_{F,C1}\widehat{\hat{f}}^2_{C1}$	0.0648	0.4758	0.6063	0.3369	0.3369	0.0992
$\beta_{F,C2}\hat{y}^2_{C2}$	1.0667	1.1640	1.0812	1.3336	0.6526	1.3336
$\alpha_{F,C2}\hat{\pi}^2_{C2}$	5.2015	3.4472	4.1563	4.6365	4.7429	4.6365
$\chi_{F,C2}\widehat{\hat{f}}^2_{C2}$	0.0648	0.4758	0.6063	0.3369	0.0992	0.3369
$\beta_{F,C3}\hat{y}^2_{C3}$	1.0667	1.1640	1.0812	0.6526	1.3336	1.3336
$\alpha_{F,C3}\hat{\pi}^2_{C3}$	5.2015	3.4472	4.1563	4.7429	4.6365	4.6365
$\chi_{F,C3}\widehat{\hat{f}}^2_{C3}$	0.0648	0.4758	0.6063	0.0992	0.3369	0.3369
$\beta_{CB}\hat{y}^2_{CB}$	1.0667	1.1640	1.0812	1.0688	1.0688	1.0688
$\alpha_{CB}\hat{\pi}^2_{CB}$	5.2015	3.4472	4.1563	4.6717	4.6717	4.6717
$\chi_{CB}\widehat{\hat{i}}^2_{U}$	0.6686	1.9034	0.2695	0.4831	0.4831	0.4831

A number of interesting conclusions can be drawn the above analysis. First of all, the relative antagonism between the CB and governments in the monetary union is an important factor which strongly determines the profitability of full fiscal coordination. In contrast to other various findings from the literature, in our model, strongly independent bank of a monetary union is not so profitable from the common perspective and more intermediate arrangements are advisable. Secondly, if bargaining power in the grand coalition do not coincide with socially optimal preferences, this regime might be counter-productive w.r.t. full-fiscal coalition, which in turn, can turn out to be optimal due to free-riding.

The analyses in Table 9 is made under the assumption that both types of authorities simultaneously change their preferences from the most conflicting to the same ones. This was rather theoretical simulation as having little chances to be realised in (the European) practice as the ECB independence is strongly safeguarded by relevant treaties. It is also interesting to study the more realistic situation in which the strong CB's focus on inflation remains unchanged while governments, at the beginning fixed only at inflation, become gradually interested in inflation.

Table 11 Sensitivity analysis, benchmark model with altered preference

$r_{CB}^{\pi/y}=0$	$J_U^*(t_0)$							
	v_{0S}^{π}	$\overline{J_U^*}$	v_{0A}^{π}	$\overline{J_U^*}$	v_{0S}^{y}	$10^2\overline{J_U^*}$	v_{0A}^{y}	$10^2\overline{J_U^*}$
$r_F^{\pi/y}=0$	$\mathbf{C}PNF$	17.16	$\check{C}645NF$	1.93	$CNPF$	4.23	$CN456F$	3.17
$r_F^{\pi/y}=0.1$	$\mathbf{C}PNF$	13.29	$\check{C}645NF$	1.53	$\check{C}PNF$	7.83	$C45N6F$	3.31
$r_F^{\pi/y}=0.2$	$\mathbf{C}PNF$	10.72	$\mathbf{C}645NF$	1.25	$\check{C}PNF$	7.77	$C45N6F$	3.33
$r_F^{\pi/y}=0.3$	$\mathbf{C}PNF$	9.35	$\mathbf{C}645NF$	1.10	$\mathbf{C}PNF$	7.65	$\check{C}45N6F$	3.32
$r_F^{\pi/y}=0.4$	$\mathbf{C}PNF$	8.63	$\mathbf{C}645NF$	1.02	$\mathbf{C}PNF$	7.50	$\mathbf{C6}45NF$	3.31
$r_F^{\pi/y}=0.5$	$\mathbf{C}PNF$	8.17	$\mathbf{C}645NF$	1.00	$\mathbf{CP}NF$	7.21	$\mathbf{C}45\mathbf{6}NF$	3.30
$r_F^{\pi/y}=0.6$	$P\mathbf{C}NF$	7.93	$\mathbf{645C}NF$	0.99	$\mathbf{CP}NF$	7.24	$\mathbf{C6}45NF$	3.29
$r_F^{\pi/y}=0.7$	$P\mathbf{C}NF$	7.66	$\mathbf{645C}NF$	0.96	$\mathbf{PC}NF$	7.13	$\mathbf{C}45\mathbf{6}FN$	3.27
$r_F^{\pi/y}=0.8$	$P\mathbf{C}NF$	7.62	$\mathbf{645C}NF$	0.94	$\mathbf{PC}NF$	7.14	$\mathbf{C}45\mathbf{6}FN$	3.27
$r_F^{\pi/y}=0.9$	$P\mathbf{C}NF$	7.41	$\mathbf{645C}NF$	0.88	$\mathbf{PCF}N$	7.14	$\mathbf{645CF}N$	3.27
$r_F^{\pi/y}=1$	$P\mathbf{C}NF$	7.29	$\mathbf{645C}NF$	0.86	$\mathbf{P\check{C}F}N$	7.15	$\mathbf{645CF}N$	3.27

More formally, we consider the case where $r_{CB}^{\pi/y}$ is kept constant at 0 where as $r_F^{\pi/y}$ changes from 0 to 1. One possible interpretation of such simulations in Table 11, which one can think of, are more and more stringent provisions of the SGP, which additionally to fiscal debt issues regulates also inflation in the EMU Member States.

