Fuzzy Methods for Demand Forecasting in Supply Chain Management

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Abstract Forecasting the future demand is crucial for supply chain planning. In this chapter, the fuzzy methods that can be used to forecast future by historical demand information are explained. The examined methods include fuzzy time series, fuzzy regression, adaptive network-based fuzzy inference system and fuzzy rule based systems. The literature review is given and the methods are introduced for the mentioned methods. Also two numerical applications using fuzzy time series are presented. In one of the examples, future enrollments of a university is forecasted using Hwang, Chen and Lee's study and in the other example a company's oil consumption is predicted using Singh's algorithm. Finally, the forecasting accuracy of the methods is determined by using Mean Absolute Error (MAE).

Keywords Fuzzy forecasting · Fuzzy time series · Fuzzy regression · Fuzzy rule based systems · Adaptive network-based fuzzy inference system

1 Introduction

Forecasting is defined as the process of predicting future events which can contain various areas such as product demand, tourism demand, climate change, health and political forecasts (Sanders 2012). Forecasting is one of the most important business activities because it drives all other actions. Decisions such as which markets to pursue, which products to produce, how much inventory to carry, and

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how many people to hire are all based upon a forecast. Planning and forecasting are two closely related actions. Planning can be defined as the process of selecting actions in anticipation of the forecast. In other words, while forecast drives the plan, plan is made in response to the forecasts. As a result, poor forecasts result in poor plans which can put an organization in an unwanted and unprepared situation. The results of poor forecasting can be in terms of loss of sales or excess inventory that cannot be sold.

Demand forecasting is the basis of all supply chain planning processes. In a push type of supply chain, the flow of products in the supply chain are performed in anticipation of customer demand, on the other all pull processes are performed in response to customer demand. For push processes, the managers in the supply chain must plan the level of activities such as production and transportation. In contrast for pull processes, the level of available capacity and inventory level should be planned. As a result, in both cases the managers should make a forecast about the future customer demands.

Although forecasting is such an important action, forecasts are rarely perfect so the forecasting studies should include both the expected value of the forecast and a measure of forecast error or demand uncertainty. The researches on forecasting activities show that aggregated forecasts are usually more accurate than individual item forecasts and short term forecasts result more accurate results when compared to long term forecasts.

Many problems in real world deal with uncertain and imprecise data so conventional approaches cannot be effective to find the best solution. In order to handle this uncertainty, the fuzzy set theory has been developed (Zadeh 1965) as an effective mathematical tool. Although humans have relatively efficient in qualitative forecasting issues, they are cannot show the same performance in making quantitative predictions (Kahraman et al. 2010). Since fuzzy linguistic models permit the translation of verbal expressions into numerical ones fuzzy logic can empower the decision making process. Especially when the decisions involve human subjectivity, fuzzy algebra provides a mathematical framework for handling the imprecision and vagueness.

The fuzzy set theory has some advantages in forecasting. Mamlook et al. (2009) state that fuzzy methods use fuzzy sets which enable the modelers to condense large amount of data into smaller set of variable rule. Another important advantage of fuzzy logic is valid for rule based systems especially, these systems are based on heuristics and therefore they are able to incorporate human intuition and experience into the forecasting process (Cirstea et al. 2002). Kahraman et al. (2010) identify one of the advantages of fuzzy time series approximations as the ability to work with a very small set of data and no requirements for the linearity assumption. Fuzzy sets offer a clear insight into the forecasting model and can be used for non-linear systems.

In this chapter an introduction to fuzzy forecasting techniques are given and different fuzzy times series methods are compared in a demand forecasting case. The rest of the study is as follows: The importance of forecasting in supply chain management is issued in Sect. 2. The possible fuzzy forecasting tools including, fuzzy time series, fuzzy regression, fuzzy rule based systems and adaptive network-based fuzzy inference system (ANFIS) are introduced in Sect. 3. The literature review of fuzzy demand forecasting techniques are given in Sect. 4. The numerical applications and comparison of the fuzzy time series methods are provided in Sect. 5. Finally Sect. 6 presents the conclusions and future research directions.

2 Forecasting in Supply Chain

Forecasting is one of the most important activities in a company because plans at different levels of the organization are made based on forecasting. Marketing department uses forecasting for size of markets, new competition future trends, emerging markets and customer demands. Finance department uses forecasting to assess financial performance and capital investment needs to set budgets. Operations department makes decisions regarding production and inventory levels based on demand forecast. Sourcing activities uses forecasts to make purchasing decisions and select suppliers. Proper planning for the future starts with a forecast (Sanders 2012).

However demand forecasting is especially critical for the entire supply chain since it affects all the plans made by each company in the chain. Forecasts that are done independently without communication between by each company in the supply chain tend to be inaccurate since each company uses the immediate buyer's data to produce the forecast instead of the final customer. The absence of communication while making the demand forecasts leads to the bullwhip effect which can be defined as the increased volatility in orders as they spread through the supply chain (Lee et al. 2004). Bullwhip affects all parties in the supply chain, inventory levels increase, working capital efficiency decrease, and production capacity is used inefficiently. In order to overcome this problems collaborative planning, forecasting and replenishment (CPRF) approach is used by supply chain members. CPRF enables companies to work together to develop forecasts and plans to optimize the supply chain by generating a consensus demand forecast (Wisner et al. 2011).

Customer demand may be affected by various factors thus in order to forecast demand, companies should first identify these factors and then ascertain the relationship between these factors and future demand (Chopra and Meindl 2012). The set of factors contains both objective factors, such as past demand, state of the economy, planned advertising; or subjective factors which include human judgments. Although the most of the forecasting methods depend on the objective data, human input is also important when they make the final forecast.

Identification of the factors is also important to choose a suitable forecasting methodology. The classical forecasting methods that can be used for demand forecasting can be classified to four groups (Chopra and Meindl 2012). (1) Qualitative methods which highly depend on human judgment and most appropriate when little historical data is available. (2) Time series method which supposes that past data is a good indicator of the future demand and uses historical demand to make a forecast. (3) Casual methods use the correlation between factors and the demand to forecast the future demand. (4) Simulation forecasting method imitates the consumer choices and other environmental issues that give rise to demand in order to forecast the demand. Since the scope of this study is fuzzy methods, the classical crisp methods are not investigated in detail.

In demand forecasting studies, just like any other forecasts, there are some steps that should be followed to ensure the credibility of the results (Sanders 2012). The first step is identifying what forecasts are needed to help us to plan the future. The second step involves analyzing available data and identifying the patterns. Identifying the patterns is critical for selecting the forecasting model. The most common data patters can be listed as; level, trend, seasonality and cycles. Level is the simplest pattern the demand data fluctuate around a constant mean. Trend is present when data exhibit an increasing or decreasing pattern over time. Seasonality is any pattern that regularly repeats itself and cycles are patterns created by economic fluctuations. As the data patterns are identified the next step is to select an appropriate forecasting model. As the model is selected the forecast is generated. At the final step, the forecasts are evaluated with the actual values in order to evaluate the performance of the forecasting method.

