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Cengiz Kahraman
Başar Öztayşi *Editors*

Supply Chain Management Under Fuzziness

Recent Developments and Techniques

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Editors

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I dedicate this book to my wife
Fatma Kahraman
and my children
Nazlı Ece, Yunus Emre, and Onur

Prof. Cengiz Kahraman

I dedicate this book to my family;
My love Senem Öztayşi
and my sweet daughter Nil

Dr. Başar Öztayşi

Preface

A supply chain is the network of all the individuals, organizations, resources, activities, and technology involved in the creation and sale of a product, from the delivery of source materials from the supplier to the manufacturer, through to its eventual delivery to the end user. Supply chain management (SCM) is the oversight of materials, information, and finances as they move in a process from supplier to manufacturer to wholesaler to retailer to consumer. The three main flows of the supply chain are the product flow, the information flow, and the finances flow. SCM involves coordinating and integrating these flows both within and among companies.

Decision making is the thought process of selecting a logical choice from the available options. This is generally made under fuzzy environment. Fuzzy decision-making is a decision process using the sets whose boundaries are not sharply defined. The aim of this book is to show how fuzzy sets and fuzzy decision-making can be used in the various stages of supply chain management.

The contents of the book were constituted by following crisp supply chain management books. Thus, our book includes all the topics, which can be found in a classical supply chain management book but under fuzziness. The handled titles of our book are supplier evaluation under fuzziness, supply chain performance measurement under fuzziness, planning, controlling, and improving supply chain under fuzziness, production and materials management under fuzziness, optimization in supply chain under fuzziness, warehouse management under fuzziness, and green and reverse logistics under fuzziness.

The authors who published many fuzzy SCM papers in the literature were selected and invited to our book. Under the above main titles, they wrote on supplier evaluation using fuzzy inference systems, multicriteria supplier selection using fuzzy PROMETHEE method, fuzzy-AHP approach to improve effectiveness of supply chain, supplier evaluation using fuzzy clustering, investigating organizational characteristics for sustainable supply chain planning under fuzziness, fuzzy multiple criteria decision making for supply chain management, supply chain performance measurement: an integrated DEMATEL and fuzzy-ANP approach, imprecise DEA models to assess the agility of supply chains, supply chain performance measurement using a SCOR-based fuzzy VIKOR approach, fuzzy estimations and system dynamics for improving manufacturing orders in VMI supply chains, fuzzy methods for demand forecasting in supply chain management flows finding in networks in fuzzy conditions, supply chain configuration as a cooperative game with fuzzy coalitions, a decentralized production and distribution planning model in an uncertain environment, a fuzzy linear programming approach for aggregate production planning, batch production plan for periodic demands with uncertain recycling rate in a closed-loop supply system, optimization models for supply chain production planning under fuzziness, recent models and solution methodologies for optimization problems in supply chain management under fuzziness, a multiple means transportation model with type-2 fuzzy uncertainty, a fuzzy set theoretic approach to warehouse storage decisions in supply chains, fuzzy c-means algorithm with fixed cluster centers for uncapacitated facility location problems: Turkish case study, a supply-chain production inventory model with warehouse facilities under fuzzy environment selection and assignment of material handling devices under uncertainty, government green procurement: a fuzzy-DEMATEL analysis of barriers, facility location selection in reverse logistics using a type-2 fuzzy decision aid method, green and reverse logistics management under fuzziness, an axiomatic design approach to the classification of reverse logistics network design studies under fuzziness, green supply chain technology: a comprehensive evaluation and justification multiattribute decision modeling approach.

The authors were invited from different countries to obtain a real international book. Among these countries, we can count Turkey, Greece, Spain, China, Russia, Iran, Canada, Taiwan, Mexico, Colombia, India, and the USA.

Finally, we thank all contributors and referees for their kind cooperation. This book would not be happening without their contributions and efforts.

Cengiz Kahraman
Başar Öztayşi

Contents

Part I
Supplier Evaluation Under Fuzziness

Supplier Evaluation Using Fuzzy Inference Systems

Atefeh Amindoust and Ali Saghafinia

Abstract Supplier selection is an important area of decision making in manufacturing and service industries, mainly for large and medium companies—either multinational (MNCs) or local. As sustainability in terms of economic, environmental, and social aspects has gained world-wide focus in supply chain management, this dimension deserves due attention in supplier selection decision. In real life applications, the importance of supplier selection criteria is different and depends on the circumstances and situations and each organization may consider its individual relative importance of the criteria. The relative importance of the criteria and also the suppliers' performance with respect to these criteria would be verified with the relevant decision makers. So, the supplier selection decision involves a high degree of vagueness and ambiguity in practice. This chapter takes the aforesaid issues into account and proposes a modular FIS method for supplier selection problem. To handle the subjectivity of decision makers' preferences, fuzzy set theory is applied. The applicability and feasibility of the proposed method are tested through a real-life supplier selection problem.

Keywords Supplier selection · Sustainability · Fuzzy set theory · Fuzzy inference system

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1 Introduction

Supply chain management (SCM) is a business term that has received a lot of attention in the last few decades. The object of SCM obviously is the supply chain which represents a network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services in the hands of the ultimate consumer (Stadtler 2005). Effective SCM has become a potentially valuable way of securing competitive advantage and improving organizational performance since competition is no longer between organizations, but among supply chains (Li et al. 2006). Moreover, sustainability issues are being discussed seriously in supply chain management (Seuring and Müller 2008; Michaelraj and Shahabudeen 2009; Pagell et al. 2010; Chaabane et al. 2010; Grzybowska 2012; Golinska and Romano 2012). Therefore, the ongoing corporate sustainability issue in the supply chain exerts pressure on all members of the chain in different components to consider sustainability in their activities.

One of the crucial challenges for purchasing department in SCM is supplier evaluation and selection. Considering the sustainability issues in the supplier selection process makes it more complicate than traditional one due to increasing the number of selection criteria. Therefore, there is a need to design an open-ended supplier selection model which can handle with large amounts of criteria. In addition, decision making in supplier selection includes a high degree of vagueness and ambiguity in practice. In fact the importance of the selection criteria is not same and depends on the situation at hand. Also, the suppliers' performance with respect to these criteria must be considered in the selection process. These two scenarios need to be verified with the purchasing managers as decision makers. They normally prefer to answer the related questions in linguistic terms instead of numerical form to express their perceptions (Amindoust et al. 2012). To cope with the subjectivity and vagueness that is being existed in their assessments, application of fuzzy logic is an appropriate tool and some researchers have used it in supplier selection issue (Awasthi et al. 2010; Bottani and Rizzi 2008; Chen 2009; Woo and Saghiri 2011; Amin et al. 2010). Also, Ordoobadi proposed a mathematical algorithm by applying fuzzy membership functions to rank the suppliers (Ordoobadi 2009). However, in case of a large number of suppliers and criteria this method is quite time consuming and the final results of ranking are very close to each other. Therefore, the ranking results from this method may not be accurate. Focusing on the said limitations and applying the FIS system to overcome the drawbacks of Ordoobadi's model can be a fertile area. Further, Carrera and Mayorga applied the FIS system for supplier selection (Carrera and Mayorga 2008). But, they did not assign the importance of weights for the selected indicators. So, this chapter intends to utilize fuzzy set theory and propose a ranking method for a modular FIS open-ended model putting significance on the relative importance of criteria and apply this method to sustainable supplier selection decision. Applying the FIS approach in supplier selection problem would be a challengeable issue in the presence of a high number of inputs. It leads to computational burden and also daunting design of the rules. Moreover, the proper selection of the membership functions plays an important role in supplier selection issue. So, the appropriate design of the FIS approach can be strengthened the supplier selection issue.

2 Sustainable Supplier Selection

Supplier selection is a vital decision in SCM and considering the sustainability issues in this process is a key scenario to achieve the success of supply chains in global competitive markets. Literature in this area shows that 20 % of the firms viewed sustainability issues as their largest supply chain risk and 25 % of the firms required suppliers to adhere to social and ecological standards in order to mitigate supply chain risks (Foerstl et al. 2010). Due to the cost-oriented outsourcing trend over the past decades, external stakeholders, such as non-governmental organizations (NGOs) and customers, expect the focal buying firms to assure socially and ecologically sound production at their supplier 'sites. Thus, the irresponsible supplier behavior of any kind may be projected to the buying firm, causing adverse publicity, reputational damage, and costly legal obligations. Thus, firms which outsource production to suppliers cannot transfer the risk related to unacceptable environmental and social standards at supplier premises, but must seek active management of the supply base for sustainability (Foerstl et al. 2010). The concept of sustainability consists of three dimensions: the protection of the natural environment, the maintenance of economic vitality, and observance of specific social considerations (Porsche et al. 2004). Going thorough literature, it is found that only few of supplier selection papers until 2008 considered environmental merits in their selection process (Ho et al. 2010). But, after this green supplier selection has been received more attention and some researchers considered environmental merits besides economic ones (Büyüközkan and Çifçi 2011; Aydın Keskin et al. 2010; Kuo et al. 2010; Humphreys et al. 2003; Hsu and Hu 2009; Mafakheri et al. 2011; Tseng and Chiu 2013). Also in recent years, social merits as an aspect of sustainability have been received attentions besides economic and environmental aspects (Bai and Sarkis 2010; Baskaran et al. 2012; Chu and Varma 2012; Amindoust et al. 2012). Therefore, the sustainable supplier selection has been focused in this chapter and the selection criteria based on the circumstances and situations at hand are combined from a sustainable point of view into three groups (economic, environmental, and social) for each selection decision in the proposed method.

3 Theoretical Background

This section briefly explains the basic theoretical background on the related theories in the proposed supplier selection method including fuzzy set theory and fuzzy inference system respectively.

3.1 Fuzzy Set Theory

Zadeh (1965) introduced fuzzy set theory to cope with the imprecision and uncertainty which is inherent to the human judgments in decision making processes through the use of linguistic terms and degrees of membership. A fuzzy set is a class of objects with grades of membership. A normalized membership function is between zero and one (Zadeh 1965). These grades present the degree of stability with which special element belongs to a fuzzy set. To express fuzzy sets on the mathematical point of view, consider a set of objects X. The set is explained as follows:

$$X = x_1, x_2, \ldots\ldots, x_n \tag{1}$$

where, x_i is an element in the set X. A membership value (μ) expresses the grade of membership related to each element x_i in a fuzzy set A, which shows a combination as below:

$$A = \mu_1(x_1), \mu_2(x_{2)}, \ldots\ldots, \mu_n(x_n) \tag{2}$$

In this chapter, fuzzy set theory is applied to consider the decision makers' preferences in relation to criteria to calculate the weights of them and also in relation to the suppliers' performance with respect to these criteria.

3.2 Fuzzy Inference System

Fuzzy inference is the process of formulating the mapping from given input(s) to an output using fuzzy logic, the mapping then provides a basis from which a decision can be made. It refers to the computational procedures used for evaluating fuzzy linguistic descriptions using concepts such as membership functions, fuzzy logic operators, and if–then rules. Because rule-based reasoning is grounded in qualitative knowledge representation, there is a need to quantify it and fuzzy logic allows us to mesh a quantitative approach with qualitative representation (Al-Najjar and Alsyouf 2003). The most common approaches to FIS are Sugeno and Mamdani approaches. Sugeno approach would be difficult to give a linguistic interpretation of the information that is described in the rule base. While, Mamdani approach is typically used in modeling human expert knowledge (Al-Najjar and Alsyouf 2003). Mamdani in 1974, investigated the feasibility of using the compositional rule of inference (Mamdani 1974). The Mamdani FIS system has four parts as shown in Fig. 1.

- Fuzzifier: the fuzzy sets of inputs are represented by membership functions to transfer crisp inputs into fuzzy inputs.
- Rules: the main part of the FIS model is "Rules". The fuzzy "if–then" rules are defined on the basis of experts' knowledge in each area. A fuzzy rule can be written as "if x_1 is a_1 and x_2 is b_1, then y is c_1" so that x_1 and x_2 are variables, y is a solution variable, and a_1, b_1, and c_1 are fuzzy linguistic terms.

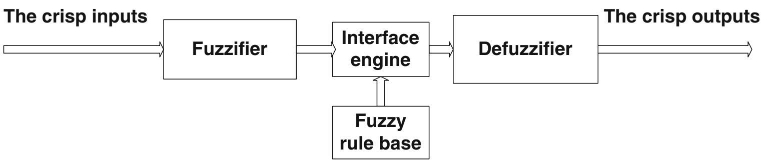


Fig. 1 The Mamdani's fuzzy inference system

- Interface engine: the fuzzy interface engine takes integrations of the identified fuzzy sets considering the fuzzy rule and allocates to integrate the related fuzzy area individually.
- Defuzzifier: transforms the fuzzy output to crisp output. Among the four parts of FIS, the defuzzification process has the most computational complexity. The defuzzifier finally identifies a numerical output value.

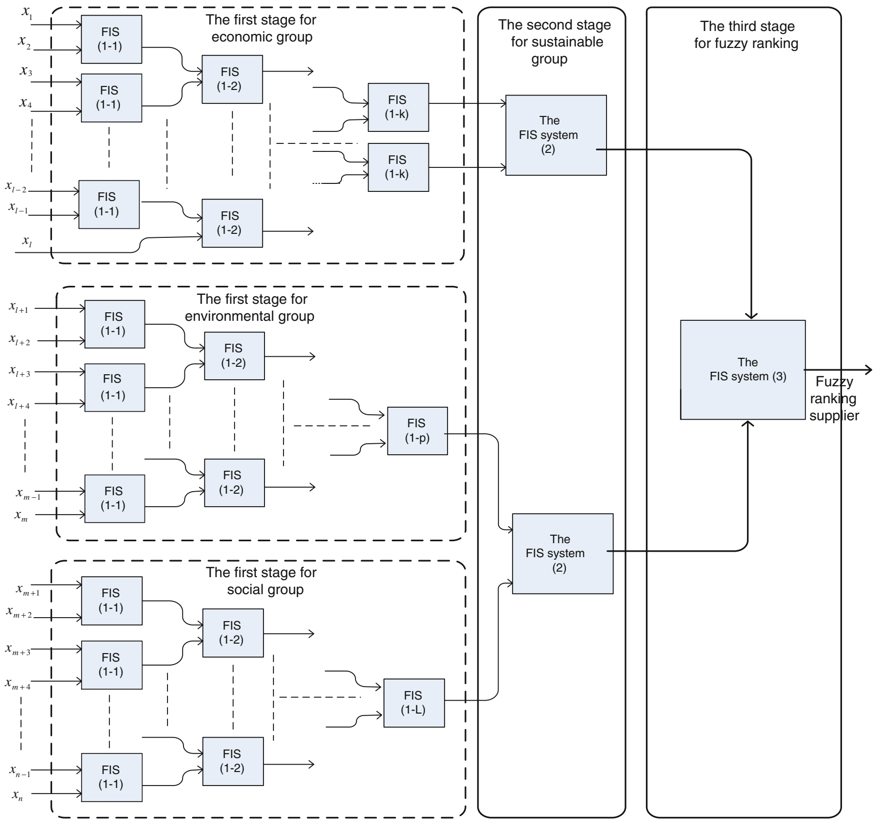
4 The Preliminaries for the Proposed Method

To design the proposed fuzzy ranking method, some fuzzy concepts must be considered. So these concepts are discussed in the next sub-sections and finally the description of the proposed method is presented in three stages and illustrated in Fig. 2.

4.1 *Fuzzy Membership Functions*

In the proposed method, the relative importance of the selection criteria and also the supplier's performance with respect to the criteria, are implemented on the basis of decision makers' opinion. Thus, two membership functions are set out, one for estimation of the criteria weights and the other for the supplier's performance with respect to the criteria. It is noted that the membership functions are applied in the triangular form in this proposed method. A triangular fuzzy number can be shown as $\tilde{w} = (a^l, a^m, a^u,)$ where, a^l, a^m, and a^u are the lower, medium, and upper amount of fuzzy number, respectively as seen in Fig. 3 and the triangular membership function is defined as Eq. (3).

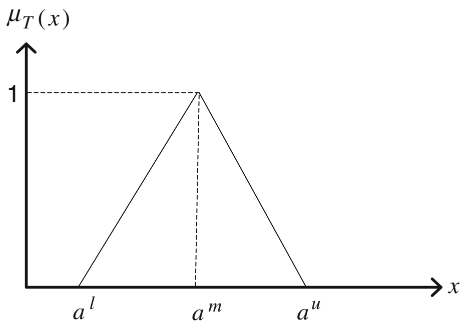
$$\mu_{\tilde{w}}(x) = \begin{cases} 0 \ if \ x < a^l \\ \dfrac{1}{a^m - a^l}(x - a^l) \ if \ a^l \le x \le a^m \\ \dfrac{1}{a^m - a^u}(x - a^u) \ if \ a^m \le x \le a^u \\ 0 \ if \ x > a^u \end{cases} \tag{3}$$



Note :the number of input variables is supposed for economic group odd and for two other groups even

Fig. 2 The methodology of the proposed method

Fig. 3 The triangular fuzzy membership function



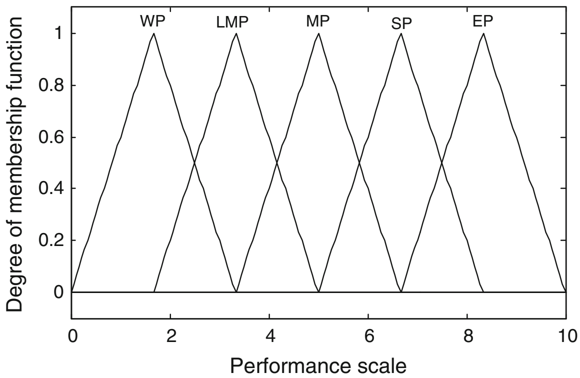


Fig. 4 The membership functions in stage 1 and 2 for the supplier's performance

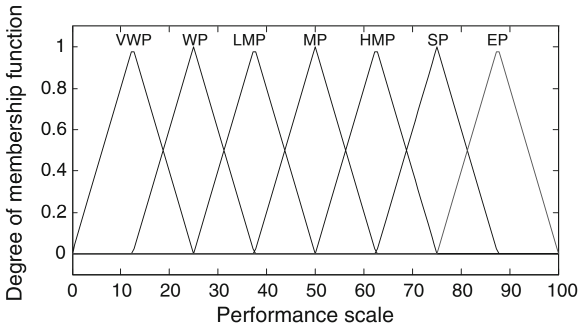


Fig. 5 The membership functions in stage 3 for ranking the suppliers

4.1.1 Membership Functions for the Suppliers 'Performance

In the first stage of the model five fuzzy sets of membership functions are applied for both inputs and outputs of the FIS systems. The fuzzy sets in the form of linguistic rating variables include weakly preferred (WP), low moderately preferred (LMP), moderately preferred (MP), strongly preferred (SP), and extremely preferred (EP) which developed by MATLAB programming as shown in Fig. 4.

Like the first stage, five fuzzy sets of membership functions for both inputs and outputs of the FIS systems are considered. In the third stage, five fuzzy sets of membership functions for inputs which are same the outputs of second stage and seven fuzzy sets of membership functions for the outputs of the FIS systems are considered. The output fuzzy sets in the form of linguistic rating variables include very weakly preferred (VWP), weakly preferred (WP), low moderately preferred (LMP), moderately preferred (MP), high moderately preferred (HMP), strongly preferred (SP), and extremely preferred (EP) which developed by MATLAB programming as shown in Fig. 5.

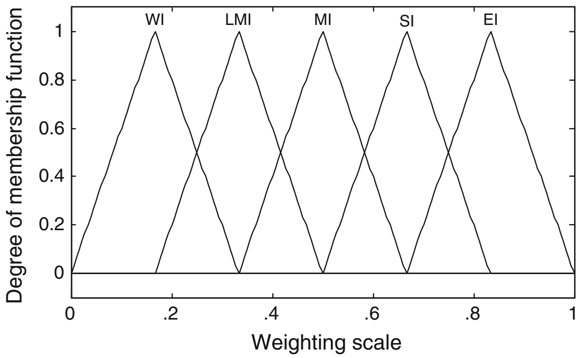


Fig. 6 The membership functions for the weights of criteria

4.1.2 Membership Functions for the Weights of Criteria

In the first stage of the proposed method, five fuzzy sets in the form of linguistic weighting variables which include weak importance (WI), low moderate importance (LMI), moderate importance (MI), strong importance (SI), and extreme importance (EI) are utilized to evaluate the relative importance of criteria which developed by MATLAB programming as shown in Fig. 6.

4.2 Fuzzy Operators

According to the definition of fuzzy numbers, suppose that $\tilde{X}$ and $\tilde{Y}$ are two triangular fuzzy numbers as,

$$\tilde{X} = (x^l, x^m, x^u) \tag{4}$$

$$\tilde{Y} = (y^l, y^m, y^u) \tag{5}$$

The basic fuzzy operators are applied in the proposed method shown below:

$$\widetilde{X} + \widetilde{Y} = \left(x^l + y^l, x^m + y^m, x^u + y^u\right) \tag{6}$$

$$\tilde{X} * \tilde{Y} = \left(x^l * y^l, x^m * y^m, x^u * y^u\right) \tag{7}$$

4.3 Applied Fuzzy Rules

A set of the fuzzy linguistic rules based on experts' knowledge are utilized to implement in the fuzzy ranking method. The rules are adjusted on the preference of decision makers to have the appropriate ranking for suppliers. Also, the rules are designed on the basis of averaging concept for each FIS system. The rules for first, second, and third stages are shown in Tables 1 and 2.

Table 1 The fuzzy rule base matrix in stage1 and stage2

		The first input				
		WP	LMP	MP	SP	EP
The second input	WP	WP	WP	LMP	LMP	MP
	LMP	WP	LMP	LMP	MP	MP
	MP	LMP	LMP	MP	MP	SP
	SP	LMP	MP	MP	SP	SP
	EP	MP	MP	SP	SP	EP

Table 2 The fuzzy rule base matrix in Stage3

		The first input				
		WP	LMP	MP	SP	EP
The second input	WP	VWP	WP	LMP	LMP	MP
	LMP	WP	LMP	LMP	MP	HMP
	MP	LMP	LMP	MP	HMP	SP
	SP	LMP	MP	HMP	SP	SP
	EP	MP	HMP	SP	SP	EP

5 Development of the Supplier Selection Method

In this section, the proposed method is described. Figure 7 shows the framework of the proposed method step by step. First, the appropriate criteria are identified based on experts' knowledge to assess the candidate suppliers. Then, decision makers evaluate suppliers by proposed modular FIS model. As mentioned in Sect. 2, the existing criteria are combined into economic, environmental, and social category to apply in the proposed method. To begin the proposed supplier selection method, the relative importance of criteria and the supplier's performance with respect to criteria must be asked from decision makers and prepared through fuzzy set theory to implement in the proposed method. First, to show the decision makers' preferences for suppliers' performance with respect to criteria, the linguistic variables are used and converted to fuzzy numbers according to Fig. 4. Suppose that there are n criteria ($j = 1, 2, \ldots g, g + 1, \ldots, h,\ h + 1, \ldots, n - 1, n$), S suppliers ($s = 1, 2, \ldots S$) and K decision makers. The decision makers' preferences for each supplier's performance with respect to criteria are solicited as,

$$sp_s = [\tilde{r}_{jk}]_{n \times K} \quad k = 1, \ldots, K \quad j = 1, 2, \ldots g, g + 1, \ldots h, h + 1, \ldots, n - 1, n \quad (8)$$

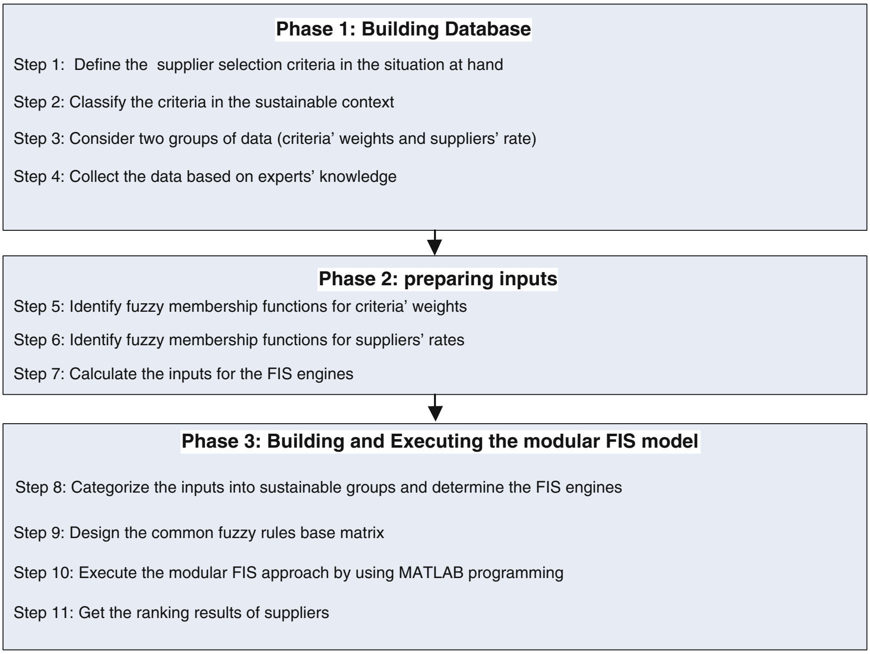


Fig. 7 The framework of the proposed FIS-based supplier selection method

To aggregate K decision makers' opinions for each group (economic, environmental, and social), the aggregated fuzzy number considering (9) can be defined as Eq. (10).

$$\widetilde{R}_p = (a_p, b_p, c_p) \qquad p = 1, 2, \ldots, q \tag{9}$$

$$\begin{aligned} &\tilde{R} = (a, b, c) \\ &st. \\ &a = \frac{1}{q}\sum_{p=1}^{q} a_p \qquad , b = \frac{1}{q}\sum_{p=1}^{q} b_p \\ &c = \frac{1}{q}\sum_{p=1}^{q} c_p \end{aligned} \tag{10}$$

By applying Eq. (10) for every row of the matrix (8) the aggregation is obtained as shown in (11). For example, the fuzzy numbers $\tilde{r}_{11}, \tilde{r}_{12,}, \ldots, \tilde{r}_{1K}$ are aggregated to $\tilde{R}_{11}$.

Table 3 The linguistic terms for criteria weights

Linguistic variables	Corresponding triangular fuzzy number
Weak importance (WI)	(0.00, 0.167, 0.334)
Low moderate importance (LMI)	(0.167, 0.334, 0.50)
Moderate importance (MI)	(0.334, 0.50, 0.667)
Strong importance (SI)	(0.50, 0. 667, 0.834)
Extreme importance (EI)	(0.667, 0.834, 1.00)

In addition, decision makers' opinions about the importance weight of criteria are considered to design the proposed method. So, the five linguistic variables are utilized to show the importance weight of criteria, as shown in Table 3.

$$SP_s = [\widetilde{r}_{jk}]_{n\times K} = \begin{bmatrix} \widetilde{r}_{11} & \widetilde{r}_{12} & \cdots & \widetilde{r}_{1K} \\ \widetilde{r}_{21} & \widetilde{r}_{22} & \cdots & \widetilde{r}_{2K} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{r}_{g1} & \widetilde{r}_{g2} & \cdots & \widetilde{r}_{gK} \\ \widetilde{r}_{(g+1)1} & \widetilde{r}_{(g+1)2} & \cdots & \widetilde{r}_{(g+1)K} \\ \widetilde{r}_{(g+2)1} & \widetilde{r}_{(g+2)2} & \cdots & \widetilde{r}_{(g+2)K} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{r}_{h1} & \widetilde{r}_{h2} & \vdots & \widetilde{r}_{hK} \\ \widetilde{r}_{(h+1)1} & \widetilde{r}_{(h+1)2} & \cdots & \widetilde{r}_{(h+1)K} \\ \widetilde{r}_{(h+2)1} & \widetilde{r}_{(h+2)2} & \cdots & \widetilde{r}_{(h+2)K} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{r}_{n1} & \widetilde{r}_{n2} & \cdots & \widetilde{r}_{nK} \end{bmatrix} \begin{matrix} \Rightarrow \\ \Rightarrow \\ \vdots \\ \Rightarrow \\ \Rightarrow \\ \Rightarrow \\ \vdots \\ \Rightarrow \\ \Rightarrow \\ \Rightarrow \\ \vdots \\ \Rightarrow \end{matrix} \begin{bmatrix} \widetilde{R}_{11} \\ \widetilde{R}_{21} \\ \vdots \\ \widetilde{R}_{g1} \\ \widetilde{R}_{(g+1)1} \\ \widetilde{R}_{(g+2)1} \\ \vdots \\ \widetilde{R}_{h1} \\ \widetilde{R}_{(h+1)1} \\ \widetilde{R}_{(h+2)1} \\ \vdots \\ \widetilde{R}_{n1} \end{bmatrix}_{n\times 1} = R_s \quad (11)$$

The decision makers express their preferences about the relative importance of each criterion in comparison with other criteria (wsc) as shown in (12).

$$wsc = [\tilde{w}sc_{jk}]_{n*K} \;\; k = 1,\ldots,K \quad j = 1,2,\ldots g, g+1,\ldots,h, h+1,\ldots,n-1,n \quad (12)$$

Similar to suppliers' performance, for aggregating K decision makers' opinions for each criterion's weights (wsc), by applying Eq. (10) for every row of matrix (12) the aggregation is obtained as shown in (13).

$$wsc = \left[\tilde{w}sc_{jk}\right]_{n*K}$$

$$= \begin{bmatrix} \tilde{w}sc_{11} & \tilde{w}sc_{12} & \ldots & \tilde{w}sc_{1K} \\ wsc_{21} & \tilde{w}sc_{22} & \ldots & \tilde{w}sc_{2K} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{w}sc_{g1} & \tilde{w}sc_{g2} & \ldots & \tilde{w}sc_{gK} \\ \\ \tilde{w}sc_{(g+1)1} & \tilde{w}sc_{(g+1)2} & \ldots & \tilde{w}sc_{(g+1)K} \\ \tilde{w}sc_{(g+2)1} & \tilde{w}sc_{(g+2)2} & \ldots & \tilde{w}sc_{(g+2)K} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{w}sc_{h1} & \tilde{w}sc_{h2} & \ldots & \tilde{w}sc_{hK} \\ \\ \tilde{w}sc_{(h+1)1} & \tilde{w}sc_{(h+1)2} & \ldots & \tilde{w}sc_{(h+1)K} \\ \tilde{w}sc_{(h+2)1} & \tilde{w}sc_{(h+2)2} & \ldots & \tilde{w}sc_{(h+2)K} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{w}sc_{n1} & \tilde{w}sc_{n2} & \ldots & \tilde{w}sc_{nK} \end{bmatrix} \begin{matrix} \Rightarrow \\ \Rightarrow \\ \vdots \\ \Rightarrow \\ \\ \\ \Rightarrow \\ \Rightarrow \\ \vdots \\ \Rightarrow \\ \\ \Rightarrow \\ \Rightarrow \\ \vdots \\ \Rightarrow \end{matrix} \begin{bmatrix} \tilde{W}SC_{11} \\ \tilde{W}SC_{21} \\ \vdots \\ \tilde{W}SC_{g1} \\ \\ \tilde{W}SC_{(g+1)1} \\ \tilde{W}SC_{(g+2)1} \\ \vdots \\ \tilde{W}SC_{h1} \\ \\ \tilde{W}SC_{(h+1)1} \\ \tilde{W}SC_{(h+2)1} \\ \vdots \\ \tilde{W}SC_{(n)1} \end{bmatrix} = WSC \quad (13)$$

where, the decision maker preferences on sub-criteria weights in *wsc* matrix are aggregated into *WSC* for each sub-criterion.

To calculate the input variables $(x_{1,}x_2, \ldots x_{g-1}, x_g, \ldots, x_{h-1}, x_h, \ldots, x_{n-1}, x_n)$ for the proposed method, the fuzzy aggregated supplier performances (11) are multiplied by the fuzzy aggregated importance weight of each criterion (13) as shown in (14).

$$\tilde{x}_{n\times 1} = \begin{bmatrix} \tilde{W}SC_{11} * \tilde{R}_{11} \\ \tilde{W}SC_{21} * \tilde{R}_{21} \\ \vdots \\ \tilde{W}SC_{g1} * \tilde{R}_{g1} \\ \tilde{W}SC_{(g+1)1} * \tilde{R}_{(g+1)1} \\ \tilde{W}SC_{(g+2)1} * \tilde{R}_{(g+2)1} \\ \vdots \\ \tilde{W}SC_{h1} * \tilde{R}_{h1} \\ \\ \tilde{W}SC_{(h+1)1} * \tilde{R}_{(h+1)1} \\ \tilde{W}SC_{(h+2)1} * \tilde{R}_{(h+2)1} \\ \vdots \\ \tilde{W}SC_{(n)1} * \tilde{R}_{n1} \end{bmatrix} \quad (14)$$

Table 4 The linguistic terms for supplier's performance with respect to sub-criteria of the FIS-based method in the first and second stages

Linguistic variables	Corresponding triangular fuzzy number
Weakly preferred (WP)	(0.00, 1.67, 3.34)
Low moderately preferred (LMP)	(1.67,3.34, 5.00)
Moderately preferred (MP)	(3.34, 5.00, 6.67)
Strongly preferred (SP)	(5.00, 6.67, 8.34)
Extremely preferred (EP)	(6.67,8.34, 10.0)

Table 5 The linguistic terms for supplier's performance with respect to sub-criteria of the FIS-based method in the third stage

Linguistic variables	Corresponding triangular fuzzy number
Very weakly preferred (VWP)	(0.00, 12.5, 25)
Weakly preferred (WP)	(12.5, 25, 37.5)
Low moderately preferred (LMP)	(25, 37.5, 50.0)
Moderately preferred (MP)	(37.5, 50.0, 62.5)
High moderately preferred (HMP)	(50.0, 62.5, 75.0)
Strongly preferred (SP)	(62.5, 75.0, 87.5)
Extremely preferred (EP)	(75.0, 87.5, 100.0)

where, $\tilde{x}_{n\times 1}$ shows the prepared inputs for the proposed method which obtained from the multiplication of supplier's performances with *WSC* matrix. Then, the obtained fuzzy numbers are defuzzified to the desired crisp numbers as input variables for the FIS systems in the first stage.

The first stage is continued and the FIS systems are applied until the number of FIS systems' outputs for economic group is equal to two and for both environmental and social groups equal to one (see Fig. 2). So, four inputs including the two outputs of economic group, the one output of environmental group, and the one output of social group are considered for two FIS systems in the second stage. In the first and second stages, five linguistic variables are utilized to show the decision makers' preferences for the supplier's performance with respect to criteria as shown in Table 4. But, in the third stage, seven membership functions are applied to show the decision makers' preferences as shown in Table 5.

This methodology (Fig. 1) must be repeated for each candidate supplier to obtain its ranking. All of the aforementioned processes are done by applying MATLAB programming software.

6 Case Study

To show the feasibility of the proposed method, a real-life supplier selection decision from palm oil industry is solved by it. The necessary data are collected from a reputed palm oil industry in Malaysia. There are three decision makers in

Table 6 Decision makers' opinions on criteria weights

Criteria	Decision makers		
	DM1	DM2	DM3
Cost (C)	EI	EI(4/6, 5/6, 1)	SI(3/6, 4/6, 5/6)
Quality (Q)	EI(4/6, 5/6, 1)	SI(3/6, 4/6, 5/6)	EI(4/6, 5/6, 1)
Delivery (D)	SI(3/6, 4/6, 5/6)	EI(4/6, 5/6, 1)	SI(3/6, 4/6, 5/6)
Technology capability (TC)	SI(3/6, 4/6, 5/6)	MI(2/6, 3/6, 4/6)	WI(0, 1/6, 2/6)
Biodiversity (B)	SI(3/6, 4/6, 5/6)	EI(4/6, 5/6, 1)	SI(3/6, 4/6, 5/6)
Waste management (WM)	EI(4/6, 5/6, 1)	SI(3/6, 4/6, 5/6)	WI(0, 1/6, 2/6)
Humane capital (HC)	SI(3/6, 4/6, 5/6)	WI(0, 1/6, 2/6)	EI(4/6, 5/6, 1)
Social responsibility (SR)	MI(2/6, 3/6, 4/6)	SI(3/6, 4/6, 5/6)	MI(2/6, 3/6, 4/6)

Table 7 Decision makers' opinions with respect to criteria for candidate suppliers

Criteria	Suppliers					
		A	B	C	D	E
C	DM1:	EP	MP	MP	SP	EP
	DM2:	EP	MP	MP	MP	SP
	DM3:	EP	MP	MP	EP	SP
Q	DM1:	EP	MP	MP	WP	SP
	DM2:	EP	MP	MP	MP	LMP
	DM3:	EP	MP	MP	LMP	MP
D	DM1:	EP	MP	MP	MP	SP
	DM2:	EP	MP	MP	EP	EP
	DM3:	EP	MP	MP	SP	SP
TC	DM1:	EP	MP	MP	LMP	MP
	DM2:	EP	MP	MP	LMP	MP
	DM3:	EP	MP	MP	LMP	MP
B	DM1:	MP	EP	MP	EP	LMP
	DM2:	MP	EP	MP	SP	WP
	DM3:	MP	EP	MP	SP	MP
WM	DM1:	MP	EP	MP	MP	LMP
	DM2:	MP	EP	MP	EP	LMP
	DM3:	MP	EP	MP	SP	LMP
HC	DM1:	MP	MP	EP	WP	SP
	DM2:	MP	MP	EP	WP	SP
	DM3:	MP	MP	EP	WP	SP
SR	DM1:	MP	MP	EP	MP	EP
	DM2:	MP	MP	EP	WP	SP
	DM3:	MP	MP	EP	LMP	EP

the company's procurement team and five suppliers as candidates. Malaysian palm oil manufacturing is composed of related parts which produce different kinds of products for their customers. To derive the vital criteria for supplier selection process, some meetings were adjusted to have face to face interviews with experts and staffs of procure activities in palm oil industry. Therefore, eight criteria in

Table 8 Validation and ranking of the final model

		Defuzzification methods					Ranking
		COA	MOM	SOM	LOM	BOA	
Suppliers	A	80.3942	75.9001	88.4765	81.7083	78.6044	1
	B	70.428	67.801	82.7952	76.7285	72.6363	2
	C	35.8409	28.3939	55.648	20.4949	34.8666	5
	D	45.5439	35	80.356	29.5617	44.1438	4
	E	66.2524	59.7556	81.4765	73.4905	65.5586	3

three sustainable groups (economic, environmental, and social) including "cost/price, quality, delivery, technology capability" (economic group), "biodiversity and waste management" (environmental group) and "human capital and social responsibility" (social group) were considered to select the suppliers. The importance weights of criteria and the suppliers' performance with respect to these criteria based on purchasing managers' perceptions must be deducted using the linguistic terms as mentioned before. This information is presented in Tables 6 and 7. It is worthy to say that the mentioned information must be averaged among three decision makers according to Eq. (11). So, the aggregate of criteria weights (12) multiplies to the aggregate of suppliers' performance (13) as inputs passing into the FIS engines to have the ranking of each supplier. The proposed method has been exerted for five suppliers and the ranking results are obtained as shown in Table 8. The order of five suppliers according to COA method is A, B, E, D, and C. To show the validity of the proposed method other defuzzification methods have been applied (Ordoobadi 2009) such as BOA, MOM, SOM, and LOM. As can be seen from Table 8. The obtained ranking results for all of the suppliers are same in different defuzzification methods and this can show the robustness of the proposed method.

7 Conclusion

This chapter introduces a fuzzy ranking method for supplier selection process in enterprises. The main conclusions of this chapter are as follows:

- A new supplier selection method developed by incorporating the sustainability issues in the selection process for manufacturing firms and service industries where the sustainability in terms of economic, environmental, and social aspects are their significant concern.
- Very often, the same relative importance of criteria is considered in the supplier selection process. But in practice, based on decision makers' preference it needs to be different from one criterion to another. Additionally, decision makers express their assessments in linguistic term instead of pure numbers normally. The proposed modular FIS method does not require the exact information from

the decision makers and the subjectivity of their opinions is kept by applying fuzzy logic.
- Expanding the number of criteria and suppliers complicates the selection issue. The proposed method utilizes the Matlab programming as computational software with high performance which provides a robust model to solve multi-criteria decision making problems with any number of suppliers and criteria in large companies.

Although many attempts have been made for the supplier selection, considering sustainable issue for this problem remains a challenge. In addition, how to assign orders to the best suppliers in the proposed method can be a subject for future research.

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Multi Criteria Supplier Selection Using Fuzzy PROMETHEE Method

Ozlem Senvar, Gülfem Tuzkaya and Cengiz Kahraman

Abstract The fundamental objective of supply chain management (SCM) is to integrate various suppliers to satisfy market demand. Supplier evaluation and selection is very important for establishing an effective supply chain. As a matter of fact, supplier selection consists of both qualitative and quantitative criteria, so it is considered as a multi criteria decision making (MCDM) problem. Under incomplete or uncertain information, the fuzzy set theory allows us make decisions with approximate reasoning. In order to overcome the uncertainty which is constituted by vague situations in supplier selection, we utilize the "extension of the PROMETHEE method in a fuzzy environment" (F-PROMETHEE). In this chapter, multi criteria supplier selection based on a fuzzy PROMETHEE method with an application to supplier selection decision problem is conducted. The main advantages of the methodology are the user friendliness coming from the linguistic evaluations, and the consideration of the vagueness or fuzziness inherent to the decision making environment. Hence, the method can be an efficient and effective methodology to be used by decision makers on supply chains. The proposed methodology can also be applied to any other selection problem.

Keywords Supply chain management (SCM) · Supplier selection · Multiple criteria decision making (MCDM) · Preference ranking organization METHod for enrichment evaluations (PROMETHEE) · Fuzzy logic

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1 Introduction

Variation in demands for production enforces outsourcing of activities. Primary problem in supply chain is control and coordinate activities (Nazeri et al. 2011). Supply Chain Management (SCM) is a process of organizing the activities from the customer's order through final delivery for speed, efficiency, and quality (Meredith 2007). SCM has an increasing importance in today's competitive business world. Companies need to have strong relationships and integrations with their suppliers for a successful SCM system. They should establish appropriate relationships with their suppliers in order to achieve their strategic goals. Therefore, supplier selection is a fundamental step of supply chain management.

Supplier evaluation process allows the selection of suitable suppliers in order to develop a supply relationship system that can rapidly react to requirements of market and to innovation dynamics (Esposito and Passaro 2009a, 2009b). Choosing an appropriate supplier considerably reduces cost, causes to competitive advantage and increases the level of customer satisfaction (Nazeri et al. 2011). Moreover, supplier selection has strategic importance in global competition for companies.

The supplier selection problem consists of the definition of models and methods to analyze and measure the performance of a set of suppliers, which are also known as vendors, in order to improve competitiveness. Supplier selection problem is a multiple criteria decision making (MCDM) problem typically having conflicting criteria that include both qualitative and quantitative measures.

Bruno et al. (2012) provided the perspective analysis of the articles about the supplier selection problem with respect to the geographic origin. Considering the country where the institution of the first author is based, they realized that USA is the main contributor to the literature with 49 articles, followed by Taiwan with 36 articles, Turkey with 27 articles, China with 21 articles, India with 16 articles, and Iran with 14 articles. This evidence testifies that the supplier selection problem is a relevant issue involving academics and practitioners of several countries, more specifically the Asian ones, where manufacturing is the prominent economic activity and/or is based on the attraction of investment by large foreign companies.

Ha and Krishnan (2008) classified researches as single models and combined models. Single models cover Mathematics, Statistics, and Artificial Intelligence.

- *Mathematics* includes multi criteria decision making methods such as analytic hierarchy process (AHP), analytic network process (ANP), analytic target cascading (ATC), game theory, data envelopment analysis (DEA), costing, and grey maths.
- *Statistics* includes process capability index (PCI), factor analysis, multivariate statistics, bootstrap, data mining, structural equations, loss functions, survey, and decision trees.
- *Artificial Intelligence* includes fuzzy set theory, simulation, expert systems, case based reasoning (CBR), vector machines, and neural networks.

Combined models cover Mathematics combined models, Artificial Intelligence combined models, Hybrid combined models.

- *Mathematics* combined models include AHP-ANP-Optimization, AHP-ANP + DEA, AHP-ANP + Grey Maths, DEA + Optimization, and DEA + Costing.
- *Artificial Intelligence* combined models include case based reasoning (CBR) + neural networks.
- *Hybrid combined models* include AHP-ANP + fuzzy set theory, AHP-ANP + simulation, AHP-ANP + loss function, AHP-ANP + quality function, costing + fuzzy set theory, DEA + neural networks, neural networks + optimization, fuzzy set theory + cluster analysis, fuzzy set theory + optimization, and simulation + optimization.

Up to now, there are many other investigations and many other publications about supplier selection have been issued. The contribution of this chapter is to utilize a method for multi criteria supplier selection problem based on fuzzy PROMETHEE which overcomes the uncertainty constituted by vague situations. Therefore, in this chapter, the "extension of the PROMETHEE method in a fuzzy environment" (F-PROMETHEE) is used for supplier selection problem. The main advantages of the methodology are user friendliness coming from the linguistic evaluations, and the consideration of the vagueness or fuzziness inherent to the decision making environment. Additionally, the utilized technique, which is known as a fuzzy version of well-known PROMETHEE outranking methodology, sustains the advantages of PROMETHEE. One of the main advantages of PROMETHEE is the simplicity of its methodology in comparison to the other outranking techniques. This is the main reason why this technique is applied to various real life problems previously. Also, PROMETHEE provides the opportunity of selection the types of preference functions. This characteristic is unique to the PROMETHEE approach and gives the opportunity of obtaining more realistic definition for the decision criteria. Hence, the method can be an efficient and effective methodology to be used by decision makers on supply chains. Although it is not very common, some versions of fuzzy PROMETHEE are applied to the supplier selection problem previously. Chen et al. (2011) used fuzzy PROMETHEE for the outsourcing decisions of Information Systems. Shirinfar and Haleh (2011) used fuzzy PROMETHEE for the supplier selection and evaluation problem. Gupta et al. (2012) used fuzzy PROMETHEE to select logistics service providers for cement industry. Tavakoli et al. (2013) applied fuzzy PROMETHEE in an fuzzy Goal Programming integrated methodology to evaluate and select suppliers. In this study, we used SCOR Level 1 performance metrics as evaluation criteria for suppliers and applied the proposed methodology to a hypothetical example.

To the best of our knowledge, in the literature, other studies using F-PROMETHEE approach can be summarized as follows. Goumas and Lygerou (2000), Bilsel et al. (2006), Geldermann et al.(2000), Chou et al. (2007), Tuzkaya et al. (2010), and Ozgen et al. (2011) have used F-PROMETHEE previously.

The organization of the rest of this chapter is as follows. Section 2 provides a brief literature review for supplier selection problem. Section 3 presents background information of PROMETHEE method and Sect. 4 gives brief information on fuzzy PROMETHEE approach. Section 5 is the application section and in the final section some concluding remarks and future research directions are given.

2 Supplier Selection Problem

Due to strategic importance of supplier selection process, extensive research has been done on supplier evaluation and selection. Particularly, more recent researches reveal that the interest devoted to this topic is increasing. In this section, a brief literature review about supplier selection problem is provided.

According to Nazeri et al. (2011) supplier selection is one of the most significant processes of product and service management for many enterprises within supply chain. Especially, in manufacturing companies the raw materials and component parts can equal up to 70 % of the product cost. In such circumstances the purchasing unit can affect in cost reduction. Supplier evaluation is one of the most fundamental issues of purchasing management. They also emphasize that the process of supplier selection and evaluation is MCDM, that is, in supplier selection many criteria may be considered during this process. Therefore, supplier selection and evaluation is a MCDM problem which includes both tangible and intangible criteria, some of which may conflict. Fundamentally, supplier selection and evaluation can be divided into two categories, which are single sourcing and multiple sourcing. In single sourcing, there are constraints, which are not considered in the supplier selection process. In other words, all suppliers can satisfy the buyer's requirements of demand, quality, delivery, and etc. The buyer only needs to make one decision, which supplier is the best. On the other hand, in multiple sourcing, there are some limitations such as supplier's capacity, quality, and delivery, which are considered in the supplier selection process. In other words, no supplier can fulfill the buyer's total requirements and the buyer needs to purchase some part of demand from one supplier and the other part of the demand from another supplier to compensate for the shortage of capacity or low quality of the first supplier. In these circumstances, buyers need to make two decisions: which suppliers are the best, and how much should be purchased from each selected supplier?

Traditional supplier evaluation and selection methods focus on the requirements of single enterprises, and fail to consider the entire supply chain. Managing the links between the suppliers and customers successfully in a supply chain necessitates their active collaboration. As a result, companies prefer to work closely with a few suppliers or dependable one supplier in order to achieve and maintain high supply chain performance. Due to strategic importance of supplier evaluation and selection process, extensive research is being done to cope with this MCDM problem. In recent years there has been a great focus on the mathematical side of the supplier selection problem. Mathematical methodologies trying to answer to the complexity of the problem, intrinsically multi-attributed.

Agarwal et al. (2011) review sixty-eight articles from 2000 to 2011 to find out the most prominent MCDM methodology followed by the researchers for supplier evaluation and selection. They report the distribution of MCDM methods used in these articles as follows: Data Envelopment Analysis (DEA): 30 %; mathematical programming models: 17 %; Analytic Hierarchy Process (AHP): 15 %; Case Based Reasoning (CBR): 11 %; Analytic Network Process (ANP): 5 %; Fuzzy Set Theory: 10 %; Simple Multi-Attribute Rating Technique (SMART): 3 %; Genetic Algorithm (GA): 2 %; and Criteria Based Decision Making Methods such as ELECTRE (ELimination Et Choix Traduisant la REalité-Elimination and choice expressing reality) and PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations): 7 %.

De Boer et al. (2001) present a review of decision methods reported in the literature for supporting the supplier selection process. They define pre-qualification as the process of reducing the set of all suppliers to a smaller set of acceptable suppliers and present categorical methods, DEA, cluster analysis, and CBR systems as the decision methods for pre-qualification of suitable suppliers. They present linear weighting models, total cost of ownership models, mathematical programming models, statistical models, and artificial intelligence-based models as the decision models for making a final choice among suitable suppliers.

Ha and Krishnan (2008) provide a classification of the employed approaches for dealing with the supplier selection problem. They also show price, quality and delivery are the three most used attributes.

Based on a literature review of 78 journal articles from 2000 to 2008 on MCDM approaches for supplier evaluation and selection, Ho et al. (2010) conclude that the most prevalent individual approach is DEA, whereas the most popular integrated approach is AHP−GP (Goal Programming); the integrated AHP approaches with other techniques include bi-negotiation, DEA, DEA and artificial neural network, GP, grey relational analysis, mixed integer non-linear programming, multi-objective programming, and fuzzy set theory. They also conclude that the most popular criterion used for evaluating the performance of suppliers is quality, followed by delivery, price/cost, manufacturing capability, service, management, technology, research and development, finance, flexibility, reputation, relationship, risk, and safety and environment.

Chen (2011) summarizes important criteria for supplier selection from the literature as price, delivery, quality, equipment and capability, geographic location, technical capability, management and organization, industrial reputation, financial situation, historical performance, maintenance service, service attitude, packing ability, production control ability, training ability, procedure legality, employment relations, communication system, mutual negotiation, previous image, business relations, previous sales, guarantee and compensation. Chen (2011) uses DEA technique to screen potential suppliers and then TOPSIS method to rank the candidate suppliers.

For supplier selection problem various researchers have studied different MCDM approaches. AHP is one of the most prominent methodologies used to address the supplier selection problem (Saaty 1980, 1994).

Although, AHP is widely used in many MCDM problems, in the conventional AHP there are some shortcomings (Ayag and Ozdemir 2006a, b);

1. the AHP method is mainly used in nearly crisp decision applications,
2. the AHP method creates and deals with a very unbalanced scale of judgement,
3. the AHP method does not take into account the uncertainty associated with the mapping of one's judgment to a number,
4. ranking of the AHP method is rather imprecise,
5. the subjective judgment, selection and preference of decision-makers have great influence on the AHP results.

In real life applications, human assessment on the relative importance of individual customer requirements is always subjective and imprecise. The linguistic terms that people use to express their feelings or judgments are generally vague. Even though the scale has the advantages of simplicity and ease of use, it does not take into account the uncertainty associated with the mapping of one's perception (or judgment) to a number (Büyüközkan et al. 2004).

Based on an extensive literature survey, the most widely preferred methodology is the combination of AHP with other methodologies, that is, different integrated AHP approaches are observed to be the most widely used. Bruno et al. (2012) conclude that AHP-based models are useful in constructing structured and formalized approaches for supplier evaluation and can be used in combination with many other approaches. For instance, AHP, and its network-based counterpart, ANP (Saaty 1980) are found to be the most utilized methods. The use of AHP/ANP with fuzzy set theory is widely accepted for dealing with qualitative evaluation attributes.

Chen et al. (2006) use the fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method for supplier selection problem.

In this study, the F-PROMETHEE technique is preferred because of the fuzzy nature of the supplier selection decision problem. In the next consecutive sections, PROMETHEE and F-PROMETHEE are explained.

3 PROMETHEE (Preference Ranking Organization METHod for Enrichment Evaluations)

PROMETHEE is the abbreviation of Preference Ranking Organization METHod for Enrichment Evaluations, which is an outranking method that initial references are prepared by Brans et al. (1984, 1986); Brans and Vincle (1985).

In PROMETHEE method, different preference functions can be defined for criteria (Dagdeviren 2008). It is a ranking method which is quite simple in conception and application compared to other methods for MCDM. It is well adapted to the problems where a finite set of alternatives are to be ranked according to several, sometimes conflicting criteria (Bilsel et al. 2006; Albadvi et al. 2007; Tuzkaya et al. 2010).

Ulengin et al. (2001) listed the advantages of PROMETHEE as follows:

1. PROMETHEE is a user friendly outranking method,
2. It has been successfully applied to real life planning problems
3. Both PROMETHEE I and PROMETHEE II allow both partial and total ranking of the alternatives while still satisfying simplicity.

The evaluation is the starting point of PROMETHEE method. In this phase, alternatives are evaluated with respect to different criteria. These evaluations involve essentially numerical data. Macharis et al. (2004) stated that the implementation of PROMETHEE requires two additional types of information, which are as follows:

- Information on the relative importance (i.e. the weights) of the criteria considered,
- Information on the decision-makers' preference function, which he/she uses when comparing the contribution of the alternatives in terms of each separate criterion.

The basic steps of the PROMETHEE algorithm can be outlined as follows (Geldermann et al. 2000; Brans et al. 1986):

Step 1. Spesify a generalized preference function $p_j(d)$ for each criterion j.

Step 2. Define a vector containing the weights, which are a measure for the relative importance of each criterion, $w_T = [w_1,\ldots,w_k]$. If all the criteria are of the same importance in the opinion of the decision maker, all weights can be taken as being equal. The normalization of the weights, $\sum_{k=1}^{K} w_k = 1$, is not necessarily required.

Step 3. Define for all the alternatives $a_t,\ a_{t'}\ \in A$ the outranking relation π:

$$\pi : \begin{cases} A \times A \to [0,\ 1] \\ \pi(a_t, a_{t'}) = \sum_{k=1}^{K} w_k\ .(p_k(f_k(a_t) - f_k(a_{t'})) \end{cases} \tag{1}$$

The preference index $\pi(a_t, a_{t'})$ is a measure for the intensity of preference of the decision maker for an alternative at in comparison with an alternative $a_{t'}$ for the simultaneous consideration of all criteria. It is basically a weighted average of the preference functions $p_k(d)$ and can be represented as a valued outranking graph.

Step 4. As a measure for the strength of alternatives $a_t \in A$, the leaving flow is calculated:

$$\Phi^+(a_t) = \frac{1}{T-1} \cdot \sum_{\substack{t'=1 \\ t' \neq t}}^{n} \pi(a_t,\ a_{t'}) \tag{2}$$

The leaving flow is the sum of the values of the arcs which leave node at and therefore yields a measure of the “outranking character” of at.

Step 5. As a measure for the weakness of the alternatives $a_t \in A$, the entering flow is calculated, measuring the “outranked character” of at (analogously to the leaving flow):

$$\Phi^{-}(a_t) = \frac{1}{T-1} \cdot \sum_{\substack{t'=1 \\ t' \neq t}}^{n} \pi(a_{t'}, a_t) \tag{3}$$

Step 6. A graphical evaluation of the outranking relation is derived: Basically, the higher the leaving flow and the lower the entering flow, the better the action. This result is graphically represented by a partial preorder (PROMETHEE I) or a complete preorder (PROMETHEE II).

In PROMETHEE I, alternative at is preferred to alternative at’ (atPat’) at least one of the elements of Eq. (4) is satisfied (Dagdeviren 2008):

$$\begin{aligned} a_t\, P\, a_{t'} \;\; if: \;\; & \Phi^{+}(a_t) > \Phi^{+}(a_{t'}) \; and \; \Phi^{-}(a_t) < \Phi^{-}(a_{t'}) \; or \\ & \Phi^{+}(a_t) > \Phi^{+}(a_{t'}) \; and \; \Phi^{-}(a_t) = \Phi^{-}(a_{t'}) \; or \\ & \Phi^{+}(a_t) = \Phi^{+}(a_{t'}) \; and \; \Phi^{-}(a_t) < \Phi^{-}(a_{t'}) \end{aligned} \tag{4}$$

PROMETHEE I evaluation allows indifference and incomparability situations. Therefore sometimes partial rankings can be obtained. In the indifference situation ($a_t I a_{t'}$), two alternatives a_t and $a_{t'}$ have the same leaving and entering flows (Dagdeviren 2008):

$$a_t\, I\, a_{t'} \;\; if: \;\; \Phi^{+}(a_t) = \Phi^{+}(a_{t'}) \; and \; \Phi^{-}(a_t) = \Phi^{-}(a_{t'}) \tag{5}$$

Two alternatives are considered incomparable, $a_t R a_{t'}$, if alternative at is better than alternative $a_{t'}$ in terms of leaving flow, while the entering flows indicate the reverse (Dagdeviren 2008):

$$\begin{aligned} a_t\, R\, a_{t'} \;\; if: \;\; & \Phi^{+}(a_t) > \Phi^{+}(a_{t'}) \; and \; \Phi^{-}(a_t) > \Phi^{-}(a_{t'}) \; or \\ & \Phi^{+}(a_t) < \Phi^{+}(a_{t'}) \; and \; \Phi^{-}(a_t) < \Phi^{-}(a_{t'}) \end{aligned} \tag{6}$$

Via PROMETHE II, the complete ranking can be obtained. For the complete ranking calculations, net flow values of alternatives can be calculated as Eq. (7). Here, if alternative a_t’s net flow is bigger that alternative $a_{t'}$’s net flow, this indicates that, alternative at outranks alternative at’.

$$\Phi^{net}(a_t) = \Phi^{+}(a_t) - \Phi^{-}(a_t) \tag{7}$$

Note that, the preference function types mentioned in Step 1 is not given in that study. Details of them can be seen in Tuzkaya et al. (2010) and Ozgen et al. (2011).

4 Fuzzy PROMETHEE (F-PROMETHEE)

In the literature, there are a few studies with respect to the fuzzy PROMETHEE (F-PROMETHEE) approach. Goumas and Lygerou (2000), Bilsel et al. (2006), Geldermann et al.(2000), Chou et al. (2007), Tuzkaya et al. (2010), and Ozgen et al. (2011) have used F-PROMETHEE previously.

In the F-PROMETHEE, the main problem arises in comparing two fuzzy numbers and the index, which corresponds to a weighted average of the fuzzy numbers, proposed from Yager (1981) is found a useful way to compare fuzzy numbers. It is determined by the center of weight of the surface representing its membership function (Goumas and Lygerou 2000; Bilsel et al. 2006). Based on the Yager's index, a triangular fuzzy number's magnitude is the value corresponding to the center of the triangle and can be expressed as in Eq. (8). The representation of a TFN here, $\tilde{F} = (n, a, b)$, is a different version of the representation used in Fig. 1 and Table 1. This is equivalent to the previous representation by $\tilde{F} = (n - a, n, n + b)$. The following fuzzy PROMETHEE formulas are based on the representation of TFN as (n, a, b).

$$\tilde{F} = (n - a, n, n + b) = (3n - a + b)/3 \tag{8}$$

In this study, PROMETHEE's linear preference function with indifference and strict preference is preferred for each criterion by (Decision Maker Team) DMT. In this preference function, two thresholds, q and p are needed to be determined. When using the fuzzy numbers in PROMETHEE, the evaluation function can be converted to Eq. (9). As mentioned in the Sect. 3, details of preference functions can be seen in Tuzkaya et al. (2010) and Ozgen et al. (2011).

$$P_j(a_t, a_{t'}) = \begin{cases} 0, & if\ n - a \leq q\ (indifference) \\ \frac{(n,a,b)-q}{p-q}, & if\ \ q \leq (n - a)\ and\ (n + b) \leq p \\ 1, & if\ n + b > p\ (strict\ preference) \end{cases} \tag{9}$$

In Eq. (9), q and p values are crisp numbers and the membership functions of the fuzzy number, $C(a_t, a_{t'}) = (n,a,b)$, is adjusted accordingly so that $n - a >= 0$ and $n + b <= 1$. In the if-statement in Eq. (9), *(n,a,b)* is a TFN which represents the differences between a_t and $a_{t'}$. The magnitude of *(n,a,b)* is calculated by using Yager Index (Eq. 8).

Similarly to the PROMETHEE approach, the leaving flow, the entering flow and the net flow notions are valid in the case of F-PROMETHEE (Bilsel et al. 2006). Outside of the abovementioned differences, F-PROMETHEE utilizes from the PROMETHEE's application steps.

In the F-PROMETHEE phase, the DMT is asked to evaluate alternatives considering each criterion. For this evaluation stage, the used linguistic scale for relative importance is given in Fig. 1 and the definitions are given in Table 1.

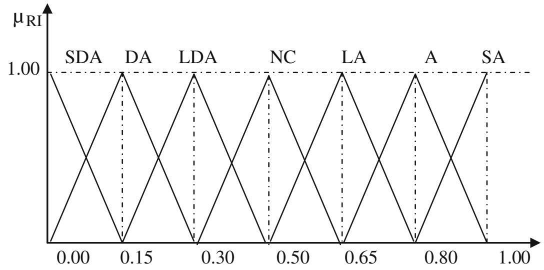

Fig. 1 Linguistic scale for evaluation (Bilsel et al. 2006)

Table 1 Linguistic scale for importance (Bilsel et al. 2006)

Linguistic scale for evaluation	Triangular fuzzy scale
Strongly disagree (SDA)	(0, 0, 0.15)
Disagree (DA)	(0, 0.15, 0.30)
Little disagree (LDA)	(0.15, 0.30, 0.50)
No comment (NC)	(0.30, 0.50, 0.65)
Little agree (LA)	(0.50, 0.65, 0.80)
Agree (A)	(0.65, 0.80, 1)
Strongly agree (SA)	(0.80, 1, 1)

5 An Application

In this study, a hypothetical example for supplier evaluation problem is performed. SCOR Level 1 performance metrics are utilized as the evaluation criteria (Supply Chain Operations Reference (SCOR) Model, Overview-Version 10.0 2013) which is presented as follows: *Reliability* (C_1), *Responsiveness* (C_2), *Agility* (C_3), *Costs* (C_4), *Assets* (C_5). For this application, a weight of each criterion is assumed to be equal and 0.20 each. Also, types of the criteria are determined as level criteria type. The values of q and p are determines as 0 and 0.6, respectively.

Four suppliers (S_1, S_2, S_3, S_4) are evaluated using the determined evaluation criteria. Table 2 shows the supplier evaluations for each criterion. For the evaluation process, linguistic preferences given in Table 1 is used.

Then the linguistic supplier evaluations are converted to triangular fuzzy numbers using the scale given in Table 1 and presented as in Table 3.

Then using the criteria evaluations in Table 3, respective Yager Index values are calculated with Eq. (8). Then Eqs. (4–6) are used to obtain preference, strict preference and indifference relations between each pair of suppliers and Table 4 is obtained with the respective positive, negative and net flow values of suppliers. Net flow values are calculated using Eq. (7).

Since “positive flow value of S_2 is greater than positive flow value of S_1” and “negative flow value of S_2 is smaller than the negative flow value of S_1”, it can be concluded that S_2 outranks S_1. Similar, analyses are realized for all other suppliers. As a result, considering the PROMETHEE I outranking conditions, S_3 outranks S_2

Table 2 Supplier evaluation results for SCOR level 1 performance metrics

	C_1	C_2	C_3	C_4	C_5
S_1	DA	SDA	LDA	DA	NC
S_2	LA	NC	LDA	LDA	NC
S_3	NC	LA	NC	A	A
S_4	SA	SA	LA	A	LA

Table 3 Supplier evaluations using triangular fuzzy numbers

	C_1			C_2			C_3		
S_1	0.00	0.15	0.30	0.00	0.00	0.15	0.15	0.30	0.50
S_2	0.50	0.65	0.80	0.30	0.50	0.65	0.15	0.30	0.50
S_3	0.30	0.50	0.65	0.50	0.65	0.80	0.30	0.50	0.65
S_4	0.80	1.00	1.00	0.80	1.00	1.00	0.50	0.65	0.80
	C_4			C_5					
S_1	0.00	0.15	0.30	0.30	0.50	0.65			
S_2	0.15	0.30	0.50	0.30	0.50	0.65			
S_3	0.65	0.80	1.00	0.65	0.80	1.00			
S_4	0.65	0.80	1.00	0.50	0.65	0.80			

Table 4 Negative, positive and net flow values of suppliers

	S_1	S_2	S_3	S_4	Q^+	Q^{net}
S_1	0.00	0.00	0.00	0.00	0.00	−1.60
S_2	0.40	0.00	0.00	0.00	0.40	−0.20
S_3	0.60	0.20	0.00	0.00	0.80	0.40
S_4	0.60	0.40	0.40	0.00	1.40	1.40
$Q-$	1.60	0.60	0.40	0.00		

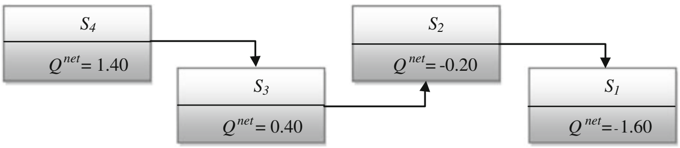

Fig. 2 PROMETHEE II complete ranking results

and S_1, S_4 outranks S_3, S_2 and S_1. There are no indifference relations between any pair of suppliers. PROMETHEE II calculations give same result as can be expected. Net flow values of suppliers shows that S_4 outranks S_3, S_3 outranks S_2, S_2 outranks S_1. Figure 2 illustrates the results of PROMETHEE II which gives the complete rankings.

6 Conclusion

In order to enhance quality and competitiveness levels, outsourcing is inevitable. Selection of the appropriate suppliers is a critical success factor for any outsourcing decision. Traditional supplier evaluation and selection methods focus on the requirements of single enterprises, and fail to consider the entire supply chain. Managing the links between the suppliers and customers successfully in a supply chain necessitates their active collaboration. As a result, companies prefer to work closely with a few suppliers or dependable one supplier in order to achieve and maintain high supply chain performance. Due to strategic importance of supplier evaluation and selection process, extensive research has been made to cope with this MCDM problem.

This study uses a fuzzy PROMETHEE method for a supplier selection problem. The objective is to select the most suitable supplier. The main advantages of the methodology are the user friendliness coming from the linguistic evaluations, and the consideration of the vagueness or fuzziness inherent to the decision making environment. Hence, the method can be an efficient and effective methodology to be used by decision makers on supply chains. The proposed methodology can also be applied to any other selection problem.

For the future researches the proposed methodology can also be easily implemented to other types of selection problems in the other application areas, more specifically in manufacturing and service sectors.

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Fuzzy-AHP Approach to Improve Effectiveness of Supply Chain

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Abstract The supply chain is an important element for the development of all industries. It can improve efficiency and effectiveness of product transfer and information sharing between complex hierarchies of all the tiers. Supplier selection is an important step in the supply chain design. In many existing decision models for supplier selection, only quantitative criteria are considered. However, supplier selection is a multi-objective problem containing quantitative as well as qualitative factors. It is difficult to map human perception to particular number or a ratio due to vagueness in the decision making process. Fuzzy Analytic Hierarchy Process (FAHP) therefore helps the decision makers to deal with imprecision and subjectiveness in pair-wise comparison process. This study aims to provide a systematic approach towards the application of FAHP to supplier selection problem. FAHP is applied to find the importance degree of each criterion as the measurable indices of the supplier. From an extensive analysis of the results, it is observed that selection of an appropriate supplier would result in improving

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effectiveness of supply chain. Thus, the overall rank ordering of alternatives as identified by fuzzy AHP seems reasonable and consistent with managerial preferences and judgments.

Keywords Supply chain · AHP · Fuzzy AHP · Alpha cut · Fuzzy set theory · Vendor selection

1 Introduction

The development of economy of any country is supported by growth of its manufacturing industries. Currently, the manufacturing industries are passing through a phase of very tough competition. The economic environment is becoming harsh. In order to survive, every industry has to strive to improve productivity in all spheres of activity. What is required is to devise new ways of improving manufacturing performance by optimally utilizing the resources. At present, industries tend to focus only on their core business and resort more and more to outsourcing several of their production functions than in the recent past. In turn, this practice has created larger and more complex supply chains. The successful management of these chains is one of the cornerstones for companies to stay competitive. Supply chain is a strategy which integrates marketing, planning, production, purchasing and finance etc., these functions creating a general plan for the organization, which satisfies the service policy, maintaining the lowest possible cost level due the incredible competition environment that they are exposed to.

A supply chain is a network of departments, which are involved in the manufacturing of a product from the procurement of raw materials to the distribution of the final products to the customer. Purchasing commands a significant position in most organization since purchased parts, components, and supplies typically represent 40–60 of the sales of its end products (Ballow 1999; Noorul and Hannan 2006). This means that relatively small cost reductions gained in the acquisition of materials can have a greater impact on profits than equal improvements in other cost-sales areas of the organization.

The purchasing function has gained great importance in the supply chain management due to factors such as globalization, increased value added in supply, and accelerated technological change. Purchasing involves buying the raw materials, supplies, and components for the organization. The activities associated with it include selecting and qualifying vendor, rating vendor performance, negotiating contracts, comparing price, quality and service, sourcing goods and service, timing purchases, selling terms of sale, evaluating the value received, predicting price, service, and sometimes demand changes, specifying the form in which goods are to be received, etc. The key and perhaps the most important process of the purchasing function is the efficient selection of vendors, because it brings significant savings for the organization. The objective of the vendor selection process is to reduce risk and maximize the total value for the buyer, and it involves considering a series of strategic variables. Some authors have identified several criteria for vendor selection, such as the net price, quality, delivery, historical supplier performance, capacity, flexibility, service, communication systems and geographic location (Dickson 1966; Dempsey 1978; Weber et al. 1991; Noorul and Kannan 2006; Sarode et al. 2008). These criteria are key issues in the supplier assessment process since it measures the performance of the suppliers.

The paper present a total ten criteria and seventy-one sub-criteria for evaluating the vendor selection for the automobile manufacturing industries located at the western part of India using the analytical hierarchy process (AHP) and the model is verified with the fuzzy AHP and α-cut-based method.

2 Literature Review

Timmerman (1986) proposed linear weighting models in which suppliers are rated on several criteria and in which these ratings are combined into a single score. These models include the categorical, the weighted point and the analytical hierarchical process (Nydick and Hill 1992). The major limitation of this approach is that it is difficult to effectively take qualitative evaluation criteria into consideration. Total cost approaches attempt to quantify all costs related to the selection of a vendor in monetary units by including cost ratio (Timmerman 1986) and total cost of ownership (Ellram 1995).

Petroni and Braglia (2000), discussed the principle component analysis (PCA) method which is multi-objective approach to vendor selection that attempts to provide a useful decision support system for purchasing manager faced with multiple vendors and trade-offs such as price, delivery, reliability, and product quality. The major limitation of this approach is that it requires the knowledge of advanced statistical technique.

Wei et al. (1997), discussed neural network for the supplier selection that saves a lot of time and money for system development comparing to conventional models for decision support system. The supplier-selecting system includes two functions: one is the function measuring and evaluating performance of purchasing (quality, quantity, timing, price, and costs) and storing the evaluation in a database to provide data sources to neural network. The other is the function using the neural network method saves money and time of system development.

Dickson (1966), reported 23 different criteria for vendor's evaluation. Of these criteria, the cost, quality and delivery times are among the most important performance measures in the selection of vendors approaching the vendor selection problem mainly from three perspectives; conceptual, empirical, and mathematical (Talluri and Narasimhan 2003).

Chan (2003), reported seven performance measures as the key elements of vendor selection including the cost, resource utilization, quality, flexibility, visibility, trust and innovativeness. Sarode et al. (2008) presented twelve measures which includes qualitative and quantitative type-quality, visibility, flexibility and responsiveness, resource utilization, cost, asset, technological capability, service and time to market apart from these twelve measure total fifty-eight items/variables identified. Noorul and Hannan (2006), identified seven performance measures- quality, delivery, production capability, service, engineering/technical capabilities, business structure and price and their thirty-two sub factors for the vendor selection.

Weber et al. (1991), presented a comprehensive review of the literature providing the most important criteria in the choice of suppliers. According to investigation, price is the most important factor in the selection process followed by lead time and quality factors. Patton (1996 sampled 1500 buyers to identify the effects of human judgment models on vendor selection. His findings suggest that it is not as much the difference in attributes between vendors that affect the outcome, but it is the type of human model used that lead to the variance in the selection of vendors. Stanley and Wisner (2001) collected data from 118 executives to study the outcome of previous research concepts. One of the important results of their study suggests that greater emphasis should be given to strategic activities in the process of suppliers' selection. Verma and Pullman (1998), proposed the supplier selection process using the two methods namely Likert scale set of questions and a discrete choice analysis (DCA) experiment. According to them quality is an important factor to select the most suitable suppliers. Lambert et al. (1998), described a method for evaluating and comparing several suppliers. A rating factor is assigned to each supplier followed by a weight to determine the importance of each factor. To make the comparison feasible, a weighted composite measure is developed by multiplying the rating factor by the weight. However, how to assign the weights has not been clearly described in the approach.

Mikhailov (2003), proposed a new approach for deriving priorities from fuzzy pairwise judgments based on α–cuts decomposition of the fuzzy judgments into a serious interval comparisons. Sheu (2004), proposed a methodology in the research that would stimulate research in the related fields of global logistics, and may help to address issues regarding the uncertainty and complexity of global logistics operations. Chan and Chung (2004), developed a multi-criteria genetic optimization for solving distribution network problems in supply chain management. In this work they combine analytic hierarchy processes with genetic algorithms to capture the capability of multi-criterion decision-making which will reduce the computation time. Vaidya and Kumar (2004), presented a literature review of the applications of the analytic hierarchy process (AHP) and also provided the various application area where the AHP is used as a multiple criteria decision-making tool. Handeld et al. (2002), integrated environment issues in supplier assessment decisions with the help of AHP.

Vanegas and Labib (2001), proposed a method to determine the weights from the AHP into fuzzy numbers using the concept of a "fuzzy line segment". Tam

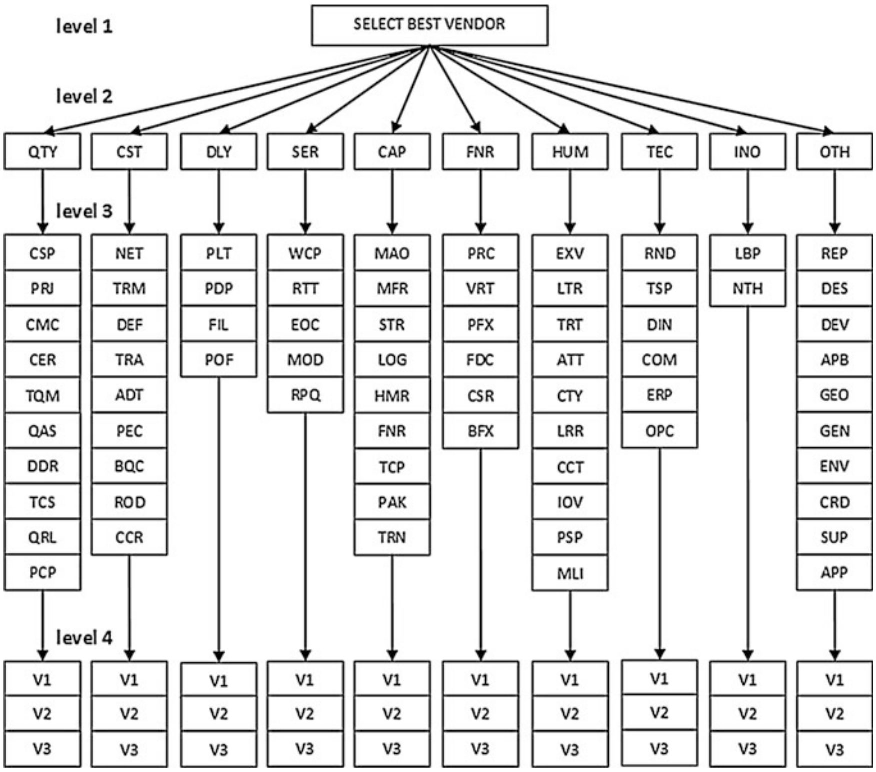


Fig. 1 Hierarchy for selection of best vendor

and Tummala (2001), discussed the vendor selection for the telecommunication systems and based on the proposed model the time taken to select the vendor has been reduced. Kahraman et al. (2004), provided comparison of catering service companies using fuzzy AHP to select best catering firm providing the highest customer satisfaction. van Laarhoven and Pedrycz (1983), presented a fuzzy method for choosing number of alternatives under conflicting decision criteria. Yang et al. (2008), proposed a vendor selection by integrating fuzzy MCDM techniques with independent and interdependent relationships using triangular fuzzy numbers to express the subjective preferences of evaluators. Lee et al. (2006) developed a web-based decision making tool that utilized fuzzy analytic hierarchy process (AHP) methodology and fuzzy set theory to solve complicated decision making problem.

Based on the above literature, most of the researchers have considered mostly four to five main factors (quality, service, price and delivery) and about 8–32 sub factors for selection of vendors. This study describes ten main factors and 71 sub factors for the vendor selection as shown in Fig. 1 and Table 1.

Table 1 Attributes and sub attributes for the vendor selection

Attributes	Abbr	Attributes	Abbr
Quality	QTY	Training resources	TRN
Conformance to your specifications	CSP	Flexibility and responsiveness	FNR
Percentage of Rejections	PRJ	Procedural compliance	PRC
Condition of machinery	CMC	Vendor response time	VRT
Certification like ISO	CER	Production flexibility	PFX
Adherence to TQM concepts	TQM	Flexibility to design changes	FDC
Quality of after sales service	QAS	Customized services	CSR
Billing flexibility	BFX	Human factors	HUM
No. of detected deficient and reworked material	DDR	Attainment of quality assurance such as certificates	TCS
Reliability in maintaining quality standards	QRL	Experience of vendors	EXV
Process capability	PCP	Long term relationship	LTR
Cost	CST	Trust	TRT
Net price of product	NET	Attitude	ATT
Terms of payment	TRM	Courtesy	CTY
Costs due to defects	DEF	Labour relations record	LRR
Transportation cost	TRA	Cultural compatibility	CCT
Additional taxes levied vendor's state	ADT	Image of vendor	IOV
Price escalation criteria	PEC	Professionalism of sales person	PSP
Bulk quantity cost	BQC	Moral/legal issues	MLI
Rate of discount	ROD	Technology	TEC
Currency conversion rates	CCR	Strong R&D	RND
Delivery Reliability	DLY	Technical support given	TSP
Product lead time	PLT	Design involvement	DIN
Compatibility with your systems like ERP	ERP	Email, video conferencing and other communication facilities	COM
Fill rate	FIL	Past delivery performance	PDP
Perfect order fulfilment	POF	Operating controls	OPC
Service	SER	Innovation	INO
Warranty and claim policies	WCP	Launch of better product	LBP
Repair turnaround time	RTT	Use of new technology	NTH
Ease of contact	EOC	Other factors	OTH
Modification support	MOD	Reputation and position in industry	REP
Repair quality	RPQ	Desire for business	DES

(continued)

Table 1 (continued)

Attributes	Abbr	Attributes	Abbr
Capacity	CAP	Development potential	DEV
Management and organization	MAO	Amount of past business	APB
Manufacturing resources	MFR	Geographic location	GEO
Storage resources	STR	General economic outlook	GEN
Logistics resources	LOG	Environment protection	ENV
Human resources	HMR	Credit rating	CRD
Financial resources	FNR	Supply variety	SUP
Technical capability	TCP	Appearance of product	APP
Packaging capability	PAK		

3 Problem Description

The company chosen for the study plans to build a supply chain for its automobile manufacturing product. Raw materials or components are planned to be outsourced to vendors. The question arisen is which vendor is to be selected for each raw material or component. The attributes and sub-attributes have to be most prevalent and important in the vendor selection process. Choosing the possible criteria for the vendor selection involves a decision making team which includes experts from the industry side (purchasing manager, purchasing director, sales manager, product manager, quality manager and production manager). The attributes and sub attributes involved in the vendor selection have been chosen by conducting a survey. A questionnaire consisting of various factors was designed for the survey. The respondents for the survey are selected randomly from different functional areas that are directly involved with the materials supplied by the vendors. Based on the survey, the major attributes and sub-attributes involved in the vendor selection are given in Table 1. The Hierarchy for selection of vendor in supply chain is shown in Fig. 1.

4 Methodology

4.1 Analytical Hierarchy Process

The analytical hierarchy process is a decision approach designed to aid in the solution of complex multiple criteria problems in a number of application domains. This method has been found to be an effective and practical that can consider complex and unstructured decisions. The analytical hierarchy process (AHP) is proposed in the research in order to handle both tangible and intangible factors and sub-factors affecting vendor selection decisions. The selection of the methodology is based on the characteristics of the problem and the consideration

Table 2 Saaty's fundamental scale (Saaty 2000)

Preference weights	Definition	Explanation
1	Equally preferred	Two activities contribute equally to the objective
3	Moderately	Experience and judgment slightly favor one activity over another
5	Strongly	Experience and judgment strongly or essentially favor one activity over another
7	Very strongly	An activity is strongly favored over another and its dominance demonstrated in practice
9	Extremely	The evidence favoring one activity over another is of the highest degree possible of affirmation
2, 4, 6, 8	Intermediate values	Used to represent compromise between the preferences listed above
Reciprocals	Reciprocals for inverse comparison	

of the advantages and drawbacks of other methodologies. The decision maker judges the importance of each criterion in pairwise comparisons. The outcome of AHP is a prioritized ranking or weighing of each decision alternative (Table 2). The research in this paper focused on formulating an AHP and fuzzy AHP-based models to select a best vendor for the automobile (manufacturing) industry.

The maximum Eigen value (λmax) is an important validating parameter in AHP (Saaty 2003). It is used as a reference index to screen information by calculating the consistency ratio (CR) of the estimated vector in order to validate whether the pair-wise comparison matrix provides a completely consistent evaluation. A measure of how far a matrix is from consistency is performed by consistency ratio. The consistency ratio is calculated as per the following steps:

1. Calculate the eigenvector or the relative weights and λmax for each matrix of order n.
2. Compute the consistency index (CI) for each matrix of order n by the formulae:

$$\mathrm{CI} = (\lambda\mathrm{max} - \mathrm{n})/(\mathrm{n} - 1) \tag{1}$$

where, n is the number of criteria.

3. The consistency ratio is then calculated using the formulae:

$$\mathrm{CR} = \mathrm{CI}/\mathrm{RI} \tag{2}$$

where, RI is random consistency index obtained from a large number of simulation runs and varies with the order of the matrix. Table 3 shows the value of the random consistency index (RI) for matrices of order 1–10 obtained by random indices using a sample size of 500.

The acceptable CR range varies according to the size of the matrix, i.e., 0.05 for a 3 by 3 matrix, 0.08 for a 4 by 4 matrix and 0.1 for all larger matrices, $n \geq 5$. If

Table 3 Average random index (RI) based on matrix size (Saaty 2000)

N	1	2	3	4	5	6	7	8	9	10
RCI	0	0	0.052	0.89	1.11	1.25	1.35	1.40	1.45	1.49

Table 4 Pairwise comparison matrix for the major criteria at level II

	QTY	CST	DLY	SER	CAP	FNR	HUM	TEC	INO	OTH	C. R.
QTY	1	3	2	2	3	4	5	5	5	6	0.058
CST	1/3	1	1	1	2	1	3	3	4	4	
DLY	1/2	1	1	1	3	3	4	4	4	5	
SER	1/2	1	1	1	3	1	3	6	6	5	
CAP	1/3	1/2	1/3	1/3	1	2	3	3	3	3	
FNR	1/4	1	1/3	1	1/2	1	2	1	2	1	
HUM	1/5	1/3	1/4	1/3	1/3	1/2	1	1/2	1/2	1/3	
TEC	1/5	1/3	1/4	1/6	1/3	1	2	1	1	1/2	
INO	1/5	1/4	1/4	1/6	1/3	1/2	2	1	1	1/2	
OTH	1/6	1/4	1/5	1/5	1/3	1	3	2	2	1	

the value of CR is equal to, or less than that value, it implies that the evaluation within the matrix is acceptable or indicates a good level of consistency in the comparative judgments represented in that matrix (Table 4). In contrast, if CR is more than the acceptable value, inconsistencies of judgments within that matrix occur and the evaluation process should be reviewed, reconsidered, and improved. The comparative judgments should be reconsidered with respect to the issues raised in the section of grouping related elements together under a more general topic. An acceptable consistency property helps to ensure decision-maker reliability in determining the priorities of a set of criteria.