In general, the outcomes in Table 11 are reasonably similar to those from Table 9 as far as main trends are considered, i.e. the less intensive conflict between authorities makes partial fiscal cooperation regimes as well as full fiscal cooperation more interesting from the social point of view. Of course, always restrictive CB makes it impossible to reach the same outcome as in the previous case. For $r_{CB}^{\pi/y}=0$ and $r_F^{\pi/y}=1$ (i.e. the last row of Table 11) the social orderings are similar $r_{F/CB}^{\pi/y}=0.5$ previously (the middle of the Table 9). Accordingly, minimal average loss obtained for the last case is now higher than when authorities' preferences were more alike. However, what is important, social losses at the end of both tables are not much different which shows that similar low social welfare can be obtained either by making preferences of fiscal and monetary authorities more parallel, or by safeguarding the CB independence and making government to be more equally oriented about inflations and output gaps. The first proposition seems to be rather unacceptable by the modern economic school, but the second one seems not only to be acceptable from this point of view, but actually implemented in the current European practice (in the form of the strongly independent ECB and the SGP, which makes governments more "inflation-aware").

The issue related to the SGP will be discussed further in this paper but first we will consider (nearly) optimal policy rules.

Table 12 Optimal losses for $\theta_\pi^M = 1.25$ and $\theta_y^F = 0, 0.1, 0.3, 0.5, 0.8, 1.0$

$\theta_\pi^U = 1.25$	-0	-0.1	-0.3	-0.5	-0.8	-1.0
$C1, C2, C3$	2.9554	5.8427	9.0471	3.3611	2.8931	2.8148
CB	6.3700	4.2528	19.2972	12.7490	12.5669	13.0460
$J_U(t_0)$	15.2364	21.7811	46.4388	22.8324	21.2463	21.4904
$J_U^*(t_0)$	**8.3235**	**9.1524**	**25.6551**	**14.1157**	**13.3184**	**13.5231**
$\beta_{F,C1}\hat{y}_{C1}^2$	1.1145	1.6788	0.8843	0.4886	0.3842	0.3234
$\alpha_{F,C1}\hat{\pi}_{C1}^2$	0.7790	0.6073	2.5124	1.8967	2.0648	2.2729
$\chi_{F,C1}\widehat{f}_{C1}^2$	1.0618	3.5565	5.6504	0.9757	0.4440	0.2184
$\beta_{CB}\hat{y}_{CB}^2$	0.2229	0.3357	0.1768	0.0977	0.0768	0.0646
$\alpha_{CB}\hat{\pi}_{CB}^2$	3.8953	3.0368	12.5621	9.4835	10.3240	11.3648
$\chi_{CB}\widehat{i}_U^2$	2.2517	0.8801	6.5582	3.1677	2.1659	1.6165

A.3 Nearly Optimal Policy Rules

In the LQDG framework it is not possible to analytically optimise certain parameters of the model, however, an approximate analyses can be performed numerically. We will use this method to study how various combinations of policy rules' parameters influence output gap and inflation volatility and which of them are likely to bring (nearly) optimal outcome from the social point of view. Due to the space constraints we will focus mainly on the symmetric inflation shock. Tables 12, 13, 14 show (optimal) losses together with their decomposition in the non-cooperative regime for different values of θ_y^i and θ_π^U. More specifically, Table 12 shows cases in which $\theta_\pi^U = 1.25$ and θ_y^i changes from 0 to -1; Table 13 cases in which $\theta_\pi^U = 1.5$ and θ_y^i as before; and, finally, Table 14 shows cases in which $\theta_\pi^U = 1.75$ and as before. Such an analysis, albeit approximate, may give us an important insight into efficiency of different policy rules' combinations.

Figure 4 compares J_U^* losses for different combinations of θ_π^U and θ_y^i. In general, from the monetary authority perspective, a rule less focused on inflation (i.e. $\theta_\pi^U = 1.25$, Table 12) results in higher losses than for the benchmark value (i.e. $\theta_\pi^U = 1.50$, Table 13), whereas a rule more focused on inflation (i.e. $\theta_\pi^U = 1.75$, Table 14) generates lower losses. The only exception from this pattern is a combination of coefficients $\theta_\pi^U = 1.50$ and $\theta_y^i = 0$ which produces the lowest social loss, i.e. is an optimal Taylor rules parameters' combination (ceteris paribus) for the non-cooperative regime. From the fiscal authority perspective the stronger reaction to output, the higher loss and vice versa. Finally, it should be mentioned that for combinations ($\theta_\pi^U = 1.25, \theta_y^i = -0.3$) and ($\theta_\pi^U = 1.25, \theta_y^i = -0.5$) strong irregularities

Table 13 Optimal losses for $\theta_\pi^M = 1.50$ and $\theta_y^F = 0, 0.1, 0.3, 0.5, 0.8, 1.0$

$\theta_\pi^U = 1.50$	−0	−0.1	−0.3	−0.5	−0.8	−1.0
$C1, C2, C3$	2.4891	2.3665	2.2435	2.1948	2.4202	2.9156
CB	4.2403	4.5948	5.4647	6.2456	8.0400	9.8299
$J_U(t_0)$	11.7078	11.6945	12.1955	12.8300	15.3007	18.5770
$J_U^*(t_0)$	**5.7327**	**5.9191**	**6.5302**	**7.1445**	**8.9168**	**11.0600**
$\beta_{F,C1}\hat{y}_{C1}^2$	1.7336	1.5703	1.2982	1.0880	0.8648	0.7523
$\alpha_{F,C1}\hat{\pi}_{C1}^2$	0.6500	0.7282	0.9183	1.0782	1.3704	1.5350
$\chi_{F,C1}\widehat{\hat{f}}_{C1}^2$	0.1054	0.0679	0.0269	0.0285	0.1848	0.6282
$\beta_{CB}\hat{y}_{CB}^2$	0.3467	0.3140	0.2596	0.2176	0.1729	0.1504
$\alpha_{CB}\hat{\pi}_{CB}^2$	3.2501	3.6410	4.5919	5.3912	6.8523	7.6753
$\chi_{CB}\widehat{\hat{i}}_U^2$	0.6434	0.6397	0.6131	0.6367	1.0147	2.0041