3 Fuzzy Forecasting Methods

3.1 Fuzzy Times Series

A time series is composed of observations x_t, each one being recorded at a specific time t. Time-series models are based on a series of discrete and equal time increments. Time series models assumes that, the predictions for the next unit time interval such as, week, month, quarter, year, are based on, and only on, the past values of the last N periods of the same time interval, of the variable we wish to forecast (Kahraman et al. 2010).

While there are various crisp times series approach such as simple exponential smoothing, trend-corrected exponential smoothing, trend and seasonality corrected exponential smoothing, after introduction of fuzzy sets by Zadeh (1965), Song and Chissom (1993a) presented the definition of fuzzy time series and outlined its model by means of fuzzy relation equations. The authors applied the model for forecasting under fuzzy environment in which historical data are of linguistic values.

The fuzzy time series are defined as follows. Let $Y(t)(t = \ldots, 0, 1, 2, 3, \ldots)$ is a subset of R1, be the universe of discourse on which fuzzy sets $f_i(t)(i = 1, 2, 3, \ldots)$ are defined and let $F(t)$ be a collection of f1(t), f2 (t),… Then, F(t) is called a fuzzy time series defined on Y(t) (t $= \ldots, 0, 1, 2, \ldots$).

Suppose $F(t)$ is caused only by $F(t-1)$ and is denoted by $F(t-1) \rightarrow F(t)$; then there is a fuzzy relationship between $F(t)$ and $F(t-1)$ and can be expressed as the relational equation where "∘" is the composition operator. The relation R is called the fuzzy relation between $F(t)$ and $F(t-1)$. And the model is called the first order model of $F(t)$:

$$F(t) = F(t-1) \circ R(t, t-1) \tag{1}$$

If for any time t, R(t, t−1) is independent of t, i.e., for any time t, R(t, t−1) = R(t, t−2), then F(t) is called a time-invariant fuzzy time series. Otherwise, it is called a time-variant fuzzy time series (Song and Chissom 1993a). Let F(t) be a fuzzy time series. If F(t) is caused by F(t − 1), F(t − 2),…, and F(t − n), then this fuzzy relationship (FLR) is represented by:

$$F(t-n), \ldots, F(t-2), F(t-1) \rightarrow F(t) \tag{2}$$

and it is called the nth order fuzzy time series forecasting model.

The traditional time series approaches require having the linearity assumption and at least 50 observations. In fuzzy time series approaches, there is not only a limitation for the number of observations but also there is no need for the linearity assumption (Kahraman et al. 2010).

Most of the existing fuzzy time series forecasting methods use the following four steps to handle forecasting problems (Chen 1996):

- Step 1: Partitioning the universe of discourse into specific intervals.
- Step 2: Fuzzifying the historical data.
- Step 3: Building the fuzzy relationships and obtaining fuzzy relationship groups.
- Step 4: Calculating the forecasted outputs.

3.2 Fuzzy Regression

Regression analysis is a statistical technique that tries to explore and model the relationship between two or more variables. Classical statistical linear regression takes the form

$$y(x) = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \varepsilon_i, i = 1, 2, \ldots, m \tag{3}$$

where y_i is the dependent variable, x_{ij} are the independent variables and $\beta_j$is the coefficients and ε_i is the random error term. All the values in the equation is crisp in the classical regression analysis. Although the classical analysis is widely used, some problems are reported in special cases such as inadequate number of observations, difficulties in verifying the distribution assumptions (Shapiro 2004).

There are various studies on Fuzzy regression (Georg 1994; Sakawa and Hitoshi 1992; Tanaka et al. 1989; Wang and Tsaur 2000). In this chapter we focus on Buckley's (2004) study, fuzzy prediction in linear regression technique which is based on confidence intervals. Buckley's crisp simple linear regression model is as follows:

$$\tilde{y}(x) = a + b(x_i - \bar{x}) + \varepsilon_i \tag{4}$$

where $\bar{x}$ is the mean value of the x_i. Initially crisp $(1-\beta)100\,\%$ confidence intervals of a, b and σ^2 are calculated. To this end the crisp estimators of the coefficients ($\hat{a}$, $\hat{b}$) should be determined. The values of the estimators are $\hat{a} = \bar{y},\ \hat{b} = \frac{B1}{B2}$ where

$$B1 = \sum\nolimits_{i=1}^{n} y_i(x_i - \bar{x}) \tag{5}$$

$$B2 = \sum\nolimits_{i=1}^{n} (x_i - \bar{x})^2 \tag{6}$$

and

$$\sigma^2 = \left(\frac{1}{n}\right) \sum\nolimits_{i=1}^{n} \left[y_i - \hat{a} - \hat{b}(x_i - \bar{x})\right]^2 \tag{7}$$

A $(1-\beta)100\,\%$ confidence interval for a and b is as follows:

$$\left[\hat{a} - t_{\beta/2}\sqrt{\frac{\hat{\sigma}^2}{(n-2)}}, \hat{a} + t_{\beta/2}\sqrt{\frac{\hat{\sigma}^2}{(n-2)}}\right] \tag{8}$$

$$\left[\hat{b} - t_{\beta/2}\sqrt{\frac{n\hat{\sigma}^2}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}, \hat{b} + t_{\beta/2}\sqrt{\frac{n\hat{\sigma}^2}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}\right] \tag{9}$$

If β is taken into account as an α-cut level, the fuzzy triangular membership function for a and b can be obtained from Eqs. (8) and (9)

The fuzzy regression equation is as follows;

$$\tilde{y}(x) = \tilde{a} + \tilde{b}(x - \bar{x}) \tag{10}$$

In the equation, $\tilde{y}(x)$, $\tilde{a}$ and $\tilde{b}$ are fuzzy numbers and x and $\bar{x}$ are real numbers. In order to predict new fuzzy values for $\tilde{y}(x)$, new values for x can be chosen.

Let $\tilde{y}(\alpha) = \left[y(x)_1(\alpha), y(x)_2(\alpha)\right]$, $\tilde{a}(\alpha) = [a_1(\alpha), a_2(\alpha)]$, and $\tilde{b}(\alpha) = [b_1(\alpha), b_2(\alpha)]$. Based on the interval arithmetic and (α)-cut operations $\tilde{y}(\alpha)$ is calculated as follows:

The (α)-cuts of $\tilde{a}$ and $\tilde{b}$ are determined using Eqs. (8), and (9) respectively.

$$\tilde{Y}[x](\alpha) = \begin{cases} y(x)_1(\alpha) = a_1(\alpha) + (x - \tilde{x})b_1(\alpha) & \\ y(x)_2(\alpha) = a_2(\alpha) + (x - \tilde{x})b_2(\alpha) & if\ (x - x) > 0 \\ y(x)_1(\alpha) = a_1(\alpha) + (x - \tilde{x})b_1(\alpha) & \\ y(x)_2(\alpha) = a_2(\alpha) + (x - \tilde{x})b_2(\alpha) & if\ (x - x) < 0 \end{cases} \tag{11}$$

3.3 Fuzzy Rule Based Systems

Fuzzy rule based systems (FRBS) is a computing framework based on concepts of fuzzy set theory, fuzzy if–then rules, and fuzzy reasoning. The term is also known as "Fuzzy inference systems", "fuzzy expert systems" and "fuzzy model" in various resources (Jang et al. 1997). The basic structure of a FRBS consists of three conceptual components: a rule base, a database and a reasoning machine. The rule base contains the fuzzy rules used in the system, database defines the membership functions used in the fuzzy rules and the reasoning mechanism performs the inference procedure based on the rules and the given facts. Block diagram of a fuzzy rule based system is given in Fig. 1.

