# Sub-channel and Power Allocation for Multiuser OFDM Systems with Proportional Rate Constraints Based on Genetic Algorithms

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**Abstract.** This paper considers sub-channel and power allocation based on genetic algorithms to maximize the overall system capacity using proportional rate constraints in multiuser orthogonal frequency division multiplexing (OFDM) systems. The proposed algorithm first performs sub-channel allocation using a rough rate constraint under the premise of equal power allocation among the sub-channels. Then power allocation proceeds based on the sub-channel allocation scheme to implement proportional fairness and maintain maximum system capacity. Owing to the separation of sub-channel and power allocation, the computational complexity can be reduced from exponential to linear. Additionally, both the sub-channel allocation and power allocation are based on genetic algorithms, so the computational complexity of the proposed resource allocation algorithm can be further reduced. Simulation results show that the proposed algorithm achieves about 95% of the maximum capacity in an eight-user system and that the ratio of data rates among users can be set freely.

**Keywords:** sub-channel allocation, power allocation, multiuser OFDM, proportional fairness, system capacity, genetic algorithm.

## 1 Introduction

Orthogonal frequency division multiplexing (OFDM), which is a key technique in 4G mobile communication systems, has recently attracted a great deal of attention owing to its ability to combat multiple path interference. The basic idea is to divide the available bandwidth into N orthogonal sub-channels [1-3]. Meanwhile, the associated resource allocation has also become a key research focus. There are two types of resource allocation schemes, fixed resource allocation and dynamic resource allocation. The former scheme allocates independent dimensions of sub-channels and power to each user. This is obviously not optimal, since channel conditions are not considered. Instead, dynamic resource allocation adaptively adjusts the allocation scheme according to the changing wireless channel environment to make full use of the limited system resource [4] [5].

In a previous resource allocation study, Jang and Lee proposed an optimized algorithm to maximize total capacity by assigning each sub-channel to the user with the

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best channel gain [6]. However, if the channel gain differences among users are large, users with higher channel gain will occupy nearly all the resource causing other users to receive no data. The max-min problem was studied in [7] to ensure that all users generate a similar data rate. However, this study ignored the fact that different users may require different data rates. The authors in [8] proposed a suboptimal allocation scheme whereby a sub-channel is first allocated to the most suitable user based on a proportional rate parameter by assuming equal power distribution, and then power allocation is employed to maintain fairness. However, the sub-channel scheme algorithm has large computational complexity.

In this paper, we focus on a new optimization problem to achieve both proportional fairness and maximum system capacity. As before, the sub-channel is first allocated using a rough rate constraint to maximize capacity under the premise of equal power allocation. Then, the power allocation algorithm is employed to maintain maximum total capacity and ensure that each user has the required proportional rate. To reduce the computational complexity, we introduce genetic algorithms to both the sub-channel allocation and power allocation.

The rest of this paper is organized as follows. Section 2 introduces the multiuser OFDM system model and derives the optimization objective function. Sub-channel allocation and power allocation based on genetic algorithms are discussed in Section 3 with simulation results presented in Section 4. Conclusions are drawn in Section 5.

## 2 System Model

Fig. 1 depicts a multiuser OFDM system. It is assumed that instantaneous channel information is available at the base station, and all channel information is sent to the resource allocation algorithm through feedback channels from all the users. Then, the resource allocation algorithm formulates the related allocation schemes and forwards these to the OFDM transmitter. The transmitter selects a different number of bits from each user to form an OFDM symbol. The resource allocation schemes change adaptively according to variation in the wireless channel.

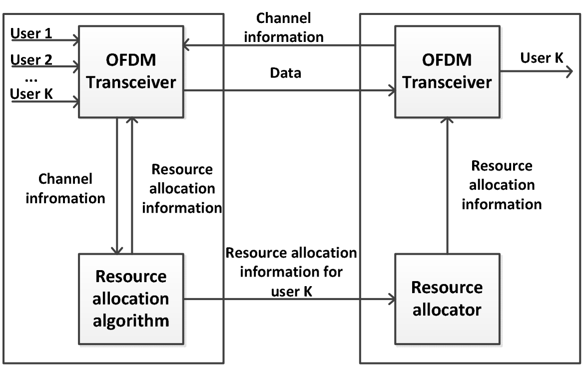


**Fig. 1.** Block diagram of multiuser OFDM system

In this paper, we consider a system with $K$ users sharing $N$ sub-channels. The objective function in the system aims to optimize sub-channel and power allocation to maximize system capacity under an aggregate power constraint. Meanwhile, owing to the introduction of proportional fairness, each user should also satisfy the related proportional rate. The benefit is that different users can achieve their expected data rate based on their different services.