Saaty (2000) believed that some uncertainty is lying in the nature of AHP method. Buckley (1985) also raised questions about certainty of the comparison ratios used in the AHP. He considered a situation in which the decision-maker can express feelings of uncertainty while ranking or comparing different alternatives or criteria. The method used to take uncertainties into account by using fuzzy numbers instead of crisp numbers in order to compare the importance between the alternatives or criteria. AHP is criticized for its inability to deal with uncertainty and imprecision of the decision maker's perceptions (Deng 1999). The major drawback of AHP is that it fails to address the uncertainty in expressing the preferences during pairwise comparison (PC). The inability of the AHP to address imprecision and uncertainty paved the way for the incorporation of fuzzy logic into the AHP (Deng 1999).

4.2 Fuzzy AHP

Though the purpose of AHP is to capture the expert's knowledge, the conventional AHP still cannot reflect the human style of thinking (Kahraman et al. 2004). Fuzzy AHP is a fuzzy extension of AHP, developed to solve the hierarchical fuzzy problems. "Fuzzy AHP" is a term used to incorporate a wide range of techniques, all of which require the initial fuzzification of a pairwise comparison matrix. The benefit of extending crisp theory and analysis methods to fuzzy techniques is the strength in solving real world problems, which inevitably entail some degree of imprecision and noise in the variables and parameters measured and processed for application. Accordingly, linguistic variables are a critical aspect of some fuzzy logic applications, where general terms such as "large", "medium" and "small" are used to capture a range of numerical values. A framework of a fuzzy AHP technique is proposed by Chang (1996) and later used by Kahraman et al. (2004).

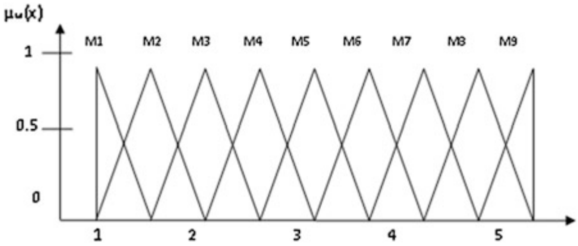
Fuzzy set theory was introduced by Zadeh in 1965 to solve problems involving the absence of sharply defined criteria (Zadeh 1965). Many decision making problems are too complex to be understood quantitatively; however people succeed by using knowledge that is imprecise rather than precise. Fuzzy set theory resembles human reasoning in its use of approximate information and uncertainty to generate decisions. It is designed to mathematically represent uncertainty and vagueness that is intrinsic to many problems. Since knowledge can be represented in a more natural way using fuzzy sets, many engineering and decision problems can be greatly simplified. This theory implements classes or groupings of data with boundaries that are not sharply defined (fuzzy). Fuzzy set theory encompasses fuzzy logic, fuzzy arithmetic, fuzzy mathematical programming, fuzzy topology, fuzzy graph theory and fuzzy data analysis, though the term fuzzy logic is often used to describe all of these (Kahraman et al. 2004).

van Laarhoven and Pedrycz (1983) presented triangular fuzzy numbers while presenting a fuzzy extension of Saaty's priority theory. A triangular fuzzy number can be denoted as M = (l, m, u), its membership function μM(x): R → [0, 1] (Fig. 2) is equal to:

$$\mu_m(x) = \begin{cases} \frac{1}{m-1}x - \frac{1}{m-1}, & x \in [l, m] \\ \frac{1}{m-u}x - \frac{1}{m-u}, & x \in [m, u] \\ 0 & otherwise \end{cases} \tag{3}$$

The fuzzy AHP method presented involves a complex process of comparison and ranking of fuzzy utilities and according to (Deng 1999) may produce unreliable results. To avoid fuzzy number comparison, the alpha cut technique to transform the fuzzy performance matrix into an interval matrix can be considered. The α-cut incorporates the decision maker's attitude towards risk. The value of α ($0 < \alpha < 1$) represents the decision maker's degree of confidence in assessment regarding criteria weights and alternative ratings. Larger α value indicates a confident decision maker. The optimism index, λ ($0 < \lambda < 1$) is used incorporating

Fig. 2 The membership function of the triangular fuzzy numbers



the decision maker's attitude towards risk and also to obtain the crisp performance matrix. Figure 3 shows the general steps involved in implementing the alpha cut technique.

Lambda function, which represents the attitude of the decision maker, converts the left and right alpha cut values into crisp values. The attitude of the decision maker may be optimistic, moderate or pessimistic. Decision maker with optimistic attitude will take the maximum values of the range: a moderate person will take the medium value and a pessimistic person will take the minimum value of the range. Here, the concept of optimism index, λ, is introduced to obtain the crisp output. Finally, the crisp values need to be normalized, because the elements of the PCM do not have the same scale. It is important to note that elements can be compared if they have uniform scale.

$$\alpha_{\text{Left}} = [\alpha \times (\text{middle fuzzy} - \text{left fuzzy})] + \textit{left fuzzy} \tag{4}$$

$$\alpha_{\text{right}} = \text{right fuzzy} - [\alpha \times (\text{right fuzzy} - \text{middle fuzzy})] \tag{5}$$

$$\textit{Crisp value} = \lambda \times \alpha_{\text{right}} + [(1 - \lambda) \times \alpha_{\text{left}}] \tag{6}$$

5 Application of AHP Models

In this section, a conceptual approach for structuring the selection of the best vendor using the AHP is introduced and the AHP decision is compared with fuzzy AHP, and fuzzy AHP with alpha cut. The above models were chosen as they can easily handle both tangible and intangible criteria (Tahriri et al. 2008; Hou et al. 2003). The model may be regarded as a feasible way for visualizing any vendor selection decision problem systematically. This decision-maker can apply this framework to structure their particular problem in selecting the best vendor for their choices in many circumstances.

Company chosen for this research study is a famous car manufacturing industry located in the western part of India. The annual turnover of the company is approximately 55 billion rupees. The overall workforce is of 3500 employees, including 800 officers. The company planned to improve the quality of the product

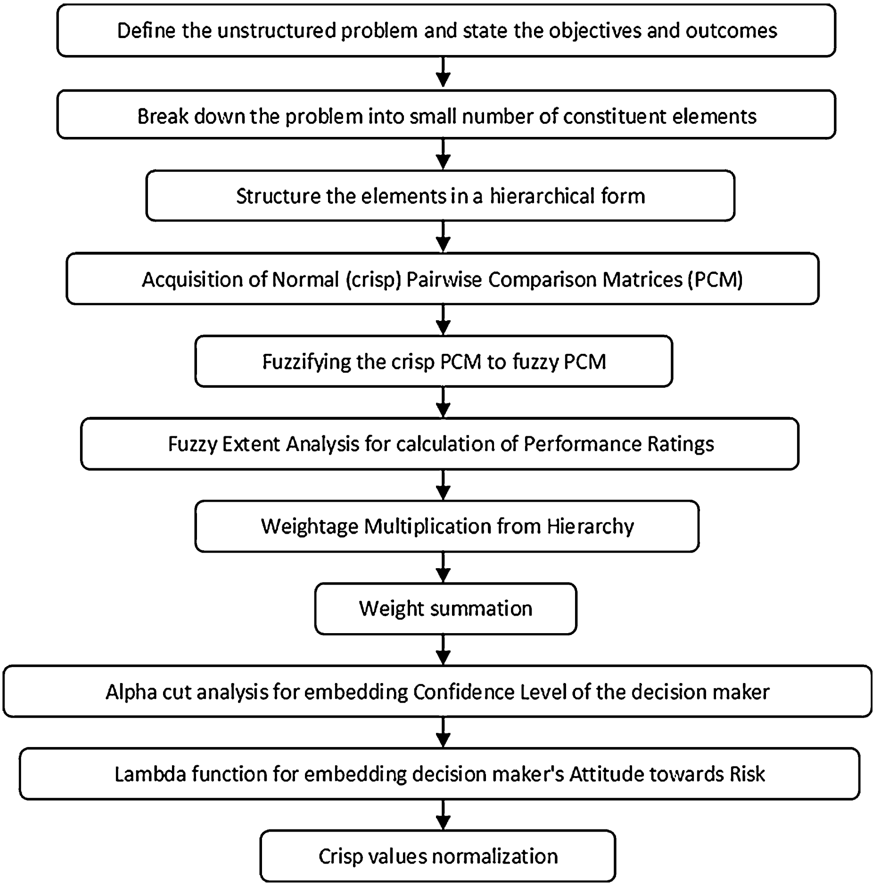


Fig. 3 Fuzzy AHP with Alpha cut and Lambda function flowchart

and to purchase the quality raw material at low cost and at a short duration of time. Instead of purchasing the material from the single vendor, three alternative vendors, namely vendor 1, vendor 2 and vendor 3 were taken into consideration.

6 Result and Discussion

A systematic approach has been applied for selecting a best vendor to supply the raw material. Table 1 shows factors and sub-factors that the decision-maker identified as being important in the vendor selection decisions. Table 4 contains the pairwise comparison matrix used to evaluate the major criteria on level 2 of the hierarchy. Table 5 shows weights of major criteria using AHP and fuzzy AHP.

Table 5 Weights of major criteria using AHP and fuzzy AHP

Relative weights using AHP	Relative weights using fuzzy AHP	Sub criteria	Relative weights using AHP	Relative weights using Fuzzy AHP	Global weights using AHP	Global weights using Fuzzy AHP
0.245	0.175	CSP	0.201	0.099	0.049	0.017
		PRJ	0.216	0.103	0.053	0.018
		CMC	0.057	0.096	0.014	0.017
		DDR	0.097	0.092	0.024	0.016
		TCS	0.044	0.099	0.011	0.017
		QRL	0.153	0.107	0.037	0.019
		PCP	0.044	0.102	0.011	0.018
0.122	0.137	NET	0.232	0.158	0.028	0.022
		TRM	0.071	0.104	0.009	0.014
		DEF	0.107	0.122	0.013	0.017
		TRA	0.080	0.104	0.010	0.014
		ADT	0.045	0.067	0.005	0.009
		PEC	0.058	0.068	0.007	0.009
		BQC	0.174	0.143	0.021	0.020
		ROD	0.155	0.139	0.019	0.019
		CCR	0.077	0.095	0.009	0.013
0.089	0.119	MAO	0.098	0.106	0.009	0.013
		MFR	0.127	0.131	0.011	0.016
		STR	0.081	0.100	0.007	0.012
		LOG	0.094	0.116	0.008	0.014
		HMR	0.066	0.087	0.006	0.010
		FNR	0.203	0.173	0.018	0.020
		TCP	0.254	0.192	0.023	0.023
		PAK	0.045	0.033	0.004	0.004
		TRN	0.032	0.062	0.003	0.007
0.069	0.083	PRC	0.267	0.210	0.018	0.018
		VRT	0.255	0.204	0.018	0.017
		PFX	0.148	0.172	0.010	0.014
		FDC	0.077	0.099	0.005	0.008
		CSR	0.106	0.149	0.007	0.012
		BFX	0.148	0.166	0.010	0.014

Table 6 shows composite relative weights of critical attributes. Tables 7 and 8 show the overall ratings of three vendors using AHP and Tables 9 and 10 show the overall ratings with fuzzy AHP respectively. Tables 11, 12, 13, 14 show the results of fuzzy AHP with alpha cut technique. Table 15 gives a summary of weights using all the three applied models.

The results show that all the three models, i.e. AHP, FAHP and FAHP with alpha cut are capable of handling a large number of tangible and intangible criteria. After comparison of overall priority of vendors and the methods applied it can seen that vendor 2 is preferred in all the methods, and hence the preferred vendor as it has the highest weight (0.362, 0.394, and 0.388) among three vendors.

Table 6 Composite relative weights of critical attributes of company

Criteria	Relative weights using AHP	Relative weights using fuzzy AHP	Sub criteria	Relative weights using AHP	Relative weights using Fuzzy AHP	Global weights using AHP	Global weights using Fuzzy AHP
Human factors	0.032	0.020	EXV	0.167	0.144	0.005	0.003
			LTR	0.091	0.109	0.003	0.002
			TRT	0.089	0.112	0.003	0.002
			ATT	0.089	0.142	0.003	0.003
			CTY	0.048	0.047	0.002	0.001
			LRR	0.065	0.084	0.002	0.002
			CCT	0.058	0.063	0.002	0.001
			IOV	0.047	0.054	0.001	0.001
			PSP	0.057	0.078	0.002	0.002
			MLI	0.244	0.165	0.008	0.003
Technology	0.041	0.050	RND	0.356	0.266	0.014	0.013
			TSP	0.197	0.206	0.008	0.010
			DIN	0.159	0.186	0.006	0.009
			COM	0.159	0.177	0.006	0.009
			ERP	0.049	0.045	0.002	0.002
			OPC	0.079	0.119	0.003	0.006
Innovation	0.036	0.034	LBP	0.167	0.484	0.006	0.017
			NTH	0.833	0.516	0.030	0.018
Other factors	0.051	0.076	REP	0.079	0.106	0.004	0.008
			DES	0.153	0.146	0.008	0.011
			DEV	0.078	0.102	0.004	0.008
			APB	0.078	0.069	0.004	0.005
			GEO	0.127	0.127	0.006	0.010
			GEN	0.031	0.044	0.002	0.003
			ENV	0.041	0.076	0.002	0.006
			CRD	0.036	0.027	0.002	0.002
			SUP	0.153	0.144	0.008	0.011
			APP	0.232	0.160	0.012	0.012

It is also found that the difference in the weights between three vendors is very less in AHP output as compared to the fuzzy techniques (0.3061, 0.3620, and 0.3319). The weight difference in the fuzzy techniques on the other hand, is far more acceptable i.e. for FAHP (0.3182, 0.3943, and 0.2875) and for FAHP with alpha cut (0.3538, 0.3880, and 0.2582).

Table 7 Overall rating of three vendors using AHP

Criteria	Weights	Sub criteria	Relative weights	Relative weights			Global weights		
				V1	V2	V3	V1	V2	V3
Quality	0.245	CSP	0.201	0.297	0.540	0.163	0.015	0.027	0.008
		PRJ	0.216	0.333	0.333	0.333	0.018	0.018	0.018
		CMC	0.057	0.240	0.550	0.210	0.003	0.008	0.003
		CER	0.057	0.143	0.714	0.143	0.002	0.010	0.002
		TQM	0.015	0.174	0.634	0.192	0.001	0.002	0.001
		QAS	0.117	0.444	0.111	0.444	0.013	0.003	0.013
		DDR	0.097	0.333	0.333	0.333	0.008	0.008	0.008
		TCS	0.044	0.250	0.500	0.250	0.003	0.005	0.003
		QRL	0.153	0.167	0.167	0.167	0.006	0.006	0.006
		PCP	0.044	0.167	0.167	0.167	0.002	0.002	0.002
Cost	0.122	NET	0.232	0.540	0.163	0.297	0.015	0.005	0.008
		TRM	0.071	0.333	0.333	0.333	0.003	0.003	0.003
		DEF	0.107	0.200	0.400	0.400	0.003	0.005	0.005
		TRA	0.080	0.333	0.333	0.333	0.003	0.003	0.003
		PEC	0.058	0.250	0.500	0.250	0.002	0.004	0.002
		BQC	0.174	0.167	0.167	0.167	0.004	0.004	0.004
		ROD	0.155	0.083	0.121	0.796	0.002	0.002	0.015
		CCR	0.077	0.333	0.333	0.333	0.003	0.003	0.003
Delivery reliability	0.158	PLT	0.351	0.415	0.086	0.500	0.023	0.005	0.028
		PDP	0.189	0.333	0.333	0.333	0.010	0.010	0.010
		FIL	0.109	0.458	0.126	0.416	0.008	0.002	0.007
		POF	0.351	0.333	0.333	0.333	0.018	0.018	0.018

(continued)

Table 7 (continued)

Criteria	Weights	Sub criteria	Relative weights	Relative weights			Global weights		
				V1	V2	V3	V1	V2	V3
Service	0.158	WCP	0.122	0.333	0.333	0.333	0.006	0.006	0.006
		RTT	0.113	0.250	0.500	0.250	0.004	0.009	0.004
		EOC	0.359	0.333	0.075	0.592	0.019	0.004	0.034
		MOD	0.155	0.143	0.571	0.286	0.004	0.014	0.007
		RPQ	0.252	0.240	0.550	0.210	0.010	0.022	0.008
Capacity	0.089	MAO	0.098	0.137	0.238	0.625	0.001	0.002	0.005
		MFR	0.127	0.216	0.682	0.103	0.002	0.008	0.001
		STR	0.081	0.333	0.333	0.333	0.002	0.002	0.002
		LOG	0.094	0.200	0.400	0.400	0.002	0.003	0.003
		HMR	0.066	0.151	0.630	0.218	0.001	0.004	0.001
		FNR	0.203	0.150	0.725	0.125	0.003	0.013	0.002
		TCP	0.254	0.333	0.333	0.333	0.008	0.008	0.008
		PAK	0.045	0.210	0.550	0.240	0.001	0.002	0.001
		TRN	0.032	0.174	0.634	0.192	0	0.002	0.001
Flexibility	0.069	PRC	0.267	0.250	0.500	0.250	0.005	0.009	0.005
		VRT	0.255	0.429	0.143	0.429	0.008	0.003	0.008
		PFX	0.148	0.167	0.667	0.167	0.002	0.007	0.002
		FDC	0.077	0.333	0.333	0.333	0.002	0.002	0.002
		CSR	0.106	0.333	0.333	0.333	0.002	0.002	0.002
		BFX	0.148	0.250	0.500	0.250	0.003	0.005	0.003

Table 8 Overall rating of three vendors using AHP

Criteria	Weights	Sub criteria	Relative weights	Relative weights			Global weights		
				V1	V2	V3	V1	V2	V3
Human factors	0.032	EXV	0.167	0.200	0.600	0.200	0.001	0.003	0.001
		LTR	0.091	0.200	0.600	0.200	0.001	0.002	0.001
		TRT	0.089	0.111	0.778	0.111	0	0.002	0
		ATT	0.089	0.333	0.333	0.333	0.001	0.001	0.001
		CTY	0.048	0.250	0.500	0.250	0	0.001	0
		LRR	0.065	0.333	0.333	0.333	0.001	0.001	0.001
		CCT	0.058	0.333	0.333	0.333	0.001	0.001	0.001
		IOV	0.047	0.131	0.661	0.208	0	0.001	0
		PSP	0.057	0.644	0.242	0.114	0.001	0	0
		MLI	0.244	0.333	0.333	0.333	0.003	0.003	0.003
Technology	0.041	RND	0.356	0.167	0.667	0.167	0.002	0.010	0.002
		TSP	0.197	0.250	0.500	0.250	0.002	0.004	0.002
		DIN	0.159	0.111	0.778	0.111	0.001	0.005	0.001
		COM	0.159	0.250	0.250	0.500	0.002	0.002	0.003
		ERP	0.049	0.333	0.333	0.333	0.001	0.001	0.001
		OPC	0.079	0.443	0.169	0.387	0.001	0.001	0.001
Innovation	0.036	LBP	0.167	0.333	0.333	0.333	0.002	0.002	0.002
		NTH	0.833	0.333	0.333	0.333	0.010	0.010	0.010

(continued)

Table 8 (continued)

Criteria	Weights	Sub criteria	Relative weights	Relative weights			Global weights		
				V1	V2	V3	V1	V2	V3
Other factors	0.051	REP	0.079	0.167	0.667	0.167	0.001	0.003	0.001
		DES	0.153	0.333	0.333	0.333	0.003	0.003	0.003
		DEV	0.078	0.333	0.333	0.333	0.001	0.001	0.001
		APB	0.078	0.250	0.500	0.250	0.001	0.002	0.001
		GEO	0.127	0.333	0.333	0.333	0.002	0.002	0.002
		GEN	0.031	0.250	0.500	0.250	0	0.001	0
		ENV	0.041	0.333	0.333	0.333	0.001	0.001	0.001
		CRD	0.036	0.333	0.333	0.333	0.001	0.001	0.001
		SUP	0.153	0.659	0.156	0.185	0.001	0.001	0.001
		APP	0.232	0.250	0.500	0.250	0.003	0.006	0.003
Overall priority							0.306	0.362	0.332
Rank							**3**	**1**	**2**

Table 9 Overall rating of three vendors using FUZZY AHP

Criteria	Weights	Sub criteria	Relative weights	Relative weights			Global weights		
				V1	V2	V3	V1	V2	V3
Quality	0.175	CSP	0.099	0.348	0.452	0.199	0.006	0.008	0.003
		PRJ	0.103	0.333	0.333	0.333	0.006	0.006	0.006
		CER	0.098	0.306	0.339	0.355	0.005	0.006	0.006
		TQM	0.097	0.260	0.573	0.167	0.004	0.010	0.003
		QAS	0.099	0.480	0.040	0.480	0.008	0.001	0.008
		DDR	0.092	0.333	0.333	0.333	0.005	0.005	0.005
		TCS	0.099	0.365	0.527	0.108	0.006	0.009	0.002
		QRL	0.107	0.246	0.704	0.050	0.005	0.013	0.001
		PCP	0.102	0.246	0.704	0.050	0.004	0.013	0.001
Cost	0.137	NET	0.158	0.452	0.199	0.348	0.010	0.004	0.008
		TRM	0.104	0.333	0.333	0.333	0.005	0.005	0.005
		DEF	0.122	0.221	0.389	0.389	0.004	0.007	0.007
		TRA	0.104	0.333	0.333	0.333	0.005	0.005	0.005
		ADT	0.067	0.333	0.333	0.333	0.003	0.003	0.003
		PEC	0.068	0.365	0.527	0.108	0.003	0.005	0.001
		BQC	0.143	0.246	0.704	0.05	0.005	0.014	0.001
		ROD	0.139	0.326	0.328	0.346	0.006	0.006	0.007
		CCR	0.095	0.333	0.333	0.333	0.004	0.004	0.004
Delivery reliability	0.152	PLT	0.306	0.314	0.320	0.366	0.015	0.015	0.017
		PDP	0.236	0.333	0.333	0.333	0.012	0.012	0.012
		FIL	0.151	0.465	0.118	0.417	0.011	0.003	0.010
		POF	0.306	0.333	0.333	0.333	0.016	0.016	0.016

(continued)

Table 9 (continued)

Criteria	Weights	Sub criteria	Relative weights	Relative weights			Global weights		
				V1	V2	V3	V1	V2	V3
Service	0.154	WCP	0.161	0.333	0.333	0.333	0.008	0.008	0.008
		RTT	0.146	0.365	0.527	0.108	0.008	0.012	0.002
		EOC	0.273	0.334	0.332	0.334	0.014	0.014	0.014
		MOD	0.174	0.117	0.529	0.354	0.003	0.014	0.009
		RPQ	0.246	0.306	0.479	0.215	0.012	0.018	0.008
Capacity	0.119	MAO	0.106	0.118	0.339	0.543	0.001	0.004	0.007
		MFR	0.131	0.318	0.349	0.333	0.005	0.005	0.005
		STR	0.100	0.333	0.333	0.333	0.004	0.004	0.004
		LOG	0.116	0.221	0.389	0.389	0.003	0.005	0.005
		HMR	0.087	0.157	0.529	0.313	0.002	0.005	0.003
		FNR	0.173	0.307	0.350	0.343	0.006	0.007	0.007
		TCP	0.192	0.333	0.333	0.333	0.008	0.008	0.008
		PAK	0.033	0.301	0.479	0.220	0.001	0.002	0.001
		TRN	0.062	0.260	0.573	0.167	0.002	0.004	0.001
Flexibility	0.083	PRC	0.210	0.360	0.527	0.108	0.006	0.009	0.002
		VRT	0.204	0.400	0.200	0.400	0.007	0.003	0.007
		PFX	0.172	0.246	0.704	0.050	0.004	0.010	0.001
		FDC	0.099	0.333	0.333	0.333	0.003	0.003	0.003
		CSR	0.149	0.333	0.333	0.333	0.004	0.004	0.004
		BFX	0.167	0.365	0.527	0.108	0.005	0.007	0.001

Table 10 Overall rating of three vendors using FUZZY AHP

Criteria	Weights	Sub criteria	Relative weights	Relative weights			Global weights		
				V1	V2	V3	V1	V2	V3
Human factors	0.020	EXV	0.144	0.295	0.497	0.208	0.001	0.001	0.001
		LTR	0.109	0.295	0.497	0.208	0.001	0.001	0
		TRT	0.112	0.307	0.351	0.342	0.001	0.001	0.001
		ATT	0.142	0.333	0.333	0.333	0.001	0.001	0.001
		CTY	0.047	0.365	0.527	0.108	0	0	0
		LRR	0.084	0.333	0.333	0.333	0.001	0.001	0.001
		CCT	0.063	0.333	0.333	0.333	0	0	0
		IOV	0.054	0.059	0.630	0.312	0	0.001	0
		PSP	0.078	0.347	0.332	0.321	0.001	0.001	0
		MLI	0.165	0.333	0.333	0.333	0.001	0.001	0.001
Technology	0.050	RND	0.266	0.246	0.704	0.050	0.003	0.009	0.009
		TSP	0.206	0.365	0.527	0.108	0.004	0.005	0.005
		DIN	0.186	0.307	0.351	0.342	0.003	0.003	0.003
		COM	0.177	0.275	0.389	0.336	0.002	0.003	0.003
		ERP	0.045	0.333	0.333	0.333	0.001	0.001	0.001
		OPC	0.119	0.389	0.221	0.389	0.002	0.001	0.001
Innovation	0.034	LBP	0.484	0.333	0.333	0.333	0.006	0.006	0.006
		NTH	0.516	0.333	0.333	0.333	0.006	0.006	0.006

(continued)

Table 10 (continued)

Criteria	Weights	Sub criteria	Relative weights	Relative weights			Global weights		
				V1	V2	V3	V1	V2	V3
Other Factors	0.076	REP	0.106	0.246	0.704	0.050	0.002	0.006	0
		DES	0.146	0.333	0.333	0.333	0.004	0.004	0.004
		DEV	0.102	0.333	0.333	0.333	0.003	0.003	0.003
		APB	0.069	0.365	0.527	0.108	0.002	0.003	0.001
		GEO	0.127	0.333	0.333	0.333	0.003	0.003	0.003
		GEN	0.044	0.365	0.527	0.108	0.001	0.002	0
		ENV	0.076	0.333	0.333	0.333	0.002	0.002	0.002
		CRD	0.027	0.333	0.333	0.333	0.001	0.001	0.001
		SUP	0.144	0.665	0.223	0.112	0.007	0.002	0.001
		APP	0.160	0.365	0.527	0.108	0.004	0.006	0.001
Overall priority							0.318	0.394	0.287
Rank							**2**	**1**	**3**

Table 11 Total weighted performance matrix

Vendor	Fuzzy value		
	Lower	Middle	Upper
V1	0.018686	0.293971	7.329715
V2	0.018079	0.383088	7.952916
V3	0.014479	0.322941	5.19414

Table 12 Alpha cut analysis with $\alpha = 0.5$

Vendor	α_{left}	α_{right}
V1	0.156329	3.811843
V2	0.200583	4.168002
V3	0.16871	2.758541

Table 13 Crisp values on applying optimism index $\lambda = 0.7$

Vendor	Crisp weights
V1	2.715189
V2	2.977776
V3	1.981591

Table 14 Normalized weights

Vendor	Normalized weights	Ranking
V1	0.353791	2
V2	0.388006	1
V3	0.258203	3

Table 15 Results summary

Vendor	Final weights		
	AHP	Fuzzy AHP	FAHP with α cut
V1	0.3061	0.3182	0.3538
V2	0.3620	0.3943	0.3880
V3	0.3319	0.2875	0.2582

7 Conclusion

The research shows that AHP and fuzzy AHP can successfully handle large a number of criteria. The major advantage is that it can be used for large number of tangible and intangible criteria. Pairwise comparison used in study reduces the dependency of the model on human judgment. The results show that the model has the capability to be flexible and can apply to different types of industries to decide vendors. As the FAHP with alpha cut technique shows a marked distance between each vendor weights, it is found to be the most preferred technique in future.

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Supplier Evaluation Using Fuzzy Clustering

Başar Öztayşi and Mine Işık

Abstract Since the suppliers have become an important determinant of a company's success, the selection and evaluation of suppliers plays a vital role in the performance of a company. Although the problem has been formulated extensively as a multi criteria decision making method in the literature, in this study we aim to use fuzzy clustering methodology to define different groups of customers based on their performances. The results of such, can be used by managers to identify similar performing suppliers and generate strategies based on these groups.

Keywords Supplier evaluation · Fuzzy sets · Cluster analysis · Supply chain management

1 Introduction

Due to recent changes, organizations become more dependent on its suppliers and the results of poor decision making about suppliers, brings many problems with it in return (Chan and Kumar 2007; Lin 2012). The supplier evaluation problem can be considered as an important area of study under supply chain management main branch. Due to its significance, it has received a lot of attention from both academicians and practitioners, but this area of concern becomes more of an issue in recent years.

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Chronologically, different approaches have been proposed to evaluate the supplier performance. Analytical Hierarchy Process (AHP), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Neural Networks, Goal Programming are the most popular methodologies and techniques from diverse fields of operations research, artificial intelligence, and decision analysis theory (Ferreira and Borenstein 2012).

Due to its nature that consists of quiet different area of c concern, supplier evaluation can be regarded as a complicated issue in supply chain management (Vahdani et al. 2012). If the supplier evaluation can be done appropriately, the positive effect of right decision making yields a right intervention throughout the whole supply chain. As a result of it, the corporate competitiveness can be increased. On the contrary, inaccurate selection and evaluation of supplier may lead to financial and operational problems (Omurca 2013).

Moreover, as an example for the application areas of supplier evaluation, field of production can be given; it plays a strategic role in the competitiveness of large manufacturing companies. As a consequence, the researchers have been dedicated to the development of different kind of methodologies to handle this problem (Bruno et al. 2012).

Various types of evaluation methodologies have been applied, examining each supplier separately or creating supplier groups that share the same qualifications are two of the main approaches. What we aimed in this study, by utilizing fuzzy clustering, are to segment suppliers in order to generate a supplier groups and to obtain a general opinion on them. By that way the right prescription can be addressed to the problems encountered by the managers. The clustering of suppliers can be considered as a prerequisite for determining the road map in which different buyer–supplier exchanges may progress, which yields in profitable partnership (Day et al. 2010).

Clustering can be defined as process of generating homogenous subsets from a set of data objects. The data in the resulting subset is expected to be similar to the other objects in the same subset and dissimilar to data in other subsets. The literature supplies various methodologies for clustering but as a general classification, these methodologies can be classified as crisp and fuzzy clustering techniques. In crisp clustering, the data is partitioned into a specified number of mutually exclusive subsets. However, in fuzzy clustering the objects can belong to several clusters simultaneously, with different degrees of membership. This property of fuzzy clustering enables the results to be more natural when compared to crisp clustering. Thus fuzzy clustering methodology is used to group the suppliers based on their performance.

The aim of this chapter is to provide insight about the potential usage of fuzzy c-means algorithm for supplier evaluation purposes. In this manner a numerical example is given using a durable goods manufacturing company. Eighty suppliers are evaluated using eight criteria and four groups of customers are identified as the result of clustering. The remaining of the chapter is organized as follows. An extended literature review about supplier evaluation and the methodologies used is given in Sect. 2. The evaluation criteria used in the literature is briefly explained in

Sect. 3. In Sect. 4 fuzzy clustering is introduced and fuzzy c-means algorithm is explained. The numerical application is given in Sect. 5 and finally the conclusion is given in Sect. 6.

2 Literature Review: Supplier Evaluation

This chapter aims to give a general structure on the recent trends on supplier evaluation techniques. Literature is investigated under 21 different main branches in order to reveal the popular areas. Moreover, the advantages and disadvantages of those popular methodologies are explained in detailed way.

As the Table 1 shows, MCDM methodology plays a dominant role in supplier evaluation framework. Among them AHP and ANP are the most commonly used methods. It is clear that, the reason of that popularity originates in the availability of evaluating qualitative and quantitative data. MCDM is followed by FST due to its ability of using fuzzy values rather than crisp values, as it is necessary in the case of supplier evaluation cause of its nature since it includes the subjective ingredients such as reputation and openness of the supplier.

In order to explain the commonly used methodologies in the literature, it is divided into the following sub groups as; DEA, Mathematical Programming Applications, MCDM Techniques, Artificial Intelligence Applications, and finally, the studies that includes Fuzzy Set Theoretic applications of supplier evaluation is given.

2.1 Data Envelopment Analysis

Data Envelopment Analysis (DEA) is a technique that utilizes non-parametric mathematical programming approach in order to evaluate the decision-making performance of homogeneous units under the condition of multiple input output case (Çelebi and Bayraktar 2008). DEA was developed by Charnes et al. (1978). It measures the relative performance of decision-making units (DMUs) by considering the observed operation practice within comparable DMUs' samples (Kuo et al. 2010a, b).

This technique can be considered as the most popular individual approach (Ho et al. 2010). It receives an increasing attention due to its structure that best fits on supplier evaluation.

This methodology is very successful in exploring the uncovered relationships, by that qualification while examining the criteria for supplier evaluation the hidden relationship within them can easily be exposed. Since supplier evaluation is a multi-faceted problem, DEA satisfies this need due to its ability to handle multiple input–output cases. Moreover, its capability of being used with any kind of input–output measurement results in no restriction of application.

As we give some examples from literature, the proposed model of Falagario et al. (2012) does not require predefined weights, the evaluation of suppliers were made by investigating them individually. Also the criteria weights are evaluated in order to maximize the supplier's efficiency. By that way suppliers can be classified as efficient and inefficient suppliers.

DEA method both finds the most favorable set of weights and also gives buyers a chance to classify suppliers into two above-mentioned groups (Kuo et al. 2010a, b).

By continuing to give examples from literature, another interesting study is also presented by Wu et al. (2007) augmented imprecise DEA for supplier selection web-based system is proposed. It is capable of handling imprecise and it has a good discriminatory power.

As mentioned before, DEA is able to overcome qualitative data. By utilizing that property supplier reputation Saen (2007) succeed to involve supplier reputation criterion into the evaluation model. The amount of know-how transfer was also measured by Saen (2006) by using DEA. As it can be seen DEA is a suitable tool to evaluate qualitative information since it is crucial for supplier evaluation.

As Ekici (2013) emphasizes that the hierarchy of criteria and the dependencies within them are ignored by DEA. Moreover, DEA requires huge amount of information about the supplier that is evaluated. So that requirement results in consumption of time and money (Çelebi and Bayraktar 2008).

2.2 Mathematical Programming

Mathematical modelling is a good tool for the complex structure problems and it is also one of the widely used for evaluation problems (Omurca 2013). Main objective of mathematical programming model is to minimize or maximize the main objective. In some studies total cost of ownership of the supplier is gathered by utilizing mathematical programming.

Goal programming is one of the most popular branches of mathematical programming used to minimize costs and maximize the preferred ability of supplier such as its quality, reliability, its technology level when evaluating suppliers (Kuo et al. 2010a, b).

Moreover, in order to utilize the advantages of both popular modeling techniques, it is very popular to use goal programming with MCDM techniques. Erdem and Gocen's (2012) study can be given as one of such examples. They develop a model integrating AHP and goal programming by that way, conducted model is able to handle order allocation by utilizing GP and evaluate both qualitative and quantitative data by means of AHP.

The complex structure of supplier evaluation and selection is satisfied by mathematical programming approach. It is a flexible tool, when an objective is point of concern. Benefits include the fact that the results can be repeated refined and adopted to different conditions.

The main drawback of Mathematical Programming is that it uses only qualitative data, so it restricts the buyer not to use subjective attributes (Keskin et al. 2010).

2.3 MDCM Techniques

Since supplier evaluation is a complex multi faceted problem, to make good judgment on supplier, diversified concepts should be reviewed such as quality, know-how, technology etc. MCDM is best fitted for this problem structure. Advantages of MCDM techniques are explained separately as follows.

AHP includes model verification by the control of consistency; this action contributes the correct building of the evaluation model and acts as a feedback mechanism for the decision makers in order to revise their judgments (Ho et al. 2010).

VIKOR is also another popular MCDM tool that aims to gather maximum group utility results from compromise solution accepted by the group (Shemshadi et al. 2011). When the evaluation process of supplier includes the view of different parties (such as production, marketing, logistic etc.) it is one of the most appropriate techniques.

In the constitution of the model, criteria diversification is crucial since it can result in different judgments upon the evaluation of supplier (Omurca 2013). So the number and type of criteria should be picked wisely otherwise the results cannot be logical. Since different types of methodologies under MCDM are used for evaluation the disadvantages will be given separately. By first explaining the disadvantages of AHP, its most crucial problem is to ignore the inter-dependencies within criteria but ANP overcomes this obstacle. Moreover, it depends heavily on human judgment so the decisions made by these models are subjective.

2.4 Artificial Intelligence

These types of models are computer-aided systems. They are trained by historical data. The models in this group do not necessitate decision-making process formalization. AI models show satisfactory performance on complexity and uncertainty rather than traditional methods. Their working mechanism is designed to mimic human judgment functioning (Kuo et al. 2010a, b). This qualification results in strong models that are able to cope better with complexity and uncertainty. It only requires the information on characteristics of current situation. The AI technologies deduce conclusions based on what they have learned from the past data (Keskin et al. 2010).

2.5 Stochastic Programming

Since supply chain management is a real world field of study, making decision upon suppliers requires data that has fluctuating structure due to the uncertainty characteristic so most of the generated models fail to notice the time dependency of the criteria's values. In this type of environment the models that are able to handle the uncertainty, bring more realistic results.

Stochastic programming is adequate tool to cope with uncertainty. Utilizing its ability to be adapted to the changing environment, its performance to acquire credible results is more preferable than deterministic programming models. For the reason that factors are not always certain in real world and beside factors are assumed to be certain in deterministic programming, stochastic programming supplies better information (Kara 2011). It also should be noted that the majority of the literature on supplier evaluation ignores the uncertainty, even if the stochastic structure gains a vital importance in recent years (Hsu et al. 2010).

2.6 FST Hybrid Studies

As it can be seen from the literature, combination of fuzzy set theory and the other methodologies become more popular since this togetherness creates a suitable environment that gives a chance to utilize both advantages of FST and another combined methodology.

AHP-FST combination is preferred to handle both quantitative and qualitative data under fuzzy environment. Moreover, in the study of Lin (2012) fuzzy analytic network process is adopted in order to identify best suppliers. ANP gives a chance to track interdependence among the considered criteria beside it also controls the consistency as in the case of AHP.

Another different hybrid application for supplier evaluation is the integration of influence diagram and fuzzy logic to evaluate suppliers (Ferreira and Borenstein 2012).

When popularity is point of concern AHP and ANP is followed by TOPSIS. Chen et al. (2006) presented a fuzzy TOPSIS model to rank the order of all suppliers.

3 Evaluation Criteria

Within the investigated literature review, the most popular criterion is quality, followed by delivery, technology, price, manufacturing capability as Ho et al. indicated in their study (2010).

Commonly used criteria are grouped under their main branches as manufacturing, facilities, technology, delivery, product properties, and sustainability. Moreover, firms' general qualifications are grouped as firm's abilities. Also another important criteria group gives the aspects related to the relationship between the supplier and the firm.

Appropriate equipment for sustainable manufacturing (Chen 2011; Vahdani et al. 2012), process/manufacturing capability management of the firm (Soroor et al. 2012; Omurca 2013; Vahdani et al. 2012), quality control system (Omurca 2013), Non-defective product rate (Bruno et al. 2012), maintenance management system (Vahdani et al. 2012), service level (Bruno et al. 2012), productivity (Khaleie et al. 2012; Ferreira and Borenstein 2012).

Moreover, flexibility (Chen and Chao 2012; Büyüközkan and Çifçi 2012; Omurca 2013), delivery performance (Kuo et al. 2010a, b; Chen and Chao 2012; Omurca 2013; Celebi and Bayraktar 2008; Chen 2011) are grouped as firm's abilities.

Beside the above mentioned criteria; price (Celebi and Bayraktar 2008; Chen 2011; Bruno et al. 2012; Soroor et al. 2012; Omurca 2013; Chen and Chao 2012; Khaleie et al. 2012), warranties and claims (Chu and Varma 2012; Bruno et al. 2012; Ha and Krishnan 2008; Omurca 2013), quality (Chen, et al. 2006; Kuo et al. 2010a, b; Celebi and Bayraktar 2008; Kumar et al. 2013; Chen and Chao 2012; Büyüközkan and Çifçi 2012; Chen 2011; Bruno et al. 2012; Soroor et al. 2012; Omurca 2013) are given as the criteria group that belongs to the product specifications.

By examining the literature, it is shown that approximately every qualification of the suppliers with wide diversity is scrutinized. In conducted case study, prominent criteria are chosen in order to evaluate the suppliers. The reason of that criteria selection is given below.

Quality It can be considered as the most popular evaluation criterion. Ho et al. (2010) indicates that 87.18 % of the investigated literature is that quality taken into account while selecting and evaluating supplier. Moreover, it can take the values 1–5 scale which is given as a final score while considering the sub-criteria such as non-defective product rate, quality control system structure, etc.

Manufacturing Capability Management Process/manufacturing capability management of the firm (Soroor et al. 2012; Omurca 2013; Vahdani et al. 2012) defines the firm abilities through the production effectiveness. Since it is the case in the real production environment, everything has a stochastic nature, so firms should adopt and be agile if and only if they have a good manufacturing capability management. In our case, the suppliers of a durable good manufacturing firm, so this makes the manufacturing capability vital for its suppliers. Manufacturing capability is also related to amount of production lot even firm can produce high qualified goods if they are not able to procure the desired amount of demand they are considered to be unsuccessful in terms of manufacturing capability.

Service Level As Bruno et al. (2012) point out that it can be considered as, punctuality of the supply and respecting all of the contract conditions. The delay of the production line can occur due to the shortage of raw material or other types of stock or services that are necessary for the survival of the production process. This creates a good reason to control the service level of the suppliers in order to evaluate them.

Geographic Location In order to decrease the lead-time and delivery costs, geographical location has a strategic position for the success of the togetherness of firm and its supplier. As it is mentioned before, due to the stochastic nature of the real time production, speed and agility become more and more significant not only for time perspective but also for money.

Willingness to Integrate SCM Kuo et al. (2010a, b) defines it as to share expertise and conflict resolution. In globalized world, recent trend of supply chain is to improve the ability and knowledge about the chain as a single body. So most of the leading firms prefer to share their knowledge with their suppliers and give an appropriate education to them. Firms are not able to accomplish such a goal unless the suppliers are willing to accept this integration. So for an improved supply chain this criterion is a must.

Delivery Performance It is one of the most popular criteria in the evaluation process. Kuo et al. (2010a, b), Chen and Chao (2012), Omurca (2013), Celebi and Bayraktar (2008), Chen (2011) are some of the studies that delivery performance is taken into account. It can be considered as the combination of some aspects, such as delivery quality, timeliness, reliability etc. It is evaluated on a percentage basis in this study. It also aims to capture the opinion of the firm about the supplier, which makes it valuable to concentrate on.

Responsiveness for environmental sustainability This criterion stands for the awareness of supplier about the environmental issues. The negative effects of the old-fashioned production conditions give a birth to environmental problems such as global warming. So firms should not be contented with their preventive actions on environmental side effects but also watch over its suppliers' actions. This criterion aims at evaluating suppliers in order to control preventive actions on their productions.

4 Methodology: Fuzzy Clustering

Clustering is the process of partitioning a set of data objects into homogenous subsets. The aim of clustering techniques is to organize the set of data such that each object in a subset, also called cluster, is similar to the other objects in the same cluster and dissimilar to objects in other clusters (Han et al. 2011). The similarity and dissimilarity is mathematically defined by distance (Babuska 2009).

Unsupervised methods are generally used in cluster analysis which means that generally unlabeled data is used and clustering is formed by measuring the distances between objects. In supervised methods, objects are identified by a label and the performance of the method can be evaluated by comparing the output of the methodology and the real label. However, since the data used by clustering techniques are unlabeled, there is no error or reward signal to evaluate a potential solution. But still literature provides some calculations, called cluster validity, for quantitative evaluation clustering algorithm's outputs (Theodoridis and Koutroumbas 2008).

There are various clustering algorithms proposed in the literature. One of the basic distinctions between the clustering algorithms can be identified as the objects membership values to the clusters, these can be crisp or fuzzy. In crisp clustering, the data is partitioned into a specified number of mutually exclusive subsets. In other words the individual objects either does or does not belong to a cluster. However in fuzzy clustering the objects can belong to several clusters simultaneously, with different degrees of membership. Since the results of fuzzy clustering ranges between 0 and 1 in most cases the results are more natural when compared to crisp clustering. This is the case for objects near to the boundaries, crisp partitioning does not make a difference in the membership value of this object, however in fuzzy clustering they are assigned membership degrees between 0 and 1 indicating their partial membership.

The data used in cluster analysis are gathered from observations with n measured variables each. An individual observation forms an n-dimensional column vector

$$\mathrm{z_k} = [\mathrm{z_{1k}}, \ldots, \mathrm{z_{nk}}]^{\mathrm{T}} \tag{1}$$

and the dataset that consists of N observations is represented as an n $\times$ N matrix. In this context, the aim of fuzzy clustering is to partition the whole data set Z into a specific number of clusters "c",

$$Z = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1N} \\ z_{21} & z_{22} & \cdots & z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nN} \end{bmatrix} \tag{2}$$

The results of clustering analysis can be represented by the partition matrix U. In this matrix, each element μ_{ij} shows the degree to which element $\mathrm{z_j}$ belongs to cluster ci. In crisp approaches the membership function μ_{ij} gets the value 0 or 1 but in the fuzzy case this value can get any real value in [0, 1].

$$U = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1N} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{c1} & \mu_{c2} & \cdots & \mu_{cN} \end{bmatrix} \tag{3}$$

Fuzzy c-means (FCM) is one of the most popular fuzzy clustering techniques. FCM is based on minimization of the following objective function:

$$J(Z, U, V) = \sum_{i=1}^{c} \sum_{j=1}^{N} (\mu_{ij})^m \left\| z_j - v_i \right\|^2 \tag{4}$$

where Z is the data set to be partitioned, U is the fuzzy partition matrix; V is the vector of cluster centers. N is the number of observations, c is the number of clusters and μ is the membership value, m is the parameter called fuzzifier, which determines the fuzziness of the resulting clusters. The fuzzifier can get values 1 and more. When m = 1 then the clusters are formed in crisp format. In the formula, $z_k - v_i$ shows the distance between observation k and the center of cluster i.

The minimization of the mentioned objective function represents a nonlinear optimization problem that can be solved by using a variety of methods such as iterative minimization, simulated annealing or genetic algorithms.

The most popular method which is known as fuzzy c-means (FCM) algorithm consists of the following steps (Babuska 2009).

Initialize U=[u_{ij}] matrix, $U^{(0)}$

At k-step: calculate the centers vectors $V^{(k)}$=[v_i] with $U^{(k)}$

$$v_i = \frac{\sum_{i=1}^{N} \mu_{ij}^{m} \cdot z_j}{\sum_{i=1}^{N} \mu_{ij}^{m}}$$

Update $U^{(k)}$, $U^{(k+1)}$

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{\| z_j \quad v_i \|}{\| z_j - v_k \|} \right)^{\frac{2}{m-1}}}$$

If $\| U^{(k+1)} - U^{(k)} \| < \delta$ then STOP; otherwise return to step 2.

In fuzzy approaches to clustering analysis, some conditions are defined to transform crisp approach to fuzzy. Initially, Ruspini (1970) defines the conditions for a fuzzy partition matrix as follows:

$$\mu_{ik} \in [0, 1], 1 \le i \le c, 1 \le k \le N, \tag{5}$$

$$\sum_{i=1}^{c} \mu_{ik} = 1, 1 \le k \le N, \tag{6}$$

$$0 < \sum_{k=1}^{N} \mu_{ik} < N, 1 \le i \le c \tag{7}$$

Equation (6) constrains the sum of each column to 1, and thus the total membership of each zk in Z equals one.

Later Krishnapuram and Keller (1993) propose by relaxing the constraint (6). The relaxed constraint ensures that each point is assigned to at least one of the fuzzy subsets with a membership greater than zero. The results may differ between fuzzy and probabilistic partition, for example in a case with two clusters ($c = 2$) an outlier observation that is far away from both cluster centers can have a membership equal to 0.5 (so that the sum of the values is equal to one). However in probabilistic case the membership for the outlier can be lower values showing that it is an outlier.

When clustering, the analyzer has to set some important parameters for getting beneficial results from the analysis. While some software packages can enable the user to setup some other parameters, most important parameters are, number of clusters, fuzzifier and termination criteria.

Number of clusters The number of clusters "c" is the parameter that influences the clustering results most. Before the clustering study, if the analyzer does not have any priori information about the structure of the data, (s)he has to make assumptions about how many clusters can exist within the data. The FCM algorithm then searches for chosen number of clusters. It is expected that when the number of clusters parameter is equal to the number of groups that actually exist in the data, the FCM will identify them correctly. But if this is not the case, misclassifications appear, and the clusters are not correctly separated. Put in another way, the algorithm finds the expected number of clusters regardless of whether they are real thus the validity of the results has to be checked. In the literature validity measures are proposed to assess the goodness of the obtained partition (Bezdek 1981; Gath and Geva 1989; Pal and Bezdek 1995). For the FCM algorithm, the Xie-Beni index (Xie and Beni 1991) has been found to perform well in practice (Han et al. 2011).

$$X(Z;U,V) = \frac{\sum_{i=1}^{c}\sum_{j=1}^{N}(\mu_{ij})^m \left\|z_j - v_i\right\|^2}{c.\min\limits_{i\neq j}\left\|v_i - v_j\right\|^2} \tag{8}$$

This index shows the ratio of the total within-group variance and the separation of the cluster centers. For different number of clusters the best is the one that minimizes the Xie-Beni index.

Another validity measure for FCM is silhouettes values defined by Kaufman and Rousseeuw (1990). The value measures how well each object has been classified by comparing its dissimilarity within its cluster to its dissimilarity with its nearest neighbor (Hintze 2007). The silhouettes value can range from minus one to one. The values of s (silhouettes) that is close to one shows a good classification, if the value is near zero, the object is between clusters two clusters and when the value is close to negative one, it means that the object is poorly classified. Using average silhouette the data miner can understand the most appropriate number of clusters. The ideal number of cluster minimizes the average silhouette.

Fuzziness Parameter (Fuzzifier) The weighting exponent m at the objective function is called the fuzziness parameter of Fuzzifier. As m approaches one from above, the partition becomes crisp and the membership values get values 0 or 1.As m gets higher values, the partition becomes completely fuzzy and the membership values goes to the limit 1/c. These limit properties are independent of the optimization method used (Pal and Bezdek 1995). Generally, the value of the fuzzifier is set to two initially.

Termination Criterion The FCM algorithm does not stop unless the difference between "U" in two consequent iterations is smaller than the termination parameter. The general choice for the criterion is 0.001; however the value 0.01 works well in most cases, while drastically reducing the computing times

5 Application

The supplier evaluation approach is applied to a firm which produces durable goods. The evaluation criteria are selected considering the conditions of the manufacturing company. Since the real data about supplier evaluations cannot be gathered due to security restrictions, the applied data are produced hypothetically. The software package NCSS (2007 Version) is used to run the fuzzy clustering algorithm (Table 1).

In this study eight different criteria is used (X1 = Quality, X2 = Manufacturing Capability, X3 = Service Level, X4 = Geographic Location, X5 = Willingness to Integrate SCM, X6 = Delivery Performance, X7 = Price, X8 = Responsiveness for environmental sustainability). The criteria, measurement scale and related definitions are given in Table 2.

In classical Fuzzy c-means algorithm, the c parameter is given by the analyzer and the clusters are determined based on this parameter. NCSS software enables trying a range of c values and analyzing the different scenarios. In this study, the c values are selected between 2 and 6. Table 1 shows the results for different clustering. As an interpretation of these results, the c value that maintain the highest average Silhouette and F(C) values and lowest D(C) values should be selected. The silhouette values between 0.7 and 1 is considered as a good separation. Analyzing the results represented in Table 3 the number of clusters is selected as four.

The clustering results (c = 4) for 80 suppliers are given in Table 4. The results show that Cluster 4 gets the biggest portion within all clusters. It is followed by cluster 3 with 27 suppliers. Based on the medoid values, each cluster is given a descriptive name as shown in Table 4.

The interpretation of the clustering results is mainly done by using the medoid table represented in Table 5. The medoid values for each cluster represents the characteristics of the cluster, thus the suppliers in each cluster can be characterized by these values and managerial decisions can be given.

According to the values shown in Table 5, the following comments can be stated for the clusters. The suppliers in Cluster 1 (Best of its kind) produce high quality goods with good manufacturing capability that results in 95 % service

Table 1 Current studies about supplier selection and methods used

Year	Authors	ANN	GA	PSO	AHP	ANP	TOPSIS	PROMETHE	ELECTRE	DT	QFD	FST	SIMULATION	TAGUCHI	C-MEANS	VIKOR	STOKASTIK	LP	MOP	GP	CM	DEA
2012	Vahdani, B. et al.	✔																				
2012	Khaleie, S. et al.				✔																	
2012	Erdem, A.S. and Göçen, E.					✔																
2012	Bruno, G. et al.											✔										
2012	Lin, R.H.											✔	✔						✔			
2012	Xiao, Z. et al.						✔		✔			✔				✔						
2012	Amindoust, A. et al.					✔																
2012	Fasangharia, E.T.					✔																
2013	Ilhan Omurca, S.							✔														
2012	Büyüközkan, G. and Cifçi, G.				✔							✔										
2012	Chu, T. and Varma, R.					✔														✔		
2012	Che, Z.H.											✔							✔			
2012	Soroor, J et al.																		✔			
2012	Ekici, A.																					✔
2011	Chen, Y.J.				✔							✔										
2011	Shemshadi, A. et al.				✔		✔															
2011	Soner Kara, S.	✔										✔										
2011	Zeydana, M. et al.											✔										
2011	Feng, B. et al.											✔										
2010	Vahdani, B. and Zandieh, M.											✔										
2010	yang et al.											✔										
2010	Kuo, R.J. et al.											✔			✔							
2010	Kuo, R.J. et al.	✔			✔																	✔
2010	Liao, C.N. and Kao, H.P.				✔		✔															
2010	Hsu, B.S. et al.				✔															✔		
2010	Awasthi, A. et al.					✔						✔									✔	
2010	Ravindran, A.R. et al.					✔	✔					✔										
2010	Keskin, G.A. et al.											✔								✔		

(continued)

Table 1 (continued)

Year	Authors	ANN	GA	PSO	AHP	ANP	TOPSIS	PROMETHE	ELECTRE	DT	QFD	FST	SIMULATION	TAGUCHI	C-MEANS	VIKOR	STOKASTIK	LP	MOP	GP	CM	DEA
2009	Boran et al.																					
2009	Wang, J.W. et al.																		✔			
2009	Ordoobadi S.M.																					✔
2009	Ming-Hung S. and Hsien-Chung W.									✔												✔
2009	Yang, G. et al.					✔						✔										
2099	Tsai, W.H. and Hung, S.J.						✔					✔										
2009	Wu, D.	✔				✔	✔					✔										
2009	Razmi, J. et al.											✔										
2009	Shih, K.H. et al.	✔																				
2008	Chan, F.T.S. et al.						✔															✔
2008	Mendoza, A. et al.											✔				✔						
2008	Talluri, S. et al.			✔								✔										
2008	Celebi, D. and Bayraktar, D.	✔		✔										✔								
2008	Mendoza, A. and Ventura, V.		✔									✔										
2007	Hou, J. et al.										✔	✔										
2007	Gencer, C. and Gürpinar, D.			✔							✔	✔										
2007	Araz, C. and Ozkarahan, I.						✔										✔					
2007	Wadhwa, V. and Ravindran, R.																					
2007	Wu, T. et al.						✔					✔										
2007	Jain, V. et al.											✔										
2007	Ramanathan, R			✔																		
2006	Amida, A. et al.																		✔			

Table 2 The criteria and related evaluation scale

	Criteria	Scale	Definition
X1	Quality	1–5 scale	5 is the best, 1 is the worst value
X2	Manufacturing capability	A-B–C-D	A is the best, D is the worst value
X3	Service level	Percentage	100 % is the best value
X4	Geographic location	A-B–C-D	A is the best, D is the worst value
X5	Willingness to integrate SCM	1–5 scale	5 is the best, 1 is the worst value
X6	Delivery Performance	Percentage	100 % is the best value
X7	Price	A-B–C-D	A is the best, D is the worst value
X8	Responsiveness for environmental sustainability	A-B–C	A is the best, C is the worst value

Table 3 Results summary

Number clusters	Average distance	Average silhouette	F(U)	Fc(U)	D(U)	Dc(U)
2	10.345109	0.770421	0.9101	0.820	0.066	0.132
3	2.110394	0.924989	0.9746	0.962	0.021	0.032
4	0.676300	0.978303	0.9815	0.975	0.006	0.008
5	0.605252	0.895564	0.9143	0.9021	0.0501	0.0573
6	0.517681	−0.150000	0.9084	0.8970	0.0596	0.0671

Table 4 Clusters and descriptive names

	# of suppliers	Descriptive name
Cluster 1	3	Best of its kind
Cluster 2	10	Watch out
Cluster 3	27	Boutique suppliers
Cluster 4	40	Logistic leaders

Table 5 Medoid values for the four clusters

	Cluster 1	Cluster 2	Cluster 3	Cluster 4
X1 = Quality	4	1	5	1
X2 = Manufacturing capability	A	B	D	A
X3 = Service level	95	75	85	90
X4 = Geographic location	C	A	C	A
X5 = Willingness to integrate SCM	1	4	4	3
X6 = Delivery performance	95	75	75	90
X7 = Price	D	A	C	A
X8 = Responsiveness for environmental sustainability	A	C	A	B

level. On the contrary they are not willing to integrate supply chain management with other firms. Since the goods that they supply have high quality so it is more costly than the average qualified goods in the sector. They can be considered as environment protectors because they show an intensive responsiveness to sustainability issues.

"Watch out" suppliers (Cluster 2) pick the firms that are not very reliable in their production and logistics but they offer the cheapest price compared with their competitors. They are willing to coordinate with their customers. Due to the geographical reasons the procurement cost is also below the sector level.

Suppliers that take place in Cluster 3 (Boutique Suppliers) produce high quality products with low manufacturing capability. So they only procure small number of orders. Boutique Suppliers have moderate performance in service with 85 %. Even they are not located in A level (which makes the logistics costs increase) 27 suppliers are grouped as boutique suppliers, due to the fact that they produce high qualified and customized goods with low lot sizes.

The most important specification of the suppliers grouped under cluster 4 is the agility of them. They can easily adopt the changes in the demand sizes and they supply desired good as fast as possible. Moreover their price is below the sector average but as a disadvantage their goods are not well qualified while comparing substitutes.

6 Conclusion

Due to recent changes, organizations become more dependent on their suppliers which makes the suppliers crucial component for the future position of the firm. This situation attaches importance to the selection and evaluation of the suppliers. In this study the literature on supplier evaluation is given and fuzzy clustering algorithm is used to analyze the performance of the suppliers. Fuzzy Set Theory comes forward due to its ability to create a suitable environment to appraise the qualitative data. Since some of the specifications of suppliers rely on qualitative data, FST gives a chance to give unbiased estimations on suppliers.

In the application section of this study the suppliers of the company that produces durable goods are evaluated. To this end first the criteria that are mostly used within the literature are investigated and the customized criteria are selected among the alternatives. Each supplier is assessed considering the selected criteria and finally four different clusters are generated as a result of fuzzy c-means methodology. By clustering a bunch of suppliers, managers are able to define suppliers that have similar performance and create general prescriptions and strategies for each cluster. As in our study, the suppliers grouped as the "Watch out" suppliers have scores below the averages but they show a promising structure in willingness to integrate SCM criteria with high score which shows that they are open for improvements and suggestions of the considered company. So manufacturing company can decide on whether to end up the relationship with these suppliers or schedule an educational program to improve their production qualifications. By doing these, both the relationship between companies is enhanced and the manufacturing company utilizes the low price with higher quality goods.

As a further study, the suppliers that belong to different supply chains (health, automotive, yacht production) can be evaluated in order to reveal the change in the importance of the considered criteria. Also, the clustering approach can be improved by integrating weights for each criterion. By this way, the distances with respect to a relatively higher importance can be emphasized which can improve the results of the clustering study.

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Investigating Organizational Characteristics for Sustainable Supply Chain Planning Under Fuzziness

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Abstract Sustainable supply chains are essential to sustain modern business growth and ensure healthy market environment. In this chapter, we address the important characteristics that constituent organizations of supply chains should possess in order to achieve the social, economic and environmental objectives of sustainability. These characteristics (criteria) are obtained using Affinity Diagram. Then, a committee of decision makers is formed to provide linguistic ratings to the candidate organizations (alternatives) against the selected criteria. The linguistic ratings are then transformed into fuzzy numbers and subject to multicriteria decision making technique called VIKOR for sustainability assessment of organizations. A numerical illustration is provided to demonstrate the applicability of the proposed approach.

Keywords Sustainable supply chain · Multicriteria decision making · Fuzzy VIKOR · Sustainability evaluation · Sustainable organizations

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1 Introduction

A supply chain is a network of organizations that are involved in different processes and activities that produce value in the form of products and services in the hands of the ultimate consumer (Christopher 1998). The Supply Chain (SC) is a metastructure (Grzybowska 2010a). A metastructure is an intermediate form between a single enterprise (microstructure) and global economy (macrostructure). The sustainable supply chain is related to the broader concept of a "sustainable economy." This view extends the idea of Total Quality Environmental Management (TQEM) beyond the boundaries of organizations and beyond the current generation of products and services. Fundamental to developing a sustainable economy is the recognition that environmental initiatives may start as operational initiatives to reduce waste and emissions, but these initiatives must grow to a point where the strategy and the vision of the company incorporates environmental issues (Walton et al. 1998).

The World Business Council for Sustainable Development defines sustainability as the "development that meets the needs of the present without compromising the ability of future generations to meet their own needs" (Peters et al. 2007). So, the fundamental question that arises is what are sustainable supply chains? According to Business for social responsibility (2007), a sustainable supply chain is a system of aligned business activities throughout the lifecycle of products that creates value to stakeholders, ensures ongoing commercial success, and improves the well-being of people and the environment. Carter and Rogers (2008) refer to sustainable supply chain as an integration of social, environmental, and economic issues in traditional supply chain. Srivastava (2007) associate the potential for reducing long term risks in a supply chain such as resource depletion, fluctuations in energy costs, product liabilities, and pollution and waste management through sustainable supply chains. According to NZBCSD (2003), "Sustainable supply chains involve management of raw materials and services from suppliers to manufacturer/service provider to customer and back with improvement of social, economic and environmental impacts explicitly considered". Therefore, in order to develop sustainable supply chains, all the involved organizations should work cohesively and constructively towards the bigger goal of achieving the triple bottom line objectives (economic, environment, social) of sustainability (Elkington 1994; Seuring and Müller 2008; Bai et al. 2012; Seuring 2013).

What characteristics or constitutive elements differentiate whether a given commercial entity or organization is a part of an eco-friendly supply chain? If one collects a group of expert opinions, it may soon turn out that their answers would greatly differ. First of all, this is a result of the complexity and vagueness of the addressed issue. But not only this, it is also a result of the lack of a definition of a complex nature (the definitions in use are either too wide or too narrow which makes them vague; the definitions do not act as protection against equivocalness and are not sufficiently clear). Finally, the discrepancies and differences in the lists of such elements may arise on the grounds of the differences between scientific or commercial environments and various interpretations of both the phenomenon itself and the term used to describe it.

These discrepancies indicate that a necessity exists to demonstrate significant constitutive elements (characteristics) of the supply chains. Hence, for the purposes of this work, an attempt has been made to list the typical elements that are common to all supply chains functioning within the market or one that will be designed or organized. At the same time, it is a typical list of constitutive elements allowing for the identification of the supply chains that differ considerably.

2 Characterizing Organizations for Developing Sustainable Supply Chains

In order to analyze organizations from sustainable supply chain perspective, we must identify the key characteristics. We have used Affinity diagram technique to identify these characteristics. Affinity Diagram is one of the seven quality management and planning tools and used to generate ideas for decision making through brainstorming, surveys, interviews etc. (Awasthi and Chauhan 2012; Foster 2008; Shafer et al. 2005). In our study, we conducted Affinity diagram exercise with several supply chain experts from Academia. The results of Affinity diagram yield two categories of characteristics that must be looked for. The first category comprises of the fundamental characteristics which are a must for any organization involved in the supply chain. The second category comprises of sustainability focused characteristics which are an essential for organizations in order to develop sustainable supply chains. Figure 1 presents the results of the Affinity Diagram.

We will now present in detail the various characteristics of the affinity diagram represented in Fig. 1.

2.1 Fundamental Characteristics

The fundamental characteristics are vital to achieving the economic goals for supply chains. A supply chain is created by a certain, specific group (set) of enterprises. One should specify the elements (factors) that determine the creation of a supply chain and therefore, the first factor is the supply chain size. The number of enterprises if two may be regarded as an extremely simple supply chain. The enterprises therein must have specific, usually complementary roles ascribed. This is the second constitutive element. Moreover, relationship and business dependencies must exist between them. Therefore, a relationship exists between enterprises, a contact resulting from roles being played and adopted status of an

Fig. 1 Affinity diagram of organizational characteristics for sustainable supply chain planning

enterprise. Such a contact affects the coherence of the supply chain thus created. Another constitutive element is communication which is essential for constructing and correctly maintaining a supply chain.

These elements may be used to create the definable characteristics. Establishing a list of common constitutive elements determining the shape of the supply chain thus seems both grounded and necessary. It is also obvious and unequivocal. If one knows the constitutive elements, it is possible to identify any (also a different) supply chain (Grzybowska 2010b).

2.1.1 The Supply Chain Size and Arrangement

A supply chain made up by three enterprises is also uncomplicated due to the number of business partners interacting with one another (relationships, contact). Such relationships refer to the business activity of these enterprises and allow for the creation of specific mutual approaches. These relationships shall be strengthened if the activity of both commercial entities is complementary. The business activity will thus mutually complement the activity of the counterparty. At this point, one should refer to the theory of sociology and social groups. According to this theory, it is possible to determine the number of interactions by applying the formula (Turowski 1993):

$$\frac{n \cdot (n-1)}{2},$$

wherein "n" denotes the number of supply chain participants (description of "n" has been modified for the need of this article). However, one should not that these are the contacts and mutual interactions of business partners operating within one supply chain.

2.1.2 The Ascribed Roles of the Supply Chain Participants

Irrespective of the number of enterprises a supply chain contains, each of its participants fulfils an ascribed role. The simplest and most common roles are the role of a supplier and recipient. One enterprise is a contractor (supplier), the other is an ordering party (recipient) and this role is adopted consciously depending on the duties performed (duty role depending on the duty ordered). However in a supply chain, one enterprise may, depending on business processes, act both as a supplier and recipient. It is a consequence of complexity of actions performed within the framework of a supply chain. These roles are definite and tend to be repetitive and foreseeable in nature. The roles of supplier-recipient are complementary roles, i.e. these roles complement one another and supplement the relationships occurring between the enterprises. This means that if one enterprise is a recipient (client) the other must be a supplier; for proper functioning, one needs the other. The enterprises in a supply chain are mutually connected since each of them performs a role upon which the counterparty depends. This results in the increasing integration of the supply chain. The enterprise (e.g. as a supplier) fulfills the role by performing given actions and accepting norms (or normative customs) relating to these actions.

2.1.3 Status of Participants of a Supply Chain

The roles ascribed to or adopted by an enterprise in a supply chain are connected with the status of a given enterprise in the said supply chain. The more comprehensive the role performed by the enterprise, the higher the status of such enterprise. Status, according to S. R. Robbins is a socially defined post (position) or ascribed importance (Robbins 2003, p. 183).

However, the status is also related to successes and achievements, both the individual success of an enterprise, as well as global successes within the entire supply chain. In the latter case, the status will refer to the contribution of the enterprise to the joint success and joint achievement of the supply chain. The status of an enterprise also determines the prestige and position the enterprise occupies within the organizational hierarchy of this supply chain, whereby the said status is not synonymous with the power, as the enterprise participating in a supply chain, e.g. logistics operator may boast high status resulting from considerable experience and high service quality, however it will still lack the power to affect the operation of other enterprises within the said supply chain.

2.1.4 Coherence of the Supply Chain

Between the enterprises functioning in a supply chain, a link of dependencies and mutual relationships develops. The strength of positive links may be referred to as the coherence. Supply chain coherence will depend upon the degree of development of relationships between the enterprises within this chain. Very coherent supply chains show strong relationships and a high degree of loyalty. As noted by J. Szczepański: each community, if it is to exist and develop, must have a link that acts as its internal binder, ensuring that individual and collective needs are satisfied, the loyalty of members towards the community is maintained as is the opposition (or cooperation) of the community against (with) other communities; in other words, each community must be internally organized and ordered (Szczepanski 1970).

2.1.5 Communication in the Supply Chain

In the course of cooperation of enterprises within a supply chain it is vital to effectively communicate. Communication is construed as the process of message transferring. As set out by Penc (2001), the nature of communication is the mutual understanding between the message sender and recipient in the scope of information transferred. It means that the meaning of the message received by the recipient must be identical with the meaning intended by the sender. Good communication consists in the transfer of proper information in the atmosphere of mutual trust (Penc 2001).

Information sharing leads to visibility in supply chains which in turn leads to cooperation among supply chain partners. According to Hahn et al. (2000) effective communication and coordination among all elements of supply chain are essential to its success. Lee and Whang (2000) suggested that information is a basic enabler for tight coordination in supply chains.

2.1.6 Coordination in the Supply Chain

Coordination has become a significant factor of the integration of various parts of the organization as well as various organizations of the supply chain. This seems to be a key factor in the success of logistics management. It is also an element allowing for a common list of tasks to be accomplished as well as common objectives to be achieved in the selected system (micro- or meta-).

Coordination is managing dependencies between activities (Malone and Crowston 1994). Coordination among independent firms, such as raw-material suppliers, manufacturers, distributors, third-party logistics providers and retailers, is the key to attaining the flexibility necessary to enable them to progressively improve logistics processes in response to rapidly changing market conditions. Poor coordination among the chain members can cause dysfunctional operational performance (Simatupang et al. 2002). Some of the negative consequences of poor coordination include higher inventory costs, longer delivery times, higher transportation costs, higher levels of loss and damage, and lowered customer service (Crowston et al. 2004).

2.1.7 Quality Management

Quality management in context of supply chains is defined as a systems-based approach to performance improvement that leverages opportunities created by upstream and downstream linkages with suppliers (Foster 2008) and customers. Modern business environment is highly competitive and as the competition moves beyond a single firm to a larger scope in the supply chain, quality management focus is no longer limited to management of internal practices alone. Instead, quality managers must integrate their firm's practices with those of customers and suppliers (Kaynak et al. 2008).

2.2 Sustainability Focused Characteristics

The sustainability focused characteristics put focus on the environmental and social goals for supply chains. This involves participation of involved organizations in environmental friendly initiatives, assuming social responsibility, adoption of environmental standards, management commitment, employee training, risk management, agility, focus on voice of the customer, technology management etc.

2.2.1 Participation in Environment Friendly Initiatives

Sustainable production and consumption will be the main characteristics of future societies to provide sustainable development and a sustainable society. Therefore, all industries are seeking to minimize their environmental impacts. Green manufacturing, which is an advanced mode of manufacturing, involves application of sustainable science to the manufacturing industry on a very wide range of topics, such as environmental consciousness, life cycle thinking, and sustainable development. Green procurement has an independent effect on the whole environmental value chain, whether only one or more companies of the chain choose to implement it (Guenther et al. 2010). According to Guenther et al. (2010) and Hamprecht (2006), Green procurement works together with suppliers, R&D and operations for designing solutions to minimize environmental impacts and address stakeholder concerns. In this capacity, it can serve to control and reduce environmental impacts within the whole life-cycle of a product, and improve life-cycle analyses as well.

2.2.2 Social Responsibility

One of the tiers of sustainability is the social side, which has been neglected by most authors. The parameter to measure social aspect is Quality of life which is an important enabler for planning sustainable supply chains. Zaklad et al. (2004)

point that people are responsible for driving at least 50 % of performance and inherently human factor is very important. It is important to build the human capabilities needed to sustain an innovative, collaborative, and integrated supply chain. Other aspects of social responsibility include ensuring labor equity, employee healthcare, safety etc. (Hutchins and Sutherland 2008).

2.2.3 Management Commitment

Management commitment is very essential for driving sustainability goals in organizations. Commitment from management includes an effort and financial backing from the upper management to implement sustainability. The most famous con of green supply chain management is that the companies do not change practice but merely advertise that they do, creating a greenwash (Greer and Bruno 1996). Therefore, the goals should be clearly set, indicated as it is to employees, and well supported by management to make sustainability a success in organizations.

2.2.4 Risk Management

Risks are associated with negative consequences or impact of different processes, activities and resources of supply chains (Christopher and Lee 2004) and supply chain (Paulson 2005; Spekman and Davis 2004). There can be various kinds of risks varying from financial to operational risks. Risk management is very crucial part of supply chain as organizations with ineffective risk management strategies will run out of business very soon.

2.2.5 Voice of the Customer

Voice of the customer involves listening to customer requirements, complaints and suggestions to improve product and service quality in organizations. Organization with effective listening to voice of the customer programs achieve higher customer satisfaction rates and can therefore remain competitive in markets for long times. Clearly, companies are most likely to improve their environmental performance when public pressure or strong regulations exist. Sometimes, companies themselves lobby for regulations if they have developed an environmentally friendly technology and believe that regulations requiring their technology would give them a competitive advantage (Kleindorfer et al. 2005).

2.2.6 Technology Management

Use of IT tools to monitor the supply chain activities, sharing of information among the partners can lead to greater visibility in supply chain, thereby providing better cooperation among different levels of the supply chain. Electronic data interchange and internet have enabled partners in supply chains to act upon same data rather than rely on distorted and noisy data that emerges in an extended supply chain (Lee and Whang 2000). Swafford et al. (2008) emphasize the role of IT integration and flexibility in achieving supply chain agility.

2.2.7 Employee Training

Employee training is vital to achieve social sustainability and also enable employees with necessary expertise to perform their tasks efficiently. A company's power comes from the physical and mental strength of their workers. Therefore, sustainability of being powerful for an organization is tied to the physical and psychological health of its employees, and their knowledge and skills. Since the importance of human resources on the organizational success has been realized, responsibility and authority of Human Resources Departments have broadened, especially in accommodation sector. Organizing Employee Trainings and maintaining Occupational Safety and Health are among the main functions of Human Resources Management departments (Sari 2009). These two functions interact and they both serve the aim of protecting employees physical, psychological and social health.

2.2.8 Adoption of Environmental Standards

Adoption of environmental standards such as ISO 14001 brings environmental gains, cost reductions, as its adoption reduces the firm‘s environmental impact and improves aspects of operational efficiency and effectiveness. Furthermore, ISO 14001 provides an external benefit through signaling the firm‘s commitment towards environmental management to its external stakeholders (Boiral and Sala 1998; Rondinelli and Vastag 2000).

2.2.9 Agility

Agility has been proposed as a response to the high levels of complexity and uncertainty in modern markets (Christopher and Jüttner 2000; Gunasekaran 1999; Yusuf et al. 1999). All the involved organizations in sustainable chains should be agile to exploit profitable opportunities in the volatile market place that is so common nowadays.

3 Addressing Uncertainty in Sustainability Assesment

In Sect. 2, we presented the fundamental and sustainability focused characteristics which organizations should possess for developing sustainable supply chains. Therefore, to qualify organizations from sustainable supply chain development perspective, they should be subject to quantitative evaluation using the proposed characteristics. However, it has been observed in general practice, that often there is almost none or very limited data available on these characteristics for organizations, thereby making the evaluation process difficult. To address this situation, we will make use of linguistics ratings for evaluation purposes. The linguistics ratings will comprise of qualitative responses such as Good, Very Good, Poor, Very Poor for assessing the characteristics (or criteria). The linguistics ratings are much easier and comfortable to use for decision makers involved in the sustainability evaluation process. The linguistic ratings of the experts will be then transformed into fuzzy numbers for further processing through multicriteria decision making methods in Sect. 4.

3.1 Preliminaries of Fuzzy Set Theory

Definition 1 A triangular fuzzy number is represented as a triplet $\tilde{a} = (\text{a1, a2, a3})$ (Fig. 2). Due to their conceptual and computation simplicity, triangular fuzzy numbers are very commonly used in practical applications (Pedrycz 1994; Klir and Yuan 1995). The membership function $\mu_{\tilde{a}}(x)$ r $\tilde{a}$ is given by:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, x \leq a_1, \\ \dfrac{x - a_1}{a_2 - a_1}, a_1 < x \leq a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, a_2 < x \leq a_3, \\ 0, x > a_3 \end{cases} \tag{1}$$

where *a1, a2, a3* are real numbers and *a1* < *a2* < *a3*. The value of x at *a2* gives the maximal grade of $\mu_{\tilde{a}}(x)$ i.e., $\mu_{\tilde{a}}(x) = 1$; it is the most probable value of the evaluation data. The value of x at *a1* gives the minimal grade of $\mu_{\tilde{a}}(x)$ i.e., $\mu_{\tilde{a}}(x) = 0$; it is the least probable value of the evaluation data. The narrower the interval [*a1,a3*], the lower is the fuzziness of the evaluation data.

Definition 2 In fuzzy set theory, conversion scales are applied to transform the linguistic terms into fuzzy numbers. In this chapter, we will use a scale of 1–9 to rate the criteria and the alternatives. Tables 1 and 2 present the linguistic variables and fuzzy ratings used for rating the alternatives and the criteria in the decision making process.

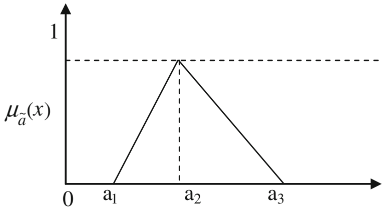

Fig. 2 Triangular fuzzy number ã

Table 1 Linguistic ratings for alternatives

Linguistic term	Membership function
Very poor (VP)	(1, 1, 3)
Poor (P)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Good (G)	(5, 7, 9)
Very good (VG)	(7, 9, 9)

Table 2 Linguistic ratings for criteria

Linguistic term	Membership function
Very low	(1, 1, 3)
Low	(1, 3, 5)
Medium	(3, 5, 7)
High	(5, 7, 9)
Very high	(7, 9, 9)

4 Multicriteria Decision Methodology for Evaluating Organizations

The proposed multicriteria decision making methodology for evaluation organizations to develop sustainable supply chains is based on the fuzzy VIKOR technique. VIKOR (in Serbian: VlseKriterijumska Optimizacija I Kompromisno Resenje) is a multicriteria decision making technique whose foundation lies in finding a compromise solution (Opricovic 1998). In other words, a feasible solution that is closest to the ideal solution. The fuzzy VIKOR technique involves fuzzy assessments of criteria and alternatives in VIKOR. The various steps of Fuzzy VIKOR are presented as follows:

Step 1: Assignment of ratings to the criteria and the alternatives.

Let us assume there are m alternatives called $A = \{A_1, A_2.., A_m\}$ which are to be evaluated against n criteria, $C = \{C_1, C_2.., C_n\}$. The criteria weights are denoted by $w_j (j = 1, 2, .., n)$. The performance ratings of decision maker $D_k (k = 1, 2, .., K)$ for each alternative $A_i (i = 1, 2, .., m)$ with respect to criteria $C_j (j = 1, 2, .., n)$ are denoted by $\tilde{R}_k = \tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk}), i = 1, 2, ..m; j = 1, 2, .., n; k = 1, 2, .., K$ with membership function $\mu_{\tilde{R}_k}(x)$.

Step 2: Compute aggregate fuzzy ratings for the criteria and the alternatives.

If the fuzzy ratings of decision makers are described by triangular fuzzy number $\tilde{R}_k = (a_k, b_k, c_k); k = 1, 2, .., K$ then the aggregated fuzzy rating is given by $\tilde{R} = (a, b, c); k = 1, 2, .., K$ where;

$$a = \min_k\{a_k\}, b = \frac{1}{K}\sum_{k=1}^{K} b_k, c = \max_k\{c_k\} \tag{2}$$

If the fuzzy rating of the kth decision maker for alternative A_i and criteria C_j are given by $\tilde{x}_{ijk} = (a_{ijk}, b_{ijk}, c_{ijk})$ and the importance weight by $\tilde{w}_{jk} = (a_{jk}, b_{jk}, c_{jk}), i = 1, 2, ..m; j = 1, 2, ..n$ respectively, then the aggregated fuzzy ratings $(\tilde{x}_{ij})$ of alternatives with respect to each criteria based on Eq. (1) are given by $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ where

$$a_{ij} = \min_k\{a_{ijk}\}, b_{ij} = \frac{1}{K}\sum_{k=1}^{K} b_{ijk}, c_{ij} = \max_k\{c_{ijk}\} \tag{3}$$

The aggregated fuzzy weights $(\tilde{w}_j)$ of each criterion are calculated as $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ where

$$w_{j1} = \min_k\{w_{jk1}\}, w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{jk2}, w_{j3} = \max_k\{w_{jk3}\} \tag{4}$$

Step 3: Compute the fuzzy decision matrix.

The fuzzy decision matrix for the alternatives $(\tilde{D})$ and the criteria $(\tilde{W})$ is constructed as follows:

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ - \\ A_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{array}, i = 1, 2, \ldots, m; j = 1, 2, .., n \tag{5}$$

$$\tilde{W} = (\tilde{w}_1, \tilde{w}_2, .., \tilde{w}_n) \tag{6}$$

Step 4: Defuzzify the elements of fuzzy decision matrix for the criteria weights and the alternatives into crisp values. A fuzzy number $\tilde{a}$ = (a1, a2, a3) can be transformed into a crisp number a by employing the below equation (Yong 2006):

$$a = \frac{a_1 + 4a_2 + a_3}{6} \tag{7}$$

Step 5: Determine the best f_j^* and the worst values f_j^- of all criteria ratings $j = 1, 2, \ldots, n$

$$f_j^* = \max_i\{x_{ij}\} \tag{8}$$

$$f_j^- = \min_i \{x_{ij}\} \tag{9}$$

Step 6: Compute the values S_i and R_i using the relations

$$S_i = \sum_{j=1}^{n} w_j \frac{f_j^* - x_{ij}}{f_j^* - f_j^-} \tag{10}$$

$$R_i = \max_j w_j \frac{f_j^* - x_{ij}}{f_j^* - f_j^-} \tag{11}$$

Step 7: Compute the values Q_i as following

$$Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1 - v) \frac{R_i - R^*}{R^- - R^*} \tag{12}$$

where

$$\begin{aligned} S^* &= \min_i S_i; \\ S^- &= \max_i S_i; \\ R^* &= \min_i R_i; \\ R^- &= \max_i R_i; \end{aligned} \tag{13}$$

And v_ϵ [0,1] is the weight for the strategy of maximum group utility and $1 - v$ is the weight of the individual regret.

Step 8: Rank the alternatives, sorting by the values S, R and Q in ascending order

Step 9: Propose as a compromise solution the alternative ($A^{(1)}$) which is the best ranked by the measure Q(minimum) if the following two conditions are satisfied

C1: acceptable advantage

$$Q(A^{(2)}) - Q(A^{(1)}) \geq DQ \tag{14}$$

where $A^{(2)}$ is the alternative with second position in the ranking list by Q and

$$DQ = 1/J - 1 \tag{15}$$

C2: Acceptable stability in decision making

The alternative $A^{(1)}$ must also be the best ranked by S or/and R. The compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when $v > 0.5$ is needed), or "by consensus $v \approx 0.5$", or "with veto" ($v < 0.5$). Please note that v is the weight of the decision making strategy of maximum group utility.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of

Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied Or

Alternatives $A^{(1)}, A^{(2)}, \ldots, A^{(M)}$ if the condition C1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M (the position of these alternatives are in closeness).

5 Numerical Application

In this section, we demonstrate the application of our approach on evaluating six organizations (A1, A2, A3, A4, A5, A6) from sustainable supply chain development perspective. Table 2 presents the details of the 12 criteria chosen for the decision making process. These are Communication (C1), Coordination (C2), Participation in environment friendly initiatives (C3), Social responsibility (C4), Management Commitment (C5), Risk Management (C6), Quality Management (C7), Voice of the customer (C8), Technology management (C9), Employee Training (C10), Adoption of environmental standards (C11), and Agility (C12). It can be seen in Table 2 that the criteria used for evaluation involve both fundamental and sustainability characteristics and are derived from Sect. 2. Please note that we have chosen these 12 criteria for illustration purposes. Interested readers can include all criteria for detailed analysis.