Fuzzy if–then rules are expressions of the form IF a THEN B, where A and B are labels of fuzzy sets characterized by appropriate membership functions. An example can be given as:

If pressure is high then volume is small.

Where pressure and volume are linguistic variables, high and small are linguistic values that are characterized by membership functions (Jang 1993).

Fuzzy inference process comprises of five parts: fuzzification of the input variables, application of the fuzzy operator (AND or OR) in the antecedent, implication from the antecedent to the consequent, aggregation of the consequents across the rules, and defuzzification.

A typical FIS can be described in four steps which are; fuzzification, fuzzy rules, fuzzy inference and defuzzification (Öztayşi et al. 2013).

Step 1: (Fuzzification) Fuzzification process involves the definition of the membership functions of input/output variables by linguistic variables.

Step 2: (Fuzzy rules) A FRBS with i-input variables has r = pi rules, where p is the number of linguistic terms per input variable. As the dimension and complexity of a system increase, the size of the rule base increases exponentially.

A sample rule can be defined as follows:

$$\text{IF } I_1 \text{ is } \widetilde{\mathrm{A}}_1^j \text{ AND } I_2 \text{ is } \widetilde{\mathrm{A}}_2^j \text{ AND} \ldots I_n \text{ is } \widetilde{\mathrm{A}}_n^j \text{ THEN } y \text{ is } \widetilde{\mathrm{B}}^j \text{ for } j = 1, 2, \ldots, r \quad (12)$$

where $I_i (i = 1, 2, \ldots, n)$ are input variables and y is the output variable, $\tilde{\mathrm{A}}_1^j, \tilde{\mathrm{A}}_2^j, \ldots, \tilde{\mathrm{A}}_n^j$ and $\tilde{B}^j$ are the linguistic terms used for the membership function of the corresponding input and output variables for the jth rule, respectively.

Step 3: (Fuzzy inference) Fuzzy inference is an inference procedure to derive a conclusion based on a set of if–then rules. In the literature different fuzzy inference models are proposed such as Mamdani's model, Sugeno's model and Tsukamoto Fuzzy Model (Mamdani and Assilian 1975; Sugeno and Kang 1988; Takagi and Sugeno 1985; Tsukamoto 1979). The Mamdani inference method is manually constructed on the basis of expert knowledge and the final model is neither trained nor optimized. The method considers fuzzy inputs and returns fuzzy outputs (Mamdani and Assilian 1975). Since Mamdani approach is not exclusively

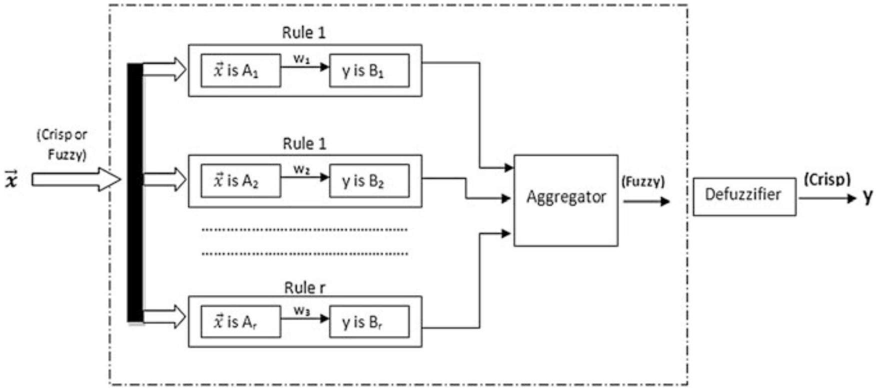

Fig. 1 Block diagram of a fuzzy rule based system (Jang et al. 1997)

dependent on a data set, with sufficient expertise on the system involved, a generalized model for effective future predictions can be obtained (Keshwani et al. 2008). The mechanism of Mamdani inference method is as follows: (1) If there is more than one input in the rule, fuzzy set operations should be applied to achieve a single membership value; (2) then implication method (min) is applied to reach each rule's conclusion; (3) the outputs obtained for each rule are combined into a single fuzzy set, using a fuzzy aggregation operator (max).

For the case where input variables $I_i(i = 1, 2, \ldots, n)$ are crisp variables and the fuzzy rules are described by Eq. (12) and Eq. (13), so for a set of disjunctive rules, where $j = 1, 2, \ldots, r.$, the output using Mamdani inference method is formulated as follows (Ross 1995);

$$\mu_{\tilde{B}}^{j}(y) = \max_{j}\left[\min\left[\mu_{\tilde{A}_1}^{j}(I_1), \mu_{\tilde{A}_2}^{j}(I_2), \ldots, \mu_{\tilde{A}_n}^{j}(I_n)\right]\right] \tag{13}$$

Step 4: (Defuzzification) The output of the fuzzy inference is a fuzzy number and can be converted into a crisp value by defuzzification. There are various defuzzification methods such as, max membership, centroid method, weighted average method, mean-max membership. Centroid method, which is also called center of area or center of gravity method, is the most prevalent and physically appealing of other defuzzification methods (Ross 1995). It is given by the algebraic expression as follows;

$$c^* = \frac{\int \mu_{\tilde{C}} \cdot c \cdot dc}{\int \mu_{\tilde{C}} dc}, c \in \tilde{C} \tag{14}$$

where $\tilde{\mathrm{C}}$ is a fuzzy set having the membership function $\mu_{\tilde{C}}$.

The graphical illustration of the introduced fuzzy rule based system is represented in Fig. 2.