Table 14 Optimal losses for $\theta_\pi^M = 1.75$ and $\theta_y^F = 0, 0.1, 0.3, 0.5, 0.8, 1.0$

$\theta_\pi^U = 1.75$	−0	−0.1	−0.3	−0.5	−0.8	−1.0
$C1, C2, C3$	3.4359	3.1001	2.8075	2.6060	2.4825	2.4264
CB	4.1017	3.9972	4.4681	5.0434	6.0023	6.5807
$J_U(t_0)$	14.4096	13.2978	12.8907	12.8614	13.4500	13.8602
$J_U^*(t_0)$	**6.5822**	**6.1072**	**6.1979**	**6.4953**	**7.1816**	**7.5970**
$\beta_{F,C1}\hat{y}_{C1}^2$	2.3617	2.1732	1.8597	1.6176	1.3284	1.1784
$\alpha_{F,C1}\hat{\pi}_{C1}^2$	0.4830	0.5555	0.7057	0.8305	1.0375	1.1745
$\chi_{F,C1}\widehat{\hat{f}}_{C1}^2$	0.5911	0.3714	0.2419	0.1578	0.1165	0.0735
$\beta_{CB}\hat{y}_{CB}^2$	0.4723	0.4346	0.3719	0.3235	0.2656	0.2356
$\alpha_{CB}\hat{\pi}_{CB}^2$	2.4154	2.7777	3.5289	4.1525	5.1879	5.8725
$\chi_{CB}\widehat{\hat{i}}_U^2$	1.2139	0.7848	0.5672	0.5673	0.5486	0.4725

emerge, explanation of which should be sought in mathematical properties of the model.[30]

Let us now focus on the individual players' perspective. Due to the irregularities mentioned above we will exclude from our analysis Table 12. For $\theta_\pi^U = 1.50$, the loss of the fiscal players is not monotonic and reaches its minimum for the

[30] Such an analysis goes far beyond the scope of this paper. It has been numerically checked that in the neighbourhood of $\theta_y^F = 0.3$ for $\theta_\pi^M = 1.25$ social loss goes to the (nearly) infinite limiting value.

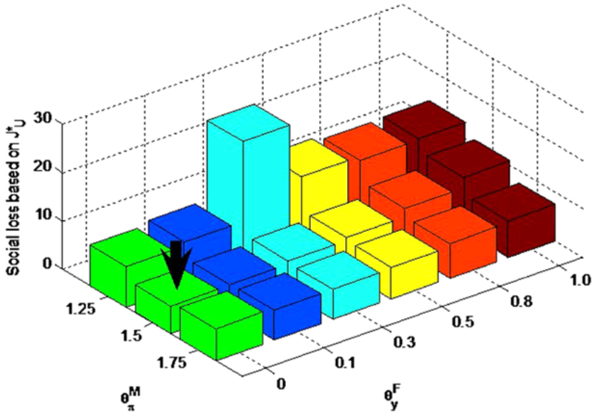

Fig. 4 Social loss for various combinations of policy rules parameters, regime *N*

benchmark parametrisation (i.e. $\theta_y^i = 0.50$). However, for $\theta_\pi^U = 1.75$ the results are more clear-cut as the stronger reaction of governments to output always leads to the lower loss. This is totally at odds with the CB losses which behave in the exactly opposite way. The reasons of this difference can be found in the decomposition of losses. Stronger reaction of fiscal debt to output gap leads to its lower volatility (in Table 14 $\beta_{F,C1}\hat{y}_{C1}^2$, and, consequently, $\beta_{CB}\hat{y}_{CB}^2$ decrease with θ_y^i), however, this positive effect is reached at the very expense of inflation volatility which grows accordingly. This is detrimental for the CB as this authority is mainly concerned about inflation under benchmark parametrisation. Overall, the conflict of interest between both types of authorities is clearly visible here. Highest value of θ_π^U without a counter-response in fiscal rule is damaging to loss of governments as the CB's strong reaction to inflation makes output gap very volatile. Thus, governments use the most of their control effort to improve the situation, however, only higher (absolute) values of θ_y^i make them increasingly better off. This pattern is robust even for θ_y^i reaching minus one. In contrast, as mentioned above, for the more moderate CB's policy rule (i.e. $\theta_\pi^U = 1.5$) and for θ_y^i higher than half, fiscal loss start to increase. This means that, if fiscal rule responses to tempered monetary rule too strongly, there is an effect of overshooting the fiscal policy rule. As a result, governments are pushed to deviate from such a rule much stronger than before because the (overshot) rule must be discretionary corrected. Consequently, $\chi_{F,C1}\widehat{\hat{f}}_{C1}^2$ grows from 0.0269 in benchmark to 0.6282 for $\theta_y^i = -1$. Comparing the CB's losses between Tables 12, 13, 14 it is evident that more reactionary stance is in the interest of the CB as its loss decreases with increasing θ_π^U due to the lower inflation deviation in the union.

To sum up, from the individual point of view, we have a situation where CB has incentives to increase θ_π^U and, at the same time, for high values of θ_π^U, governments have incentives to increase θ_y^i. When both types of authorities do it at the same time, the economies end up in a position which is not only suboptimal from the social point of view (right-down corner in Fig. 4), but also from individual ones. For $\theta_\pi^U = 1.75$ and $\theta_y^i = -1.0$ governments obtain loss of 2.4264 and the CB of 6.5807, which is the outcome *Pareto*-dominated by other combinations, e.g. the benchmark. Unfortunately, the socially optimal combination ($\theta_\pi^U = 1.5, \theta_y^i = 0$) does not *Pareto*-dominate combination ($\theta_\pi^U = 1.75, \theta_y^i = -1.0$) so the mutual agreement to move toward the optimum seems unlikely to be obtained.