After selecting the criteria, a committee of three decision makers (D1, D2, and D3) is formed to weight the criteria and the alternatives. The decision makers provide linguistic ratings to the criteria and the alternatives (organizations) using Tables 1 and 2. The results of these ratings are presented in Tables 3 and 4.

The linguistic ratings are transformed into fuzzy triangular numbers using values provided in Tables 1 and 2 respectively. Then, we calculate the aggregated fuzzy weights (w_{ij}) of criterion using Eq. (3) and aggregate ratings for alternatives using Eq. (2). For example, for criteria C1, the aggregated fuzzy weight is given by $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$ where

$$w_{j1} = \min_k(7, 7, 5), w_{j2} = \frac{1}{3}(9 + 9 + 7), w_{j3} = \max_k(9, 9, 7)$$
$$\tilde{w}_j = (5, 8.33, 9)$$

Likewise, we compute the aggregate weights for the remaining criteria. The results for aggregate weights of the 12 criteria are presented in Table 5. To transform the aggregated fuzzy weights $\tilde{w}_j$ into crisp number w_j, we use Eq. (6). Therefore, for $\tilde{w}_j = (5, 8.33, 9)$, we have $w_j = \frac{1*5+(4*8.33)+9)}{6} = 7.89$.

Likewise, the aggregate fuzzy scores and respective crisp values of the six alternatives are computed using Eq. (2). Table 6 presents the aggregate fuzzy decision matrix for the alternatives.

Then, we compute the best f_j^* and the worst values f_j^- of the 12 criteria using Eq. (7). The results can be seen in last two columns of Table 7.

Table 3 Linguistic assessments for the 12 criteria

Criteria	D1	D2	D3
Communication (C1)	VH	VH	H
Coordination (C2)	H	H	M
Participation in environment friendly initiatives (C3)	H	H	H
Social responsibility (C4)	H	VH	H
Management commitment (C5)	VH	VH	VH
Risk management (C6)	M	M	M
Quality management (C7)	M	L	L
Voice of the customer (C8)	VH	H	VH
Technology management (C9)	VH	VH	VH
Employee training (C10)	H	H	VH
Adoption of environmental standards (C11)	VH	VH	VH
Agility (C12)	H	VH	H

Table 8 presents the S_i, R_i and Q_i values for the six alternatives computed using Eqs. (8, 9). The values of S* = 7.534, S- = 66.982, R* = 2.179, R- = 8.667 are obtained using Eq. (10) (Table 8).

Table 9 ranks the six alternatives, sorting by the values S_i, R_i and Q_i in ascending order.

It can be seen from the results of Table 9 that alternative A4 is the best ranked by the measure Q_i(minimum). We now check it for the following two conditions.

(1) C1: acceptable advantage.
Using Eq. (11), DQ = 1/12 − 1 = 1/11 = 0.0909. Applying Eq. (10), we find Q(A5) − Q(A4) = 0.602 − 0 = 0.602 > 0.0909, hence the condition $Q(A^{(2)}) - Q(A^{(1)}) \geq DQ$ is satisfied.

(2) C2: Acceptable stability in decision making

Since alternative A4 is also best ranked by S_i and R_i (considering the "by consensus rule $v \approx 0.5$"), therefore it is finally chosen as the best organization from sustainable supply chain development perspective.

6 Conclusions and Future Work

In this book chapter, we address the problem of evaluating organizations from a sustainably supply chain development perspective and present various sustainability focused and fundamental criteria that can be used by decision makers for evaluation purposes. Examples of these criteria are Communication, Coordination, Participation in environment friendly initiatives, Social responsibility, Management Commitment, Risk Management, Quality Management, Voice of the customer, Technology management, Employee Training, Adoption of environmental standards, and Agility.

Table 4 Linguistic assessments for the six alternatives

Criteria	D1						D2						D3					
	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6	A1	A2	A3	A4	A5	A6
C1	VH	M	H	VH	H	H	VH	L	VH	VH	H	M	VH	VL	VH	H	H	M
C2	VH	VL	M	H	M	L	VH	VL	H	H	L	VL	VH	L	VH	H	VL	L
C3	VL	VH	H	VH	H	VH	L	VH	H	VH	VH	VH	VL	VH	H	VH	H	VH
C4	VH	M	VH	VH	H	H	VH	H	VH	VH	M	VH	H	M	H	VH	M	H
C5	M	M	M	VH	H	H	H	H	H	VH	VH	M	VH	H	H	H	H	H
C6	L	VL	L	L	VL	VL	M	VL	VL	M	L	L	H	VL	L	L	VL	VL
C7	M	VL	VL	L	VL	VL	L	VL	L	VL	VL	L	M	L	VL	L	VL	VL
C8	M	M	VH	H	H	H	H	L	H	VH	H	H	M	M	VH	H	H	M
C9	M	H	H	VH	VH	H	M	H	H	H	H	H	M	M	H	VH	VH	H
C10	L	VL	M	M	M	M	M	L	M	M	L	VL	H	VL	L	M	M	L
C11	VL	VL	H	VH	H	VH	L	VL	VH	H	VH	H	VL	L	H	VH	H	H
C12	H	VH	H	VH	H	H	H	VH	H	VH	VH	VH	M	H	VH	H	VH	H

Table 5 Aggregate fuzzy criteria weights

Criteria	D1	D2	D3	Fuzzy	Crisp
C1	(7, 9, 9)	(7 ,9, 9)	(5 ,7, 9)	(5, 8.333,9)	7.89
C2	(5, 7, 9)	(5, 7, 9)	(3, 5, 7)	(3, 6.333, 9)	6.22
C3	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)	(5, 7, 9)	7
C4	(5, 7, 9)	(7, 9, 9)	(5, 7, 9)	(5, 7.67, 9)	7.44
C5	(7, 9, 9)	(7, 9, 9)	(7, 9, 9)	(7, 9, 9)	8.67
C6	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	(3, 5, 7)	5
C7	(3, 5, 7)	(1, 3, 5)	(1, 3, 5)	(1, 3.67, 7)	3.78
C8	(7, 9, 9)	(5, 7, 9)	(7, 9, 9)	(5, 8.34, 9)	7.89
C9	(7, 9, 9)	(7, 9, 9)	(7, 9, 9)	(7, 9, 9)	8.67
C10	(5, 7, 9)	(5, 7, 9)	(7, 9, 9)	(5, 7.67, 9)	7.44
C11	(7, 9, 9)	(7, 9, 9)	(7, 9, 9)	(7, 9, 9)	8.67
C12	(5, 7, 9)	(7, 9, 9)	(5, 7, 9)	(5, 7.67, 9)	7.44

Table 6 Aggregate fuzzy decision matrix for alternatives

Criteria	A1	A2	A3	A4	A5	A6
C1	(7 ,9, 9)	(1, 3, 7)	(5, 8.34, 9)	(5, 8.34, 9)	(5, 7, 9)	(3, 5.67, 9)
C2	(7, 9, 9)	(1, 1.67, 5)	(3, 7, 9)	(5, 7, 9)	(1, 3, 7)	(1, 2.34, 5)
C3	(1, 1.67, 5)	(7, 9, 9)	(5, 7, 9)	(7, 9, 9)	(5, 7.67, 9)	(7, 9, 9)
C4	(5, 8.34, 9)	(3, 5.67, 9)	(5, 8.34, 9)	(7, 9, 9)	(3, 5.67, 9)	(5, 7.67, 9)
C5	(3, 7, 9)	(3, 6.34, 9)	(3, 6.34, 9)	(5, 8.34, 9)	(5, 7.67, 9)	(3, 6.34, 9)
C6	(1, 5, 9)	(1, 1, 3)	(1, 2.34, 5)	(1, 3.67, 7)	(1, 1.67, 5)	(1, 1.67, 5)
C7	(1, 4.34, 7)	(1, 1.67, 5)	(1, 1.67, 5)	(1, 2.34, 5)	(1, 1, 3)	(1, 1.67, 5)
C8	(3, 5.67, 9)	(1, 4.34, 7)	(5, 8.34, 9)	(5, 7.67, 9)	(5, 7, 9)	(3, 6.34, 9)
C9	(3, 5, 7)	(3, 6.34, 9)	(5, 7, 9)	(5, 8.34, 9)	(5, 8.34, 9)	(5, 7, 9)
C10	(1, 5, 9)	(1, 1.67, 5)	(1, 4.34, 7)	(3, 5, 7)	(1, 4.34, 7)	(1, 3, 7)
C11	(1, 1.67, 5)	(1, 1.67, 5)	(5, 7.67, 9)	(5, 8.34, 9)	(5, 7.67, 9)	(5, 7.67, 9)
C12	(3, 6.34, 9)	(5, 8.34, 9)	(5, 7.67, 9)	(5, 8.34, 9)	(5, 8.34, 9)	(5, 7.67, 9)

Table 7 Alternative Crisp ratings, f_j^* and the worst values f_j^- for the 12 criteria

Criteria	Criteria weight	Crisp alternative ratings						f_j^*	f_j^-
		A1	A2	A3	A4	A5	A6		
C1	7.89	8.667	3.333	7.889	7.889	7	5.778	8. 667	3. 333
C2	6.22	8.667	2.111	6.667	7	3.333	2.556	8. 667	2. 111
C3	7	2.111	8.667	7	8.667	7.444	8.667	8. 667	2. 111
C4	7.44	7.889	5.778	7.889	8.667	5.778	7.444	8. 667	5. 778
C5	8.67	6.667	6.222	6.222	7.889	7.444	6.222	7. 889	6. 222
C6	5	5	1.333	2.556	3.778	2.111	2.111	5	1. 333
C7	3.78	4.222	2.111	2.111	2.556	1.333	2.111	4. 222	1. 333
C8	7.89	5.778	4.222	7.889	7.444	7	6.222	7. 889	4. 222
C9	8.67	5	6.222	7	7.889	7.889	7	7. 889	5
C10	7.44	5	2.111	4.222	5	4.222	3.333	5	2. 111
C11	8.67	2.111	2.111	7.444	7.889	7.444	7.444	7. 889	2. 111
C12	7.44	6.222	7.889	7.444	7.889	7.889	7.444	7. 889	6.222

Table 8 S_i, R_i and Q_i values for the alternatives

	A1	A2	A3	A4	A5	A6
S_i	44.679	66.98	28.91	7.534	30.88	41.789
R_i	8.666	8.666	8.666	2.179	7.444	8.6666
Q_i	0.812	1	0.679	0	0.602	0.7881

Table 9 Alternative rankings

S_i	A4	A3	A5	A6	A1	A2
R_i	A4	A5	A3	A6	A1	A2
Q_i	A4	A5	A3	A6	A1	A2

The main strength of our approach is the investigation of organizational characteristics for sustainable supply chain planning. Another advantage is the ability to evaluate organizational performance from sustainable supply chain development perspective under partial or lack of quantitative information. Organizations can be evaluated for these characteristics in terms of linguistic preferences and use of multicriteria decision making methods. We have used fuzzy VIKOR technique because of its ability to evaluate alternatives based on their closeness to ideal solution.

The next step of our work involves designing sustainable supply chain networks considering the different characteristics (or criteria) proposed in this chapter.

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Part II
Supply Chain Performance Measurement Under Fuzziness

Fuzzy Multiple Criteria Decision Making for Supply Chain Management

Yuh-Wen Chen and Moussa Larbani

Abstract Supply Chain Management (SCM) problem can be simply described as if an enterprise is requested to provide adequate commodities to its customers on time, it should be able to design its own appropriate purchase/production/transportation network at the lowest-cost level in time. Modeling SCM by fuzzy mathematical programming is an innovative and a popular issue, this chapter introduces fuzzy multiple attribute decision making (FMADM) and fuzzy multiple objective programming (FMOP) for the solutions of SCM.

Keywords Supply chain · Fuzzy · Multiple attribute decision making · Multiple objective programming

1 Introduction

Recently, the global market schemes have generated new concepts in various economic and industrial sectors. Supply Chain Management (SCM) optimally integrates the operational networks from material suppliers to end customers, which is the most popular issue since 2000 (Chen and Tzeng 2002; Zarandi et al. 2002; Zhou et al. 2008).

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Fuzzy models are also popular in the field of SCM. The advantages and disadvantages of fuzzy models are:
Advantages

- Flexibility
- Convenient user interface
- Easy computation
- Learning ability
- Quick validation
- Ambiguousness
- Combination with existed models.

Disadvantages

- Insufficient experimental evidence
- Many manual setting parameters
- Unclear options
- Dimensionality/complexity of building models for beginners.

Readers should be aware of the limitations of fuzzy models in advance. In addition, some academic fields are against the fuzzy models. This is why in the literature review most of previous models are crisp, rather than fuzzy. This chapter is dedicated to Fuzzy Multiple Criteria Decision Making (FMCDM) methods for SCM. The method of FMCDM is considering the conflicts/trade-off among multiple criteria in order to make the optimal decision (Chen and Hwang 1992).

Supply Chain Management could be simply defined as if an enterprise is requested to provide adequate commodities to its customers on time, it should be able to design its own appropriate purchase/production/transportation network at the lowest-cost level in time (Chopra and Meindl 2010; Dobrila 2001; Dobrila et al. 1998). This idea is simply illustrated in Fig. 1.

The important issues of managing supply chain summarized by Chopra and Meindl (2010) are:

- Forecasting
- Aggregate planning
- Inventory control
- Level of availability
- Network design: transportation and location
- Information technology (IT) and e-business.

Considering the published papers strongly related to FMCDM, only the topics of fuzzy multi-objective programming (FMOP) and fuzzy multi-attribute decision making (FMADM) are focused in this chapter. In such a case, not all important

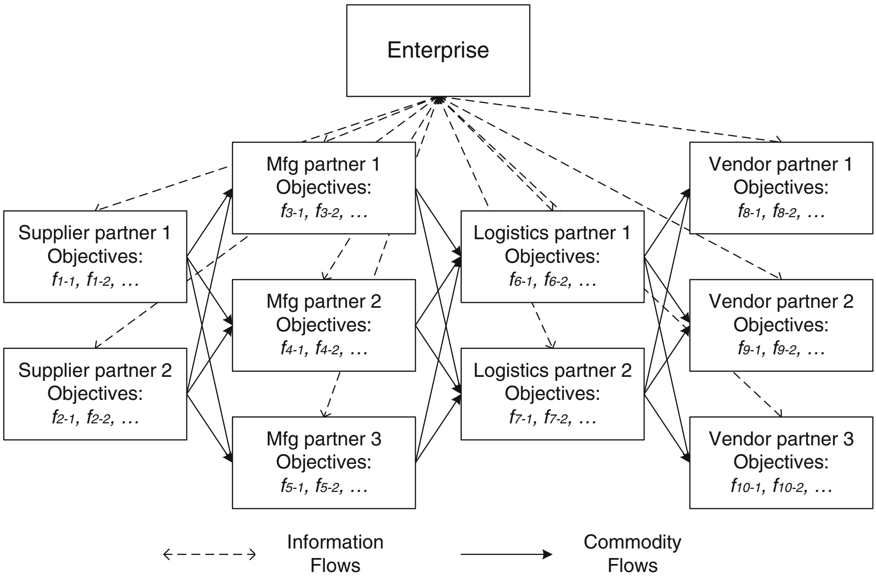


Fig. 1 Framework of supply chain

SCM issues above will be presented. The chapter is organized as follows: Sect. 2 is used to present the basics of fuzzy multi-objective programming and fuzzy multi-attribute decision making, i.e., a fuzzy ranking method. Section 3 gives the game model with FMOP and FMADM. Section 4 proposes the Data Envelopment Analysis (DEA) by FMOP. Finally, conclusions and recommendations are available in Sect. 5, some advanced issues are also discussed here.

Modeling SCM by fuzzy mathematical programming is an interesting, innovative and a popular issue, here the fuzzy art of modeling SC is summarized by some categories in Table 1, which includes the major studying areas of modeling SC by fuzzy sets.

Generally speaking, it is easy to find the SCM articles of aggregate planning than the other categories, mathematical programming is the most popular technique. But the number of using FMCDM methods is comparatively less.

Table 1 Fuzzy models for SCM

Subjects of SCM	Articles
1. Forecasts	Forecasting energy demand using fuzzy seasonal time series (Sarı and Öztayşi 2012), hybrid demand forecasts to improve SCM (Aburto and Weber 2007), analyzing demand variability by fuzzy regression (Tozan and Vayvay 2007)
2. Aggregate Planning	Modelling and simulation of a supply chain in an uncertain environment (Chen and Chang 2006; Gunasekaran et al. 2006; Liang 2007; Yang 2007), fuzzy-genetic approach to aggregate production–distribution planning (Aliev et al. 2007), fuzzy goal approach (Jamalnia and Soukhakian 2008; Selim et al. 2006), adaptive formulation (Lou and Si 2006)
3. Inventory control	Managing the inventory level by fuzzy supply and demand (Giannaoccaro et al. 2003; Gupta et al. 2000), fuzzy inventory control (Xiong and Koivisto 2003)
4. Vendor selection	Selecting the vendor by fuzzy multi-objective approach (Amid et al. 2006), vendor selection by integrated fuzzy MCDM techniques (Yang et al. 2008), vendor selection by fuzzy goal programming approach (Kumar et al. 2004), fuzzy multi-objective vendor selection with lean procurement (Yu et al. 2012), fuzzy synthetic evaluation and fuzzy ANP to select the vendor (Pang and Bai 2013), using linguistic variables to develop the multi-criteria group decision-making approach for vendor selection (Shahgholian et al. 2012)
5. Transportation and location	A random fuzzy design of multi-objective supply chain networks (Ning et al. 2006), fuzzy transportation problems for SCM (Liu and Kao 2004), fuzzy programming for production/transportation planning (Sakawa et al. 2001), fuzzy approach to select the location of the distribution center (Chen 2001), a fuzzy system for facility location selection(Bhatnager and Sohal 2005; Chou et al. 2008; Uno et al. 2012)
6. Fuzzy game of supply chain	Fuzzy cooperation in a supply chain (Hua and Li 2008; Smirnov et al. 2004), two echelon fuzzy game (Zhou et al. 2008), fuzzy coalition (Pan et al. 2006), fuzzy MADM game (Chen and Larbani 2006), a fuzzy game with alliances (Chen et al. 2010)

2 Fuzzy MCDM

The basics of FMOP and FMADM will be clearly illustrated here.

2.1 *Fuzzy Multi-objective Planning*

Zimmermann's fuzzy linear programming with i linear objective functions is defined as follows (Zimmerman 1985):

$$\begin{aligned}&\text{Max}\, f(x) = (f_1(x), f_2(x), \ldots, f_i(x))^T \\ &st \\ &\mathbf{A}x \le b,\ x \ge \mathbf{0}\end{aligned} \tag{1}$$

$f_i(x)$ The objective function, $f_i(x) = c_i x, \quad i = 1, 2, \ldots, p$;
$\boldsymbol{x}$ the decision variable, $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T$;
$\boldsymbol{b}$ the Right Hand Side (RHS) value, $\boldsymbol{b} = (b_1, b_2, \ldots, b_m)^T$;
A the coefficient matrix, $A = [\alpha_{i,j}]_{m \times n}$.

The advantages and disadvantages of FMOP are:

Advantages

- Multiple objectives are considered at one time
- Easy computation.

Disadvantages

- Membership functions should be set first: each objective has an individual setting
- Many computations for one problem.

For each of the objective function $f_i(x)$, $i = 1, 2, \ldots, p$; of this problem, assuming that the decision maker has a fuzzy goal, e.g., maximizing the profit; thus, the corresponding linear membership function $\mu_i^L(f_i(x))$ is defined as:

$$\mu_i^L(f_i(x)) = \begin{cases} 0 & ; \quad f_i(x) \le f_i(x)^- \\ \frac{f_i(x) - f_i(x)^-}{f_i(x)^+ - f_i(x)^-} & ; \quad f_i(x)^- \le f_i(x) \le f_i(x)^+ \\ 1 & ; \quad f(x) \ge f_i(x)^+ \end{cases} \tag{2}$$

$f_i(x)^-$ denotes the objective value of pessimistic expectation by a decision maker, and $f_i(x)^+$ denotes the objective value of optimistic expectation by a decision maker. His membership function is shown in Fig. 2 (Zimmerman 1985).

Using such a linear membership function $\mu_i^L(f_i(x))$, $i = 1, 2, \ldots, p$; and apply the min operator, the original problem can be changed as in Eq. (3) by interpreting the auxiliary variable λ:

$$\begin{aligned} &\text{Max } \lambda \\ &st \\ &\lambda \le \mu_i^L(f_i(x)), \quad i = 1, 2, \ldots, p \\ &\mathbf{A}x \le \boldsymbol{b}, \ \boldsymbol{x} \ge 0 \end{aligned} \tag{3}$$

Equation (1.3) is the fuzzy transformation for general uses. A supply chain game to show the aggregate planning is available in Sect. 3.

(1) Fuzzy Multi-Attribute Decision Making

Here two MADM techniques: Fuzzy Analytical Hierarchy Process (FAHP) and FMADM game are presented.

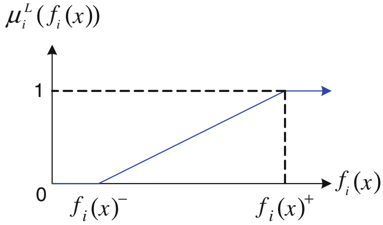


Fig. 2 Achievement level/aspiration degree for each fuzzy goal

FAHP

Thomas L. Saaty, professor in Pittsburgh University in U.S.A., developed AHP method in 1971 and it is applied popularly recently among economics, society, management field, etc. to dealing with complicated policy decision (Chen and Hwang 1992). The advantages and disadvantages of AHP are:

Advantages

- Easy understanding for users
- Easy computation.

Disadvantages

- Consistency test is complicated
- Questionnaire consumes much time because of the pair-wise comparison.

However, in real situation, the recognition of the interviewee is often fuzzy, thus “capital” criteria “much” more important than “secure sanitary management, and If the evaluation scale which Saaty offered was expressed, the definition of “much more” maybe just 1/7, 1/8, 1/9, in other words, there exits some differences between the pair comparative values and the real recognition cognition of the interviewees. For expressing the feeling of the interviewees more accurately, the following adopts fuzzy theory to handling the linguistic scale problems.

(i) Triangular Fuzzy Number

A triangular fuzzy number $\tilde{A}$ whose value point is (a_1, a_2, a_3) (Fig. 3), and the membership function will be defined as Eq. (4):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases} \tag{4}$$

(ii) Fuzzy Number Calculating

Now there are two fuzzy numbers

$$\tilde{A} = (a_1, a_2, a_3), \tilde{B} = (b_1, b_2, b_3),$$

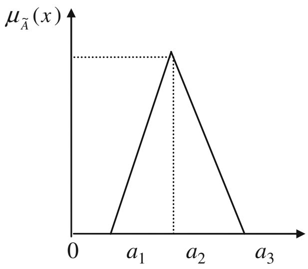


Fig. 3 Triangular fuzzy number Ã

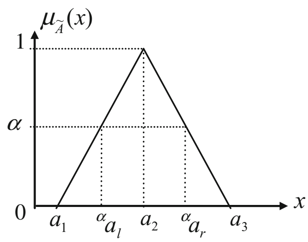


Fig. 4 α - cut

then

$$\begin{aligned}(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \\ (a_1, a_2, a_3) \otimes (b_1, b_2, b_3) &= (a_1 b_1, a_2 b_2, a_3 b_3) \\ \tilde{A}^{-1} = (a_1, a_2, a_3)^{-1} &\cong (\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1})\end{aligned} \tag{5}$$

(iii) α-Cut (Fig. 4)

$$\begin{aligned}\forall \alpha \in [0, 1], \tilde{A} \text{ of } \alpha{-}cut \text{ shows } {}^{\alpha}\tilde{A}, \text{ and} \\ {}^{\alpha}\tilde{A} = [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3] = [{}^{\alpha}a_l, {}^{\alpha}a_r]\end{aligned} \tag{6}$$

(iv) Fuzzy AHP

FAHP (Fuzzy Analytic Hierarchy Process) is offered by Buckley in 1985. The method makes the pair comparative value in AHP offered by Saaty, and calculates the fuzzy weight with Geometric Mean Method. The theory and methodology are as follow. Consider a fuzzy orthogonal matrix $\tilde{A} = [\tilde{a}_{ij}]$, and $\tilde{a}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij})$ is a trapezium fuzzy number. Taking Saaty's max-λ method as base and considering:

$$\tilde{A} \otimes \tilde{w} = \tilde{\lambda} \otimes \tilde{w} \tag{7}$$

In which $\tilde{w}^T = (\tilde{w}_1, \cdots, \tilde{w}_m)$, $\tilde{w}_i = (\tilde{\varepsilon}_i, \tilde{\xi}_i, \tilde{\eta}_i, \tilde{\theta}_i)$, $\tilde{\lambda} = (\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3, \tilde{\lambda}_4)$ are all fuzzy numbers. Where $A = [\alpha_{ij}], B = [\beta_{ij}],\ C = [\gamma_{ij}], D = [\delta_{ij}]$.

Let $A = [\alpha_{ij}], B = [\beta_{ij}],\ C = [\gamma_{ij}], D = [\delta_{ij}]$, then
$\mathbf{x}^1 = (\varepsilon_1, \cdots, \varepsilon_m)^T, \mathbf{x}^2 = (\xi_1, \cdots, \xi_m)^T, \mathbf{x}^3 = (\eta_1, \cdots, \eta_m)^T, \mathbf{x}^4 = (\theta_1, \cdots, \theta_m)^T$
Then Eq. (7) will be adapted as

$$A\mathbf{x}^1 = \lambda_1\mathbf{x}^1, B\mathbf{x}^2 = \lambda_2\mathbf{x}^2, C\mathbf{x}^3 = \lambda_3\mathbf{x}^3, D\mathbf{x}^4 = \lambda_4\mathbf{x}^4,$$

In such a case, there will be four sets of max-λ and eigenvalues, so they cannot be coped with the problem with Saaty's max-λ. Therefore Buckley led in one method for calculating fuzzy weight and fuzzy utilities.

(v) Fuzzy Weight

Hypothesizing $A = [a_{ij}]$ as a positive reciprocal matrix, and listing the geometric mean value

$$r_i = \left(\prod_{j=1}^{m} a_{ij}\right)^{1/m}, w_i = r_i/(r_1 + r_2 + \ldots + r_m)$$

If m = 3, the result is the same as Saaty's max-λ, If m > 3, the two results of both methods are pretty close.

Now if assuming $\tilde{A} = [\tilde{a}_{ij}]$, $\tilde{a}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij})$ as the attribute (j = 1,2,…, m) of pair comparison matrix, then the fuzzy weight of the i-th attribute is:

$$\tilde{r}_i = (\tilde{a}_{i1} \otimes \cdots \otimes \tilde{a}_{im})^{1/m},\ \tilde{w}_i = \tilde{r}_i \otimes (\tilde{r}_1 \oplus \cdots \oplus \tilde{r}_m)^{-1} \tag{8}$$

Fuzzy MADM Game

Multiple Attribute Decision Making (MADM) is a management science technique, popularly used to rank the priority of alternatives with respect to their competing attributes in a crisp or a fuzzy environment (Chen and Hwang 1992; Chen and Larbani 2006).

$$\tilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \ldots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \ldots & \tilde{a}_{2n} \\ \cdots & & & \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \ldots & \tilde{a}_{mn} \end{bmatrix} \end{array} \tag{9}$$

The advantages and disadvantages of FMADM game are:

Advantages

- No pair-wise comparison is needed: data collection and data input are simple
- Friendly user interface: only a decision matrix is required.

Disadvantages

- Computation is complicated
- Users are encouraged to understand the game theory.

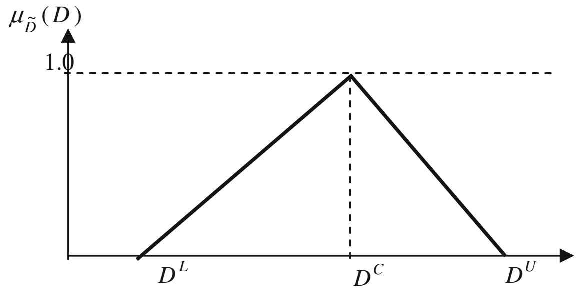


Fig. 5 Triangular fuzzy decision matrix $\tilde{D}$

The FMADM game is shown as follows: considering a fuzzy MADM problem with the fuzzy decision matrix (9)

FMADM game is a two-person zero-sum game. Here a DM is player A, who has m alternatives $(A_i, i = 1, 2, \ldots, m)$ with respect to n attributes $(C_j, j = 1, 2, \ldots, n)$; the normalized weight of A_i is x_i, the normalized weight of C_j is y_j, $\tilde{a}_{ij}$ represents the evaluation of alternative i with respect to attribute j, i = 1,2,..., m and $\tilde{a}_{ij} \geq 0$; j = 1,2,..., n. Nature is player B, who gives the fuzzy decision matrix (9). This fuzzy MADM problem defined as the DM chooses the best alternative according to the available $\tilde{D}$ as a fuzzy matrix with triangular membership function, i.e. $\tilde{D} = (D^L,\ D^C,\ D^U)$. The membership function of $\tilde{D}$ is assumed in Fig. 5 and Eq. (10).

$$\mu_{\tilde{D}}(D) = \mu_{\tilde{D}}(\lambda D^U + (1-\lambda)D^L) = \mu(\lambda) = \begin{cases} 0, & \lambda \leq 0 \\ \frac{\lambda - 0}{\bar{\lambda} - 0}, & 0 < \lambda < \bar{\lambda} \\ 1, & \lambda = \bar{\lambda} \\ \frac{1-\lambda}{1-\bar{\lambda}}, & \bar{\lambda} < \lambda < 1 \\ 0, & \lambda \geq 1 \end{cases} \tag{10}$$

Thus, the $\tilde{D}$'s behavior can be described by various α-cuts:

$$\tilde{D}_\alpha = [D^U_\alpha, D^L_\alpha] = \{D^U_\alpha + (1-)D^L_\alpha, [0,1]\} \tag{11}$$

A vector x in IR^m is a mixed strategy of player A if it satisfies the following probability condition:

$$x^t e_m = 1 \tag{12}$$

where the components of $x = [x_1, x_2, \ldots, x_m]^t$ are greater than or equal to zero; e_m is an $m \times 1$ vector, where each component is equal to 1. Similarly, a mixed strategy of player B is defined by $y = [y_1, y_2, \ldots, y_n]^t$ and $y^t e_n = 1$. If the mixed strategies x and y, are proposed by players A (decision maker) and B (Nature) respectively, then the fuzzy expected payoff of player A is defined by

$$x^t \tilde{D} y = \sum_{j=1}^{n} \sum_{i=1}^{m} \tilde{a}_{ij} x_i y_j \tag{13}$$

The Eq. (13) is player A's objective and should be maximized. Considering the two-person zero-sum game (9), x^* and y^* are optimal strategies under the Nash equilibrium: if $x^t \tilde{D} y^* \leq x^{*t} \tilde{D} y^*$ and $x^{t*} \tilde{D} y \geq x^{*t} \tilde{D} y^*$, for any mixed strategies x and y. Player A's objective is to maximize his pay-off over all possible x when player B chooses his best strategy y^*. Player B's objective is to minimize his pay-off over all possible y when player A chooses his best strategy x^*.

The solution for the two-person zero-sum game is (9) a given α-cut derives from the optimal solutions of the following pair of optimization problems (14)–(15):

$$\begin{aligned} &\operatorname*{Max}_{x} \; v_A \\ &st \quad x^t D_\alpha^U \geq v_A \, e_n^t \\ &\qquad x^t D_\alpha^L \geq v_A \, e_n^t \\ &\qquad x^t e_m = 1, \; x \geq 0 \end{aligned} \tag{14}$$

$$\begin{aligned} &\operatorname*{Min}_{y} \; v_B \\ &st \quad D_\alpha^U y \leq v_B e_m \\ &\qquad D_\alpha^L y \leq v_B e_m \\ &\qquad y^t e_n = 1, \; y \geq 0. \end{aligned} \tag{15}$$

Moreover, the fuzzy score of each alternative is computed by the following interval:

$$ES(A_i) = [\, x_{i,\alpha}^* \sum_{j=1}^{n} a_{\alpha\, ij}^L y_{j,\alpha}^*, \; x_{i,\alpha}^* \sum_{j=1}^{n} a_{\alpha\, ij}^U y_{j,\alpha}^* \,] \tag{16}$$

The alternative with higher score is more preferred. Any de-fuzzy method can be used to decide the final rank of these alternatives.

Example 1

Experienced experts from various vendors and customers of this logistics company are invited to rank eleven candidate warehouse locations in Fig. 6 for Taipei. Multiple attributes for appropriately ranking the location of warehouse are collected—these attributes are land cost (C_1), labor cost (C_2), traffic congestion (C_3), accessibility to the metropolitan (C_4), accessibility to the industrial park (C_5), accessibility to the international airport (C_6) and accessibility to the international harbor (C_7). These experienced logistics managers are asked to provide their evaluations of the locations with respect to attributes. These fuzzy values are ranged within the quality interval from 1 to 10 from the beneficial side, where "1" means the lowest degree and "10" means the highest degree.

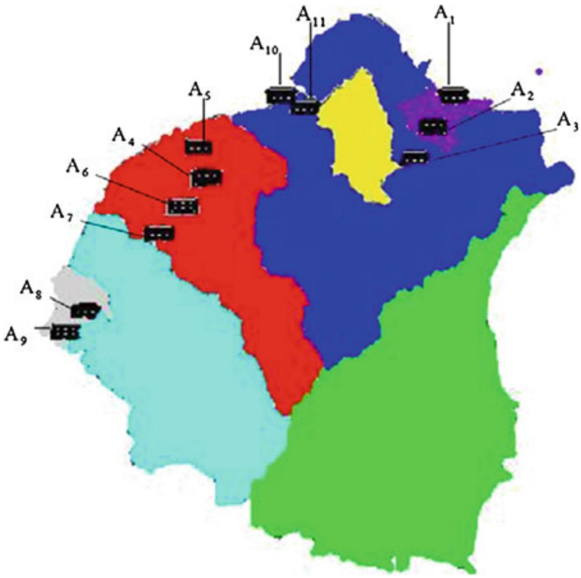


Fig. 6 Candidate locations around the taipei metropolitan (*yellow district*)

Fig. 7 Ranking results by excel

Score	Lower	Upper
A1	0	0
A2	0	0
A3	0.153483	0.191706
A4	2.658324	3.320341
A5	0	0
A6	0.282409	0.352738
A7	0	0
A8	0	0
A9	1.964581	2.453831
A10	1.358595	1.698777
A11	0	0

An Excel interface with Visual Basic Application (VBA) is proposed to facilitate the use of fuzzy MADM game. The ranking results are available in Fig. 7. In addition, the fuzzy decision matrix is available in Table 2. According to the computational results and defuzzification by choosing the median between the lower bound and the upper bound for each alternative, the top three (most preferred) alternatives are: $A_4 > A_9 > A_{10}$. Readers should recognize that only one fuzzy decision matrix: Table 2 is needed for the computation of Example 1, this is much simpler than the pair-wise comparison in FAHP. The ranking method provides here is appropriate to solve any priority problem in SCM.

Table 2 Fuzzy decision matrix for location decision

Alternatives/Attributes	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	5, 6, 7	7, 8, 9	5, 6, 7	2, 4, 5	3, 4, 5	6, 6, 7	3, 3, 4
A_2	6, 7, 8	7, 9, 10	6, 8, 9	3, 4, 5	4, 5, 5	6, 7, 7	3, 4, 4
A_3	8, 9, 10	7, 9, 10	6, 8, 9	4, 5, 6	5, 5, 6	6, 6, 7	4, 5, 6
A_4	7, 9, 10	4, 5, 6	7, 8, 9	8, 9, 10	8, 9, 10	7, 8, 9	6, 8, 9
A_5	8, 8, 9	3, 4, 5	5, 6, 7	6, 7, 8	7, 8, 8	7, 7, 8	6, 7, 8
A_6	8, 8, 9	5, 6, 8	7, 8, 8	6, 7, 8	7, 7, 8	5, 6, 7	6, 7, 8
A_7	5, 6, 8	6, 7, 7	7, 8, 8	7, 7, 8	7, 8, 9	5, 5, 6	6, 7, 8
A_8	8, 8, 10	4, 5, 5	7, 8, 9	5, 6, 7	4, 5, 5	3, 4, 5	8, 8, 9
A_9	7, 8, 9	8, 9, 10	4, 5, 6	5, 6, 7	4, 5, 6	4, 5, 6	7, 8, 9
A_{10}	3, 4, 5	7, 8, 8	8, 9, 9	4, 5, 6	6, 7, 8	8, 9, 10	4, 4, 5
A_{11}	3, 4, 5	7, 8, 8	8, 9, 9	6, 7, 8	6, 7, 8	7, 7, 8	4, 5, 6

3 Supply Chain Game by FMOP

This section is designed to illustrate using FMOP on Supply Chain Game. The SC game will be deduced step by step so that readers are able to use or develop some advanced fuzzy games of their own.

3.1 Supply Chain Game

Game theory is concerned with the actions (strategies) of decision makers, who are aware that their actions affect each other (Rasmusen 1989). In addition to the Table 1 of literature review in Sect. 2, Nagarajan and Sošić (2008) mentioned about the cooperation analysis in SC game; in addition, Huang and Li (2001), and Li et al. (2002) also analyzed the SC performance from the game aspect. Interested readers may find the literature above for further reading. However, their formulations are crisp rather than using FMOP.

The advantages and disadvantages of game models are:

Advantages

- Rigid deduction process
- Strong proofs in mathematics
- Extension with existed models.

Disadvantages

- Users are encouraged to have sufficient background in mathematics
- Complicated symbols for beginners because of formulations and extensions are very various and abstract.

A two-person zero-sum game is the simplest case of game theory with only two players. Such a game is resolved by assuming that both players propose pure (discrete), mixed (probability) or continuous strategies. The strategies proposed here for each partner will be its capacity to meet the maximal satisfaction: both from the micro scope and macro scope.

The degree of cooperation (or non-cooperation) between players is assumed to be vague in this study: the cooperation degree won't be measured in this study; actually, it is an abstract idea. Let us consider the following n-person non-cooperative game in normal form (Rasmusen 1989):

$$\langle I, X, f(x) \rangle \tag{17}$$

$I = \{1, 2, \ldots, n\}$ is the set of players, X is the set of situations of the game, X_i is the set of strategies of the i-th player, $i = 1, 2, \ldots, n$; $f = (f_1, f_2, \ldots, f_n)$, f_i is the objective function of the i-th player; $x = (x_1, x_2, \ldots, x_n) \in X$ is a situation of the game, $x_i \in X_i$ is the strategy of the i-th player, $i = 1, 2, \ldots, n$.

Definition 1 The game (17) is in the normal form if it is played one time.

Definition 2 The game (17) is non-cooperative if players cannot make enforceable agreements.

Definition 3 $x^0 \in X$ is called Nash equilibrium of the game (17) if $\forall i \in I, \quad \forall x_i \in X_i, \quad f_i(x^0//x_i) \leq f_i(x^0)$.

$(x^0//x_i)$ is the issue obtained from the issue x^0 by substituting the i-th component of the vector x^0 for x_i.

Definition 4 Suppose that in the game (17) there are n players, the pay-off function of each player is f_i and $I = \{1, 2, \ldots, n\}$. Here the game is not necessarily non-cooperative. The relation between players is represented by the following $n \times n$ matrix:

$$\mathbf{C} = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,n} \\ \alpha_{2,1} & \alpha_{2,2} & \cdots & \alpha_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ \alpha_{n,1} & \alpha_{n,2} & \cdots & \alpha_{n,n} \end{bmatrix}_{n \times n} \tag{18}$$

Thus, the Nash equilibrium of the game: $\left\langle I, X, g(x) = \mathrm{C} \times [f_i]_{n \times 1} \right\rangle$ is defined as A-Nash equilibrium (Aliged Nash equilibrium). Here $\alpha_{i,j} \in [-1, \quad 1]$, which represents the degree of cooperation between player i and player j or more precisely between two players. C is named as the "alliance matrix".

Remark 1 If a coefficient $\alpha_{i,j}$ is positive, it is easy to show that there is cooperation between player i and player j because their pay-offs are united. If $\alpha_{i,j}$ is negative then it means that the player i is in competition with player j resulting from their interests are antagonistic. If $\alpha_{i,j} = 0$ then the player i is neutral according to player

j. It is easy to formulate the Definition 4 for the general case of n-person game. Let us briefly illustrate our ideas of alliance matrix of a SC as follows:

(1) each partner in a SC is playing the cooperative or non-cooperation game;
(2) the cooperation degree $\alpha_{i,j}$ between partners can be regarded as their various alliances, e.g., $\alpha_{i,j} \in [-1,\ 1]$;
(3) such alliances among partners can be described by alliance matrix: A. Thus, consider n players in a SC, each partner's objective is, e.g., $f_1, f_2, \ldots, f_n$, etc., their integrated objectives from the micro level can be expressed by the following equation:

$$A \times f(x) = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \ldots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \ldots & \alpha_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \alpha_{n1} & \alpha_{n2} & \ldots & \alpha_{nn} \end{bmatrix} \times \begin{bmatrix} f_1 \\ f_2 \\ \ldots \\ f_n \end{bmatrix} \tag{19}$$

where $\alpha_{i,j}$ represents the cooperation degree between partner i and partner j; $\alpha_{i,j} = 1$ if $i = j$ and $\alpha_{i,j} \in [-1,\ 1]$. The cooperation degree is arbitrarily decided in this study; however, exploring the measurement of $\alpha_{i,j}$ would be an interesting issue for readers.

3.2 Formulation and Resolution

In this section, a simple example is illustrated for SC game. Now Fig. 1 in Sect. 2 is used as the model formulation. The objective and constraints of each partner in the SC will be established. The symbols are shown in Table 3.

(1) Supplier partner's objective and constraints

The supplier partner's objective is assumed to maximize its own net profits. And constraints are available storage space and working time.

$$\begin{aligned} &\text{Max} \quad f_{s,t} = \sum_{m\in M}\sum_{e\in E} p_s^e x_{sm,t}^e - \sum_{m\in M}\sum_{e\in E} c_s^e x_{sm,t}^e, \forall s \in S, \forall t \in T \\ &st \\ &\sum_{m\in M} x_{sm,t}^e \le space_s, \forall s \in S, \forall t \in T, \\ &\sum_{m\in M}\sum_{e\in E} wt_s^e x_{sm,t}^e \le awt_s, \forall s \in S, \forall t \in T, \end{aligned} \tag{20}$$

Table 3 Symbol notations

Symbol	Notation
p_s^e	The sale price for material e of supplier s
p_m	The sale price of merchandise for Mfg. partner m
p_{lv}	The sale price from logistics partner l to vendor v
pa_m^e	The consumed quantity of material e when one unit $x_{ml,t}$ is produced
$x_{sm,t}^e$	The shipped quantity of material e from supplier s to Mfg. partner m at time t, decision variable
$x_{ml,t}$	The shipped quantity of merchandise from Mfg. partner m to logistics partner l at time t, decision variable
c_s^e	The unit cost of material e for supplier s
C_{lv}	The transportation cost from logistics partner l to vendor v
$space_s$	The available storage space of supplier s
$space_m$	The available storage space of Mfg. partner m
$space_l$	The available storage space of logistics partner l
awt_s	The available working time for supplier s
awt_m	The available working time for Mfg. partner m
wt_s	The unit working time for producing material e by supplier s
wt_m	The unit working time for producing merchandise by Mfg. partner m
$d_{v,t}$	The demand of vendor v at time t
$tc_{sm,t}$	The unit transportation cost from supplier s to Mfg. partner m
$tc_{ml,t}$	The unit transportation cost from Mfg. partner m to logistics partner l
$tc_{lv,t}$	The unit transportation cost from logistics partner l to vendor v
M	The set of all manufacturing partners, $M = \{1, 2\}$
S	The set of all suppliers, $S = \{1, 2, 3\}$
T	The set of planning horizon, $T = \{1, 2, 3\}$
L	The set of all logistics partners, $L = \{1, 2\}$
V	The set of all vendors, $V = \{1, 2, 3\}$
E	The set of materials, $E = \{x, y\}$

(2) Manufacturing partner's objective and constraints

The Mfg. partner's objective is similarly assumed to maximize its own net profits. And constraints are available storage space and working time. In addition, the manufacturing ability of each Mfg. partner is assumed various in the last constraint.

$$
\begin{aligned}
&\text{Max}\quad f_{m,t} = \sum_{l \in L} p_m x_{ml,t} - \sum_{l \in L} c_m x_{ml,t}, \forall m \in M, \forall t \in T \\
&st \\
&\sum_{l \in L} x_{ml,t} \le space_m, \forall m \in M, \forall t \in T, \\
&\sum_{l \in L} wt_m x_{ml,t} \le awt_m, \forall m \in M, \forall t \in T, \\
&x_{ml,t} = \sum_{e \in E} \sum_{s \in S} p\, a_m^e x_{sm,t}^e, \forall m \in M, \forall t \in T,
\end{aligned}
\tag{21}
$$

(3) Logistics partner's objective and constraints

The logistics partner also achieves to maximize its own net profits. And constraints are available storage (the first one) space and constant flow (the last one).

$$\begin{aligned}
&\text{Max}\, f_{l,t} = \sum_{v\in V} p_{lv}x_{lv,t} - \sum_{v\in V} c_{lv}x_{lv,t}, \forall l \in L, \forall t \in T \\
&st \\
&\sum_{l\in L} x_{lv,t} \le space_l, \forall v \in V, \forall t \in T, \\
&\sum_{l\in L} wt_m x_{ml,t} \le awt_m, \forall m \in M, \forall t \in T, \\
&\sum_{v\in V} x_{lv,t} = d_{v,t}, \forall l \in L, \forall t \in T,
\end{aligned} \tag{22}$$

Finally, the following constraints of globally constant flow should be satisfied:

$$\begin{aligned}
\sum_{m\in M}\sum_{l\in L} x_{ml,t} = \sum_{l\in L}\sum_{v\in V} x_{lv,t} = \sum_{v\in V} d_{v,t} \quad \forall t \in T \\
\sum_{m\in M} x_{ml,t} = \sum_{v\in V} x_{lv,t}, \quad \forall l \in L,\ \forall t \in T,
\end{aligned} \tag{23}$$

Therefore, the micro objective of SC game is presented as follows:

$$\text{Max}\ A \times f(x) \equiv \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{17} \\ \vdots & \ddots & \vdots \\ \alpha_{71} & \cdots & \alpha_{77} \end{bmatrix} \times \begin{bmatrix} f_{s=1} \\ \vdots \\ f_{l=2} \end{bmatrix} \tag{24}$$

And the macro objective is

$$\text{Min} \sum_{s\in S}\sum_{m\in M} tc_{sm,t}x_{sm,t} + \sum_{m\in M}\sum_{l\in L} tc_{ml,t}x_{ml,t} + \sum_{l\in L}\sum_{v\in V} tc_{lv,t}x_{lv,t}, \quad \forall t \in T \tag{25}$$

The optimization problem above is a vector optimization problem by considering the constraints of each partner simultaneously: i.e., this is a multi-objective optimization problem. And it is resolved by the fuzzy multi-objective approach (3) of Sect. 2.

Example 2
The model parameters of partners are arbitrarily set as follows.

(1) Supplier Partner 1 ($s = 1$)

$$\begin{aligned}
&\text{Max} \quad f_{s=1,t} = 3x + 4y - x - y, \forall t \in T, \\
&st \quad x + y \le 400, \forall t \in T \\
&\quad 2x + y \le 600, \forall t \in T,
\end{aligned} \tag{26}$$

(2) Supplier Partner 2 ($s = 2$)

$$\begin{aligned} &\text{Max} \quad f_{s=2,t} = 4x + 3y - 2x - y, \forall t \in T, \\ &\text{st} \quad x + y \leq 600, \forall t \in T \\ &\quad x + 2y \leq 500, \forall t \in T, \end{aligned} \tag{27}$$

(3) Mfg. Partner 1 ($m = 1$)

$$\begin{aligned} &\text{Max} \quad f_{m=1,t} = 2z - z, \forall t \in T \\ &\text{st} \quad \mathrm{z} = 2x + y, \forall t \in T \\ &\quad z \leq 200, \forall t \in T, \\ &\quad 5z \leq 900, \forall t \in T, \end{aligned} \tag{28}$$

(4) Mfg. Partner 2 ($m = 2$)

$$\begin{aligned} &\text{Max} \quad f_{m=2,t} = 4z - 2z, \forall t \in T \\ &\text{st} \quad \mathrm{z} = x + 2y, \forall t \in T \\ &\quad z \leq 600, \forall t \in T, \\ &\quad 2z \leq 600, \forall t \in T, \end{aligned} \tag{29}$$

(5) Mfg. Partner 3 ($m = 3$)

$$\begin{aligned} &\text{Max} \quad f_{m=3,t} = 3z - 2z, \forall t \in T \\ &\text{st } \mathrm{z} = x + y, \forall t \in T \\ &\quad z \leq 300, \forall t \in T, \\ &\quad 3z \leq 1000, \forall t \in T, \end{aligned} \tag{30}$$

(6) Logistics Partner 1 ($l = 1$)

$$\begin{aligned} &\text{Max}\, f_{l=1,t} = 6x_{l=1,v=1} + 7x_{l=1,v=2} + 9x_{l=1,v=3} - x_{l=1,v=1} - x_{l=1,v=2} - 2x_{l=1,v=3}, \forall t \in T \\ &\text{st} \\ &\quad x_{l=1,v=1} + x_{l=1,v=2} + x_{l=1,v=3} \leq 500 \end{aligned} \tag{31}$$

(7) Logistics Partner 2($l = 2$)

$$\begin{aligned} &\text{Max} \;\; f_{l=2,t} = 7x_{l=2,v=1} + 6x_{l=2,v=2} + 7x_{l=2,v=3} - 3x_{l=2,v=1} - x_{l=2,v=2} - x_{l=2,v=3}, \forall t \in T \\ &\text{st} \\ &\quad x_{l=2,v=1} + x_{l=2,v=2} + x_{l=2,v=3} \leq 1000 \end{aligned} \tag{32}$$

(8) Transportation Cost (Table 4).

Table 4 Transportation cost

From\To	$m = 1$	$m = 2$	$m = 3$	$l = 1$	$l = 2$	$v = 1$	$v = 2$	$v = 3$
$s = 1$	1	2	1	–	–	–	–	–
$s = 2$	2	1	2	–	–	–	–	–
$m = 1$	–	–	–	3	4	–	–	–
$m = 2$	–	–	–	4	6	–	–	–
$m = 3$	–	–	–	5	3	–	–	–
$l = 1$	–	–	–	–	–	2	3	1
$l = 2$	–	–	–	–	–	2	3	2

Table 5 Computational results of various alliances

Results\Alliance	Ideal cooperation	Extreme competition	Stackelberg competition
Global achievement level (λ)	1.00	0.64	0.51
Global profit	14,700	13,799	13,900
Global transportation Cost	9,700	9,836	9,697
Objective value of $f_{s=1,t}$	2,100	0	1,050
Objective value of $f_{s=2,t}$	0	1,400	700
Objective value of $f_{m=1,t}$	1,000	862	1,200
Objective value of $f_{m=1,t}$	0	1,017	750
Objective value of $f_{m=3,t}$	3,600	2,520	2,200
Objective value of $f_{l=1,t}$	2,000	2,000	2,000
Objective value of $f_{l=2,t}$	6,000	6,000	6,000

3.3 Results and Discussions

Three scenarios: ideal cooperation (all partner are joined as a big union with only one objective), Stackberg competition (every partner maximizes its own pay-off by ignoring the pay-offs of others) and extreme competition (every partner maximizes its own pay-off by minimizing the pay-offs of others) are simulated for Example 2. Discussions are also presented in the end of this section.

The vendors' demands are given first for each planning period, after that the problem is resolved by the fuzzy multi-objective approach (3) of Sect. 2. The first alliance matrix is the ideal cooperation case, the elements of are assumed as all ones. The second alliance matrix is the extreme competition case, the elements in A are all negative ones except the diagonal elements are positive ones. The third case is stackelberg competition case, the elements in A are all zeros, except the diagonal elements are ones. The global profit is defined as the sum of each partner's profit. The computational results are summarized in Table 5.

According to the computational results above, discussions are proposed as follows:

According to the simulation results, it is clear that the global achievement level: λ value is Ideal cooperation > Extreme competition > Stackberg competition Extreme competition. This is beyond our previous imagination that: Stackberg competition > Extreme competition. Thus, from the macro scope, cooperation seems to add the global achievement level because the ideal cooperation case has the largest λ.

However, ideal cooperation doesn't guarantee the maximal profit of each partner: especially satisfying the individual objective optimum of partner from the micro scope. This hints satisfying the allocation of global profit to each partner would be a challenging problem in the ideal cooperation case. If a partner feels unsatisfied for its individual objectives, then this partner may not be willing to join this supply chain. In short, globally maximal satisfaction doesn't guarantee locally maximal satisfaction, and vice versa.

According to the simulation results, using the fuzzy multi-objective game theory for modeling SC is an interesting idea. A new and simple concept of alliance matrix is introduced, which is designed to describe the cooperation degree between partners. Simulation results reflect greater realities and show that ideal cooperation is the best from the macro scope; however, extreme competition could have better individual performance of partner from the micro scope. Because of these conflicts and selfishness of partners, ideal cooperation is not easy to survive in practices. About the future studies, our new model could be used to explore the real alliance between partners. This means, readers are encouraged to extend and modify the SC model proposed here in order to meet their customized needs. A more complicated and advanced game via FMOP is available in the paper of Chen et al. (2010).

4 Fuzzy Data Envelopment Analysis for Supply Chain Management

This section is designed to illustrate the basic concepts of DEA by using FMOP. The extension from basic form will be deduced step by step so that readers are able to use or develop some advanced DEA by FMOP.

4.1 Basic DEA

Data envelopment analysis (DEA) defines mathematical programming of the outputs/inputs ratio as the index of production efficiency, developed by Charnes, et al. (1978), and followed by many others (Chen et al. 2009; Karsak and Ahiska 2007; Seiford 1996). The advantages and disadvantages of DEA are:

Advantages

- Ratio concept is easy for users
- Easy computation.

Disadvantages

- Not all inputs and outputs can be quantified
- Many decision making units (DMUs) could have the same and the highest scores, i.e., one (low discrimination power)
- Dual form of DEA is complicated.

The DEA model, developed by Charnes, et al. (1978), is changing the fractional programming problem to a linear mathematical programming model, which is able to handle several inputs and outputs. This model assumes n decision-making units (DMUs), with m inputs and p outputs, where the efficiency evaluation model of the k-th DMU can be defined as in Eq. (33).

$$\begin{aligned}
\text{Max} \quad & f_k = \frac{\sum_{r=1}^{p} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}} \\
\text{s.t.} \quad & \frac{\sum_{r=1}^{p} u_r y_{rl}}{\sum_{i=1}^{m} v_i x_{il}} \leq 1, \; l = 1, 2, \ldots, n; \\
& u_r \geq \varepsilon, \; r = 1, 2, \ldots, p; \\
& v_i \geq \varepsilon, \; i = 1, 2, \ldots, m.
\end{aligned} \tag{33}$$

where

x_{il} the i-th input value for l-th DMU;
y_{rl} the r-th output value for the l-th DMU;
u_r the weight values of the r-th output;
v_i the weight values of the i-th input i,
ε a very small positive value.

Obtaining the solution from Eq. (33) is difficult because it is a nonlinear programming problem. Charnes et al. transformed Eq. (33) into a linear programming problem by assuming $\sum_{i=1}^{m} v_i x_{ik} = 1$.

4.2 DEA with Fuzzy Inputs and Outputs

There are many available models for fuzzy DEA, which are based on various assumptions and deductions. The idea with fuzzy inputs and outputs here (Chen 2002) is modified from the model of Nagano et al. (1995).

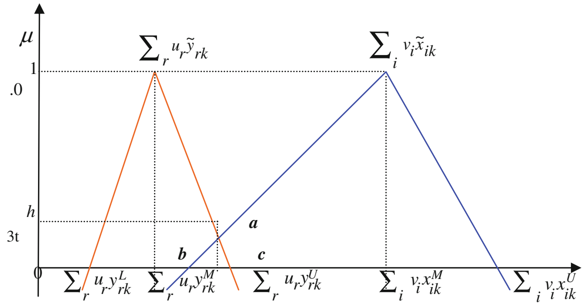
First, considering the firm n as the reference point in a DEA model, i.e., $\sum_r v_i x_{in}^M = 1$. Let $\tilde{\theta}_n = (\theta_n^L, \theta_n^M, \theta_n^U) = \left(\sum_r u_r y_{rn}^L, \sum_r u_r y_{rn}^M, \sum_r u_r y_{rn}^U\right)$. Thus, there are two desired objectives for this DEA model with fuzzy data:

(1) the fuzzy width of $\tilde{\theta}_n$ should be minimized—this situation is shown in Fig. 8,
(2) the overlap area in Fig. 8 should be minimized—the bounded area of triangle abc in Fig. 8 should be minimized.

The triangular fuzzy inputs and outputs are analyzed as in Fig. 8 with more details.

The first intersection type of $\sum_r u_r \tilde{y}_{rk}$ and $\sum_i v_i \tilde{x}_{ik}$ is analyzed as follows: considering two fuzzy numbers, the weighted sum of fuzzy outputs: $\sum_r u_r \tilde{y}_{rk}$, which is denoted by $\tilde{Y}$; and the weighted sum of fuzzy inputs: $\sum_i v_i \tilde{x}_{ik}$ which is denoted by $\tilde{X}$. $\tilde{X}$ and $\tilde{Y}$ may have some overlap area (intersection) in actuality—which will cause the vagueness of $\frac{\tilde{Y}}{\tilde{X}}$. Since the fuzzy efficiency score is defined by $\frac{\tilde{Y}}{\tilde{X}}$ and the unclear degree of $\frac{\tilde{Y}}{\tilde{X}}$ is the maximal $\mu_{\frac{\tilde{Y}}{\tilde{X}}} = \sup_{\frac{\tilde{Y}}{\tilde{X}}} \min(\mu_{\tilde{Y}}, \mu_{\tilde{X}}) = h_{3t}$, the lower the h_{3t} (e.g., $h_{3t} = 0$ means the computational result of $\frac{\tilde{Y}}{\tilde{X}}$ is very clear instead of fuzzy), the more reliability level of $\frac{\tilde{Y}}{\tilde{X}}$—the maximal reliability level of $\frac{\tilde{Y}}{\tilde{X}}$ is $1 - h_{3t}$. Therefore, the following concept can be deduced: the larger the overlap area, the lower reliability level when viewing the final efficiency scores of firms. If the overlap area between the weighted sum of fuzzy inputs and outputs can be reduced as small as possible—the optimal case is no overlap area; thus, the evaluated scores of firms by a DEA will be closer to the actuality with higher reliability. Furthermore, the weighted sum of outputs has no chance to be greater than the weighted sum of outputs and resulting in: $\sum_r u_r \tilde{y}_{rk}$ is less than or equal to $\sum_i v_i \tilde{x}_{ik}$ in a traditional DEA model with crisp data. However, the weighted sum of outputs almost all fall down the left side of point ***b***-except the overlap area between $\sum_r u_r \tilde{y}_{rk}$ and $\sum_i v_i \tilde{x}_{ik}$ (see Fig. 8) in a fuzzy condition. The overlapping degree: h_{3t} can be regarded as the degree of DMUs going outside the enveloped efficiency frontier by the modified DEA model. The efficiency scores of these un-enveloped DMUs are possibly greater than 1 in the extended DEA model. Of course, this h_{3t} should be reduced as small as possible so as to reflect more actuality and maximize the reliability of efficiency scores—all DMUs can be enveloped within the efficiency frontier if $h_{3t} = 0$. In addition to the first type of intersection between $\sum_r u_r \tilde{y}_{rk}$ and $\sum_i v_i \tilde{x}_{ik}$, the second intersection type is explained as follows: let the weighted sum of fuzzy outputs has a triangular fuzzy membership function of firm n like that in Fig. 8. Consider the fuzzy number: $\tilde{X}$ again, which is intersected with $\tilde{Y}$; moreover, h_{1n} and h_{2n} are created by the intersection points between $\tilde{X}$ and $\tilde{Y}$ (see Fig. 9). These two heights: h_{1n} and h_{2n}, represent the reliability levels for the weighted sum of fuzzy outputs for the reference point: n^{th} DMU, where the objective function of maximizing the fuzzy efficiency score can be obtained—

Fig. 8 Fuzzy inputs and outputs



because that $\mu_{\frac{\tilde{Y}}{\tilde{X}}} = \sup_{\frac{\tilde{Y}}{\tilde{X}}} \min(\mu_{\tilde{Y}}, \mu_{\tilde{X}}) = \max(h_{1n}, h_{2n})$ in such an intersection case of $\tilde{X}$ and $\tilde{Y}$. The concept of this objective is shown as in Fig. 9—both h_{1n} and h_{2n} should be maximized so as to force the $\tilde{\theta}_n$ within the minimal width of fuzzy interval.

Finally, an extended DEA model is proposed as follows:

$$
\begin{aligned}
&\text{Max} \;\; h_{1n} = \frac{\sum_{r=1}^{p} u_r y_{rn}^L}{1 - \left(\sum_{r=1}^{p} u_r y_{rn}^M - \sum_{r=1}^{p} u_r y_{rn}^L\right)} \\
&\text{Max} \;\; h_{2n} = \frac{\sum_{r=1}^{p} u_r y_{rn}^U}{1 + \left(\sum_{r=1}^{p} u_r y_{rn}^U - \sum_{r=1}^{P} u_r y_{rn}^M\right)} \\
&\text{Min } h_{3t} = \frac{\sum_{r=1}^{p} u_r y_{rt}^U - \sum_{i=1}^{m} v_i x_{it}^M}{\sum_{r=1}^{p} u_r y_{rt}^U - \sum_{r=1}^{p} u_r y_{rt}^M} \quad (t = 1, 2, \ldots, k;\; t \neq n;) \\
&\text{s.t.} \quad \sum_{i=1}^{m} v_i x_{in}^M = 1; \\
&0 \leq h_{1n} \leq 1; \\
&0 \leq h_{2n} \leq 1; \\
&0 \leq h_{3t} \leq 1, \;\; t = 1, 2, \ldots, k, t \neq n; \\
&u_r \geq \varepsilon, \;\; r = 1, 2, \ldots, p; \\
&v_i \geq \varepsilon, \;\; i = 1, 2, \ldots, m; \\
&n \in \{1, 2, \ldots, k\}.
\end{aligned}
\tag{34}
$$

Here, h_{1n} and h_{2n} are the reliability levels for the weighted sum of fuzzy outputs for the reference point: n^{th} DMU. Furthermore, h_{3t} denotes the degree of some

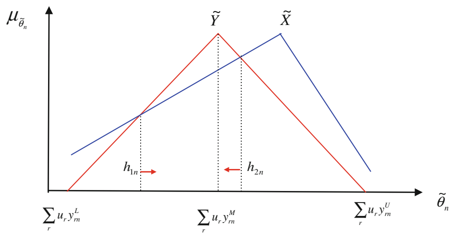


Fig. 9 The first objective

DMUs going outside the piece-wise frontier only when $t \neq n$; $\sum_{i=1}^{m} v_i x_{in}^M = 1$ implies that the i^{th} input resource of n^{th} DMU is limited. Moreover, h_{1n}, h_{2n} and h_{3t} must be between 0 and 1 for normalized fuzzy sets.

It is clear that Eq. (34) is a multi-objective problem; thus, this problem can be translated to a fuzzy multi-objective problem in Eq. (35) by the general λ transformation.

$$
\begin{aligned}
&\text{Max } \lambda \\
&\text{st.} \quad h_{1n} = \frac{\sum_{r=1}^{p} u_r y_{rn}^L}{1 - \left(\sum_{r=1}^{p} u_r y_{rn}^M - \sum_{r=1}^{p} u_r y_{rn}^L \right)} \geq \lambda \\
&\qquad h_{2n} = \frac{\sum_{r=1}^{p} u_r y_{rn}^U}{1 + \left(\sum_{r=1}^{p} u_r y_{rn}^U - \sum_{r=1}^{P} u_r y_{rn}^M \right)} \geq \lambda \\
&\qquad 1 - h_{3t} = 1 - \frac{\sum_{r=1}^{p} u_r y_{rt}^U - \sum_{i=1}^{m} v_i x_{it}^M}{\sum_{r=1}^{p} u_r y_{rt}^U - \sum_{r=1}^{p} u_r y_{rt}^M} \geq \lambda \ (t = 1, 2, \ldots, k;\ t \neq n;) \\
&\qquad \sum_{i=1}^{m} v_i x_{in}^M = 1; \\
&\qquad 0 \leq \lambda \leq 1; \\
&\qquad 0 \leq h_{1n} \leq 1; \\
&\qquad 0 \leq h_{2n} \leq 1; \\
&\qquad 0 \leq h_{3t} \leq 1,\ t = 1, 2, \ldots, k, t \neq n; \\
&\qquad u_r \geq \varepsilon,\ r = 1, 2, \ldots, p; \\
&\qquad v_i \geq \varepsilon,\ i = 1, 2, \ldots, m; \\
&\qquad n \in \{1, 2, \ldots, k\}.
\end{aligned}
\tag{35}
$$

where λ can be regarded as the global reliability level of viewing the final efficiency scores, the higher the λ value, the less vagueness in the final results.

Table 6 Assumed data of Example 3

	Firm 1	Firm 2	Firm 3	Firm 4	Firm 5	Firm 6
Input x_1	10	20	(4, 5, 6)	15	10	(13, 15, 17)
Input x_2	40	62	25	65	50	50
Output y_1	12	23	(6, 8, 9)	(12, 13, 15)	15	(17, 18, 19)

Table 7 Computational results

λ = 0.9774	Weight v_1	Weight v_2	Weight u_1	Scores = $\frac{\sum_r u_r y_{rk}}{\sum_i v_i x_{ik}}$
Firm 1	0.0001	0.0161	0.0424	0.7901
Firm 2				0.9770
Firm 3				(0.6321, 0.8428, 0.9481)
Firm 4				(0.4861, 0.5268, 0.6079)
Firm 5				0.7900
Firm 6				(0.8955, 0.9481, 1.0000)

Equation (35) is a non-linear programming problem, which should be resolved by the LINGO software. The predicted efficiency score will have a lower and upper bound at a global reliability level λ:

$$\tilde{\theta}_n = \left(\theta_n^L, \theta_n^M, \theta_n^U\right) = \left(\sum_r u_r y_{rn}^L, \sum_r u_r y_{rn}^M, \sum_r u_r y_{rn}^U\right) \tag{36}$$

Example 3
A simple example will be illustrated in this section so as to validate this extended DEA model with fuzzy data. These assumed crisp/fuzzy data are shown in Table 6.

After taking the Firm 2 as the reference point and inputting all the available data in Table 6, final results are obtained by LINGO in Table 7. The approach proposed here is suitable for taking the fuzzy input/output data into account. However, the fuzzy score in Table 7 may vary when the reference point is changed. Thus, some scholars try to find the common weight for DEA: maximizing the efficiency of each DMU simultaneously.

4.3 DEA of FMOP

The traditional DEA model is optimized for one single objective of the referred DMU (reference point). Many scholars from MCDM seek to optimize the performance of each DMU at the same time (Golany 1988; Kao and Hung 2005; Li and Reeves 1999), which is called as the common weight approach for DEA.

Chiang and Tzeng (2000) proposed the following FMOP approach to DEA. This method provided a common weight (μ^*, ω^*)for all DMUs, which were evaluated on an equal standard. By employing the FMOP approach, all DMUs can be treated at the same time. Hence it is effective for large numbers of DMUs. Considering the efficiency ratio of all DMUs, it can establish the multiple objective programming model as shown in Model (37):

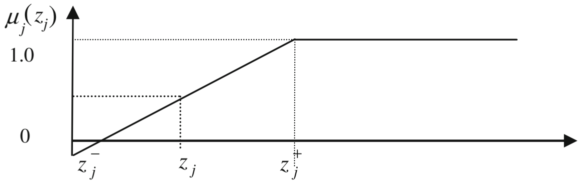
$$
\begin{aligned}
&\text{Max } z_1 = \frac{\sum_{r=1}^{s} \mu_r \cdot y_{r1}}{\sum_{i=1}^{m} \omega_i \cdot x_{i1}} \\
&\text{Max } z_2 = \frac{\sum_{r=1}^{s} \mu_r \cdot y_{r2}}{\sum_{i=1}^{m} \omega_i \cdot x_{i2}} \\
&\qquad \vdots \\
&\text{Max } z_n = \frac{\sum_{r=1}^{s} \mu_r \cdot y_{rn}}{\sum_{i=1}^{m} \omega_i \cdot x_{in}} \\
&\text{st} \\
&\quad \frac{\sum_{r=1}^{s} \mu_r \cdot y_{rk}}{\sum_{i=1}^{m} \omega_i \cdot x_{ik}} \le 1, \quad k = 1, 2, \ldots, n \\
&\quad \mu_r, \omega_i \ge \varepsilon > 0, \forall r, i,
\end{aligned}
\tag{37}
$$

where

- y_{rk} the observed amount of output of the r^{th} $(r = 1, 2, \ldots, s)$ type for the k^{th} DMU$(k = 1, \ldots, n)$;
- x_{ik} the observed amount of input of the $i^{th}(i = 1, 2, \ldots, m)$ type for the k^{th} DMU$(k = 1, \ldots, n)$;
- ω_i the multiplier or weight of the i^{th} input;
- μ_r the multiplier or weight of the r^{th} output;
- ε non-Archimedean quantity.

Model (37) is a multiple objective programming (MOP). In this model, we try to solve a common weight, which makes all the DMU's efficiency maximal at the same time. It can be solved by the FMOP approach proposed by Zimmermann in Eq. (3) of Sect. 1. The concept of FMOP utilizing the membership function transfers multi-objective function into one objective function. The membership function is as follows:

Fig 10 Linear membership function of z_j



$$\mu_j(z_j) = \begin{cases} 0 & ; \quad z_j \le z_j^- \\ \frac{z_j - z_j^-}{z_j^+ - z_j^-} & ; \quad z_j^- \le z_j \le z_j^+ \\ 1 & ; \quad z_j \ge z_j^+ \end{cases} \tag{38}$$

where z_j^- and z_j^+ are the negative ideal solution and the positive ideal solution respectively for the value of the objective function z_j such that the degree of membership function is [0, 1]. The geometric view of the linear membership function is shown in Fig. 10.

The degree of membership function of z_j in $\mu(z_j)$ refers to the achievement level of the efficiency ratio for the DMU$_j$. The problem of obtaining the maximum decision is to choose (μ^*, ω^*) such that

$$\begin{aligned} &\underset{\mu,\,\omega}{\text{Max}}\ \underset{\text{j}}{\text{Min}} \left\{ \mu_{\text{j}}(\text{z}_\text{j}) \mid \text{j} = 1, 2, \ldots, \text{n} \right\} \\ &\text{st} \\ &\quad \frac{\sum_{r=1}^{s} \mu_r \cdot y_{rk}}{\sum_{i=1}^{m} \omega_i \cdot x_{ik}} \le 1, \quad j = 1, 2, \ldots, n \\ &\quad \mu(z_j) \ge \alpha \\ &\quad \mu_r,\ \omega_i \ge\ \varepsilon > 0,\ \forall r,\ i \end{aligned} \tag{39}$$

Then, the achievement level of objective functions for Model 1 should be a larger level such as:

$$\alpha = \frac{z_j - z_j^-}{z_j^+ - z_j^-} \tag{40}$$

Equation (40) is variable transformation, the model has transformed $z_j = \alpha \cdot z_j^+ + (1 - \alpha) \cdot z_j^-$. Where z_j is a convex combination of z_j^- and z_j^+, Eq. (38) can be rewritten as model of Eq. (40). According to the concept of multiple objective linear programming, the common weight$(\mu*, \omega*)$ should satisfy all DMUs restrictions. The weight $(\mu*, \omega*)$ from all of the DMUs is the common weight to all DMUs which are evaluated on a consist standard for ranking. We may apply LINGO for resolution to solve the model (39).

Table 8 Assumed Data of Example 4

DMU\Inputs or Outputs	x_1	x_2	y_1	y_2
A	5	3	5	1
B	1	6	4	5
C	3	3	6	4

$$\begin{aligned}
&\underset{\mu,\omega}{Max}\,\alpha \\
&\text{st} \\
&\quad \sum_{r=1}^{s}\mu_r\cdot y_{rk}-\sum_{i=1}^{m}\omega_i\cdot x_{ik}\le 0,\quad k=1,\cdots,n \\
&\quad \sum_{r=1}^{s}\mu_r\cdot y_{rj}-\alpha\cdot\sum_{i=1}^{m}\omega_i\cdot x_{ij}\ge 0,\quad j=1,\cdots,n \\
&\quad \mu_r,\omega_i\ge\varepsilon>0
\end{aligned}\tag{41}$$

Employing Model (41), a common weight (μ^*,ω^*) is determined for all DMUs and the efficiency score of each DMUj is defined as the following:

$$e_j=\frac{\sum_{r=1}^{s}\mu_r^*\cdot y_{rj}}{\sum_{i=1}^{m}\omega_i^*\cdot x_{ij}}\tag{42}$$

Example 4

Consider three firms with two inputs and two outputs as follows (Table 8).

Using the programming problem (41); therefore, the following problem (43) is derived and resolved:

$$\begin{aligned}
&\underset{\mu,\omega}{\text{Max}}\ \alpha \\
&\text{st} \\
&\mu_1\times 5+\mu_2\times 1-\omega_1\times 2-\omega_2\times 3\le 0 \\
&\mu_1\times 4+\mu_2\times 5-\omega_1\times 1-\omega_2\times 6\le 0 \\
&\mu_1\times 6+\mu_2\times 4-\omega_1\times 3-\omega_2\times 3\le 0 \\
&\mu_1\times 5+\mu_2\times 1-\alpha\times(\omega_1\times 2+\omega_2\times 3)\ge 0 \\
&\mu_1\times 4+\mu_2\times 5-\alpha\times(\omega_1\times 1+\omega_2\times 6)\ge 0 \\
&\mu_1\times 6+\mu_2\times 4-\alpha\times(\omega_1\times 3+\omega_2\times 3)\ge 0 \\
&\mu_1,\mu_2,\omega_1,\omega_2\ge 10^{-5}
\end{aligned}\tag{43}$$

Therefore the following results are computed by LINGO:

$$\alpha=0.61,\mu_1=28.9,\mu_2=10^{-5},\omega_1=31.6,\omega_2=26.3,$$

Table 9 The weights of fuzzy DEA model

Weight\Year	2003
Manpower of environmental protection (v_1)	4.82×10^{-4}
Budget of environmental protection (v_2)	4.35×10^{-4}
Advertisement of environmental protection (v_3)	1.00×10^{-4}
Harmful emission (u_1)	1.00×10^{-4}
Number of noise event (u_2)	1.00×10^{-4}
Ratio of qualified water (u_3)	1.00×10^{-4}
Recycle quantity from wastes (u_4)	1.00×10^{-4}
Number of malodorous air event (u_5)	1.00×10^{-4}

The efficiency score of each firm is shown as follows:

$$e_A = 0.60, e_B = 0.61 \text{ and } e_C = 0.99.$$

The model (41) is nonlinear and could result in some computational difficulties. In the next section, a linear model with FMOP is developed. Readers should distinguish the difference between model (35) and model (41). The model (41) is fuzzy multi-objective and only able to compute crisp data; however, the model (35) is also fuzzy multi-objective and is able to compute fuzzy data. Decision maker should choose the model that meets his/her requirements.

4.4 DEA of FMOP by Difference Between Inputs and Outputs

This section is presented to some readers, who are interested in advanced forms in DEA by FMOP. The presented model is based on the computation of efficiency via the difference between inputs and outputs (Chen et al. 2009) rather than the fractional model in tradition. Consider the problem (33) again. Assume that

$$\sum_{i=1}^{m} v_i x_{il} > 0, \ l = 1, 2, \ldots, n$$

Then the first n constraints of the problem (33) are equivalent to the following respectively

$$\sum_{r=1}^{p} u_r y_{rl} - \sum_{i=1}^{m} v_i x_{il} \leq 0, \ l = 1, 2, \ldots, n. \tag{44}$$

Moreover, from the constraints of the problem (33) we deduce that

$$0 < f_k = \frac{\sum\limits_{r=1}^{p} u_r y_{rk}}{\sum\limits_{i=1}^{m} v_i x_{ik}} \leq 1$$

Thus, the maximum value that the efficiency f_k of a DMU k can ideally reach is 1. For DMU k, consider the function

$$g_k = \sum_{i=1}^{m} v_i x_{ik} - \sum_{r=1}^{p} u_r y_{rk},$$

From (33) we deduce that

$$g_k = \sum_{i=1}^{m} v_i x_{ik} - \sum_{r=1}^{p} u_r y_{rk} \geq 0,$$

then the smallest value that g_k can ideally reach is 0. Moreover when g_k, $\sum_{i=1}^{m} v_i x_{ik} = \sum_{r=1}^{p} u_r y_{rk}$, which means $f_k = 1$. Consider now the following linear programming problem. Problem (45) is formulated as a minimization problem of the g_k, for $k = 1, 2, \ldots, n$ as follows:

$$\begin{aligned}
&\text{Min} \quad g_1 = \sum_{i=1}^{m} v_i x_{i1} - \sum_{r=1}^{p} u_r y_{r1} \\
&\text{Min } g_2 = \sum_{i=1}^{m} v_i x_{i2} - \sum_{r=1}^{p} u_r y_{r2} \\
&\ldots \\
&\text{Min } g_n = \sum_{i=1}^{m} v_i x_{in} - \sum_{r=1}^{p} u_r y_{rn} \\
&st \sum_{r=1}^{p} u_r y_{rk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0,\ k = 1, 2, \ldots, n; \\
&\quad u_r \geq \varepsilon,\ r = 1, 2, \ldots, p; \\
&\quad v_i \geq \varepsilon,\ i = 1, 2, \ldots, m.
\end{aligned} \tag{45}$$

Now the optimistic expectation of g_k is assumed as zero, the pessimistic expectation of g_k is assumed as ξ, the ξ is a small positive value subjectively determined by the decision maker. When g_k is zero, this also implies that the k-th DMU satisfies that: $\sum\limits_{i=1}^{m} v_i x_{ik} = \sum\limits_{r=1}^{p} u_r y_{rk}$. Thus, according to the fuzzy transformation in Fig. 2, the achievement level for each objective/DMU is:

Table 10 Normalized Input/Output data 2003

District	Input			Output				
	x_1	x_2	x_3	y_1	y_2	y_3	y_4	y_5
A	1.000	1.000	0.131	0.952	0.001	0.925	1.000	0.366
B	0.669	0.809	0.044	0.852	0.569	0.951	0.671	0.167
C	0.562	0.740	0.026	0.958	0.414	0.924	0.430	0.518
D	0.394	0.385	0.045	0.756	0.717	0.935	0.583	0.504
E	0.385	0.611	0.100	0.917	0.820	0.696	0.465	0.061
F	0.325	0.423	0.029	0.727	0.787	0.922	0.350	0.652
G	0.364	0.386	0.087	0.787	0.689	0.948	0.395	0.134
H	0.374	0.625	0.104	0.902	0.753	0.674	0.453	0.270
I	0.302	0.392	0.046	0.873	0.758	0.412	0.499	0.344
J	0.442	0.437	0.046	0.809	0.849	0.953	0.477	0.490
K	0.383	0.324	0.041	0.740	0.895	0.975	0.530	0.159
L	0.368	0.388	0.031	0.746	0.814	0.991	0.334	0.362
M	0.364	0.432	0.093	0.870	0.797	0.928	0.368	0.190
N	0.414	0.602	0.051	0.798	0.777	0.877	0.501	0.406
O	0.364	0.438	0.009	0.828	0.750	0.977	0.208	0.495
P	0.411	0.548	0.036	0.538	0.600	0.927	0.667	0.542
Q	0.473	0.565	0.232	0.667	0.693	0.984	0.489	0.615
R	0.626	0.541	1.000	0.001	0.810	0.978	0.715	0.697
S	0.541	0.544	0.090	0.589	0.725	0.976	0.659	0.883
T	0.419	0.748	0.412	0.674	0.355	0.923	0.502	0.437
U	0.476	0.447	0.135	0.832	0.647	0.934	0.890	0.463
V	0.472	0.584	0.150	0.320	0.195	1.000	0.303	0.530
W	0.460	0.503	0.226	0.806	0.610	0.967	0.671	0.001

$$\mu_k(g_k) = \begin{cases} 0 & ; \quad g_k \geq \xi \\ \frac{\xi - g_k}{\xi - 0} & ; \quad 0 \leq g_k < \xi \\ 1 & ; \quad g_k < 0 \end{cases} \tag{46}$$

Here $g_k = \sum_{i=1}^{m} v_i x_{ik} - \sum_{r=1}^{p} u_r y_{rk}$. Then resolving the problem (45) by fuzzy multi-objective approach is identical to solve the optimization problem (47):

$$\begin{aligned} &\text{Max} \quad \gamma \\ &st \quad \gamma \leq \mu_k(g_k),\ k = 1, 2, \ldots, n; \\ &\sum_{r=1}^{p} u_r y_{rk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0,\ k = 1, 2, \ldots, n; \\ &\quad u_r \geq \varepsilon,\ r = 1, 2, \ldots, p; \\ &\quad v_i \geq \varepsilon,\ i = 1, 2, \ldots, m. \end{aligned} \tag{47}$$

Table 11 Efficiency Score

District/Year	2003	
DMU	Score	Rank
A	0.349	23
B	0.473	21
C	0.544	18
D	0.966	3
E	0.641	17
F	0.999	2
G	0.838	10
H	0.660	15
I	0.899	5
J	0.876	7
K	1.000	1
L	0.930	4
M	0.845	9
N	0.719	13
O	0.887	6
P	0.743	12
Q	0.694	14
R	0.502	20
S	0.756	11
T	0.508	19
U	0.861	8
V	0.472	22
W	0.659	16

Example 5

Environmental protection issues are attracting attentions from both governments and academics in the field of environmental economics. Furthermore, environmental protection performance is also becoming a major concern for green SCM (Vachon and Klassen 2008; Wu et al. 2007). The fuzzy DEA above is used to analyze the partner performance in a green supply chain. The input/output data from 23 district governments of Taiwan in 2003 are collected, district governments are encoded from "A" to "W". The inputs are defined as: manpower of environmental protection (v_1), budget of environmental protection (v_2) and advertisement of environmental protection (v_3); in addition, the outputs are defined as: the reduced amount of harmful emission (u_1), the reduced number of noise event (u_2), ratio of qualified water (u_3), recycle quantity from wastes (u_4) and the reduced number of malodorous air event (u_5). These data are normalized in Table 10.

Here ε is assumed as 10^{-5} and ξ is set to 0.1 in this study, the computed results for are available in Tables 9 and 11.

According to the computational results, the district K is the most efficient DMU (less inputs and more outputs); on the contrary, the district A is the least efficient DMU (more inputs and less outputs). These reports are valuable to push the district government competing for further improvements of environmental protection. The model (47) is linear, fuzzy multi-objective and appropriate for crisp data.

5 Conclusions and Future Studies

According to the simulation results and examples in previous sections, readers are encouraged to use fuzzy MCDM: FMOP and FMADM for solving problems of SCM and develop/extend the fuzzy model in this chapter further. These two methods: FMOP and FMADM are valuable for developing new and advanced approaches in the near future. In addition, FMOP validates its general use for various optimization models of SC. For example, it could be useful in network design, aggregate planning, vehicle routing problem, production scheduling problem, …, etc.

In Sect. 2, the simple framework of SC is proposed; furthermore, FMOP and FMADM are both presented. A new and simple game of alliance matrix for simulating SC performance is illustrated in Sect. 3, which is designed to describe the cooperation degree between partners. Simulation results reflect greater realities and show that ideal cooperation is the best from the macro scope; however, extreme competition could have better individual performance of partner from the micro scope. In Sect. 4, the fuzzy DEA model and its extensions are presented by FMOP. In this section, some possible studies for future are provided. Readers are encouraged to develop their own applications and advanced models from this beginning.