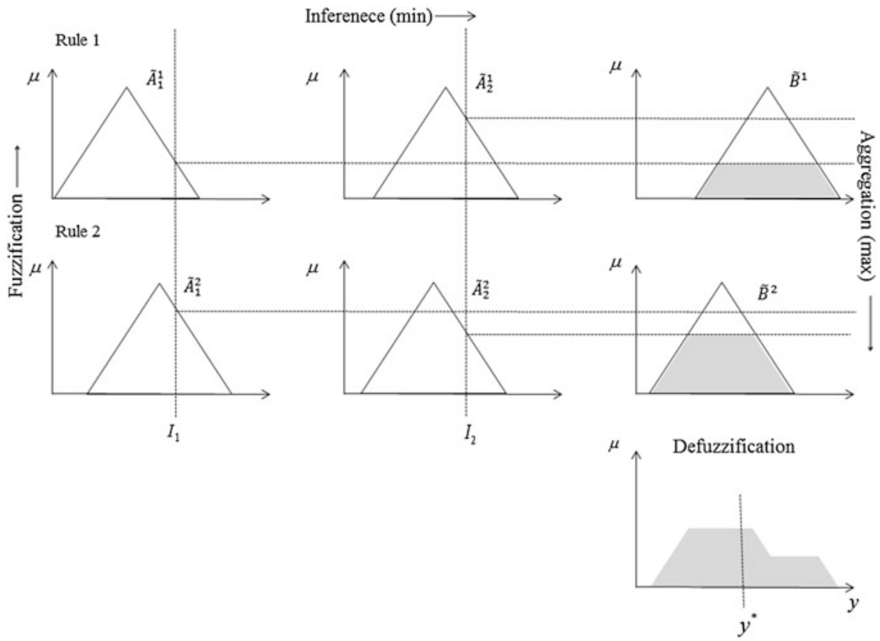

Fig. 2 Graphical Mamdani (max–min) inference method (Öztayşi et al. 2013)

3.4 Adaptive-Network-Based Fuzzy Inference System

Adaptive network-based fuzzy inference system (ANFIS) is a fuzzy inference system implemented in the framework of adaptive networks. By using a hybrid learning procedure, ANFIS can construct an input–output mapping using both human knowledge and predetermined input–output data set (Jang 1993). ANFIS is a fuzzy inference system based on the Sugeno model. It incorporates the self-learning ability of ANN with the linguistic expression function of fuzzy inference (Yun et al. 2008). Using a given input/output data set, ANFIS constructs a fuzzy inference system whose membership function parameters are adjusted using various algorithms. This adjustment allows the fuzzy systems to learn from the data (Matlab 2012).

The model of the ANFIS changes according to the number of input, output and rules employed. For the simplicity, the fuzzy inference system under consideration is assumed to have two inputs (x and y) and one output (z). For a first order Sugeno fuzzy model, a common rule set with two fuzzy if–then rules are as follows:

$$\text{Rule 1: If x is } A_1 \text{ and y is } B_1 \text{ then } f_1 = p_1x + q_1y + r_1 \tag{15}$$

$$\text{Rule 1: If x is } A_2 \text{ and y is } B_2 \text{ then } f_2 = p_2x + q_2y + r_2. \tag{16}$$

where A_i and B_i are the fuzzy sets, f_i is the output set within the fuzzy region specified by the fuzzy rule p_i and q_i and r_i are the design parameters that are determined during

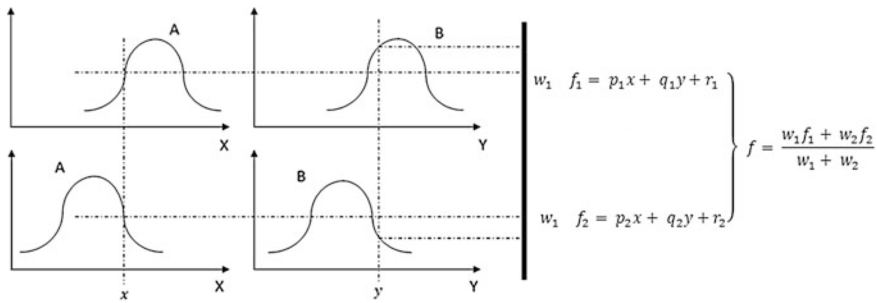

Fig. 3 The reasoning mechanism for the given Sugeno model (1988)

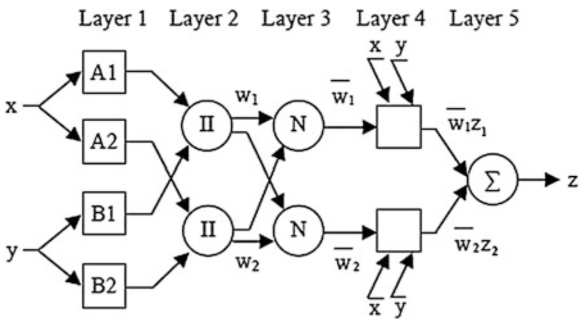

Fig. 4 ANFIS architecture (Jang 1993)

the training process. Figure 3 illustrates the reasoning mechanism for the given Sugeno model, and Fig. 4 represents the corresponding equivalent ANFIS architecture.

ANFIS is composed of five layer feed forward neural network. The node functions in the same layer are of the same function family as described below (Jang 1993):

Layer 1: Every node I in this layer is an adaptive node with a node function:

$$O_{1,i} = \mu_{A_i}(x), \text{for } i = 1, 2 \text{ or} \tag{17}$$

$$O_{1,i} = \mu_{B_{i-2}}(y), \text{for } i = 3, 4, \tag{18}$$

where x or y are the input to node I and A_i or $B_{i\text{-}2}$ is a linguistic label; such as "small" or "large"; associated with this node. O_1, I refers to the membership degree of a fuzzy set A and it specifies the degree to which the given input x or y satisfies the quantifier A. The membership function A can be any appropriate parameterized membership function such as the generalized bell function:

$$\mu_A(x) = \frac{1}{1 + \left|\frac{x - c_i}{a_i}\right|^{2b}}, \tag{19}$$

where a_i, b_i, c_i are the parameters. The parameters in this layer are call premise parameters.

Layer 2: Every node in this layer is a fixed node whose output is the product of all the incoming signals:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), i = 1, 2 \tag{20}$$

Each node output represents the firing strength of a rule.

Layer 3: Every node in this layer is a fixed node labeled N. The ith node calculates the ratio of the ith rule's firing strength to the sum of all rules' firing strengths:

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2}, i = 1, 2 \tag{21}$$

The outputs of this layer are called normalized firing strengths.

Layer 4: Every node I in this layer is an adaptive node with a node function as follows:

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i(p_i x + q_i y + r_i) \tag{22}$$

where $\bar{w}_i$ is a normalized firing strength from layer 3 and pi, qi ri are the parameter set for this node. Parameters in this layer are referred to as consequent parameters.

Layer 5: The signal node in this layer is a fixed node labeled Σ, which computes the overall output as the summation of all incoming signals.

$$overall\ output = O_{5,1} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \tag{23}$$

ANFIS learns the premise and consequent parameters for the membership functions and the rules. Jang et al. (1997) propose the hybrid learning algorithm which uses a combination of Steepest Descent and Least Squares Estimation (LSE). In this approach ANFIS uses a two pass learning algorithm: In the foreword pass the premise (nonlinear) parameters are unmodified and consequent (linear) parameters are computed using a LSE algorithm. In the backward pass, the consequent (linear) parameters are unmodified and premise (nonlinear) parameters are computed using a gradient descent algorithm such as back propagation.

4 Literature Review

The literature provides various studies that employ fuzzy techniques for demand forecasting. These studies can be classified into four groups which are introduced in Sect. 3.