A.4 SGP Analysis

Within our model we can also investigate the effects of the major policy-surveillance institution of the EMU, namely the SGP. The SGP imposes a framework of fiscal stringency and coordination measures that aim at securing the implementation of the BEPGs. In our model the effects of various levels of the SGP stringency can be studied by considering (i) different levels of the countercyclical parameter θ_y^i in the fiscal rule; and (ii) different weights associated with the domestic fiscal deficit, χ_i, in the objective functions of the fiscal players. We compare the following three cases, each characterised by stricter SGP provisions than the other:[31]

I. In the fiscal rule the coefficient measuring a countercyclical reaction of fiscal debt to deviation of output gap is two times smaller than in the benchmark, i.e.: $\theta_y^{i,new} = -0.25$;
II. As above, but, additionally, deviations from the rule are more costly (i.e. are more severely punished by the SGP provisions), $\chi_i^{new,II} = 1.5\chi_i$;
III. As in point I, but, additionally, $\chi_i^{new,III} = 3\chi_i$.

It is expected that smaller countercyclical reaction of the fiscal rule is going to force fiscal authorities to deviate stronger from the rule than in the benchmark. On the other hand, more costly deviations from the rule in cases II and III are likely to diminish the use of fiscal instrument w.r.t. case I. As far as individual losses of players are concerned, it is possible to directly compare new cases to the benchmark, however, it is not exactly obvious whether we can do so with the social loss. Whereas governments, as public authorities, might be bound by tougher SGP provisions, it does not have to lead to an automatic increase of the social loss. In the benchmark we assumed that $\chi_U = \chi_i$. Now, we are going to compute "adjusted" social loss of the entire union $J_U^*(t_0)$, denoted $J_U^{*A}(t_0)$, by assuming that cost of the deviation of the fiscal instrument from the rule is unchanged, i.e. equals to $\chi_U = \chi_i$ as in the benchmark, instead of $\chi_U = \chi_i^{new,II}$ or $\chi_U = \chi_i^{new,III}$. By doing so we will see

[31]Naturally, there is problem with interpretation of the SGP as well as other issues related to the control variables caused by the (linear-) quadratic form of the loss functions. In reality, a negative deviation of fiscal debt from the rule, i.e. more restrictive budgetary policy, is not likely to be considered so "bad" or "undesirable" as the same positive deviation which, eventually, is going to increase public debt. It could be possible to partially take into account such issues also in the quadratic loss function but in the much complex model, which is far our of the scope of this paper.

Table 15 Regimes N, C and F for different levels of the SGP stringency

(sc_1, v_{0S}^{π})	N	N^{I}	N^{II}	N^{III}	C	C^{I}	C^{II}	C^{III}
$C1, C2, C3$	2.19	4.22	6.18	4.73	2.12	2.23	2.22	2.31
CB	6.24	8.65	16.82	12.88	5.33	3.66	3.64	3.65
$J_U(t_0)$	12.80	21.30	35.36	27.07	11.70	10.35	10.31	10.57
$J_U^*(t_0)$	7.14	11.60	21.67	16.05	6.37	5.02	4.99	5.05
$J_U^{A*}(t_0)$	**7.14**	**11.60**	**20.48**	**14.84**	**6.37**	**5.02**	**4.93**	**4.89**
$\beta_{F,C1}\hat{y}_i^2$	1.09	1.57	1.60	1.71	1.06	1.47	1.47	1.47
$\alpha_{F,C1}\hat{\pi}_i^2$	1.08	0.94	1.01	1.22	0.88	0.58	0.57	0.61
$\chi_{F,C1}\widehat{\hat{f}}_i^2$	0.03	1.71	2.38	0.60	0.19	0.18	0.12	0.08
$\chi_{F,C1}^{new,I/II}\widehat{\hat{f}}_i^2$	–	–	3.57	1.80	–	–	0.18	0.23
$\beta_{CB}\hat{y}_{CB}^2$	0.22	0.31	0.32	0.34	0.22	0.29	0.29	0.29
$\alpha_{CB}\hat{\pi}_{CB}^2$	5.40	4.72	5.06	6.10	4.37	2.89	2.87	3.05
$\chi_{CB}\widehat{\hat{i}}_U^2$	0.64	3.62	11.44	6.43	0.76	0.47	0.47	0.30

what is the contribution of a change in use of a fiscal instrument in the total change of the loss from stabilisation effort.[32]

Players' losses in the first three regimes in the case of a symmetric inflation shock and benchmark parametrisation are shown in Tables 15 and 16. In spite of vast differences between cases a few general conclusions can be drawn. First of all, lower counter-cyclical reaction of fiscal debt (case I) always makes the losses of fiscal players higher than in the benchmark, which means that the assumed value $\theta_y^i = -0.5$ was chosen relatively well for the initial simulations and which confirms our findings from the previous section.[33] Secondly, in all the new cases higher SGP stringency leads to increasing losses of fiscal players from output gap volatility. This is natural as fiscal authorities refrain from using the more expensive control instruments. Interestingly, in different regimes we obtain different relationships between SGP stringency and the amount of the control instrument used. Whereas in all regimes with any form of cooperation (i.e. C, F, and P-regimes) the higher χ_i, the less control instrument is used (compare cases II to I and III to II), then under non-cooperation this relationship is highly non-linear. For $\chi_i^{new,II}$ governments decide

[32] A change in total loss from stabilisation effort caused by a change in the value of the relevant preference parameters can be decomposed into two effects. First is the change in the use of the stabilisation instrument as it becomes more/less expensive w.r.t. to other elements of the loss. Second change is directly caused by the increased/decreased cost.