About the future studies, all these fuzzy models presented in this chapter could be integrated with IT technologies nowadays. This means: all optimization models should be computed on line or accept transmitted data by internet for real-time decision making. These efforts will extend the ability of fuzzy models for SCM. The basic idea of cloud computing is simply introduced here, the implementation of IT framework, issues of green supply chain and other trends are summarized as follows:

(1) Cloud Computing

Cloud computing is the delivery of computing as a service, whereby shared resources, software, and information over an internet (Buyya et al. 2008). Today, the latest paradigm to emerge is that of Cloud computing which promises reliable services delivered through next-generation data centers that are built on compute and storage virtualization technologies. Consumers will be able to access applications and data from a "Cloud" anywhere in the world on demand. Cloud computing is simply shown as follows in Fig. 11 for better understanding. Actually, some scholars are starting to study SCM issues by setting a cloud. The articles

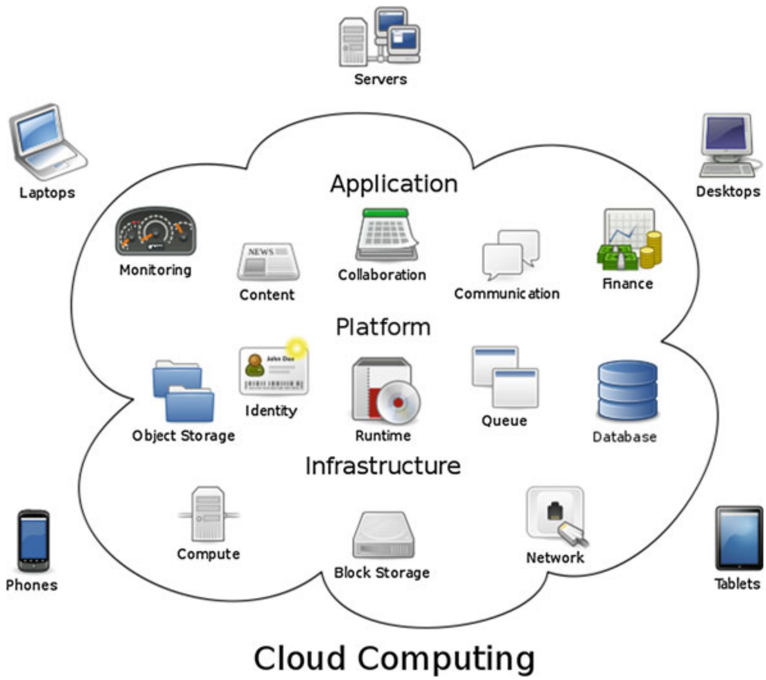


Fig. 11 Simple model of cloud computing. *Source* Wikimedia commons by Johnston S

of Lindner et al. (2010), and Celesti et al. (2010) provide visions for integrating Information Technology (IT) and SCM in the very near future.

Enterprises currently are eager to employ cloud services in order to improve the scalability of their services and to deal with bursts in resource demands. With the cloud service, consumers are able to use the services by internet anytime and anywhere. Although there are many papers to talk about the cloud framework from the theoretical view, the number of practical implementations/applications for SCM are still less in academic papers.

(2) IT Framework

The model concept to integrate SCM and the optimization module is simple; eventually, a Decision Support System (DSS) should be developed. The data from the demand side and the supply side are considered simultaneously to make the best decision for resource allocation. For example, ranking the suppliers by FMADM approach via collecting the attribute data on line is an interesting idea. For example, Chen et al. use the fuzzy MADM for selecting the appropriate hospital to transfer patients (Chen et al. 2012), the fuzzy resolution approach for any SCM problem could be implemented by a cloud service by the IT framework of Fig. 12.

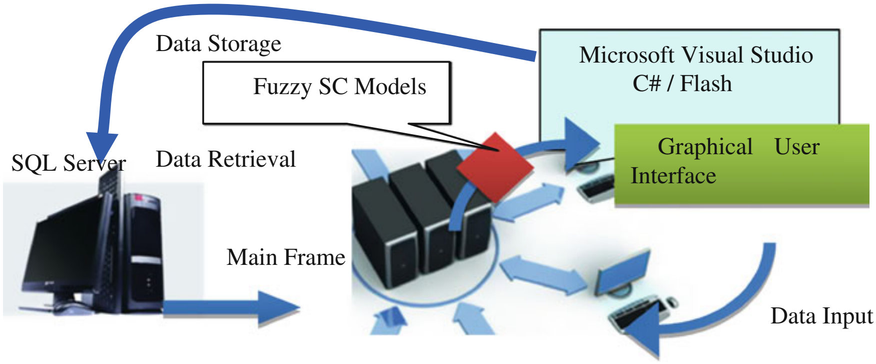


Fig. 12 IT framework

In addition to the IT platform of C#, SQL and Flash, the Java + MySQL platform is also popular. Once the IT framework is set up, decision maker can make mobile decisions by smart devices: e.g., phones, computers, anytime and anywhere. Readers interested in real applications are encouraged to refer the research as follows: Yong and Zhang (2008) propose fuzzy evaluating method for channel selection (IT platform), Balan et al. (2007) reduce the Bullwhip effect in a supply chain with fuzzy logic, Harnisch and Buxmann (2013) use FAHP to evaluate cloud services. Fang et al. (2002) propose the DSS for SCM in textile industry, and Radio Frequency Identification (RFID) is used to integrate the multi-objective model for dispatching patients to hospitals for emergency by Chen et al. (2011). DSS usually has three elements: graphical user interface, model and database. Fuzzy MCDM could play important roles in the model construction.

(3) Green Supply Chain Management

Green SCM focuses influence and relationships between supply-chain management and environmental thinking into supply-chain management, including product design (concept), material sourcing and selection, manufacturing processes, delivery of the final product to the consumers as well as end-of-life management of the product after its useful life (Srivastava 2007), which is illustrated in Fig. 13.

The literature here is simply classified following categories:

(i) Green Design

Understanding of how design decisions affect a product's environmental compatibility is concerned in this field; for example, Madu et al. (2002) present a very useful hierarchic framework for environmentally conscious design. Interested readers can also find the literature existing on design for material and product recovery (He et al. 2004; Krikke et al. 2003).

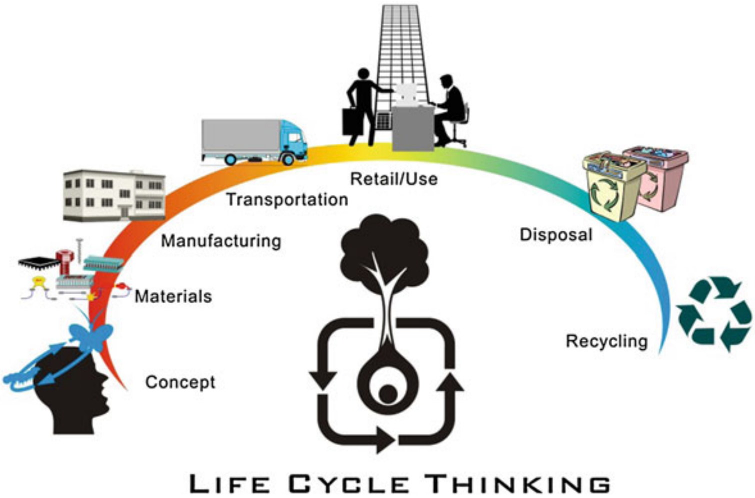


Fig. 13 Green supply chain management. *Source* www.dlinkgreen.com

(ii) Green Operations

Three main streams are popularly discussed here. The first stream is recycling. Remanufacturing is defined as recycling-integrated manufacturing. Industries that apply remanufacturing typically include automobiles, electronics (Hsu and Hu 2008) and tires (Hoshino et al. 1995). Product recovery refers to the broad set of activities designed to reclaim value from a product at the end of its useful life. A model for evaluating recovery strategies for the product without violating the physical and economical feasibility constraints is proposed by Krikke et al. (2003). The second stream is reverse logistics and network design. Reverse logistics networks have some generic characteristics related to the coordination requirement of two markets, supply uncertainty, returns disposition decisions, postponement and speculation (Yalabik et al. 2005). As a result, they extend the traditional network design to a more wide consideration. The final stream is waste management: disposal has always been a compelling problem and has led to green consciousness. Teunter and Vlachos (Teunter and Vlachos 2002) focus on the necessity of a disposal option for remanufacturable items.

Some scholars mentioned the use of MOP/MADM on green SCM (Wu et al. 2007). Paksoy et al. (2012) use fuzzy multi-objective model by including environmental hazards. Chen et al. (2008) proposed the route planning for transportation of nuclear waste by Geographical Information System (GIS). Lin (2013) uses the fuzzy DEMATEL to evaluate the management practices. In addition, fuzzy and crisp AHP models are also popular here (Peng 2012; Wang et al. 2011). It is important to note that many issues of green supply chain often include social justice inside, e.g., the issues of handling environmental hazards, nuclear waste, toxic material,… etc. Interested readers may study further by these aforementioned articles.

(iii) Others

The concept of SCM is not only useful in manufacturing industries, but also valuable in service industries. Readers are encouraged to explore more SCM applications by FMOP/FMADM in addition to the traditional applications. Especially when considering the service industries, Pramod and Banwet (2013) use fuzzy ISM on the SCM issues of telecom service, Cho et al. (2012) use FAHP on catering enterprises and Chen et al. (2012) employ FMOP on dispatching patients to hospitals for Emergency Medicine (EM). The service industry is an interesting, developing and attractive area for SCM models with IT, FMOP and FMADM in the very near future.

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Supply Chain Performance Measurement: An Integrated DEMATEL and Fuzzy-ANP Approach

Ozlem Senvar, Umut Rifat Tuzkaya and Cengiz Kahraman

Abstract Supply chain performance measurement is vital for the continuous improvement of supply chain management. Effective supply chain performance measurement is one of the most important aspects for supply chain management in which decision makers can analyze the historical performance and current status, and set future performance targets. This chapter provides a conceptual point of view to supply chain performance measurement. Inevitably, quantification of the values with precision in a complex supply chain performance measurement system is difficult. The supply chain performance measurement under fuzziness can consider the uncertainty and ambiguity surrounding the supply chain performance measurement. The aim of this chapter is to present a fuzzy decision making approach to deal with the performance measurement in supply chain systems. In this chapter, DEMATEL method is adapted to model complex interdependent relationships and construct a relation structure using measurement criteria for evaluation. F-ANP is performed to overcome the problem of dependence and feedback among each measurement criteria. The integrated DEMATEL and F-ANP approach provides an effective decision tool for the supply chain performance measurement.

Keywords Suppy chain management · Performance management · DEMATEL · ANP · Multi criteria decision making · Fuzzy logic

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1 Introduction

Supply chain is an important component in logistics development for all industries. It can improve efficiency and effectiveness of not only product transfer, but also information sharing between the complex hierarchies of all the tiers. Waters and Waters (2007) emphasized that supply chain comprises a key element in corporate competitiveness; some firms have come to view this function as the cornerstone of their differentiation strategy. The supply chain is a continuous process, from raw materials to finished goods. It contains different functions such as process design, products design, manufacturing, distribution, sales, purchasing, marketing and forecasting. Supply chain management defines a network of interdependent partners which are working extremely close together in order to accomplish a common goal of customer satisfaction. The success in the flow of supply chain management produces products of high quality at low cost and a good customer service (El-Baz 2011). Supply chain management involves integrating all key operational processes at any level between the final users and original suppliers of the products, services and information that offer added value to customers and other stakeholders (Cooper and Lambert 2000). Supply chain management creates value for customers, companies, and stakeholders interacting throughout a supply chain (Estampe et al. 2013). Inevitably, it is important to measure the performance of the complete supply chain as well as the individual processes. The performance measurement system should be based on the strategy, value drivers and important goals of the companies and the whole supply chain. Supply chain performance measurement is necessary for the continuous improvement of supply chain management (Chan 2003). In other words, supply chain performance measurement is essential for a company in order to survive in today’s competitive business environment. Successful supply chain performance measurement relies on appropriate metrics that capture the entire essence of the supply chain process. Supply chain performance measurement should be a business-critical process, driven by metrics and supported by business intelligence. With increasing competition and changing market forces, tapping into this critical asset is essential in sustaining competitive advantage in the global space. Unfortunately, performance measurement in the supply chain field has not kept pace with today’s world of interdependent business relationships. What companies need is a new performance measurement system that unifies different business elements, concepts, technologies and tools (Stefanović and Stefanović 2011).

For measuring performance the most widely used method is the balanced scorecard. Kaplan and Norton (1992) presented balanced scorecard model in order to evaluate corporate performance in four types of approaches: the financial, the internal business process, the customer as well as learning and growth. Balanced scorecard method has been widely used in strategy formulation with clearly defined missions, targets, suitable performance measures and metrics (Gunasekaran and Kobu 2007). Several researchers have proposition using balanced scorecard for measuring supply chain management capability (Forker et al. 1997; Yamin et al. 1999; Brewer and Speh 2000; Lapide 2000; Gunasekaran et al. 2001; Mehrjerdi 2009).

Inevitably, quantification of the values with precision in a complex supply chain performance measurement system is difficult. As a matter of fact, fuzzy logic is a technique suitable for dealing with uncertainty and subjectivity. Hence, the supply chain performance measurement under fuzziness can be a new direction in measuring the uncertainty and ambiguity surrounding the supply chain performance measurement.

From this standpoint, the aim of this chapter is to present a fuzzy decision making approach to deal with the performance measurement in supply chain systems. The complex supply chain performance measurement system can be partitioned into separate subsystems in order to facilitate the evaluation of each partition. In this chapter, a decision-making trial and evaluation laboratory (DEMATEL) method is used to develop interrelations among each measurement criterion. That is, DEMATEL method is adapted to model complex interdependent relationships and construct a relation structure using measurement criteria for evaluation. Afterwards, the weight of each criterion is evaluated using fuzzy analytic network process (F-ANP). The F-ANP is performed in order to overcome the problem of dependence and feedback among each measurement criteria.

The rest of the chapter can be summarized as follows: Sect. 2 reviews the selected studies regarding fuzzy supply chain performance measurement in brief. Section 3 explains the methodology used in this chapter. Section 4 provides an illustrative example. Section 5 provides conclusion, discussions as well as recommendations for further studies.

2 Fuzzy Supply Chain Performance Measurement

The supply chain performance measurement under fuzziness can be a new direction in measuring the uncertainty and ambiguity surrounding supply chain performance measurement.

Chen (2002) proposed an algorithm for external performance evaluation of distribution centers in logistics from retailers' viewpoint under fuzzy environment. In this regard, the concepts of factor analysis, eigenvector method, fuzzy Delphi method, fuzzy set theory, and multi criteria decision making method have been adopted.

Lau et al. (2002) considered a framework of supply chain management that involves the principles of fuzzy logic for analysis and monitoring performance of suppliers based on the criteria of product quality and delivery time. The proposed system recommends the quantity should be placed in the next purchase order by identifying the possible issues to be considered prior to final confirmation with the relevant suppliers.

Chan and Qi (2003) proposed an innovative performance measurement method for supply chain management. They employed process-based systematic perspective in order to build an effective method for measuring holistic performance of complex supply chains. They used fuzzy set theory to address the real situation in judgment and evaluation processes.

Chang et al. (2006) proposed a fuzzy multiple attribute decision making (FMADM) method based on the fuzzy linguistic quantifier. They tried to ensure that the evaluation results satisfy the current product competition strategies, and also improve the effectiveness and efficiency of the entire supply chain. They used the fuzzy concept to both the ordinal and cardinal information. Furthermore, they used the fuzzy linguistic quantifier guided order-weighted aggregation (FLQG-OWA) operator to satisfy the enterprise product development strategy based on different phases of product life cycle.

Kahraman et al. (2007) constructed a multi-attribute decision making model for evaluation and selection of logistic information technologies consisting of 4 main and 11 sub criteria. They developed a hierarchical fuzzy TOPSIS method to solve the complex selection problem with vague and linguistic data.

Ganga and Carpinetti (2011) proposed a supply chain performance model based on fuzzy logic to predict performance based on causal relationships between metrics, which are performance metrics levels 1 and 2 of the Supply Council Operations Reference model (SCOR) model. They adopted a prediction model based on fuzzy logic and on metrics of the SCOR model that seems to be a feasible technique to help managers in the decision making process of managing performance of supply chains.

Performance measurement is based on different quantitative and qualitative factors. Some of these factors may have a larger effect on the performance measure than others. Units of measure of the quantitative factors are different such as time, money, percentage, ratio, and counts. El-Baz (2011) presents a performance measurement approach based on fuzzy set theory and the pair-wise comparison of Analytical Hierarchy Process (AHP), which ensures the consistency of the designer's assignments of importance of one factor over another to find the weight of each of the manufacturing activity in the departmental organization. In the proposed model, various input factors have been selected, and treated as a linear membership function of fuzzy type. The fuzzy decision making approach provided an effective tool for the performance measurement in supply chain systems of manufacturing environment.

Seyedhosseini et al. (2011) developed a systematic and logical method for the auto part manufacturing organizations to enable them to extract and set leanness criteria for being lean by using the concept of balance scorecard. For determining the lean performance measurement through the company's lean strategy map, a set of objectives should be driven based on the balanced scorecard concept. To determine the company's lean strategy map, they used DEMATEL approach to identify the cause and effect relationships among objectives as well as their priorities. In addition, by combining this method and other group decision making methods such as Delphi, Nominal Group Technique, they come up with a cause and effect relationship among the objectives and draw a lean strategy map for the organization, which can improve the criteria selection strategy by using the higher weighted lean objectives indicating the degree of improved leanness in the manufacturing or service operations. Their study may be a reference point for auto part manufacturing companies to identify their production weaknesses, and help them to focus on their improvement based on their most important and suitable selected objectives and criteria.

Buyukozkan and Ciftci (2012) examined green supply chain management as well as capability dimensions to propose an evaluation framework for green suppliers. Since the nature of supplier selection is known as a complex multi-criteria problem including both quantitative and qualitative factors which may be in conflict and may also be uncertain, they integrated the identified components into a novel hybrid fuzzy multiple criteria decision making (MCDM) model combining the fuzzy Decision Making Trial and Evaluation Laboratory Model (DEMATEL), the Analytical Network Process (ANP), and Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) in a fuzzy context.

Yang and Tzeng (2011) proposed an integrated multiple criteria decision making (MCDM) techniques combining with the decision making trial and evaluation laboratory (DEMATEL) and a novel cluster-weighted with ANP method in which the DEMATEL method is used to visualize the structure of complicated causal relationships between criteria of a system and obtain the influence level of these criteria. Then, they adopted these influence level values as the base of normalization supermatrix for calculating ANP weights to obtain the relative importance.

3 Methodology

3.1 DEMATEL Method

The Decision Making Trial and Evaluation Laboratory (DEMATEL) method is developed by Science and Human Affairs Program of the Battelle Memorial Institute through Geneva Research Centre between 1972 and 1976 (Tzeng and Huang 2011). It is used for evaluating complicated and intertwined multi-criteria decision problems (Wu et al. 2010). DEMATEL uses a graph theory to discover mutual impressible and effective relations of elements. Additionally, in this method, the importance and weight of each element are influenced by all factors such as upstream and downstream (Herat et al. 2012).

The following steps can be followed to apply DEMATEL method (Tzeng and Huang 2011; Hung 2011; Herat et al. 2012; Tuzkaya et al. 2012).

Step 1: Calculating the direct-relation matrix.

Pairwise comparisons between each *i* factor/criterion and each *j* factor/criterion should be done by giving integer numbers range from 0 to 4 which imply no influence, low influence, medium influence, high influence and very high influence, respectively. Then, a direct-relation $n \times n$ matrix denoted by X_{ij} is formed and it implies the effect of criterion i on criterion j.

Step 2: Calculating the normalized direct-relation matrix.

The normalized direct-relation matrix can be computed by normalizing the direct-relation matrix X.

$$Y = k \cdot X \tag{1}$$

where

$$k = min\left[\frac{1}{\max\limits_{1<i<n} \sum_{i=1}^{n} x_{ij}}, \frac{1}{\max\limits_{1<j<n} \sum_{j=1}^{n} x_{ij}}\right] \quad i,j = 1, 2, \ldots, n \tag{2}$$

In direct-relation matrix, the diagonal is assigned to zero.

Step 3: Calculating total-relation matrix.

The total-relation matrix T, which is the infinite series of direct and indirect impacts of each factor, can be calculated by this formula:

$$T = Y + Y^2 + Y^3 + \ldots + Y^m = Y(I - Y)^{-1} \tag{3}$$

Where, I represents identity matrix.

Step 4: Obtaining R and C values.

$$T = \left[t_{ij}\right]_{n \times n} \quad i, j = 1, 2, \ldots, n \tag{4}$$

$$R = [R_i]_{n \times 1} = \sum_{j=1}^{n} t_{ij} \tag{5}$$

which represents row sum of matrix T and lower than 1.

$$C = \left[C_j\right]_{1 \times n} = \sum_{i=1}^{n} t_{ij} \tag{6}$$

which represents column sum of matrix T and lower than 1.

In T matrix, rows point out direct and indirect impacts over other criteria and columns point out influences from other criteria. $j = i(r_i + c_i)$ represents the "influence" or degree of which ith factor/criterion affects or is affected by jth factor/criterion. R_i–C_j represents the effect of factors/criteria on the system. When R–C is positive, the criterion/factor affects their criteria/factors and assigned to "cause" group. If R–C is negative, the criterion/factor is affected by the other criteria/factors and assigned to "effect" group.

Step 5: Setting a threshold value and obtaining impact-digraph diagram.

Impact-digraph diagram should be obtained in order to determine structural relationship among criteria. This helps to reduce the complexity of the system. Moreover, a threshold value should be assigned by experts or decision makers. Higher values than this threshold value are chosen and included in impact-digraph-map. The other ones are eliminated (Tzeng and Huang 2011; Hung 2011). When the threshold value is too low, many elements are included in the impact-digraph-map. This result in a complex map and essential information may not be differentiated. When the threshold value is too high, many factors are not represented in the map. Therefore, it is vitally important to determine an appropriate threshold value to apply DEMATEL method efficiently (Tzeng and Huang 2011).

3.2 The ANP Method

The ANP allows both interaction and feedback within clusters of elements (inner dependence) and between clusters (outer dependence). Such feedback best captures the complex effects of interplay in human society, especially when risk and uncertainty are involved. The elements in a cluster may influence other elements in the same cluster and those in other clusters with respect to each of several properties. The main objective is to determine the overall influence of all the elements. In that case, first of all properties or criteria must be organized and they must be prioritized in the framework of a control hierarchy. Then the comparisons must be performed and synthesized to obtain the priorities of these properties. Additionally, the influence of elements in the feedback system with respect to each of these properties must be derived. Finally, the resulting influences must be weighted by the importance of the properties and added to obtain the overall influence of each element (Saaty 1996, 2003; Onut et al. 2011).

Before performing pairwise comparisons, all criteria and clusters compared are linked to each other. There are three types of connections, namely one-way, two way and loop. The pairwise comparisons are made depending on the 1–9 scale recommended by Saaty.

All of these relations are evaluated as pairwise comparisons. To obtain global priorities, the local priority vectors are entered in the appropriate columns of a matrix of influence among the elements, known as a supermatrix. The supermatrix is raised to limiting powers to calculate the overall priorities and consequently the cumulative influence of each element on every other element with which it interacts is determined (Saaty and Vargas 1998). The supermatrix representation of a hierarchy with three levels is given as follows

$$W = \begin{array}{c} \\ Goal(G) \\ Criteria(C) \\ Alternatives(A) \end{array} \begin{array}{c} \begin{array}{ccc} G & C & A \end{array} \\ \begin{pmatrix} 0 & 0 & 0 \\ W_{21} & 0 & 0 \\ 0 & W_{32} & I \end{pmatrix} \end{array} \quad (7)$$

where W_{21} is a vector that represents the impact of the goal on the criteria, W_{32} is a vector that represents the impact of the criteria on each of the alternatives, and I is the identity matrix. W is referred to as a supermatrix because its entries are matrices. For example, if the criteria are dependent among themselves, then the (2, 2) entry of W given by W_{22} would be nonzero.

$$W = \begin{pmatrix} 0 & 0 & 0 \\ W_{21} & W_{22} & 0 \\ 0 & W_{32} & I \end{pmatrix} \tag{8}$$

The general form of the supermatrix is described in Eq. (9). C_m is the mth cluster, e_{mn} is the nth element in mth cluster, and W_{ij} is the principal eigenvector of the influence of the elements compared in the jth cluster to the ith cluster. If the jth cluster has no influence to the ith cluster, then $W_{ij} = 0$. The influence of a set of elements belonging to a cluster, on any element from another component, can be represented as a priority vector by applying pairwise comparisons. All priority vectors in the network are combined into appropriate positions in a supermatrix, in which each entry indicates the influence of the row element on the column element (Chung et al. 2005).

$$W = \begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 \quad \cdots & C_m \\ e_{11} \cdots e_{1n_1} & e_{21} \cdots e_{2n_2} \cdots & e_{m1} \cdots e_{mn_n} \end{array} \\ \begin{array}{cc} C_1 & \begin{array}{c} e_{11} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots & \vdots \\ C_m & \begin{array}{c} e_{m1} \\ \vdots \\ e_{mn_m} \end{array} \end{array} & \begin{pmatrix} W_{11} & W_{12} & \cdots & W_{1m} \\ W_{21} & W_{22} & \cdots & W_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ W_{m1} & W_{m2} & \cdots & W_{mm} \end{pmatrix} \end{array} \tag{9}$$

Since W is a column stochastic matrix, it is known that the synthesis of all the interactions among the elements of this system is given by W^{∞}. Limiting priorities of the supermatrix depend on the reducibility, primitivity, and cyclicity of that matrix. But there are different forms of the limit depending on the multiplicity of its principal eigenvalue, which must be equal to one or is a complex root of one, and on whether the matrix is reducible and cycles or not (Saaty 2004). If the matrix is irreducible and primitive, the limiting value is obtained by raising W to powers (Saaty and Vargas 1998).

$$W^{\infty} = \lim_{k \to \infty} W^k \tag{10}$$

In this situation, the limit is unique, and there is a column vector w^{∞} for W^{∞}. If there are other roots of unity and the supermatrix has the effect of cyclicity (irreducible and imprimitive), the limiting supermatrix is not only one. There are two or more limiting supermatrices in this situation and the Cesaro sum would be calculated to get the average priority.

$$W^{\infty} = \lim_{k \to \infty} \left(\frac{1}{N} \right) \sum_{j=1}^{N} W_j^k \tag{11}$$

where W_j is the jth limiting supermatrix. The Cesaro sum is mostly used for taking the limits when they are not unique. Otherwise, the supermatrix would be raised to large powers to get the priority weights (Yu and Tzeng 2006). In another words, it must be computed the limit priorities of the stochastic supermatrix according to whether it is irreducible or it is reducible with one being a simple or a multiple root and whether the system cyclic or not. If the matrix is reducible, then the multiplicity of the roots (m_i) of the principal eigenvalue has to be considered to obtain limit priorities from a reducible stochastic matrix with the principal eigenvalue being a multiple root. As an illustration, when $m_i = 1$, W^{∞} for a hierarchy with three levels is given by (Saaty and Vargas 1998; Onut et al. 2011):

$$W^{\infty} = \lim_{k \to \infty} \begin{pmatrix} 0 & 0 & 0 \\ W_{22}^k W_{21} & W_{22}^k 0 & 0 \\ W_{32} \left(\sum_{h=0}^{k-2} W_{22}^h \right) W_{21} & W_{32} \left(\sum_{h=0}^{k-1} W_{22}^h \right) & I \end{pmatrix} \tag{12}$$

Because $(W_{22})^k$ tends to zero as k tends to infinity for $|W_{22}| < 1$, W^{∞} is found as follows.

$$W^{\infty} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ Z = W_{32}(I - W_{22})^{-1} W_{21} & W_{32}(I - W_{22})^{-1} & I \end{pmatrix} \tag{13}$$

Thus, the impact of the goal on the ranking of the alternatives is given by the (3, 1) entry of W^{∞}. According to Neumann series, if $\lim_{k \to \infty} W^k = 0$, then $I - W$ is non-singular and

$$(I - W_{22})^{-1} = I + W_{22} + W_{22}^2 + W_{22}^3 + \cdots = \sum_{k=0}^{\infty} W_{22}^k. \tag{14}$$

It provides approximations of $(I - W_{22})^{-1}$ when W_{22} has entries of suitable magnitude. If the first several terms of Neumann series are approximately substituted to the Z,

$$Z = W_{32} \left(I + W_{22} + W_{22}^2 + W_{22}^3 + \cdots \right) W_{21} \tag{15}$$

Hence the vector Z can be used for evaluating and ordering the alternatives. In another words, after forming the supermatrix, if it is column stochastic, we can simply raise it to powers to obtain an answer. Otherwise, the weighted supermatrix is generated first and then raised it to limiting powers to get the global priority vector. Because the supermatrix is not column stochastic in generally, the limiting matrix does not exist. Hence, stochasticity of the supermatrix can be saved by additional normalization of the columns of the sub-matrices (Ramik 2006). For this reason, this normalization approach can be used to obtain new sub-matrices as mentioned in Eqs. (12–15) and especially with the vector Z, fuzzy evaluations of the alternatives can be executed effectively. The detailed discussion of the mathematical processes of the ANP can refer to Saaty (1996), Saaty and Vargas (1998), and Ramik (2006).

3.3 Fuzzy ANP

The fuzzy set theory introduced by Zadeh (1965, 1976) is suitable for dealing with the uncertainty and imprecision associated with information concerning various parameters (Tuzkaya and Onut 2008). Human judgment is generally characterized by vague language, like 'equally', 'moderately', 'strongly', 'very strongly', 'extremely' and a 'significant degree'. Using such language, decision makers quantify uncertain events and objects. Generally, the fuzzy sets are defined by the membership functions. The fuzzy sets represent the grade of any element x of X that have the partial membership to A. The degree to which an element belongs to a set is defined by the value between 0 and 1. An element x really belongs to A, if $\mu A(x) = 1$ and clearly not, if $\mu A(x) = 0$. Higher is the membership value, $\mu A(x)$, greater is the belongingness of an element x to a set A. Some main arithmetic operations can be extended to fuzzy numbers by the extension principle in the case of triangular fuzzy numbers (Chen et al. 1992).

Fuzzy-ANP method has been used to solve the problem of supply chain performance measurement. It is convenient in situations where there is a high degree of interdependence between various attributes of the alternatives. In this approach, pair-wise comparison matrices are formed between various attributes of each level with the help of triangular fuzzy numbers. Fuzzy-ANP can easily accommodate the interrelationships existing among the functional activities (Mohanty et al. 2005). The concept of supermatrices is employed to obtain the composite weights that overcome the existing interrelationships. Most of the supply chain performance measurement studies generally employ crisp data for evaluation of criteria and alternatives. However, a large amount of uncertainty is associated with various parameters of supply chain performance measurement models, and thus there is a need for fuzzy theory. The values of parameters such as supplier rejection rate, delivery performance, quality of delivered goods, capacity utilization, etc. are transformed into triangular fuzzy numbers and are used to calculate fuzzy values.

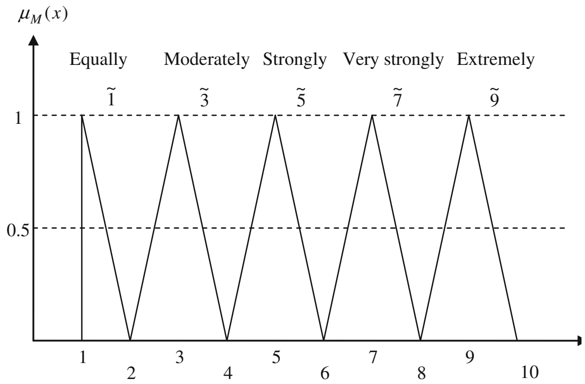

Fig. 1 Fuzzy membership function scale

In the pairwise comparison of attributes, decision maker can use triangular fuzzy numbers to state their preferences. Even though the discrete scale of 1–9 has the advantages of simplicity and easiness for use, it does not consider the uncertainty associated with the mapping of one's perception or judgment to a number. For these reasons a scale of $\tilde{1} - \tilde{9}$ can be defined for triangular fuzzy numbers instead of the scale of 1–9, When comparing attribute i with attribute j, $\tilde{1}$, $\tilde{3}$, $\tilde{5}$, $\tilde{7}$ and $\tilde{9}$ indicate equal importance among the compared attributes, moderate importance of i over j, strong importance of i over j, very strong importance of i over j and extreme importance of i over j, respectively, where $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$. This scale is shown in Fig. 1. To evaluate of the decision maker preferences, pairwise comparison matrices are structured by using triangular fuzzy numbers (l, m, u). The $m \times n$ triangular fuzzy matrix can be given as follows (Tuzkaya and Onut 2008).

$$\tilde{A} = \begin{pmatrix} (a_{11}^l, a_{11}^m, a_{11}^u) & (a_{12}^l, a_{12}^m, a_{12}^u) & \cdots & (a_{1n}^l, a_{1n}^m, a_{1n}^u) \\ (a_{21}^l, a_{21}^m, a_{21}^u) & (a_{22}^l, a_{22}^m, a_{22}^u) & \cdots & (a_{2n}^l, a_{2n}^m, a_{2n}^u) \\ \vdots & \vdots & \vdots & \vdots \\ (a_{m1}^l, a_{m1}^m, a_{m1}^u) & (a_{m2}^l, a_{m2}^m, a_{m2}^u) & \cdots & (a_{mn}^l, a_{mn}^m, a_{mn}^u) \end{pmatrix} \tag{16}$$

The element a_{mn} represents the comparison of component m (row element) with component n (column element). If $\tilde{A}$ is a pairwise comparison matrix, it is assumed that it is reciprocal, and the reciprocal value, i.e. $1/a_{mn}$, is assigned to the element a_{nm}

$$\tilde{A} = \begin{pmatrix} (1,1,1) & (a_{11}^l, a_{11}^m, a_{11}^u) & \cdots & (a_{1n}^l, a_{1n}^m, a_{1n}^u) \\ (\frac{1}{a_{11}^u}, \frac{1}{a_{11}^m}, \frac{1}{a_{11}^l}) & (1,1,1) & \cdots & (a_{2n}^l, a_{2n}^m, a_{2n}^u) \\ \vdots & \vdots & \vdots & \vdots \\ (\frac{1}{a_{1n}^u}, \frac{1}{a_{1n}^m}, \frac{1}{a_{1n}^l}) & (\frac{1}{a_{2n}^u}, \frac{1}{a_{2n}^m}, \frac{1}{a_{2n}^l}) & \cdots & (1,1,1) \end{pmatrix} \tag{17}$$

$\tilde{A}$ is also a triangular fuzzy pairwise comparison matrix. There are several methods for getting estimates for fuzzy priorities $\tilde{w}_i$, where $\tilde{w}_i = \left(w_i^l, w_i^m, w_i^u\right)$, $i = 1, 2, \ldots, n$, from the judgment matrix $\tilde{A}$ which approximate the fuzzy ratios $\tilde{a}_{ij}$ so that $\tilde{a}_{ij} \approx \tilde{w}_i / \tilde{w}_j$. One of these methods, logarithmic least squares method (Chen et al. 1992), is reasonable and effective, and it is used in this study. Hence the triangular fuzzy weights for the relative importance of the criteria, the feedback of the criteria and the alternatives according to the individual criteria can be calculated. The logarithmic least squares method for calculating triangular fuzzy weights can be given as follows:

$$\tilde{w}_k = \left(w_k^l, w_k^m, w_k^u\right) \qquad k = 1, 2, 3, \cdots, n. \tag{18}$$

where

$$w_k^s = \frac{\left(\prod_{j=1}^{n} a_{kj}^s\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}^m\right)^{1/n}}, \quad \mathrm{s} \in \{l, m, u\}. \tag{19}$$

After calculating triangular fuzzy weights, the aggregated triangular fuzzy evaluations of the alternatives are obtained using the approximate Eq. (15) which is applied to the triangular fuzzy matrices as follows (Ramik 2006):

$$\widetilde{Z} = \widetilde{W}_{32}\left(I \,\tilde{+}\, \widetilde{W}_{22} \,\tilde{+}\, \widetilde{W}_{22}^2 \,\tilde{+}\, \widetilde{W}_{22}^3 \,\tilde{+} \cdots\right) \widetilde{W}_{21} \tag{20}$$

After finding the alternatives described as triangular fuzzy numbers, they must be ordered from the best to the worst using the one of the ordering methods. The ordering methods transform the fuzzy numbers to crisp numbers by defuzzification. There are different defuzzification methods such as, centroid average, maximum center average, mean of maximum, smallest of maximum, largest of maximum. Here, the centroid average method is used because of its easiness and being one of the most commonly used defuzzification techniques. This method determines the centre of the area of the aggregated membership functions.

4 An Illustrative Example

As a common methodology for identifying the production or service performance criteria, the performance measurement systems (PMS) are considered and developed to some extent. Reviewing the state-of-the arts, various systems and approaches used for measuring performance of supply chains have been examined. In this study, the concept of balanced scorecard approach has been extended for

Table 1 Main and sub-criteria for supply chain performance evaluation

C1	Financial criteria	F1	Supplier rejection rate
		F2	Buyer–supplier partnership level
		F3	Variations against budget
C2	Customer criteria	C1	Level of customer perceived value of product
		C2	Range of products and services
		C3	Flexibility of service systems to meet particular customer needs
C3	Business criteria	B1	Total supply chain cycle time
		B2	Capacity utilization
		B3	Total cash flow time
C4	Innovation and learning criteria	I1	Supplier assistance in solving technical problems
		I2	Supplier ability to respond to quality problems
		I3	Supplier's booking in procedures
C5	Logistics criteria	L1	Delivery performance
		L2	Responsiveness to urgent deliveries
		L3	Total distribution cost
		L4	Total inventory cost as:
C6	Planning criteria	P1	Effectiveness of distribution planning schedule
		P2	Effectiveness of master production schedule
		P3	Accuracy of forecasting techniques
C7	Quality criteria	Q1	Quality of delivered goods
		Q2	Delivery reliability
		Q3	Achievement of defect free deliveries

determining and selecting the above main and sub-performance criteria given in Table 1. Textile sector is chosen as the application area and two alternative supply chain types are considered to measure their supply chain performance. First alternative has a producer that relatively small, procures raw material to stock and also produce to stock and tries to pump it to the distribution channel. In the second supply chain, all the actors try to apply just-in-time approach from supplier to point of sales. The production facilities produce according to the order quantities and also supply raw material and components considering the production schedule.

In the first step of the illustrative example, DEMATEL is applied to determine cause and effect groups by categorizing the influencing factors. It starts with calculating the initial direct relation matrix which is given in Table 2. And then the normalized direct relation matrix is given in Table 3.

After generating the total-relation matrix given in Table 4, the cause and effect groups are determined (Table 5) and the threshold value, 0.64, is adopted. It means that the row criteria, which have an under threshold value, are not strongly affecting the column criteria. Therefore, these values of the total-direct matrix can be eliminated in the Fuzzy-ANP evaluation process.

Thanks to the DEMATEL method, the number of Fuzzy-ANP evaluations decreases to prevent intractably complex systems. Considering the results of DEMATEL, Fuzzy-ANP initial supermatrix for supply chain performance measurement is prepared. All paired comparisons are carried out by using the

Table 2 The initial direct relation matrix

	C1	C2	C3	C4	C5	C6	C7
C1	0.0	1.5	3.5	1.5	2.5	3.0	4.0
C2	3.0	0.0	1.5	1.5	3.5	3.5	4.0
C3	3.5	1.0	0.0	2.0	2.0	3.5	3.0
C4	1.0	3.0	1.5	0.0	1.5	1.5	3.5
C5	2.0	3.5	2.5	2.5	0.0	2.5	2.5
C6	3.5	1.5	3.5	3.5	3.5	0.0	1.0
C7	1.0	3.5	1.5	2.0	1.5	2.5	0.0

Table 3 The normalized direct relation matrix

	C1	C2	C3	C4	C5	C6	C7
C1	0.00	0.08	0.19	0.08	0.14	0.17	0.22
C2	0.17	0.00	0.08	0.08	0.19	0.19	0.22
C3	0.19	0.06	0.00	0.11	0.11	0.19	0.17
C4	0.06	0.17	0.08	0.00	0.08	0.08	0.19
C5	0.11	0.19	0.14	0.14	0.00	0.14	0.14
C6	0.19	0.08	0.19	0.19	0.19	0.00	0.06
C7	0.06	0.19	0.08	0.11	0.08	0.14	0.00

Table 4 The total-relation matrix

	C1	C2	C3	C4	C5	C6	C7
C1	0.571	0.652	0.740	0.620	0.709	0.805	0.881
C2	0.750	0.618	0.692	0.657	0.797	0.869	0.928
C3	0.706	0.597	0.550	0.615	0.658	0.790	0.803
C4	0.488	0.593	0.508	0.412	0.530	0.585	0.706
C5	0.662	0.732	0.682	0.652	0.582	0.772	0.813
C6	0.757	0.670	0.763	0.725	0.773	0.682	0.786
C7	0.506	0.621	0.524	0.525	0.547	0.642	0.552

Table 5 The sum of the influences on criteria

	Di	Ri	$Di + Ri$	$Di - Ri$
C1	4.98	4.44	9.42	0.54
C2	5.31	4.48	9.79	0.83
C3	4.72	4.46	9.18	0.26
C4	3.82	4.21	8.03	−0.39
C5	4.89	4.60	9.49	0.30
C6	5.16	5.14	10.30	0.01
C7	3.92	5.47	9.39	−1.55

triangular fuzzy numbers. The calculation details of local priority vectors (W_{21}) and matrices (W_{22} and W_{32}) which are the parts of the supermatrix are explained below.

Table 6 Normalized fuzzy weights of the sub-criteria according to the goal

	Lower	Mean	Upper
F1	0.039	0.077	0.116
F2	0.029	0.058	0.087
F3	0.034	0.068	0.101
C1	0.080	0.123	0.216
C2	0.018	0.027	0.048
C3	0.062	0.096	0.168
B1	0.023	0.042	0.060
B2	0.018	0.032	0.047
B3	0.010	0.018	0.027
I1	0.009	0.020	0.038
I2	0.010	0.023	0.044
I3	0.001	0.003	0.005
L1	0.036	0.045	0.081
L2	0.051	0.063	0.114
L3	0.029	0.036	0.065
L4	0.022	0.027	0.049
P1	0.031	0.042	0.084
P2	0.027	0.036	0.072
P3	0.022	0.030	0.060
Q1	0.018	0.044	0.097
Q2	0.015	0.038	0.084
Q3	0.020	0.051	0.111

W_{21} comparisons are related with the top of the hierarchy in the network. There is not any feedback or inner loop in these comparisons. Firstly the main criteria are compared by the decision maker and the weights of the clusters are obtained. Then, the sub-criteria in each cluster are compared with respect to the goal to determine their own weights. At last, weights of the sub- criteria are multiplied by the main criteria weights to get a column stochastic vector. The obtained vectors are given in Table 6 which is constituted from the fuzzy triangular numbers.

The evaluation time of the W_{22} matrix is decreased by eliminating some of the criteria evaluations that are belong to the same or different clusters. There are 22 sub-criteria and 484 comparisons in W_{22}. However, the number of evaluations is decreased to the 295 according to the chosen threshold value in DEMATEL. The normalized version of result matrix (W_{22}) with triangular fuzzy numbers is given as an appendix, since it is too large.

The last part of the Fuzzy-ANP supermatrix (W_{32}) is the comparison of the alternative supply chains according to the each sub-criterion. After these comparisons are realized by the decision makers, the last weights of the alternatives are calculated by Eq. (15). Table 7 shows the fuzzy weights of the alternatives with respect to the sub-criteria.

In the aggregating stage of the parts of supermatrix, W_{22} and W_{32} matrices will have importance values denoted w_1 and w_2, respectively. It is assumed that both

Table 7 Corresponding fuzzy weight matrix of the alternatives according to the sub-criteria

	Financial criteria											
	F1			F2			F3					
	L	M	U	L	M	U	L	M	U			
Alt 1	0.375	0.667	1.500	0.281	0.500	1.125	0.188	0.333	0.750			
Alt 2	0.188	0.333	0.750	0.281	0.500	1.125	0.375	0.667	1.500			
	Customer criteria											
	C1			C2			C3					
	L	M	U	L	M	U	L	M	U			
Alt 1	0.300	0.500	0.625	0.400	0.667	0.867	0.200	0.333	0.417			
Alt 2	0.300	0.500	0.625	0.200	0.333	0.433	0.400	0.667	0.833			
	Business criteria											
	B1			B2			B3					
	L	M	U	L	M	U	L	M	U			
Alt 1	0.300	0.429	0.643	0.280	0.400	0.600	0.280	0.400	0.600			
Alt 2	0.400	0.571	0.857	0.420	0.600	0.900	0.420	0.600	0.900			
	Innovation and learning criteria											
	I1			I2			I3					
	L	M	U	L	M	U	L	M	U			
Alt 1	0.250	0.333	0.467	0.150	0.200	0.280	0.250	0.333	0.467			
Alt 2	0.500	0.667	0.933	0.600	0.800	1.120	0.500	0.667	0.933			
	Logistics criteria											
	L1			L2			L3			L4		
	L	M	U	L	M	U	L	M	U	L	M	U
Alt 1	0.286	0.571	1.086	0.214	0.429	0.814	0.071	0.143	0.271	0.429	0.857	1.629
Alt 2	0.214	0.429	0.814	0.286	0.571	1.086	0.429	0.857	1.629	0.071	0.143	0.271
	Planning criteria											
	P1			P2			P3					
	L	M	U	L	M	U	L	M	U			
Alt 1	0.250	0.500	1.050	0.167	0.333	0.700	0.150	0.300	0.630			
Alt 2	0.250	0.500	1.050	0.333	0.667	1.400	0.350	0.700	1.470			
	Quality criteria											
	Q1			Q2			Q3					
	L	M	U	L	M	U	L	M	U			
Alt 1	0.375	0.625	0.781	0.400	0.667	0.833	0.343	0.571	0.714			
Alt 2	0.225	0.375	0.469	0.200	0.333	0.417	0.257	0.429	0.536			

the sub-matrices have an equal importance as $w_1 = 0.5$ and $w_2 = 0.5$. A column stochastic matrix is obtained by multiplying w_1 with W_{22} and w_2 with W_{32}, respectively. The results are denoted by W_{22}* and W_{32}* matrices and inserted to the supermatrix. Then the approximation formula of Neumann series (Eq. 19) is used for the synthesis calculations and aggregated triangular fuzzy weights of the performance of supply chain alternatives are obtained (Table 8).

Table 8 Aggregated triangular fuzzy weights of the alternative supply chains' performance

Alternatives	Weights		
	Lower	Mean	Upper
Supply chain alternative 1	0.10776	0.39871	1.83697
Supply chain alternative 2	0.12254	0.43675	2.02579

The fuzzy weights give an idea about the performance values of the supply chain alternatives. However, defuzzification of the results and determining the rank of results is important. In this example, the centroid average method is used since it is easy applicable and practical one. At the end the defuzzified and normalized weights or performance values for alternative 1 and alternative 2 are determined as 47.5 % and 52.5 %, respectively.

5 Conclusion

As a matter of fact, supply chain is the upstream fraction of the value chain activities. The right materials, services as well as technologies should be purchased from the right sources at the right time and in the right quality. For this purpose, it is necessary to have good monitoring scheme for supply chain. Hence, effective supply chain performance measurement is the key issue towards efficient supply chain management. Current supply chain performance measurement systems still suffer from being too inward looking and not considering external environmental factors that might affect the overall supply chain performance. An effective overall supply chain performance evaluation model is necessary for suppliers as well as manufacturers to assess their companies under different supply chain strategies.

This chapter presents a fuzzy decision making approach to deal with the performance measurement in supply chain systems. In this chapter, DEMATEL method is adapted to model complex interdependent relationships and construct a relation structure using measurement criteria for evaluation. F-ANP is performed to overcome the problem of dependence and feedback among each measurement criteria.

For future directions, proposed integrated DEMATEL and Fuzzy-ANP approach can also be considered with different or extended criteria for the same problem. Furthermore, other convenient hybrid methodologies can be used for evaluating the same criteria, which are determined in this study. The comparison of these methodologies may be helpful for the decision makers.

Appendix

Normalized fuzzy weights of W22 part of the supermatrix

	Fnancial perspective									Customer perspective								
	F1			F2			F3			C1			C2			C3		
	L	M	U	L	M	U	L	M	U	L	M	U	L	M	U	L	M	U
F1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.12	0.16	0.08	0.13	0.17	0.07	0.12	0.16
F2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.06	0.08	0.05	0.09	0.11	0.02	0.04	0.05
F3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.08	0.10	0.03	0.04	0.06	0.06	0.10	0.13
C1	0.07	0.12	0.28	0.07	0.13	0.30	0.06	0.11	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C2	0.05	0.10	0.22	0.06	0.11	0.25	0.03	0.05	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C3	0.06	0.11	0.25	0.05	0.09	0.20	0.09	0.16	0.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B1	0.04	0.08	0.17	0.01	0.02	0.05	0.03	0.05	0.10	0.03	0.05	0.06	0.02	0.04	0.05	0.04	0.06	0.08
B2	0.02	0.03	0.06	0.02	0.04	0.09	0.02	0.04	0.09	0.02	0.04	0.05	0.04	0.06	0.08	0.01	0.02	0.03
B3	0.01	0.02	0.04	0.03	0.06	0.14	0.02	0.04	0.08	0.02	0.03	0.04	0.01	0.02	0.03	0.02	0.04	0.05
I1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.01	0.02	0.03	0.02	0.03	0.03
I2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.02	0.03	0.04	0.02	0.03	0.04
I3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.02	0.00	0.01	0.01	0.00	0.00	0.00
L1	0.04	0.08	0.17	0.03	0.05	0.11	0.03	0.05	0.12	0.04	0.07	0.09	0.05	0.09	0.11	0.04	0.06	0.08
L2	0.03	0.05	0.11	0.04	0.07	0.15	0.02	0.03	0.07	0.04	0.07	0.08	0.04	0.07	0.09	0.04	0.07	0.09
L3	0.03	0.06	0.13	0.05	0.08	0.18	0.05	0.08	0.19	0.03	0.06	0.07	0.03	0.05	0.06	0.03	0.06	0.07
L4	0.02	0.04	0.10	0.02	0.03	0.07	0.04	0.06	0.14	0.02	0.03	0.04	0.01	0.02	0.03	0.02	0.04	0.05
P1	0.04	0.07	0.16	0.03	0.05	0.11	0.02	0.04	0.09	0.04	0.07	0.08	0.03	0.05	0.06	0.04	0.06	0.07
P2	0.02	0.03	0.06	0.02	0.04	0.08	0.03	0.05	0.11	0.03	0.04	0.05	0.03	0.06	0.07	0.03	0.05	0.06
P3	0.02	0.04	0.10	0.03	0.06	0.13	0.03	0.05	0.12	0.02	0.03	0.04	0.02	0.04	0.05	0.02	0.04	0.04
Q1	0.03	0.06	0.13	0.03	0.06	0.13	0.03	0.05	0.11	0.04	0.07	0.09	0.04	0.06	0.07	0.04	0.07	0.09
Q2	0.03	0.05	0.12	0.02	0.04	0.09	0.03	0.06	0.13	0.03	0.05	0.06	0.02	0.03	0.04	0.03	0.05	0.06
Q3	0.04	0.07	0.15	0.04	0.08	0.18	0.04	0.07	0.15	0.04	0.06	0.07	0.05	0.09	0.11	0.04	0.06	0.07

(continued)

Normalized fuzzy weights of W22 part of the supermatrix

	Business perspective									Innovation and learning perspective								
	B1			B2			B3			I1			I2			I3		
	L	M	U	L	M	U	L	M	U	L	M	U	L	M	U	L	M	U
F1	0.12	0.16	0.25	0.11	0.15	0.23	0.08	0.11	0.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F2	0.03	0.04	0.05	0.05	0.07	0.10	0.05	0.07	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F3	0.09	0.13	0.19	0.08	0.11	0.16	0.11	0.15	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L1	0.05	0.07	0.11	0.08	0.11	0.17	0.07	0.09	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L2	0.06	0.08	0.13	0.02	0.03	0.04	0.04	0.06	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L3	0.05	0.07	0.10	0.04	0.06	0.08	0.05	0.08	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L4	0.04	0.06	0.08	0.06	0.08	0.13	0.03	0.05	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P1	0.06	0.09	0.13	0.04	0.06	0.09	0.05	0.07	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P2	0.02	0.03	0.05	0.05	0.07	0.10	0.04	0.06	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P3	0.04	0.05	0.08	0.03	0.05	0.07	0.03	0.04	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q1	0.06	0.09	0.14	0.05	0.08	0.12	0.03	0.05	0.07	0.28	0.38	0.53	0.28	0.38	0.53	0.13	0.17	0.23
Q2	0.04	0.05	0.08	0.03	0.04	0.06	0.05	0.07	0.11	0.22	0.29	0.41	0.22	0.29	0.41	0.38	0.50	0.70
Q3	0.05	0.07	0.11	0.07	0.10	0.15	0.07	0.10	0.14	0.25	0.33	0.47	0.25	0.33	0.47	0.25	0.33	0.47

(continued)

Normalized fuzzy weights of W22 part of the supermatrix

	Logistics perspective												Planning perspective					
	L1			L2			L3			L4			P1			P2		
	L	M	U	L	M	U	L	M	U	L	M	U	L	M	U	L	M	U
F1	0.06	0.11	0.21	0.06	0.11	0.21	0.03	0.07	0.13	0.04	0.08	0.16	0.05	0.09	0.19	0.04	0.08	0.18
F2	0.02	0.05	0.09	0.03	0.06	0.12	0.02	0.04	0.08	0.03	0.06	0.12	0.02	0.05	0.09	0.02	0.05	0.10
F3	0.04	0.09	0.16	0.04	0.07	0.14	0.07	0.13	0.25	0.05	0.10	0.19	0.03	0.07	0.14	0.04	0.07	0.15
C1	0.07	0.13	0.25	0.06	0.12	0.23	0.09	0.17	0.32	0.07	0.15	0.28	0.04	0.08	0.17	0.04	0.08	0.17
C2	0.02	0.05	0.09	0.01	0.02	0.04	0.02	0.04	0.08	0.02	0.05	0.09	0.03	0.07	0.14	0.05	0.10	0.20
C3	0.06	0.12	0.22	0.08	0.16	0.30	0.04	0.09	0.16	0.05	0.10	0.19	0.05	0.10	0.21	0.03	0.07	0.14
B1	0.03	0.06	0.11	0.02	0.05	0.09	0.02	0.05	0.09	0.03	0.06	0.12	0.02	0.05	0.10	0.02	0.03	0.06
B2	0.01	0.02	0.04	0.01	0.03	0.05	0.01	0.02	0.05	0.01	0.01	0.02	0.01	0.03	0.05	0.02	0.04	0.08
B3	0.02	0.04	0.07	0.02	0.04	0.07	0.02	0.04	0.07	0.02	0.04	0.08	0.01	0.02	0.04	0.01	0.03	0.05
I1	0.01	0.02	0.04	0.01	0.02	0.04	0.01	0.03	0.05	0.01	0.03	0.05	0.01	0.02	0.05	0.01	0.02	0.04
I2	0.01	0.02	0.04	0.01	0.03	0.05	0.01	0.02	0.04	0.01	0.02	0.04	0.01	0.02	0.04	0.01	0.02	0.03
I3	0.01	0.01	0.03	0.00	0.01	0.02	0.00	0.01	0.01	0.00	0.01	0.02	0.00	0.01	0.01	0.01	0.01	0.02
L1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.06	0.12	0.03	0.05	0.11
L2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.07	0.02	0.05	0.10
L3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.06	0.13	0.02	0.04	0.08
L4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.02	0.03	0.07
P1	0.03	0.05	0.10	0.02	0.04	0.08	0.03	0.05	0.10	0.03	0.06	0.11	0.03	0.05	0.11	0.03	0.05	0.11
P2	0.02	0.03	0.06	0.02	0.04	0.07	0.02	0.04	0.07	0.02	0.03	0.06	0.02	0.04	0.08	0.02	0.04	0.08
P3	0.02	0.05	0.09	0.03	0.05	0.10	0.02	0.04	0.08	0.02	0.04	0.08	0.01	0.02	0.04	0.01	0.02	0.04
Q1	0.02	0.05	0.09	0.03	0.05	0.10	0.04	0.07	0.14	0.04	0.08	0.15	0.02	0.04	0.09	0.02	0.04	0.09
Q2	0.02	0.04	0.08	0.02	0.04	0.08	0.02	0.04	0.07	0.01	0.03	0.05	0.02	0.04	0.08	0.02	0.04	0.08
Q3	0.04	0.07	0.14	0.03	0.07	0.13	0.03	0.05	0.10	0.03	0.05	0.10	0.03	0.05	0.11	0.02	0.05	0.10

(continued)

Normalized fuzzy weights of W22 part of the supermatrix

				Quality Perspective								
	P3			Q1			Q2			Q3		
	L	M	U	L	M	U	L	M	U	L	M	U
F1	0.03	0.06	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F2	0.03	0.06	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F3	0.04	0.08	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C1	0.05	0.10	0.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C2	0.01	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C3	0.06	0.12	0.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B1	0.02	0.04	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B2	0.02	0.03	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B3	0.01	0.02	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I1	0.01	0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I2	0.01	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
I3	0.01	0.02	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L1	0.03	0.05	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L2	0.02	0.05	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L3	0.02	0.04	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
L4	0.02	0.03	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P1	0.03	0.05	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P2	0.02	0.04	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P3	0.01	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q1	0.02	0.04	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q2	0.02	0.04	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q3	0.03	0.05	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

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Imprecise DEA Models to Assess the Agility of Supply Chains

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Abstract In this chapter the concept of agility in supply chain is introduced. The criteria of agile supply chain (ASC) are introduced through a conceptual model. The ambiguity and vagueness of ASC criteria are investigated. Afterward, the significance of efficiency of a supply chain in making agility is introduced. Fuzzy Data Envelopment Analysis (DEA) models are developed in order to assess the efficiency of agility of supply chain processes in uncertain situations. Two patterns for agility of supply chains are introduced and the associated models are developed. The properties of the models are discussed. Finally, a real case study is provided to illustrate the application of proposed procedure and conclusion remarks are drawn.

Keywords Uncertainty · Supply chain assessment · Fuzzy DEA · Two-stage DEA · Agility

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1 Introduction

Agility is defined as "the ability to cope with unexpected challenges as opportunities" (Sharifi and Zhang 1999). Research works in this area have emphasized that firm's ability to respond is a key measure of agility (Ovebye et al. 2000; Dove 2001). Companies have recognized that agility is crucial for their survival and competitiveness. Agility is "the ability to detect opportunities for innovation and seize those competitive market opportunities by assembling requisite assets, knowledge and relationships with speed and surprise" (Sambamurthy 2003). Since the introduction of agility paradigm, the potential benefits of it were widely recognized by researchers and industries. The agility is recognized as a winning competitive advantage (Christopher 2000; Christopher and Towill 2000; Goldman et al. 1995; Kidd 1994; Naylor 1999; Swafford et al. 2006; Van Hoak 2005; White et al. 2005).

Agility in supply chain is the ability to rapidly align the network and its operations to dynamic and turbulent requirements of the customers (Ismail and Sharifi 2005). Agile Supply Chain (ASC) is seen as a winning strategy for companies wishing national and international leadership (Yusuf 1999). The design and development of ASCs has become an essential step in acquiring various distinguishing capabilities to respond to the changing environments (Christopher 2000). Sharp et al. (1999) and Christopher (2000) have identified these capabilities as Responsiveness; Competency; Flexibility; Quickness. An ASC requires some enablers like Collaborative relationship, Process integration, Information integration, and finally Customer/marketing sensitivity (Sharifi and Zhang 1999).

After embracing ASC or even while making the supply chain agile, some important questions may be raised. How companies can measure efficiency of processes in which the supply chain is being agile? If a supply chain is not successful in making agility, what are the main inefficiency reasons? What are the practical benchmarks for inefficient supply chains? Answering these will help finding a ratio in which the ASC converts inputs of agility (i.e. agility providers) to outputs of agility (i.e. agility capabilities). So a gap analysis can be made base on existent efficiency of agility and the desired one. These also provide more informative and reliable information for decision making about inefficient supply chains which are not able to properly transform the agility providers into capabilities of agility. Therefore, this chapter attempts to answer aforementioned questions through representing a systematic approach for measuring performance of agility in supply chain.

To our best knowledge, although a large number of frameworks have been reported for introduction and classification of agility indices in SCM but there are just few systematic approaches for measuring the efficiency of process in which the providers of agility change into capabilities of agility. Among them, few researches consider about ambiguity and multi possibility associated with mapping of agility criteria. Due to the qualitative and ambiguous attributes linked to the agility measurements in different levels, most measures are assumed to be described subjectively using linguistic terms, and cannot be handled effectively using crisp scales. Besides, agility is assumed to have different level in a supply chain.

Providers of agility supply potential agility of a supply chain while capabilities of agility are assumed as emergent agility of a supply chain. Both providers and capabilities of agility have direct/indirect effects on overall agility of supply chain.

Lack of an efficient tool to consider the aforementioned aspects of agility in supply chain made us to develop a procedure to supply them. A framework for measuring different aspects of agility, including providers of agility and capabilities of agility is provided. The existing relations between providers of agility and capabilities of agility in supply chain made us to supply DEA approach to measure the efficiency transformation process in context of agility for a given supply chain. The imprecise nature of attributes for associated concepts persuaded us to supply linguistic terms parameterized through fuzzy sets in favor of developing fuzzy DEA.

The following sections of the chapter are organized as follows. Section 2 is allocated to introduce brief literature of past works. In Sect. 3, two conceptual models are proposed for agility levels and goals of supply chain. In the first model the provider of agility is converted into capabilities of agility. In second model, a serial relation is considered among providers of agility, capabilities of agility, and goals of supply chains. In Sect. 4, fuzzy DEA models are proposed to measure the relative efficiency of agility levels in supply chain. The models are introduced in association with each of the conceptual models. The properties of the proposed models are also investigated through theorems and lemmas. The case of top-twenty Iranian dairy industry is represented in Sect. 5. Experimental results are discussed in Sect. 5.3. Section 6 has been assigned to represent the conclusion remarks.

2 Literature Review

In this section, the relevant literature of DEA, two-stage DEA, and fuzzy DEA is briefly introduced. Finally, the applications of DEA in supply chains are reviewed.

2.1 DEA Literature

DEA is a widely used technique that was originally developed by Charnes et al. (1978) and was extended by Banker et al. (1984) to include variable returns to scale. DEA generalizes the Farrell (1957) single-input single-output technical efficiency measure to the multiple-input multiple-output case to evaluate the relative efficiency of peer Decision Making Units (DMUs) (Charnes et al. 1994; Cooper et al. 2006). Unlike parametric methods which require detailed knowledge of the process, DEA is non-parametric and does not require an explicit functional form relating inputs and outputs (Cooper et al. 2006; Cook and Seiford 2009). Numerous applications in recent years have been accompanied by new extensions and developments in the concept and methodology of DEA (Seiford 1997; Emrouznejad et al. 2008).

2.2 Two-Stage DEA Literature

In a two-stage DEA model a DMU consists of two serial sub-DMUs. The sub-DMUs are related through a series relation and all the outputs of the first stage are used as inputs in the second stage. The outputs of the first stage are named as intermediate measures and treated as inputs in the second stage to produce the outputs of the second stage (Cook et al. 2010). Chen and Zhu (2004) proposed a two-stage DEA model to measure the indirect impact of information technology on the firms' performance. Chen et al. (2006) developed a non-linear programming DEA model to evaluate the impact of information technology on multiple stages of a business process to maximize the efficiency of the information technology-related resources. Kao and Hwang (2008) divided the efficiency of a DMU into two sub-DMUs and used a conventional DEA model to identify the causes of inefficiency for each sub-DMU independently. Cook et al. (2010) classified the solution procedures of the two-stage DEA models into four categories: the standard DEA approach, efficiency decomposition, network DEA, and game theoretic.

2.3 Fuzzy DEA Literature

The observed values of the input and output data in real-life problems are often imprecise or vague. Fuzzy logic and fuzzy sets can represent ambiguous, uncertain or imprecise information in DEA by formalizing inaccuracy in decision making (Zadeh 1978; Zimmermann 1996). Kao and Liu (2000) developed a procedure to measure the efficiencies of the DMUs with fuzzy observations. Their basic idea was based on transforming a fuzzy DEA model to a family of conventional crisp DEA models by applying the α-cut approach. Guo and Tanaka (2001) proposed a fuzzy DEA model to deal with the efficiency evaluation problem with fuzzy input and output data. Lertworasirikul et al. (2003) developed DEA models using imprecise data represented by fuzzy sets. They also showed that fuzzy DEA models take the form of fuzzy linear programming which typically was solved with the aid of some methods to rank fuzzy sets. Hatami-Marbini et al. (2011) have presented a comprehensive review of the FDEA methods in the literature. They proposed a classification scheme with four primary categories, namely, the tolerance approach, the α-level based approach, the fuzzy ranking approach and the possibility approach.

2.4 Applications of DEA in Supply Chains

The DEA models used in supply chain studies can be grouped into deterministic and uncertain categories as follows:

Deterministic Methods

Liang et al. (2006) developed several DEA-based approaches for characterizing and measuring supply chain efficiency when intermediate measures were incorporated into the performance evaluation. Dong and Zhi-Ping (2006) used a DEA-based approach to survey the performance of a reverse logistic in a supply chain integration project. Wong and Wong (2007) developed the technical efficiency and the cost efficiency model to help supply chain managers in resource planning decisions. Chen (2009) developed a DEA model to address some important issues concerning the evaluation and design of supply chain operations focusing on evaluation of operational performance of processes in a dynamic setting, and system design under risks and uncertainty.

Saranga and Moser (2010) developed a performance measurement framework for purchasing and supply management using the classical and two-stage value chain DEA models. Halkos et al. (2011) surveyed and classified supply chain DEA models which investigated the internal structures of a DMU. Amirteimoori and Khoshandam (2011) developed a DEA model for measuring the performance of suppliers and manufacturers in supply chain operations. Chen and Yan (2011) proposed an alternative network DEA model to embody the internal structure of a supply chain performance evaluation. Efficiency analysis including the relationship between supply chain and divisions, and the relationship among the three different organization mechanisms were discussed. Mishra (2012) proposed a DEA-based approach to measure the performance of pharmacological supply chain in India.

Uncertainty Methods

Xu et al. (2009) studied the supply chain performance evaluation of the Chinese furniture manufacture industry. They identified the main uncertainty factors affecting the evaluation process, and then modeled and analyzed those using rough DEA models. Abtahi and Khalili-Damghani (2011) proposed a mathematical formulation for measuring the performance of agility in supply chains using single-stage fuzzy DEA. Khalili-Damghani et al. (2011) applied the proposed formulation of Abtahi and Khalili-Damghani (2011) to measure the efficiency of agility in supply chains and used a simulation-based approach to rank the interval efficiency scores. Khalili-Damghani and Taghavifard (2012a) proposed a fuzzy two-stage DEA approach for agility performance measurement in supply chain. Khalili-Damghani and Taghavifard (2012b) proposed a three-stage fuzzy DEA approach to measure the performance of a serial process including just-in-time (JIT) practices, agility indices, and the overall goals in a supply chain. Khalili-Damghani et al. (2012) modeled the ordinal Likert-based data in a new two-stage DEA approach for agility performance in supply chain and illustrated the efficacy of their approach in a real-life supply chain. Khalili-Damghani and Taghavifard (2013) performed sensitivity and stability analysis in two-stage DEA models with fuzzy data. They proposed several fuzzy models to calculate the stability radius, in which an efficient DMU will not alter from efficient to inefficient or vice versa. Khalili-Damghani and Tavana (2013) proposed a new network DEA model for measuring the performance of agility in supply chains. The uncertainty of the input and output data were modeled with linguistic terms and the proposed model was used to measure the performance of agility in a real-life case study in the dairy industry. Tavana et al. (2013) developed a fuzzy group data envelopment analysis model for high-technology project selection at NASA.

3 Proposed Conceptual Models of Agility Levels in Supply Chain

Sharifi and Zhang (1999) proposed a framework for agility measurement through drivers, providers, and capabilities. Lin et. al (2006) proposed a conceptual framework for agile enterprise based on agility drivers, agility capabilities, and agility enablers. We consider two types of factors affecting agility of supply chain as capabilities and providers of agility. The selected indices are supplied in Table 1.

3.1 Providers of Agility (Potential Agility)

The providers are assumed to measure the potential level of agility in supply chain. More obvious, providers can be treated as input oriented factors of agility. The level of success of supply chain in conversion of providers to emergent agility (i.e. capabilities) can be assumed as efficiency of agility.

Collaborative Relationship: Strategic relationship with customers, lasting relationship with suppliers, and close relationship with suppliers.

Process Integration: Concurrent execution of activities and enterprise integration.

Information Integration: Information accessible to employees/suppliers/customers.

Customer/Market Sensitivity: New product introduction, customer driven innovations, and response to market changes.

3.2 Capabilities of Agility (Emergent Agility)

The capabilities are assumed to represent and measure the current and existing level of agility in supply chain. More formally, capabilities represent the emergent level of agility which is outcome of some input oriented indices of agility.

Flexibility: Product volume flexibility, product model/configuration flexibility, organization and organizational issues, flexibility, and people flexibility.

Table 1 Capabilities and providers of agile supply chain

Attribute	References
Capabilities	
Flexibility	Sharp et al. (1999), Christopher (2000), Swafford et al. (2006), Sharifi and Zhang (1999), Lin et al. (2006)
Responsiveness	Sharifi and Zhang (1999), Goldman et al. (1995), kidd (1994), Lin et al. (2006)
Competency	Lin et al. (2006), Sharif and Zhang (1999)
Cost	Swafford et al. (2006), Sharifi and Zhang (1999), Van Hoak et al. (2005), Goldman et al. (1995)
Providers	
Innovation	Sharp et al. (1999), Christopher (2000), Swafford et al. (2006), Sharifi and Zhang (1999), Lin et al. (2006)
People	Sharifi and Zhang (1999), Goldman et al. (1995), kidd (1994), Lin et al. (2006)
Technology	Lin et al. (2006), Sharif and Zhang (1999)
Organization	Swafford et al. (2006), Sharifi and Zhang (1999), Van Hoak et al. (2005), Goldman et al. (1995)

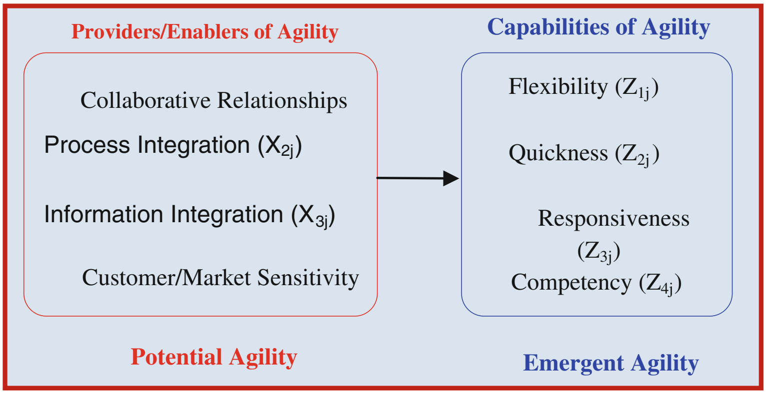

Fig. 1 Transformation process of potential agility into emergent agility

Quickness: Develop new products quickly for the market, products and services delivery quickness and timeliness, and fast operation time.

Responsiveness: Sensing, perceiving and anticipating changes, immediate reaction to changes by incorporating them into the system, and recovery from change.

Competency: Developing business practices difficult to copy such as strategic actions, product/services quality, cost effectiveness, and a high rate of new product development.

Based on providers and capabilities of agility, the conceptual framework for measurement of efficiency of ASC is represented in Fig. 1. The overall agility of supply chain is a function of capabilities of agility and providers of agility. The efficiency of agility is the amount of success which is gained in changing inputs (providers of agility) into outputs (capabilities of agility).

3.3 Supply Chain Goals

It has been shown that the main goals of supply chains are as follows (Gunasekaran et al. 2001; Gunasekaran et al. 2004; Gunasekaran and Kobu 2007):

Cost: Providing products and services with a competitive price by utilizing efficient cost management strategies.

Time: Production and technology preparation time, period of manufacturing, speed of products design, and short development cycle time.

Quality: Quality over product life, first time right decision, and products and services with high information and value-added contents.

Service Level: Customer satisfaction, employee satisfaction, and customer enrichment.

Based on the aforementioned goals of supply chains a serial process in which providers of agility are changed into capabilities of agility and then, the capabilities of agility are changed into goals of supply chain, can be defined. This serial process will result in second conceptual model of agility in supply chains. Figure 2 represents this conceptual model.

4 Proposed Fuzzy DEA Models

The aforementioned existing relations between providers of agility, capabilities of agility, and overall goals in supply chain persuaded us to propose DEA models to measure the efficiency of both levels of agility and overall performance of supply chain. In next section a fuzzy DEA model and a fuzzy two-stage DEA model are proposed for first and second conceptual models, respectively.

4.1 First Model: Single Stage Fuzzy DEA Approach

As mentioned, the providers and capabilities can be assumed as inputs and outputs of a DEA model to measure the relative efficiency of agility for a given supply chain. Linguistic terms parameterized through the fuzzy sets have been applied to model the vagueness of qualitative indices of proposed framework in Fig. 1. Without loss of generality, triangular fuzzy numbers (TrFNs) were supplied through the chapter. The idea of imprecise DEA model in Despotis and Smirlis (2002) is customized for measuring fuzzy efficiency of Agility in Supply Chains. Remind the CCR DEA model of Charnes et al. (1978) as follows.

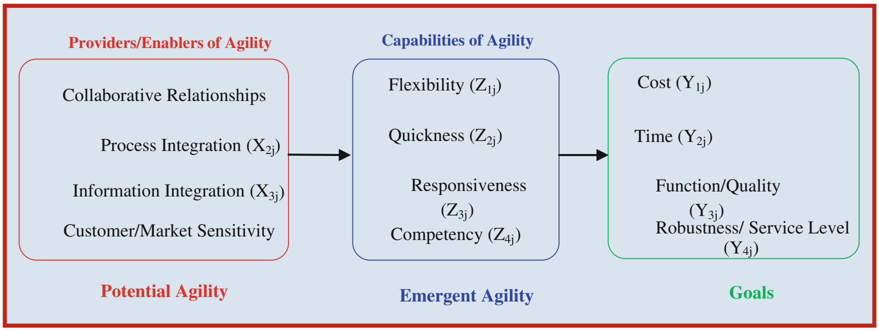

Fig. 2 Levels of agility and performance measures in supply chain

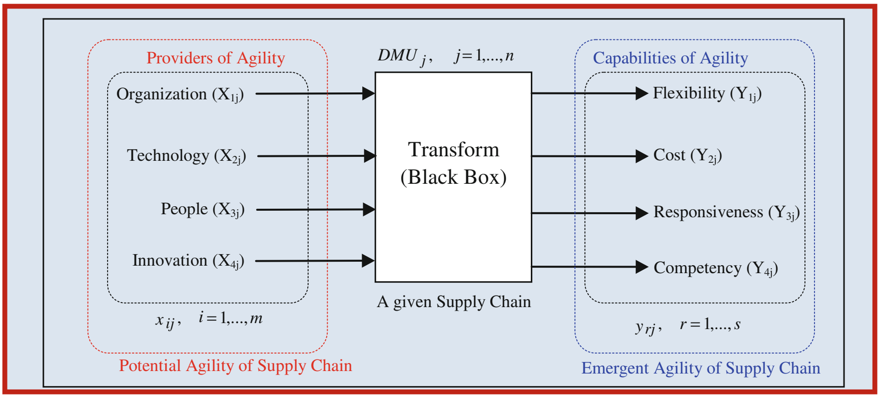

Fig. 3 A given supply chain as a DMU

$$\begin{aligned}
&Max\ \ E=\sum_{r=1}^{s}u_r y_{ro}\\
&s.t.\ \ \sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\ \le\ 0\quad j=1,2,\ldots,n,\\
&\qquad \sum_{i=1}^{m}v_i x_{io}=1,\\
&\qquad v_i\ge\varepsilon,\quad i=1,2,\ldots,m;\quad u_r\ge\varepsilon,\quad r=1,2,\ldots,s.
\end{aligned}\tag{1}$$

We consider a supply chain as a DMU which consumes agility drivers in order to produce agility capabilities. Figure 3 represents the schematic view of a supply chain as a DMU.

As the criteria for agility levels are mixed with a considerable amount of uncertainty, so we consider TrFNs in left and right spread format as inputs and outputs of n DMU. Each DMUj (j = 1, 2,…, n) consumes m fuzzy providers of agility $\tilde{x}_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4),\ i = 1, 2, \ldots, m$ to produce s capabilities of agility $\tilde{y}_{rj} = (y_{rj}^1, y_{rj}^2, y_{rj}^3, y_{rj}^4),\ r = 1, 2, \ldots, s$. For an arbitrary α-cut for each TrFNs the lower and upper bound of the membership functions of inputs/outputs are calculated as (2)–(5).

$$(x_{ij}^L)_{\alpha_i} = x_{ij}^1 + \alpha_i(x_{ij}^2 - x_{ij}^1), \quad \alpha_i \in [0, 1],\ i = 1, \ldots, m;\ j = 1, \ldots, n \tag{2}$$

$$(x_{ij}^U)_{\alpha_i} = x_{ij}^4 - \alpha_i(x_{ij}^4 - x_{ij}^3), \quad \alpha_i \in [0, 1],\ i = 1, \ldots, m;\ j = 1, \ldots, n \tag{3}$$

$$(y_{rj}^L)_{\alpha_r} = y_{rj}^1 + \alpha_r(y_{rj}^2 - y_{rj}^1), \quad \alpha_r \in [0, 1],\ r = 1, \ldots, s;\ j = 1, \ldots, n \tag{4}$$

$$(y_{rj}^U)_{\alpha_r} = y_{rj}^4 - \alpha_r(y_{rj}^4 - y_{rj}^3), \quad \alpha_r \in [0, 1],\ r = 1, \ldots, s;\ j = 1, \ldots, n \tag{5}$$

Replacing Equations (2)–(5) in model (1) derives a pair nonlinear model associated with upper and lower bound of efficiency score for an arbitrary α-cut level. Let us consider $\lambda_i = \alpha_i v_i, i = 1, \ldots, m$ where $0 \le \lambda_i \le v_i$ and $\eta_r = \alpha_r u_r, r = 1, \ldots, s$ where, $0 \le \eta_r \le u_r$ for all inputs and outputs measures, respectively. Note that, these conversions are essential due to warrant the linearity of resultant models. This variable exchange results in a pair of linear and also independent of α-cutmodels. The model (6) and model (7) are free to select the optimum values of λ_i and η_r the optimum values of $\alpha_i, i = 1, \ldots, m$ and $\alpha_r, r = 1, \ldots, s$ are easily determined for all inputs and outputs variables. So there is no need to solve the model for different α-cut levels.

Model (6) represents the upper bound of efficiency score for DMU under consideration. More formally model (6) is reserved for optimistic situation in which the DMU under consideration produces the maximum outputs and consume minimum inputs while other DMUs produce minimum outputs and consume maximum inputs. Model (7) represents the lower bound of efficiency score for DMU under consideration. More formally model (7) is reserved for pessimistic situation in which the DMU under consideration produces the minimum outputs and consume maximum inputs while other DMUs produce maximum outputs and consume minimum inputs.

$$\begin{aligned}
&Max\ \ E^U = \sum_{r=1}^{s} u_r y_{ro}^4 - \eta_r (y_{ro}^4 - y_{ro}^3) \\
&s.t.\ \ \sum_{r=1}^{s} u_r y_{rj}^1 + \eta_r (y_{rj}^2 - y_{rj}^1) - \sum_{i=1}^{m} v_i x_{ij}^4 - \lambda_i (x_{ij}^4 - x_{ij}^3) \le 0 \quad j = 1,2,\ldots,n, \quad j \ne o, \\
&\qquad \sum_{r=1}^{s} u_r y_{ro}^4 - \eta_r (y_{ro}^4 - y_{ro}^3) - \sum_{i=1}^{m} v_i x_{io}^1 + \lambda_i (x_{io}^2 - x_{io}^1) \le 0, \\
&\qquad \sum_{i=1}^{m} v_i x_{io}^1 + \lambda_i (x_{io}^2 - x_{io}^1) = 1, \\
&\qquad v_i \ge \varepsilon, \quad i = 1,2,\ldots,m; \quad u_r \ge \varepsilon, \quad r = 1,2,\ldots,s, \\
&\qquad 0 \le \lambda_i \le v_i, \quad i = 1,2,\ldots,m; \quad 0 \le \eta_r \le u_r, \quad r = 1,2,\ldots,s.
\end{aligned} \tag{6}$$

$$\begin{aligned}
&Max\ \ E^L = \sum_{r=1}^{s} u_r y_{ro}^1 + \eta_r (y_{ro}^2 - y_{ro}^1) \\
&s.t.\ \ \sum_{r=1}^{s} u_r y_{rj}^4 - \eta_r (y_{rj}^4 - y_{rj}^3) - \sum_{i=1}^{m} v_i x_{ij}^1 + \lambda_i (x_{ij}^2 - x_{ij}^1) \le 0, \quad j = 1,2,\ldots,n, \quad j \ne o, \\
&\qquad \sum_{r=1}^{s} u_r y_{ro}^1 + \eta_r (y_{ro}^2 - y_{ro}^1) - \sum_{i=1}^{m} v_i x_{io}^4 - \lambda_i (x_{io}^4 - x_{io}^3) \le 0, \\
&\qquad \sum_{i=1}^{m} v_i x_{io}^4 - \lambda_i (x_{io}^4 - x_{io}^3) = 1, \\
&\qquad v_i \ge \varepsilon, \quad i = 1,2,\ldots,m; \quad u_r \ge \varepsilon, \quad r = 1,2,\ldots,s, \\
&\qquad 0 \le \lambda_i \le v_i, \quad i = 1,2,\ldots,m; \quad 0 \le \eta_r \le u_r, \quad r = 1,2,\ldots,s.
\end{aligned} \tag{7}$$

Properties of Proposed Models

Theorem 1 Model (6) is always feasible and bounded. Its optimal objective function is equal to unit.

Proof Defining proper dual variables, the dual form of model (6) can be written as follows:

$$
\begin{aligned}
Min \quad & z = \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
s.t. \quad & \sum_{\substack{j=1 \\ j \neq 0}}^{n} y_{rj}^1 \alpha_j + y_{ro}^4 \alpha_o - \mu_r - s_r^+ \geq y_{ro}^4, \quad r = 1, 2, \ldots, s, \\
& \sum_{\substack{j=1 \\ j \neq 0}}^{n} \left(y_{rj}^2 - y_{rj}^1\right)\alpha_j - \left(y_{ro}^4 - y_{ro}^3\right)\alpha_0 - \gamma_r + \mu_r \geq -\left(y_{ro}^4 - y_{ro}^3\right), \quad r = 1, 2, \ldots \\
& -\sum_{\substack{j=1 \\ j \neq 0}}^{n} x_{ij}^4 \alpha_j - x_{io}^1 \alpha_o + x_{io}^1 \theta - \varphi_i - s_i^- \geq 0, \quad i = 1, 2, \ldots, m, \\
& \sum_{\substack{j=1 \\ j \neq 0}}^{n} \left(x_{ij}^4 - x_{ij}^3\right)\alpha_j - \left(x_{io}^2 - x_{io}^1\right)\alpha_o + \left(x_{io}^2 - x_{io}^1\right)\theta - \beta_i + \varphi_i \geq 0, \; i = 1, 2, \ldots, \\
& \alpha_j \geq 0, \; j = 1, 2, \ldots, n, \\
& \varphi_i, \; \beta_i, \; s_i^- \geq 0, \; i = 1, 2, \ldots, m, \\
& \mu_r, \; \gamma_r, \; s_r^+ \geq 0, \; r = 1, 2, \ldots, s, \\
& \theta \quad free
\end{aligned}
$$

Consider a solution for dual of model (6) as follows:

$$
\begin{aligned}
& \alpha_j = 0, \quad j = 1, 2, \ldots, n, j \neq o \\
& \alpha_0 = 1 \\
& \varphi_i = \beta_i = s_i^- = 0, \quad i = 1, 2, \ldots, m, \\
& \mu_r = \gamma_r = s_r^+ = 0, \quad r = 1, 2, \ldots, s, \\
& \theta = 1
\end{aligned}
$$

It is obvious that the above solution is a feasible solution for dual model. So, independent of inputs and outputs variables, there always exists at least one feasible solution for dual and primal models. So the optimum value of objective function of dual model is definitely less than or equal to unit (i.e., $Z^* \leq 1$). By virtue of duality theorem in linear programming the objective function of dual and primal are equal at optimal solution (i.e., $Z^* = E_o^{*U}$). So, it can be concluded that $E_o^{*U} \leq 1$ is always true. Hence, the model (6) is always bounded. This completes the proof.

Theorem 2 Model (7) is always feasible and bounded. Its optimal objective function is equal to unit.