Mathematically, the optimization problem discussed in this paper is formulated as

$$\max_{p_{k,n},\rho_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{\rho_{k,n}}{N} \log_2\left(1+\frac{|h_{k,n}|^2 p_{k,n}}{N_0 \frac{B}{N}}\right)$$

subject to $$\sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} \le P_{total} \quad p_{k,n} \ge 0 \text{ for all } k \text{ and } n$$

$$\sum_{k=1}^{K} \rho_{k,n} = 1,\ \rho_{k,n} = \{0,1\} \text{ for all } n$$

$$R_1 : R_2 : \ldots : R_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K$$

where K is the number of users, N is the number of sub-channels, and $h_{k,n}$ and $p_{k,n}$ are the channel gain and power allocated to user k in sub-channel n, respectively. Further, $\rho_{k,n}$, which is either 0 or 1, indicates whether sub-channel n is assigned to user k, $N_0$ is the power spectral density of additive white Gaussian noise (AWGN), and B is the available bandwidth and $P_{total}$ is the total power. Here, $\{\gamma_i\}_{i=1}^{K}$ is a set of values indicating the data rate ratio among users.

# 3 Sub-channel Allocation and Power Allocation

## 3.1 Sub-channel Allocation

The sub-channel is first allocated under the assumption of equal power allocation to all the sub-channels based on a genetic algorithm to maximize system capacity using a rough rate constraint. A genetic algorithm is an evolutionary intelligent search technique that has been successfully used to solve many troublesome optimization problems. Fig. 2 depicts the flow chart, while each of the procedures is described below.

1) The number in each cell denotes which user is occupying the associated sub-channel. For example, in the chromosome shown in Fig. 3, cell number 3 contains the number 5, which means that sub-channel 3 is assigned to user 5. Different sub-channel allocation schemes can be represented by different chromosomes. Initially, W chromosomes are randomly generated.

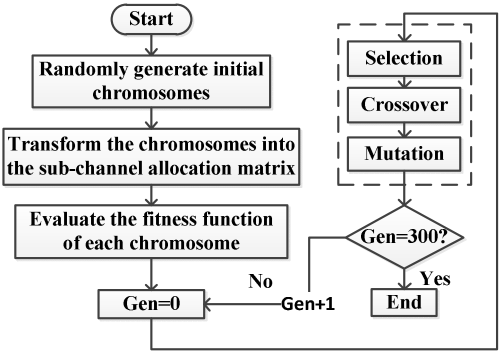


**Fig. 2.** Flow chart for sub-channel allocation

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 7 | 5 | 4 | 8 | …… | 6 | 3 |

**Fig. 3.** A chromosome

2)Transform the chromosomes into a sub-channel allocation matrix: Since the generated chromosome cannot be used directly to evaluate the fitness function, it must first be transformed into an allocation matrix. The chromosome shown in Fig. 2 can be represented as a $8 \times 64$ matrix, with the elements in the matrix set to 0 or 1. Rows represent user numbers, while each column represents a sub-channel number. For example, if the element in the 5th row and 12th column is 1, it means that sub-channel 12 is assigned to user 5.

3)Evaluate the fitness function of each chromosome: The fitness function used is given by (1) and the target is to achieve maximum system capacity and a rough rate constraint for each user. Thus, the fitness of chromosomes is ranked based on the value of the fitness function. A larger value has a higher rank while the chromosome beyond the rate constraint requirement has the lowest rank.

4)Generate a new population by the following steps:

a)Selection: In the first generation, W chromosomes are randomly generated and assigned a fitness rank by step 3. In this step, the chromosomes with the lowest fitness ranks based on the selection probability Ps are abandoned; in other words, W*Ps chromosomes with the lowest fitness ranks are discarded.

b)Crossover: The remaining chromosomes are randomly grouped into pairs. For each pair, two new chromosomes are created using one-point crossover with crossover probability Pc, and the new individuals are called children.

c)Mutation: The mutation operator is used to alter one or more genes in the chromosome from its initial state to maintain individual diversity with mutation probability Pm. By following the above steps, a new generation with W chromosomes is formed.

5)Gen=300?: Gen defines the maximum number of generations. If the number of cycle operations satisfies the predefined number, the genetic algorithm terminates and returns the optimal solution.