4.1 Fuzzy Time Series

The most widely used fuzzy forecasting technique is the fuzzy time series (FTS) forecasting. Time series approach assumes that the predictions for the next period are based on the past values of the last periods. The fuzzy extensions of time series are initially proposed by Song and Chissom (1993a, b). Chen (1996) studies on how the forecasting model can be improved with lower error levels with a basic model and presents a new method to forecast university enrollments. The robustness of the proposed method is tested and it is shown that the technique can make robust forecasts when the historical data are not accurate. Wong et al. (2009) compare multivariate Fuzzy Time Series models with Traditional Time Series models for the forecasting accuracy. In this chapter, it is stated that when the data with longer time trend the traditional time series model has good pattern fitting. Also, when the period of data is short or indefinite, fuzzy time series model relatively exceeds the time series pattern.

In FTS, partitioning the universe of discourse into specific intervals is the first step of the studies. Huargn (2001) focuses on the effective length of intervals, in order to generate more accurate forecasting. In another study in this area, Li and Chen (2004) dwell on partitioning the intervals in FTS and propose a novel approach that can partition the universe of discourse step by step. Huarng and Yu (2006) work on exploring ways of determining the useful lengths of intervals between the ranges. The results of the study show that that the ratio-based lengths of intervals can improve the FTS forecasting. Jilani and Burney (2008) propose a method that uses heuristic approach to define frequency-density-based partitions of the universe of discourse. Davari et al. (2009) use a modified version of particle swarm optimization for the definition of suitable partitions of FTS forecasting. They propose a method that improves the forecasting accuracy for tuning the length of forecasting intervals. Lin (2009) also studies on intervals of fuzzy time series in order to increase the forecasting accuracy. The universe of discourse is partitioned into subintervals are employed to fuzzify the time series into fuzzy time series and the midpoints of two adjacent cluster centers generated. Chen et al. (2012) propose a new model which incorporates the concept of the equal frequency partitioning and fast fourier transform algorithm. The source is actual trading data from TAIEX. The model is compared with Chen (1996), Yu (2005), and Chang et al. (2011) and the proposed model and it presents better results.

Another approach to improve forecasting accuracy is to integrate other techniques with FTS. Yu (2005) use FTS for forecasting recurrence and weighting of fuzzy logical groups. In the proposed model, different weights are given to various fuzzy relationships and the model is compared with local regression models. Fuzzy relations in fuzzy time series are analyzed by Tsaur et al. (2005). This study proposes an analytical approach and its aim is finding the steady state of fuzzy relation matrix to revise the logic forecasting process. Pai (2006) proposes a new FTS called hybrid elipsoidal fuzzy system for time series forecasting (HEFST) and apply it electricity data. The results of the comparison among HEFST, ANN and regression models show that the proposed model gives the best results. Huarng et al. (2007) propose a heuristic function integrated FTS model which can handle multiple variables to improve forecasting results and avoid complicated computations due to the inclusion of multiple variables. Cheng et al. (2007) propose a model that improves FTS with fuzzy logic relation which is identified using rough set theory. The model implements different linguistic values in order to determine the most accurate linguistic value in order to increases the forecasting accuracy. Cheng et al. (2008) propose using fuzzy clustering integrated with fuzzy time series to improve the accuracy level. The forecasting results show that the proposed method can multiple-attribute data effectively and outperform former methods. Liu (2009) studies in short-time load forecasting. The proposed forecasting method adjusts an analysis slide window of FTS to train the trend predictor in the training phase. Later the trend predictor is used to generate forecasting values. Tsaur and Kuo (2011) propose an Adaptive FTS model for forecasting Taiwan's tourism demand. In the study, FTS data is transferred to the fuzzy logic group and the weights are assigned to periods. Chen and Chen (2011) proposed a new method that is based on FTS and fuzzy variation groups. Daily Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) is issued for forecasting. They proposed a method that uses both fuzzy variation groups, where the main input factor is the previous day's TAIEX, and the secondary factor is either the Dow Jones, the NASDAQ, or their combination, and fuzzy logical relationship groups data for forecasting the TAIEX.

4.2 Fuzzy Regression

Regression analysis is one of the widely used approach for relationship identification and forecasting for both univariate and multivariate cases. Similar to this, fuzzy regression is also used to define fuzzy relationships and fuzzy forecasting. Heshmaty and Kandel (1985) use fuzzy regression models in sales forecasting under uncertainty. In their chapter, two different sales forecasting techniques are issued. The first technique consists of non-fuzzy abstract methods of linear regression and econometrics. The second sales forecasting technique uses fuzzy linear regression. Fuzzy linear regression is used to forecast in uncertain environments. Feng and Guang (1993) propose a forecasting model of fuzzy self-regression. In the model, the awaiting estimated parameters and the dependent variables are fuzzy numbers of M–N form. Liang and Cheng (2000) propose an integrated approach that consists of multilinear regression and fuzzy inference system has been presented for short-term load forecasting. The multilinear regression model is applied to find a preliminary load forecast and the fuzzy inference system is used for load correction from historical information.

Song et al. (2005) handle fuzzy regression analysis concept that is issued in the short-term forecasting to reduce the load forecasting error. The fuzzy linear regression model is made from the load data of the previous 3 years and the coefficients of the model are established as a result of the model. Khashei et al. (2008) propose a model that consists of the artificial neural networks (ANNs) and fuzzy regression, for forecasting in financial markets. By using the fuzzy regression models, the limitation of big amount of historical data is lifted. In the same year, Chen and Dang (2008) propose a three-phase method to construct the fuzzy regression model with variable spreads. In the first phase, the membership functions of the least-squares estimates of regression coefficients are constructed. In the second phase, the coefficients are defuzzified to obtain crisp values. In the last phase, the error terms of the model are determined. Al-Hamadi (2011) shows a long-term electric load forecasting technique that is based on fuzzy linear regression. This technique uses long term annual growth factors in order to forecast the model's parameters. In this chapter, the objective of the linear optimization problem is set as to minimize the spread of fuzzy regression parameters. Kazemi et al. (2012) develop an energy demand prediction model for Iran using socio-economic indicators. The approach is structured as a multi-level model fuzzy linear regression and used for predicting the industry energy demand from 2011 to 2020.