Proof Defining proper dual variables, the dual form of model (7) can be written as follows:

$$
\begin{aligned}
Min \quad & Z = \theta - \left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \right) \\
s.t. \quad & \sum_{\substack{j=1 \\ j \neq 0}}^{n} y_{rj}^4 \alpha_j + y_{ro}^1 \alpha_o - \mu_r - s_r^+ \geq y_{ro}^1, \quad r = 1, 2, \ldots, s, \\
& \sum_{\substack{j=1 \\ j \neq 0}}^{n} \left(y_{rj}^4 - y_{rj}^3 \right) \alpha_j - \left(y_{ro}^2 - y_{ro}^1 \right) \alpha_o - \gamma_r + \mu_r \geq - \left(y_{ro}^2 - y_{ro}^1 \right), \quad r = 1, 2, \ldots, s \\
& - \sum_{\substack{j=1 \\ j \neq 0}}^{n} x_{ij}^1 \alpha_j - x_{io}^4 \alpha_o + x_{io}^4 \theta - \varphi_i - s_i^- \geq 0, \quad i = 1, 2, \ldots, m, \\
& \sum_{\substack{j=1 \\ j \neq 0}}^{n} \left(x_{ij}^2 - x_{ij}^1 \right) \alpha_j - \left(x_{io}^4 - x_{io}^3 \right) \alpha_o + \left(x_{io}^4 - x_{io}^3 \right) \theta - \beta_i + \varphi_i \geq 0, \quad i = 1, 2, \ldots, n \\
& \alpha_j \geq 0, \quad j = 1, 2, \ldots, n, \\
& \varphi_i, \beta_i, s_i^- \geq 0, \quad i = 1, 2, \ldots, m, \\
& \mu_r, \gamma_r, s_r^+ \geq 0, \quad r = 1, 2, \ldots, s, \\
& \theta \quad free
\end{aligned}
$$

Consider a solution for dual of model (7) as follows:

$$
\begin{aligned}
& \alpha_j = 0, \quad j = 1, 2, \ldots, n, \quad j \neq o \\
& \alpha_0 = 1 \\
& \varphi_i = \beta_i = s_i^- = 0, \quad i = 1, 2, \ldots, m, \\
& \mu_r = \gamma_r = s_r^+ = 0, \quad r = 1, 2, \ldots, s, \\
& \theta = 1
\end{aligned}
$$

It is obvious that the above solution is a feasible solution for dual model. So, independent of inputs and outputs variables, there always exists at least one feasible solution for dual and primal models. So the optimum value of objective function of dual model is definitely less than or equal to unit (i.e., $Z^* \leq 1$). By virtue of duality theorem in linear programming the objective function of dual and primal are equal at optimal solution (i.e., $Z^* = E_o^{*L}$). So, it can be concluded that $E_o^{*L} \leq 1$ is always true. Hence, the model (7) is always bounded. This completes the proof.

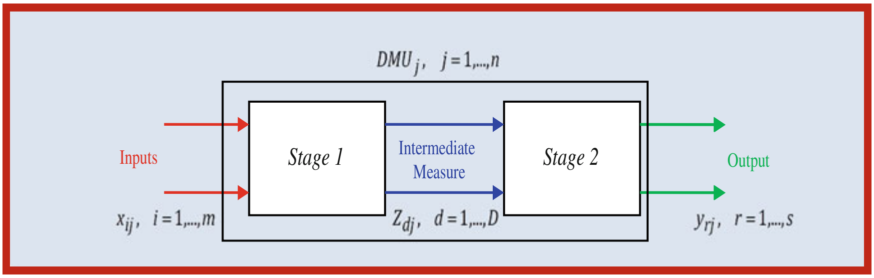

Fig. 4 Two-stage process

4.2 Second Model: Two Stage Fuzzy DEA Approach

As our proposed fuzzy two-stage model is based on the model proposed by Kao and Hwang (2008), we revisit Kao and Hwang (2008) approach to make a better sense. The schematic procedure of a two-stage process has been represented in Fig. 4.

Considering usual notation of two-stage DEA in literature, assume each DMUj (j = 1, 2, …, n) consumes m inputs xij (i = 1, 2, …, m) to produce D outputs zdj (d = 1, 2, …, D) in first stage. All these D outputs then treat as the inputs to the second stage and are assumed as intermediate measures to produce s outputs yrj (r = 1, 2, .., s) of second stage. For a given DMUj, the ej, e1j, e2j are reserved for overall efficiency score, efficiency of first stage, and efficiency of second stage, respectively. The model (8) was proposed by Kao and Hwang (2008):

$$
\begin{aligned}
& e_o = Max \sum_{r=1}^{s} u_r y_{ro} \\
& subject \quad to \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1, 2, \ldots, n \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1, 2, \ldots, n \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& w_d \ge \varepsilon, \quad d = 1, 2, \ldots, D \\
& v_i \ge \varepsilon, \quad i = 1, 2, \ldots, m \\
& u_r \ge \varepsilon, \quad r = 1, 2, \ldots, s
\end{aligned}
\tag{8}
$$

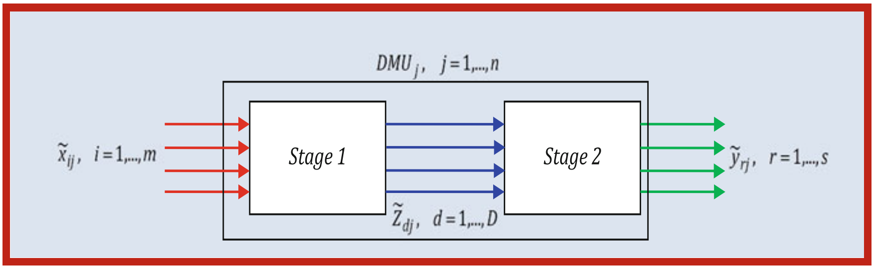

Fig. 5 Two-stage process with fuzzy parameters

The model (8) is the Kao and Hwang (2008) which represents the overall efficiency of the two-stage process. Supposing a unique solution for model (8), the e1j, and e2j can be calculated easily.

Proposed Practical Fuzzy Two-Stage DEA Approach

Without loss of generality, trapezoidal fuzzy numbers (TrFNs) are also supplied to develop a practical fuzzy two-stage DEA approach. Figure 5 represent the considered two-stage process with fuzzy inputs, intermediates, and outputs.

The idea of imprecise single stage DEA model by Despotis and Smirlis (2002), two-stage DEA model by Kao and Hwang (2008), and fuzzy two-stage DEA approach by Kao and Liu (2011) are recombined to develop a practical fuzzy two-stage DEA approach.

Consider TrFNs in left and right spread format as inputs, intermediate measures, and outputs of n DMU with two-stage process. Each DMUj (j = 1, 2, …, n) consumes m fuzzy inputs $\tilde{x}_{ij} = (x^1_{ij}, x^2_{ij}, x^3_{ij}, x^4_{ij}),\ i = 1, 2, \ldots, m$ to produce D intermediate measures $\tilde{z}_{dj} = (z^1_{dj}, z^2_{dj}, z^3_{dj}, z^4_{dj}),\ d = 1, 2, \ldots, d$ in first stage. All these D intermediate measures treat as the inputs to the second stage to produce s outputs $\tilde{y}_{rj} = (y^1_{rj}, y^2_{rj}, y^3_{rj}, y^4_{rj}),\ r = 1, 2, \ldots, s$. For an arbitrary α-cut, the lower and upper bound of the membership functions of each input, and output are calculated as (2)–(5). For an arbitrary α-cut, the lower and upper bound of the membership functions of intermediate measure are calculated as (9)–(10).

$$(z^L_{dj})_{\alpha_r} = z^1_{dj} + \alpha_d(z^2_{dj} - z^1_{dj}), \quad \alpha_d \in [0, 1],\ d = 1, \ldots, D;\ j = 1, \ldots, n \tag{9}$$

$$(z^U_{rj})_{\alpha_r} = z^4_{dj} - \alpha_d(z^4_{dj} - z^3_{dj}), \quad \alpha_d \in [0, 1],\ d = 1, \ldots, D;\ j = 1, \ldots, n \tag{10}$$

Upper Bound of Efficiency Values of Main DMU

For an arbitrary α-cut level, model (11) is proposed for calculating the upper (e^U_o) bound of efficiency values of main DMUs.

$$
\begin{aligned}
Max \quad & e_o^U = \sum_{r=1}^{s} u_r (y_{ro}^U)_{\alpha_r} \\
s.t. \quad & \sum_{r=1}^{s} u_r (y_{rj}^L)_{\alpha_r} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1, 2, \ldots, n, \quad j \neq o \\
& \sum_{r=1}^{s} u_r (y_{ro}^U)_{\alpha_r} - \sum_{d=1}^{D} w_d z_{do} \le 0 \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i (x_{ij}^U)_{\alpha_i} \le 0 \quad j = 1, 2, \ldots, n, \quad j \neq o \\
& \sum_{d=1}^{D} w_d z_{do} - \sum_{i=1}^{m} v_i (x_{io}^L)_{\alpha_i} \le 0 \\
& \sum_{i=1}^{m} v_i (x_{io}^L)_{\alpha_i} = 1, \\
& v_i \ge \varepsilon, \quad i = 1, 2, \ldots, m, \\
& w_d \ge \varepsilon, \quad d = 1, 2, \ldots, D, \\
& u_r \ge \varepsilon, \quad r = 1, 2, \ldots, s.
\end{aligned} \tag{11}
$$

In model (11), the input variables take lower bound and output variables take upper bound for main DMU under consideration. For all other DMUs, the input variables take upper bound and output variables take lower bound. As the intermediate measures emerge in all sets of constraint of the model (11) so, they cannot be determined using a single level optimization model.

Hence, as proposed by Kao and Liu (2011), the two-level optimization model (12) is suggested for determining the optimum values of intermediate measures in which the objective function of model (11) is at the highest possible values for an arbitrary α-cut level.

$$
\underset{(z_{dj}^L)_{\alpha_d} \le z_{dj} \le (z_{dj}^U)\alpha_d, \forall j,\, d}{Max}
\left\{
\begin{aligned}
Max \quad & e_o^U = \sum_{r=1}^{s} u_r (y_{ro}^U)_{\alpha_r} \\
s.t. \quad & \sum_{r=1}^{s} u_r (y_{rj}^L)_{\alpha_r} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1, 2, \ldots, n, \quad j \neq o \\
& \sum_{r=1}^{s} u_r (y_{ro}^U)_{\alpha_r} - \sum_{d=1}^{D} w_d z_{do} \le 0 \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i (x_{ij}^U)_{\alpha_i} \le 0 \quad j = 1, 2, \ldots, n, \quad j \neq o \\
& \sum_{d=1}^{D} w_d z_{do} - \sum_{i=1}^{m} v_i (x_{io}^L)_{\alpha_i} \le 0 \\
& \sum_{i=1}^{m} v_i (x_{io}^L)_{\alpha_i} = 1, \\
& v_i \ge \varepsilon, \quad i = 1, 2, \ldots, m, \\
& w_d \ge \varepsilon, \quad d = 1, 2, \ldots, D, \\
& u_r \ge \varepsilon, \quad r = 1, 2, \ldots, s.
\end{aligned}
\right. \tag{12}
$$

As $Z_{dj}, \forall j,\ d$ are assumed as decision variables for outer optimization level and perceived as constant multipliers for inner optimization level so, model (12) cannot be solved in current format. model (12) should be reduced to a single level optimization. Fortunately, as the orientation of both objective functions in model (12) are maximization so model (12) can be replaced with a single optimization model such as (13).

$$
\begin{aligned}
& Max\ e_o^U = \sum_{r=1}^{s} u_r (y_{ro}^U)_{\alpha_r} \\
& s.t. \quad \sum_{r=1}^{s} u_r (y_{rj}^L)_{\alpha_r} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1, 2, \ldots, n, \quad j \ne o \\
& \qquad \sum_{r=1}^{s} u_r (y_{ro}^U)_{\alpha_r} - \sum_{d=1}^{D} w_d z_{do} \le 0 \\
& \qquad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i (x_{ij}^U)_{\alpha_i} \le 0 \quad j = 1, 2, \ldots, n, \quad j \ne o \\
& \qquad \sum_{d=1}^{D} w_d z_{do} - \sum_{i=1}^{m} v_i (x_{io}^L)_{\alpha_i} \le 0 \\
& \qquad \sum_{i=1}^{m} v_i (x_{io}^L)_{\alpha_i} = 1, \\
& \qquad (z_{dj}^L)_{\alpha_d} dj \le (z_{dj}^U)_{\alpha_d},\ j = 1, 2, \ldots, n,\ j \ne o;\ d = 1, 2, \ldots, D \\
& \qquad (z_{do}^L)_{\alpha_d} \le z_{do} \le (z_{do}^U)_{\alpha_d},\ d = 1, 2, \ldots, D \\
& \qquad v_i \ge \varepsilon, \quad i = 1, 2, \ldots, m, \\
& \qquad w_d \ge \varepsilon, \quad d = 1, 2, \ldots, D, \\
& \qquad u_r \ge \varepsilon, \quad r = 1, 2, \ldots, s.
\end{aligned}
\tag{13}
$$

It is notable that the model (13) is a non-linear mathematical programming so its global optimum may not be found easily. Moreover, the model (13) is dependent to α-cut. So it should be solved for different α-cut levels with a pre-determined step size. This may yield the computational problems in practice (Kao and Liu 2011). Replacing the values of equations (2)–(5) and (9) and (10) in model (13) will result in model (14).

$$
\begin{aligned}
&Max\;\; e_o^U = \sum_{r=1}^{s} u_r(y_{ro}^4 - \alpha_r(y_{ro}^4 - y_{ro}^3)) \\
&s.t.\;\; \sum_{r=1}^{s} u_r(y_{rj}^1 + \alpha_r(y_{rj}^2 - y_{rj}^1)) - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1,2,\ldots,n, \quad j \ne o \\
&\quad \sum_{r=1}^{s} u_r(y_{ro}^4 - \alpha_r(y_{ro}^4 - y_{ro}^3)) - \sum_{d=1}^{D} w_d z_{do} \le 0 \\
&\quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i(x_{ij}^4 - \alpha_i(x_{ij}^4 - x_{ij}^3)) \le 0 \quad j = 1,2,\ldots,n, \quad j \ne o \\
&\quad \sum_{d=1}^{D} w_d z_{do} - \sum_{i=1}^{m} v_i(x_{io}^1 + \alpha_i(x_{io}^2 - x_{io}^1))_{\alpha_i} \le 0 \\
&\quad \sum_{i=1}^{m} v_i(x_{io}^1 + \alpha_i(x_{io}^2 - x_{io}^1)) = 1, \\
&\quad (z_{dj}^1 + \alpha_d(z_{dj}^2 - z_{dj}^1)) \le z_{dj} \le (z_{dj}^4 - \alpha_d(z_{dj}^4 - z_{dj}^3)), \quad j = 1,2,\ldots,n, \quad j \ne o;\; d = 1,2,\ldots,D \\
&\quad (z_{do}^1 + \alpha_d(z_{do}^2 - z_{do}^1)) \le z_{do} \le (z_{do}^4 - \alpha_d(z_{do}^4 - z_{do}^3)), \quad d = 1,2,\ldots,D \\
&\quad v_i \ge \varepsilon, \quad i = 1,2,\ldots,m, \\
&\quad w_d \ge \varepsilon, \quad d = 1,2,\ldots,D, \\
&\quad u_r \ge \varepsilon, \quad r = 1,2,\ldots,s.
\end{aligned} \tag{14}
$$

Using proper variable interchange as $\lambda_i = \alpha v_i, i = 1,\ldots,m$, where $0 \le \lambda_i \le v_i$, and $\eta_r = \beta u_r, r = 1,\ldots,s$, where $0 \le \eta_r \le u_r$ we have:

$$
\begin{aligned}
&e_o^U = Max \sum_{r=1}^{s} (u_r y_{ro}^4 - \eta_r(y_{ro}^4 - y_{ro}^3)) \\
&\textit{subject to} \\
&\sum_{r=1}^{s} (u_r y_{rj}^1 + \eta_r(y_{rj}^2 - y_{rj}^1)) - \sum_{d=1}^{D} w_d z_{dj} \le 0 \; j = 1,2,\ldots,n,\; j \ne o \\
&\sum_{r=1}^{s} (u_r y_{ro}^4 - \eta_r(y_{ro}^4 - y_{ro}^3)) - \sum_{d=1}^{D} w_d z_{do} \le 0 \\
&\sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} (v_i x_{ij}^4 - \lambda_i(x_{ij}^4 - x_{ij}^3)) \le 0 \; j = 1,2,\ldots,n,\; j \ne o \\
&\sum_{d=1}^{D} w_d z_{do} - \sum_{i=1}^{m} (v_i x_{io}^1 + \lambda_i(x_{io}^2 - x_{io}^1)) \le 0 \\
&\sum_{i=1}^{m} (v_i x_{io}^1 + \lambda_i(x_{io}^2 - x_{io}^1)) = 1 \\
&(z_{dj}^1 + \alpha_d(z_{dj}^2 - z_{dj}^1)) \le z_{dj} \le (z_{dj}^4 - \alpha_d(z_{dj}^4 - z_{dj}^3)),\; j = 1,2,\ldots,n,\; j \ne o;\; d = 1,2,\ldots,D \\
&w_d \ge \varepsilon,\; d = 1,2,\ldots,D \\
&v_i \ge \lambda_i \ge 0,\; i = 1,2,\ldots,m \\
&u_r \ge \eta_r \ge 0,\; r = 1,2,\ldots,s
\end{aligned} \tag{15}
$$

The model (15) is also non-linear due to terms $w_d z_{dj}$, and $w_d z_{do}$. It is also dependent to α-cut type variables α_d. Resolving the aforementioned issues, the following procedure is proposed.

Multiply sides of set of inequalities concerning intermediate measures by the positive value w_d. So we have.

$$w_d(z_{dj}^1 + \alpha_d(z_{dj}^2 - z_{dj}^1))_d z_{dj} \le w_d(z_{dj}^4 - \alpha_d(z_{dj}^4 - z_{dj}^3)),\ j = 1, 2, \ldots, n;\ d = 1, 2, \ldots, D$$

The variable interchanges $\theta_d = \alpha_d w_d,\ d = 1, 2, \ldots, D$ where $0 \le \theta_d \le w_d$ and $\bar{z}_{dj} = w_d z_{dj}, d = 1, 2, \ldots, D; j = 1, 2, \ldots, n$ is accomplished. The model (16) is achieved as follows:

$$\begin{aligned}
&e_o^U = Max \sum_{r=1}^{s} (u_r y_{ro}^4 - \eta_r(y_{ro}^4 - y_{ro}^3)) \\
&subject \quad to \\
&\sum_{r=1}^{s} (u_r y_{rj}^1 + \eta_r(y_{rj}^2 - y_{rj}^1)) - \sum_{d=1}^{D} \bar{z}_{dj} \le 0 \quad j = 1, 2, \ldots, n, \quad j \ne o \\
&\sum_{r=1}^{s} (u_r y_{ro}^4 - \eta_r(y_{ro}^4 - y_{ro}^3)) - \sum_{d=1}^{D} \bar{z}_{do} \le 0 \\
&\sum_{d=1}^{D} \bar{z}_{dj} - \sum_{i=1}^{m} (v_i x_{ij}^4 - \lambda_i(x_{ij}^4 - x_{ij}^3)) \le 0 \quad j = 1, 2, \ldots, n, \quad j \ne o \\
&\sum_{d=1}^{D} \bar{z}_{do} - \sum_{i=1}^{m} (v_i x_{io}^1 + \lambda_i(x_{io}^2 - x_{io}^1)) \le 0 \\
&\sum_{i=1}^{m} (v_i x_{io}^1 + \lambda_i(x_{io}^2 - x_{io}^1)) = 1 \\
&w_d z_{dj}^1 + \theta_d(z_{dj}^2 - z_{dj}^1)) \le \bar{z}_{dj} \le w_d z_{dj}^4 - \theta_d(z_{dj}^4 - z_{dj}^3)), j = 1, 2, \ldots, n;\ d = 1, 2, \ldots, D \\
&w_d \ge \theta_d \ge 0, \quad d = 1, 2, \ldots, D; v_i \ge \lambda_i \ge 0, \quad i = 1, 2, \ldots, m; u_r \ge \eta_r \ge 0, \\
&\quad r = 1, 2, \ldots, s; \bar{z}_{dj} \ge 0, \quad d = 1
\end{aligned} \tag{16}$$

model (16) is a practical single stage linear optimization model which is independent of α-cut type variables. Hence, its global optimum solution can be found easily. The problems of proposed procedure by Kao and Liu (2011) such as high volume of computational efforts, determination of a proper step size for α-cut type variables, conflictive interval efficiency scores, and conflictive ranking of DMUs does not exist in model (16).

Lower Bound of Efficiency Values of Main DMU.

For an arbitrary α-cut level, model (17) is proposed for calculating the lower (e_o^L) bound of efficiency values of main DMUs.

$$
\begin{aligned}
Max\ \ & e_o^L = \sum_{r=1}^{s} u_r (y_{ro}^L)_{\alpha_r} \\
s.t.\ \ & \sum_{r=1}^{s} u_r (y_{rj}^U)_{\alpha_r} - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad j = 1, 2, \ldots, n, \quad j \ne o \\
& \sum_{r=1}^{s} u_r (y_{ro}^L)_{\alpha_r} - \sum_{d=1}^{D} w_d z_{do} \le 0 \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i (x_{ij}^L)_{\alpha_i} \le 0 \quad j = 1, 2, \ldots, n, \quad j \ne o \\
& \sum_{d=1}^{D} w_d z_{do} - \sum_{i=1}^{m} v_i (x_{io}^U)_{\alpha_i} \le 0 \\
& \sum_{i=1}^{m} v_i (x_{io}^U)_{\alpha_i} = 1, \\
& v_i \ge \varepsilon, \quad i = 1, 2, \ldots, m, \\
& w_d \ge \varepsilon, \quad d = 1, 2, \ldots, D, \\
& u_r \ge \varepsilon, \quad r = 1, 2, \ldots, s.
\end{aligned} \tag{17}
$$

Similar to the procedure for upper bound of efficiency score, the final model for lower bound of efficiency score is developed as follows:

$$
\begin{aligned}
Min \quad & z = \theta - \varepsilon \left(\sum_{i=1}^{m} s_i^- + \sum_{d=1}^{D} \varphi_d + \sum_{r=1}^{s} s_r^+ \right) \\
s.t. \quad & \sum_{\substack{=1 \\ j \ne o}}^{n} y_{rj}^4 \alpha_j + y_{ro}^1 \alpha_o - \mu_r - s_r^+ ro^1, \quad r = 1, 2, \ldots, s, \\
& - \sum_{\substack{j=1 \\ j \ne o}}^{n} \left(y_{rj}^4 - y_{rj}^3 \right) \alpha_j + \left(y_{ro}^2 - y_{ro}^1 \right) \alpha_o - \rho_r + \mu_r \ge \left(y_{ro}^r - y_{ro}^1 \right), \quad r = 1, 2, \ldots, s, \\
& - \sum_{\substack{j=1 \\ j \ne 0}}^{n} x_{ij}^1 \tau_j - x_{io}^4 \tau_o + x_{io}^4 \theta - s_i^- - \beta_i \ge 0, \quad i = 1, 2, \ldots, m, \\
& - \sum_{\substack{j=1 \\ j \ne 0}}^{n} \left(x_{ij}^2 - x_{ij}^1 \right) \tau_j + \left(x_{io}^4 - x_{io}^3 \right) \tau_o - \left(x_{io}^4 - x_{io}^3 \right) \theta - \sigma_i + \beta_i \ge 0, \quad i = 1, 2, \ldots, m, \\
& - \sum_{j=1 \ne 0}^{n} z'_{dj} - z'_{do} + \sum_{j=1 \ne o}^{n} z''_{dj} + z''_{do} - \varphi_d \ge 0, \quad d = 1, 2, \ldots, D, \\
& w_d z_{dj}^1 + \theta_d \left(z_{dj}^2 - z_{dj}^1 \right) \le \bar{z}_{dj} d z_{dj}^4 - \theta_d \left(z_{dj}^4 - z_{dj}^3 \right), \quad j = 1, 2, \ldots, n; \quad d = 1, 2, \ldots, D \\
& z'_{dj}, z''_{dj} \ge 0, \quad j = 1, 2, \ldots, n; \quad d = 1, 2, \ldots, D, \\
& \sigma_i,\ \beta_i,\ s_i^- \ge 0, \quad i = 1, 2, \ldots, D, \\
& w_d,\ \varphi_d \ge 0, \quad d = 1, 2, \ldots, D, \\
& \mu_r,\ \rho_r,\ s_r^+ \ge 0, \quad r = 1, 2, \ldots, S, \\
& \theta \quad free \\
& \bar{Z}_{dj} \ge 0,\ d = 1, 2, \ldots, D; \quad j = 1, 2, \ldots, n
\end{aligned} \tag{18}
$$

Model (18) is linear and independent of α-cut variables. So it can achieve the lower bound of efficiency values of DMUs.

The Maximum Achievable Value of Efficiency of Sub-DMUs.

According to proposed procedure in modeling of main DMUs, the procedure of modeling of the maximum achievable value of upper and lower bound of efficiency of sub-DMUs is straightforward and is not represented here for sake of brevity.

5 Case Study

As most food industries, fresh food industries are characterized by "repetitive production operations carrying out specific physical (e.g. blending or milling) or chemical reactions" (Günther and Van Beek 2003). Process industries usually show a higher complexity than discrete manufacturing, which is caused by factors such as the perishability of products, the high number of end products, a great variety of possible productions paths, special storage equipment, co- and by-products, or variable recipes. Most production systems in fresh food industries—as well as in the food industry in general—contain "Processing" and "Packaging". The number of products involved increases with each production step. Out of a limited number of raw materials (e.g. raw milk), a still moderate number of intermediate products are produced within the processing step. High product complexity typically occurs at the packaging level due to different tastes and packaging formats. As clear supply, production, distribution, and delivery of fresh foods are accomplished through co-operating of different companies in chains. According to aforementioned properties of fresh food industries which are accomplished in supply chains these days, the main processes of sourcing, making, and delivery are required to have different levels of agility in order to achieve proper levels of final goal in supply chain of this industry.

As mentioned, aforementioned properties of fresh food productions reveal that different levels of agility indices are essential in such supply chains. We applied the proposed Fuzzy DEA models in assessment of DUMs and Sub-DMUs of top-twenty dairy companies in Iran which supply dairy products through chains. Each dairy company has been assumed as an independent supply chain with aforementioned structure in the context of different levels of agility and performance goals.

5.1 Measurement Scales

As mentioned, providers of agility, capabilities of agility and goals of supply chain are assumed to be subjective, qualitative, and mixed with a large amount of vagueness so, without loss of generality, linguistic terms parameterized through TrFNs are supplied to measure them. Figure 6 shows the membership functions of

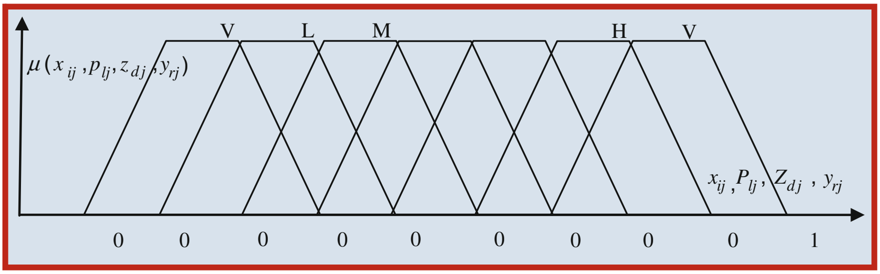

Fig. 6 Membership functions of linguistic terms

Table 2 Linguistic terms and associated TrFNs

Linguistic terms	TrFN
Very Low (VL)	(0.1, 0.2, 0.3, 0.4)
Low (L)	(0.2, 0.3, 0.4, 0.5)
Medium Low (ML)	(0.3, 0.4, 0.5, 0.6)
Medium (M)	(0.4, 0.5, 0.6, 0.7)
Medium High (MH)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.6, 0.7, 0.8, 0.9)
Very High (VH)	(0.7, 0.8, 0.9, 1)

TrFNs. Table 2 represents the linguistic terms and associated TrFNs used in proposed fuzzy DEA models.

5.2 Data Gathering

We used the experimental experiences of managers of selected diary supply chains to determine the values of indices for providers of agility, capabilities of agility, and goals of supply chain. The data collected from experts of Iranian diary supply chains through questionnaires. The experts who filled the questionnaires were experienced managers working for diary supply chains. These managers had 10 years of experience on average.

A set of 20 diary supply chains was selected and a manager of a given supply chain was requested to rate the affecting factor for all supply chains. These managers were left free to use linguistic terms of Table 1 in their judgments. The aggregation of opinions of other supply chain managers forms the values of indices for a given supply chain (i.e. main DMUs, and Sub-DMUs) have distinctively been summarized in Table 3.

It is notable that as the selected supply chains produce different dairy products, in order to make a homogenous assessment of similar DMU the milk, cheese, and cream are considered for evaluation. The experts were also requested to rate the indices according to these similar products in all supply chains.

Table 3 DMs' aggregate opinions

Brand	DMU	Providers of agility (Inputs)				Capabilities of agility (Intermediate measures)				Goals of supply chain (Outputs)			
		X_1	X_2	X_3	X_4	Z_1	Z_2	Z_3	Z_4	Y_1	Y_2	Y_3	Y_4
Pak	1	M	V	V	M	H	H	M	M	M	M	M	M
Mihan	2	M	V	V	M	M	M	M	H	M	H	H	M
Pegah Fars	3	V	H	H	H	M	M	M	L	L	M	L	M
Pegah-Gilan	4	V	V	M	V	M	M	L	V	H	M	V	L
Pegah-Khuzestan	5	V	V	M	L	L	L	L	V	M	L	M	L
Pegah-Golpaygan	6	M	H	V	M	V	V	M	M	L	M	M	M
Bistoon	7	M	H	M	V	M	M	L	M	H	M	M	L
Kaleh	8	M	L	M	M	L	L	H	M	V	V	M	H
Ta'rif	9	M	M	L	M	L	L	V	H	V	L	V	V
Maadi-Mimas	10	H	V	M	M	M	M	M	M	V	V	M	M
Arak Dairy	11	V	M	M	L	L	M	M	M	H	M	V	M
Barekat	12	V	M	M	V	H	V	M	H	M	M	V	H
Kamel-Novin	13	H	H	M	M	L	L	L	M	M	V	H	L
Alborz Laban	14	M	V	L	M	V	V	H	M	M	V	V	V
Pars-Pooyan Zagros	15	V	M	L	M	L	V	M	L	L	V	V	V
AzarnooshSharq	16	V	M	M	L	M	V	L	M	V	M	H	M
Aban-Shir Ardabil	17	M	V	L	H	M	M	H	M	M	M	H	M
Abshar-Sepid Shiraz	18	M	M	H	M	M	M	V	V	L	M	L	M
AzarShiraneh	19	V	M	V	M	M	M	V	L	L	M	M	V
Aryan-ShirAlborz	20	M	M	M	M	V	M	V	V	M	H	V	M

5.3 Experimental Results

The proposed fuzzy DEA models were coded using LINGO 11.0 software. The codes of proposed mathematical models were executed on a Pentium IV portable PC with Core 2 due CPU, 2 GHz, and Windows XP using 1 GB of RAM.

Calculating Interval Efficiency Scores and Interpreting the Results

Running fuzzy DEA models for case study yielded unique interval efficiency scores for all DMUs. The relative efficiency and the classification of DMUs and sub-DMUs are represented in Table 4.

Contents of Table 4 reveal the source of inefficiency in each sub-DMU. It is notable that using the proposed structure for a DMU it is possible to find the inefficiency each level of providers of agility, and capabilities of agility distinctively. More clearly, the sub-processes, in which the process has a problem, are determined. This can be helpful for improving overall performance of supply chains in context of providers of agility and capabilities of agility.

More formally, the proposed approach can be utilized to distinguish the relative efficiency scores of sub-processes of a two-stage process. This achieves a deeper view into the performance of sub-processes which are involved in a real process in context of agility. To interpret the achieved efficiency scores Fig. 7 has been supplied.

Table 4 Interval efficiency scores

Main-DMU$_1$			Sub-DMU$_1$		Sub-DMU$_2$	
	e^U	e^L	$[e^{+1}]^U$	$[e^{+1}]^L$	$[e^{+2}]^U$	$[e^{+2}]^L$
DM	0.880838	0.634846	0.865305	0.861246	0.757344	0.733731
DM	0.847384	0.601363	0.945375	0.898326	0.744415	0.657903
DM	0.9034	0.525735	0.864271	0.828932	0.750225	0.689498
DM	0.873592	0.681795	0.894978	0.821955	0.741142	0.604107
DM	0.829303	0.717116	0.970115	0.850776	0.77752	0.760859
DM	0.755942	0.516534	0.932681	0.820724	0.859506	0.779202
DM	0.851553	0.733009	0.890655	0.773594	0.775183	0.584163
DM	0.797853	0.662563	0.875868	0.821923	0.869798	0.550475
DM	0.792007	0.685066	0.897102	0.885936	0.863265	0.585702
DM	0.906853	0.575242	0.896762	0.872199	0.819263	0.611162
DM	0.773672	0.665702	0.957151	0.899822	0.825549	0.780233
DM	0.791772	0.723011	0.989897	0.779611	0.792535	0.615332
DM	0.797866	0.761317	0.963526	0.84232	0.797492	0.791526
DM	0.78475	0.670532	0.990081	0.823469	0.76868	0.75698
DM	0.796339	0.590458	0.991776	0.848643	0.831916	0.61725
DM	0.892117	0.805598	0.992019	0.799134	0.874873	0.682008
DM	0.751774	0.69566	0.911657	0.796758	0.868976	0.569521
DM	0.794939	0.505473	0.889176	0.864619	0.737541	0.733789
DM	0.806897	0.579079	0.998217	0.800467	0.731156	0.718082
DM	0.865285	0.793658	0.858332	0.831542	0.739226	0.652239

As shown in Fig. 7 the upper bound of efficiency scores for all DMUs and Sub-DMUs are greater than or equal to lower bound of efficiency scores. This validates homogeneity and discrimination power of our proposed approach. In order to more investigation of efficiency scores the mean and range of scores for DMUs and Sub-DMUs are calculated and presented in Table 5.

We can determine the most and least efficient DMUs and Sub-DMUs based on mean efficiency score measure. The main DMU16 has the maximum mean efficiency score. So, it can be assumed as the most efficient main DMU considering the mean measurement which is calculated based on average of upper bound and lower bound of efficiency scores of each DMU and Sub-DMU. The main DMU6 has the minimum mean efficiency score and is assumed to be most inefficient main DMU among the others.

In first stage Sub-DMU11 has the maximum mean efficiency score. So, it can be assumed as the most efficient Sub-DMU in first stage. Based on this interpretation, Sub-DMU8 has the minimum mean efficiency score and is assumed to be most inefficient Sub-DMU in the first stage. In second stage Sub-DMU6 has the maximum mean efficiency score. So, it can be assumed as the most efficient Sub-DMU in second stage. Based on this interpretation, Sub-DMU7 has the minimum mean efficiency score and is assumed to be most inefficient Sub-DMU in the second stage. Other main DMUs and Sub-DMUs can also be ranked based on their mean efficiency score in Table 5.

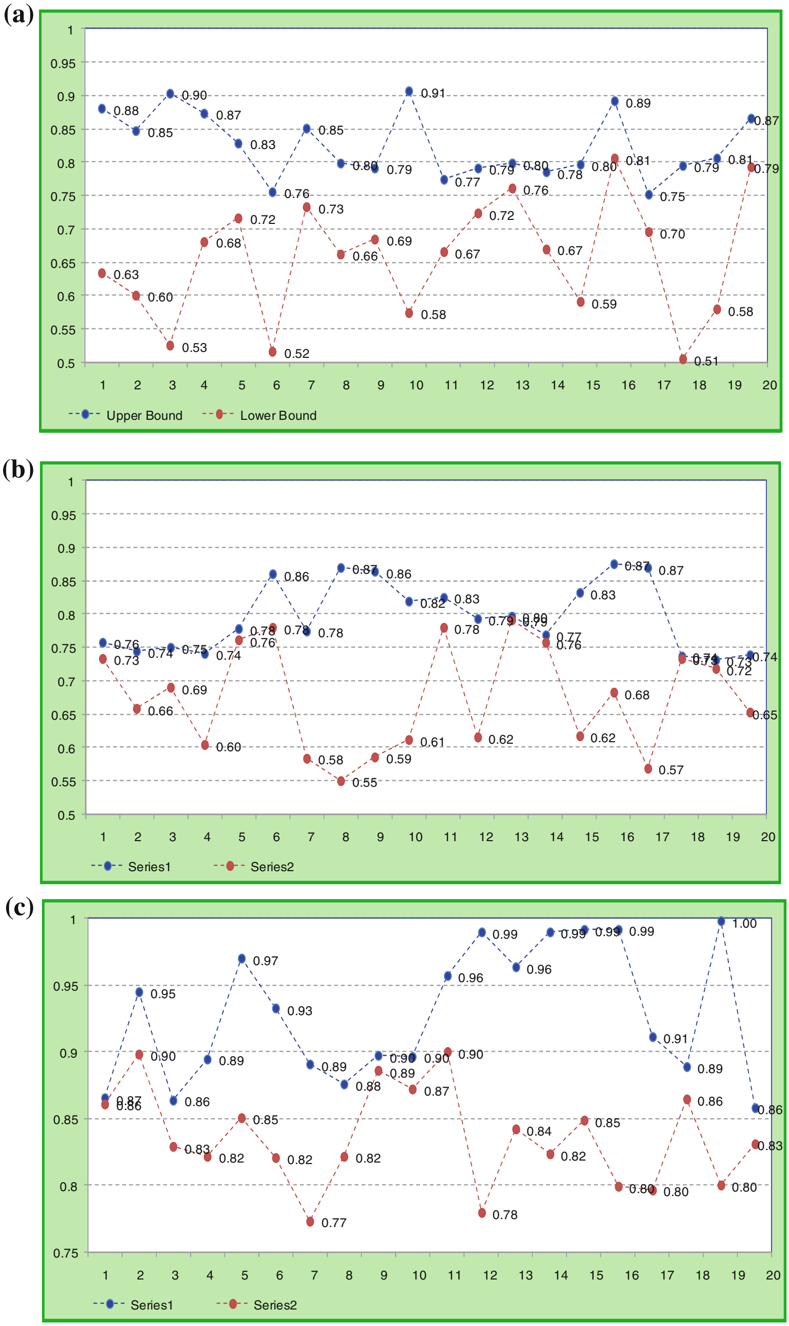

Fig. 7 Upper and lower bound of efficiency scores, **a** Efficiency scores of main DMUs, **b** Efficiency scores of first sub-DMUs, **c** Efficiency scores of second sub-DMUs

Table 5 Mean and range of efficiency scores

DMU	Main-DMU		Sub-DMU$_1$		Sub-DMU$_2$	
	Mean	Range	Mean	Range	Mean	Range
DM	0.757842	0.245992	0.8632755	0.004059	0.7455375	0.023613
DM	0.7243735	0.246021	0.9218505	0.047049	0.701159	0.086512
DM	0.7145675	0.377665	0.8466015	0.035339	0.7198615	0.060727
DM	0.7776935	0.191797	0.8584665	0.073023	0.6726245	0.137035
DM	0.7732095	0.112187	0.9104455	0.119339	0.7691895	0.016661
DM	0.636238	0.239408	0.8767025	0.111957	0.819354	0.080304
DM	0.792281	0.118544	0.8321245	0.117061	0.679673	0.19102
DM	0.730208	0.13529	0.8488955	0.053945	0.7101365	0.319323
DM	0.7385365	0.106941	0.891519	0.011166	0.7244835	0.277563
DM	0.7410475	0.331611	0.8844805	0.024563	0.7152125	0.208101
DM	0.719687	0.10797	0.9284865	0.057329	0.802891	0.045316
DM	0.7573915	0.068761	0.884754	0.210286	0.7039335	0.177203
DM	0.7795915	0.036549	0.902923	0.121206	0.794509	0.005966
DM	0.727641	0.114218	0.906775	0.166612	0.76283	0.0117
DM	0.6933985	0.205881	0.9202095	0.143133	0.724583	0.214666
DM	0.8488575	0.086519	0.8955765	0.192885	0.7784405	0.192865
DM	0.723717	0.056114	0.8542075	0.114899	0.7192485	0.299455
DM	0.650206	0.289466	0.8768975	0.024557	0.735665	0.003752
DM	0.692988	0.227818	0.899342	0.19775	0.724619	0.013074
DM	0.8294715	0.071627	0.844937	0.02679	0.6957325	0.086987
Max	0.848858	0.377665	0.928487	0.210286	0.819354	0.319323
Min	0.636238	0.036549	0.832125	0.004059	0.672625	0.003752

Although, the mean efficiency score is a suitable measure for ranking the DMUs but it cannot consider the deviation of efficiency score of each DMU and Sub-DMU. For example, a DMU with high value of mean efficiency score and high values of range of efficiency has a possibility of low efficiency score in practice. So, considering the range of efficiency score we can determine the most and least reliable DMUs and Sub-DMUs.

Hence, DMU13 can be assumed as the most reliable DMU in presence of uncertain inputs, outputs, and intermediate measures. DMU3 has the maximum range for upper bound and lower bound of efficiency score. So, DMU3 can be assumed as the least reliableDMU in presence of uncertain inputs, outputs, and intermediate measures.

Moreover, in first stage the Sub-DMU1 has the minimum range for upper bound and lower bound of efficiency score. So, Sub-DMU1 in first stage can be assumed as the most reliable Sub-DMU. Again in first stage, Sub-DMU12 has the maximum range for upper bound and lower bound of efficiency score. So, Sub-DMU12 in first stage can be assumed as the least reliable DMU.

Finally, in second stage the Sub-DMU18 has the minimum range for upper bound and lower bound of efficiency score. So, Sub-DMU18 in second stage can

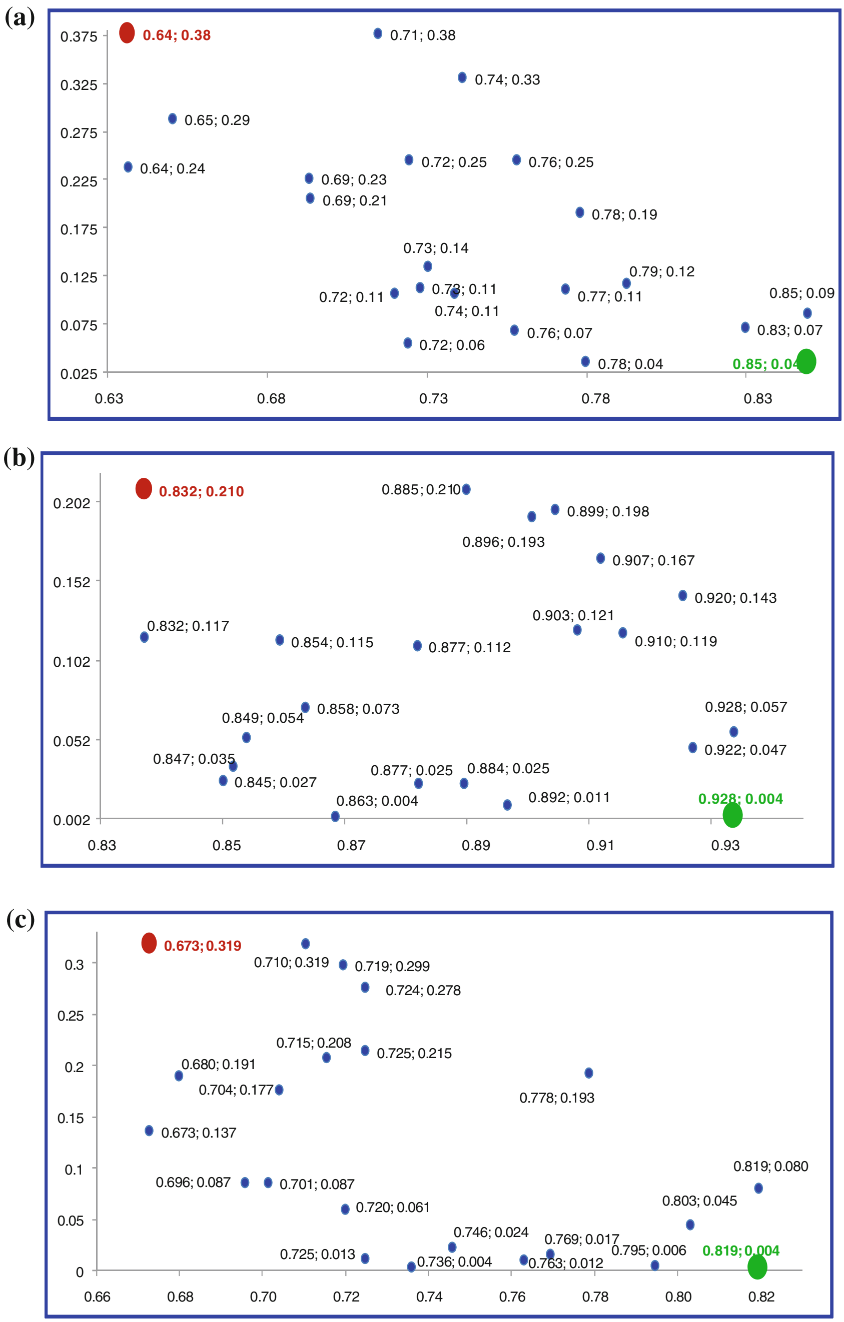

Fig. 8 Range-mean plot of DMUs and sub-DMUs **a** Reliability-efficiency plot for main DMUs **b** Reliability-efficiency plot for first sub-DMUs **c** Reliability-efficiency plot for second sub-DMUs

Table 6 Distances and final ranking of DMUs and sub-DMUs

DMU	Main-DMU				Sub-DMU1				Sub-DMU2			
	Distance$^+$	Distance$^-$	Ratio	Rank	Distance$^+$	Distance$^-$	Ratio	Rank	Distance$^+$	Distance$^-$	Ratio	Rank
DM	0.228364	0.179235	0.439734	14	0.065211	0.208566	0.76181	6	0.076442	0.304566	0.79937	5
DM	0.243669	0.158424	0.393997	15	0.043499	0.186272	0.810684	2	0.144289	0.234553	0.619132	10
DM	0.366598	0.07833	0.17605	20	0.087656	0.175545	0.666961	8	0.114651	0.262875	0.696309	9
DM	0.170781	0.233573	0.577645	12	0.098279	0.139768	0.587143	10	0.198227	0.182288	0.479056	12
DM	0.106975	0.29873	0.736323	6	0.116683	0.120023	0.507055	11	0.051799	0.317693	0.859811	3
DM	0.293869	0.138257	0.319946	17	0.119681	0.107962	0.47426	13	0.076552	0.280463	0.785578	7
DM	0.09962	0.302478	0.75225	5	0.14851	0.093225	0.38565	16	0.233624	0.128496	0.354845	15
DM	0.154362	0.259954	0.62743	11	0.093933	0.157238	0.626021	9	0.333936	0.037512	0.100988	20
DM	0.130865	0.289407	0.688618	8	0.037644	0.20779	0.846621	1	0.289781	0.066583	0.186839	18
DM	0.314141	0.114481	0.267092	18	0.048548	0.192962	0.79898	3	0.229356	0.119097	0.341788	16
DM	0.147601	0.28231	0.656671	10	0.05327	0.18078	0.772399	5	0.044706	0.303396	0.871573	2
DM	0.096972	0.331813	0.773844	4	0.210813	0.05263	0.199776	20	0.208344	0.145528	0.411245	14
DM	0.069266	0.370014	0.842319	3	0.119904	0.113788	0.486915	12	0.024943	0.336227	0.930937	1
DM	0.143965	0.278853	0.659511	9	0.163997	0.086488	0.345282	17	0.05708	0.320576	0.848857	4
DM	0.229871	0.181044	0.440588	13	0.13932	0.110763	0.442905	14	0.231228	0.116845	0.335692	17
DM	0.04997	0.360518	0.878267	2	0.191672	0.065795	0.255546	19	0.193488	0.16489	0.4601	13
DM	0.126661	0.333238	0.72459	7	0.133427	0.09791	0.423234	15	0.312188	0.050681	0.139667	19
DM	0.321604	0.089298	0.217322	19	0.055512	0.191049	0.774855	4	0.083689	0.321806	0.793613	6
DM	0.246737	0.160233	0.393722	16	0.195871	0.068376	0.258759	18	0.095193	0.310631	0.765434	8
DM	0.040078	0.361937	0.900306	1	0.086586	0.183943	0.679937	7	0.149031	0.233482	0.61039	11

Distance$^+$: Distance from Ideal Case
Distance$^-$: Distance from Anti-Ideal Case
Ratio: Distance$^-$/(Distance$^-$ + Distance$^+$)

be assumed as the most reliable Sub-DMU. Again in second stage, Sub-DMU8 has the maximum range for upper bound and lower bound of efficiency score. So, Sub-DMU8 in second stagecan be assumed as the least reliable DMU.

It is obvious that efficient and reliable DMUs and Sub-DMUs are different, so a final ranking is not retrievable in this situation. We plot the DMUs and Sub-DMUs based on their mean (horizontal axis) and range (vertical axis) measures as shown in Fig. 8. The mean and range measures can be assumed as proper indices for efficiency and reliability of a DMU, respectively.

The red and green points in each plot present the Anti-Ideal and Ideal cases, respectively. The ideal case has the biggest mean value and smallest range value while Anti-Ideal case has the smallest mean value and biggest range value among the other DMUs. These ideal and Anti-Ideal cases have been achieved considering the existing real blue points in each plot. By the aforementioned definition all DMUs and Sub-DMUs are ranked based on their distance from Ideal and Anti-Ideal cases. Under this condition a DMU which is far from Anti-Ideal and near to Ideal cases simultaneously takes the best rank and the DMU which is near to Anti-Ideal and far from to Ideal cases simultaneously takes the worst rank. The final ranking of DMUs and Sub-DMUs based on this procedure is presented in Table 6.

6 Conclusion Remarks

In this chapter, fuzzy DEA models were proposed for assessing relative interval efficiency score of overall and segments of supply chains in context of agility as providers of agility and capabilities of agility. The proposed structure was associated to a DMU containing two serial sub-DMUs with uncertain inputs and outputs. The efficiency score of a DMU decomposed into efficiency scores of its Sub-DMUs. As the inputs, intermediate measures, and outputs of DMUs and Sub-DMUs were mainly qualitative and mixed with uncertainty in real life problems, so linguistic terms parameterized using fuzzy sets were applied.

A fuzzy single stage DEA model and a fuzzy two-stage DEA model were proposed to calculate the relative efficiencies. The optimistic and pessimistic situations were considered to calculate an interval efficiency score for each DMU and sub-DMUs.

The proposed models served linear mathematical models for the efficiency calculation which could imply the global optimum solutions in practice. The proposed approach served α-cut independent models which are not need to be solved for different α-cuts. So, the volume of a full fuzzy DEA analysis extremely decreases in real-life problems. Hence, there is no need to determine the best step-size for the α-cut values. A real case of top-twentyIranian dairy supply chains was surveyed. The results were promising and computations were straight forward.

The proposed procedure can be used in other management and engineering problems which have two-stage structures. As the Just In Time practices seems to have considerable effects on agility of supply chain, JIT practices can be joint with the proposed structure of this chapter to elaborate the proposed models. Different types of uncertainty such as robust optimization and stochastic modeling can be assumed as proper mechanism for modeling the uncertainty of the problems for the future researches.

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Supply Chain Performance Measurement Using a SCOR Based Fuzzy VIKOR Approach

Başar Öztayşi and Özge Sürer

Abstract Supply Chain performance measurement is a vital issue for supply chain management. Both from the academia and professional life, various models are proposed for this subject. In this chapter, the literature is investigated for current performance measurement models and a multi-criteria decision making approach is proposed for supply chain performance measurement. In this study, SCOR model is used for structuring the problem, Fuzzy Analytic Hierarchy Process (AHP) is used to determine the importance weights of the criteria and finally Fuzzy VIKOR is used to rank the alternatives based on expert evaluations.

Keywords Supply chain · Performance measurement · Fuzzy VIKOR · Fuzzy AHP

1 Introduction

The term performance is a combination of goals, and relational models that enable the company to accomplish these goals on time. Since it is affected by goals and conditions, the definition of company performance may vary depending on the time and the place (Lebas 1995). Meyer (2002) denotes that performance should be related both to the action, and the consequence of that action. Both the action and the consequences should be benchmarked to a standard in order to make a reference to a degree of achievement. Folan et al. (2007) define three key concepts, relation,

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goal and characteristics about performance measurement. Relation highlights the relation of the company with its environment. The second concept "goal" that expresses the performance of a company is about what it wants to achieve. And the last concept, characteristics define that the performance measurement should be composed of summarized, related characteristics of a company, such as cost, quality and flexibility. It is also stated that, in order to measure the performance; the mentioned characteristics should be numerically expressed and measured by performance indicators.

Supply chain is defined as integrated process where in various business entities such as, suppliers, manufacturers, distributors, and retailers work together in an effort to acquire raw materials/components, convert these raw materials into specified final products, and deliver these final products to retailers (Beamon 1998). In this context, supply chain management is defined as the use of information technology to endow automated intelligence to the planning and control of the flow of supply chain to speed time to market, reduce inventory levels, lower overall costs and, ultimately, enhance customer service and satisfaction (Wang et al. 2004).

Performance measurement in supply chains has gained attention both in the academic and professional world. Various supply chain performance models are proposed such as Balanced Score Card (Kaplan and Norton 1992), Activity Based Costing (Schulze et al. 2012; Qian and Ben-Arieh 2008; Tsai and Hung 2009), Gunasekaran's Model (Gunasekaran et al. 2004). One of the most extended studies is the Supply Chain Operations Reference model—SCOR of the Supply Chain Council (SCC 2010). The SCOR model proposes metrics to manage performance on multiple dimensions in a hierarchical structure defined based on the causal relationships (Garg and Carpinetti 2011). The SCOR model uses supply chain asset, reliability and responsiveness performance measurement perspectives as the main perspectives for performance measurement. Quantification of SC performance is a multicriteria decision making (MCDM) problem since the evaluations have to be done from different perspectives and criteria. In the literature there are various MCDM techniques such as; analytical hierarchy process (AHP) (Elgazzar et al. 2012), analytical network process (ANP) (Ravi et al. 2005) and fuzzy set theory which are used in the field of performance measurement. While traditional MCDM techniques use crisp numbers, new approaches such as grey and fuzzy sets are recently integrated with the current techniques in order to handle uncertainty.

Fuzzy set theory (Zadeh 1965) has emerged as a powerful way of representing quantitatively and manipulating the imprecision in decision-making problems. Using the fuzzy sets, unquantifiable information, incomplete information or non-obtainable information can be used in a decision model. Human preferences and judgments are often vague and thus cannot be estimated with exact numerical values. In order to overcome this problem, linguistic assessments can be used instead of numerical values, such as ratings and weights of the criteria in the problem (Kulak and Kahraman 2005). Fuzzy multi criteria decision making is recently used for performance assessment (Yu et al. 2011; Yalcin et al. 2012; Moussa et al. 2012). Also fuzzified versions of other techniques can also be proposed and used in the performance measurement area such as fuzzy Data Envelopment analysis (Zhou et al. 2012), fuzzy linear regression (Pan et al. 2009), and fuzzy rule based systems (El-Baz 2011). In addition, the hybrid fuzzy methods are used such as fuzzy AHP (Gou et al. 2013), fuzzy DEMATEL (Lin 2013).

The aim of the chapter is to represent the potential application of fuzzy analytical hierarchy process (FAHP) integrated with Fuzzy VIKOR in supply chain performance measurement, to this end a decision model is proposed based on SCOR model, the weights of the criteria are determined using Fuzzy AHP and the experts' linguistic variables are used in Fuzzy VIKOR method to determine the best performing supply chain among the alternatives. This paper is organized as follows: Sect. 2 briefly gives the current literature review about supply chain performance measurement. First the SC performance measurement models are explained and then decision making techniques in this area are given. Section 3 describes the proposed measurement model, starting with SCOR model, FAHP and Fuzzy VIKOR techniques are introduced. Section 4 presents the performance evaluation criteria which consist of five criteria and 16 sub-criteria. Next, Sect. 5 presents a numerical application in which performance of five alternative supply chains are compared. Final considerations about this research work are made in Sect. 6.

2 Literature Review

In this section, literature review about supply chain performance evaluation models are given, later the current studies that focus performance measurement through decision making approach are denoted.

2.1 Supply Chain Performance Evaluation Models

Measuring the efficiency of supply chain systems is a need for organizations. Organizations should employ a performance measurement and improvement projects for all processes in order to achieve their goals and improve their processes. There are number of frameworks about the evaluation of supply chain in the literature. Estampe et al. (2013) analyze the different evaluation models. Instead of giving a unique model for organizations, they emphasize the special characteristics of the models. By the way, the managers can choose the appropriate model in order to measure their supply chain performance. Forme et al. (2007) identify 5 main supply chain performance models. Cagnazzo et al. (2010) make a literature review of performance measurement systems in supply chains. The readers who are interested in these frameworks can find information in these papers. In order to enhance the supply chain processes, we need a systemic performance evaluation models. In this section, we explain supply chain performance measurement models shortly.

Activity-Based Costing (ABC): The aim of supply chain operations is cost reduction. Therefore, managers need an effective tool for the evaluation of cost consequences of supply chain processes (Schulze et al. 2012). The ABC method takes into consideration all the activities for producing the product in order to estimate the production cost. The ABC is more accurate than the traditional cost estimation models (Qian and Ben-Arieh 2008). It is designed for eliminating the non-value added activities in the organizations by a systematic way (Tsai and Hung 2009). In this study, activities are divided into four layers: primary, secondary, and long-term strategic and non-value added activities, and finally composite performance indexes of suppliers are obtained. Baykasoglu and Kaplanoglu (2008) assert that although the determination of activities is hard, they show that there is a considerable difference between the current cost systems of the company and activity-based costing.

Balanced Score Card (BSC): In order to overcome the disadvantages of the performance measurement system that examines only the financial point of view, BSC measures supply chain in a balanced way by utilizing financial and non-financial measures (Bhagwat and Sharma 2007a). The BSC uses four perspectives: financial, customer, business, innovation and learning perspectives (Kaplan and Norton 1992). Bhagwat and Sharma (2007a), Brewer and Speh (2000), Ravi et al. (2005) apply BSC method for evaluating performance. In some studies such as in (Naini et al. 2011), BSC approach is combined with other methods in order to focus on multi-dimensional performance indicators.

Supply Chain Operation Reference Model (SCOR): It is developed by Supply Chain Council (www.supply-chain.org). Based on generic supply chain processes (planning, sourcing, production, delivering, and return activities), SCOR identifies how a best supply chain processes should be at the three levels of it (Forme et al. 2007). The studies by Ganga and Carpinetti (2011), Li et al. (2005) employ SCOR model.

Gunasekaran's Model: They propose supply chain performance metrics based on four major processes of a supply chain (plan, source, make, deliver) at the strategic, tactical and the operational levels (Gunasekaran et al. 2004).

Cooper's Model (GSCF framework): Business processes, management components and the structure of the supply chain are included by supply chain management framework. There are seven business processes: customer relationship management, customer service management, demand management, order fulfillment, manufacturing flow management, supplier relationship management, product development and commercialization, and returns management (Cooper et al. 1997).

ASLOG Audit: It is a European procedure proposed by ASLOG association and aims to logistics improvement by analyzing the management, strategies and planning, product conception and projects, sourcing, production, moving, stock, sales, return and maintenance, management of indicators, and permanent progress in the supply chain processes (Forme et al. 2007; Estampe et al. 2013). Companies can evaluate their performance based on the ASLOG questionnaire.

Gilmour's Model: It evaluates firm's supply chain performance both from an operational and from a strategic viewpoint. A framework for supply chain operations consist of three main capabilities which are process, technology and organization capabilities and 11 sub-capabilities (six process, two IT and three organizational). In order to measure the performance; questions, which evaluate the key performance indicators used to measure these 11 capabilities, are asked to the companies (Gilmour 1999).

Odette's Logistic Evaluation (EVALOG): There are six main categories: strategy and improvement, organization, production plan and availability, customer relationship, product and process control, supplier relationship. Questions or criteria are evaluated under each category based on specific weighing system (Odette 2013).

Efficient Consumer Response (ECR): It exists as a grocery-industry strategy in which distributors and supplier work with in order to maximize customer satisfaction and minimize cost. It aims to transform supply chain from a "push system" to a "pull system" (Harris and Swatman 1999). In four areas, there are efficiency initiatives: efficient store assortment, efficient promotion, efficient product introduction, efficient product replenishment. In order to achieve ECR's ultimate goal, firms apply number of programs and enabling technologies and electronic commerce (EC) technologies. (Lohtia et al. 2004; Harris and Swatman 1999; Kurnia and Johnston 2001) propose models based on ECR.

Quality Models: They basically focus on the quality factors of supply chain. The EFQM (Business Excellence Model) is a famous one that can apply for supply chain performance measurement (Cagnazzo et al. 2010). It is a tool for organizations for self-assessment based on eight principles: customer focus, leadership, definition of objectives, process-based management, staff involvement, continuous innovation process, development of partnerships and society responsibility.

Strategic Profit Model: It mainly focuses on the financial results of logistics processes. Net profit, asset turnover and financial leverage are the three important factors. Based on these factors, it employs two important ratios: Return on assets and return on net worth (Stapleton et al. 2002).

In addition to these models, there are different frameworks for SC performance measurement. Lambert and Pohlen (2001) propose a framework for developing supply chain metrics. Felix and Chan (2003) develop a process-based model and define measures such as costs, time, capacity, capability, effectiveness, reliability, availability, flexibility, productivity, utilization and outcome.

2.2 SCM Performance Evaluation by Decision Making

In order to measure a performance of a system, we need to quantify it to see the results of managerial actions. We see in Sect. 2.1 that the decision process for supply chain performance evaluation models include both quantitative and qualitative criteria. Therefore, the supply chain performance evaluation is a Multi-

Criteria Decision Making (MCDM) problem. Before constructing the model, the measures that evaluate the efficiency of a process should be defined. In this process, one of the frameworks proposed in Sect. 2.1 is chosen based on a company structure and goals. Then, the suitable decision making method is applied to the measures of the performance. We make a comprehensive research about the application of fuzzy theory and AHP/ANP methods, since they are the most common techniques in decision theory and supply chain performance evaluation. In addition, a detailed research is made related to fuzzy VIKOR method and is chosen as a best suitable MCDM method for this study.

AHP and ANP methods are the mostly used multi-criteria techniques in supply chain performance measurement. As AHP is used as a single technique for supplier performance evaluation, it is also used as integrated with other methods. Elgazzar et al. (2012) offer a method by linking supply chain processes performance and a company's financial performance by using Dempster Shafer/Analytical Hierarchy Processes (DS/AHP). The main contribution of this study is that managers can develop a new supply chain strategies based on the financial evaluation of a company. Chan (2003) develops an AHP-based model based on seven criteria in order to measure supply chain for each company in a multidimensional way. Najmi and Makui (2010) extract the supply chain performance of a company based on flexibility, reliability, responsiveness, quality, and asset management metrics by applying the AHP and DEMATEL method. They obtain a performance score by comparing benchmark chain with the ideal chain. Bhagwat and Sharma (2007b) reveal which decision level performance (strategic, operational, tactical) plays the most important role in overall performance measurement and give priority to different BSC perspectives by using AHP method. In addition to this, Bhagwat and Sharma (2009) propose a hybrid model that integrates the AHP method and pre-emptive goal programming (PGP). In this study, AHP is used to assign weight to the qualitative selection criteria; in the final selection process DEA and NN are used. Tsai and Hung (2009) offer a decision model by using fuzzy goal programming and AHP by utilizing ABC in order to determine performance measures. Theeranuphattana et al. (2012) integrate three different MCDM methods: the multi-attribute value theory (MAVT), the swing weighting method and the eigenvector procedure that rely on SCOR model. In some cases, the AHP method is insufficient to evaluate the performance, since it considers only the criteria and sub-criteria in a hierarchal structure. In order to overcome the disadvantages of AHP, ANP method that considers the feedback and interactions between clusters and factors is proposed. In the paper by Ravi et al. (2005), the dimensions of the ANP model are the perspectives of the BSC for the selection of an alternative for the reverse logistics operations (Table 1).

The nature of supply chain evaluation process is complex and unstructured. In decision making process, fuzzy set theory can be applied in order to overcome the uncertainty. El-Baz (2011) offers a framework for evaluating the performance of the different departments of the company by using fuzzy set theory and AHP. The weights of important factors are found by using AHP and then the weight and input factors are fuzzified. In addition to this, Fuzzy-AHP approach is used in

Table 1 Various studies by using AHP/ANP

Studies by	Methods	Evaluation criteria
Elgazzar et al. (2012)	DDS/AHP	SCOR metrics
Chan (2003)	AHP	Cost, resource utilization, quality, flexibility, visibility, trust, innovation
Bhagwat and Sharma (2007b)	AHP	BSC and strategic, tactical, operational level criteria
Najmi and Makui (2010)	AHP with DEMATEL method	Flexibility, reliability, responsiveness, quality, asset management and their metrics
Theeranuphattana et al. (2012)	MAVT, the swing weighting method and the eigenvector method of the AHP	SCOR-Level 1 metrics
Bhagwat and Sharma (2009)	AHP with PGP	Strategic level (Total cash flow time, Rate of return on investment, Flexibility to meet particular customer needs, Delivery lead time, Total cycle time, Buyer supplier partnership level, Customer query time), Tactical level (Extent of cooperation to improve quality, Total transportation cost, Truthfulness of demand predictability/forecasting methods, Product development cycle time), Operational level (Manufacturing cost, Capacity utilization, Information carrying cost, Inventory carrying cost)
Ravi et al. (2005)	ANP	The dimensions of BSC (customer, internal business, innovation and learning, and finance), The determinants (economic factors, legislation, corporate citizenship, environment and green issues)

(Gou et al. 2013) to evaluate the performance of service-oriented catering supply chain based on the six dimensions of supply chain and their indicators. Zhihong et al. (2013) propose a triangular Fuzzy-AHP method for the green supply chain performance evaluation model to consider both qualitative and quantitative factors and fuzzy characteristics of the factors. The fuzzy set theory and decision making trial and evaluation laboratory (DEMATEL) model is proposed by Lin (2013) to reflect the cause and effect relationships among criteria for the green supply chain management. By using fuzzy DEMATEL approach, they simplify the complex decision making procedure by dividing a set of complex factors into cause and effect groups. Ganga and Carpinetti (2011) propose a quantitative model by utilizing fuzzy logic approach and the measures based on the SCOR level 1 and 2 metrics. Chan and Qi (2003) focus on the all business aspects that affect supply chain processes and use a fuzzy set theory (Table 2).

MCDM model based on fuzzy sets theory and VIKOR method is generally applied in the field of supplier selection. Chen and Wang (2009); Sanayei et al. (2010); Shemshadi et al. (2011) rate the suppliers under a fuzzy environment by using VIKOR method. To our knowledge, although there are numerous applications of on fuzzy sets theory and VIKOR method into supplier selection subject, this research is the first application that measures the supply chain performance using a SCOR based Fuzzy VIKOR approach.

3 Methodology

In this section the supply chain performance evaluation approach is introduced. To this end, initially the SCOR model is introduced, then Fuzzy AHP techniques which are used to identify the weights of the criteria are explained and finally Fuzzy VIKOR, which is directly used to evaluate the performance of alternative supply chains, is explained.

3.1 SCOR Model

The Supply Chain Operations Reference (SCOR®) model was developed by Supply Chain Council, a global nonprofit organization. The SCOR Model is a strategic planning tool for organizations in order to manage their supply chain processes. This regularly updated model is the world's most widely accepted framework for evaluating and comparing supply chain activities and performance. Every organization can apply the SCOR model to analyze supply chain performance in a systematic way (SCC 2010).

The model contains multi-level performance metrics, processes and practices. The performance section of SCOR consists of two types of elements: Performance Attributes and Metrics. Performance attribute is used to set a strategic goal for

Table 2 Various studies by using fuzzy sets

Studies by	Methods	Evaluation criteria
El-Baz (2011)	Fuzzy set theory and AHP	Engineering, planning, production, customer service dimensions and their performance indicators
Ganga and Carpinetti (2011)	Fuzzy logic approach	SCOR metrics (Delivery reliability, Cost, Flexibility and responsiveness, Assets), Level 1–2 metrics
Gou et al. (2013)	Fuzzy AHP	Agility, logistics capability, customer satisfaction, cooperation level of supplier, information ability, stability and their indicators
Zhihong et al. (2013)	Triangular Fuzzy-AHP	Financial profitability, market strength, customer service evaluation, innovation and learning abilities, environmental protection and their indicators
Lin (2013)	Fuzzy DEMATEL	Green purchasing, green design, supplier/customer collaboration, recovery and reuse of used products, environmental performance, economic performance, regulation and stakeholder pressures
Chan and Qi (2003)	Fuzzy set theory	Supplying, inbound logistics, core manufacturing, outbound logistics, marketing and sales

Table 3 Definitions of SCOR attributes

Attributes	Definition
Reliability	The ability to perform tasks as expected
Responsiveness	The speed at which a supply chain provides the products to customers
Agility	The ability to respond to market changes in order to gain or maintain its competitive advantage
Cost	The costs associated with operating the supply chain
Asset	The efficiency of an organization in managing its assets to meet demand

organizations. In order to achieve these strategic attributes, metrics are standards used to measure the performance of a supply chain processes.

The SCOR-model specifies five supply chain performance attributes in two categories: customer-facing attributes that include reliability, responsiveness, and agility, and the internal-facing attributes that include cost and asset management. At Table 3, the definitions of these attributes can be seen based on Supply Chain Council's definitions.

SCOR identifies three levels of predefined metrics. SCOR Level 1 metrics are strategic, high-level measures in order to evaluate the whole supply chain performance. Each level 1 metric consists of a number of more detailed level 2 metrics. Level 2 metric consists of more-detailed level 3 metrics.

Based on SCOR Model, a process is a unique activity performed to meet predefined outcomes. From level 1 to 3, SCOR identifies the processes and these processes are applicable across all industries. From level 4 to below, organizations and industries develop their own processes. Based on SCOR Model, supply chain

Table 4 SCOR processes

SCOR processes	Definition
Plan	The planning activities associated with operating a supply chain for supply and demand planning
Source	The ordering and receipt of goods and services in order to meet actual demand
Make	Processes that convert products to finished state
Deliver	The creation, maintenance, fulfillment and shipment of customer orders
Return	The processes related to receiving returned product

management is the combination of five level 1 distinct processes which are shown in Table 4: Plan- Source- Make- Deliver- Return.

In this study, SCOR Model is chosen since it contains well-defined and standardized processes and metrics for performance measurement for whole supply chain processes.

3.2 Fuzzy Analytic Hierarchy Process

Analytic Hierarchy Process, developed by Saaty (1980), is a technique that structures a decision problem as a hierarchy with an overall goal, a group of alternatives, and of a group of criteria which link the alternatives to the goal. Pairwise comparisons are classically carried out by asking the decision maker how valuable a criterion (C1) when compared to another criterion (C2) with respect to overall goal. Also the alternatives are pairwise compared by asking the comparison of an alternative A with alternative B with respect to a specified criterion.

In classical AHP, the comparisons are done using a scale which contains crisp numbers. However, fuzzy extensions of AHP are proposed in the literature to handle the uncertainty in linguistic variables in a better way. Laarhoven and Pedrycz (1983) proposed the first algorithm in fuzzy AHP by describing compared fuzzy ratios with triangular fuzzy membership functions. Buckley (1985) presented fuzzy priorities of comparison ratios whose membership functions are trapezoidal. He also extended Saaty's AHP method to incorporate fuzzy comparison ratios. To overcome the calculation difficulties, Buckley used the geometric mean method to derive fuzzy weights and performance scores. Chang (1996) proposed a methodology with the use of triangular fuzzy numbers for pairwise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pairwise comparisons.

In this chapter, Buckley's fuzzy AHP method is used and the steps of this method are given in the following:

Table 5 Linguistic scale used for evaluations (Chang 1996)

Linguistic terms		Triangular fuzzy numbers
Absolutely Strong	A	3.5, 4, 4.5
Very strong	VS	2.50, 3, 3.5
Fairly strong	FS	1.50, 2, 2.5
Weak	W	0.67, 1, 1.5
Equal	E	1, 1, 1

Step 1. The decision model is structured and pairwise comparison matrices are constructed. The pairwise comparison matrices are form as shown in Eq. (1) where each element $\left(\tilde{a}_{ij}\right)$ is a linguistic term. The pairwise comparison matrix is given by;

$$\tilde{A} = \begin{vmatrix} 1 & \tilde{a}_{12} & \ldots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \ldots & \tilde{a}_{2n} \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \ldots & 1 \end{vmatrix} \tag{1}$$

When triangular fuzzy numbers are use, Eq. (1) is rewritten as follows:

$$\tilde{A} = \begin{vmatrix} 1 & (a12l, a12m, a12u) & \ldots & (a1nl, a1nm, a1nu) \\ (a21l, a21m, a21u) & 1 & \ldots & (a2nl, a2nm, a2nu) \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ (an1l, an1m, an1u) & (an2l, an2m, an2u) & \ldots & 1 \end{vmatrix} \tag{2}$$

The linguistic scale used in the evaluation procedure is given in Table 5.

Step 2. The consistency of the fuzzy pairwise comparison matrices are examined in the next step. In order to check the consistency of the fuzzy pairwise comparison matrices, pairwise comparison values are defuzzified. Assuming $\tilde{A} = \left[\tilde{a}_{ij}\right]$ is a fuzzy positive reciprocal matrix and $A = \left[a_{ij}\right]$ is its defuzzified positive reciprocal matrix. If the result of the comparisons of $A = \left[a_{ij}\right]$ is consistent, then it can imply that the result of the comparisons of $\tilde{A} = \left[\tilde{a}_{ij}\right]$ is also consistent (Buckley 1985).

If the pairwise comparisons are not consistent, experts must reevaluate the pairwise comparisons.

Step 3. The fuzzy geometric mean for each row of matrices is computed in order to weigh the criteria and alternatives. First the geometric mean of the first parameters of triangular fuzzy numbers in each row is calculated:

$$a_{1l} = [1 \times a_{12l} \times \ldots \times a_{1nl}]^{1/n}$$
$$a_{2l} = [a_{21l} \times 1 \times \ldots \times a_{2nl}]^{1/n}$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$a_{il} = [a_{n1l} \times a_{n2l} \times \ldots \times 1]^{1/n}$$

Then the geometric mean of the second and third parameters of triangular fuzzy numbers in each row is calculated respectively:

$$b_{1m} = [1 \times b_{12m} \times \ldots \times b_{1nm}]^{1/n}$$
$$b_{2m} = [b_{21m} \times 1 \times \ldots \times b_{2nm}]^{1/n}$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$b_{im} = [b_{n1m} \times b_{n2m} \times \ldots \times 1]^{1/n}$$

and

$$c_{1u} = [1 \times c_{12u} \times \ldots \times c_{1nu}]^{1/n}$$
$$c_{2u} = [c_{21u} \times 1 \times \ldots \times c_{2nu}]^{1/n}$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$c_{iu} = [c_{n1u} \times c_{n2u} \times \ldots \times 1]^{1/n}$$

Assuming the sums of the geometric mean values in the row is a_{1s} for lower parameters; a_{2s} for medium parameters; and $a_{3s}$for upper parameters. Finally $\tilde{r}_{ij}$ matrix is obtained by using a_{ij} values obtained above:

$$\tilde{r}_{ij} = \left|\begin{array}{cccc} \text{Criterion 1} & \text{Criterion 2} & & \text{Criterion j} \\ \left(\frac{a_{1l}}{a_{3s}}, \frac{b_{1m}}{a_{2s}}, \frac{c_{1u}}{a_{1s}}\right), & \left(\frac{a_{1l}}{a_{3s}}, \frac{b_{1m}}{a_{2s}}, \frac{c_{1u}}{a_{1s}}\right), & \ldots, & \left(\frac{a_{1l}}{a_{3s}}, \frac{b_{1m}}{a_{2s}}, \frac{c_{1u}}{a_{1s}}\right), \\ \left(\frac{a_{2l}}{a_{3s}}, \frac{b_{2m}}{a_{2s}}, \frac{c_{2u}}{a_{1s}}\right), & \left(\frac{a_{2l}}{a_{3s}}, \frac{b_{2m}}{a_{2s}}, \frac{c_{2u}}{a_{1s}}\right), & \ldots, & \left(\frac{a_{2l}}{a_{3s}}, \frac{b_{2m}}{a_{2s}}, \frac{c_{2u}}{a_{1s}}\right), \\ \ldots & \ldots & & \ldots \\ \left(\frac{a_{il}}{a_{3s}}, \frac{b_{im}}{a_{2s}}, \frac{c_{iu}}{a_{1s}}\right), & \left(\frac{a_{il}}{a_{3s}}, \frac{b_{im}}{a_{2s}}, \frac{c_{iu}}{a_{1s}}\right), & \ldots, & \left(\frac{a_{il}}{a_{3s}}, \frac{b_{im}}{a_{2s}}, \frac{c_{iu}}{a_{1s}}\right), \end{array}\right| \quad (3)$$

Step 4. The fuzzy weights and fuzzy performance scores are aggregated as follows:

$$\tilde{U}_i = \sum_{j=1}^{n} \tilde{w}_j \tilde{r}_{ij}, \forall i. \quad (4)$$

Where U_i is the fuzzy utility of alternative i, w_j is the weight of the criterion j, and r_{ij}

Step 5. Defuzzification of fuzzy numbers in order to determine the importance ranking of the criteria and alternatives. The Center of Area (COA or Center Index, CI) method can be used for defuzzification in this step. The COA method for a triangular fuzzy number $\tilde{A} = (l, m, u)$ can be calculated as follows;

$$BNP_i = \frac{(u_i - l_i) + (m_i - l_i)}{3} + l_i, \forall i \tag{5}$$

where BNP means best nonfuzzy performance.

Step 6. The best alternative id determined based on the defuzzified alternative scores.

3.3 Fuzzy VIKOR

VIKOR method is developed as a multi-criteria decision making method to solve a discrete decision problem with non-commensurable and conflicting criteria (Opricovic and Tzeng 2004). This method determines compromise solution for a problem with conflicting criteria. The multicriteria measure for compromise ranking is developed from the LP—metric used as an aggregating function in a compromise programming method Yu (1973) and Zeleny (1982). The methodology simply works on the principle that each alternative can be evaluated by each criterion function; the compromise ranking will be realized by comparing the degrees of closeness to the ideal alternative.

VIKOR method is also extended using fuzzy approaches. In fuzzy VIKOR it is suggested that decision makers use linguistic variables to evaluate the ratings of alternatives with respect to criteria. The steps of Fuzzy VIKOR are given in the following:

Step 1: Fuzzy decision matrix is formed for n criteria and m alternatives.

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \cdots & \tilde{x}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

where $\tilde{x}_{ij}$ is the score of ith alternative with respect to jth criterion and

$$W = [\tilde{w}_1, \tilde{w}_2, \ldots\ldots \tilde{w}_n]$$

W is the weights matrix and $\tilde{w}_j$ denotes the weight of the jth criterion.

Step 2: The fuzzy best value $(\tilde{f}_j^*)$ and the fuzzy worst value $\left(\tilde{f}_j^-\right)$ is determined for each criterion.

$$\tilde{f}_j^* = \max_i \tilde{x}_{ij} \tag{6}$$

$$\tilde{f}_j^- = \min_i \tilde{x}_{ij} \tag{7}$$

Step 3: The fuzzy separation values $\tilde{R}_i$ and $\tilde{S}_i$ are computed using the following equations:

$$\tilde{S}_i = \sum_{j=1}^{n} \tilde{w}_j \frac{\left(\tilde{f}_j^* - \tilde{x}_{ij}\right)}{\left(\tilde{f}_j^* - \tilde{f}_j^-\right)} \tag{8}$$

$$\tilde{R}_i = \max_j \left[\tilde{w}_j \frac{\left(\tilde{f}_j^* - \tilde{x}_{ij}\right)}{\left(\tilde{f}_j^* - \tilde{f}_j^-\right)} \right] \tag{9}$$

Step 4: $\tilde{S}^*$, $\tilde{S}^-$, $\tilde{R}^*$, $\tilde{R}^-$ and $\tilde{Q}_i$ values are calculated.

$$\tilde{S}^* = \min_i \tilde{S}_i, \quad \tilde{S}^- = \max_i \tilde{S}_i \tag{10}$$

$$\tilde{R}^* = \min_i \tilde{R}_i, \quad \tilde{R}^- = \max_i \tilde{R}_i \tag{11}$$

$$\tilde{Q}_i = \frac{v\left(\tilde{S}_i - \tilde{S}^*\right)}{\left(\tilde{S}^- - \tilde{S}^*\right)} + \frac{(1-v)\left(\tilde{R}_i - \tilde{R}^*\right)}{\left(\tilde{R}^- - \tilde{R}^*\right)} \tag{12}$$

The indices $\min_i \tilde{S}_i$ and $\min_i \tilde{R}_i$ are related to a maximum majority rule, and a minimum individual regret of an opponent strategy, respectively. The parameter v is defined as a weight for the strategy of maximum group utility, whereas $1-v$ defines the weight of the individual regret. In general v is usually assumed to be 0.5.

Step 5: Next step is the defuzzification of the triangular fuzzy number $\tilde{Q}_i$. The Center of Area (COA or Center Index, CI) method given in Eq. 5 can be used for defuzzification in this step.

Step 6: The alternative are sorted in descending order according to their $\tilde{Q}_i$ value. The alternative with the minimum value is determined as the best alternative.

4 Performance Evaluation Criteria

The performance evaluation criteria used in this study is based on the SCOR 10.0 model (SCC 2010). Five performance attributes are defined in the model which constitutes the main criteria in this study. For each criterion, metrics from different

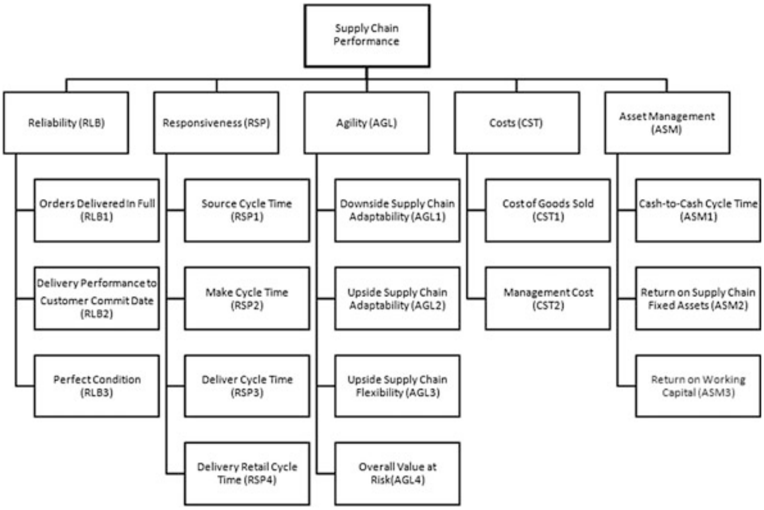

Fig. 1 Supply chain performance evaluation criteria

detail levels are also defined. In this study only metrics from first two levels are considered. The proposed performance evaluation model is represented in Fig. 1.

The definitions of the criteria and sub-criteria are given in SCOR 10 as follows (SCC 2010):

Supply Chain Reliability: The performance of the supply chain in delivering: the correct product, to the correct place, at the correct time, in the correct condition and packaging, in the correct quantity, with the correct documentation, to the correct customer. The criteria contain the sub-criteria:

- **Orders delivered in full:** Percentage of orders which all of the items are received by customer in the quantities committed.
- **Delivery performance to customer commit date:** The percentage of orders that are fulfilled on the customer's originally scheduled or committed date.
- **Perfect condition:** Percentage of orders delivered in an undamaged state that meet specification, have the correct configuration, are faultlessly installed (as applicable), and accepted by the customer.

Supply Chain Responsiveness: The speed at which a supply chain provides products to the customer. The criteria contain the sub-criteria:

- **Source cycle time:** The average time associated with source processes.
- **Make cycle time:** The average time associated with make processes.
- **Delivery cycle time:** The average time associated with deliver processes.
- **Delivery retail cycle time:** The average cycle time of the processes used to acquire, merchandise, and sell finished goods at a retail store.

Supply Chain Flexibility: The agility of a supply chain in responding to marketplace changes to gain or maintain competitive advantage. The criteria contain the sub-criteria:

- **Downside supply chain adaptability:** The reduction in quantities ordered sustainable at 30 days prior to delivery with no inventory or cost penalties.
- **Upside supply chain adaptability:** The maximum sustainable percentage increase in quantity delivered that can be achieved in 30 days.
- **Upside supply chain flexibility:** The number of days required to achieve an unplanned sustainable 20 % increase in quantities delivered.
- **Overall Value-at-Risk:** It is a category of risk metrics that describe probabilistically the market risk of a trading portfolio (McCormack et al. 2008).

Supply Chain Costs: The costs associated with operating the supply chain. The criteria contain the sub-criteria:

- **Cost of goods sold:** The cost associated with buying raw materials and producing finished goods. This cost includes direct costs (labor, materials) and indirect costs.
- **Cost of management:** The sum of the costs associated with the processes to plan, source, deliver, and return.

Supply Chain Asset Management: The effectiveness of an organization in managing assets to support demand satisfaction. This includes the management of all assets: fixed and working capital.

- **Cash-to-cash cycle time:** The time it takes for an investment made to flow back into a company after it has been spent for raw materials. For services, this represents the time from the point where a company pays for the resources consumed in the performance of a service to the time that the company received payment from the customer for those services.
- **Return on supply chain fixed assets:** Measures the return an organization receives on its invested capital in supply chain fixed assets. This includes the fixed assets used in plan, source, make, deliver, and return.
- **Return on working capital:** Return on working capital is a measurement which assesses the magnitude of investment relative to a company's working capital position verses the revenue generated from a supply chain. Components include accounts receivable, accounts payable, inventory, supply chain revenue, cost of goods sold and supply chain management costs.