[33] If the absolute value of θ_y^i is too high, the counter-cyclical output gap stabilisation effort can be overshot, i.e. output gap can be (ceteris paribus) more volatile than for lower values of θ_y^i, and would probably require additional pro-cyclical (and costly) control effort from fiscal authorities (see previous section for more details).

Table 16 Regimes N, C and F for different levels of the SGP stringency

(sc_1, v_{0S}^{π})	F	F^I	F^{II}	F^{III}
$C1$	9.30	13.48	9.05	5.61
$C2$	9.30	13.48	9.05	5.61
$C3$	9.30	13.48	9.05	5.61
CB	22.00	29.30	14.20	6.66
$J_U(t_0)$	50.00	69.73	41.30	23.50
$J_U^*(t_0)$	**30.00**	**41.84**	**22.30**	**11.30**
$J_U^{A*}(t_0)$	**30.00**	**41.48**	**20.70**	**20.01**
$\beta_{F,C1}\hat{y}_{C1}^2$	1.06	1.48	1.49	1.50
$\alpha_{F,C1}\hat{\pi}_{C1}^2$	1.00	0.63	0.65	0.67
$\chi_{F,C1}\widehat{\hat{f}}_{C1}^2$	7.00	11.37	4.61	1.15
	–	–	6.91	3.44
$\beta_{CB}\hat{y}_{CB}^2$	0.20	0.30	0.30	0.30
$\alpha_{CB}\hat{\pi}_{CB}^2$	5.00	3.17	3.27	3.36
$\chi_{CB}\widehat{\hat{i}}_U^2$	16.00	25.83	10.60	3.00

to use $\frac{\chi_{C1}\widehat{\hat{f}}_i^2}{\chi_{C1}} = \widehat{\hat{f}}_i^2 = 1190$ which is nearly 40 % more (not less) than for χ_i in case I, however when χ_i increases to $\chi_i^{new,III}$ they contract substantially control action to $\frac{\chi_{C1}\widehat{\hat{f}}_i^2}{\chi_{C1}} = \widehat{\hat{f}}_i^2 = 300$. The SGP regulating the use of control instrument influences also the use of interest rate by the CB of the union. In many cases when control action of fiscal authorities is diminished the response of the CB also fades out, i.e. conflict between both types of authorities is hampered. In relative terms, the biggest reductions in the control effort of the monetary authority is obtained under F^{III} where cost of the control effort is lowered from 25.83 to 3.00. On the other hand, under N^{III} we also witness quite a reduction in the fiscal control effort w.r.t. N^I, but the main driving force in this case is a free-riding of fiscal players, which forces the CB to increase its engagement in the union economy not stabilised enough by national governments. As the loss from the CB's control instrument is an important part of $J_U^*(t_0)$ and $J_U^{A*}(t_0)$, this leads to higher social loss under N^{III} than under F^{III}.

To summarise, we established the third factor (next to the degree of backward-lookingness and loss functions' preferences) which heavily determined the results obtained for the benchmark parametrisation of the model. The increased SGP stringency reduces incentives of fiscal players to use control instruments, therefore, in situations where high social losses where driven by the conflict between authorities (notably regime F), such a firmer stance is beneficial to the union-wide economic interest. However, in situations in which free-riding is present (notably regime N under benchmark) increased SGP stringency may lead to more extensive free-riding of governments as undertaking any actions become more costly. This, in turn, makes the CB to intervene and increases social loss of the union. In other words, the stringent SGP has both positive and negative effects in the context of this paper and is able to make unprofitable regime to become profitable.

Similar analysis has been performed for 3 other shocks. Since the conflict under v_{0A}^{π} is less eminent also the social gains from higher SGP stringency are lower. As before, both output shocks are characterised by the lower variability of losses between different regimes, however, still SGP stringency is able to make non-cooperation inferior to fiscal cooperation, at least, in the case of the symmetric shock.

A.5 Model Derivations

A.5.1 Reduced Form of the Model

Defining

$$K_y := \begin{bmatrix} \kappa_{1,y} & 0 & \cdots & 0 \\ 0 & \kappa_{2,y} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \kappa_{n,y} \end{bmatrix}, \qquad K_\pi := \begin{bmatrix} \kappa_{1,\pi} & 0 & \cdots & 0 \\ 0 & \kappa_{2,\pi} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \kappa_{n,\pi} \end{bmatrix},$$

$$G := \begin{bmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \gamma_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \gamma_n \end{bmatrix}, \qquad E := \begin{bmatrix} \eta_1 & 0 & \cdots & 0 \\ 0 & \eta_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \eta_n \end{bmatrix},$$

$$\Xi := \begin{bmatrix} \xi_1 & 0 & \cdots & 0 \\ 0 & \xi_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \xi_n \end{bmatrix}, \qquad B := \begin{bmatrix} \beta_1 & 0 & \cdots & 0 \\ 0 & \beta_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \beta_n \end{bmatrix},$$

$$R := \begin{bmatrix} 0 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 0 & \cdots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1} & \rho_{n2} & \cdots & 0 \end{bmatrix},$$

$$D := \begin{bmatrix} \delta_{12} & \delta_{13} & \cdots & \delta_{1n} \\ -\sum_{j\in F/2}\delta_{2j} & \delta_{23} & \cdots & \delta_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \delta_{n2} & \delta_{n3} & \cdots & -\sum_{j\in F/n}\delta_{nj} \end{bmatrix},$$