4.3 Fuzzy Rule-Based Forecasting

Fuzzy rule based systems (FRBS) are composed of if–then rules and use these rules to make inference and decisions. FRBSs are also used in forecasting area. Liu (2006) study, fuzzy rule-based classifier for electrical load pattern classification is established. Multi-objective genetic algorithms are applied to prefer a pattern classification system. Cardoso and Gomide (2007) study on newspapers demand for customer's need using fuzzy clustering and fuzzy rules. The method produces more accurate results when compared with neural network-based predictors, and autoregressive forecasters. Chang et al. (2007) propose a model which integrates the wavelet and TakagiSugeno-Kong (TSK) FRBS for financial time series data prediction. The wavelet in the model is used to decrease the noises in the data. The proposed method is used to forecast the future stock. Dimitriou et al. (2008) suggest an adaptive hybrid fuzzy rule-based system for forecasting traffic flow. Univariate and multivariate data structures are used in the model and online and offline fuzzy rule-based system is considered. In Chang et al. (2008) study, a case based clustering TSK fuzzy rule system for stock price predictions in Taiwan Stock Exchange Corporation is presented. The model is integrated by a case based reasoning technique, a TSK Fuzzy Rule based system, and Simulated Annealing (SA). Chen and Chang (2010) propose a method for multi-variable fuzzy forecasting. The model composed of fuzzy clustering and fuzzy rule interpolation techniques. Fuzzy rules are created by training samples and the fuzzy rule corresponds to a given cluster. Pratondo (2010) proposes a FRBS based on uncertain environment conditions to enhance demand forecasting. In Zhang and Liu's (2010) study, a new method is presented for mid-long term load forecasting using fuzzy rules and genetic algorithms. The genetic algorithms are based on Takagi–Sugeno Fuzzy Logic System. The system is proposed for electricity forecasting with its computation speed. Cheikhrouhou et al. (2011) propose using knowledge from forecasters combined with mathematical forecasts. In the proposed model, the mathematical forecasts are adjusted by the knowledge from different forecasters. In Ivette and Rosangela's (2011) study, data-driven approach applied to the long term prediction of daily time series is presented. Daily samples are aggregated to build weekly time series. The results are validated using multiple time series. Moreover, the results are compared with obtained using daily models. Yanfei and Yinbo (2011) focus on short term load forecasting with a model that consists of ANN and FRBS. The first part is the basic load component and the second part is the temperature and the holiday load component. Initially the ANN processes and then fuzzy rules are completed. The results of the study show that using ANN process while applying FRBS improves the model's sensitivity.

4.4 Adaptive Neuro Fuzzy Inference System

Adaptive Neuro Fuzzy Inference System (ANFIS) is a kind of neural network that is based on fuzzy inference system. ANFIS's inference system corresponds to a set of fuzzy IF–THEN rules that have learning capability to approximate nonlinear functions. Padmakumai et al. (1999) propose a hybrid fuzzy neural technique which combines neural network and fuzzy logic modeling, and present an application for long term land use based distribution load forecasting. ANFIS is used for forecasting in different areas. Atsalakis and Valavanis (2009) develop a neuro-fuzzy adaptive control system in order to forecast next day's stock price trends. For fuzzifying the system inputs, Gaussian-2 shaped membership functions are used. In Efendigil's (2009) study, ANFIS techniques and artificial neural networks is compared. A new model for forecasting the uncertain customer demand under fuzziness is proposed for better accuracy of model. In Moreno's (2009) study, ANFIS is used for monthly ideal generation of a hydraulic plant considering different factors like weather conditions ant the plant's reservoir level. In Chabaa's et al. (2009) study, a set of input and output data of internet traffic time series is forecasted.

In Azadeh's et al. (2010) study on short-term natural gas prediction using ANFIS. The obtained results are compared with ANN and proposed model out performs ANN. In Chen's et al. (2010) study, tourist arrivals to Taiwan is forecasted by ANFIS and the proposed model gives more accurate results when compared with FTS, Grey Model and Markov Residual Modified model. Mohamad et al. (2010) make a case study to compare Back Propagation Neural Network (BPNN) and ANFIS. The testing errors show that ANFIS perform better than BPNN. Ho and Tsai (2011) use ANFIS and structural equation modeling that are compared in new product development. In their study, the authors show that ANFIS gives better forecasting results and can explain nonlinear relationships. In Wei's (2011) study, the model incorporates an autoregressive model into an ANFIS.

The model is employed in earning per share time series data of shares in Taiwan. Kisi et al. (2012) use ANFIS for forecasting the intermittent stream flows using ANFIS, ANN and Support Vector Machine (SVM). In the result part, ANFIS and ANN give good results using the data from two stations, Uzunkopru and Babaeski. Wei and Cheng (2012) use Taiwan Stock Exchange Index that is forecasted in a volatile environment. Four models including Chen's model, Yu's model, Huarng's model, are compared and the proposed model is superior to the listing methods in terms of the root mean squared error. In Zahedi et al. (2013) study, electricity demand forecasting modeled by ANFIS. Inputs of model are employment, gross domestic product, population, dwelling count and two meteorological parameters. In conclusion, the employment is found as the most important input for demand. Azadeh et al. (2013) present ANFIS-fuzzy data envelopment analysis (FDEA) algorithm. Two types of ANFIS are used for forecasting the natural gas demand. In conclusion, fuzzy one performed well with a lower error.

5 Applications

In this section two fuzzy time series method are introduced and relevant numerical applications are presented.

5.1 *Fuzzy Time Series Using Hwang, Chen, Lee's Method (Hwang et al. 1998)*

Let's think that we know the demand. We are going to find the demand with fuzzy demand forecasting. Let U be the universe of discourse U = {u_1, u_2,….. u_n}. A fuzzy set (Zadeh 1965) A of U, is defined by;

$$A = \mu_a(u_1)/u_1 + \mu_a(u_2)/u_2 + \ldots + \mu_a(u_n)/u_n \quad (24)$$

Firstly we find the variations. Let's take the years as; if the first year is t, the second year is t + 1. The first variation is the need of t + 1 minus the need of t. For example, the customer need in 1996 is 25.552 and the need in 1997 is 25.996. The variation of year 1997 is; 25996−25552 = 444. In this series, we can easily find the minimum increase $D_{\min}$ and maximum increase $D_{\max}$. After that, the universe of discourse U is defined, $U = [D_{min} - D_1, D_{max} + D_2]$, where the D_1 and D_2 are suitable numbers. $D_{\min} = -376$ and $D_{\max} = 1399$. D_1 and D_2 are positive numbers. We select the $D_1 = 24$ and $D_2 = 1$. So, U can be represented as $U = [-400, 1400]$. The universe of discourse is partition off into six intervals, where $U_1 = [-400, -100]$, $U_2 = [-100, 200]$, $U_3 = [200, 500]$, $U_4 = [500, 800]$, $U_5 = [800, 1100]$, $U_6 = [1100, 1400]$. Now, the next step is to define the fuzzy sets on the universe of discourse U. We determined some linguistic values. Seven fuzzy sets that are defined as; $A1$ = Decrease, $A2$ = No Change, $A3$ = Little Increase,

$A4$ = Increase, $A5$ = Big Increase, $A6$ = Too Big Increase. Then, Fuzzy sets on the Universe Of Discourse are defined as follows;

$$A1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \tag{25}$$

$$A2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \tag{26}$$

$$A3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 \tag{27}$$

$$A4 = 0/u_1 + 0\,/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 \tag{28}$$

$$A5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 \tag{29}$$

$$A6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 \tag{30}$$

After that, the historical data are being fuzzified.