5 Numerical Application

In this chapter we aim to represent an application of integrated fuzzy AHP and VIKOR methodology on supply chain performance evaluation case. The aim of the application is to assess the performance of five different supply chains based on the model described in Sect. 4 and by using linguistic variables. The integrated approach consists of two phases; in the first phase the relative importance weights of the criteria are determined using Fuzzy AHP method and in the second phase Fuzzy VIKOR is used to rank the supply chains based on their performance.

5.1 Determining the Weights of the Evaluation Criteria

In this stage, the weights of the five criterion and 16 sub-criteria are determined using fuzzy AHP. To this end, Buckley's fuzzy AHP method is used as explained in Sect. 3.2. According to the performance evaluation model described in Sect. 4, there are five main criteria namely, reliability (RLB), responsiveness (RSP), agility (AGL), costs (CST) and Asset Management (ASM). As the first step, these five criteria are pairwise compared with each other with respect to their importance in overall performance evaluation. The linguistic scale represented in Table 5 is used for the pairwise comparison matrices. The pairwise comparisons of the criteria are represented in Table 6.

The linguistic evaluations shown in Table 7 are then transformed to triangular fuzzy numbers for further calculations.

Following the steps defined in Sect. 3.2 the fuzzy weights for the criteria are calculated and represented in Table 8. The triangular fuzzy numbers are later used in Fuzzy VIKOR operations but for a better interpretation the defuzzified weights are also shown in Table 8.

The same procedure is applied to all sub-criteria, but this time the pairwise comparisons are done with respect to the related criterion. Tables 9, 10, 11, 12, 13 represent the pairwise comparisons of the sub-criteria with respect to the related criteria. The weights of each sub-criteria are calculated using same steps and represented in the same tables.

The calculated weights show the relative importance of each sub-criterion with respect to the related criteria, however for fuzzy VIKOR operations, we need the global weights. In order to determine the global weights of each sub-criterion, their weights are multiplied with the weight of the related criterion. Table 14 represents the global weights of all sub-criteria.

The triangular fuzzy weights, shown in Table 9 are later used for determining the performance evaluation.

5.2 Obtaining the Relative Performance

In this section, five supply chains are compared using the model defined in Sect. 4 and weights calculated by Fuzzy AHP. For the determining the relative performance, each supply chain is evaluated by three experts. The experts use a

Table 6 Pairwise comparison of criteria with respect to the goal

	RLB	RSP	AGL	CST	ASM
RLB	E	FS	1/FS	1/FS	A
RSP	1/FS	E	1/VS	1/FS	W
AGL	FS	VS	E	1/VS	1/E
CST	FS	FS	VS	E	VS
ASM	1/A	1/W	E	1/A	E

Table 7 Fuzzy evaluation matrix

	RLB	RSP	AGL	CST	ASM
RLB	(1, 1, 1)	(1.5, 2, 2.5)	(0.4, 0.5, 0.67)	(0.4, 0.5, 0.67)	(3.5, 4, 4.5)
RSP	(0.4, 0.5, 0.67)	(1, 1, 1)	(0.29, 0.33, 0.4)	(0.4, 0.5, 0.67)	(0.67, 1, 1.5)
AGL	(1.5, 2, 2.5)	(2.5, 3, 3.5)	(1, 1, 1)	(0.29, 0.33, 0.4)	(1, 1, 1)
CST	(1.5, 2, 2.5)	(1.5, 2, 2.5)	(2.5, 3, 3.5)	(1, 1, 1)	(2.5, 3, 3.5)
ASM	(0.22, 0.25, 0.29)	(0.67, 1, 1.5)	(1, 1, 1)	(0.22, 0.25, 0.29)	(1, 1, 1)

Table 8 Fuzzy and crisp weights of the criteria with respect to the goal

	Triangular fuzzy weights	Crisp weights
RLB	0.15, 0.21, 0.30	0.21
RSP	0.08, 0.11, 0.16	0.11
AGL	0.16, 0.21, 0.27	0.21
CST	0.26, 0.37, 0.51	0.37
ASM	0.08, 0.10, 0.14	0.10

Table 9 Pairwise comparison of sub-criteria with respect to the reliability

	RLB1	RLB2	RLB3	Triangular fuzzy weights	Crisp weights
RLB1	E	VS	FS	0.40, 0.55, 0.74	0.54
RLB2	1/VS	E	1/W	0.15, 0.21, 0.30	0.21
RLB3	1/FS	W	E	0.17, 0.24, 0.36	0.25

Table 10 Pairwise comparison of sub-criteria with respect to the responsiveness

	RSP1	RSP2	RSP3	RSP4	Triangular fuzzy weights	Crisp weights
RSP1	E	E	E	E	0.25, 0.25, 0.25	0.25
RSP2	E	E	E	E	0.25, 0.25, 0.25	0.25
RSP3	E	E	E	E	0.25, 0.25, 0.25	0.25
RSP4	E	E	E	E	0.25, 0.25, 0.25	0.25

linguistic scale shown in Table 15 for the evaluations. Different from the AHP approach, the absolute evaluations are used instead of pairwise comparisons.

Table 16 represents the performance evaluations of three experts from sixteen different perspectives. According to the table, the experts' evaluations for

Table 11 Pairwise comparison of sub-criteria with respect to the agility

	AGL1	AGL2	AGL3	AGL4	Triangular fuzzy weights	Crisp weights
AGL1	E	E	1/FS	1/VS	0.11, 0.14, 0, 18	0.14
AGL2	E	E	1/FS	1/VS	0.11, 0.14, 0, 19	0.14
AGL3	FS	FS	E	1/FS	0.19, 0.26, 0.37	0.27
AGL4	VS	VS	FS	E	0.34, 0.46, 0.60	0.45

Table 12 Pairwise comparison of sub-criteria with respect to the costs

	CST1	CST2	Triangular fuzzy weights	Crisp weights
CST1	E	E	0.25, 0.25, 0.25	0.25
CST2	E	E	0.25, 0.25, 0.25	0.25

Table 13 Pairwise comparison of sub-criteria with respect to the assets management

	ASM1	ASM2	ASM3	Triangular fuzzy weights	Crisp weights
ASM1	E	1/FS	1/FS	0.16, 0.20, 0.27	0.20
ASM2	FS	E	E	0.33, 0.40, 0.48	0.40
ASM3	FS	E	E	0.33, 0.40, 0.48	0.40

Table 14 The fuzzy and crisp global weights of the sub-criteria

Criteria	Triangular fuzzy weights	Crisp weights
RLB1	0.059, 0.114, 0.219	0.121
RLB2	0.022, 0.043, 0.089	0.047
RLB3	0.024, 0.049, 0.106	0.054
RSP1	0.019, 0.027, 0.041	0.027
RSP2	0.019, 0.027, 0.041	0.027
RSP3	0.019, 0.027, 0.041	0.027
RSP4	0.019, 0.027, 0.041	0.027
AGL1	0.017, 0.029, 0.05	0.029
AGL2	0.017, 0.029, 0.05	0.029
AGL3	0.029, 0.054, 0.1	0.056
AGL4	0.052, 0.094, 0.165	0.095
CST1	0.13, 0.185, 0.254	0.176
CST2	0.13, 0.185, 0.254	0.176
ASM1	0.012, 0.02, 0.038	0.021
ASM2	0.025, 0.041, 0.067	0.041
ASM3	0.025, 0.041, 0.067	0.041

Table 15 Linguistic scale for SCM performance evaluations

Linguistic terms		Fuzzy score
Very poor	(VP)	(0, 0, 1)
Poor	(P)	(0, 1, 3)
Medium poor	(MP)	(1, 3, 5)
Fair	(F)	(3, 5, 7)
Medium good	(MG)	(5, 7, 9)
Good	(G)	(7, 9, 10)
Very good	(VG)	(9, 9, 10)

Table 16 Evaluation scores of the five compared supply chains

		RLB1	RLB2	RLB3	RSP1	RSP2	RSP3	RSP4	AGL1
Alt1	Exp1	G	F	G	F	MG	F	F	MG
	Exp2	MG	F	MG	MG	G	MG	F	G
	Exp3	G	G	G	G	G	G	G	MG
Alt2	Exp1	MG	P	MP	F	MG	P	F	F
	Exp2	MG	MP	F	MG	G	MP	F	F
	Exp3	G	F	F	G	G	F	G	G
Alt3	Exp1	P	F	G	F	P	P	F	F
	Exp2	MP	P	MG	G	MP	MP	G	F
	Exp3	P	P	G	G	F	F	G	G
Alt4	Exp1	VP	MP	F	F	F	P	MP	MG
	Exp2	MP	F	G	G	MG	MP	F	MG
	Exp3	MP	F	G	G	G	F	F	MG

(continued)

Table 16 (continued)

		RLB1	RLB2	RLB3	RSP1	RSP2	RSP3	RSP4	AGL1
Alt5	Exp1	MG	G	F	F	F	MG	F	MG
	Exp2	G	MG	G	G	G	G	F	F
	Exp3	G	G	G	G	G	G	G	F

		AGL2	AGL3	AGL4	CST1	CST2	ASM1	ASM2	ASM3
Alt1	Exp1	F	F	F	F	F	F	MG	VG
	Exp2	G	G	G	G	F	G	F	F
	Exp3	G	G	F	F	MG	G	F	G
Alt2	Exp1	F	F	F	G	P	F	MG	G
	Exp2	G	G	G	G	VP	G	G	MG
	Exp3	G	G	F	MG	P	F	G	MG
Alt3	Exp1	MP	F	G	MG	G	P	MG	P
	Exp2	F	F	G	G	G	MP	G	MP
	Exp3	MP	G	MG	G	MG	F	G	P
Alt4	Exp1	MG	F	F	F	G	G	MG	F
	Exp2	G	G	G	G	G	G	G	G
	Exp3	G	G	F	F	MG	MG	G	MG
Alt5	Exp1	F	G	MG	F	G	F	MG	MG
	Exp2	F	G	G	G	G	G	G	MG
	Exp3	G	MG	G	F	VG	F	G	F

Table 17 Fuzzy evaluation matrix for the alternatives

	RLB1	RLB2	RLB3	RSP1	RSP2	RSP3
Alt1	6.33, 8.33, 9.66	4.33, 6.33, 8	6.33, 8.33, 9.66	5, 7, 8.66	6.33, 8.33, 9.66	5, 7, 8.66
Alt2	5.66, 7.66, 9.33	1.33, 3, 5	2.33, 4.33, 6.33	5, 7, 8.66	6.33, 8.33, 9.66	1.33, 3, 5
Alt3	0.33, 1.66, 3.66	1, 2.33, 4.33	6.33, 8.33, 9.66	5.66, 7.66, 9	1.33, 3, 5	1.33, 3, 5
Alt4	0.66, 2, 3.66	2.33, 4.33, 6.33	5.66, 7.66, 9	5.66, 7.66, 9	5, 7, 8.66	1.33, 3, 5
Alt5	6.33, 8.33, 9.66	6.33, 8.33, 9.66	5.66, 7.66, 9	5.66, 7.66, 9	5.66, 7.66, 9	6.33, 8.33, 9.66
	RSP4	AGL1	AGL2	AGL3	AGL4	CST1
Alt1	4.33, 6.33, 8	5.66, 7.66, 9.33	5.66, 7.66, 9	5.66, 7.66, 9	4.33, 6.33, 8	4.33, 6.33, 8
Alt2	4.33, 6.33, 8	4.33, 6.33, 8	5.66, 7.66, 9	5.66, 7.66, 9	4.33, 6.33, 8	6.33, 8.33, 9.66
Alt3	5.66, 7.66, 9	4.33, 6.33, 8	1.66, 3.66, 5.66	4.33, 6.33, 8	6.33, 8.33, 9.66	6.33, 8.33, 9.66
Alt4	2.33, 4.33, 6.33	5, 7, 9	6.33, 8.33, 9.66	5.66, 7.66, 9	4.33, 6.33, 8	4.33, 6.33, 8
Alt5	4.33, 6.33, 8	3.66, 5.66, 7.66	4.33, 6.33, 8	6.33, 8.33, 9.66	6.33, 8.33, 9.66	4.33, 6.33, 8
	CST2	ASM1	ASM2	ASM3		
Alt1	3.66, 5.66, 7.66	5.66, 7.66, 9	3.66, 5.66, 7.66	6.33, 7.66, 9		
Alt2	0, 0.66, 2.33	4.33, 6.33, 8	6.33, 8.33, 9.66	5.66, 7.66, 9.33		
Alt3	6.33, 8.33, 9.66	1.33, 3, 5	6.33, 8.33, 9.66	0.33, 1.66, 3.66		
Alt4	6.33, 8.33, 9.66	6.33, 8.33, 9.66	6.33, 8.33, 9.66	5, 7, 8.66		
Alt5	7.66, 9, 10	4.33, 6.33, 8	6.33, 8.33, 9.66	4.33, 6.33, 8.33		

Table 18 Separation measures of the alternatives

	$\tilde{S}_i$	$\tilde{R}_i$
Alt1	0.32, 0.47, 0.71	0.13, 0.18, 0.25
Alt2	0.3, 0.49, 0.81	0.13, 0.18, 0.25
Alt3	0.23, 0.38, 0.69	0.05, 0.11, 0.21
Alt4	0.34,0.53,0.87	0.13, 0.18, 0.25
Alt5	0.18, 0.26, 0.39	0.13, 0.18, 0.25

Table 19 $\tilde{S}^*, \tilde{S}^-, \tilde{R}^*, \tilde{R}^-$ values

$\tilde{S}^*$	(0.18, 0.26, 0.39)
$\tilde{S}^-$	(0.34, 0.53, 0.87)
$\tilde{R}^*$	(0.05, 0.11, 0.21)
$\tilde{R}^-$	(0.13, 0.18, 0.25)

Table 20 Integrated fuzzy VIKOR-AHP analysis results

	$\tilde{Q}_i$	Q_i	Rank
Alt1	(0.94, 0.89, 0.83)	0.893	3
Alt2	(0.89, 0.92, 0.93)	0.918	4
Alt3	(0.16, 0.23, 0.31)	0.237	1
Alt4	(1, 1, 1)	1	5
Alt5	(0.5, 0.5, 0.5)	0.5	2

Alternative1 are Good (G), Medium Good (MG) and Good (G) respectively according to the "Orders Delivered in Full" criteria.

As the linguistic evaluations are completed, these linguistic values are transformed to fuzzy values and the evaluations are consolidated using arithmetic mean operations. For example the evaluations for Alt1 from RLB1 perspective is G, MG, G which can be defined as (7, 9, 10) (5, 7, 9) and (7, 9, 10). To determine the consolidated value arithmetic mean is applied: (7 + 5 + 7)/3, (9 + 7 + 9)/3, (10 + 9 + 10)/3 which equals to (6.33, 8.33, 9.66). The Fuzzy evaluation matrix is represented in Table 17.

Then, separation measures from the fuzzy best value $(\tilde{S}_i)$ and the fuzzy worst value $(\tilde{R}_i)$ are computed and given in Table 18.

Next, $\tilde{S}^*$, $\tilde{S}^-$, $\tilde{R}^*$, $\tilde{R}^-$ values are calculated using Eqs. (10, 11) and represented in Table 19.

Finally assuming v as 0.5, $\tilde{Q}_i$ values are computed for each alternative using Eq. (12). Table 20 gives the fuzzy and defuzzified values.

Based on the crisp Q_i values the alternatives are ranked. The alternative with the lowest Q_i value is ranked as the best performing supply chain. According to the results the ranking of the supply chains in descending order are Alt3, Alt5, Alt1, Alt2 and Alt4.

6 Conclusion

As a result of increasing importance of supply chain management, supply chain performance measurement has become a very critical issue for benchmarking and improving the current supply chains. Performance measurement is done based on the characteristics or metrics that are related to the supply chain. The literature provides various studies that focus on the criteria that can be used for performance measurement.

In the proposed approach, SCOR model is used for selection of the criteria and metrics. Fuzzy AHP is used to determine the weights of these criteria and metrics. Finally Fuzzy VIKOR method is used in order to evaluate and rank the alternatives according to their overall performance. In this study, five alternative supply chains are compared using 16 subcriteria clustered under five criteria, namely reliability, responsiveness, agility, costs and asst management and. Linguistic evaluations are used as an input, and converted to triangular fuzzy numbers to be used in further methods.

As further study the same evaluation data can be examined with other mentioned fuzzy and non-fuzzy multicriteria decision making method such as TOPSIS, COPRAS, and MACBETH and the results can be compared with the results of this study.

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Part III
Planning, Controlling, and Improving Supply Chain Under Fuzziness

Fuzzy Estimations and System Dynamics for Improving Manufacturing Orders in VMI Supply Chains

Francisco Campuzano-Bolarín, Josefa Mula and David Peidro

Abstract In this chapter, we evaluate the behavior of fuzzy estimations of demand for releasing manufacturing orders in a Vendor-Managed Inventory (VMI) supply chain, which is based on a collaborative deal between retailer and manufacturer, and focuses on the interchange of information about demand and inventory levels. The supply chain considered consists of an end consumer, a retailer and a manufacturer. A system dynamics model with fuzzy estimations of demand has been constructed for supply chain simulation. Fuzzy numbers are used to model fuzzy estimations of demand. With a numerical example, we show that the bullwhip effect can be effectively reduced at the level where fuzzy orders exist and that the fill rate reached improves at the retailer level.

Keywords Supply chain · Vendor-managed inventory · Fuzzy estimations · Bullwhip effect · System dynamics · Simulation

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1 Introduction

The VMI is a process in which a supplier generates orders for its customer based on the demand information that this customer sends. During this process, the supplier is guided to fulfill the objectives of the inventory levels and transaction costs. Evidently, an agreement is reached beforehand among partners, and it is a collaborative technique for this reason. The customer sends to its supplier the warehouse stocks to be replenished and information about its consumptions, irrespectively of it being a distribution centre or a point of sales. The supplier analyzes the products consumed, supply times, the possible amendments made to demand, the agreed days of maximum stock, etc., and decides how much it must replenish. Therefore, the supplier replenishes directly; that is to say, it generates an internal order to prepare products and sends it to the customer. In other words, the products that the supplier decides to replenish reach the customer's distribution centre or point of sales in order to always achieve the level of service agreed on. This chapter explores the use of fuzzy estimations of demand for generating manufacturing orders in VMI multi-level supply chains. We consider a collaborative supply chain formed by three levels: an end consumer, a retailer and a manufacturer. The main contribution of this paper is the validation of the fuzzy estimation approach based on dynamic systems in a VMI multi-level supply chain. We develop two simulation models. Firstly, retailer and manufacturer have a VMI collaboration deal in which the retailer's inventory levels are previously agreed and exponential smoothing for demand forecasting is considered. Fuzzy estimations of demand are used for generating manufacturing orders. Exponential smoothing for generating retailer replenishment orders based on the up-to-level order (S, s) is also employed.

This chapter is organized as follows. Firstly, Sect. 2 presents a literature review section. In Sect. 3, the models formulation is described. Section 4 describes the measurement of the bullwhip effect. Section 5 evaluates the proposed models with a numerical example. Finally, Sect. 6 provides conclusions and further research lines.

2 Literature Review

Magee (1958) was a precursor of the Vendor-Managed Inventory (VMI) concept, which is characterized by the agreement reached between the distributor and retailer (or between the manufacturer and distributor). The distributor knows not only the end consumer's sales because it receives information about them, but also the retailer's inventory status because it manages the level of this inventory, which is always around the level agreed on by both parties. According to Disney et al. (Disney and Towill 2001; Disney et al. 2004), the bullwhip effect can be reduced by employing collaborative techniques, which imply the use of new information technologies and electronic data interchange (EDI) among supply chain members. Among these collaborative strategies, it is worth stressing the Electronic Point of Sales (EPOS) and VMI structures. The main characteristic of supply chains in which the EPOS system is employed is that the end consumer's sales information is sent to all the supply chain members. In this way, each member will know the real demand of those products ordered by the end consumer during each period. In any case, different forecasting methods, and making the most of opportunities when purchasing raw materials at low prices, can lead to placing strange orders which distort the information and can bring about the bullwhip effect. The VMI has evolved toward the Collaborative Planning Forecasting and Replenishment (CFPR) (Holmstrom et al. 2002), which includes planning demand.

Both the VMI and CFPR are an advance on the traditional supply chain, which avoid the disappearance of retailers, wholesalers and other distribution centers, and they manage to reduce the bullwhip effect. Forrester (1961) showed that this effect is a result of industrial dynamics, time varying behavior or industrial companies, and proposed a methodology for the simulation of dynamic models, industrial dynamics, which is the origin of system dynamics (Sterman 2000). In general, the main objective of system dynamics is to understand the structural causes that bring about the behavior of a system. Campuzano and Mula (2011) show readers how to simulate a multi-level supply chain by using the system dynamics methodology.

Lee et al. (1997a, b) identify how the sales-related demand distortion due to the Forrester effect is amplified even more because of the following effects, which may even show simultaneously in the supply chain: order sizing, product price fluctuation, rationing and lack of finished products. The combination of these four elements leads to amplification of variance in product demand. This amplification of demand, which increases upstream within the supply chain, is called the bullwhip effect, can be used to measure supply chain management efficiency. Based on the work by Lee et al. (1997b), Carlsson and Fullér (2001) show how the bullwhip effect can be essentially reduced through a fuzzy version based on the possibility theory setting (Dubois and Prade 1988) of a single-item, multi-period inventory model with non stationary demand in which demand forecasts are updated from past demands. In Campuzano et al. (2010), the behavior of fuzzy estimations of demand instead of exponential smoothing for demand forecasts in a two-level, single-item, multi-period supply chain is evaluated.

A system dynamics model with fuzzy estimations of demand was constructed for supply chain simulation. Fuzzy numbers were used to model fuzzy demand estimations. With a numerical example, we indicate how the bullwhip effect and the amplification of the inventory variance can be effectively reduced. Then, in Campuzano-Bolarin et al. (2013), it is extended our model to multi-level supply chains by also using Gaussian and autoregressive demand patterns.

Other studies, which have used fuzzy approaches for improving supply chain ordering or reducing the bullwhip effect can be found in Xiong and Helo (2006), Balan et al. (2007), Zarandi et al. (2008), Lin et al. (2010), Wangphanich et al. (2010), Cannella and Ciancimino (2010), Kristianto et al. (2012) and Cannella et al. (2012).

3 Simulation Model

The dynamic model used herein is based on system dynamics and includes the variables needed to characterize the demand management process (inventory levels, replenishment orders, manufacturing, forecasting, etc.). This model considers capacity constraints, management of backlogged orders, the fill rate, and measurement of the bullwhip effect. It can be used to recreate VMI strategies to measure the impact of these strategies on the demand amplification upstream of the supply chain.

In this model, the manufacturer in a three-level VMI relationship manages the retailer's inventory. The manufacturer receives information on the retailer's sales and inventory levels. Here, the retailer does not place orders with the manufacturer; instead the manufacturer dispatches the adequate amounts of products to ensure that there is enough inventory at the retailer's to avoid stockout periods.

The replenishment policy in this structure used by the manufacturer to meet the retailer's demand is the order-up-to level (*S, s*) inventory control policy (Silver et al. 1988). When using this policy, replenishment orders are carried out for the purpose of taking the inventory position to an *S* level whenever this reaches or is below order point *s*. It has been called so because when an inventory level reaches a previously defined amount, the replenishment or manufacturing order is released. Moreover, two variables are introduced, the maximum and minimum inventory allowed in the retailer's warehouse, to ensure that the retailer delivers an appropriate service to the customer, thus avoiding stockout periods.

The behavior of the model under study is analyzed by a simulation model based on system dynamics methodology principles. The main characteristics of this model are summarized in the following points:

- The retailer and manufacturer ship goods immediately upon receiving the order if there is enough on-hand inventory. We considered a pull planning strategy.
- Orders may be partially fulfilled (each order to be delivered includes current demand and backlogged orders, if any), and unfulfilled orders are backlogged.
- Shipped goods arrive with a transit lead time, and they are also delayed because of the information lead time.
- The manufacturer receives raw materials from an infinite source and manufactures finished goods under capacity constraints. In this work, capacity constraints do not influence the size of manufacturing orders since manufacturing capacity was set high enough to prevent those constraints from having an impact on the proposed analysis.
- The variables used to create the three-level supply chain causal diagram have been selected by taking the APIOBPCS (Automatic Pipeline, Inventory and Order-Based Production Control System) model as a reference (John et al. 1994).

3.1 A VMI Multi-level Supply Chain Model Without Fuzzy Estimations

The variables used for this model are the following:

(a) End consumer demands and demands from one level toward the level situated immediately upstream. Normal distribution is used for creating the end consumer demand signal.
(b) Firm orders (manufacturer and retailer).
(c) Backlogged orders (manufacturer and retailer).
(d) On-hand inventory (manufacturer and retailer).
(e) Demand forecasting (manufacturer and retailer). Forecasts have been made by using exponential smoothing.
(f) Inventory position (manufacturer and retailer).
(g) Orders to manufacture (manufacturer). The ordering policy used is up-to-level (S, s).
(h) Manufacturing lead time (manufacturer).
(i) Lead time (retailer).
(j) A transit lead time between the retailer and end consumer is not considered.
(k) On-order products (manufacturer and retailer).
(l) Manufacturing capacity (manufacturer).
(m) Fill rate (manufacturer and retailer).
(n) Maximum inventory retailer levels.
(o) Minimum inventory retailer levels.

The difference between the solid arrows and dashed arrows in Fig. 1 highlights both those variables which allow to configure VMI orders and replenishment orders from retailer to manufacturer.

This model has been dubbed as a VMI supply chain. Figure 1 shows the causal diagram associated with the formulation of this model.

The formulation of the variable corresponding to the replenishment orders from the manufacturer level is the most outstanding point in this diagram. This replenishment order for the retailer's warehouse is conditioned by a maximum and a minimum level (corresponding to the forecasted safety stock) of the pre-established inventory. Therefore, by knowing the sales that the retailer sends to the end consumer and the forecast (exponential smoothing) of these sales, attempts will be made so that the retailer's warehouse does not go too far below or over the set limits.

3.2 A VMI Supply Chain Model with Fuzzy Estimations

In this section, we propose a fuzzy model of a VMI supply chain in order to face demand uncertainty which could arise from volatile demand or inaccurate forecasts based on historical data. In those cases in which statistical data are unreliable,

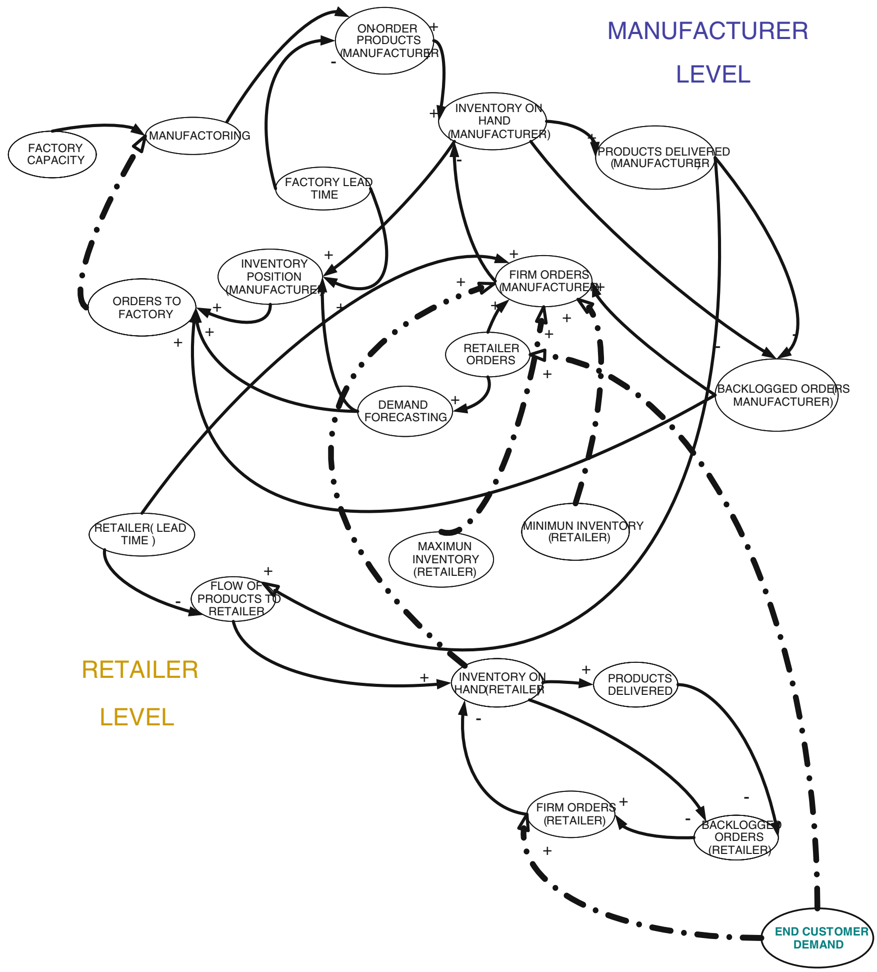

Fig. 1 The causal loop of the VMI supply chain model

or are not even available, models based on the determination of probability distributions might not be the best choice while the possibility theory (Dubois and Prade 1988) could provide an alternative approach for dealing with supply chain demand uncertainties.

The following assumptions are considered: a single-item, multi-level, multi-period VMI supply chain model is considered where demand is non stationary over time; past demands are not used for forecasting, only the forecasting of previous periods; re-supply is infinite with a fixed lead time; excess demand is backlogged; inventory data is considered crisp; the manufacturer uses fuzzy estimations for releasing manufacturing orders and exponential smoothing for supplying the retailer through the up-to-level inventory policy; and the fuzzy estimations for manufacturing orders are modeled by fuzzy numbers. Thus, a VIM model has been created in which, according to the end consumer's demand, fuzzy estimations are generated at the manufacturer level, which are subsequently employed to manufacture products and to replenish its warehouse. In parallel, and depending on the customer's demand forecasting (exponential smooth forecasting) and on the retailer's inventory levels, which must fluctuate in an interval whose minimum and maximum levels have been agreed and set, the manufacturer generates a replenishment order, which is sent to the retailer whenever necessary.

The fuzzy numbers considered are fuzzy trapezoidal numbers defined by $\tilde{A} = (a, b, \alpha, \beta)$ (Fig. 2), where $a - \alpha$ represents the smallest possible value, a and b are the main values, and $b + \beta$ depicts the largest possible value according to Carlsson and Fullér (2000).

The considered membership function for a fuzzy trapezoidal number is:

$$\mu_{\tilde{A}}(t) = \begin{cases} 1 - \frac{a-t}{\alpha} \; if \; a - \alpha \leq t \leq a \\ \quad 1 \;\; if \; a \leq t \leq b \\ 1 - \frac{t-b}{\beta} \; if \; b \leq t \leq b + \beta \\ \quad 0 \; otherwise \end{cases} \tag{1}$$

It is worth highlighting that the model has been generally considered to work with fuzzy trapezoidal numbers. Nonetheless, should $a = b$, then a fuzzy triangular number will be considered (Fig. 3).

The considered membership function for fuzzy triangular numbers is:

$$\mu_{\tilde{A}}(t) = \begin{cases} 1 - \frac{a-t}{\alpha} \; if \; a - \alpha \leq t \leq a \\ 1 - \frac{t-a}{\beta} \; if \; a \leq t \leq \alpha + \beta \\ \quad 0 \; otherwise \end{cases} \tag{2}$$

All the fuzzy numbers in the model must fulfill the following conditions: $\alpha > 0$, $\beta > 0$, $a \leq b$ and $a > \alpha$.

As the demand data and the orders to be generated are fuzzy, but the inventory and backorder data are considered crisp, the fuzzy model uses the mean and standard deviation of a fuzzy number as a defuzzification method.

Dubois and Prade (1987) establish the mean value of a fuzzy number as a closed interval bound by the expectations calculated from its upper and lower distribution functions. They also show that this expectation remains additive in the sum of fuzzy numbers.

Based on the principles introduced into Dubois and Prade (1987) and the possibilistic interpretation of the ordering proposed by Goetschel and Voxman (1986), Carlsson and Fullér (2001) introduce the notations of lower possibilistic and upper possibilistic mean values, and they define the interval-valued possibilistic mean, the crisp possibilistic mean value and the crisp (possibilistic) variance of a continuous possibility distribution, which are consistent with the extension principle and the definitions of expectation and variance in probability theory. The authors prove that the proposed concepts "behave properly" (similarly to their probabilistic counterparts).

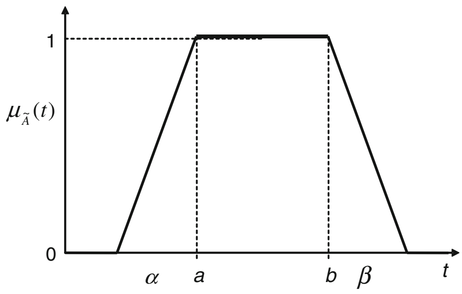

Fig. 2 Fuzzy trapezoidal number

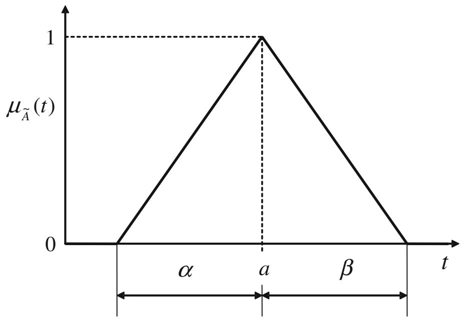

Fig. 3 Fuzzy triangular number

Thus, given a γ-level set of a fuzzy number defined by $[\tilde{A}]^{\gamma} = [a_1(\gamma),\ a_2(\gamma)]$, where $a_1(\gamma)$ denotes the left-hand side and $a_2(\gamma)$ denotes the right-hand side of the γ-cut, Carlsson and Fullér (2000, 2001) use the Goetschel-Voxman defuzzification method to define $E(\tilde{A})$ the mean or expected value of a triangular fuzzy number $\tilde{A} = (a, \alpha, \beta)$ by

$$E(\tilde{A}) = \frac{\int_0^1 \gamma \frac{a_1(\gamma)+a_2(\gamma)}{2} d\gamma}{\int_0^1 \mathsf{F}\gamma} = \int_0^1 \gamma(a_1(\gamma) + a_2(\gamma))d\gamma, \tag{3}$$

i.e., the weight of the arithmetic mean of $a_1(\gamma)$ and $a_2(\gamma)$ is only γ. If $\tilde{A} = (a, \alpha, \beta)$ is a triangular fuzzy number, then

$$E(\tilde{A}) = \int_0^1 \gamma[a - (1-\gamma)\alpha + a + (1-\gamma)\beta]d\gamma = a + \frac{\beta - \alpha}{6} \tag{4}$$

Especially when $\tilde{A} = (a, \alpha)$ is a symmetric triangular fuzzy number, $E(\tilde{A}) = a$. When $\tilde{A} = (a, b, \alpha, \beta)$ is a trapezoidal fuzzy number, then

$$E(\tilde{A}) = \frac{a+b}{2} + \frac{\beta - \alpha}{6} \tag{5}$$

Other properties of fuzzy numbers can be found in Carlsson and Fullér (2001).

The following data and variables are also considered for an n-level supply chain:

Parameters

N	Set of levels ($n = 1, 2,\ldots, N$)
$D_n(0)$	Initial demand during period 0 at n. Fuzzy value ($D_n(0)a$, $D_n(0)b$, $D_n(0)\alpha$, $D_n(0)\beta$)
d_n	Basic or granted demand at n, which is constant to avoid negative demand. Fuzzy value ($d_n a$, $d_n b$, $d_n \alpha$, $d_n \beta$)
$S_n(0)$	Initial inventory during period 0 at n
υ_n	Lead time at n. It must be >=0 and integer
ρ_n	The correlation coefficient of demands at $n \in [-1, 1]$
σ_n^2	The variance to calculate estimation of demand must be significantly less than d at n

Variables

$O_n(t)$	Amount of order during period t at n. Fuzzy value ($O_n(t)a$, $O_n(t)b$, $O_n(t)\alpha$, $O_n(t)\beta$)
$D_n(t)$	Demand during period t at n. Fuzzy value ($D_n(t)a$, $D_n(t)b$, $D_n(t)\alpha$, $D_n(t)\beta$)
$B_n(t)$	Backorders during period t at n
$u_n(t)$	Normally, independently and identically distributed with a zero mean and a variance of $\sigma^2 = 1$ at n

In this case, with a manufacturer, a retailer and an end consumer, the quantity to manufacture, $O_n(t)$, is based on the previous works by Heyman and Sobel (1984), Kahn (1987), Lee et al. (1997b), Carlsson and Fullér (2001), Campuzano et al. (2010) and Campuzano-Bolarin et al. (2013). See, for instance, Appendix A by Campuzano-Bolarin et al. (2013).

Products arrive at the retailer after the lead time, υ_n. The available inventory is used to satisfy demand.

This model has been dubbed as a fuzzy VMI supply chain.

4 Bullwhip Effect Measures

The bullwhip effect refers to the scenario where orders to the supplier tend to display greater fluctuations than sales to the buyer, and this distortion increasingly spirals upstream in a supply chain (Lee et al. 1997a, b).

According to Fransoo and Woters (2000), we measure the bullwhip effect at a particular level in a multi-level supply chain as the quotient between the coefficient of demand variation at the level where the bullwhip effect is measured and the coefficient of demand variation is received at this level.

$$Bullwhip = \frac{C_{out}}{C_{in}} \tag{6}$$

where

$$C_{out} = \frac{\sigma(O_n(t, t+T))}{\mu(O_n(t, t+T))} \tag{7}$$

and

$$C_{in} = \frac{\sigma(D_n(t, t+T))}{\mu(D_n(t, t+T))} \tag{8}$$

The total bullwhip effect along the supply chain, and measured at the manufacturer level, is the coefficient of variation of the production plan, divided by the coefficient of variation of end consumer demand.

$$Total \quad Bullwhip = \frac{C_{out1} \cdot C_{out2} \cdot C_{out3}}{C_{in1} \cdot C_{in2} \cdot C_{in3}} = \frac{C_{out1}}{C_{in3}} \tag{9}$$

where Level 2 is the retailer and Level 1 is the manufacturer.

5 Computational Experiment

The Vensim® program is used as a simulation software for system dynamics. The initial values assigned to the corresponding variables were the following:

- Simulation was carried out over a period of 365 days in order to avoid the transitional state and to stabilize the model.
- The initial inventory level for both the manufacturer and retailer levels was set at 15 units.
- The demand pattern followed a normal distribution with a mean of 12 and a standard deviation of 1.
- Manufacturer capacity was set at 160 units a day.
- The manufacturing lead time was set at 1 day for the manufacturing time and at 1 day for the transit time at the manufacturer level.
- Maximum and minimum inventory levels were established after an optimization process with the Vensim® DSS software optimization module, which ensures lower inventory levels and higher fill rates at the retailer level. These values are [17, 46].
- The fill rate factor, k, which corresponds to the safety factor at the fixed order up-to-level, for each level is analyzed for $k = 2$ in the traditional supply chain model. This factor is fixed to minimize the bullwhip effect by raising the fill rate

Table 1 VMI fuzzy supply chain model parameters

Values for time $t = 0$	Values for $t > 0$
D(0)a: 12 units	*da*: 15 units
D(0): 0 units	*d*: 10 units
D(0)β: 0 units	*dβ*: 5 units
ρ: The correlation constant of demands $\in [-1, 1]$ is set at –0.5	
u(t): Normally, independently and identically normally distributed with a zero mean and a variance of $\sigma^2 = 1$	

but, in contrast, by also raising the inventory holding costs (Dejonckheere et al. 2002).

- The forecast adjustment factor is 2 for smooth forecasting $\alpha = 0.5$.

The VMI fuzzy supply chain model parameters at the manufacturer level are presented in Table 1.

After simulating the two models (the VMI supply chain and the VMI fuzzy supply chain), the bullwhip effect was calculated with (9) for each level. Figure 4 shows the total bullwhip effect at the manufacturer level. The bullwhip effect is seen to be lower in the model in which the manufacturer uses fuzzy estimations for manufacturing orders. This is due mainly to the reduction of the distortion of the manufacturing orders generated as these are directly related to the end consumer demand instead of to the retailer forecast information (characteristic of VMI systems), along with fuzziness, which is inherent to the demand nature and is provided by the manufacturing order function used.

Now we go on to compare the results provided in this chapter with those provided by Campuzano-Bolarin et al. (2013), where the use of fuzzy estimations for demand is analyzed instead of demand forecasts based on exponential smoothing in a three-level, single-item, multi-period traditional supply chain context. Four simulation models were developed: retailer and manufacturer using exponential smoothing for demand forecasting (traditional supply chain); retailer and manufacturer using fuzzy estimations of demand (fuzzy supply chain); retailer using exponential smoothing and manufacturer using fuzzy estimations of demand (fuzzy supply chain scenario 1 LF); retailer using fuzzy estimations of demand and manufacturer using exponential smoothing (fuzzy supply chain scenario 2 FL). The main objective is to compare the bullwhip effect obtained in the VMI structures modeled for this research work with the traditional structures proposed in Campuzano-Bolarin et al. (2013) in their fuzzy and deterministic versions (Fig. 5).

VMI structures provide good (but not the best) results in terms of the bullwhip effect measurement if compared with the different simulated scenarios of traditional supply chains. Logically, these results are susceptible to vary depending on the maximum and minimum inventory levels agreed on but, as mentioned earlier, these values have been optimized to reduce inventory levels with high levels of service. In line with all this, Table 2 provides the accumulated fill rate achieved by

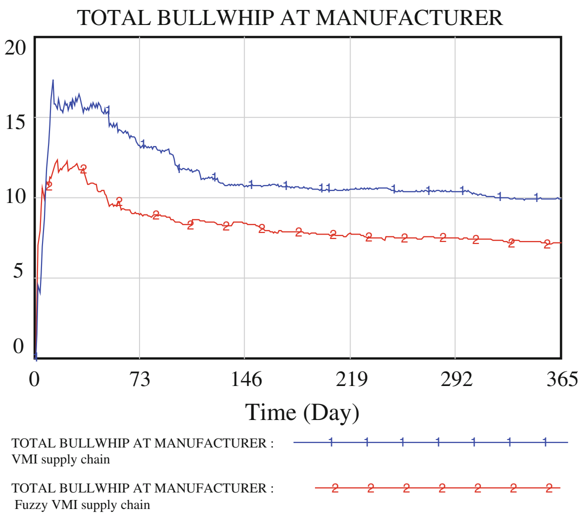

Fig. 4 Total bullwhip effect at the manufacturer level

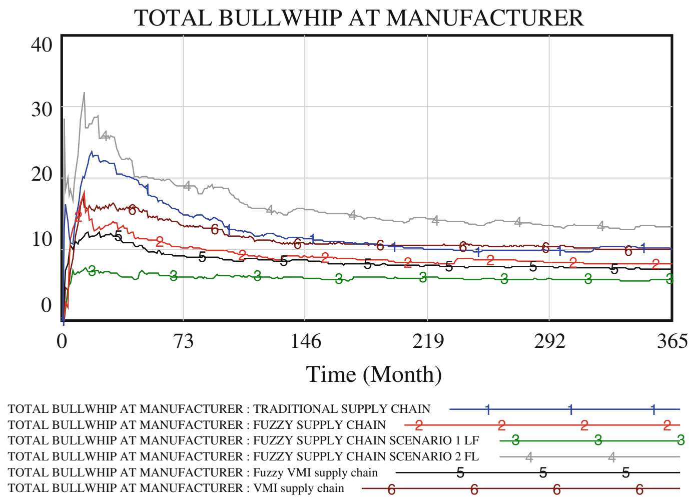

Fig. 5 Total bullwhip effect at the manufacturer level in several scenarios

each model at the retailer level, which is the interesting level to fulfill with higher fill rates and lower inventory levels (Fig. 6) in the VMI context considered.

The VMI scenarios satisfactorily fulfill the demand required by the end consumer, and they reach a level of service for both of almost 100 % with lower inventory levels (Fig. 6). This is because a larger number of orders is generated, which adapts to the maximum and minimum inventory levels in accordance with

Table 2 Accumulated fill rate

	Fill rate at the retailer level (%)
VMI supply chain	99.85
Fuzzy VMI supply chain	99.55
Traditional supply chain model	88.93
Fuzzy supply chain model	92.69
Fuzzy supply chain scenario 1 LF model	87.27
Fuzzy supply chain scenario 2 FL model	92.47

Fig. 6 On-hand inventory at the retailer level

the end consumer's forecasted demand levels. The fuzzy VMI supply chain model further reduced the bullwhip effect with similar fill rates and inventory levels to the VMI supply chain.

6 Conclusions

In this chapter, we have analyzed the use of fuzzy estimations for creating manufacturing orders in a three-level, single-item, multi-period supply chain. We have developed two simulation models based on system dynamics: retailer and manufacturer in a VMI supply chain using exponential smoothing forecasting; and retailer and manufacturer using fuzzy estimations for computing manufacturing orders. The fuzzy models use fuzzy numbers based on the possibility theory to represent demand and orders. Despite the increased complexity of the fuzzy model formulation, the results improve in terms of the bullwhip effect, with similar fill rates and inventory levels to traditional VMI supply chains.

VMI supply chain structures can provide better results than traditional ones in terms of the bullwhip effect, fill rates and inventory levels. As regards fuzzy VMI supply chain models, we conclude that fuzzy estimations for generating manufacturing orders can reduce the bullwhip effect, aligned to Carlsson and Fullér (2000), with high fill rates and low inventory levels.

Future research will address: (i) using fuzzy numbers to represent the minimum and maximum inventory levels for considering fuzzy inventories; and (ii) simulation with other collaborative supply chain strategies; (iii) simulating operational costs as order, inventory holding and backorder costs; and (iii) testing in a real world application.

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Fuzzy Methods for Demand Forecasting in Supply Chain Management

Başar Öztayşi and Eda Bolturk

Abstract Forecasting the future demand is crucial for supply chain planning. In this chapter, the fuzzy methods that can be used to forecast future by historical demand information are explained. The examined methods include fuzzy time series, fuzzy regression, adaptive network-based fuzzy inference system and fuzzy rule based systems. The literature review is given and the methods are introduced for the mentioned methods. Also two numerical applications using fuzzy time series are presented. In one of the examples, future enrollments of a university is forecasted using Hwang, Chen and Lee's study and in the other example a company's oil consumption is predicted using Singh's algorithm. Finally, the forecasting accuracy of the methods is determined by using Mean Absolute Error (MAE).

Keywords Fuzzy forecasting · Fuzzy time series · Fuzzy regression · Fuzzy rule based systems · Adaptive network-based fuzzy inference system

1 Introduction

Forecasting is defined as the process of predicting future events which can contain various areas such as product demand, tourism demand, climate change, health and political forecasts (Sanders 2012). Forecasting is one of the most important business activities because it drives all other actions. Decisions such as which markets to pursue, which products to produce, how much inventory to carry, and

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how many people to hire are all based upon a forecast. Planning and forecasting are two closely related actions. Planning can be defined as the process of selecting actions in anticipation of the forecast. In other words, while forecast drives the plan, plan is made in response to the forecasts. As a result, poor forecasts result in poor plans which can put an organization in an unwanted and unprepared situation. The results of poor forecasting can be in terms of loss of sales or excess inventory that cannot be sold.

Demand forecasting is the basis of all supply chain planning processes. In a push type of supply chain, the flow of products in the supply chain are performed in anticipation of customer demand, on the other all pull processes are performed in response to customer demand. For push processes, the managers in the supply chain must plan the level of activities such as production and transportation. In contrast for pull processes, the level of available capacity and inventory level should be planned. As a result, in both cases the managers should make a forecast about the future customer demands.

Although forecasting is such an important action, forecasts are rarely perfect so the forecasting studies should include both the expected value of the forecast and a measure of forecast error or demand uncertainty. The researches on forecasting activities show that aggregated forecasts are usually more accurate than individual item forecasts and short term forecasts result more accurate results when compared to long term forecasts.

Many problems in real world deal with uncertain and imprecise data so conventional approaches cannot be effective to find the best solution. In order to handle this uncertainty, the fuzzy set theory has been developed (Zadeh 1965) as an effective mathematical tool. Although humans have relatively efficient in qualitative forecasting issues, they are cannot show the same performance in making quantitative predictions (Kahraman et al. 2010). Since fuzzy linguistic models permit the translation of verbal expressions into numerical ones fuzzy logic can empower the decision making process. Especially when the decisions involve human subjectivity, fuzzy algebra provides a mathematical framework for handling the imprecision and vagueness.

The fuzzy set theory has some advantages in forecasting. Mamlook et al. (2009) state that fuzzy methods use fuzzy sets which enable the modelers to condense large amount of data into smaller set of variable rule. Another important advantage of fuzzy logic is valid for rule based systems especially, these systems are based on heuristics and therefore they are able to incorporate human intuition and experience into the forecasting process (Cirstea et al. 2002). Kahraman et al. (2010) identify one of the advantages of fuzzy time series approximations as the ability to work with a very small set of data and no requirements for the linearity assumption. Fuzzy sets offer a clear insight into the forecasting model and can be used for non-linear systems.

In this chapter an introduction to fuzzy forecasting techniques are given and different fuzzy times series methods are compared in a demand forecasting case. The rest of the study is as follows: The importance of forecasting in supply chain management is issued in Sect. 2. The possible fuzzy forecasting tools including, fuzzy time series, fuzzy regression, fuzzy rule based systems and adaptive network-based fuzzy inference system (ANFIS) are introduced in Sect. 3. The literature review of fuzzy demand forecasting techniques are given in Sect. 4. The numerical applications and comparison of the fuzzy time series methods are provided in Sect. 5. Finally Sect. 6 presents the conclusions and future research directions.

2 Forecasting in Supply Chain

Forecasting is one of the most important activities in a company because plans at different levels of the organization are made based on forecasting. Marketing department uses forecasting for size of markets, new competition future trends, emerging markets and customer demands. Finance department uses forecasting to assess financial performance and capital investment needs to set budgets. Operations department makes decisions regarding production and inventory levels based on demand forecast. Sourcing activities uses forecasts to make purchasing decisions and select suppliers. Proper planning for the future starts with a forecast (Sanders 2012).

However demand forecasting is especially critical for the entire supply chain since it affects all the plans made by each company in the chain. Forecasts that are done independently without communication between by each company in the supply chain tend to be inaccurate since each company uses the immediate buyer's data to produce the forecast instead of the final customer. The absence of communication while making the demand forecasts leads to the bullwhip effect which can be defined as the increased volatility in orders as they spread through the supply chain (Lee et al. 2004). Bullwhip affects all parties in the supply chain, inventory levels increase, working capital efficiency decrease, and production capacity is used inefficiently. In order to overcome this problems collaborative planning, forecasting and replenishment (CPRF) approach is used by supply chain members. CPRF enables companies to work together to develop forecasts and plans to optimize the supply chain by generating a consensus demand forecast (Wisner et al. 2011).

Customer demand may be affected by various factors thus in order to forecast demand, companies should first identify these factors and then ascertain the relationship between these factors and future demand (Chopra and Meindl 2012). The set of factors contains both objective factors, such as past demand, state of the economy, planned advertising; or subjective factors which include human judgments. Although the most of the forecasting methods depend on the objective data, human input is also important when they make the final forecast.

Identification of the factors is also important to choose a suitable forecasting methodology. The classical forecasting methods that can be used for demand forecasting can be classified to four groups (Chopra and Meindl 2012). (1) Qualitative methods which highly depend on human judgment and most appropriate when little historical data is available. (2) Time series method which supposes that past data is a good indicator of the future demand and uses historical demand to make a forecast. (3) Casual methods use the correlation between factors and the demand to forecast the future demand. (4) Simulation forecasting method imitates the consumer choices and other environmental issues that give rise to demand in order to forecast the demand. Since the scope of this study is fuzzy methods, the classical crisp methods are not investigated in detail.

In demand forecasting studies, just like any other forecasts, there are some steps that should be followed to ensure the credibility of the results (Sanders 2012). The first step is identifying what forecasts are needed to help us to plan the future. The second step involves analyzing available data and identifying the patterns. Identifying the patterns is critical for selecting the forecasting model. The most common data patters can be listed as; level, trend, seasonality and cycles. Level is the simplest pattern the demand data fluctuate around a constant mean. Trend is present when data exhibit an increasing or decreasing pattern over time. Seasonality is any pattern that regularly repeats itself and cycles are patterns created by economic fluctuations. As the data patterns are identified the next step is to select an appropriate forecasting model. As the model is selected the forecast is generated. At the final step, the forecasts are evaluated with the actual values in order to evaluate the performance of the forecasting method.

3 Fuzzy Forecasting Methods

3.1 Fuzzy Times Series

A time series is composed of observations x_t, each one being recorded at a specific time t. Time-series models are based on a series of discrete and equal time increments. Time series models assumes that, the predictions for the next unit time interval such as, week, month, quarter, year, are based on, and only on, the past values of the last N periods of the same time interval, of the variable we wish to forecast (Kahraman et al. 2010).

While there are various crisp times series approach such as simple exponential smoothing, trend-corrected exponential smoothing, trend and seasonality corrected exponential smoothing, after introduction of fuzzy sets by Zadeh (1965), Song and Chissom (1993a) presented the definition of fuzzy time series and outlined its model by means of fuzzy relation equations. The authors applied the model for forecasting under fuzzy environment in which historical data are of linguistic values.

The fuzzy time series are defined as follows. Let $Y(t)(t = \ldots, 0, 1, 2, 3, \ldots)$ is a subset of R1, be the universe of discourse on which fuzzy sets $f_i(t)(i = 1, 2, 3, \ldots)$ are defined and let $F(t)$ be a collection of f1(t), f2 (t),… Then, F(t) is called a fuzzy time series defined on Y(t) (t $= \ldots, 0, 1, 2, \ldots$).

Suppose $F(t)$ is caused only by $F(t-1)$ and is denoted by $F(t-1) \rightarrow F(t)$; then there is a fuzzy relationship between $F(t)$ and $F(t-1)$ and can be expressed as the relational equation where "$\circ$" is the composition operator. The relation R is called the fuzzy relation between $F(t)$ and $F(t-1)$. And the model is called the first order model of $F(t)$:

$$F(t) = F(t-1) \circ R(t, t-1) \tag{1}$$

If for any time t, R(t, t−1) is independent of t, i.e., for any time t, R(t, t−1) = R(t, t−2), then F(t) is called a time-invariant fuzzy time series. Otherwise, it is called a time-variant fuzzy time series (Song and Chissom 1993a). Let F(t) be a fuzzy time series. If F(t) is caused by F(t − 1), F(t − 2),…, and F(t − n), then this fuzzy relationship (FLR) is represented by:

$$F(t-n), \ldots, F(t-2), F(t-1) \rightarrow F(t) \tag{2}$$

and it is called the nth order fuzzy time series forecasting model.

The traditional time series approaches require having the linearity assumption and at least 50 observations. In fuzzy time series approaches, there is not only a limitation for the number of observations but also there is no need for the linearity assumption (Kahraman et al. 2010).

Most of the existing fuzzy time series forecasting methods use the following four steps to handle forecasting problems (Chen 1996):

- Step 1: Partitioning the universe of discourse into specific intervals.
- Step 2: Fuzzifying the historical data.
- Step 3: Building the fuzzy relationships and obtaining fuzzy relationship groups.
- Step 4: Calculating the forecasted outputs.

3.2 Fuzzy Regression

Regression analysis is a statistical technique that tries to explore and model the relationship between two or more variables. Classical statistical linear regression takes the form

$$y(x) = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_k x_{ik} + \varepsilon_i, i = 1, 2, \ldots, m \tag{3}$$

where y_i is the dependent variable, x_{ij} are the independent variables and $\beta_j$is the coefficients and ε_i is the random error term. All the values in the equation is crisp in the classical regression analysis. Although the classical analysis is widely used, some problems are reported in special cases such as inadequate number of observations, difficulties in verifying the distribution assumptions (Shapiro 2004).

There are various studies on Fuzzy regression (Georg 1994; Sakawa and Hitoshi 1992; Tanaka et al. 1989; Wang and Tsaur 2000). In this chapter we focus on Buckley's (2004) study, fuzzy prediction in linear regression technique which is based on confidence intervals. Buckley's crisp simple linear regression model is as follows:

$$\tilde{y}(x) = a + b(x_i - \bar{x}) + \varepsilon_i \tag{4}$$

where $\bar{x}$ is the mean value of the x_i. Initially crisp $(1-\beta)100\,\%$ confidence intervals of a, b and σ^2 are calculated. To this end the crisp estimators of the coefficients ($\hat{a}$, $\hat{b}$) should be determined. The values of the estimators are $\hat{a} = \bar{y},\ \hat{b} = \frac{B1}{B2}$ where

$$B1 = \sum\nolimits_{i=1}^{n} y_i(x_i - \bar{x}) \tag{5}$$

$$B2 = \sum\nolimits_{i=1}^{n} (x_i - \bar{x})^2 \tag{6}$$

and

$$\sigma^2 = \left(\frac{1}{n}\right) \sum\nolimits_{i=1}^{n} \left[y_i - \hat{a} - \hat{b}(x_i - \bar{x})\right]^2 \tag{7}$$

A $(1-\beta)100\,\%$ confidence interval for a and b is as follows:

$$\left[\hat{a} - t_{\beta/2}\sqrt{\frac{\hat{\sigma}^2}{(n-2)}}, \hat{a} + t_{\beta/2}\sqrt{\frac{\hat{\sigma}^2}{(n-2)}}\right] \tag{8}$$

$$\left[\hat{b} - t_{\beta/2}\sqrt{\frac{n\hat{\sigma}^2}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}, \hat{b} + t_{\beta/2}\sqrt{\frac{n\hat{\sigma}^2}{(n-2)\sum_{i=1}^{n}(x_i - \bar{x})^2}}\right] \tag{9}$$

If β is taken into account as an α-cut level, the fuzzy triangular membership function for a and b can be obtained from Eqs. (8) and (9)

The fuzzy regression equation is as follows;

$$\tilde{y}(x) = \tilde{a} + \tilde{b}(x - \bar{x}) \tag{10}$$

In the equation, $\tilde{y}(x)$, $\tilde{a}$ and $\tilde{b}$ are fuzzy numbers and x and $\bar{x}$ are real numbers. In order to predict new fuzzy values for $\tilde{y}(x)$, new values for x can be chosen.

Let $\tilde{y}(\alpha) = \left[y(x)_1(\alpha), y(x)_2(\alpha)\right]$, $\tilde{a}(\alpha) = [a_1(\alpha), a_2(\alpha)]$, and $\tilde{b}(\alpha) = [b_1(\alpha), b_2(\alpha)]$. Based on the interval arithmetic and (α)-cut operations ỹ(α) is calculated as follows:

The (α)-cuts of $\tilde{a}$ and $\tilde{b}$ are determined using Eqs. (8), and (9) respectively.

$$\tilde{Y}[x](\alpha) = \begin{cases} y(x)_1(\alpha) = a_1(\alpha) + (x - \tilde{x})b_1(\alpha) & \\ y(x)_2(\alpha) = a_2(\alpha) + (x - \tilde{x})b_2(\alpha) & \text{if } (x - x) > 0 \\ y(x)_1(\alpha) = a_1(\alpha) + (x - \tilde{x})b_1(\alpha) & \\ y(x)_2(\alpha) = a_2(\alpha) + (x - \tilde{x})b_2(\alpha) & \text{if } (x - x) < 0 \end{cases} \tag{11}$$

3.3 Fuzzy Rule Based Systems

Fuzzy rule based systems (FRBS) is a computing framework based on concepts of fuzzy set theory, fuzzy if–then rules, and fuzzy reasoning. The term is also known as "Fuzzy inference systems", "fuzzy expert systems" and "fuzzy model" in various resources (Jang et al. 1997). The basic structure of a FRBS consists of three conceptual components: a rule base, a database and a reasoning machine. The rule base contains the fuzzy rules used in the system, database defines the membership functions used in the fuzzy rules and the reasoning mechanism performs the inference procedure based on the rules and the given facts. Block diagram of a fuzzy rule based system is given in Fig. 1.

Fuzzy if–then rules are expressions of the form IF a THEN B, where A and B are labels of fuzzy sets characterized by appropriate membership functions. An example can be given as:

If pressure is high then volume is small.

Where pressure and volume are linguistic variables, high and small are linguistic values that are characterized by membership functions (Jang 1993).

Fuzzy inference process comprises of five parts: fuzzification of the input variables, application of the fuzzy operator (AND or OR) in the antecedent, implication from the antecedent to the consequent, aggregation of the consequents across the rules, and defuzzification.

A typical FIS can be described in four steps which are; fuzzification, fuzzy rules, fuzzy inference and defuzzification (Öztayşi et al. 2013).

Step 1: (Fuzzification) Fuzzification process involves the definition of the membership functions of input/output variables by linguistic variables.

Step 2: (Fuzzy rules) A FRBS with i-input variables has r = pi rules, where p is the number of linguistic terms per input variable. As the dimension and complexity of a system increase, the size of the rule base increases exponentially.

A sample rule can be defined as follows:

$$\text{IF } I_1 \text{ is } \widetilde{\text{A}}_1^j \text{ AND } I_2 \text{ is } \widetilde{\text{A}}_2^j \text{ AND}\ldots I_n \text{ is } \widetilde{\text{A}}_n^j \text{ THEN } y \text{ is } \widetilde{\text{B}}^j \text{ for } j = 1, 2, \ldots, r \quad (12)$$

where $I_i (i = 1, 2, \ldots, n)$ are input variables and y is the output variable, $\tilde{\text{A}}_1^j, \tilde{\text{A}}_2^j, \ldots, \tilde{\text{A}}_n^j$ and $\tilde{B}^j$ are the linguistic terms used for the membership function of the corresponding input and output variables for the jth rule, respectively.

Step 3: (Fuzzy inference) Fuzzy inference is an inference procedure to derive a conclusion based on a set of if–then rules. In the literature different fuzzy inference models are proposed such as Mamdani's model, Sugeno's model and Tsukamoto Fuzzy Model (Mamdani and Assilian 1975; Sugeno and Kang 1988; Takagi and Sugeno 1985; Tsukamoto 1979). The Mamdani inference method is manually constructed on the basis of expert knowledge and the final model is neither trained nor optimized. The method considers fuzzy inputs and returns fuzzy outputs (Mamdani and Assilian 1975). Since Mamdani approach is not exclusively

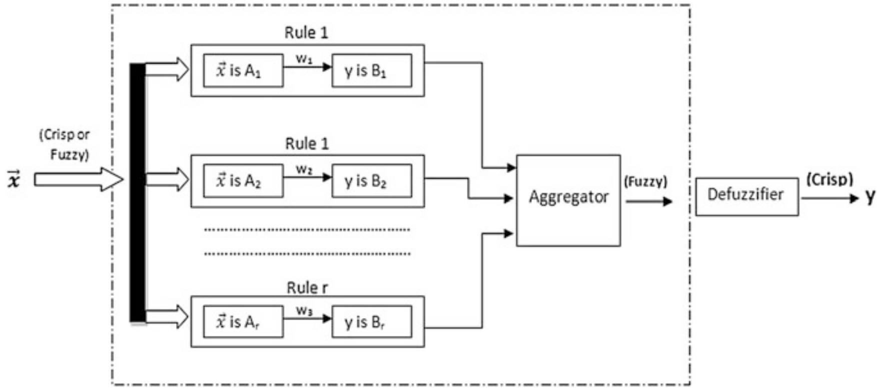

Fig. 1 Block diagram of a fuzzy rule based system (Jang et al. 1997)

dependent on a data set, with sufficient expertise on the system involved, a generalized model for effective future predictions can be obtained (Keshwani et al. 2008). The mechanism of Mamdani inference method is as follows: (1) If there is more than one input in the rule, fuzzy set operations should be applied to achieve a single membership value; (2) then implication method (min) is applied to reach each rule's conclusion; (3) the outputs obtained for each rule are combined into a single fuzzy set, using a fuzzy aggregation operator (max).

For the case where input variables $I_i(i = 1, 2, \ldots, n)$ are crisp variables and the fuzzy rules are described by Eq. (12) and Eq. (13), so for a set of disjunctive rules, where $j = 1, 2, \ldots, r.$, the output using Mamdani inference method is formulated as follows (Ross 1995);

$$\mu^j_{\tilde{B}}(y) = \max_j \left[\min\left[\mu^j_{\tilde{A}_1}(I_1), \mu^j_{\tilde{A}_2}(I_2), \ldots, \mu^j_{\tilde{A}_n}(I_n)\right]\right] \tag{13}$$

Step 4: (Defuzzification) The output of the fuzzy inference is a fuzzy number and can be converted into a crisp value by defuzzification. There are various defuzzification methods such as, max membership, centroid method, weighted average method, mean-max membership. Centroid method, which is also called center of area or center of gravity method, is the most prevalent and physically appealing of other defuzzification methods (Ross 1995). It is given by the algebraic expression as follows;

$$c^* = \frac{\int \mu_{\tilde{C}} \cdot c \cdot dc}{\int \mu_{\tilde{C}} dc}, c \in \tilde{C} \tag{14}$$

where $\tilde{C}$ is a fuzzy set having the membership function $\mu_{\tilde{C}}$.

The graphical illustration of the introduced fuzzy rule based system is represented in Fig. 2.

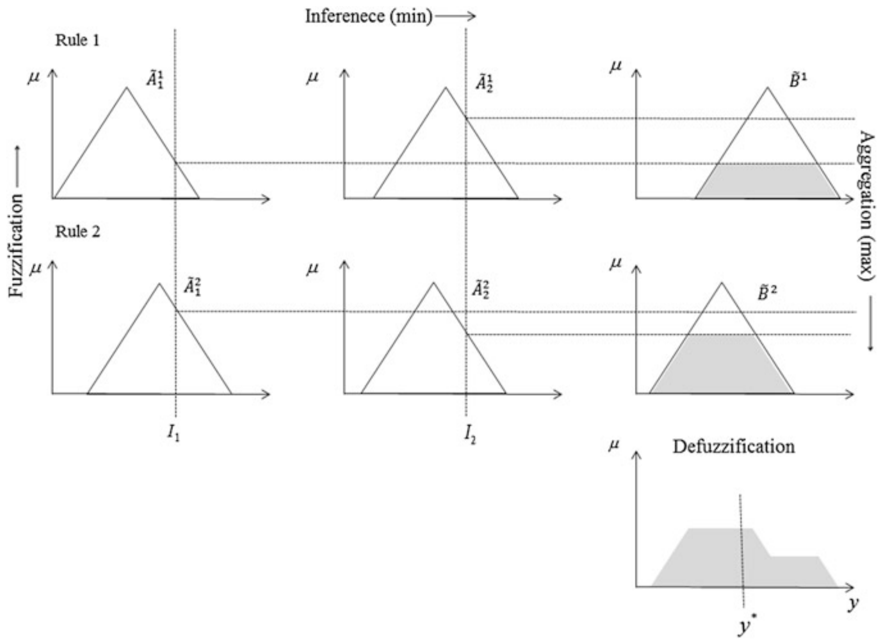

Fig. 2 Graphical Mamdani (max–min) inference method (Öztayşi et al. 2013)

3.4 Adaptive-Network-Based Fuzzy Inference System

Adaptive network-based fuzzy inference system (ANFIS) is a fuzzy inference system implemented in the framework of adaptive networks. By using a hybrid learning procedure, ANFIS can construct an input–output mapping using both human knowledge and predetermined input–output data set (Jang 1993). ANFIS is a fuzzy inference system based on the Sugeno model. It incorporates the self-learning ability of ANN with the linguistic expression function of fuzzy inference (Yun et al. 2008). Using a given input/output data set, ANFIS constructs a fuzzy inference system whose membership function parameters are adjusted using various algorithms. This adjustment allows the fuzzy systems to learn from the data (Matlab 2012).

The model of the ANFIS changes according to the number of input, output and rules employed. For the simplicity, the fuzzy inference system under consideration is assumed to have two inputs (x and y) and one output (z). For a first order Sugeno fuzzy model, a common rule set with two fuzzy if–then rules are as follows:

$$\text{Rule 1}: \text{ If x is } A_1 \text{ and y is } B_1 \text{ then } f_1 = p_1x + q_1y + r_1 \tag{15}$$

$$\text{Rule 1}: \text{ If x is } A_2 \text{ and y is } B_2 \text{ then } f_2 = p_2x + q_2y + r_2. \tag{16}$$

where A_i and B_i are the fuzzy sets, f_i is the output set within the fuzzy region specified by the fuzzy rule p_i and q_i and r_i are the design parameters that are determined during

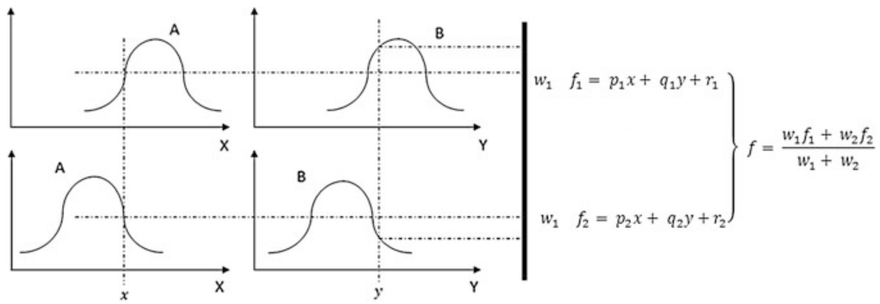

Fig. 3 The reasoning mechanism for the given Sugeno model (1988)

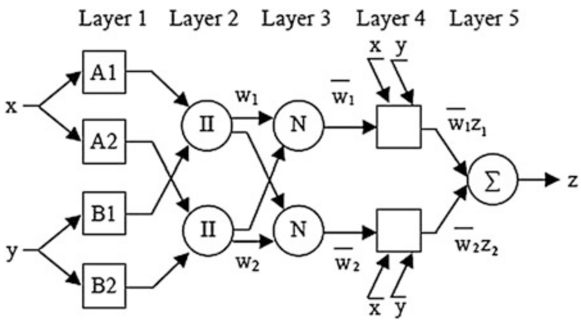

Fig. 4 ANFIS architecture (Jang 1993)

the training process. Figure 3 illustrates the reasoning mechanism for the given Sugeno model, and Fig. 4 represents the corresponding equivalent ANFIS architecture.

ANFIS is composed of five layer feed forward neural network. The node functions in the same layer are of the same function family as described below (Jang 1993):

Layer 1: Every node I in this layer is an adaptive node with a node function:

$$O_{1,i} = \mu_{A_i}(x), \text{for } i = 1, 2 \text{ or} \tag{17}$$

$$O_{1,i} = \mu_{B_{i-2}}(y), \text{for } i = 3, 4, \tag{18}$$

where x or y are the input to node I and A_i or B_{i-2} is a linguistic label; such as "small" or "large"; associated with this node. O_1, I refers to the membership degree of a fuzzy set A and it specifies the degree to which the given input x or y satisfies the quantifier A. The membership function A can be any appropriate parameterized membership function such as the generalized bell function:

$$\mu_A(x) = \frac{1}{1 + \left|\frac{x - c_i}{a_i}\right|^{2b}}, \tag{19}$$

where a_i, b_i, c_i are the parameters. The parameters in this layer are call premise parameters.

Layer 2: Every node in this layer is a fixed node whose output is the product of all the incoming signals:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), i = 1, 2 \tag{20}$$

Each node output represents the firing strength of a rule.

Layer 3: Every node in this layer is a fixed node labeled N. The ith node calculates the ratio of the ith rule's firing strength to the sum of all rules' firing strengths:

$$O_{3,i} = \bar{w}_i = \frac{w_i}{w_1 + w_2}, i = 1, 2 \tag{21}$$

The outputs of this layer are called normalized firing strengths.

Layer 4: Every node I in this layer is an adaptive node with a node function as follows:

$$O_{4,i} = \bar{w}_i f_i = \bar{w}_i(p_i x + q_i y + r_i) \tag{22}$$

where $\bar{w}_i$ is a normalized firing strength from layer 3 and pi, qi ri are the parameter set for this node. Parameters in this layer are referred to as consequent parameters.

Layer 5: The signal node in this layer is a fixed node labeled Σ, which computes the overall output as the summation of all incoming signals.

$$overall\ output = O_{5,1} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \tag{23}$$

ANFIS learns the premise and consequent parameters for the membership functions and the rules. Jang et al. (1997) propose the hybrid learning algorithm which uses a combination of Steepest Descent and Least Squares Estimation (LSE). In this approach ANFIS uses a two pass learning algorithm: In the foreword pass the premise (nonlinear) parameters are unmodified and consequent (linear) parameters are computed using a LSE algorithm. In the backward pass, the consequent (linear) parameters are unmodified and premise (nonlinear) parameters are computed using a gradient descent algorithm such as back propagation.

4 Literature Review

The literature provides various studies that employ fuzzy techniques for demand forecasting. These studies can be classified into four groups which are introduced in Sect. 3.

4.1 Fuzzy Time Series

The most widely used fuzzy forecasting technique is the fuzzy time series (FTS) forecasting. Time series approach assumes that the predictions for the next period are based on the past values of the last periods. The fuzzy extensions of time series are initially proposed by Song and Chissom (1993a, b). Chen (1996) studies on how the forecasting model can be improved with lower error levels with a basic model and presents a new method to forecast university enrollments. The robustness of the proposed method is tested and it is shown that the technique can make robust forecasts when the historical data are not accurate. Wong et al. (2009) compare multivariate Fuzzy Time Series models with Traditional Time Series models for the forecasting accuracy. In this chapter, it is stated that when the data with longer time trend the traditional time series model has good pattern fitting. Also, when the period of data is short or indefinite, fuzzy time series model relatively exceeds the time series pattern.

In FTS, partitioning the universe of discourse into specific intervals is the first step of the studies. Huargn (2001) focuses on the effective length of intervals, in order to generate more accurate forecasting. In another study in this area, Li and Chen (2004) dwell on partitioning the intervals in FTS and propose a novel approach that can partition the universe of discourse step by step. Huarng and Yu (2006) work on exploring ways of determining the useful lengths of intervals between the ranges. The results of the study show that that the ratio-based lengths of intervals can improve the FTS forecasting. Jilani and Burney (2008) propose a method that uses heuristic approach to define frequency-density-based partitions of the universe of discourse. Davari et al. (2009) use a modified version of particle swarm optimization for the definition of suitable partitions of FTS forecasting. They propose a method that improves the forecasting accuracy for tuning the length of forecasting intervals. Lin (2009) also studies on intervals of fuzzy time series in order to increase the forecasting accuracy. The universe of discourse is partitioned into subintervals are employed to fuzzify the time series into fuzzy time series and the midpoints of two adjacent cluster centers generated. Chen et al. (2012) propose a new model which incorporates the concept of the equal frequency partitioning and fast fourier transform algorithm. The source is actual trading data from TAIEX. The model is compared with Chen (1996), Yu (2005), and Chang et al. (2011) and the proposed model and it presents better results.

Another approach to improve forecasting accuracy is to integrate other techniques with FTS. Yu (2005) use FTS for forecasting recurrence and weighting of fuzzy logical groups. In the proposed model, different weights are given to various fuzzy relationships and the model is compared with local regression models. Fuzzy relations in fuzzy time series are analyzed by Tsaur et al. (2005). This study proposes an analytical approach and its aim is finding the steady state of fuzzy relation matrix to revise the logic forecasting process. Pai (2006) proposes a new FTS called hybrid elipsoidal fuzzy system for time series forecasting (HEFST) and apply it electricity data. The results of the comparison among HEFST, ANN and regression models show that the proposed model gives the best results. Huarng et al. (2007) propose a heuristic function integrated FTS model which can handle multiple variables to improve forecasting results and avoid complicated computations due to the inclusion of multiple variables. Cheng et al. (2007) propose a model that improves FTS with fuzzy logic relation which is identified using rough set theory. The model implements different linguistic values in order to determine the most accurate linguistic value in order to increases the forecasting accuracy. Cheng et al. (2008) propose using fuzzy clustering integrated with fuzzy time series to improve the accuracy level. The forecasting results show that the proposed method can multiple-attribute data effectively and outperform former methods. Liu (2009) studies in short-time load forecasting. The proposed forecasting method adjusts an analysis slide window of FTS to train the trend predictor in the training phase. Later the trend predictor is used to generate forecasting values. Tsaur and Kuo (2011) propose an Adaptive FTS model for forecasting Taiwan's tourism demand. In the study, FTS data is transferred to the fuzzy logic group and the weights are assigned to periods. Chen and Chen (2011) proposed a new method that is based on FTS and fuzzy variation groups. Daily Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) is issued for forecasting. They proposed a method that uses both fuzzy variation groups, where the main input factor is the previous day's TAIEX, and the secondary factor is either the Dow Jones, the NASDAQ, or their combination, and fuzzy logical relationship groups data for forecasting the TAIEX.

4.2 Fuzzy Regression

Regression analysis is one of the widely used approach for relationship identification and forecasting for both univariate and multivariate cases. Similar to this, fuzzy regression is also used to define fuzzy relationships and fuzzy forecasting. Heshmaty and Kandel (1985) use fuzzy regression models in sales forecasting under uncertainty. In their chapter, two different sales forecasting techniques are issued. The first technique consists of non-fuzzy abstract methods of linear regression and econometrics. The second sales forecasting technique uses fuzzy linear regression. Fuzzy linear regression is used to forecast in uncertain environments. Feng and Guang (1993) propose a forecasting model of fuzzy self-regression. In the model, the awaiting estimated parameters and the dependent variables are fuzzy numbers of M–N form. Liang and Cheng (2000) propose an integrated approach that consists of multilinear regression and fuzzy inference system has been presented for short-term load forecasting. The multilinear regression model is applied to find a preliminary load forecast and the fuzzy inference system is used for load correction from historical information.

Song et al. (2005) handle fuzzy regression analysis concept that is issued in the short-term forecasting to reduce the load forecasting error. The fuzzy linear regression model is made from the load data of the previous 3 years and the coefficients of the model are established as a result of the model. Khashei et al. (2008) propose a model that consists of the artificial neural networks (ANNs) and fuzzy regression, for forecasting in financial markets. By using the fuzzy regression models, the limitation of big amount of historical data is lifted. In the same year, Chen and Dang (2008) propose a three-phase method to construct the fuzzy regression model with variable spreads. In the first phase, the membership functions of the least-squares estimates of regression coefficients are constructed. In the second phase, the coefficients are defuzzified to obtain crisp values. In the last phase, the error terms of the model are determined. Al-Hamadi (2011) shows a long-term electric load forecasting technique that is based on fuzzy linear regression. This technique uses long term annual growth factors in order to forecast the model's parameters. In this chapter, the objective of the linear optimization problem is set as to minimize the spread of fuzzy regression parameters. Kazemi et al. (2012) develop an energy demand prediction model for Iran using socio-economic indicators. The approach is structured as a multi-level model fuzzy linear regression and used for predicting the industry energy demand from 2011 to 2020.

4.3 Fuzzy Rule-Based Forecasting

Fuzzy rule based systems (FRBS) are composed of if–then rules and use these rules to make inference and decisions. FRBSs are also used in forecasting area. Liu (2006) study, fuzzy rule-based classifier for electrical load pattern classification is established. Multi-objective genetic algorithms are applied to prefer a pattern classification system. Cardoso and Gomide (2007) study on newspapers demand for customer's need using fuzzy clustering and fuzzy rules. The method produces more accurate results when compared with neural network-based predictors, and autoregressive forecasters. Chang et al. (2007) propose a model which integrates the wavelet and TakagiSugeno-Kong (TSK) FRBS for financial time series data prediction. The wavelet in the model is used to decrease the noises in the data. The proposed method is used to forecast the future stock. Dimitriou et al. (2008) suggest an adaptive hybrid fuzzy rule-based system for forecasting traffic flow. Univariate and multivariate data structures are used in the model and online and offline fuzzy rule-based system is considered. In Chang et al. (2008) study, a case based clustering TSK fuzzy rule system for stock price predictions in Taiwan Stock Exchange Corporation is presented. The model is integrated by a case based reasoning technique, a TSK Fuzzy Rule based system, and Simulated Annealing (SA). Chen and Chang (2010) propose a method for multi-variable fuzzy forecasting. The model composed of fuzzy clustering and fuzzy rule interpolation techniques. Fuzzy rules are created by training samples and the fuzzy rule corresponds to a given cluster. Pratondo (2010) proposes a FRBS based on uncertain environment conditions to enhance demand forecasting. In Zhang and Liu's (2010) study, a new method is presented for mid-long term load forecasting using fuzzy rules and genetic algorithms. The genetic algorithms are based on Takagi–Sugeno Fuzzy Logic System. The system is proposed for electricity forecasting with its computation speed. Cheikhrouhou et al. (2011) propose using knowledge from forecasters combined with mathematical forecasts. In the proposed model, the mathematical forecasts are adjusted by the knowledge from different forecasters. In Ivette and Rosangela's (2011) study, data-driven approach applied to the long term prediction of daily time series is presented. Daily samples are aggregated to build weekly time series. The results are validated using multiple time series. Moreover, the results are compared with obtained using daily models. Yanfei and Yinbo (2011) focus on short term load forecasting with a model that consists of ANN and FRBS. The first part is the basic load component and the second part is the temperature and the holiday load component. Initially the ANN processes and then fuzzy rules are completed. The results of the study show that using ANN process while applying FRBS improves the model's sensitivity.

4.4 Adaptive Neuro Fuzzy Inference System

Adaptive Neuro Fuzzy Inference System (ANFIS) is a kind of neural network that is based on fuzzy inference system. ANFIS's inference system corresponds to a set of fuzzy IF–THEN rules that have learning capability to approximate nonlinear functions. Padmakumai et al. (1999) propose a hybrid fuzzy neural technique which combines neural network and fuzzy logic modeling, and present an application for long term land use based distribution load forecasting. ANFIS is used for forecasting in different areas. Atsalakis and Valavanis (2009) develop a neuro-fuzzy adaptive control system in order to forecast next day's stock price trends. For fuzzifying the system inputs, Gaussian-2 shaped membership functions are used. In Efendigil's (2009) study, ANFIS techniques and artificial neural networks is compared. A new model for forecasting the uncertain customer demand under fuzziness is proposed for better accuracy of model. In Moreno's (2009) study, ANFIS is used for monthly ideal generation of a hydraulic plant considering different factors like weather conditions ant the plant's reservoir level. In Chabaa's et al. (2009) study, a set of input and output data of internet traffic time series is forecasted.

In Azadeh's et al. (2010) study on short-term natural gas prediction using ANFIS. The obtained results are compared with ANN and proposed model out performs ANN. In Chen's et al. (2010) study, tourist arrivals to Taiwan is forecasted by ANFIS and the proposed model gives more accurate results when compared with FTS, Grey Model and Markov Residual Modified model. Mohamad et al. (2010) make a case study to compare Back Propagation Neural Network (BPNN) and ANFIS. The testing errors show that ANFIS perform better than BPNN. Ho and Tsai (2011) use ANFIS and structural equation modeling that are compared in new product development. In their study, the authors show that ANFIS gives better forecasting results and can explain nonlinear relationships. In Wei's (2011) study, the model incorporates an autoregressive model into an ANFIS.

The model is employed in earning per share time series data of shares in Taiwan. Kisi et al. (2012) use ANFIS for forecasting the intermittent stream flows using ANFIS, ANN and Support Vector Machine (SVM). In the result part, ANFIS and ANN give good results using the data from two stations, Uzunkopru and Babaeski. Wei and Cheng (2012) use Taiwan Stock Exchange Index that is forecasted in a volatile environment. Four models including Chen's model, Yu's model, Huarng's model, are compared and the proposed model is superior to the listing methods in terms of the root mean squared error. In Zahedi et al. (2013) study, electricity demand forecasting modeled by ANFIS. Inputs of model are employment, gross domestic product, population, dwelling count and two meteorological parameters. In conclusion, the employment is found as the most important input for demand. Azadeh et al. (2013) present ANFIS-fuzzy data envelopment analysis (FDEA) algorithm. Two types of ANFIS are used for forecasting the natural gas demand. In conclusion, fuzzy one performed well with a lower error.

5 Applications

In this section two fuzzy time series method are introduced and relevant numerical applications are presented.

5.1 Fuzzy Time Series Using Hwang, Chen, Lee's Method (Hwang et al. 1998)

Let's think that we know the demand. We are going to find the demand with fuzzy demand forecasting. Let U be the universe of discourse $U = \{u_1, u_2,\ldots.. u_n\}$. A fuzzy set (Zadeh 1965) A of U, is defined by;

$$A = \mu_a(u_1)/u_1 + \mu_a(u_2)/u_2 + \ldots + \mu_a(u_n)/u_n \tag{24}$$

Firstly we find the variations. Let's take the years as; if the first year is t, the second year is t + 1. The first variation is the need of t + 1 minus the need of t. For example, the customer need in 1996 is 25.552 and the need in 1997 is 25.996. The variation of year 1997 is; 25996−25552 = 444. In this series, we can easily find the minimum increase $D_{\min}$ and maximum increase $D_{\max}$. After that, the universe of discourse U is defined, $U = [D_{min} - D_1, D_{max} + D_2]$, where the D_1 and D_2 are suitable numbers. $D_{\min} = -376$ and $D_{\max} = 1399$. D_1 and D_2 are positive numbers. We select the $D_1 = 24$ and $D_2 = 1$. So, U can be represented as $U = [-400, 1400]$. The universe of discourse is partition off into six intervals, where $U_1 = [-400, -100]$, $U_2 = [-100, 200]$, $U_3 = [200, 500]$, $U_4 = [500, 800]$, $U_5 = [800, 1100]$, $U_6 = [1100, 1400]$. Now, the next step is to define the fuzzy sets on the universe of discourse U. We determined some linguistic values. Seven fuzzy sets that are defined as; $A1$ = Decrease, $A2$ = No Change, $A3$ = Little Increase,

$A4$ = Increase, $A5$ = Big Increase, $A6$ = Too Big Increase. Then, Fuzzy sets on the Universe Of Discourse are defined as follows;

$$A1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \tag{25}$$

$$A2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \tag{26}$$

$$A3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 \tag{27}$$

$$A4 = 0/u_1 + 0\,/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 \tag{28}$$

$$A5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 \tag{29}$$

$$A6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 \tag{30}$$

After that, the historical data are being fuzzified.