$$\Psi_y := \begin{bmatrix} \psi_{1,y} & 0 & \cdots & 0 \\ 0 & \psi_{2,y} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \psi_{n,y} \end{bmatrix}$$

$$V := \begin{bmatrix} \varsigma_{12} & \varsigma_{13} & \cdots & \varsigma_{1n} \\ -\sum_{j\in F/2}\varsigma_{2j} & \varsigma_{23} & \cdots & \varsigma_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \varsigma_{n2} & \varsigma_{n3} & \cdots & -\sum_{j\in F/n}\varsigma_{nj} \end{bmatrix},$$

$$\Psi_\pi := \begin{bmatrix} \psi_{1,y} & 0 & \cdots & 0 \\ 0 & \psi_{2,y} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \psi_{n,y} \end{bmatrix}$$

$$\Psi := \begin{bmatrix} \Psi_y & 0 \\ 0 & \Psi_\pi \end{bmatrix}, \quad \text{and} \quad v_t := \begin{bmatrix} v_t^y \\ v_t^\pi \end{bmatrix},$$

$$\iota_n := \begin{bmatrix} 1 \\ 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix}_n, \qquad S := \begin{bmatrix} -\iota_{(n-1)} & I_{n-1} \end{bmatrix},$$

$$\Theta_\pi^F := \begin{bmatrix} \theta_\pi^1 & \theta_\pi^2 & \cdots & \theta_\pi^n \end{bmatrix}^T \quad \text{and} \quad \Theta_y^F := \begin{bmatrix} \theta_y^1 & \theta_y^2 & \cdots & \theta_y^n \end{bmatrix}^T$$

the SNKM model can be rewritten as:

$$\begin{aligned} y_t = {} & K_y E_t y_{t+1} + (I_n - K_y) y_{t-1} - G(\iota_n i_t - E\pi_{t+1}) + E f_t \\ & - K_y R E_t y_{t+1} + R y_t - (I_n - K_y) R y_{t-1} - K_y D E_t s_{t+1} \\ & + D s_t - (I_n - K_y) D s_{t-1} + v_t^y, \end{aligned} \tag{12}$$

$$\pi_t = K_\pi B E_t \pi_{t+1} + (I_n - K_\pi) B \pi_{t-1} + \Xi y_t + \Xi V s_t + v_t^\pi, \tag{13}$$

$$s_{t+1} = s_t + S\pi_{t+1}, \tag{14}$$

$$v_{t+1} = \Psi v_t + \varepsilon_{t+1}, \tag{15}$$

where $\mathbf{I}_m$ is $m \times m$ identity matrix $(m = n-1, n)$, $s_t := [s_{12,t} \cdots s_{1n,t}]$ and y_t, π_t, v_t^y and v_t^π are appropriately defined vectors of size n each. In particular, it can be shown that every $s_{ij,t} := p_{j,t} - p_{i,t}$ $(j \neq i)$ can be expressed in terms of $s_{12,t}, \ldots, s_{1n,t}$. For example, in a three-country monetary union we have six bilateral real exchange rates: $s_{12,t} = p_{2,t} - p_{1,t}$, $s_{13,t} = p_{3,t} - p_{1,t}$, $s_{21,t} = p_{1,t} - p_{2,t}$, $s_{23,t} = p_{3,t} - p_{2,t}$, $s_{31,t} = p_{1,t} - p_{3,t}$, and $s_{32,t} = p_{2,t} - p_{3,t}$. Clearly, the last four variables can be expressed as a combination of the first two, i.e. $s_{21,t} = -s_{12,t}$, $s_{23,t} = s_{13,t} - s_{12,t}$, $s_{31,t} = -s_{13,t}$ and $s_{32,t} = s_{12,t} - s_{13,t}$.

Defining fiscal and monetary policy rule vectors as:

$$f_t := \Theta_\pi^F \pi_t + \Theta_y^F y_t, \quad \text{and} \tag{16}$$

$$i_t := \theta_\pi^U \omega^T \pi_t + \theta_y^U \omega^T y_t, \tag{17}$$

substituting them into system (12)–(15) and rearranging we get:

$$\begin{aligned}
&-K_y(I_n-R)E_t y_{t+1} - GE_t\pi_{t+1} + K_y DE_t s_{t+1} \\
&\quad = -\big(I_n - E\Theta_y^F + G\iota_n\theta_y^U\omega^T - R\big)y_t - \big(G\iota_n\theta_\pi^U\omega^T - E\Theta_\pi^F\big)\pi_t \\
&\qquad + (I_n-K_y)(I_n-R)y_{t-1} + Ds_t - (I_n-K_y)Ds_{t-1} + I_n\upsilon_t^y, \qquad (18)
\end{aligned}$$

$$-K_\pi BE_t\pi_{t+1} = \Xi y_t - \pi_t + (I_n-K_\pi)B\pi_{t-1} + \Xi Vs_t + I_n\upsilon_t^\pi, \qquad (19)$$

$$s_{t+1} - SE_t\pi_{t+1} = s_t, \qquad (20)$$

$$\upsilon_{t+1} = \Psi\upsilon_t + \varepsilon_{t+1}. \qquad (21)$$

Introducing three additional vectors of variables $a_{t+1} := y_t$, $b_{t+1} := \pi_t$ and $c_{t+1} := s_t$ we may rewrite system (18)–(21) as:

$$\begin{aligned}
&-K_y(I_n-R)E_t y_{t+1} - GE\pi_{t+1} + K_y Ds_{t+1} \\
&\quad = -\big(I_n - E\Theta_y^F + G\iota_n\theta_y^U\omega^T - R\big)y_t - \big(G\iota_n\theta_\pi^U\omega^T - E\Theta_\pi^F\big)\pi_t \\
&\qquad + Ds_t + (I_n-K_y)(I_n-R)a_t - (I_n-K_y)Dc_t + I_n\upsilon_t^y \qquad (22)
\end{aligned}$$