Applying the above procedures will generate an optimal sub-channel allocation scheme with a rough rate constraint. The simulation results are presented in Section 4.

### 3.2 Power Allocation

After sub-channel allocation, the system has an optimal sub-channel scheme. Next, power allocation is carried out to ensure proportional fairness. The basic strategy is to assign power to the users according to the optimal solution, while power for the sub-channels owned by each user is allocated equally. The basic process flow, shown in Fig. 4, is similar to that depicted in Fig. 3. The optimization problem can be simplified as follows:

$$\max_{p_{k,n},\rho_{k,n}} \sum_{k=1}^{K} \sum_{n \in A_K} \frac{1}{N} \log_2 \left(1 + \frac{|h_{k,n}|^2 p_{k,n}}{N_0 \frac{B}{N}}\right)$$

subject to $\sum_{k=1}^{K} \sum_{n \in A_k} p_{k,n} \le P_{total}$, $p_{k,n} \ge 0$ for all $k$ and $n$

$$R_1 : R_2 : \ldots : R_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K$$

where $A_k$ is the sub-channel distribution for the k-th user.

Compared with sub-channel allocation, power allocation is a multiple object optimization problem that includes maintaining system capacity, ensuring that the total power is consumed, and achieving proportional fairness.

Randomly generate initial chromosomes: A series of K elements, representing the power values for the users, with values ranging from 0 to 1 is randomly generated.

Evaluate the fitness function of each chromosome: There are two fitness functions: one is for system capacity, and the other is for proportional fairness. The chromosomes should, therefore, be divided into two groups, to which the respective fitness function is applied. The number of chromosomes allocated to each group is based on the weight. Here, the process focuses on the realization of proportional fairness. The specific method first calculates the user rates according to the power and sub-channel distributions. Next, it generates the user proportional rate by dividing the user rate by the proportion $\gamma_k$. Finally, the difference between the maximum and minimum user proportional rates is obtained, and if the absolute value of the difference is optimized to 0, proportional fairness has been achieved. So, the principle of ranking fitness considers the maximum system capacity and the minimum of the absolute value of the difference between the maximum and minimum user proportional rates.

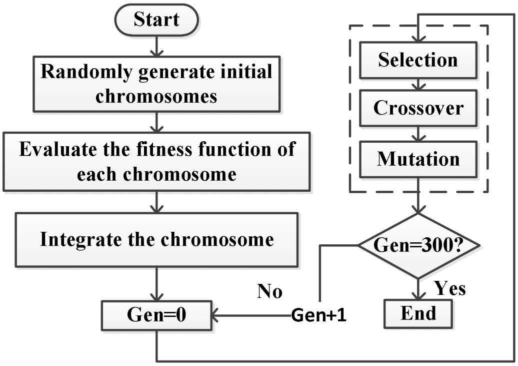


**Fig. 4.** Flow chart for power allocation

Integrate the chromosome: Owing to the separation of the fitness function calculation, the chromosome must be integrated after the evaluation terminates, to be included in the next procedure.

This step is the same as that for sub-channel allocation, including the operations of selection, crossover, and mutation.

When the algorithm terminates, system power has been rationally allocated among the users and the allocation scheme satisfies proportional fairness among users and maximum system capacity. The simulation results are presented in Section 4.

## 4 Simulation Results

In this section, we present our simulation results to validate the effectiveness of the proposed resource allocation strategy. A system with 64 sub-channels and 8 users was used. The wireless channel was modeled as a 6-tap frequency-selective Rayleigh channel employed in [8], and the total power $P_{total}$ and available bandwidth were set to 1 Watt and 1 MHz, respectively. The power spectral density for noise was set to -80 dB/Hz. The parameters in the genetic algorithms for sub-channel allocation and power allocation were the same, that is, W=100, Ps=0.9, Pc=0.7, Pm=0.035, and Gen=300.

Fig. 5 shows the total system capacity in terms of sub-channel allocation, power allocation, and maximum capacity. Owing to the fact that the sub-channel and power are both assigned to the user with the best channel gain, the maximum capacity method achieves maximum capacity. However, it also results in other users in the system having a zero data rate as shown in Fig. 6. The method with sub-channel allocation achieves slightly less than maximum capacity, but it introduces a rough rate constraint that ensures that all users can acquire sub-channels. The system capacity after power allocation is close to the sub-channel allocation. Since the method needs to realize proportional fairness at the expense of capacity loss, it has little effect on system capacity performance.