Now, we are going to choose a suitable window basis, w. Let's calculate the operation matrix $O^w(t)$ and the criterion matrix C(t). t is the year that we want to forecast. In this example, we can select w = 5. So we can set a 4 × 6 operation matrix $O^5(t)$ and the criterion matrix C(t) as follows.

$$\begin{aligned} O^5(2002) &= \begin{bmatrix} \textit{Fuzzy variation of the enrollment of } 2000 \\ \textit{Fuzzy variation of the enrollment of } 1999 \\ \textit{Fuzzy variation of the enrollment of } 1998 \\ \textit{Fuzzy variation of the enrollment of } 1997 \end{bmatrix} = \begin{bmatrix} A2 \\ A2 \\ A1 \\ A3 \end{bmatrix} \\ &= \begin{bmatrix} 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 1 & 0.5 & 0 & 0 \end{bmatrix} \\ C(2002) &= \textit{Fuzzy variation of the enrollment of } 2001 = [A6] \\ [A6] &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.5 \quad 1] \end{aligned} \tag{31}$$

Calculated relation matrix $R(t)$ by $R(t)\ [i,\ j] = O^w(t)\ [i,\ j]\ XC(t)\ [J]$, where $1 \leq i \leq 4$, and $1 \leq j \leq 6$. We can get;

$$R(2002) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Next, the column's maximum values selected (Table 1).

$$F(2002) = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0)$$

After that, we are ready to defuzzify process. There are some principles to defuzzify the fuzzified forecasted variations.

Table 1 Membership functions of forecasted variations (under w = 5)

Years	U1	U2	U3	U4	U5	U6
2002	0	0	0	0	0	0
2003	0	0	0.5	1	0.25	0
2004	0	0	0.5	0.5	0.25	0
2005	0.25	1	0.5	0	0	0
2006	0	0.25	1	0.5	0	0
2007	0.5	1	0.25	0	0	0
2008	0.25	1	0.25	0	0	0
2009	0.5	0.5	0	0	0	0
2010	0.25	1	0.25	0	0	0

Table 2 Actual and forecasted values of enrollments

Years	Actual needs	Variations	Fuzzified variations	Actual	Forecasted
1996	25552	–	–	–	–
1997	25996	444	A3	–	–
1998	25620	−376	A1	–	–
1999	25745	125	A2	–	–
2000	25870	125	A2	–	–
2001	26120	250	A3	–	–
2002	27519	1399	A6	27519	26120
2003	28245	726	A4	28245	28169
2004	28807	562	A4	28807	29045
2005	28919	112	A2	28919	28857
2006	29388	469	A3	29388	29269
2007	29433	45	A2	29433	29438
2008	29497	64	A2	29497	29483
2009	29145	−352	A1	29145	29397
2010	29163	18	A2	29163	29195

a. If the value of memberships all 0, the variation of forecasting is 0.
b. If the numbers memberships in the Table 1 have only one maximum u_i, the forecasted variation is m_i is the midpoint of u_i.
c. If there is more than one maximum value of membership, then the midpoints are taken in average, like $(m_1 + m_2 + m_3 + \cdots + m_k)/k$.

Actual number in 2001 is 26,120, and the forecasted value of 2002 is 26120 + 0 = 26120 (Table 2).

The MAE is 244, 1

$$MAE = \frac{1}{n}\sum\nolimits_{i=1}^{n} |fi - yi| \quad (32)$$

fi Forecasted value
yi Actual Value.

5.2 Fuzzy Time Series Using Singh's Method

Singh (2008) proposes a method on forecasting of enrollments of Alabama University. Steps of the computational algorithm of proposed method for fuzzy time series forecasting is given as follows;

1. Defining the Universe of discourse (U).

$U = [D_{min} - D_1, D_{max} + D_2]$ where D_1 and D_2 are two proper positive numbers.

2. Partition the Universe of discourse into equal length of intervals: u1, u2,…, u_m. The number of intervals will be in accordance with the number of linguistic variables (fuzzy sets) $A1$, $A2$,…, Am to be considered.
3. Constructing the fuzzy sets Ai in accordance with the intervals and apply the triangular membership rule to each intervals in each fuzzy set so constructed.
4. Fuzzifying the historical data and establish the fuzzy logical relationships by the rule: If Ai is the fuzzy production of year n and A_j is the fuzzify production of year $n + 1$, then the fuzzy logical relation is denoted as $Ai \rightarrow Aj$. Here Ai is showed current state and Aj is next state.
5. Rules for forecasting. The notations used are defined as;

- $[*A_j]$ is corresponding interval u_j for which membership in A_j is Supremum
- $L\ [*A_j]$ is the lower bound of interval u_j.
- $U\ [*A_j]$ is the upper bound of interval u_j.
- $l\ [*A_j]$ is the length of the interval u_j whose membership in A_j is Supremum
- $M\ [*A_j]$ is the midvalue of the interval u_j having Supremum value in A_j.

For a fuzzy logical relation $Ai \rightarrow Aj$:

- A_i is the fuzzified enrollments of year n.
- A_j is the fuzzified enrollments of year $n + 1$.
- E_i is the actual enrollments of year n.
- E_{i-1} is the actual enrollments of year $n - 1$.
- E_{i-2} is the actual enrollments of year $n - 2$.
- F_j is the crisp forecasted enrollments of the year $n + 1$.

This model of order three utilizes the historical data of years $n - 2$, $n - 1$, n for framing rules to implement on fuzzy logical relation, $A_i \rightarrow A_j$, is fuzzified enrollments of year $n + 1$. The proposed method, is explained below step by step, for forecasting is mentioned as rule for generating the relations between the time series data of years $n - 2$, $n - 1$, n for forecasting the enrollment of year $n + 1$.

Computational Algorithm: Forecasting enrollments F_j for year $n + 1$ and onwards

for k to ….. K

Obtained fuzzy logical relation for year k to k + 1

$Ai \rightarrow Aj$

R = 0 and S = 0

(where K shows end of time series data)
Compute

$$D_i = ||(E_i - E_{i-1})| - |(E_{i-1} - E_{i-2})|| \quad (33)$$

$$X_i = Ei + D_i/2 \quad (34)$$

$$XX_i = E_i - D_i/2 \quad (35)$$

$$Yi = Ei + D_i \quad (36)$$

$$YYi = Ei - D_i \quad (37)$$

$$Pi = E_i + D_i/4 \quad (38)$$

$$PP_i = E_i - D_i/4 \quad (39)$$

$$Q_i = E_i + 26^* D_i \quad (40)$$

$$QQ_i = E_i - 2^* D_i \quad (41)$$

$$G_i = E_i + D_i/6 \quad (42)$$

$$GG_i = E_i - D_i/6 \quad (43)$$

$$H_i = E_i + 3^* D_i \quad (44)$$

$$HH_i = E_i - 3^* D_i \quad (45)$$

$$\begin{array}{l}\text{If} X_i \geq L\left[{}^*A_j\right] \text{ and } X_i \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + X_i \text{ and } S = S + 1\end{array} \quad (46)$$

$$\begin{array}{l}\text{If } XX_i \geq L\left[{}^*A_j\right] \text{ and } XX_i \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + XX_i \text{ and } S = S + 1\end{array} \quad (47)$$

$$\begin{array}{l}\text{If } Y_i \geq L\left[{}^*A_j\right] \text{ and } Yi \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + Y_i \text{ and } S = S + 1\end{array} \quad (48)$$

$$\begin{array}{l}\text{If } YY_i \geq L\left[{}^*A_j\right] \text{ and } YY_i \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + YYi \text{ and } S = S + 1\end{array} \quad (49)$$

$$\begin{array}{l}\text{If } P_i \geq L\left[{}^*A_j\right] \text{ and } P_i \leq U\left[{}^*A_j\right] \\ \text{Then } R = R + P_i \text{ and } S = S + 1\end{array} \quad (50)$$

$$\begin{aligned}&\text{If } PP_i \geq L\left[{}^{*}A_j\right] \text{ and } PP_i \leq U\left[{}^{*}A_j\right]\\&\text{Then } R = R + PPi \text{ and } S = S + 1\end{aligned} \tag{51}$$

$$\begin{aligned}&\text{If } Q_i \geq L\left[{}^{*}A_j\right] \text{ and } Q_i \leq U\left[{}^{*}A_j\right]\\&\text{Then } R = R + Q_i \text{ and } S = S + 1\end{aligned} \tag{52}$$

$$\begin{aligned}&\text{If } QQ_i \geq L\left[{}^{*}A_j\right] \text{ and } QQ_i \leq U\left[{}^{*}A_j\right]\\&\text{Then } R = R + Q_i \text{ and } S = S + 1\end{aligned} \tag{53}$$

$$\begin{aligned}&\text{If } QQ_i \geq L\left[{}^{*}A_j\right] \text{ and } QQ_i \leq U\left[{}^{*}A_j\right]\\&\text{Then } R = R + QQ_i \text{ and } S = S + 1\end{aligned} \tag{54}$$

$$\begin{aligned}&\text{If } G_i \geq L\left[{}^{*}A_j\right] \text{ and } G_i \leq U\left[{}^{*}A_j\right]\\&\text{Then } R = R + GG_i \text{ and } S = S + 1\end{aligned} \tag{55}$$

$$\begin{aligned}&\text{If } H_i \geq L\left[{}^{*}A_j\right] \text{ and } H_i \leq U\left[{}^{*}A_j\right]\\&\text{Then } R = R + H_i \text{ and } S = S + 1\end{aligned} \tag{56}$$

$$\begin{aligned}&\text{If } HH_i \geq L\left[{}^{*}A_j\right] \text{ and } HH_i \leq U\left[{}^{*}A_j\right]\\&\text{Then } R = R + HH_i \text{ and } S = S + 1\end{aligned} \tag{57}$$

$$\begin{aligned}&F_j = \left(R + M\left({}^{*}A_j\right)\right)/(S + 1)\\&\text{Next k}\end{aligned} \tag{58}$$

Considering the rules and the algorithm, one company's oil consumption is forecasted. Universe of discourse is defined as; U = [11000, 29000]. The partition of universe of discourse U in the six intervals are shown as: U_1 = [11000, 14000], U_2 = [14000, 17000], U_3 = [17000, 20000], U_4 = [20000, 23000], U_5 = [23000, 26000], U_6 = [26000, 29000]. After that, the next step is defining six fuzzy sets A1, A2,…, A6 as linguistic variables on the universe of discourse U. These fuzzy variables are defined as; A1 : Poor Consumption, A2 : Below Average Consumption, A3 : Average Consumption, A4 : Good Consumption, A5 : Very Good Consumption, A6 : Excellent Consumption. Also the membership grades to these fuzzy sets of linguistic values are defined as;

$$A1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \tag{59}$$

$$A2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \tag{60}$$

$$A3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 \tag{61}$$

$$A4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6 \tag{62}$$

$$A5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6 \tag{63}$$

Table 3 The results of Singh's method

Years	Actual oil consumption	Linguistic variables	Actual	Forecasted
1980	11379	A1	11379	–
1981	12933	A1	12933	–
1982	14933	A2	14933	–
1983	13155	A1	13155	13048
1984	17517	A3	17517	18500
1985	17884	A3	17884	18101
1986	19073	A3	19073	18606
1987	20081	A4	20081	21252
1988	26415	A6	26415	27500
1989	23957	A5	23957	24716
1990	22421	A4	22421	21647
1991	20429	A4	20429	22011

$$A6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6 \quad (64)$$

The historical time series data of oil consumption are fuzzified using the triangular membership function to obtain the enrollments in terms of linguistic variables.

In order to forecast the consumptions, the algorithm is explained step by step. For forecast the year 1985, the algorithm runs as follows;

$D = D_i = //(17517 - 13155)/ - /(13155 - 14933)// = 2584$

$X_i = 17517 + 2584/2 = 18809$, $XX_i = 17517 - 2584/2 = 16225$, $Yi = 17517 + 2584 = 20101$,

$YY_i = 17517 - 2584 = 14933$, $P_i = 17517 + 2584/4 = 18163$, $PP_i = 17517 - 2584/4 = 16871$,

$Q_i = 17517 + 2 * 2584 = 22685$, $QQ_i = 17517 - 2 * 2584 = 12349$,

$G_i = 17517 + 2584/6 = 17948$, $GG_i = 17517 - 2584/6 = 17086$,

$H_i = 17517 + 3 * 2584 = 25269$, $HH_i = 17517 - 3 * 2584 = 9765$

The values which are between $U_1 = [17000, 20000]$ are considered for finding the forecast. The X_i, P_i, G_i and GG_i are between intervals of U_1. Because of that, the forecasted value of 1985 is,

$$F1985 = (18809 + 18163 + 17948 + 17086 + 18500)/(5) = 18101$$

The other forecasted values are found in the same way. They are given in the Table 3.

The Mean Absolute Error (MAE) is 793, 9

$$MAE = \frac{1}{n}\sum\nolimits_{i=1}^{n} |f_i - y_i|, \quad (65)$$

f_i = Forecasted value, y_i = Actual value.

6 Conclusion

Since all plans from different management levels build their plans based on forecasts, the process of forecasting is a vital activity in a supply chain. Departments such as marketing, finance, operations, and purchasing directly use forecasts in their processes. Forecasting is especially important for supply chains since the results affect all the parties in the chain. Thus, collaborative planning, forecasting and replenishment approach is used by supply chain members in order to develop forecasts and plans to optimize the supply chain.

The literature provides different techniques including time series, regression, ARIMA, simulation, artificial neural networks, particle swarm optimization, genetic algorithm and fuzzy methods, to build demand forecasting models. Fuzzy set theory can handle uncertainity and incorporate human intuition and experience into the forecasting process thus fuzzy set theory provides advantages to modellers in forecasting process.

In this study, mostly used fuzzy demand forecasting methods including, fuzzy time series, fuzzy regression, fuzzy rule based systems and adaptive neuro fuzzy inference system are explained briefly and a literature review is supplied for each methodology. Additionaly, two fuzzy time serries methods are applied to two different demand forecasting problem. Also, the models' performance measurement (MAE) are calculated. The methods results are shown in table. The two examples include two different time series data. If we want to compare the two methods, the method that gives low MAE is the best.

As further study the same forecasting problem can be examined with other mentioned fuzzy and non-fuzzy techniques and the prediction accuracy of each technique can be benchmarked.

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Flows Finding in Networks in Fuzzy Conditions

Alexandr Bozhenyuk and Evgeniya Gerasimenko

Abstract The following chapter deals with flow problems in transportation networks in terms of fuzziness. Literature review considering flows and basic problem statements is given. The task of maximum flow finding in transportation network with lower flow bounds in fuzzy conditions is described and solved. The necessity of considering dynamic transportation networks is explained. The task of maximum flow finding with lower flow bounds in fuzzy conditions in dynamic network is solved. Peculiarity of the considered task is in fuzzy and transit nature of the network parameters.

Keywords Maximum flow · Dynamic fuzzy transportation network · Lower flow bounds

1 Introduction

The relevance of the flows finding tasks is that the economic development of any country and any region is caused by the presence of roads or routes. Due to the process of urbanization the number of vehicles has increased, but the quality of roads remains poor; adequate policy regarding the construction of new roads and repairing of existed ones is not pursued. Currently the problem of traffic management, especially in the large cities is relevant. The increasing number of vehicles either personal or public leads to congestion of city roads, "traffic jams" and increasing number of accidents.

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Science deals with developing and solving of various optimization problems on transportation networks, in particular, the tasks of identifying of congested areas on the maps, making optimal routes, determining of routes with minimal cost. But these achievements are not practically implemented on real railways, air- and sea roads.

Otherwise, even if we have the implementation of researches in practice, the complexity of factors influencing the time parameters, costs and capacities of the roads, in particular, their uncertainty is not taken into account.

Hence, it becomes necessary to investigate the flow problems in transportation networks in fuzzy conditions. The aim of our investigation is proposing of flow algorithms in fuzzy networks, in particular, maximum flow finding algorithm in fuzzy network with lower flow bounds. Another goal is to consider dynamic graphs in fuzzy conditions and associated flow algorithms, i.e. algorithm of maximum flow finding with lower flow bounds in fuzzy dynamic graph.

Present chapter is organized as follows. The problem statement of maximum flow finding in transportation networks and its variation—the task of maximum flow finding with lower flow bounds is presented in Sect. 2. The necessity of arc capacities representation in a fuzzy form is explained in Sect. 3. The algorithm of maximum flow finding with lower flow bounds in fuzzy conditions is given in Sect. 4. The review of problems in dynamic networks, in particular, the task of maximum flow finding in dynamic network and its variation—the task of maximum flow finding with lower flow bounds and is presented in Sect. 5. The algorithm of maximum flow finding with lower flow bounds in fuzzy conditions in dynamic networks is given in Sect. 6. Numerical example is illustrated in Sect. 7. We summarize our conclusions and observe the future studies in Sect. 8.

2 The Problem Statement of Maximum Flow Finding in Transportation Networks

One of the first fundamental works devoted to consideration of transportation networks was "Flows in Networks" by Ford and Fulkerson (1962). Transportation network is called a finite connected directed graph $G = (X,\ A)$ without loops, where $X = \{x_1, x_2 \ldots,\ x_n\}$ is the set of nodes, $A = \{(x_i,\ x_j)\},\ \ i,\ j \in I = \overline{1,n}$ is the set of arcs. Each arc from A has nonnegative arc capacity $u(x_i,\ x_j)$, determining the maximum rate of flow, which can pass along the arc and there are two nodes: the source $x_0 = s$, which no arc goes to, and the sink $x_n = t$, which no arc goes from.

Present chapter discusses various optimization problems in transportation networks, in particular the problem of the maximum flow finding in the network, first formulated by Dantzig in 1951. The authors stated and proved the theorem of maximum flow and minimum cut, which claims that the maximum amount of flow from the source to the sink in the network is equal to the minimal capacity of the cut. They propose "labeling algorithm" to solve this problem. A formal problem statement of the maximum flow task can be represented as follows (Christofides 1975):

$$v = \sum_{x_j \in \Gamma(s)} \xi_{sj} = \sum_{x_k \in \Gamma^{-1}(t)} \xi_{kt} \to \max, \tag{1}$$

$$\sum_{x_j \in \Gamma(x_i)} \xi_{ij} - \sum_{x_k \in \Gamma^{-1}(x_i)} \xi_{ki} = \begin{cases} v, \ x_i = s, \\ -v, \ x_i = t, \\ 0, \ x_i \neq s, \ t, \end{cases} \tag{2}$$

$$\xi_{ij} \le u_{ij}, \ \forall (x_i, \ x_j) \in A, \tag{3}$$

where ξ_{ij}—the amount of flow, passing along the arc (x_i, x_j); v—the maximum amount of flow in the network; s—initial node (source); t—final node (sink); $\Gamma(x_i)$—the set of nodes, arcs from $x_i \in X$ go to, $\Gamma^{-1}(x_i)$—the set of nodes, arcs from $x_i \in X$ go from; u_{ij}—the maximum amount of flow, which can pass along the arc (x_i, x_j) (arc capacity). The Eq. (1) means that we maximize the flow of the value v, which is equal to the total flow, going from the source $\sum_{x_j \in \Gamma(s)} \xi_{sj}$ and the flow entering the sink $\sum_{x_k \in \Gamma^{-1}(t)} \xi_{kt}$. The Eq. (2) indicates that maximum flow v, leaving the source is equal to the flow entering the sink. The amount of flow $\sum_{x_j \in \Gamma(x_i)} \xi_{ij}$, leaving x_i is equal to the flow $\sum_{x_k \in \Gamma^{-1}(x_i)} \xi_{ki}$ entering x_i for any intermediate node except s и t. Inequality (3) shows that the flows ξ_{ij} for any arcs must not exceed arc capacities u_{ij} along the corresponding arcs.

The relevance of this problem lies in the fact that the determination of the maximum flow under the restrictions on the arc capacities allows to find parts of the roads with saturated traffic and redistribute it.

In the studies of the authors Minieka (1978), Hu (1970) the solution of the maximum flow problem in the transportation network by the "labeling technique" is proposed.

There are various modifications of the Ford-Fulkerson's "labeling technique". Among them there are algorithm proposed in 1972 by Edmonds and Karp (1972), where the shortest augmenting path from the source to the sink in the residual network at each step is chosen. The shortest route can be found by the "breadth first search". Other scholars, such as Diniz, Karzanov, Cherkasky also worked to improve the running time of the algorithm and reduction of complexity. The most modern modification of the Ford-Fulkerson's algorithm is Goldberg-Rao's algorithm proposed in the 1997 (Goldberg and Rao 1998).

A variation of the maximum flow problem in transportation network is the problem of the maximum flow determining, taking into account the existence of lower and upper flow bounds for arcs, where upper arc flow bound is its arc capacity. Let's consider railway networks or sea roads. Thus freight trains go at a certain level of load, which is not less than threshold of profitability, and transport planes fly at a specific given level of load. Therefore, it is necessary to introduce lower flow bounds for such problems. The introduction of the lower flow bound makes this problem different from the task of maximum flow finding, as this restriction makes the existence of feasible solution not obvious (Bozhenyuk et al. 2012).

The formal problem statement of the maximum flow finding task taking into account lower bounds can be represented as follows:

$$v = \sum_{x_j \in \Gamma(s)} \xi_{sj} = \sum_{x_k \in \Gamma^{-1}(t)} \xi_{kt} \rightarrow \max, \tag{4}$$

$$\sum_{x_j \in \Gamma(x_i)} \xi_{ij} - \sum_{x_k \in \Gamma^{-1}(x_i)} \xi_{ki} = \begin{cases} v, \ x_i = s, \\ -v, \ x_i = t, \\ 0, \ x_i \neq s, \ t, \end{cases} \tag{5}$$

$$l_{ij} \leq \xi_{ij} \leq u_{ij}, \ \forall (x_i, x_j) \in A, \tag{6}$$

where l_{ij}—lower flow bound for the arc (x_i, x_j). Equation (4) means that we maximize flow of the value v, which is equal to the total flow leaving the source $\sum_{x_j \in \Gamma(s)} \xi_{sj}$ and the total flow entering the sink $\sum_{x_k \in \Gamma^{-1}(t)} \xi_{kt}$. Equation (5) indicates that maximum flow leaving the source v is equal to the flow entering the sink. The amount of flow $\sum_{x_j \in \Gamma(x_i)} \xi_{ij}$, leaving x_i is equal to the flow $\sum_{x_k \in \Gamma^{-1}(x_i)} \xi_{ki}$ entering x_i for any intermediate node except s и t. Inequality (6) shows that the flows ξ_{ij} for any arcs must not exceed upper flow bounds u_{ij} and must not be less that lower flow bounds l_{ij} along the corresponding arcs.

The task of maximum flow finding with lower flow bound was not widely reflected in the literature. In particular, the authors Christofides (1975), Mutry (1992) consider the problem of maximum flow determining in the transportation network, taking into account the lower and upper flow bounds. Thus, N. Christofides considers the problem of feasible flow finding, i.e. does not actually solves the maximum flow problem with lower and upper flow bounds in the graph.

This problem was not considered in the literature in fuzzy conditions, which is not true in general case. Let us examine in details the issue of the need to specify capacities in a fuzzy way.

3 Selecting of the Arc Capacities

Thus, arc capacities are the parameters that limit the flows passing along the arcs of the network. Road capacity defines the maximum number of vehicles that can pass along the considered road at a unit of time. In fact, capacities are fundamental parameter in determining the maximum flow. Therefore it is necessary to turn to the procedures of capacities determining.

Capacities depend on many factors: the road conditions (the width of the roadway, longitudinal slope, curve radii, visibility distance, etc.), the flow of vehicles, the availability of control resources, climatic conditions, the ability to maneuver vehicles across the width of the roadway, psycho-physiological characteristics of drivers and vehicle design. Changing these factors leads to significant fluctuations in capacities during the day, month, season and year. There are significant variations in the speed, resulting in the large number of vehicles moving in groups, as well as reducing the average speed of the flow with the frequent location of noises on the road.

Capacities are considered to divide into theoretical, practical and computational. Theoretical capacity u_{theor} is capacity of the road or its part in terms of constant intervals between cars, homogeneous composition of vehicles (e.g., if the composition contains only passenger cars). If we consider a highway, the theoretical capacity of its strip is 2900 passenger cars per hour.

Practical capacity u_{pract} is a parameter provided in real traffic conditions. There are two types of it: the maximal practical $u_{\max}$ and practical u_{pract} in real driving conditions. Maximal practical capacity is capacity of the part of road under reference conditions. Practical capacity in specific road conditions corresponds to the areas of roads with the worst road conditions in compared with the reference site.

The computational flow capacity u_{comp} determines economically reasonable number of vehicles that can pass along the road (its area) in the specific road conditions and the particular organization of movement. Computational flow capacity can be calculated by the equation:

$$u_{comp} = k_n u_{theor} \tag{7}$$

In (7) k_n—the transition coefficient from the theoretical capacity to the computational, which is determined depending on the type of road, road category and terrain.

In real capacities calculations the formula for calculating of practical capacity in real traffic conditions can be used besides Eq. (7):

$$u_{pract} = Bu_{\max} \tag{8}$$

In (8) $u_{\max}$ is a constant depending on the number of lanes of the road. The coefficient B is the final reduction factor of capacity, equals to the product of partial factors $\beta_1 \ldots \beta_{15}$. The coefficients are constant for certain values of factors.

For example, the coefficient β_1 is a table relationship between the number of lanes on the road, lanes' or highway's width. β_2 depends on the roadside's width, β_3 represents the relationship between the distance from the edge of the carriageway to the obstacle and width of the lane, β_4 shows the relationship between the number of trains in the flow in % and the number of passenger cars and average trucks in %, β_5 represents a ratio indicating the relationship between the longitudinal slope in %, the length of ascent and number of trains in the flow in %. Coefficient β_6 depends on the visibility distance in meters, β_7 depends on the radius of the curve in the plan, β_8 represents a coefficient reflecting a speed limit as a sign, the ratio β_9 depends on the three parameters: the number of vehicles turning to the left in %, type of intersection and the width of the carriageway of the main road, the coefficient β_{10} depends on strengthening of the roadside, β_{11} depends on the type of road surface, β_{12} changes its value in the presence of gas stations, rest areas with a complete separation from the main road and the presence of special lanes for entry or the same attributes in the presence or absence of stripping, β_{13}

depends on the type of marking, β_{14} depends on the speed limit (similar to β_8) and the presence of signs of lanes, β_{15} reveals the relationship between the number of buses in a stream in % and the number of cars in the flow in %.

The following difficulties can appear while determining of the practical capacity according to the described schemes:

1. The mentioned formulas of calculating the capacities of the road (or its part) don't take into account weather conditions, such as snowfalls, sleet, etc. In fact, in most cases it is only controlling of capacities for roads conditions in given terms, and not recalculation.
2. Repair works, traffic jams on the roads are not taken into account. Neglecting these figures leads to incorrect interpretation of the indicators included in the partial factors (lanes, highway and roadside's width).
3. Some of the indicators that make up the partial factors can be unknown due to the lack of statistical data, the unique formulations of the problems (in particular, the indicators of the number of cars in the stream, the number of vehicles turning to the left, etc.). The lack of statistical data may be caused by the construction of new roads, repairing of existed ones. It makes data collection of the number of vehicles impossible.
4. Some of the indicators included in the combined coefficients, can be misinterpreted because of the variable structure of the road system. For example, if you have statistical data about the width of the roadway, roadside, number of lanes, type of cover, the strengthening of the roadside, speed limits and carrying out repairing activities in the areas of roads, accidents, weather disasters, the information cannot be used because it is out of date and incorrect.

Thus, in spite of the existing methods of evaluating of the roads capacities, we cannot set this parameter in a crisp way due to the specificity of transportation networks, in particular the influence of weather conditions, the intervention of human activity, the errors in measurements, or lack of data about road's conditions. Consequently, the capacities of the transportation networks in below described algorithms should be represented in a fuzzy way.

4 Algorithm of the Maximum Flow Finding with Lower Flow Bounds in Transportation Networks in Fuzzy Conditions

Let us consider the problem statement of the maximum flow finding task with lower flow bounds in transportation network in fuzzy terms:

$$\tilde{v} = \sum_{x_j \in \Gamma(s)} \tilde{\xi}_{sj} = \sum_{x_k \in \Gamma^{-1}(t)} \tilde{\xi}_{kt} \rightarrow \max, \tag{9}$$

$$\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij} - \sum_{x_k \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ki} = \begin{cases} \tilde{v}, \; x_i = s, \\ -\tilde{v}, \; x_i = t, \\ 0, \; x_i \neq s, \; t, \end{cases} \tag{10}$$

$$\tilde{l}_{ij} \leq \tilde{\xi}_{ij} \leq \tilde{u}_{ij}, \; \forall (x_i, x_j) \in \tilde{A}, \tag{11}$$

where $\tilde{\xi}_{ij}$—fuzzy flow value passing along the arc (x_i, x_j), $\tilde{v}$—maximum fuzzy flow value in transportation network, $\tilde{u}_{ij}$—the upper fuzzy flow bound for the arc (x_i, x_j), $\tilde{l}_{ij}$—the lower fuzzy flow bound for the arc (x_i, x_j). Equation (9) means that we maximize fuzzy flow of the value $\tilde{v}$, which is equal to the total fuzzy flow leaving the source $\sum_{x_j \in \Gamma(s)} \tilde{\xi}_{sj}$ and the total fuzzy flow entering the sink $\sum_{x_k \in \Gamma^{-1}(t)} \tilde{\xi}_{kt}$. Equation (10) indicates that maximum fuzzy flow leaving the source $\tilde{v}$ is equal to the flow entering the sink. The fuzzy amount of flow $\sum_{x_j \in \Gamma(x_i)} \tilde{\xi}_{ij}$, leaving x_i is equal to the fuzzy flow $\sum_{x_k \in \Gamma^{-1}(x_i)} \tilde{\xi}_{ki}$ entering x_i for any intermediate node except s и t. Inequality (11) shows that the flows $\tilde{\xi}_{ij}$ for any arcs must not exceed upper fuzzy flow bounds $\tilde{u}_{ij}$ and must not be less that lower fuzzy flow bounds $\tilde{l}_{ij}$ along the corresponding arcs.

Let's represent a formal algorithm that implements the solution of considered problem (Bozhenyuk et al. 2012).

Step 1. Let us define if the initial graph $\tilde{G} = (X, \tilde{A})$ has a feasible flow. Introduce artificial source s^* and sink t^* and turn to the new graph $\tilde{G}^* = (X^*, \tilde{A}^*)$ without lower flow bounds according to the method, described in (Christofides 1975). Introduce the arc (t, s) in the new graph with $\tilde{u}^*_{ts} = \infty$, $\tilde{l}^*_{ts} = \tilde{0}$. For each node (x_i, x_j) in $\tilde{G}$ with $\tilde{l}_{ij} \neq \tilde{0}$: 1) decrease $\tilde{u}_{ij}$ to $\tilde{u}^*_{ij} = \tilde{u}_{ij} - \tilde{l}_{ij}$, $\tilde{l}_{ij}$ to $\tilde{0}$. 2) Introduce arcs (s^*, x_j) and (x_i, t^*) with capacities equal to $\tilde{u}^*_{s^* x_j} = \tilde{u}^*_{x_i t^*} = \tilde{l}_{ij}$, $\tilde{l}^*_{s^* x_j} = \tilde{l}^*_{x_i t^*} = \tilde{0}$. Arcs without lower flow bounds are the same for $\tilde{G}^*$: for any arc (x_i, x_j) with $\tilde{l}_{ij} = 0$ is $\tilde{u}^*_{ij} = \tilde{u}_{ij}$.

Step 2. Find maximum flow in $\tilde{G}^*$ between artificial nodes according to Edmonds and Karp's algorithm (Edmonds and Karp 1972). Build a fuzzy residual network $\tilde{G}^{*\mu} = (X^{*\mu}, \tilde{A}^{*\mu})$ starting with zero flows according to the rule: if $\tilde{\xi}^*_{ij} < \tilde{u}^*_{ij}$, then $\tilde{u}^{*\mu}_{ij} = \tilde{u}^*_{ij} - \tilde{\xi}^*_{ij}$. If $\tilde{\xi}^*_{ij} > 0$, then $\tilde{u}^{*\mu}_{ji} = \tilde{\xi}^*_{ij}$ and turn to maximum flow finding in such a network.

Step 3. Search the shortest path $\tilde{P}^{*\mu}$ in terms of the number of arcs from the artificial source s^* to the artificial sink t^* in the constructed fuzzy residual network according to Edmonds-Karp's algorithm (Edmonds and Karp 1972) starting with zero flow values. The choice of the shortest path is according to the breadth-first search.

(I) If the $\tilde{P}^{*\mu}$ is found, go to the step 4.

(II) The flow value $\tilde{\phi} < \sum_{\tilde{l}_{ij} \neq 0} \tilde{l}_{ij}$ is obtained, which is the maximum flow in $\tilde{G}^*$, if the path is failed to find. It means that it is impossible to pass any unit of flow, but not all the artificial arcs are saturated. Therefore, initial graph $\tilde{G}$ has no feasible flow and the task has no solution. Exit.

Step 4. Pass the minimum from the arc capacities $\tilde{\delta}^{*\mu} = \min[\tilde{u}_{ij}^{*\mu}]$ along the path $\tilde{P}_p^{*\mu}$.

Step 5. Update the fuzzy flow values in the graph $\tilde{G}^*$: replace the fuzzy flow $\tilde{\xi}_{ji}^*$ along the corresponding arcs (x_j, x_i) from $\tilde{G}^*$ by $\tilde{\xi}_{ji}^* - \tilde{\delta}^{*\mu}$ for arcs (x_i^μ, x_j^μ) in $\tilde{G}^{*\mu}$ and replace the fuzzy flow $\tilde{\xi}_{ij}^*$ along the arcs (x_i, x_j) from $\tilde{G}^*$ by $\tilde{\xi}_{ij}^* + \tilde{\delta}^{*\mu}$ for arcs (x_i^μ, x_j^μ) in $\tilde{G}^{*\mu}$. Replace $\tilde{\xi}_{ij}^*$ by $\tilde{\xi}_{ij}^* + \tilde{\delta}^{*\mu} \times P^{*\mu}$.

Step 6 (I) If the flow vector $\tilde{\xi}_{ij}^* + \tilde{\delta}^{*\mu} \times P^{*\mu}$ of the value $\tilde{\sigma}^*$is less than $\sum_{\tilde{l}_{ij} \neq 0} \tilde{l}_{ij}$, i.e. not all artificial arcs become saturated, go to the **step 2**.

(II) If the flow value $\tilde{\xi}_{ij}^* + \tilde{\delta}^{*\mu} \times P^{*\mu}$ is equal to $\sum_{\tilde{l}_{ij} \neq 0} \tilde{l}_{ij}$, i.e. all arcs from the artificial source to the artificial sink become saturated, then the value $\tilde{\xi}_{ij}^* + \tilde{\delta}^{*\mu} \times P^{*\mu}$ is required value of maximum flow $\tilde{\sigma}^*$ in $\tilde{G}^*$ In this case the flow $\tilde{\xi}_{ts}^*$ passing along the artificial arc (t, s) in $\tilde{G}^*$ determines the feasible flow in the initial graph $\tilde{G}$ of the value $\tilde{\sigma} = \tilde{\xi}_{ts}^*$. Turn to the graph $\tilde{G}$ from the graph $\tilde{G}^*$ as following: reject artificial nodes and arcs, connecting them with other nodes. The feasible flow vector $\tilde{\xi} = (\tilde{\xi}_{ij})$ of the value $\tilde{\sigma}$ is defined as: $\tilde{\xi}_{ij} = \tilde{\xi}_{ij}^* + \tilde{l}_{ij}$, where $\tilde{\xi}_{ij}^*$—the flows, going along the arcs of the graph $\tilde{G}^*$ after deleting all artificial nodes and connecting arcs. The network $G(\tilde{\xi})$ is obtained. Go to the **step 7**.

Step 7. Construct the residual network $G(\tilde{\xi}^\mu)$ taking into account the feasible flow vector $\tilde{\xi} = (\tilde{\xi}_{ij})$: for all arcs, if $\tilde{\xi}_{ij} < \tilde{u}_{ij}$ then include the corresponding arc in $G(\tilde{\xi}^\mu)$ with the arc capacity $\tilde{u}_{ij}^\mu = \tilde{u}_{ij} - \tilde{\xi}_{ij}$. For all arcs, if $\tilde{\xi}_{ij} > \tilde{l}_{ij}$, then include the corresponding arc in $G(\tilde{\xi}^\mu)$ with the arc capacity $\tilde{u}_{ji}^\mu = \tilde{\xi}_{ij} - \tilde{l}_{ij}$.

Step 8. Define the shortest path $\tilde{P}_p^\mu$ according to the Edmonds and Karp's algorithm from the artificial source to the artificial sink in the constructed residual network $G(\tilde{\xi}^\mu)$

(I) Go to the step 9 if the augmenting path $\tilde{P}^\mu$ is found.

(II) The maximum flow $\tilde{\xi}_{ij} + \tilde{\delta}^\mu \times \tilde{P}^\mu = \tilde{v}$ in $\tilde{G}$ is found if the path is failed to find, then stop.

Step 9. Pass $\tilde{\delta}^\mu = \min[\tilde{u}_{ij}^\mu]$ along the found path.

Step 10. Update the fuzzy flow values in the graph $\tilde{G}$: replace the fuzzy flow $\tilde{\xi}_{ji}$ along the corresponding arcs (x_j, x_i) from $\tilde{G}$ by $\tilde{\xi}_{ji} - \tilde{\delta}^\mu$ for arcs (x_i^μ, x_j^μ)

in $G(\tilde{\xi}^{\mu})$ and replace the fuzzy flow $\tilde{\xi}_{ij}$ along the arcs (x_i, x_j) from $\tilde{G}$ by $\tilde{\xi}_{ij} + \tilde{\delta}^{\mu}$ for arcs (x_i^{μ}, x_j^{μ}) in $G(\tilde{\xi}^{\mu})$ and replace the flow value in $\tilde{G}$: $\tilde{\xi}_{ij} \to \tilde{\xi}_{ij} + \tilde{\delta}^{\mu} \times \tilde{P}^{\mu}$ and turn to the **step 7** starting from the new flow value along the arcs.

Thus the described algorithm allows to find maximum flow in networks with lower flow bounds in fuzzy conditions or show that the feasible flow doesn't exist.

5 Review of Flow Tasks in Dynamic Networks

Conventional tasks of maximum flow finding assume the instant flow, passing along the arcs of the graph, what certainly, is simplification of the real life. Such tasks are called static flow tasks. In fact, it turns out that the flow spends certain time passing along the arcs of the graph. Then, we turn to dynamic networks, in which each flow unit passes from the source to the sink for a period of time less than given. Dynamic network is a network $G = (X, A)$, where $X = \{x_1, x_2 \ldots, x_n\}$—the set of nodes, $A = \left\{(x_i,\ x_j)\right\},\ i,\ j \in I = \overline{1, n}$—the set of arcs. Each arc of the dynamic graph (x_i, x_j) is set two parameters: transit time τ_{ij} (Melkonian 2007) and arc capacity u_{ij}. The time horizon $T = \{0, 1, \ldots, p\}$ determining that all flow units sent from the source must arrive in the sink within the time p is given (Ford and Fulkerson 1962). Let τ_{ij} be a positive number. Let $\Gamma(x_i)$ is the set of nodes, arcs from x_i go to, $\Gamma^{-1}(x_i)$ is the set of nodes, arcs from x_i go from. Thus, not more than u_{ij} units of flow can be sent along the arc (x_i, x_j) at each time period in dynamic networks. Let x_j be the final node and x_i is the initial node of the arc (x_i, x_j), then the flow leaving x_i at $\theta \in T$ will enter x_j at time period $\theta + \tau_{ij}$ (Bozhenyuk et al. 2012). There are various problem statements in dynamic graphs: the maximum flow finding in dynamic graphs, the minimum cost flow finding, etc.

Dynamic networks describe complex systems, problems of decision-making, models, which parameters can vary over time. Such models can be found in communication systems, economic planning, transportation systems and many other applications, so they have a wide practical application.

Historically, the maximum flow finding in dynamic graphs was the first task in dynamic graphs, described in the literature. The notion "dynamic flow" was proposed by Ford and Fulkerson (Ford and Fulkerson 1962) as a task of maximum dynamic flow finding in a network. This problem is in finding of maximum flow, passing from the source (s) to the sink (t), $s,\ t \in X$ in the network for p discrete time periods, starting from zero period of time. All flow units leaving the source must arrive at the sink not later than at p. Let $v(p)$ be the total number of flow units leaving the source and entering the sink for time periods $0, \ldots, p$. This task can be formulated as follows (Ford and Fulkerson 1962):

$$\text{maximize } v(p) \tag{12}$$

$$\sum_{\theta=0}^{p}\sum_{x_j\in X}[\xi_{sj}(\theta)-\xi_{js}(\theta-\tau_{js})]-v(p)=0, \tag{13}$$

$$\sum_{x_j\in X}[\xi_{ij}(\theta)-\xi_{ji}(\theta-\tau_{ji})]=0,\ x_i\neq s,\ t;\ \theta\in T, \tag{14}$$

$$\sum_{\theta=0}^{p}\sum_{x_j\in X}[\xi_{tj}(\theta)-\xi_{jt}(\theta-\tau_{jt})]+v(p)=0, \tag{15}$$

$$0\leq\xi_{ij}(\theta)\leq u_{ij},\ \forall(x_i,x_j)\in A,\ \theta\in T. \tag{16}$$

Equation (12) means that we maximize the amount of flow v for p periods of time. Expressions (13) and (15) show that the maximum flow v for p periods of time leaving the source $\sum_{\theta=0}^{p}\xi_{sj}(\theta)$ is equal to the flow value entering the sink $\sum_{\theta=0}^{p}\xi_{jt}(\theta-\tau_{jt})$. The flow amount $\sum_{\theta=0}^{p}\xi_{js}(\theta-\tau_{js})$ entering the source is equal to flow amount leaving the sink $\sum_{\theta=0}^{p}\xi_{tj}(\theta)$ and is equal to zero. The amount of flow units $\xi_{ji}(\theta-\tau_{ji})$ entering the node x_i at time $(\theta-\tau_{ji})$ equals to the amount of flow units $\xi_{ij}(\theta)$ leaving the node x_i at time θ for each period of time θ and for each node x_i, except the source and the sink, as stated in (14). Inequality (16) indicates that the flows for all time periods $\xi_{ij}(\theta)$ should be less than arc capacities of the corresponding arcs.

The task of maximum dynamic flow finding was widely reported in the literature. Ford and Fulkerson (1962) proposed two methods for its solution: the first is based on the constructing of the time-expanded network, and the second—implements the algorithm of minimum cost flow finding considering the flow transit time along the arc equals to the arc cost of the corresponding arc, applying the shortest path algorithm. Time-expanded static graph corresponding to the original dynamic graph is constructed by expanding the original network in the time dimension by making a separate copy of every node $x_i\in X$ at every time $\theta\in T$.

Let $G_p=(X_p,A_p)$ be the time-expanded static graph of the initial dynamic graph G. The set of nodes X_p of the graph G_p is given by $X_p=\{(x,\theta):\ (x_i,\theta)\in X\times T\}$.The set of arcs A_p consists of arcs from every node-time pair $(x_i,\ \theta)\in X_p$ to every node-time pair $(x_j,\theta+\tau_{ij})$, where $x_j\in\Gamma(x_i)$ and $\theta+\tau_{ij}\leq p$. Arc capacities, connecting (x_i,θ) to $(x_j,\theta+\tau_{ij})$ are equal to u_{ij} (Chabini and Abou-Zeid 2003).

The approach operating with the time-expanded network (Nasrabadi and Hashemi 2010) is more widespread despite the density of the second method. This is true due to the fact that algorithm based on flow decomposition doesn't consider dynamic structure of the transportation network (only flow transit times are taken into account), while the arc capacities and flow transit times are defined by constants. This fact allows to consider the model proposed by Ford-Fulkerson as "stationary-dynamic" one. In the real life parameter of flow departure is crucial, because it influences the arc capacities and flow transit times of the network. For example, in the morning roads are not loaded, so they have high capacities, and therefore flow transit times are small. In the evening the roads are loaded, hence, their capacities are small and flow transit times become large. Consequently, we come to notion of "dynamic network", i.e. such a network, which parameters can vary over time.

Minieka (1978) studied "stationary-dynamic" maximum flows in addition to Ford and Fulkerson (1962). In particular, author simulated a situation, in which some of the arcs of the dynamic graph may not be available at some time periods (Minieka 1978). Powell et al. (1995) considered the problems of maximum flow determining in the dynamic graphs either in the case of discrete time periods or continuous ones. Fonoberova and Lozovanu (2004) investigated the problem of admissibility of flow existence in the case, when arcs of the dynamic network have either lower flow bounds or upper ones.

Consider a transportation network consisting of railways. The vertices of the network represent the stations, and the arcs are the roads connecting these stations. It is known that freight trains go at a certain level of load, therefore, the capacity is the maximum possible number of flow units (load), which can be carried along the road connecting stations. The lower flow bound determines the minimum number of flow units which can be carried along the road (depending on the profitability of transmission). The amount of time periods, i.e. moments of trains departure is also given. It is necessary to define the maximum amount of cargo which can be transported on the roads with the lower and upper flow bounds for the given number of time periods. The described model of the problem is also valid for the mixed type of transportation networks, i.e. those that include various road networks: sea, air, road and railways.

Consequently, we come to the problem formulation of the maximum flow finding task in a dynamic transportation network, which arcs can have lower flow bounds:

$$\text{maximaze } \; v(p), \tag{17}$$

$$\sum_{\theta=0}^{p}\sum_{x_j\in X}[\xi_{sj}(\theta)-\xi_{js}(\theta-\tau_{js})]-v(p)=0, \tag{18}$$

$$\sum_{x_j\in X}[\xi_{ij}(\theta)-\xi_{ji}(\theta-\tau_{ji})]=0, \;\; x_i\neq s,\; t; \;\; \theta\in T, \tag{19}$$

$$\sum_{\theta=0}^{p}\sum_{x_j\in X}[\xi_{tj}(\theta)-\xi_{jt}(\theta-\tau_{jt})]+v(p)=0, \tag{20}$$

$$l_{ij}\leq\xi_{ij}(\theta)\leq u_{ij}, \; \forall(x_i,\; x_j)\in A, \;\theta\in T. \tag{21}$$

Equation (17) means that we maximize the amount of flow v for p periods of time. Expressions (18) and (20) show that the maximum flow v for p periods of time leaving the source $\sum_{\theta=0}^{p} \xi_{sj}(\theta)$ is equal to the flow value entering the sink $\sum_{\theta=0}^{p} \xi_{jt}(\theta - \tau_{jt})$. The flow amount $\sum_{\theta=0}^{p} \xi_{js}(\theta - \tau_{js})$ entering the source is equal to flow amount leaving the sink $\sum_{\theta=0}^{p} \xi_{tj}(\theta)$ and is equal to zero. The amount of flow units $\xi_{ji}(\theta - \tau_{ji})$ entering the node x_i at time $(\theta - \tau_{ji})$ equals to the amount of flow units $\xi_{ij}(\theta)$ leaving the node x_i at time θ for each period of time θ and for each node x_i, except the source and the sink, as stated in (19). Inequality (21) indicates that the flows for all time periods $\xi_{ij}(\theta)$ should be less than upper flow bounds and more than lower flow bounds of the corresponding arcs.

This problem variation of the maximum dynamic flow finding is not considered in the literature. All studies were carried out in crisp conditions and on static graphs. This problem was not discussed in the literature in fuzzy conditions. Hence, the necessity to solve this problem in fuzzy environment appears.

6 Algorithm for Maximum Flow Finding with Lower Flow Bounds in Dynamic Networks in Terms of Fuzziness

Let us turn to considering of the maximum flow finding problem with lower flow bounds in the dynamic networks in fuzzy terms. We'll take into account the true dynamic nature of the network parameters: the lower and upper flow bounds and parameters of transit times may depend on the departure time:

$$\text{maximaze } \tilde{v}(p), \tag{22}$$

$$\sum_{\theta=0}^{p} \sum_{x_j \in X} [\tilde{\xi}_{sj}(\theta) - \tilde{\xi}_{js}(\theta - \tau_{js}(\theta))] - \tilde{v}(p) = 0, \tag{23}$$

$$\sum_{x_j \in X} [\tilde{\xi}_{ij}(\theta) - \tilde{\xi}_{ji}(\theta - \tau_{ji}(\theta))] = \tilde{0}, \quad i \neq s,\ t; \quad \theta \in T, \tag{24}$$

$$\sum_{\theta=0}^{p} \sum_{x_j \in X} [\tilde{\xi}_{tj}(\theta) - \tilde{\xi}_{jt}(\theta - \tau_{jt}(\theta))] + \tilde{v}(p) = \tilde{0}, \tag{25}$$

$$\tilde{l}_{ij}(\theta) \leq \tilde{\xi}_{ij}(\theta) \leq \tilde{u}_{ij}(\theta),\ for\ \theta:\ \theta + \tau_{ij}(\theta) \leq p,\ \theta \in T. \tag{26}$$

Equation (22) means that we maximize the amount of flow $\tilde{v}$ for p periods of time. Expressions (23) and (25) show that the maximum flow $\tilde{v}$ for p periods of time leaving the source $\sum_{\theta=0}^{p} \tilde{\xi}_{sj}(\theta)$ is equal to the flow value entering the sink $\sum_{\theta=0}^{p} \tilde{\xi}_{jt}(\theta - \tau_{jt})$. The flow amount $\sum_{\theta=0}^{p} \tilde{\xi}_{js}(\theta - \tau_{js})$ entering the source is equal to flow amount leaving the sink $\sum_{\theta=0}^{p} \tilde{\xi}_{tj}(\theta)$ and is equal to $\tilde{0}$. The amount of flow units $\tilde{\xi}_{ji}(\theta - \tau_{ji})$ entering the node x_i at time $(\theta - \tau_{ji})$ equals to the amount of flow units $\tilde{\xi}_{ij}(\theta)$ leaving the node x_i at time θ for each period of time θ and for each node x_i, except the source and the sink, as stated in (24). Inequality (26) indicates that the flows $\tilde{\xi}_{ij}(\theta)$ for time periods $\theta: \ \theta + \tau_{ij}(\theta) \le p, \theta \in T$ should be less than upper flow bounds $\tilde{u}_{ij}(\theta)$ and more than lower flow bounds $\tilde{l}_{ij}(\theta)$ of the corresponding arcs.

The algorithm for solving this problem is also reduced to expanding of the original dynamic graph to "time-expanded" static version of the original graph and the implementation of the algorithm of maximum flow finding in a fuzzy graph with lower and upper flow bounds.

Step 1. Go to the time-expanded fuzzy static graph $\tilde{G}_p$ from the given fuzzy dynamic graph $\tilde{G}$ by expanding the original dynamic graph in the time dimension by making a separate copy of every node $x_i \in X$at every time $\theta \in T$. Let $\tilde{G}_p = (X_p, \tilde{A}_p)$ represent fuzzy time-expanded static graph of the original dynamic fuzzy graph. The set of nodes X_p of the graph $\tilde{G}_p$ is defined as $X_p = \{(x_i, \theta) : (x_i, \theta) \in X \times T\}$. The set of arcs $\tilde{A}_p$ consists of arcs from each node-time pair $(x_i, \theta) \in X_p$ to every node-time pair $(x_j, \theta + \tau_{ij}(\theta))$, where $x_j \in \Gamma(x_i)$ and $\theta + \tau_{ij}(\theta) \le p$. Fuzzy upper flow bounds $\tilde{u}(x_i, x_j, \theta, \theta + \tau_{ij}(\theta))$ joining (x_i, θ) with $(x_j, \theta + \tau_{ij}(\theta))$ are equal to $\tilde{u}_{ij}(\theta)$ and fuzzy lower flow bounds $\tilde{l}(x_i, x_j, \theta, \theta + \tau_{ij}(\theta))$ joining $(x_i,\ \theta)$ with $(x_j, \theta + \tau_{ij}(\theta))$ are equal to $\tilde{l}_{ij}(\theta)$.

Step 2. Determine, if the time-expanded fuzzy graph $\tilde{G}_p$, corresponding to the initial dynamic graph $\tilde{G}$, has a feasible flow. Introduce the artificial source s^* and sink t^* in the graph $\tilde{G}_p$ and turn to the graph $\tilde{G}_p^* = (X_p^*, \tilde{A}_p^*)$ without lower flow bounds according to the method, described in (Christofides 1975). The set X_p^* consists of the nodes from the set X_p and the artificial nodes s^* and t^*. Introduce the arcs, connecting the node-time pair (t, $\forall \theta \in$ T) and (s, $\forall \theta \in$ T) with upper fuzzy flow bound $\tilde{u}^*(t, s,\ \forall \theta \in T,\ \forall \theta \in T) = \infty$, lower fuzzy flow bound $\tilde{l}^*(t,\ s,\ \forall \theta \in T,\ \forall \theta \in T) = \tilde{0}$ in the graph $\tilde{G}_p^*$. It means that every node t in each time period from p is connected with every node s at all time periods in the graph $\tilde{G}_p^*$. Introduce the following modification for each arc connecting the node-time pair (x_i, ϑ) with the node-time pair $(x_j, \theta = \vartheta + \tau_{ij}(\vartheta))$ with nonzero lower fuzzy flow bound $\tilde{l}(x_i, x_j, \vartheta, \theta) \neq \tilde{0}$: 1) reduce $\tilde{u}(x_i, x_j, \vartheta, \theta)$ to $\tilde{u}^*(x_i, x_j, \vartheta, \theta) = \tilde{u}(x_i, x_j, \vartheta, \theta) - \tilde{l}(x_i, x_j, \vartheta, \theta)$, $\tilde{l}(x_i, x_j, \vartheta, \theta)$ to $\tilde{0}$. 2) Introduce the arcs connecting s^* with (x_j, θ), and the arcs connecting t^* with (x_i, ϑ) with upper fuzzy flow bounds equal to lower fuzzy flow bounds $\tilde{u}^*_{s*x_j(\theta)} = \tilde{u}^*_{x_i(\vartheta)t^*} = \tilde{l}(x_i, x_j, \vartheta, \theta)$, zero lower fuzzy flow bounds $\tilde{l}^*_{s*x_j(\theta)} = \tilde{l}^*_{x_i(\vartheta)t^*} = \tilde{0}$.

Step 3. Build a fuzzy residual network $\tilde{G}_p^{*\mu}$ depending on the flow values going along the arcs of the graph $\tilde{G}_p^*$. Fuzzy residual network $\tilde{G}_p^{*\mu} = (X_p^{*\mu}, \tilde{A}_p^{*\mu})$ is constructed according to the time-expanded fuzzy static graph $\tilde{G}_p^*$ without lower fuzzy flow bounds depending on the flow values $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$ going along it as follows: each arc in the fuzzy residual network $\tilde{G}_p^{*\mu}$, connecting the node-time pair (x_i^μ, ϑ) with the node-time pair (x_j^μ, θ), which the flow $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$ is sent along at each period of time $\vartheta \in T$ has fuzzy residual capacity $\tilde{u}^{*\mu}(x_i, x_j, \vartheta, \theta) = \tilde{u}^*(x_i, x_j, \vartheta, \theta) - \tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$ with transit time $\tau^{*\mu}(x_i, x_j, \vartheta, \theta) = \tau^*(x_i, x_j, \vartheta, \theta)$ and a reverse arc connecting the node-time pair (x_j^μ, θ) with (x_i^μ, ϑ) with residual fuzzy arc capacity $\tilde{u}^{*\mu}(x_j, x_i, \theta, \vartheta) = \tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$ and transit time $\tau^{*\mu}(x_j, x_i, \theta, \vartheta) = -\tau^*(x_i, x_j, \vartheta, \theta)$.

Step 4. Search the augmenting shortest path (in terms of the number of arcs) $\tilde{P}_p^{*\mu}$ from the artificial source s^* to the artificial sink t^* in the constructed fuzzy residual network according to the Edmonds and Karp's algorithm from zero flow values (Edmonds and Karp 1972).

(I) Go to the step 5 if the augmenting path $\tilde{P}_p^{*\mu}$ is found.

(II) The flow value $\tilde{\phi}^* < \sum_{\tilde{l}(x_i, x_j, \vartheta, \theta) \neq \tilde{0}} \tilde{l}(x_i, x_j, \vartheta, \theta)$ is obtained, which is the maximum flow in $\tilde{G}_p^*$, if the path is failed to find. It means that it is impossible to pass any unit of flow, but not all the artificial arcs are saturated. Therefore, the time-expanded graph $\tilde{G}_p$ has no feasible flow as the initial dynamic fuzzy graph $\tilde{G}$and the task has no solution. Exit.

Step 5. Pass the minimum from the arc capacities $\tilde{\delta}_p^{*\mu} = \min[\tilde{u}^{*\mu}(x_i, x_j, \vartheta, \theta)]$, $(x_i, x_j) \in \tilde{P}_p^{*\mu}$, included in the path $\tilde{P}_p^{*\mu}$ along this path.

Step 6. Update the fuzzy flow values in the graph $\tilde{G}_p^*$: replace the fuzzy flow $\tilde{\xi}^*(x_j, x_i, \vartheta, \theta)$ along the corresponding arcs going from (x_j, ϑ) to (x_i, θ) from $\tilde{G}_p^*$ by $\tilde{\xi}^*(x_j, x_i, \vartheta, \theta) - \tilde{\delta}_p^{*\mu}$ for arcs connecting node-time pair (x_i^μ, θ) with (x_j^μ, ϑ) in $\tilde{G}_p^{*\mu}$ and replace the fuzzy flow $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$ along the arcs going from (x_i, ϑ) to (x_j, θ) from $\tilde{G}_p^*$ by $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{*\mu}$ for arcs connecting node-time pair (x_i^μ, ϑ) with (x_j^μ, θ) in $\tilde{G}_p^{*\mu}$. Replace $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$ by $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{*\mu} \times \tilde{P}_p^{*\mu}$.

Step 7. (I) If the flow value $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{*\mu} \times \tilde{P}_p^{*\mu}$ is less than $\sum_{\tilde{l}(x_i, x_j, \vartheta, \theta) \neq \tilde{0}} \tilde{l}(x_i, x_j, \vartheta, \theta)$, i.e. not all artificial arcs become saturated, go to the step 3.

(II) If the flow value $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{*\mu} \times \tilde{P}_p^{*\mu}$ is equal to $\sum_{\tilde{l}(x_i, x_j, \vartheta, \theta) \neq \tilde{0}} \tilde{l}(x_i, x_j, \vartheta, \theta)$, i.e. all arcs from the artificial source to the artificial sink become saturated, then the value $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{*\mu} \times \tilde{P}_p^{*\mu}$ is required value of maximum flow $\tilde{\sigma}^*$ In this case the total flow along the artificial arcs connecting the node-time pairs $(t,\ \forall\theta \in \mathrm{T})$ with $(s,\ \forall\theta \in \mathrm{T})$, which is equal to $\sum_{\theta=0}^{p} \tilde{\xi}^*(t, s,\ \forall\theta \in T, \forall\theta \in T)$ in $\tilde{G}_p^*$ determines the feasible flow in time-expanded graph $\tilde{G}_p$ with the flow value $\sum_{\theta=0}^{p} \tilde{\xi}^*(t, s,\ \forall\theta \in T, \forall\theta \in T) = \tilde{\sigma}$. Turn to the graph $\tilde{G}_p$ from the graph $\tilde{G}_p^*$ as following: reject artificial nodes and arcs, connecting them with other nodes. The feasible flow vector $\tilde{\xi} = (\tilde{\xi}(x_i, x_j, \vartheta, \theta))$ of the value $\tilde{\sigma}$ is defined as: $\tilde{\xi}(x_i, x_j, \vartheta, \theta) = \tilde{\xi}^*\ (x_i, x_j, \vartheta, \theta) + \tilde{l}(x_i, x_j, \vartheta, \theta)$, where $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$—the flows, going along the arcs of the graph $\tilde{G}_p^*$ after deleting all artificial nodes and connecting arcs. The network $\tilde{G}(\tilde{\xi})$ is obtained. Go to the step 8.

Step 8. Construct the residual network $G(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$ taking into account the feasible flow vector $\tilde{\xi} = (\tilde{\xi}(x_i, x_j, \vartheta, \theta))$ in $\tilde{G}_p$ adding the artificial source and sink and the arcs with infinite arc capacity, connecting s' with true sources and t' with true sinks according to the following rules: for all arcs, if $\tilde{\xi}(x_i, x_j, \vartheta, \theta) < \tilde{u}(x_i, x_j, \vartheta, \theta)$, then include the corresponding arc in $G(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$ with the arc capacity $\tilde{u}^{\mu}(x_i, x_j, \vartheta, \theta) = \tilde{u}(x_i, x_j, \vartheta, \theta) - \tilde{\xi}(x_i, x_j, \vartheta, \theta)$. For all arcs, if $\tilde{\xi}(x_i, x_j, \vartheta, \theta) > \tilde{l}(x_i, x_j, \vartheta, \theta)$, then include the corresponding arc in $G(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$ with the arc capacity $\tilde{u}^{\mu}(x_j, x_i, \theta, \vartheta) = \tilde{\xi}(x_i, x_j, \vartheta, \theta) - \tilde{l}(x_i, x_j, \vartheta, \theta)$.

Step 9. Define the shortest path $\tilde{P}_p^{\mu}$ according to the Edmonds and Karp's algorithm (Edmonds and Karp 1972) from s' to t' in the constructed residual network $G(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$.

(I) Go to the step 10 if the augmenting path $\tilde{P}_p^{*\mu}$ is found.

(II) The maximum flow $\tilde{\xi}(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{\mu} \times \tilde{P}_p^{\mu} = \tilde{v}(p)$ in $\tilde{G}(\tilde{\xi})$ is found if the path is failed to find, then the maximum flow in "time-expanded" static fuzzy graph can be found at the step 12.

Step 10. Pass the flow value $\tilde{\delta}_p^{\mu} = \min[\tilde{u}^{\mu}(x_i, x_j, \vartheta, \theta)], (x_i, x_j) \in \tilde{P}_p^{\mu}$ along the found path.

Step 11. Update the flow values in the graph $\tilde{G}_p$: replace the flow $\tilde{\xi}(x_j, x_i, \vartheta, \theta)$ by $\tilde{\xi}(x_j, x_i, \vartheta, \theta) - \tilde{\delta}_p^{\mu}$ along the corresponding arcs, going from (x_j, ϑ) to (x_i, θ) from $\tilde{G}_p$ for arcs, connecting node-time pair (x_i^{μ}, θ) with (x_j^{μ}, ϑ) in $\tilde{G}(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$ and replace the flow $\tilde{\xi}(x_i, x_j, \vartheta, \theta)$ by $\tilde{\xi}(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{\mu}$ along the corresponding arcs, going from (x_i, ϑ) to (x_j, θ) from $\tilde{G}_p$ for arcs, connecting node-time pair (x_i^{μ}, ϑ) with (x_j^{μ}, θ) in $\tilde{G}(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$ and replace the flow value in $\tilde{G}_p$: $\tilde{\xi}(x_i, x_j, \vartheta, \theta) \to \tilde{\xi}(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{\mu} \times \tilde{P}_p^{\mu}$.

Step 12. If the maximum flow $\tilde{\xi}(x_i, x_j, \vartheta, \theta) + \tilde{\delta}_p^{\mu} \times \tilde{P}_p^{\mu} = \tilde{v}(p)$ from the artificial source to the artificial sink in $\tilde{G}(\tilde{\xi})$ is found, we can define maximum flow $\tilde{v}(p)$ in $\tilde{G}_p$ rejecting the artificial nodes and arcs with flows, connecting them with artificial nodes and finding the total flow from the set of sources to the set of sinks for all time periods not later than p.

Step 13. Turn to the initial dynamic graph $\tilde{G}$ from the time-expanded static graph $\tilde{G}_p$ as follows: the maximum dynamic flow in the graph $\tilde{G}$ for p time periods is equal to the flow, leaving the set of sources for all time periods and entering the set of sinks for all time periods not later than p. Each path, connecting the node-time pairs (s, ϑ) with $(t, \varsigma = \vartheta + \tau_{st}(\vartheta))$, $\zeta \in T$, with the flow $\tilde{\xi}(s, t, \vartheta, \varsigma)$ passing along it in $\tilde{G}_p$ corresponds to the flow $\tilde{\xi}_{st}(\vartheta)$ in $\tilde{G}$.

7 Numerical Example

Let us consider a numerical example illustrating the implementation of the described algorithm. Transportation network is the part of the railway network and presented as a fuzzy directed graph, obtained from GIS « Object Land » (Fig. 1).

The node x_1 is the source, the node x_5 is the sink. Fuzzy lower and upper flow bounds and transit times depending on the flow departure time are represented in Tables 1 and 2. It is necessary to find maximum flow in dynamic graph, taking into account lower and upper flow bounds for 4 periods of time. We turn to the static graph, "time-expanded" for p periods of time from initial dynamic graph (Fig. 2). Add artificial nodes and arcs connecting them with artificial nodes according to the step 2 and turn to the graph without lower flow bounds $\tilde{G}_p^*$ (Fig. 3).

Connectors, which have the same shape (for example, ▲) link the corresponding pair of nodes in Fig. 3. Every arc, going from x_5 for all time periods to the nodes x_1 for all time periods have infinite upper flow bounds.

Find the first augmenting path $\tilde{P}_1^{*\mu}$ according to the Edmonds-Karp's algorithm in residual network $\tilde{G}_p^{*\mu}$: $\tilde{P}_1^{*\mu} = s^*, (x_4, 3),\ (x_5, 4),\ (x_1, 0),\ t^*$. Pass min $[(10,\ 1.5, 2),\ (25,\ 4,\ 5),\ \infty,\ (5,\ 0.5,\ 0.5)]$, i.e. $(5,\ 0.5,\ 0.5)$ flow units along the path $\tilde{P}_1^{*\mu} = s^*,\ (x_4, 3),\ (x_5, 4),\ (x_1, 0),\ t^*$, i.e. then flow $\tilde{0}$ turns to $(5,\ 0.5,\ 0.5) \times \tilde{P}_1^{*\mu}$. The flow value $(5,\ 0.5,\ 0.5) \times \tilde{P}_1^{*\mu}$is less than $\sum_{\tilde{l}(x_i,\, x_j,\, \vartheta,\, \theta) \neq \tilde{0}} \tilde{l}(x_i, x_j, \vartheta, \theta)$, so turn to the constructing of residual network with new flow value and find the second augmenting path $\tilde{P}_2^{*\mu}$ according to the Edmonds-Karp's algorithm in $\tilde{G}_p^{*\mu}$: $\tilde{P}_2^{*\mu} = s^*,\ (x_4, 3),\ (x_5, 4),\ (x_1, 1),\ (x_3, 2),\ t^*$.

Pass $[(5, 0.5, 0.5), (20, 3, 4), \infty, (15, 3, 2), (10, 1.5, 2)]$, i.e. $(5, 0.5, 0.5)$ flow units along $\tilde{P}_2^{*\mu} = s^*, (x_4, 3), (x_5, 4), (x_1, 1), (x_3, 2), t^*$ and the flow $(5, 0.5, 0.5) \times \tilde{P}_1^{*\mu}$ turns to $(5, 0.5, 0.5) \times \tilde{P}_1^{*\mu} + (5, 0.5, 0.5) \times \tilde{P}_2^{*\mu}$.

Fig. 1 Initial fuzzy graph $\tilde{G}$

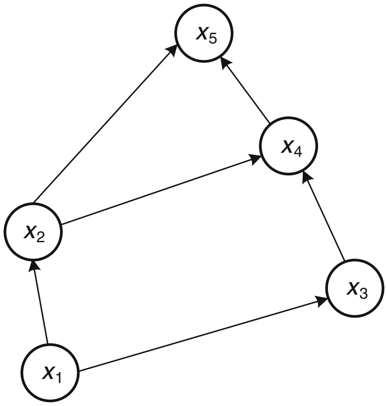


Table 1 Fuzzy lower and upper flow bounds depending on the flow departure time

(x_i, x_j)	$\theta = 0$	$\theta = 1$	$\theta = 2$	$\theta = 3$	$\theta = 4$
(x_1, x_2)	(5, 0.5, 0.5), (10, 1.5, 2)	(8, 1, 1)	(8, 1, 1)	(5, 0.5, 0.5), (10, 1.5, 2)	(18, 3, 3)
(x_1, x_3)	(18, 3, 3)	(15, 3, 2)	(18, 3, 3)	(18, 3, 3)	(15, 3, 2)
(x_2, x_4)	(25, 4, 5)	(20, 3, 4)	(25, 4, 5)	(8, 1, 1), (18, 3, 3)	(25, 4, 5)
(x_2, x_5)	(30, 5, 6)	(25, 4, 5)	(25, 4, 5)	(30, 5, 6)	(30, 5, 6)
(x_3, x_4)	(25, 4, 5)	(25, 4, 5)	(10, 1.5, 2), (20, 3, 4)	(8, 1, 1), (20, 3, 4)	(30, 5, 6)
(x_4, x_5)	(30, 5, 6)	(18, 3, 3) (25, 4, 5)	(18, 3, 3)	(25, 4, 5)	(25, 4, 5)

Table 2 Transit times depending on the flow departure time

(x_i, x_j)	$\theta = 0$	$\theta = 1$	$\theta = 2$	$\theta = 3$	$\theta = 4$
(x_1, x_2)	1	2	3	2	2
(x_1, x_3)	5	1	3	1	1
(x_2, x_4)	6	1	3	3	3
(x_2, x_5)	2	4	3	2	2
(x_3, x_4)	5	4	1	3	1
(x_4, x_5)	5	4	1	1	2

The flow value $(5, 0.5, 0.5) \times \tilde{P}_1^{*\mu} + (5, 0.5, 0.5) \times \tilde{P}_2^{*\mu}$ is less than $\sum_{\tilde{l}(x_i,\, x_j,\, \vartheta,\, \theta) \neq \tilde{0}} \tilde{l}(x_i, x_j, \vartheta, \theta)$, so turn to the constructing of residual network with new flow value and find the third augmenting path $\tilde{P}_3^{*\mu}$ according to the Edmonds-Karp's algorithm in $\tilde{G}_p^{*\mu}$: $\tilde{P}_3^{*\mu} = s^*, (x_2, 1), (x_4, 2), (x_5, 3), (x_1, 1), (x_3, 2), t^*$. Pass $[(5, 0.5, 0.5), (20, 3, 4), (18, 3, 3), \infty, (10, 1.5, 2), (5, 0.5, 0.5)]$, i.e. $(5, 0.5, 0.5)$ flow units along $\tilde{P}_3^{*\mu} =$

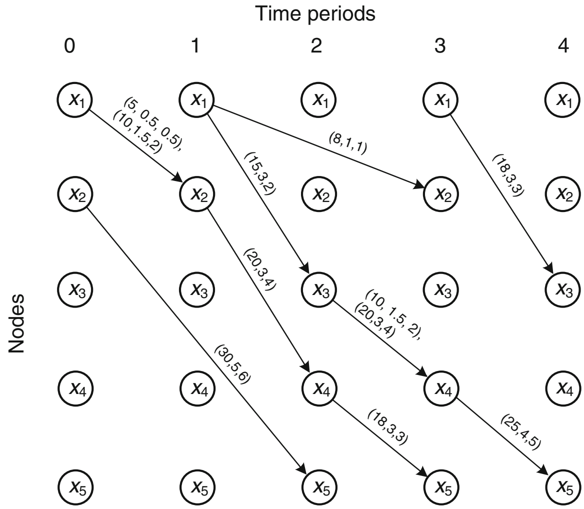


Fig. 2 $\tilde{G}_p$—time-expanded version of the graph $\tilde{G}$

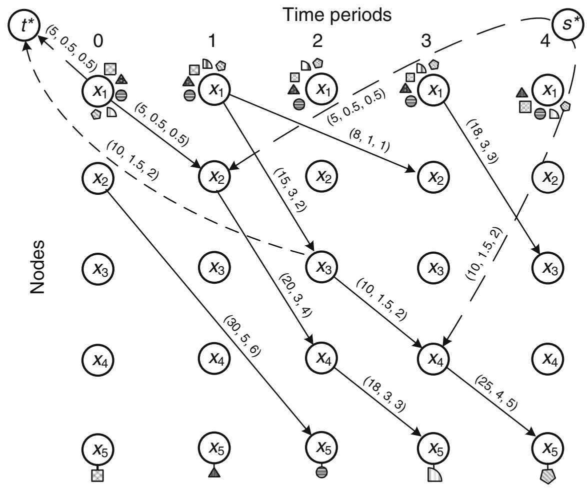


Fig. 3 Graph $\tilde{G}_p^*$ without lower flow bounds with artificial arcs

$s^*, (x_2, 1), (x_4, 2), (x_5, 3), (x_1, 1), (x_3, 2), t^*$ and the flow $(5, 0.5, 0.5) \times \tilde{P}_1^{*\mu} + (5, 0.5, 0.5) \times \tilde{P}_2^{*\mu}$ turns to $(5, 0.5, 0.5) \times \tilde{P}_1^{*\mu} + (5, 0.5, 0.5) \times \tilde{P}_2^{*\mu} + (5, 0.5, 0.5) \times \tilde{P}_3^{*\mu}$.

The flow value $(5, 0.5, 0.5) \times \tilde{P}_1^{*\mu} + (5, 0.5, 0.5) \times \tilde{P}_2^{*\mu} + (5, 0.5, 0.5) \times \tilde{P}_3^{*\mu}$ is equal to $\sum_{\tilde{l}(x_i,\, x_j,\, \vartheta,\, \theta) \neq \tilde{0}} \tilde{l}(x_i, x_j, \vartheta, \theta)$, so the maximum flow in "time-expanded" graph $\tilde{G}_p^*$ is found, which is equal to the sum of the lower flow bounds, going along the artificial arcs, i.e. $(10, 1.5, 2) + (5, 0.5, 0.5) = (15, 3, 2)$.

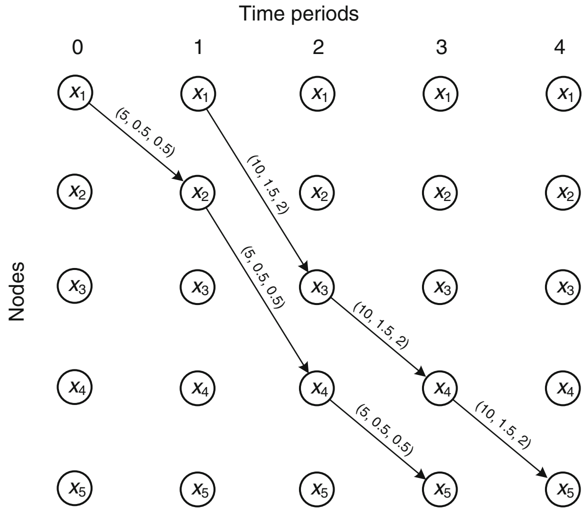


Fig. 4 Graph $\tilde{G}(\tilde{\xi})$ with the feasible flow $\tilde{\xi} = (\tilde{\xi}(x_i, x_j, \vartheta, \theta))$

Therefore, the feasible flow exist in $\tilde{G}_p$ and it is equal to the total flow, passing along the reverse arcs, connecting the nodes $(x_5,\ \forall\theta \in T)$ with $(x_1,\ \forall\theta \in T)$for all time periods, i.e. $(15, 3, 2)$.units. Construct the network with flow $\tilde{G}_p(\tilde{\xi})$, deleting artificial nodes and arcs and taking into account, that the feasible flow vector $\tilde{\xi} = (\tilde{\xi}(x_i, x_j, \vartheta, \theta))$ of the value $\tilde{\sigma}$ is defined as $\tilde{\xi}(x_i, x_j, \vartheta, \theta) = \tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$ $+\tilde{l}(x_i, x_j, \vartheta, \theta)$, where $\tilde{\xi}^*(x_i, x_j, \vartheta, \theta)$—the flows, passing along the arcs of the graph $\tilde{G}_p^*$ after deleting the artificial nodes and connected arcs. Construct a network with the feasible flow, as shown in the Fig. 4.

Introduce the artificial source and sink, connecting them with the true sources and sinks by the arcs with infinite arc capacities and construct the residual network for the graph in Fig. 4, as shown in Fig. 5.

Find the first augmenting path $\tilde{P}_1^\mu$ according to the Edmonds-Karp's algorithm in the residual network $\tilde{G}(\tilde{\xi}^\mu(x_i, x_j, \vartheta, \theta))$: $\tilde{P}_1^\mu = s', (x_1, 0), (x_2, 1), (x_4, 2), (x_5, 3),$ t'. Pass min $[\infty, (5, 0.5, 0.5), (15, 3, 2), (13, 2.4, 2), \infty]$, i.e. $(5, 0.5, 0.5)$ flow units along the path $\tilde{P}_1^\mu = s', (x_1, 0), (x_2, 1), (x_4, 2), (x_5, 3),\ t'$ and the flow $\tilde{\xi}(x_i, x_j, \vartheta, \theta)$ turns to $\tilde{\xi}(x_i, x_j, \vartheta, \theta) + (5, 0.5, 0.5) \times \tilde{P}_1^\mu$. Find the second augmenting path $\tilde{P}_2^\mu$according to the Edmonds-Karp's algorithm in the residual network $\tilde{G}(\tilde{\xi}^\mu(x_i, x_j, \vartheta, \theta))$: $\tilde{P}_2^\mu = s', (x_1, 1), (x_3, 2), (x_4, 3), (x_5, 4),\ t'$.

Pass min $[\infty, (5, 0.5, 0.5), (10, 1.5, 2), (15, 3, 2), \infty]$, i.e. $(5, 0.5, 0.5)$ flow units along the path $\tilde{P}_2^\mu = s', (x_1, 1), (x_3, 2), (x_4, 3), (x_5, 4),\ t'$ and the flow $\tilde{\xi}(x_i, x_j, \vartheta, \theta) +$ $(5, 0.5, 0.5) \times \tilde{P}_1^\mu$ turns to $\tilde{\xi}(x_i, x_j, \vartheta, \theta) + (5, 0.5, 0.5) \times \tilde{P}_1^\mu + (5, 0.5, 0.5) \times \tilde{P}_2^\mu$ (Fig. 6).

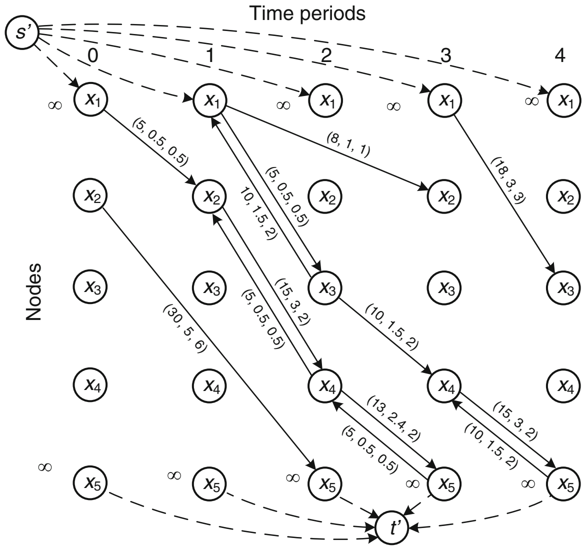


Fig. 5 Residual network $\tilde{G}(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$ for $\tilde{G}(\tilde{\xi})$

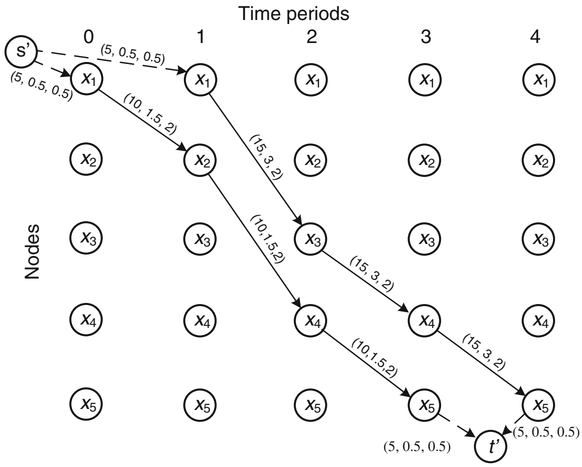


Fig. 6 Graph $\tilde{G}(\tilde{\xi})$ with the flow value c $\tilde{\xi} = (\tilde{\xi}(x_i, x_j, \vartheta, \theta))$

Construct the residual network $\tilde{G}(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$ for the graph in Fig. 6, as shown in Fig. 7. There is no augmenting path in the residual network in Fig. 7, therefore, the maximum flow in $\tilde{G}(\tilde{\xi})$ is found. Thus, the maximum flow in "time-expanded" graph $\tilde{G}_p$ can be found deleting artificial nodes and arcs with flows, connecting them with artificial nodes, as shown in Fig. 8. Turning to dynamic graph $\tilde{G}$ from expanded static graph $\tilde{G}_p$, we come to the conclusion, that given

Fig. 7 Residual network $\tilde{G}(\tilde{\xi}^{\mu}(x_i, x_j, \vartheta, \theta))$ for the graph $\tilde{G}(\tilde{\xi})$

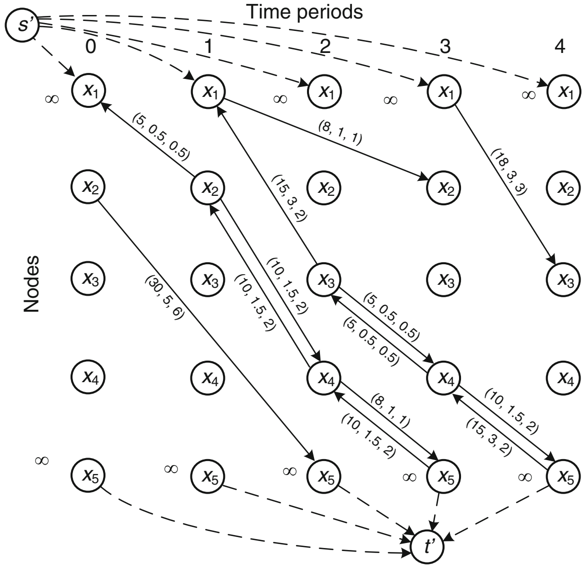
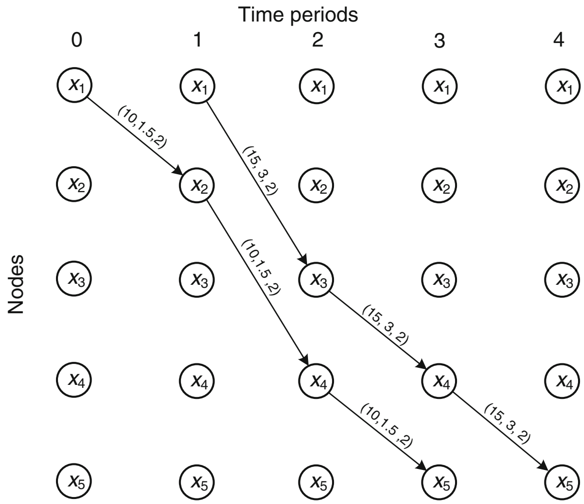


Fig. 8 Maximum flow in $\tilde{G}_p$



flow value for 4 time periods is equal to the flow, leaving from the "node-time" pairs $(x_1, 0)$ and $(x_1, 1)$ and entering the "node-time" pairs $(x_5, 3)$ and $(x_5, 4)$ i.e. $(25, 4, 5)$ flow units, which defined by a path $x_1 \to x_2 \to x_4 \to x_5$ which departs at $\theta = 0$ and arrives at the sink at $\theta = 3$ and by a path $x_1 \to x_3 \to x_4 \to x_5$ which departs at $\theta = 1$ and arrives at the sink $\theta = 4$.

8 Conclusion and Future Studies

This chapter describes the optimization flow problems that arise when considering transportation networks in fuzzy environment. Literature review considering flows and basic problem statements is given. The necessity of representing network parameters in a fuzzy form is justified. The present paper describes the problem statement and solution of the maximum flow problem with lower flow bounds in fuzzy conditions. Literature analysis in the field of dynamic graphs is presented and the necessity of considering such types of roads is explained. The task of maximum flow finding with lower flow bounds in fuzzy conditions in dynamic network is solved. The relevance of the described problem formulations is that algorithms of the tasks solving can be imbedded in the real road networks when it is necessary to find maximum amount of traffic that must be transferred from the source to the sink in the real road and weather conditions as well as the necessity to introduce profitability factor. The field of our future researches is various optimization tasks in dynamic fuzzy networks, in particular, developing of minimum cost finding algorithms and minimum cost finding algorithms with lower flow bounds in fuzzy dynamic networks. These algorithms have important practical value and allow to find transportation routes of optimal cost in different types of roads.