Now, we are going to choose a suitable window basis, w. Let's calculate the operation matrix $O^w(t)$ and the criterion matrix C(t). t is the year that we want to forecast. In this example, we can select w = 5. So we can set a 4 × 6 operation matrix $O^5(t)$ and the criterion matrix C(t) as follows.

$$\begin{aligned} O^5(2002) &= \begin{bmatrix} \textit{Fuzzy variation of the enrollment of } 2000 \\ \textit{Fuzzy variation of the enrollment of } 1999 \\ \textit{Fuzzy variation of the enrollment of } 1998 \\ \textit{Fuzzy variation of the enrollment of } 1997 \end{bmatrix} = \begin{bmatrix} A2 \\ A2 \\ A1 \\ A3 \end{bmatrix} \\ &= \begin{bmatrix} 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 1 & 0.5 & 0 & 0 \end{bmatrix} \\ C(2002) &= \textit{Fuzzy variation of the enrollment of } 2001 = [A6] \\ [A6] &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.5 \quad 1] \end{aligned} \tag{31}$$

Calculated relation matrix $R(t)$ by $R(t)\ [i,\ j] = O^w(t)\ [i,\ j]\ XC(t)\ [J]$, where $1 \leq i \leq 4$, and $1 \leq j \leq 6$. We can get;

$$R(2002) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Next, the column's maximum values selected (Table 1).

$$F(2002) = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$$

After that, we are ready to defuzzify process. There are some principles to defuzzify the fuzzified forecasted variations.

Table 1 Membership functions of forecasted variations (under w = 5)

Years	U1	U2	U3	U4	U5	U6
2002	0	0	0	0	0	0
2003	0	0	0.5	1	0.25	0
2004	0	0	0.5	0.5	0.25	0
2005	0.25	1	0.5	0	0	0
2006	0	0.25	1	0.5	0	0
2007	0.5	1	0.25	0	0	0
2008	0.25	1	0.25	0	0	0
2009	0.5	0.5	0	0	0	0
2010	0.25	1	0.25	0	0	0

Table 2 Actual and forecasted values of enrollments

Years	Actual needs	Variations	Fuzzified variations	Actual	Forecasted
1996	25552	–	–	–	–
1997	25996	444	A3	–	–
1998	25620	−376	A1	–	–
1999	25745	125	A2	–	–
2000	25870	125	A2	–	–
2001	26120	250	A3	–	–
2002	27519	1399	A6	27519	26120
2003	28245	726	A4	28245	28169
2004	28807	562	A4	28807	29045
2005	28919	112	A2	28919	28857
2006	29388	469	A3	29388	29269
2007	29433	45	A2	29433	29438
2008	29497	64	A2	29497	29483
2009	29145	−352	A1	29145	29397
2010	29163	18	A2	29163	29195

a. If the value of memberships all 0, the variation of forecasting is 0.
b. If the numbers memberships in the Table 1 have only one maximum u_i, the forecasted variation is m_i is the midpoint of u_i.
c. If there is more than one maximum value of membership, then the midpoints are taken in average, like $(m_1 + m_2 + m_3 + \cdots + m_k)/k$.

Actual number in 2001 is 26,120, and the forecasted value of 2002 is 26120 + 0 = 26120 (Table 2).

The MAE is 244, 1

$$MAE = \frac{1}{n}\sum\nolimits_{i=1}^{n} |fi - yi| \tag{32}$$

fi Forecasted value
yi Actual Value.

5.2 Fuzzy Time Series Using Singh's Method

Singh (2008) proposes a method on forecasting of enrollments of Alabama University. Steps of the computational algorithm of proposed method for fuzzy time series forecasting is given as follows;

1. Defining the Universe of discourse (U).

$U = [D_{min} - D_1, D_{max} + D_2]$ where D_1 and D_2 are two proper positive numbers.

2. Partition the Universe of discourse into equal length of intervals: u1, u2,…, u_m. The number of intervals will be in accordance with the number of linguistic variables (fuzzy sets) $A1$, $A2$,…, Am to be considered.
3. Constructing the fuzzy sets Ai in accordance with the intervals and apply the triangular membership rule to each intervals in each fuzzy set so constructed.
4. Fuzzifying the historical data and establish the fuzzy logical relationships by the rule: If Ai is the fuzzy production of year n and A_j is the fuzzify production of year $n + 1$, then the fuzzy logical relation is denoted as $Ai \rightarrow Aj$. Here Ai is showed current state and Aj is next state.
5. Rules for forecasting. The notations used are defined as;

- $[*A_j]$ is corresponding interval u_j for which membership in A_j is Supremum
- $L\ [*A_j]$ is the lower bound of interval u_j.
- $U\ [*A_j]$ is the upper bound of interval u_j.
- $l\ [*A_j]$ is the length of the interval u_j whose membership in A_j is Supremum
- $M\ [*A_j]$ is the midvalue of the interval u_j having Supremum value in A_j.

For a fuzzy logical relation $Ai \rightarrow Aj$:

- A_i is the fuzzified enrollments of year n.
- A_j is the fuzzified enrollments of year $n + 1$.
- E_i is the actual enrollments of year n.
- E_{i-1} is the actual enrollments of year $n - 1$.
- E_{i-2} is the actual enrollments of year $n - 2$.
- F_j is the crisp forecasted enrollments of the year $n + 1$.

This model of order three utilizes the historical data of years $n - 2$, $n - 1$, n for framing rules to implement on fuzzy logical relation, $A_i \rightarrow A_j$, is fuzzified enrollments of year $n + 1$. The proposed method, is explained below step by step, for forecasting is mentioned as rule for generating the relations between the time series data of years $n - 2$, $n - 1$, n for forecasting the enrollment of year $n + 1$.

Computational Algorithm: Forecasting enrollments F_j for year $n + 1$ and onwards

for k to ….. K

Obtained fuzzy logical relation for year k to k + 1

$Ai \rightarrow Aj$

R = 0 and S = 0

(where K shows end of time series data)
Compute

$$D_i = ||(E_i - E_{i-1})| - |(E_{i-1} - E_{i-2})|| \tag{33}$$

$$X_i = Ei + D_i/2 \tag{34}$$

$$XX_i = E_i - D_i/2 \tag{35}$$

$$Yi = Ei + D_i \tag{36}$$

$$YYi = Ei - D_i \tag{37}$$

$$Pi = E_i + D_i/4 \tag{38}$$

$$PP_i = E_i - D_i/4 \tag{39}$$

$$Q_i = E_i + 26^* D_i \tag{40}$$

$$QQ_i = E_i - 2^* D_i \tag{41}$$

$$G_i = E_i + D_i/6 \tag{42}$$

$$GG_i = E_i - D_i/6 \tag{43}$$

$$H_i = E_i + 3^* D_i \tag{44}$$

$$HH_i = E_i - 3^* D_i \tag{45}$$

$$\begin{array}{l}\text{If} X_i \geq L\left[{}^*A_j\right] \text{ and } X_i \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + X_i \text{ and } S = S + 1\end{array} \tag{46}$$

$$\begin{array}{l}\text{If } XX_i \geq L\left[{}^*A_j\right] \text{ and } XX_i \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + XX_i \text{ and } S = S + 1\end{array} \tag{47}$$

$$\begin{array}{l}\text{If } Y_i \geq L\left[{}^*A_j\right] \text{ and } Yi \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + Y_i \text{ and } S = S + 1\end{array} \tag{48}$$