$$-K_\pi BE\pi_{t+1} = \Xi y_t - \pi_t + \Xi Vs_t + (I_n-K_\pi)Bb_t + I_n\upsilon_t^\pi, \qquad (23)$$

$$-S\pi_{t+1} + s_{t+1} = s_t, \qquad (24)$$

$$a_{t+1} = y_t, \qquad (25)$$

$$b_{t+1} = \pi_t, \qquad (26)$$

$$c_{t+1} = s_t, \qquad (27)$$

$$\upsilon_{t+1} = \Psi\upsilon_t + \varepsilon_{t+1}. \qquad (28)$$

Defining: $A_{11} = -K_y(I-R)$, $A_{12} = -G$, $A_{13} = K_yD$, $B_{11} = -(I - E\Theta_y^F + G\iota_n\theta_y^U\omega^T - R)$, $B_{12} = -(G\iota_n\theta_\pi^U\omega^T - E\Theta_\pi^F)$, $B_{13} = D$, $B_{14} = (I_n-K_y)(I_n-R)$, $B_{16} = -(I_n-K_y)D$, $B_{17} = [0_{n\times n}\ I_n]$, $A_{22} = -K_\pi B$, $B_{21} = \Xi$, $B_{22} = -I_n$, $B_{23} = \Xi V$, $B_{25} = (I_n-K_\pi)B$, $B_{27} = [I_n\ 0_{n\times n}]$, $A_{32} = -S$, $B_{77} = \Psi$, the system (22)–(28) in state-space form as:

$$E_t z_{t+1} = A^{-1}Bz_t + F\upsilon_t, \qquad (29)$$

where $z_t := [y_t^T\ \pi_t^T\ s_t^T\ a_t^T\ b_t^T\ c_t^T\ \upsilon_t^T]^T$ or $z_t := [z_{1,t}^T\ z_{2,t}^T\ \upsilon_t^T]^T$ with $z_{1,t} := [a_t^T\ b_t^T\ c_t^T]^T$, $z_{2,t} := [y_t^T\ \pi_t^T\ s_t^T]^T$,

$$\upsilon_t := \big[0_{1\times n}\quad 0_{1\times n}\quad 0_{1\times(n-1)}\quad 0_{1\times n}\quad 0_{1\times n}\quad 0_{1\times(n-1)}\quad \varepsilon_t^T\big]^T,$$

$$A := \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0_1 & 0_1 & 0_2 & 0_3 \\ 0_1 & A_{22} & 0_2 & 0_1 & 0_1 & 0_2 & 0_3 \\ 0_4 & A_{32} & I_{(n-1)} & 0_4 & 0_4 & 0_5 & 0_6 \\ 0_1 & 0_1 & 0_1 & I_n & 0_1 & 0_2 & 0_3 \\ 0_1 & 0_1 & 0_1 & 0_1 & I_n & 0_2 & 0_3 \\ 0_4 & 0_4 & 0_5 & 0_4 & 0_4 & I_{(n-1)} & 0_6 \\ 0_7 & 0_7 & 0_8 & 0_7 & 0_7 & 0_8 & I_{2n} \end{bmatrix},$$

$$B := \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} & 0_1 & B_{16} & B_{17} \\ B_{21} & B_{22} & B_{23} & 0_1 & B_{25} & 0_2 & B_{27} \\ 0_4 & 0_4 & I_{(n-1)} & 0_4 & 0_4 & 0_5 & 0_6 \\ 0_1 & 0_1 & 0_1 & I_n & 0_1 & 0_2 & 0_3 \\ 0_1 & 0_1 & 0_1 & 0_1 & I_n & 0_2 & 0_3 \\ 0_4 & 0_4 & 0_5 & 0_4 & 0_4 & I_{(n-1)} & 0_6 \\ 0_7 & 0_7 & 0_8 & 0_7 & 0_7 & 0_8 & \Psi \end{bmatrix},$$

$$F := \begin{bmatrix} 0_1 & 0_1 \\ 0_1 & 0_1 \\ 0_4 & 0_4 \\ 0_1 & 0_1 \\ 0_1 & 0_1 \\ 0_4 & 0_4 \\ I_n & 0_1 \\ 0_1 & I_n \end{bmatrix},$$

where 0_1, 0_2, 0_3, 0_4, 0_5, 0_6, 0_7, 0_8, 0_9 are zero matrices of dimensions $n \times n$, $n \times (n-1)$, $n \times 2n$, $(n-1) \times n$, $(n-1) \times (n-1)$, $n \times 2n$, $2n \times n$, $2n \times (n-1)$ and $2n \times 1$.

In order to obtain LQDG NKM, we assume that $E_t z_{1,t+1} = z_{1,t+1}$, i.e. that economic agents in the deterministic NKM make neither systematic nor random errors when predicting the future. Furthermore, substituting monetary and fiscal rules (6)–(7) in which deviation is possible into system (12)–(15) in the way presented above and performing similar transformations we obtain the system:

$$\begin{aligned} -K_y(I_n - R)E_t y_{t+1} - GE\pi_{t+1} + K_y D s_{t+1} \\ = -\big(I_n - E\Theta_y^F + G\iota_n\theta_y^U\omega^T - R\big)y_t - \big(G\iota_n\theta_\pi^U\omega^T - E\Theta_\pi^F\big)\pi_t \\ + Ds_t + (I_n - K_y)(I_n - R)a_t - (I_n - K_y)Dc_t + I_n v_t^y + E\hat{f}_t - G\hat{\imath}_t \end{aligned} \tag{30}$$