Fig. 6 shows the normalized ergodic capacity distribution among users for the different methods. Here, the data rate ratio is $\gamma_1 = \gamma_2 = 4$ and $\gamma_3 = \gamma_4 = \dots = \gamma_8 = 1$. It can be seen that the method with maximum capacity assigns all the resource to user 1. Static TDMA tends to allocate equal capacity to each user based on the principle that all users have the same opportunities to transmit. However, this method does not satisfy the requirement of proportional fairness owing to the lack of a fairness control mechanism. Meanwhile, consider the method using only sub-channel allocation. Although the user rate distribution has no strict regulation, it satisfies a rough rate constraint and the total system capacity is close to maximum capacity. Using the proposed sub-channel and power allocation algorithms, the capacity is distributed fairly among users according to the rate ratio.

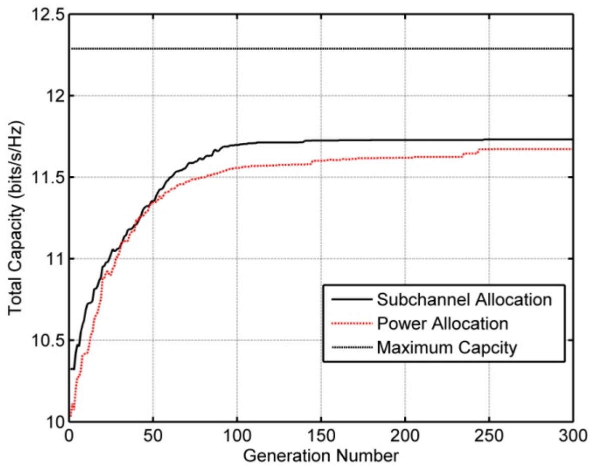


**Fig. 5.** Total system capacity for different numbers of generations

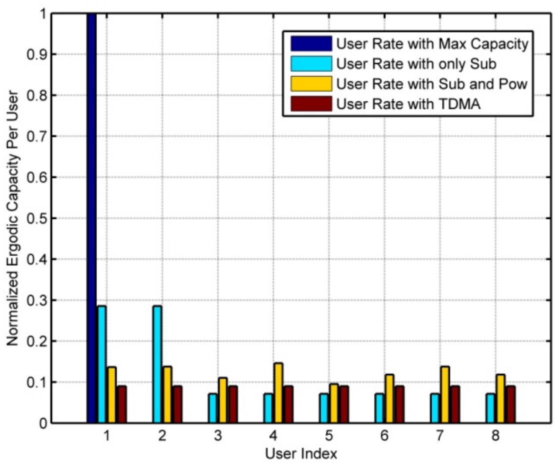


**Fig. 6.** Table 1. Fig. 6 Normalized ergodic capacity among users

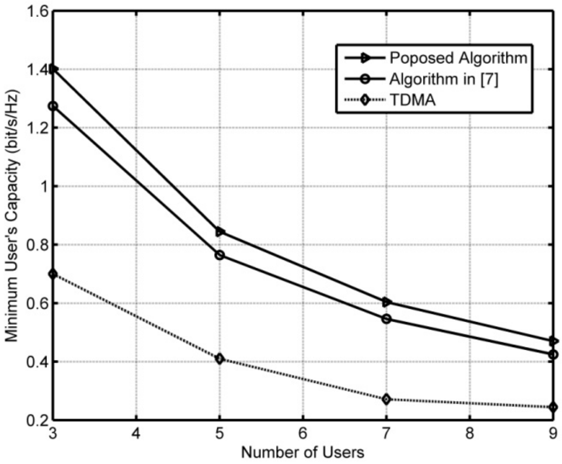


**Fig. 7.** Minimum user capacity with a different number of users

Fig. 7 shows the minimum user capacity for an increasing number of users in an OFDM system. We can see that the adaptive algorithms, including the proposed algorithm and the algorithm given in [7], show significant improvement over the non-adaptive TDMA method. The proposed algorithm with optimal sub-channel and power allocation achieves even higher capacity than the method proposed in [7].

## 5 Conclusion

In this paper we presented an algorithm based on genetic algorithms for adaptive resource allocation in a multiuser OFDM system. The algorithm first carries out sub-channel allocation using a rough rate constraint to achieve maximum capacity. Using the sub-channel allocation solution, optimal power allocation is implemented to achieve proportional fairness and maintain maximum system capacity. Simulation results show that the proposed algorithm not only effectively satisfies the requirements of maximum capacity and proportional fairness, but also reduces computational complexity.

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