$$\begin{array}{l}\text{If } YY_i \geq L\left[{}^*A_j\right] \text{ and } YY_i \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + YYi \text{ and } S = S + 1\end{array} \tag{49}$$

$$\begin{array}{l}\text{If } P_i \geq L\left[{}^*A_j\right] \text{ and } P_i \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + P_i \text{ and } S = S + 1\end{array} \tag{50}$$

$$\begin{aligned}&\text{If } PP_i \geq L\left[{}^*A_j\right] \text{ and } PP_i \leq U\left[{}^*A_j\right]\\&\text{Then } R = R + PPi \text{ and } S = S + 1\end{aligned} \tag{51}$$

$$\begin{aligned}&\text{If } Q_i \geq L\left[{}^*A_j\right] \text{ and } Q_i \leq U\left[{}^*A_j\right]\\&\text{Then } R = R + Q_i \text{ and } S = S + 1\end{aligned} \tag{52}$$

$$\begin{aligned}&\text{If } QQ_i \geq L\left[{}^*A_j\right] \text{ and } QQ_i \leq U\left[{}^*A_j\right]\\&\text{Then } R = R + Q_i \text{ and } S = S + 1\end{aligned} \tag{53}$$

$$\begin{aligned}&\text{If } QQ_i \geq L\left[{}^*A_j\right] \text{ and } QQ_i \leq U\left[{}^*A_j\right]\\&\text{Then } R = R + QQ_i \text{ and } S = S + 1\end{aligned} \tag{54}$$

$$\begin{aligned}&\text{If } G_i \geq L\left[{}^*A_j\right] \text{ and } G_i \leq U\left[{}^*A_j\right]\\&\text{Then } R = R + GG_i \text{ and } S = S + 1\end{aligned} \tag{55}$$

$$\begin{aligned}&\text{If } H_i \geq L\left[{}^*A_j\right] \text{ and } H_i \leq U\left[{}^*A_j\right]\\&\text{Then } R = R + H_i \text{ and } S = S + 1\end{aligned} \tag{56}$$

$$\begin{aligned}&\text{If } HH_i \geq L\left[{}^*A_j\right] \text{ and } HH_i \leq U\left[{}^*A_j\right]\\&\text{Then } R = R + HH_i \text{ and } S = S + 1\end{aligned} \tag{57}$$

$$\begin{aligned}&F_j = \left(R + M\left({}^*A_j\right)\right)/(S+1)\\&\text{Next k}\end{aligned} \tag{58}$$

Considering the rules and the algorithm, one company's oil consumption is forecasted. Universe of discourse is defined as; U = [11000, 29000]. The partition of universe of discourse U in the six intervals are shown as: U_1 = [11000, 14000], U_2 = [14000, 17000], U_3 = [17000, 20000], U_4 = [20000, 23000], U_5 = [23000, 26000], U_6 = [26000, 29000]. After that, the next step is defining six fuzzy sets A1, A2,…, A6 as linguistic variables on the universe of discourse U. These fuzzy variables are defined as; A1 : Poor Consumption, A2 : Below Average Consumption, A3 : Average Consumption, A4 : Good Consumption, A5 : Very Good Consumption, A6 : Excellent Consumption. Also the membership grades to these fuzzy sets of linguistic values are defined as;

$$A1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \tag{59}$$

$$A2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \tag{60}$$

$$A3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 \tag{61}$$

$$A4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 \tag{62}$$

$$A5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 \tag{63}$$

Table 3 The results of Singh's method

Years	Actual oil consumption	Linguistic variables	Actual	Forecasted
1980	11379	A1	11379	–
1981	12933	A1	12933	–
1982	14933	A2	14933	–
1983	13155	A1	13155	13048
1984	17517	A3	17517	18500
1985	17884	A3	17884	18101
1986	19073	A3	19073	18606
1987	20081	A4	20081	21252
1988	26415	A6	26415	27500
1989	23957	A5	23957	24716
1990	22421	A4	22421	21647
1991	20429	A4	20429	22011

$$A6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 \tag{64}$$

The historical time series data of oil consumption are fuzzified using the triangular membership function to obtain the enrollments in terms of linguistic variables.

In order to forecast the consumptions, the algorithm is explained step by step. For forecast the year 1985, the algorithm runs as follows;

$D = D_i = //(17517 - 13155)/ - /(13155 - 14933)// = 2584$
$X_i = 17517 + 2584/2 = 18809$, $XX_i = 17517 - 2584/2 = 16225$, $Yi = 17517 + 2584 = 20101$,
$YY_i = 17517 - 2584 = 14933$, $P_i = 17517 + 2584/4 = 18163$, $PP_i = 17517 - 2584/4 = 16871$,
$Q_i = 17517 + 2 * 2584 = 22685$, $QQ_i = 17517 - 2 * 2584 = 12349$,
$G_i = 17517 + 2584/6 = 17948$, $GG_i = 17517 - 2584/6 = 17086$,
$H_i = 17517 + 3 * 2584 = 25269$, $HH_i = 17517 - 3 * 2584 = 9765$

The values which are between $U_1 = [17000, 20000]$ are considered for finding the forecast. The X_i, P_i, G_i and GG_i are between intervals of U_1. Because of that, the forecasted value of 1985 is,

$$F1985 = (18809 + 18163 + 17948 + 17086 + 18500)/(5) = 18101$$

The other forecasted values are found in the same way. They are given in the Table 3.

The Mean Absolute Error (MAE) is 793, 9

$$MAE = \frac{1}{n}\sum\nolimits_{i=1}^{n} |f_i - y_i|, \tag{65}$$

f_i = Forecasted value, y_i = Actual value.

6 Conclusion

Since all plans from different management levels build their plans based on forecasts, the process of forecasting is a vital activity in a supply chain. Departments such as marketing, finance, operations, and purchasing directly use forecasts in their processes. Forecasting is especially important for supply chains since the results affect all the parties in the chain. Thus, collaborative planning, forecasting and replenishment approach is used by supply chain members in order to develop forecasts and plans to optimize the supply chain.

The literature provides different techniques including time series, regression, ARIMA, simulation, artificial neural networks, particle swarm optimization, genetic algorithm and fuzzy methods, to build demand forecasting models. Fuzzy set theory can handle uncertainity and incorporate human intuition and experience into the forecasting process thus fuzzy set theory provides advantages to modellers in forecasting process.

In this study, mostly used fuzzy demand forecasting methods including, fuzzy time series, fuzzy regression, fuzzy rule based systems and adaptive neuro fuzzy inference system are explained briefly and a literature review is supplied for each methodology. Additionaly, two fuzzy time serries methods are applied to two different demand forecasting problem. Also, the models' performance measurement (MAE) are calculated. The methods results are shown in table. The two examples include two different time series data. If we want to compare the two methods, the method that gives low MAE is the best.

As further study the same forecasting problem can be examined with other mentioned fuzzy and non-fuzzy techniques and the prediction accuracy of each technique can be benchmarked.

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