$$-K_\pi B E\pi_{t+1} = \Xi y_t - \pi_t + \Xi V s_t + (I_n - K_\pi)Bb_t + I_n v_t^\pi \tag{31}$$

$$-S\pi_{t+1} + s_{t+1} = s_t \tag{32}$$

$$a_{t+1} = y_t \tag{33}$$

$$b_{t+1} = \pi_t \tag{34}$$

$$c_{t+1} = s_t \tag{35}$$

$$v_{t+1} = \Psi v_t, \tag{36}$$

which, compared to the system (22)–(28), has two additional vectors of control variables $\hat{f}_t$ and $\hat{\imath}_t$. System (30)–(36) in state-space form can we written as:

$$z_{t+1} = A^{-1}Bz_t + A^{-1}Cu_t, \tag{37}$$

where $u_t := [\hat{f}_t^T \ \hat{\imath}_t]^T$ and

$$C := \begin{bmatrix} E & -G\iota_n \\ 0_1 & 0_1 \\ 0_1 & 0_1 \\ 0_1 & 0_1 \\ 0_4 & 0_4 \\ 0_7 & 0_9 \end{bmatrix}.$$

A.5.2 Initial Condition Derivation

Initial condition z_0 should position the system on the saddle-path so that the model would converge to the equilibrium. We propose two alternative ways of deriving this initial condition:

1. One way to obtain z_0 which positions the system on the saddle-path is to solve the RE version of the model and then use the initial state obtained. This initial state, by definition (if RE-model is stable), meets the required condition because it positions the system on the saddle path. In particular, at $t = 0$ vector of endogenous non-predetermined variables $z_{1,t}$ will "jump" to a saddle path whereas vector of endogenous state (predetermined) variables $z_{2,t}$ will have a value of 0. The initial value of shock vector v_t should follow the same assumptions made while solving the RE SNKM, i.e. its initial value should equal to standard deviation of ε_t. A number of freeware applications is available to solve RE model with DYNARE by Juillard (1996) being probably the most famous.[34]
2. Another method to position the system on the saddle-path is to calculate the orthogonal projection of the shock v_t onto the stable subspace at time $t = 1$. This method will be described below in more details.

Let

$$z_{t+1} = \bar{A}z_t, \quad z(0) = z_0, \tag{38}$$

be the deterministic NKM, where $\bar{A} := A^{-1}B$.

Now, let $\bar{A} = SJS^{-1}$ be a Jordan decomposition of $\bar{A}$ such that $J = \text{diag}(\Lambda_S, \Lambda_U)$ and $S = [S_S \ S_U]$, where Λ_S contains all stable eigenvalues of $\bar{A}$ and Λ_U all unstable eigenvalues of $\bar{A}$ and S_S (S_U) is the with Λ_S (Λ_U) corresponding stable

[34] For DYNARE website with the most current version of the software see: www.dynare.org.

(unstable) subspace of $\bar{A}$. Then, if z_0 belongs to S_S we have $z(0) = S_S y$ for some y ($y = (S_S^T S_S)^{-1} S_S^T z_0$).[35] In that case we may write:

$$z(t) = e^{\bar{A}t} z(0) = S e^{Jt} S^{-1} S_{\bar{S}} y$$
$$= S e^{Jt} \begin{bmatrix} I \\ 0 \end{bmatrix} y = S \begin{bmatrix} e^{\Lambda_S t} \\ 0 \end{bmatrix} y = S \begin{bmatrix} e^{\Lambda_S t} (S_S^T S_S)^{-1} S_S^T z_0 \\ 0 \end{bmatrix},$$

which is the solution for $t \geq 0$. In our simulations we always consider such orthogonal projection of **z** onto the stable subspace at time $t = 1$ as the initial condition $\tilde{z}_0$.

A.5.3 Change from a Discrete- to a Continuous-Time Model

Following Kwakernaak (1976) let the continuous time system be:

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du.$$

Under the assumptions that $u(t) = u(t_i)$, $t_i \leq t \leq t_{i+1}$ and $\Delta = t_{i+1} - t_i$ the equivalent discrete-time system is:

$$x(i+1) = A_{Cl} x(i) + B_d u(i) \tag{39}$$
$$y(i) = C_d x(i) + D_d u(i), \tag{40}$$

where

$$A_{Cl} = e^{A\Delta},$$
$$B_d = \left(\int_0^{\Delta} e^{A\tau} d\tau \right) B,$$
$$C_d = C e^{A\Delta} \quad \text{and}$$
$$D_d = C \left(\int_0^{\Delta} e^{A\tau} d\tau \right) B + D.$$

Assuming $\Delta = 1$ we may rewrite the continuous time system in terms of discrete time system matrices as:

$$A = \log A_{Cl} = \log(I + A_{Cl} - I) \approx A_{Cl} - I, \tag{41}$$
$$B = \left[\int_0^1 e^{A\tau} d\tau \right]^{-1} B_d = \left(e^A - I \right)^{-1} A B_d, \tag{42}$$

[35] In case x_0 does not belong to S_A, vector $y = (S_S^T S_S)^{-1} S_S^T x_0$ is such that the distance between S_S and x_0 is minimal (y is the least-squares solution of $x_0 = S_S y$, i.e. $\|x_0 - S_S y\| \leq \|x_0 - S_S \tilde{y}\|$ for all $\tilde{y}$).

$$C = C_d e^{-A}, \tag{43}$$

$$\begin{aligned} D &= D_d - C\left(\int_0^{\Delta} e^{A\tau}\,d\tau\right)B \\ &= D_d - C\left[A^{-1}\left(e^{A} - I\right)\right]B. \end{aligned} \tag{44}$$

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