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Supply Chain Configuration as a Cooperative Game with Fuzzy Coalitions

Leonid B. Sheremetov and Alexander V. Smirnov

Abstract The chapter considers the problem of flexible supply chains (FSCs) configuring in a highly dynamic economic environment. A novel coalition formation mechanism is proposed, which helps to resolve conflictions between the objectives of the FSC participants and to agree upon effective solutions. This mechanism is based on a generalized model of the core of a cooperative game with fuzzy coalitions. The implementation of the proposed model for configuring of an automotive FSC is described. Simulation results are discussed.

Keywords Cooperative game · Core · Supply chain · Configuring

1 Introduction

Nowadays agility, reactivity, flexibility and adaptivity of a supply chain play a key role for the success of an enterprise in gaining competitiveness. As a consequence, new organizational forms of enterprise integration emerge to address these challenges resulting in more agile structures of federated enterprises known as adaptive, agile and open supply chains and networks (Surana et al. 2005; Garavelli 2003). These organizations are based on the principles of partnership between the enterprises, agile network structures instead of linear chains and are driven by novel business strategies based on the product demand (Fig. 1). A self-organizing

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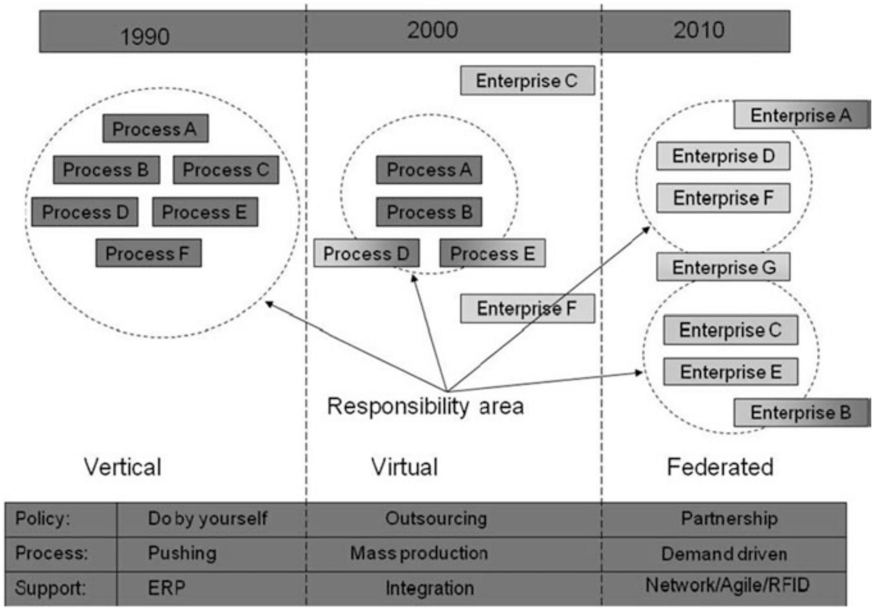

Fig. 1 Organizational forms of enterprise integration (adopted from McBeath et al. 2010)

Flexible Supply Chain (FSC) is an interconnected network of multiple entities (or self-interested agents) that exhibit adaptive action in response to changes in both the environment and the system of entities itself (Choi et al. 2001). A good example of such adaptivity is Build-to-Order (BTO) supply chains strategy, when customer orders are introduced prior to production, so that production channels have to be dynamically configured under demand (Gunasekarana and Ngaib 2005; Kathawala and Wilgen 2005). In contrast to conventional supply chains, FSCs are characterized by:

- availability of alternative providers,
- availability of alternative configurations meeting order's specifications,
- expediency of dynamic configuration and reconfiguration of the network depending on the order stream and economic benefit of every enterprise,
- conflicting objectives of each organization and non-integrated decision making processes.

A FSC belongs to the class of systems with dynamically changing structures, which means that once a new order comes, a new configuration emerges. Thus FSC configuring can be considered one of the main supply chain management tasks (Chandra and Grabis 2007). Traditionally, configuring has been solved in a two-stage fashion: (i) a structure of a network is formed at a strategic level and (ii) its behavior is optimized at tactical and operational levels based on demand forecast. Being suitable for vertical and even virtual enterprises (Fig. 1), such practices, unfortunately, do not meet the requirements of a highly dynamic environment (Smirnov 1999). One of the consequences is a so-called bullwhip effect, when even small demand fluctuations in a loosely balanced forecast-driven distribution systems lead to increased inventories, and, as a consequence, spatial constraints, unused capital, obsolete inventories and so on (Suckya 2009). Fuzzy estimations of demand forecasts and the use of collaborative management strategies can effectively reduce the bullwhip effect and the amplification of inventory variance (Net Stock Amplification) in a two-level (manufacturer and end customer) and three-level supply chains for different demand patterns (Campuzano et al. 2010).

Federated enterprises composed of self-interested entities with probably conflicting goals require flexible dynamic configurations. Unfortunately, adoption of more flexible and dynamic practices, like constraint satisfaction, auctions and knowledge-based approaches, which offer the prospect of better matches between suppliers and customers as market conditions change, has faced difficulties, due to the complexity of many supply chain relationships and the difficulty of effectively supporting dynamic trading practices (Campuzano et al. 2010; Sandkuhl et al. 2007; Smirnov et al. 2006). Due to the conflictions among the objectives of each organization and non-integrated decision making processes, there has been a need for new mechanisms, which could help to resolve those conflictions and to agree upon effective solutions.

For most of the approaches to configuring supply chains it is typical to concentrate on the processes of the chain forming without taking into account the profit obtained as a result of collaboration within the chain (McBeath 2010; Beamon 1998; Michalewicz 2007; Olhager and Rudberg 2002; Rudberg 2004). The characteristic feature of the chain is that during such collaboration the partners sustain the relations both of competency and cooperation. That is why, the cooperative game theory is related to cooperative integrated enterprises and flexible supply chains, where the key question is the selection of the appropriate partner guaranteeing the efficient work of the whole system (Guo 2008). The theory of cooperative games provides a formal approach to solving this task.

For advanced demand-driven or build-to-order (BTO) supply chains business strategies, a task of configuring of virtual production channels can be defined as a coalition formation task (Smirnov and Sheremetov 2012). The benefit distribution among the FSC members has proved to be fuzzy, uncertain, and ambiguous (Roth 1995; Hosam and Khaldoun 2006). Using the theory of fuzzy cooperative games (FCGs), the uncertainty can be processed by means of the introduction of a fuzzy benefit concept through the bargaining process to the conclusion about the corresponding fuzzy distribution of individual benefits among the coalition members. A game-theoretic approach is used to form coalitions among the FCS partners. A class of FCG with core solution concept is considered.

The basic definition of the fuzzy core was proposed by Mareŝ (2001). This chapter integrates the results obtained by the authors in the past decade (Smirnov et al. 2006; Sheremetov and Romero-Cortes 2003; Smirnov et al. 2004; Sheremetov 2009), where they (a) develop a framework for FSC configuring based on FCG, (b) generalize the core definition by introducing fuzzy individual payments and binary values φ_{ij} to form the structure of effective coalitions, (c) use membership functions (MF) to represent the player's degree of satisfaction of the payoffs, (d) develop a negotiation mechanism for the partners to form a core and (e) propose an effective solution method based on genetic algorithms.

The rest of the chapter is structured as follows. In Sect. 2, a FSC configuring task is defined as a problem of coalition formation. In Sect. 3, different approaches to coalition formation in cooperative games theory are analyzed. In Sect. 4, the mathematical structure of the model is described; it is shown that the model represents an extension of the model proposed in Mareŝ (2001). Section 5 describes a negotiation algorithm developed to construct a core of the game by the players and to distribute the obtained solution. A case study applying the proposed approach is discussed in Sect. 6. A prototype consisting of seven enterprises and a structure of three coalitions is considered. Finally, the obtained results are discussed in Sect. 7.

2 Supply Chain Configuring as a Coalition Formation Problem

The setting of the configuring task in the context of coalition formation is defined by the principles of the FSC forming. It is supposed that a new production channel is created each time a new order enters the system and there is no central control unit that could influence upon a suppliers selection of other units. Thus, each node is responsible for selecting its partners to fulfill an order.

2.1 A Generic Configuration Pattern

Such conceptualization permits to define a generic pattern of a FSC, which can be further used to define a configuring task. When a production system (such as a supply chain) is considered, each new demand to be fulfilled assumes that there is some amount of work to be done and some facilities which can perform this work (FSC nodes with associated resources). The work consists of several operations (parallel and/or sequential tasks), which should convert the raw materials into a product. Supply chain consists of production units capable to perform a number of tasks. Every node (agent) of FSC is described as a set of competencies of a certain capacity and associated attributes/properties. Both products and units with the associated competencies are described in the application domain ontology.

Configuring deals with creating configuration solutions and selecting components and ways to configure these. As shown in Fig. 2, in FSC each unit forms a production channel with its direct suppliers ("first-level suppliers"). Example unit

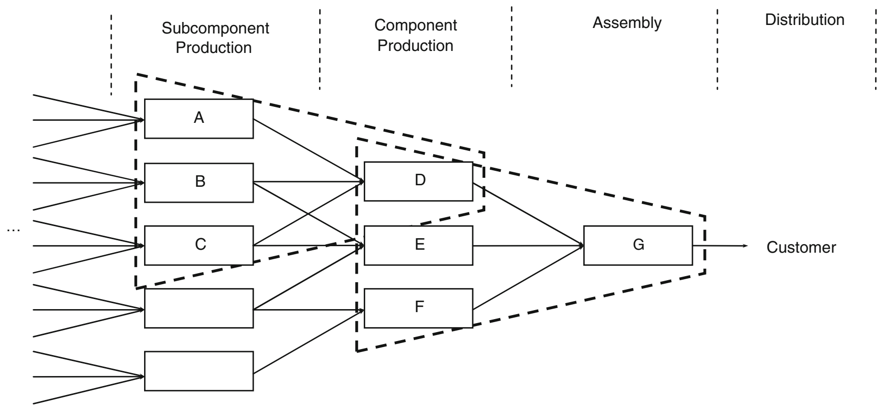

Fig. 2 Structure of a generic FSC pattern

D can choose its suppliers from units A, B and C, and unit G can choose its suppliers from units D, E and F. In this case, a generic FSC pattern can be defined (shown with dashed lines) as a group consisting of a unit with its first-level suppliers, trying to maximize the profit obtained as a result of collaboration within the channel. All the units have uncertain expectations of their possible profits. This allows considering this generic FSC pattern as a configuration pattern.

Our approach to the FSC configuring is based on the following assumptions:

- Each FSC partner is represented by an agent characterized by a set of available competencies necessary to complete the demand. Agents join their capacity of competencies to satisfy the requirements of an order forming coalitions.
- All agents are responsible for forming their preferences expressed in terms of membership functions (MF). These functions define their expectations of a payoff assigned as a reward for participating in a coalition fulfilling an order or a task according to some benefit-distribution strategy.
- Each coalition is represented by an agent as well. Any coalition agent acts on behalf of the coalition members including the negotiation, resource assignment and benefit distribution according to the signed contract. Coalition agent controls the interactions among the agents within a coalition initiating negotiation process.
- Coalition agent is also characterized by a membership function which integrates the MFs of the potential coalition members.
- Integrator agent obtains the solution of a game and distributes it among the players of all coalitions. This solution defines a structure of affective coalitions and rational distribution of the payoffs among the players. A structure of effective coalitions corresponds to an optimal configuration of a FSC.

2.2 Formal Definition of Configuring as a Coalition Formation Problem

The task of configuring can be defined as a selection of those agents (enterprises), which have available competencies to complete the demand/order, and joining them together in the most efficient structure according to the selected criteria. The main components of the configuring task are: order, resource and configuration. Let us consider that to fulfill the order *T, I* tasks should be executed: $T = \{T_1, T_2, \ldots, T_I\}$. Each task T_i $(i = 1, 2, \ldots, I)$, is defined by a tuple $\langle\{B_{T_i}\}, \{\mathrm{Pref}_{Ti}\}\rangle$, where B_{T_i}—is a vector of numerical values of dimensionality r: $B_{T_i} = (b^1_{T_i}, b^2_{T_i}, \ldots, b^r_{T_i})$, $b^k_{T_i} \geq 0$, characterizing a capacity on each competency $k = 1, 2, \ldots, r$, required to perform a task T_i. If the tasks are ordered, then $T_1 \preceq T_2 \preceq \ldots \preceq T_l \preceq T_m \preceq \ldots \preceq T_I$, where $T_l \preceq T_m$ means that T_l precedes the task T_m. The preferences vector Pref_{Ti} may include additional parameters like the preferred lot size, penalties for backorders, etc. Fulfillment of each order T and each task T_i implies a payoff: *Payoff*(T_i).

Example. Suppose that the order is to produce 100 products (cars of a specific model) per week. A car consists of four basic components: (1) the body T_1 (14 external tubes $b^1_{T_1}$ and 5 exterior sheets $b^2_{T_1}$); (2) the interior T_2 (a dash board $b^1_{T_2}$, 3 seats $b^2_{T_2}$: two front and one rear); (3) the chassis T_3 (4 wheels $b^1_{T_3}$, 2 axles $b^2_{T_3}$: a front and a rear, 4 dampers $b^3_{T_3}$: two front and two rear); (4) the power train T_4 (a motor $b^1_{T_4}$ and a transmission $b^2_{T_4}$). In other words: $\mathrm{T} = \{\mathrm{T}_1, \mathrm{T}_2, \mathrm{T}_3, \mathrm{T}_4\}$, $B_{T_1} = (1400, 500)$, $B_{T_2} = (100, 300)$, $B_{T_3} = (400, 200, 400)$, $B_{T_4} = (100, 100)$.

The enterprises of the supply chain represent resources. Depending upon their role in the FSC, these resources can be suppliers of raw materials and components, assembly plants or warehouses. They are modeled as active autonomous entities with purposeful actions and, thus, may be called agents. Let us consider a finite set of agents $Agent = \{A_1, A_2, \ldots, A_N\}$. Then each agent $A_j \in Agent$ $(j = 1, 2, \ldots, N)$ is defined as a tuple $\langle\{B_{A_j}\}, \{\mathrm{Pref}_{A_j}\}\rangle$. For simplicity let's designate A_j as j. Then B_j—is a vector of numerical values of the dimension r: $B_j = (b^1_j, b^2_j, \ldots, b^r_j)$, $b^{k'}_j \geq 0$, characterizing agent's available capacity on each competency $k' = 1, 2, \ldots, r$. The preferences vector $Pref_k$ denotes agents' preferences on the lot size, orders time lag, etc. Please note that for the model of the cooperative game no preferences for tasks and agents are considered, Pref_{Ti} and $Pref_k = 0$.

Finally, a configuration is such a set of agents (resources) $C_T \subseteq Agent$ that their joint capacity of competencies satisfies the requirements of an order T. To solve the configuring task for the case when the agents' and tasks' competencies coincide ($b^k_{T_i}$ and $b^{k'}_j$ mean the capacities on the same competency) means to assign resources to the tasks in such a way that the order T is fulfilled. Each agent $A_j \in Agent$ may be assigned to a task T_i iff it has available capacities $\exists k \in \{1, 2, \ldots, r\}$, $b^k_j \geq 0$. Being self-interested, each agent will try to optimize this assignment according to one of the following criteria:

- Maximize the use of his capacities $\sum_{k=1}^{r}\left(b_j^k - b_{T_i}^k\right) \rightarrow \min$;
- Get the most profitable task (to increase the payoff) $\sum_{k=1}^{r} g(b_{T_i}^k) - \sum_{k=1}^{r} f_{T_i}(b_j^k) \rightarrow \max$, where $g(b_{T_i}^k)$—is a reward function associated with the payoff. $Payoff(T_i)$, $f_{T_i}(b_j^k)$—a cost of agent's $j \in Agent$ capacity b_j^k required to fulfill the task T_i;
- Reduce the task T_i fulfillment time: $\sum_{k=1}^{r} t_{T_i}\left(b_j^k\right) \rightarrow \min$, where $t_{T_i}\left(b_j^k\right)$—time of fulfillment of the task T_i by agent $k \in Agent$ using his capacity b_j^k.

Agents can form coalitions to execute tasks. The notion of coalition is widely used in organizational systems. A coalition can be defined as a group of self-interested agents that by means of negotiation protocols decide to cooperate in order to solve a problem or to achieve a goal (Gasser 1991). Within the context of this chapter, a coalition is defined as a group of agents joining their capacities for task T_i fulfillment. A coalition is described by a tuple: $\langle K_{T_i}, alloc_{T_i}, u_{T_i}\rangle$, where $K_{T_i} \subseteq Agent$ and $K_{T_i} \neq \emptyset$; $alloc_{T_i}$—an allocation function assigning each task i a group of m agents such that $alloc_{T_i} = K_{T_i}$, if $\sum_m b_m^k \geq b_{T_i}^k$. If for each competency k, $b_j^k \geq b_{T_i}^k$, K_{T_i} may consist of a single agent $j \in K_{T_i}$, then $alloc_{T_i} = j$. The coalition of all agents involved in the order's T execution is called grand coalition K_T. The utility of a coalition is defined by a characteristic function: $v(K_{T_i}) = Payoff(T_i) - \sum_k \sum_j f_{T_i}(b_j^k) \cdot \varphi(T_i, k, j)$, where φ is a binary variable that determines agent's participation in task completion with its capacity b_j^k:

$$\varphi(T_i, k, j) = \begin{cases} 1, & \text{if the agent executes } b_j^k \\ 0, & \text{otherwise.} \end{cases}$$

The coalition's utility $v(K_{T_i})$ is distributed between the coalition members according to the vector of payment distribution $u_{T_i} = \left\{u_{T_i}^1, u_{T_i}^2, \ldots, u_{T_i}^{|K_{T_i}|}\right\}$, where $u_{T_i}^j$ is a payment to agent $j \in Agent$, and $u_{T_i}^{|K_{T_i}|}$ is a payment to the coalition. If within a coalition K_{T_i}, an agent j provides several competencies then $u_{T_i}^j = \sum_k g_j(b_{T_i}^k)$, $g_j(b_{T_i}^k) = \frac{b_{T_i}^k}{\sum_{l=1}^{r} b_{T_i}^l} v(K_{T_i}) \cdot \varphi(T_i, k, j)$ is satisfied.

The grand coalition K_T, joining together all the agents participating in the order's fulfillment corresponds to the configuration of the supply chain C_T. Thus to form a coalition means to find the appropriate coalition structure which permits to maximize the payoff for all agents belonging to this structure.

3 Coalition Formation in Cooperative Game Theory

Until recently, in the domain of supply chains and networks management, non-cooperative game theory was usually used for modeling of the competing enterprises as zero-sum (strictly competitive) and non zero-sum games (Cachon and Netessine 2004). In that context, all the players are considered being self-interested trying to optimize their own profits. The main purpose of such a game is to find the optimal strategy for each player and determine if the obtained strategy coordinates the supply chain, i.e. maximizes the global profit.

A competitive game has also been defined in fuzzy settings both for strategies (for different levels of significance and intensity) and a payoff function (in terms of excellent, good, or sufficiently reliable, durable, resistant). Unfortunately, the lack of operationalization had not allowed them to become practically used until several solution methods were proposed. In the paper by Campos et al. (1992), a general method of solving a matrix game with fuzzy pay-offs was presented. The above method may be used when players choose their fuzzy number ranking procedures in a wide class represented by linear ranking functions. The authors studied both the case when the players used the same criterion to rank fuzzy numbers, and when each player used different criteria. Bector and Chandra (2005) considered a problem of solving a matrix game with fuzzy pay-offs based on the principle of duality in linear programming. Peldschus and Zavadskas (2005) combined fuzzy sets and matrix game theories for multi-criteria decision-making. They defined a fuzzy set for the set of strategies of each player.

In the book by Sakawa and Nishizaki (2009), the authors applied different solution concepts like fuzzy programming, multiobjective programming, stochastic programming, and genetic algorithms to noncooperative and cooperative decision making in hierarchical organizations, using multiobjective and two-level linear programming. The discussed applications in supply chain management range from a work force assignment and transportation problems to inventory and production management.

The cooperative nature of federated enterprises causes necessity of considering FSC within the context of cooperative game theory in order to model and understand the behavior of cooperating partners. The principal difference between both approaches lies in different assumptions about the nature of the game and of the rational behavior of the players. In other words, cooperative games are considered in those cases when the players can form coalitions. In the context of FSC configuring, the theory of cooperative games offers results that show the structure of possible interaction between partners and the conditions required for it. N-person cooperative games (coalition games) were proposed in 1944 by von Neumann and Morgenstern (1944); since then a variety of models has been developed. The main questions they try to answer are: what coalitions will be formed, how the common wealth will be distributed among them and if the obtained coalition structure is stable. Once coalitions are formed and they have a feasible set of payoffs available to its members, the question is the identification of final payoffs awarded to each player. That is, given a collection of feasible sets of payoffs, one for each coalition, can one predict or recommend a payoff (or set of payoffs) to be awarded to each player?

The models are usually classified based upon the type of the environment and the principles of the payoff distribution (Fig. 3). The environment of the game can de superadditive and subadditive. Usually, coalitions joining together can increase the wealth of their players. If they form a single coalition (grand coalition), the only question is to find acceptable distributions of the payoff of the grand coalition. But in the latter case, at least one coalition does not meet this condition. The payoff distribution should guarantee the stability of the coalition structure when no one player has an intention to leave a coalition because of the expectation to increase its payoff. Moreover, profit distribution can be fuzzy, uncertain, and ambiguous (Mareŝ 2001). Using the theory of fuzzy cooperative games (FCGs), one can process the uncertainty and pass from the introduction of a fuzzy profit concept through the bargaining process to the conclusion about the corresponding fuzzy distribution of individual payoffs (Aubin 1981).

Due to the model complexity, most of the models of cooperative games have been developed for superadditive environments and, for fuzzy settings, allowing to consider only linear membership functions. Nevertheless, for realistic applications additive environments and the absence of the restrictions on the type of membership functions is a time challenge.

The predictions or recommendations of payment distribution are embodied in different solution concepts. According to Kahan and Rapoport (1984), cooperative games can be divided into two classes based on the way a solution of the game is obtained: games with a solution set and games with a single solution. To the former class belong the approaches of the stable sets (Von Neumann and Morgenstern 1944), the core (Gillies 1953), the kernel (Davis and Machler 1965) and bargaining set (Aumann and Maschler 1964). To the latter—Shapley value (Shapley 1953), τ value in the TU-games (Tijs 1981) and the nucleolus (Schmeidler 1969).

Core and Stable sets are two widely used mechanisms for analyzing the possible set of stable outcomes of cooperative games with transferable utilities. The concept of a core is attractive since it tends to maximize the so called social wealth, i.e. the sum of coalition utilities in the particular coalition structure. Such imputations are called C-stable. The core of a game with respect to a given coalition structure is defined as a set of such imputations that prevent the players from forming small coalitions by paying off all the subsets an amount which is at least as much they would get if they form a coalition (we proceed with a formal definition of a core in the following section). Thus the core of a game is a set of imputations which are stable. The problem of the core is that, on the one hand, the computational complexity of finding the optimal structure is high since for the game with n players at least $2^{|n|-1}$ of the total $|n|^{|n|/2}$ coalition structures should be tested. On the other hand, for particular classes of the game a core can be empty. Because of these problems, using the C-stable coalition structures was quite unpopular in practical applications (Klusch and Gerber 2002) and only recently

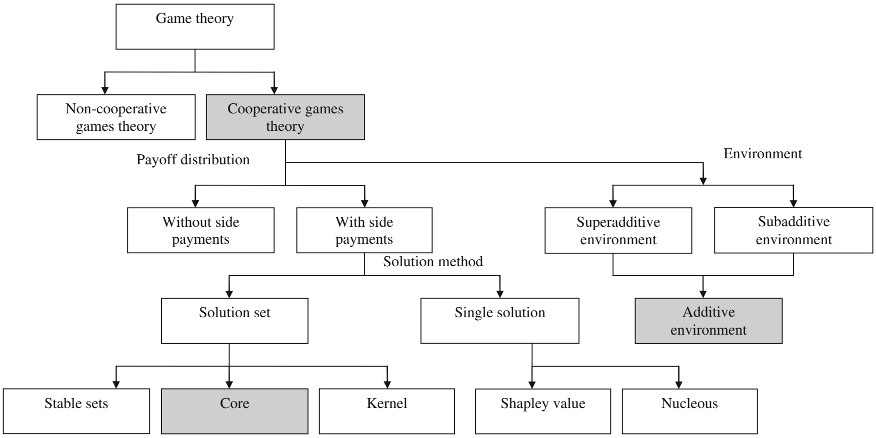

Fig. 3 Cooperative games' taxonomy

has attracted more attention of the researchers (Marchi et al. 2009; Shen and Gao 2010).

According to Serrano (2009), the core and Shapley value, as probably two most widely used solution concepts, can be seen also as two different ways of understanding of a prediction mechanism of the likely outcome of the interaction among the players, and hence, the resulting payoff. In the former case, the payoff distribution is understood as the natural consequence of the forces at work in the system. While in the latter, it reflects a normative or prescriptive approach, setting up a number of normative goals and trying to derive their logical implications. In certain respects, these two approaches to the payoff allocation conflict with each other, but also there is an overlap or agreement between them (Gilles 2010). As a consequence, there are situations in which the Shapley value is not a core allocation, while in other situations the value is a central allocation in the core. For more details see Gilles (2010).

In the following sections, it is shown that most of the problems of the core can be solved in a proposed generalized model.

4 Generalized Model of the Core of a Fuzzy Cooperative Game

A FCG is defined as a pair $(Agent, w)$, where $Agent$ is nonempty and finite set of players, subsets of $Agent$ joining together to fulfil some task T_i are called coalitions K, and w is called a characteristic function of the game, being $w: 2^n \rightarrow \Re^+$ a mapping connecting every coalition $K \subset Agent$ with a fuzzy quantity $w(K) \in \Re^+$, with a membership function $\mu_K : R \rightarrow [0, 1]$. A modal value of $w(K)$ corresponds to the characteristic function of the crisp game $v(K)$: max $\mu_K(w(K)) = \mu_K(v(K))$.

For an empty coalition $w(\varnothing) = 0$. A fuzzy core for the game $(Agent, w)$ with the imputation $X = (x_{ij})_{i \in I, j \in Agent} \in \Re^+$ as a fuzzy subset C_F of $\Re^+$:

$$C_F = \left\{ x_{ij} \in \Re^+ : v \succ= \left(w(Agent), \sum_{\substack{i \in I, \\ j \in Agent}} x_{ij}\varphi_{ij} \right), \min_{\substack{K_i \in \bar{k} \\ j \in Agent}} \left(v \succ= \left(\sum_{j \in K_i} x_{ij}\varphi_{ij}, w(K_i) \right) \right) \right\}, \tag{1}$$

where x_{ij} is the fuzzy payment of an agent j participating in a coalition i, $i = 1, 2, \ldots, I$, $j = 1, 2, \ldots, N$, $\bar{k} = [K_1, K_2, \ldots, K_l]$ is the ordered structure of effective coalitions; $\succ=$—is a fuzzy partial order relation *w*ith a membership function $v \succ= : R \times R \to [0, 1]$, and φ_{ij} is a binary variable such that:

$$\varphi_{ij} = \begin{cases} 1, & \text{if an agent } j \text{ participates in a coalition } i; \\ 0 & \text{otherwise.} \end{cases}$$

This variable can be considered as a result of some agent's strategy on joining a coalition. A fuzzy partial order relation is defined as follows (for more details see Zadeh (1971)).

Definition 1. Let *a, b* be fuzzy numbers with membership functions μ_a and μ_b respectively, then the possibility of partial order $a \succ= b$ is defined as $v \succ= (a, b) \in [0, 1]$ as follows: $v \succ= (a, b) = \sup\limits_{\substack{x, y \in R \\ x \geq y}} (\min(\mu_a(x), \mu_b(y)))$.

The core C_F is the set of possible distributions of the total payment achievable by the coalitions, and none of coalitions can offer to its members more than they can obtain accepting some imputation from the core. The first argument of the core C_F indicates that the payments for the grand coalition are less than the characteristic function of the game. The second argument reflects the property of group rationality of the players, that there is no other payoff vector, which yields more to each player. The membership function $\mu_{C_F} : R \to [0, 1]$, is defined as:

$$\mu_{C_F}(x) = \min \left\{ v \succ= \left(w(Agent), \sum_{\substack{i \in I \\ j \in Agent}} x_{ij}\varphi_{ij} \right), \min_{\substack{K_i \in \bar{k} \\ j \in Agent}} \left(v \succ= \left(\sum_{j \in K_i} x_{ij}\varphi_{ij}, w(K_i) \right) \right) \right\} \tag{2}$$

With the possibility that a non-empty core C_F of the game $(Agent, w)$ exists:

$$\gamma_{C_F}(Agent, w) = \sup\left(\mu_{C_F}(x) : x \in \Re^n\right) \tag{3}$$

The solution of a cooperative game is a coalition configuration *(S, x)* which consists of (i) a partition *S* of *Agent*, the so-called coalition structure, and (ii) an efficient payoff distribution *x* which assigns each agent in *Agent* its payoff out of the utility of the coalition it is member of in a given coalition structure S. A coalition configuration *(S, x)* is called stable if no agent has an incentive to leave its coalition in S due to its assigned payoff x_i.

A game $(Agent, w)$ is defined as superadditive, subadditive, or simply additive for any two coalitions $K, L \subset Agent,\ K \cap L = \varnothing$ as follows:

$$\begin{aligned} w(K \cup L) &\succ= w(K) \oplus w(L)\text{---superadditive,} \\ w^*(K \cup L) &\prec= w^*(K) \oplus w^*(L)\text{---subadditive,} \\ w^*(K \cup L) &= w^*(K) \oplus w^*(L)\text{---additive,} \end{aligned} \tag{4}$$

where $\oplus$—is a sum of fuzzy numbers with a membership function defined as: $\mu_{a \oplus b}(x) = \sup\limits_{x,y \in R}(\min(\mu_a(y), \mu_b(x - y)))$, * defines superoptimal values of the corresponding coalitions (Mareŝ 2001).

The properties of the game are defined in three lemmas and two theorems (Sheremetov and Romero-Cortes 2003). One of them proves that the fuzzy set of coalition structures forming the game core represents a subset of the fuzzy set formed by the structure of effective coalitions. In turn, this inference allows us to specify the upper possibility bound for the core, which is a very important condition for the process of solution searching, because in this case, the presence of a solution that meets the efficiency condition may serve as the signal to terminate the search algorithm.

Definition 2. A coalition K is called effective if it can't be eliminated from the coalition structure by a subcoalition $L \subset K$. A set of effective coalitions is called a coalition structure. A possibility that a coalition K is effective is defined as follows: $\sup_{x \in R^n}(\min(\mu_k(x), \mu_l^*(x) : L \subset K))$.

Theorem. Let $(Agent, w)$ be a fuzzy coalition game. Then for some structure of effective coalitions $\bar{k}$, its possibility is at least equal to the possibility of forming the core.

Proof of the theorem. From formula (2), if all φ_{ij} are equal to 1, then we obtain the structure of coalitions that belong to the core; otherwise, the coalition structure corresponds to the generalized model. In addition, the inequality $v \succ= (\sum\limits_{j \in K_i} x_{ij}, \sum\limits_{j \in K_i} x_{ij}\varphi_{ij},\ \ i \in I)$ holds with positive possibility and, consequently, the possibility of the structure is higher for the generalized model than for the basic one.

It should be noted that the above statements take into account only the characteristics of the game $(Agent, w)$; therefore, any real argument can be introduced into the fuzzy core. For example, such restrictions as a number of agents in each coalition and those defining coalitions to be overlapping or not or regulating the tasks order are admissible. This feature is very important for the application of the model for FSC configuration management.

5 Implementation of the Model of Fuzzy Coalition Game

The model is implemented in the three-level architecture, where the problem domain agents work out the proposed configuring model based on FCGs (Fig. 4). Note that the game purpose is to generate an effective structure of agent coalitions for executing some production order. In turn, the generated structure of agent coalitions represents the optimal FSC configuration. In this setting, each partner of the FSC was supplied with a domain Agent playing one of the following roles: Supplier Agent (for the suppliers of the raw materials and components), Coalition Agent (for each head of the configuration pattern like the suppliers of the components, called Body, Motor and Transmission respectively in the case study) and Integrator Agent (called Assembly in the presented case study). Coalition Agents have been generated for each component every time a new demand occurred. The Integrator Agent (simulating the grand coalition) was the one who initiated negotiation protocols and decided the final configuration based on the computations performed by the game solvers.

All these agents are equipped with the contract net negotiation protocol (CNP) used to collect the biddings for the demand. They also have access to the techniques of solution search (linear programming, genetic algorithm, fuzzy number adder, functions of fuzzy nonlinear regression, etc.), which are represented with the modules of legacy software. These techniques are used to determine the game players on the basis of the domain model, to generate the individual membership functions of agents, to form the membership functions of coalitions, and, finally, to search for the FCG solutions. Genetic algorithms are implemented using the Evolver software (2001), accessed through the Excel Wrapper agent, while Fuzzy summator is a MATLAB component.

The steps of the algorithm that implements the FCG model described in Sect. 4 are listed below.

Step 1. *Order Specification.* At this stage, the parameters of the product's order are specified. The user can perform this operation by means of the configurator of the FSC simulation software or directly through the Integrator agent's API, which allows one to load the order's parameters and run the simulation to get results. Configurator also serves for testing the system, keeping, recovering and even modifying the tests. As soon as the order is generated, the integrator agent chooses the components required to satisfy it using the ontology, where all the order components can be found.

Step 2. *Identification of possible suppliers.* Every order component has a corresponding agent, which forms a coalition capable to produce this component. In other words, the agent is responsible for receiving all the sub-products needed to form the component. This agent is called the coalition agent. It receives an order for the components from the integrator agent and consults the domain ontology and the order specification. As soon as the agent gathers all the required information, it starts the FIPA contract net protocol (FIPA-CNP) to choose the suppliers for every sub-product. All supplier agents capable to produce the component

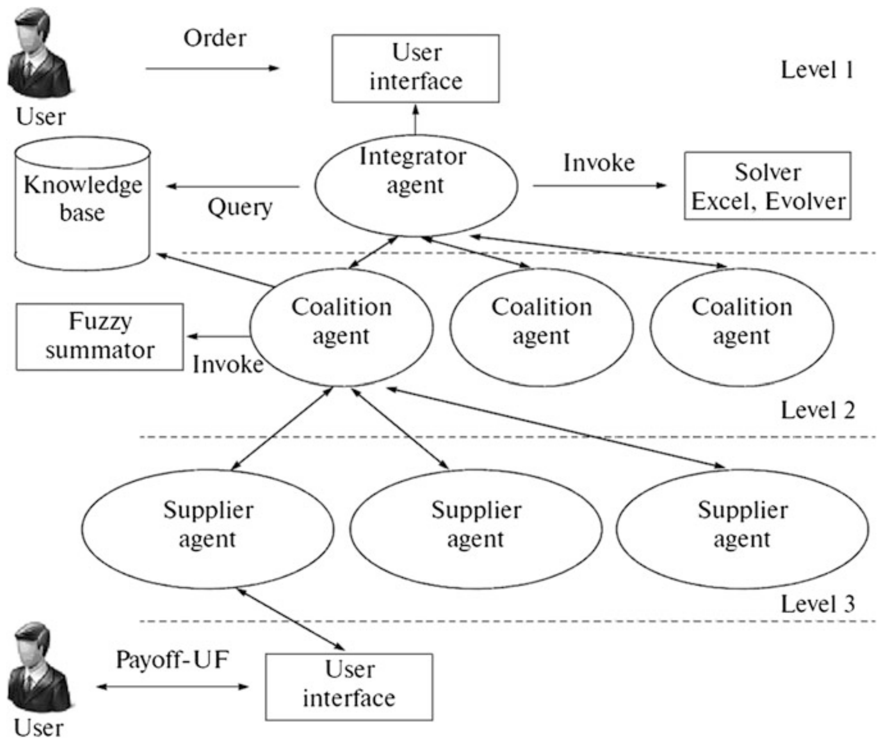

Fig. 4 Three-level architecture of multi-agent environment that implements FCG model

receive a message "call for proposals" (CFP) from the coalition agent and, if interested in the order, return a proposal with the utility function. The membership functions can be either generated automatically on the basis of the order history using the technique of nonlinear fuzzy regression to estimate the parameters of utility functions for player payments or assigned by the user. The supplier agents offer a corresponding graphical interface to specify their parameters.

Step 3. *Generation of coalition proposals.* After all proposals are received by the coalition agent, one for each sub-product, it calculates the membership function for the coalition by calling the fuzzy set summator. Thus, the algorithm of fuzzy number summation represents an important element of the model. The sum operation is based on Zadeh extension principle (Zadeh 1971) for fuzzy numbers a and b (which are convex sets normalized in R):

$$\mu_{a(*)b}(z) = \sup_{z = x^* y} \min(\mu_a(x),\ \mu_b(y)), \tag{5}$$

where * can designate the sum $\oplus$ or the product • of fuzzy numbers. Each fuzzy set is decomposed into two segments, a non-decreasing and non-increasing one. The operation * is performed for every group of n segments (one segment for each fuzzy set) that belong to the same class (non-decreasing or non-increasing one).

Thus, a fuzzy set is generated for every group of n segments. The summation result is derived as superposition of these sets, which gives the membership function as the sum of n fuzzy numbers. The obtained membership function is sent to the integrator agent.

Step 4. *Game solution.* Having received utility functions from every coalition and supplier agent, the integrator agent determines the best game solution. To find the analytical (exact) solution of the FCG, it is necessary to determine the fuzzy super-optimum and the fuzzy relation of domination (Mareŝ 2001), which is extremely difficult in real applications. Therefore, it is proposed to find solutions that are close to the optimal one using genetic algorithms (GA) in the context of fuzzy logic. It is equivalent to binary encoding of the fuzzy core with the fitness function equal to the supremum of all minimums of the membership function. Application of GA allows one to obtain an approximate solution for the games with a large number of players and a membership function of any type. Being an anytime algorithm that steadily improves the solution, the GA can find the best solution under the time constraints. Evolver component implements a genetic algorithm.

Step 5. *Distribution of results and FSC configuration selection.* In the case, a feasible solution was found at the previous step, it is sent to the coalition and supplier agents. As soon as all suppliers receive the payment proposal, they can decide whether it meets their interests or not. It is necessary to note that the derived decision certainly meets their wishes represented with the membership functions and the generated coalitions are effective (see Sect. 4). At the same time, the system is able to search for the decisions that increase an individual payment to the agent. If all agents accept the payment distribution, then "accept" messages are generated and the FSC configuration is formed. Otherwise, a "reject" message is sent to all system agents and a new attempt to configure the FSC is made either by simply replaying the same game to analyze another feasible solution or by choosing another game configuration (e.g., by changing the utility functions).

6 Case Study: A Cooperative Game for 3-Echelons Automotive FSC

The developed model of a cooperative game was used for configuring of an FSC's production channel for a specific car's model. The instantiation of the generic configuration patter for the case study is shown in Fig. 5. The case study deals with production of a hypothetic vehicle (a *Car*). The production process consists of the following two phases: *Component Production* and *Car Assembly*. *Component Production* consists of three parallel operations: *Body Production*, *Motor Production*, and *Transmission Production* (Smirnov et al. 2006).

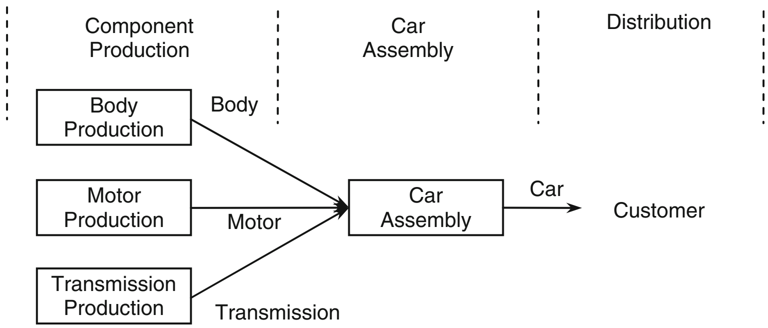

Fig. 5 Structure of the production process

The demand is represented by a uniform distribution around the linear trend:

$$d_t = a + b \cdot t + \sigma \cdot \mu, \tag{6}$$

where t—time, d—demand (d_t corresponds to time interval [t−1, t]), a—basis value, b—trend (equals 0 for a demand without trend), μ—random noise uniformly distributed within [0, 1], and σ—distribution amplitude. For the demand forecasting *Simple Moving Average (SMA)* is used:

$$f_{t+2} = f_{t+1} = \frac{\sum_{i=t-n+1}^{t} d_i}{n} \tag{7}$$

where f—forecast, n—forecast base.

Suppose that the FSC contains several enterprises capable of satisfying the demand both in components' production and vehicle's assembly. The configuring task can be defined as follows: to select an effective configuration of a production channel (both the enterprises and the demand's distribution between them) such that an ordered quantity of vehicles ($a = 100$) can be produced on five consecutive week intervals ($n = 5$) with a low noise ($\sigma = 5$) and without fluctuations associated with storing and delivery of the final and intermediate products. The enterprises pursuit a goal of maximizing their payoffs. The following parameters are considered: production capacity (units per week), production cost (per unit), stocking costs (per unit per week) and penalties for backorders (per unit per week). Stocks are unlimited. Payoffs for each component are fuzzy variables defined, for simplicity, by a uniform positive ramp membership function. The forecasting model for the demand is the following:

$$100 + 5t + 5\mu,\ for\ \ t = 1, \ldots, 5, \tag{8}$$

Component production can be performed by 6 enterprises, each with different competencies (Table 1). For simplicity, the competencies are restricted to the task level. The payoff for the assembled car is $20,000.

Table 1 Input data for the fuzzy cooperative game for automotive FSC configuring

Enterprise	Capacity (units per week)	Competency	Membership function (MF)	Component's price (parameters of the MF) ($)	Production cost ($)	Stocking cost (per unit per week) ($)	Penalties for backorders (per unit per week) ($)	Associated variable
1	100	Body	Positive ramp (+)	(6500–7000)	4500	250	400	$x_{11},\ \varphi_{11}$
2	100	Motor	Positive ramp (+)	(4500–5000)	3500	150	300	$x_{22},\ \varphi_{22}$
3	100	Transmission	Positive ramp (+)	(3800–4000)	2500	50	250	$x_{33},\ \varphi_{33}$
4	300	Body	Positive ramp (+)	(6500–7000)	4900	300	400	$x_{14},\ \varphi_{14}$
		Motor	Positive ramp (+)	(4500–5000)	3800	200	300	$x_{24},\ \varphi_{24}$
		Transmission	Positive ramp (+)	(3800–4000)	2700	80	250	$x_{34},\ \varphi_{34}$
5	100	Motor	Positive ramp (+)	(4500–5000)	3600	170	300	$x_{15},\ \varphi_{15}$
		Transmission	Positive ramp (+)	(3800–4000)	2600	60	250	$x_{25},\ \varphi_{25}$
6	200	Body	Positive ramp (+)	(6500–7000)	4700	270	400	$x_{16},\ \varphi_{16}$
		Motor	Positive ramp (+)	(4500–5000)	3600	170	300	$x_{26},\ \varphi_{26}$
7	150	Assembly	Positive ramp (+)	(2000–4000)	1500	750	1500	$x_{47},\ \varphi_{47}$

Table 2 The structure of the core of the cooperative game

Core's component	Definition
$C = \{2500x_{11t} + 2100x_{14t} + 2300x_{16t} + 1500x_{22t} + 1200x_{24t} + 1400x_{25t} +$ $1400x_{26t} + 1500x_{33t} + 2500x_{47t} + 1300x_{34t} + 1400x_{35t} \geq (100 + 5t + 5\mu)$ $w(Agent),$	Constraint on the grand coalition
$2500x_{11t} + 2100x_{14t} + 2300x_{16t} \leq (100 + 5t + 5\mu)\, w(k_1)$ $1500x_{22t} + 1200x_{24t} + 1400x_{25t} + 1400x_{26t} \leq (100 + 5t + 5\mu)\, w(k_2)$ $1500x_{33t} + 1300x_{34t} + 1400x_{35t} \leq (100 + 5t + 5\mu)\, w(k_3)$ $2500x_{47t} \leq (100 + 5t + 5\mu)\, w(k_4)$	Constraints on the components' coalitions
$x_{11t} + x_{14t} + x_{16t} \leq 100 + 5t + 5\mu$ $x_{22t} + x_{24t} + x_{25t} + x_{26t} \leq 100 + 5t + 5\mu$ $x_{33t} + x_{34t} + x_{35t} \leq 100 + 5t + 5\mu$ $x_{47t} \leq 100 + 5t + 5\mu$	Constraints on the forecasted demand for each component
$x_{11t} \leq 100 \quad x_{14t} \leq 300 \quad x_{16t} \leq 200$ $x_{22t} \leq 100 \quad x_{24t} \leq 300 \quad x_{25t} \leq 100 \quad x_{26t} \leq 200$ $x_{33t} \leq 100 \quad x_{34t} \leq 300 \quad x_{35t} \leq 100$ $x_{47t} \leq 150$	Capacity constraints on the payoffs
$x_{ijt} \in R^+,\ i = 1, \ldots, 4;\ j = 1, \ldots, 7\ \ t = 1, \ldots, 5\},$	

The order is decomposed into tasks which correspond to each car component's assembly. As a result, an effective structure of three coalitions (according to the number of the components) is to be formed considering capacity constraints. The structure of the core of the cooperative game is shown in Table 2. Additional constraints define the viability of the obtained solution.

The following notation is used: x_{ijt}—the quantity of the i component to be produced by agent j in time t, $w(I)$—fuzzy payoff per unit for car production, $w(k_1)$—fuzzy payoff per unit for Body Production, $w(k_2)$—fuzzy payoff per unit for Motor Production, $w(k_3)$—fuzzy payoff per unit for Transmission Production, $w(k_4)$—fuzzy payoff per unit for car assembly, and μ—uniform random variable in [0, 1]. The solution of the game obtained using Evolver package and genetic algorithms is shown in Table 3.

The total FSC payoffs per car obtained for each time interval are equal to 7578.46, 7578.22, 7578.22, 7577.84, 7577.84 respectively. The payoffs (p) of the participating enterprises per car/component are as follows: $p_1 = 2500$; $p_2 = 1500$; $p_3 = 1500$; $p_4 = 0$; $p\ _5 = 1400$ (motor and transmission); $p_6 = 2300$ (body); $p_6 = 1400$ (motor); $p_7 = 2500$. The same gross payoffs per enterprise were obtained for each time interval for each component: $w(I) = 20{,}000$; $w(k_1) = 7{,}000$; $w(k_2) = 5{,}000$; $w(k_3) = 4{,}000$; $w(k_4) = 4{,}000$. The possibility of the fuzzy game $\gamma_c(I, w) = 1.00$ (because of the simplicity of the case study), though the imputation obtained took into account the subjective estimations of the players defined by their fuzzy payments.

The analysis of the obtained solution shows the following. The constraint capacity of the first 3 units though having minimal production costs, does not permit them to satisfy all the demand. That is why, while demand is increasing,

Table 3 The coalition structure and the number of produced components for five time intervals

t	x_{11t}	x_{22t}	x_{33t}	x_{14t}	x_{24t}	x_{34t}	x_{25t}	x_{35t}	x_{16t}	x_{26t}	x_{47t}
1	100	100	100	0	0	0	5.299	5.3	5.3	0.001	105.3
2	100	100	100	0	0	0	8.399	11.5	11.5	3.101	111.5
3	100	100	100	0	0	0	10.25	15.2	15.2	4.95	115.2
4	100	100	100	0	0	0	14.50	23.7	23.7	9.20	123.7
5	100	100	100	0	0	0	16.75	28.2	28.2	11.45	128.2

other enterprises are involved in the production. In the case of Motor Production (k_2), the incrementing production of this component is assigned to both enterprises 5 and 6 (Table 3). If we compare the parameters of these enterprises (Table 2), it can be seen that they are the same both for the production cost ($3,600 per motor) and for stocking ($170 per motor/week). That means that the solution strategy looks for a balanced final solution.

In the conducted experiments on model complexity the number of iterations needed to approach the optimal solution served as the investigated variable with the following factors: the number of agents and coalitions, the accuracy, and the order of fuzzy payments. Results show that the number of iterations (computation time) decreases or remains constant when the number of agents increases. In other words, it takes less time to form coalitions. On the other hand, the results demonstrate almost linear relation between the numbers of coalitions and agents. On the whole, the experiments justified that all factors are highly significant; the only surprise was that the order of fuzzy payments substantially influence the number of iterations (the convergence time).

7 Conclusions

In this chapter, the approach to FSC configuration based on formation of enterprise coalitions as a result of a fuzzy cooperative game was considered. Uncertainty in realistic cooperation models occurs in two cases: when players participate in several coalitions, and when there exist fuzzy expectations of player and coalition benefits. The presented approach is mostly aimed at the latter case. This uncertainty of the agent payments may be caused by such dynamic events as production failures, changes in confidence estimations and reputations of potential coalition partners, and receiving unclear or even incomplete information and data during the task performance and negotiation.

The proposed model considers the coalitions' efficiency by introducing binary variables φ_{ij} into the fuzzy core. This permits not only to increase individual benefits for players but also the possibility to find an effective and stable agreement. Initially, all suppliers forming part of the general structure of the FSC are qualified to participate in the game. But the advantage of the proposed algorithm is that the structure of efficient (the best) coalitions is formed as a result of the game among the participants. So an inherent property of the model is that the best (for the particular task) subset of suppliers is chosen. Using the constraints of the application domain the number of viable coalitions can be significantly decreased, thus reducing the algorithmic complexity of the problem.

Though in the case study a positive ramp membership function was used (to be able to use also conventional Excel solver), the general solution method (applying genetic algorithms) permits the use of function of any type (linear or nonlinear, universal or not). Obviously, there is no guaranty that the obtained solution corresponds to a global optimum, but for a game with side payments, there is no algorithm to obtain the optimal solution.

An illustrative case study has been provided to show the applicability of the model. For automotive industry, where the suppliers form a strategic alliance (e.g. in case of Toyota company they even share the best technological practices), they can be considered cooperative. In case of competitive suppliers, a non-cooperative game model with non zero-sum could be used.

The fields of FCGs and dynamic coalition formation are still in their infancy and require further research efforts. For example, the notions of a superadditive FCG and a "stable" distribution of fuzzy payments in the games using fuzzy extension of the core and Shapley values were examined in Mareŝ (2001). Some aspects of application of the coalition game models to the development of dynamic coalition formation schemes were considered in Shapley (1953). Nevertheless, sub-additive fuzzy games and the notions of "uncertain" stability and effective algorithms for FCGs represent the subjects for current research. In the future work, the development of algorithms for dynamic formation of fuzzy coalitions seems to be the promising and challenging problem in the field of self-organizing system research.

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Part IV
Production and Materials Management Under Fuzziness

A Decentralized Production and Distribution Planning Model in an Uncertain Environment

Johannes Hegeman, David Peidro, María del Mar Alemany and Manuel Díaz-Madroñero

Abstract Distributed Decision Making (DDM) is a discipline of decision theory in which decision making power is distributed among several decision making units. Supply Chain planning problems usually involve multiple decision makers, making DDM highly suitable for realistic modelling. Furthermore, due to the complexity and dynamism of supply chain environments, accounting for uncertainty is important when modelling a supply chain planning problem. This chapter contributes to existing knowledge on the one hand with a brief literature review of DDM systems developed in the recent past. On the other hand, it contributes a proposed DDM coordination mechanism for a supply chain planning problem with two distributed decision makers, in a multi-echelon context, with multiple product levels. The DDM system's performance is evaluated under demand uncertainty by applying a fuzzy approach. Computational results show that the proposed distributed model closely approximated the optimal solutions generated by the centralised model, strengthening the evidence for DDM's applicability to real problems. Finally, the fuzzy approach is shown to be a useful tool for decision makers in evaluating risk in their supply chain planning decisions.

Keywords Supply chain · Production planning · Optimization · Uncertainty · Fuzziness

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1 Introduction

A Supply Chain (SC) can be defined as a system of organizations, people, technology, activities, information and resources involved together in the creation of value for an end customer by moving a product or service to that customer. It is imperative that some form of collaboration exists between supply chain members to coordinate their activities and plans for better results. This coordination or alignment as it is also known is referred to as collaborative planning. Formally defined, "collaborative planning is a joint decision making process for aligning plans of individual SC members with the aim of achieving coordination in light of information asymmetry" (Stadtler 2009).

Key elements of this definition are that collaborative planning is a decision making process, and that it is done in light of information asymmetry. The latter simply means that not all SC members have access to the same information. Jung et al. (2008) found that most supply chain planning approaches involve some form of centralized supply chain environment, in which the decision maker has all the required information. However, exactly that is what is lacking in a collaborative planning environment according to Stadtler's definition.

Distributed Decision Making (DDM) is a discipline of decision theory in which decision making power is distributed among several decision making units. These decisions are interrelated because one decision affects the outcome of another. How to structure these distributed decision problems into a coordinated problem is the central question in Distributed Decision Making. "DDM can therefore be characterized as the design and coordination of connected decisions" (Schneeweiss 2003).

Application of DDM theory to supply chain planning problems started over two decades ago. New approaches however, continue to be developed in the scientific community. As part of the Quantitative Modelling Techniques, DDM will be applied to a centralized supply chain planning model. The centralized model that serves as the basis for this work is the Production and Distribution Planning Model developed by Park (2005). Jung et al.'s (2008) work serves as the basis for creating the distributed model.

The real world complexity and dynamism of Supply Chain environments also imply there is usually a degree of uncertainty regarding SC planning decisions. This uncertainty can greatly influence the effectiveness of decisions taken, meaning it is valuable to consider it in the decision making process. Davis (1993) recognises three main types of uncertainty, supplier uncertainty, process uncertainty and demand uncertainty. Supply uncertainty results from variability in suppliers' performance. Process uncertainty results from unreliability issues in the production process. The most important type of uncertainty, according to Davis, is demand uncertainty which arises from volatile demand or inaccurate forecasts. Coincidently, demand uncertainty was also required to be included in the Distributed Decision Making model.

The objective of this chapter is therefore twofold:

1. To convert a centralized supply chain planning model into a distributed decision making model and compare the performance of both models,
2. To apply fuzzy logic theory (possibility theory approach) to the distributed model to incorporate demand uncertainty and comment on its performance and use.

Although the adaptation of Park's centralised model towards a distributed model is to be done using a similar method to that of Jung et al., many other methods exist in the literature. A review of the literature will be performed to identify existing DDM systems. Especially the coordination mechanisms they employ are interesting since this determines to great extent how the system works. A classification of DDM systems based on characteristics taken from earlier classifications will be attempted to gain a more structured view on the existing body of work. In addition to simply creating this overview, it is hoped that insights will be gained as to how to create a DDM model out of Park's centralized model.

The following steps are of a more practical nature. The centralized model will have to be decomposed into distributed models after which a coordination mechanism can be designed. Both model's performance will then be compared with the aid of a commercial modelling program and solver, MLP and CPLEX. The last step to achieve the second objective is model the uncertainty of demand by applying fuzzy possibility theory.

The rest of this section starts with a description of the search methodology and corresponding literature review. The overview of existing DDM systems finishes Sect. 2, and with it the more theoretical half of this report. Section 3 first introduces the generic centralised model before presenting the hands on problem that is to be solved by all the mathematical models. The centralised model is then decomposed into distributed models in Sect. 4, after which the coordination mechanism is also presented. Section 5 has a more elaborate introduction on fuzzy logic theory and presents the distributed model under uncertainty. Section 6 presents the computational results for all three models and contains a discussion of these results, after which Sect. 7 offers some final conclusions and future work.

2 Literature Review

This section contains the theoretical part of this research project. It starts with the search methodology, followed by the discussion of important Distributed Decision Making characteristics and the actual analysis of the existing literature on DDM and mathematical programming.

Table 1 Search terms devised prior to consulting scientific databases

Search terms			
Operations planning	Distributed decision making	Mathematical model	Review
Production planning	Decentralized decision making	MILP	Survey
Supply chain planning	Collaborative planning	ILP	Body of knowledge
Network planning	Coordination mechanisms		State of art
Distribution planning	Minimal information sharing		

2.1 Search Methodology

This section outlines the methodology used during the search for relevant scientific literature regarding DDM. First the search terms used are presented, followed by the scientific databases that were consulted. The section is concluded by a representation of the obtained results.

2.1.1 Search Terms

Distinguishing search terms had to be devised to find relevant material. First of all, prior work had to preferably be related to some form of operations, production or supply chain planning. That would fit closer to the problem treated later in the applied part of this research. Second, it was imperative that the mathematical models were distributed or decentralized. Because collaborative planning uses DDM extensively, and coordination mechanisms are essential elements these search terms were also chosen. To increase the possibility of finding mathematical models, abbreviations were added to the former. Finally, reviews, surveys and bodies of knowledge were queried for because they could offer a good starting point for more articles and search terms. The search terms devised before starting the search are presented in Table 1.

2.1.2 Scientific Databases and Search Strategy

Four databases of scientific articles were consulted in order to find relevant literature. These were the following: ScienceDirect, Scopus, Emerald Insight and IEEE Explorer. The article titles, abstracts and keywords were queried for matching results. The results for these queries are presented in Sect. 2.1.3. Furthermore, cross checking of oft cited articles was performed to find other relevant material. This was particularly fruitful for the review and survey type results. For the subsequent analysis, priority was generally given to those articles that were most recent and/or cited often.

Table 2 Number of articles found with combination of search terms, sorted for scientific database

Search terms combination	Science direct	Scopus	Emerald insight	IEEE explorer
"Operations planning" "distributed decision making"	1	1	2	0
"Production planning" "distributed decision making"	3	9	2	5
"Production planning" "decentralized decision making"	2	3	5	1
"Decentralized decision making" "review"	4	23	73	5
"Decentralized decision making" "survey"	2	18	43	2
"Distributed decision making" "review"	4	12	34	5
"Distributed decision making" "survey"	4	11	13	1
"Decentralized decision making" "production planning" "review"	0	0	4	63
"Distributed decision making" "production planning" "review"	0	0	2	67
"Mathematical models" "distributed decision making"	2	10	1	2
"Mathematical models" "operations planning" "distributed decision making"	0	2	1	3
"Mathematical models" "coordination mechanisms" "distributed decision making"	0	0	0	4
"Mathematical models" "coordination mechanisms" "decentralized decision making"	0	1	0	0
"Coordination mechanisms" "distributed decision making"	4	4	3	2
"Coordination mechanisms" "decentralized decision making"	1	14	3	0
"Collaborative planning" "distributed decision making"	30	5	2	2
"Collaborative planning" "decentralized decision making"	34	2	1	0

2.1.3 Obtained Results

Table 2 shows the number of articles found for a particular combination of search terms, for different scientific databases. Differences are accountable to the different search algorithms the databases employ. IEEE explorer e.g., returned many articles when 'production planning' was used. Practically no results were left when the term was taken out.

Table 3 shows the amount of articles that were eventually selected as reference articles, sorted by the journal in which they were published. Only the European Journal of Operational Research provided more than one reference article, with all others providing a single article. The wide range, from chemical engineering to computer engineering shows that distributed decision making is applied in a broad field of disciplines.

Table 3 Reference articles sorted by journal of publication

Journal of publication	Number of reference articles	Percentage (%)
Computers and Chemical Engineering	1	9
Computers and Industrial Engineering	1	9
Computers in Industry	1	9
European Journal of Operational Research	3	27
International Journal of Production Research	1	9
International Journal of Production Economics	1	9
Journal of Engineering and Technology Management	1	9
OR Spectrum	1	9
Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics	1	9

Table 4 Reference articles sorted by year of publication

Year of publication	Number of reference articles	Percentage (%)
2003	1	9
2005	1	9
2006	1	9
2007	1	9
2008	2	18
2009	3	27
2010	1	9
2012	1	9

The final representation of literature search results is given in Table 4. Not many conclusions can be drawn from this table, because not enough articles were analysed to offer a comprehensive picture on the publication dates of articles on DDM in mathematical modelling.

2.2 *Presentation of DDM Characteristics for Analysis of Literature*

Due to the sheer variety in mathematic models developed, it should come as no surprise that there exists a similar variety of Distributed Decision Making systems. Various authors have tried to classify those using different distinguishing characteristics. Three of those efforts are discussed here, after which the most relevant characteristics are chosen for the classification in this research.

2.2.1 Review of Earlier Taxonomies and Classification Attempts

Schneeweiss (2003) developed taxonomy to classify and formally describe various hierarchical DDM systems in a unified way. It is important to note first, that

Schneeweiss distinguishes between a Top level and a Base level within the hierarchy. The Top level is regarded as the leader in the hierarchy that makes the first decision and the Base level follows the Top level's instruction. The Base level then engages in its own local decision making. Now, Schneeweiss identified three key characteristics that can be used to characterize DDM systems.

The first characteristic is the state of information. This can be either symmetric, which means that the same information is known to all decision makers in the system, or asymmetric, which means that certain information may be known to one, e.g., Top, but unknown to another, Base e.g.

The second characteristic is the grade of anticipation that decision makers show in their decision making. Two options exist. A reactive anticipation means that the Top level considers a possible reaction of the base-level with respect to a top-level's possible instructions. For non-reactive anticipation, on the other hand, no specific reaction is taken into account. Reactive anticipation can be perfect, meaning that the Top level has full knowledge of the Base level's model and thus its reaction. It can also be approximate, when the Base level's model is approximately known by the Top level. The last possibility is implicit anticipation where only a part of the base level is anticipated. The grade of anticipation employed shows how much 'bottom-up' influence in the decision making at Top level there is within the hierarchy.

The final characteristic is defined as the configuration of criteria. Coupling equations of criteria are used to demonstrate the degree of coupling between Top and Base levels. For any form of Top level criterion in which the Base level's criteria are integrated in-, added to- or even make up entirely the Top level criterion, a DDM is said to be team based. This is because the value of the Top level criterion depends on the Base level through its definition. Of course, the Base level has to comply to the Top level's instructions so a Base level criterion need not take Top level criteria into account. The three configurations of criteria mentioned are all common in team based DDM systems.

The other possible configuration is non-team. This happens when a Top level's criterion completely ignores the Base level. In this case, each level is thus completely self-interested and will show what is known as opportunistic behaviour. The goal is always to increase one's own benefit, even when detrimental to the global solution. Based on these criteria, Schneeweiss classifies DDM systems into three main types. They are shown in Table 5. He notes that many variations to these general types may occur.

Another notable effort is a framework developed by Stadtler (2009), which is meant to classify Collaborative Planning approaches along various characteristics. Three main groups of characteristics are identified, (1) the supply chain structure and the relationships within the supply chain, (2) the decision situation, or which decisions take place, when, with which objectives and with which information, and (3) the characteristics of the collaborative planning schemes. Only the characteristics relevant to DDM models and in particular coordination mechanisms will be discussed here.

Within the relationships between supply chain members involved in CP, their behaviour is important. It can be team, opportunistic (non-team) or somewhere in

Table 5 Three types of DDM systems as identified by Schneeweiss (2003)

DDM type	Information status	Grade of anticipation	Configuration of criteria
Top-down hierarchy	Symmetric or asymmetric	Non-reactive anticipation	Base criterion internalized in top level criterion (Team)
Tactical-operational hierarchy	Asymmetric	Reactive anticipation	Base criterion added to top level criterion (Team)
Standard principal agent model	Asymmetric	Reactive anticipation	Base criterion ignored by top level criterion (Non-team)

between, and coincides with one of Schneeweiss's characteristics. However, Schneeweiss mathematically formalized team and non-team behaviour with the aid of coupling equations and criterions for Top and Base level, whilst Stadtler merely pointing out its importance. Nevertheless, its importance is now clearer than ever and a coordination mechanism must take possible opportunism into account. The required solution is also important for the coordination mechanism. It can be limited to aligning flows of materials, or merely finding a feasible solution. Perhaps an optimal solution for the supply chain as a whole is required, or one step further, a fair solution for all members involved. This was not part of Schneeweiss's analysis.

Within the decision situation the models that are employed, in which phase there is collaboration and which objectives are employed are all important. However, the most important aspect here is referred to as information status. Which information is shared, how certain can we be of its correctness and is certain information hidden? The latter corresponds to symmetry or asymmetry. Recall that Stadtler by definition regards collaborative planning to involve asymmetric information.

The final group of characteristics is in my opinion the most important with respect to DDM and coordination mechanisms. The presence of a mediator could significantly alter the dynamics of collaboration. How the initial solution is defined is also important. This is mostly done by upstream planning according to Stadtler (2009), but downstream planning or random initial solutions can also be used. The number of plans exchanged between levels, consisting of the number of rounds allowed to reach a solution and the number of offers sent per round can also change how a system works. Finally, with respect to the final results, being able to check optimality or not, and the allowing of side payments could affect a coordination mechanism. The latter could e.g., be used to make a solution fairer to all members.

The final research used for the development of taxonomy is a review on collaborative supply chain planning by Frayret (2009). For collaborative planning, there are three challenges to be dealt with: "the design of a coordination process, …, the design of local decision making processes; and the design and utilization of Advanced Planning and Scheduling systems (APS)" (Frayret 2009). The former is conveniently the main focal point of his classifications. The local decision making processes correspond to what Stadtler referred to as the decision models. APS systems are inherently linked to coordination techniques according to Frayret (2009), but will not be considered further as they are outside the scope of this project.

The coordination processes can once again be divided further, into three groups. The first group is the Coordination Heuristics, which consists of:

(a) Greedy heuristics and information sharing,
(b) Distributed local search,
(c) Distributed search with constraint propagation.

Greedy heuristics and information sharing are the most simple coordination mechanisms. They include upstream planning and variations of it to improve performance. Other examples are when more information is shared, even up to the point where more centralized solving is possible. Distributed local search generally involves an iterative exchange of information between supply chain partners, during which the local levels adjust their own initial plans by searching for local optima. Distributed search occurs when more than one search process is carried out simultaneously. In the distributed local search the members take their local searches in turn. In the distributed search this is not the case, making the search faster.

The second group is Agent Based Coordination, which consists of:

(a) Knowledge-based coordination,
(b) Market-based coordination.

Agent Based Coordination is based in agent technology, which uses artificial intelligence (AI) to develop coordination approaches. Agents are pieces of software that represent a certain interest. They can thus be used to represent members of a supply chain. In knowledge-based coordination, a set of agents use protocols that tell them how to interact with other agents. These protocols define all possible actions and model the outcomes of interactions. The protocols thus govern the coordination. In some cases, additional information known as arguments are sent to other agents, with the aim of influencing their actions. The other agent based coordination technique is Market-based coordination. Basically, like in an auction proposals are sent and received by agents. The contents of these proposals are modified to increase or decrease benefits, according to their prior success or failure respectively. Here, the learning or AI aspect of agents becomes clear.

The final coordination technique identified by the author is Mathematical decomposition (6). He claims the main decomposition approach is Lagrangean decomposition. Its general idea is that an originally distributed model is turned into a centralized model, by relaxing the 'complicating' constraints where local variables of two or more coupled decision makers appear. A penalty is assigned to violating these relaxed constraints by using Lagrangean multipliers. Then, a distributed and synchronous iterative process is developed to adjust the penalties until the model converges on a feasible solution. One could view it this way. The 'local' models (bear in mind that the model is centralized) communicate through these Lagrange multipliers. Values that increase other local decision makers' penalties are communicated if one's own non relaxed constraints are violated. The contrary is true when there is slack in those constraints. This is done until all hard constraints are satisfied and penalties are preferably at a minimum. Details on the exact working of mathematical decomposition can be found in works by Nishi et al. (2007), Walther et al. (2008) and Lu et al. (2012).

2.2.2 Selection of Characteristics for Analysis

The state of information is identified by both Schneeweiss and Stadtler, so it seems very important to include that. Stadtler added the degree of uncertainty and the type of information (products, costs, other KPI's) to the existing characteristic of (a) symmetry of information. Type of information is very specific, so it is ignored. Degree of uncertainty will also be considered next to symmetry, because the end goal of this research is a model that accounts for demand uncertainty.

The grade of anticipation of one level's criteria by another is also interesting since it greatly influences a Top level's instruction. The last of Schneeweiss's characteristics is the configuration of criteria. How a top level decision maker in a hierarchy takes the base criteria into account, is not extremely important. Most important is to know whether it happens or not, because it interests us to know whether decision makers exhibit team- or opportunistic behaviour in a DMM system. This was coincidently Stadtler's only distinction. The exact configuration is therefore dropped. All three characteristics will be used, but the type identification as performed in (Schneeweiss 2003) will not be employed. The reason is that the characteristics themselves reveal more than a type.

Distinguishing between DDM systems that look for feasible solutions, optimal supply chain solutions or even fair solutions is very interesting. First, optimality is much more difficult to achieve than just a feasible solution. Second, requiring a fair solution has strong implications for the coordination mechanism because the initial mechanism might not produce a fair solution. Also, "computational tests showed that fair solutions sacrifice 37.15 % on average in solution quality" (Stadtler 2009). Related to that is the allowing of side payments at the final solution, as they could diminish that sacrifice. Therefore, these will also be taken into account. Other characteristics identified by Stadtler that will be looked out for is the presence of a mediator, solely because it could completely change how coordination works, how the initial solution is computed, and the number of rounds and offers used in the communication process. Few or many rounds e.g., determine whether or not a system can be operated manually or must be fully automatic.

To conclude, the coordination mechanism distinction from Frayret will also be included in the analysis. Although a certain coordination mechanism may imply one of the earlier characteristics, the actual mechanism will greatly set the studied works apart. It will also be useful for a reader to see which general coordination mechanism is employed to decide whether it interests him/her.

2.3 Analysis of Literature

The articles studied for this project were not all designed for use in supply chain contexts. However, they do all represent some form of distributed decision making system. It proved quite difficult to find distributed decision making systems within the time constraints, so non supply chain systems were also included. Their techniques are what matter most, not only the application area. Each system was analysed for identification of the characteristics chosen in Sect. 2.2. The respective characteristics for each system are summarized in Table 6. The numbers below the characteristics point out their source, and N/A means that information was not provided or not applicable.

The first DDM system analysed by Cao and Chen (2006) was a decentralized facility location problem. They changed a decentralized two level nonlinear programming model into an equivalent linear single level model. The result was a hierarchical model with a coordination mechanism resembling the upstream planning approach in a supply chain context.

A more advanced system used a distributed local search for local optima. Jung et al. (2008) developed a decentralized supply chain planning framework based on minimal-information sharing between the manufacturer and a third party logistics provider. Each used its own model and kept private information. The coordination mechanism ensured local solutions converged towards a feasible solution, although the levels did not cooperate as a team. Each level strived for local optimisation. However, opportunistic behaviour was not demonstrated as the information they exchanged was truthful.

While the different levels in Jung et al.'s model had to wait for input from the other level before proceeding to search for their new local optimum, Gaudreault et al. (2009) developed a system wherein levels concurrently evaluate other level's earlier decisions instead of one local optimum being processed at any given moment. The authors call this a "distributed discrepancy search procedure" and it is categorized as a distributed search with constrain propagations. The procedure is illustrated in Fig. 1. The top level (agent A, closest to the customer) takes lower levels (tiers further away) into account. This is because the lower levels communicate their locally optimal plans upwards. The optimal solution is thus known to agent A but not to agents B or C. The distributed search for the optimal solution is like a tree. Each agent computes its optimal solution based on the request by the agent directly above him (one tier closer to the customer). It is thus possible that agent C is working on a local solution based on what agent B sent him, whilst at the same time agent A is computing a new solution based on the locally optimal response it got from agent B. More than one solution is thus evaluated at a time.

An example of agent based coordination was found in the work of Wernz and Denshmukh (2010). The specific application was intra-organizational, but the techniques were interesting nonetheless. First of all, the Top level agent and Base level agent are in a hierarchical relationship, but the agents make decisions simultaneously instead of sequentially. There is also two way interaction through

Table 6 Classification of studied works

Authors	State of information[a,b]	Grade of anticipation[a]	Supply chain member behavior	Required supply chain solution[b]	Mediator[b]	Initial solution[b]	Number of plans Exchanged[b]	Final result[b]	Coordination mechanism[c]
Cao and Chen (2006)	Asymmetric information fully certain	Non-reactive anticipation	Team	Search for SC optimum, but not fair solution	No	None	1 round, 1 offer	No side payments	Greedy heuristics and information sharing
Jung et al. (2008)	Asymmetric information fully certain	Non-reactive anticipation	Non-team	Search for SC optimum, but not fair solution	No	Upstream planning	n $<$ 25, 1 offer	No side payments	Distributed local search
Lu et al. (2012)	Asymmetric information fully certain	Non-reactive anticipation	Team	Search for SC optimum, but not fair solution	No	N/A	n large, 1 offer	No side payments	Mathematical decomposition through Lagrangean relaxation
Nishi et al. (2007)	Asymmetric information fully certain	Non-reactive anticipation	Team	Search for SC optimum, but not fair solution	No	N/A	n large, 1 offer	No side payments	Mathematical decomposition through Lagrangean relaxation
Walther et al. (2008)	Asymmetric information fully certain	Non-reactive anticipation	Team	Search for SC optimum, but not fair solution	No	N/A	n large, 1 offer	No side payments	Mathematical decomposition through Lagrangean relaxation

(continued)

Table 6 (continued)

Authors	State of information[a,b]	Grade of anticipation[a]	Supply chain member behavior	Required supply chain solution[b]	Mediator[b]	Initial solution[b]	Number of plans Exchanged[b]	Final result[b]	Coordination mechanism[c]
Wernz and Deshmukh (2010)	Asymmetric information fully certain	Implicit anticipation	Non-team	Search for SC optimum, but not fair solution	No	N/A	1 round, 1 offer	No side payments	Agent-based coordination: knowledge-based protocol-based coordination
Gaudreault et al. (2009)	Asymmetric information fully certain	Non-reactive anticipation	Team	Search for SC optimum, but not fair solution	No	Upstream planning	n large, multiple offers	No side payments	Distributed search with constraint propagation

[a] From Schneeweiss
[b] From Stadtler
[c] From Frayret

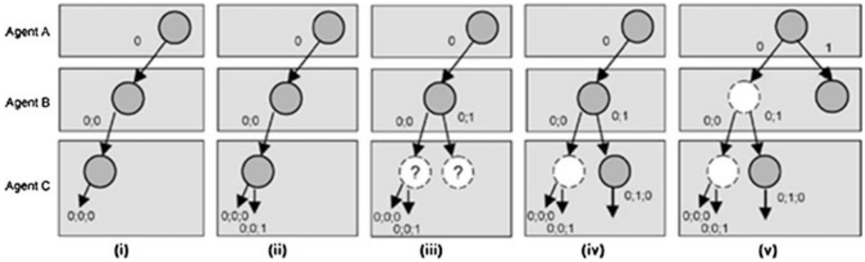


Fig. 1 Illustration of distributed discrepancy search procedure, *Source* (Gaudreault et al. 2009)

reward and influence, which is not seen in any of the other studied works. This anticipation is merely implicit. It would be characterized as a principal agent system according to the definition by Schneeweiss (2003).

The remaining three studied systems employed mathematical decomposition through Lagrangean relaxation of constraints. The first of these by Nishi et al. (2007) was developed to determine the production scheduling and distribution planning for a single stage production system with parallel distributed production units. The novelty is in their use of quadratic penalty terms in the objective function. Walther et al.'s (2008) mathematical decomposition is applied to a supply chain problem, that of a recycling supply chain looking to assign optimal quantities of mass for recycling. The mathematical decomposition of the initial centralized model is performed to create the negotiation mechanism between a head firm and several recycling companies. In these two systems, a master problem serves as a top level coordinator. The sub-problems communicate their local solutions to the master problem to eventually find the optimum. Lu et al.'s (2012) approach also involves Lagrangean relaxation, but they do not introduce a master problem to server as a coordinator of the decomposed original central problem. "Instead, the resulted sub-problems are equally ranked, and a novel self-coordination scheme is developed which enables the solving of sub-problems is coordinated through peer-to-peer communication, rather than communication between each sub-problem and the master problem" Lu et al. (2012).

Interesting similarities between all studied DDM systems is that all deal with asymmetric information, the objective is always to find the SC optimum and never a fair solution, and none of the systems employs a mediator. The biggest differences are found in the team or opportunistic behaviour demonstrated, and of course the coordination mechanisms used. The reader is reminded that the overview of the classification can be found in Table 6.

3 Centralized Deterministic Model

This section presents the centralised deterministic model used as the basis for the applied part of this research project. The model is an adapted form of the production and distribution planning model by Park (2005). First the general model formulation is given, followed by an explanation and the specific configuration of the planning problem. Computational results are given in Sect. 6.

3.1 General Model Formulation

The centralised planning problem considers a supply chain of manufacturing plants and retailers, with a planning horizon of five time periods. The manufacturing plants produce multiple items with a limited production capacity. For every item that is produced in a given time period, a plant dependent fixed set-up cost is incurred that is independent of the lot size. Excess production may be stored at the plant at a holding cost, for which there is no storage capacity limit. The items are structured in a three level bill of materials (BOM). Those items at level two and three of the BOM are consumed for the production of higher level items, according to amounts defined in the BOM. Only the items at level one, which are the final products, are delivered to the retailers.

Plants are capable of producing only a given set of items, with the items distributed over the different plants. Therefore, plants also act as suppliers to each other for the delivery of items used as subcomponents. Only items that are consumed as subcomponents are delivered in between plants. Consequently, the planning problem is a multi-stage problem, with the plants capable of being at various stages, dependent on the items they produce and those items' positions in the BOM.

Delivery between plants is regarded to be free of charge and free of capacity constraints. Delivery from plants to retailers is performed by means of a fleet of homogeneous vehicles with similar capacities and usage costs. Sending a vehicle from any plant to a retailer incurs a fixed cost (depreciation of vehicle, insurance, driver wages) and a variable cost dependent on the transported item, its quantity and the route (plant—retailer combination). Any one vehicle can only transport one item type and travel one route per time period. The amount of vehicles used can change without incurring costs.

The demand for the final products (level one items) is expressed as a 'core demand' and a 'forecasted demand'. The 'core demand' may be considered as the demand by a retailer's loyal customer base, and must be satisfied. The 'forecasted demand' contains the 'core demand' and is the total amount of final products that can be sold in a given time period. In the centralised deterministic model, these demands are known with certainty. Any unsatisfied forecasted demand is considered a stock-out, for which a stock-out opportunity cost is incurred.

Backordering is not allowed. Every retailer is allowed to keep a finite amount of final products in inventory, for which an inventory cost is incurred.

The objective of the centralised planning problem is to maximise profits over the five time periods. The decision maker has all the data (demand, inventories, production costs etc.) available to him and plans the production and distribution of final product items and subcomponent items. A mixed-integer model is used to solve the centralised production and distribution planning problem. First, the notations used are presented, followed by the model.

Indices

$$i = plants, i \in (1, \ldots, I)$$
$$j = retailers, j \in (1, \ldots, J)$$
$$k = items, k \in (1, \ldots, K)$$
$$t = timeperiods, t \in (1, \ldots, T)$$

Parameters

$$c_{ik} = unit\ processing\ cost\ ofitem\ k\ at\ plant\ i$$
$$s_{ik} = setup\ cost\ for\ item\ k\ at\ plant\ i$$
$$o_{ik} = processing\ time\ for\ item\ k\ at\ plant\ i$$
$$u_{ik} = setup\ time\ for\ item\ k\ at\ plant\ i$$
$$h^p_{ik} = inventory\ holding\ cost\ of\ item\ k\ at\ plant\ i\ per\ period\ t$$
$$\lambda_{ik} = \begin{cases} 1 & if\ plant\ i\ can\ produce\ item\ k \\ 0 & if\ plant\ i\ can\ NOT\ product\ item\ k \end{cases}$$
$$\beta_{ik'k} = required\ cuantity\ of\ item\ k\ for\ the\ production\ of\ one\ item\ k'at\ plant\ i$$
$$L_i = production\ capacity\ of\ plant\ in\ time$$
$$d_{ijk} = unit\ transportation\ cost\ of\ item\ k\ between\ plant\ i\ and\ retailer\ j$$
$$g = fixed\ cost\ per\ vehicle$$
$$B = fixed\ capacity\ per\ vehicle$$
$$E_{jkt} = demand\ for\ item\ k\ at\ retailer\ j\ in\ period\ t\ that\ must\ be\ filled$$
$$F_{jkt} = total\ forecast\ demand\ for\ item\ k\ at\ retailer\ j\ in\ period\ t, E_{jkt} is\ part\ of\ F_{jkt}$$
$$p_{jk} = unit\ selling\ price\ of\ item\ k\ at\ retailer\ j$$
$$h^r_{jk} = inventory\ holding\ cost\ of\ item\ k\ at\ retailer\ j\ per\ period\ t$$
$$W^r_j = capacity\ for\ units\ of\ inventory\ at\ retailer\ j$$
$$v_{jk} = stockout\ cost\ per\ unit\ of\ item\ kat\ retailer\ j$$

Decision Variables

x_{ikt} = *quantity of item k produced in plant i in period t*

$y_{ikt} = \begin{cases} 1 & \textit{if setup must be performed at plant i for item k in period t} \\ 0 & \textit{otherwise} \end{cases}$

a^p_{ikt} = *level of inventory of item k at plant i in period t*

C_{ikt} = *quantity of item k consumed as subcomponent at plant i in period t*

$qi_{ii'kt}$ = *quantity of components k shipped from plant i to plant i′in period t*

qj_{ijkt} = *quantity of item k transported from plant i to retailer j in period t*

z_{ijt} = *number o vehicles required for distribution from plant i to retailer j in period t*

$\mathbf{Z_{jkt}}$ = ***supply shortage volume of item k for retailer j in; period t***

$\mathbf{Si_{jkt}}$ = ***outcome variable with available supply to be sent to Retailers model***

Model

Objective function

$$\begin{aligned} & Max \sum\nolimits_j \sum\nolimits_k p_{jk} \sum\nolimits_t \left(a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt} \right) \\ & - \left(\sum\nolimits_i \sum\nolimits_k \sum\nolimits_t c_{ik} x_{ikt} + \sum\nolimits_i \sum\nolimits_k \sum\nolimits_t s_{ik} y_{ikt} + \sum\nolimits_i \sum\nolimits_k \sum\nolimits_t h^p_{ik} a^p_{ikt} \right) \\ & - \left(\sum\nolimits_j \sum\nolimits_k \sum\nolimits_t h^r_{jk} a^r_{jkt} + \sum\nolimits_j \sum\nolimits_k \sum\nolimits_t v_{jk} \left(F_{jkt} - \left(a^r_{jkt-1} + \sum\nolimits_i qj_{ijkt} - a^r_{jkt} \right) \right) \right) \\ & - \left(\sum_i \sum_j \sum_t g * z_{ijt} + \sum_i \sum_j \sum_k \sum_t d_{ijk} qj_{ijkt} \right) \end{aligned} \tag{1}$$

Subject to

$$\left(\sum\nolimits_k x_{ikt} * o_{ik} + y_{ikt} * u_{ik} \right) \leq L_i \quad \forall i \forall t \tag{2}$$

$$x_{ikt} \leq M * y_{ikt} \quad \forall i \forall k \forall t \tag{3}$$

$$x_{ikt} \leq M * \lambda_{ik} \quad \forall i \forall k \forall t \tag{4}$$

$$C_{ikt} = \sum\nolimits_{k'} \beta_{ik'k} * x_{ik't} \quad \forall i \forall k \forall t \tag{5}$$

$$C_{ikt} = \sum\nolimits_{i'} qi_{i'ikt} \quad \forall i \forall k \forall t \tag{6}$$

$$a^p_{ikt} = a^p_{ikt-1} + x_{ikt} - \sum\nolimits_j qj_{ijkt} - \sum\nolimits_{i'} qi_{ii'kt} \quad \forall i \forall k \forall t \tag{7}$$

$$a^r_{jkt-1} + \sum\nolimits_i qj_{ijkt} - a^r_{jkt} \geq E_{jkt} \quad \forall j \forall k \forall t \tag{8}$$

$$a^r_{jkt-1} + \sum\nolimits_i qj_{ijkt} - a^r_{jkt} \leq F_{jkt} \quad \forall j \forall k \forall t \tag{9}$$

$$\sum\nolimits_k a^r_{jkt} \leq W^r_j \quad \forall j \forall t \tag{10}$$

$$\sum\nolimits_k qj_{ijkt} \leq B * z_{ijt} \quad \forall i \forall j \forall t \tag{11}$$

$$a^p_{ik0} = 0, a^r_{jk0} = 0, \quad \forall i \forall j \forall k \tag{12}$$

$$\begin{aligned} &x_{ikt} \geq 0, a^p_{ikt} \geq 0, y_{ikt} \in \{0, 1\}, C_{ikt} \geq 0, qi_{ii'kt} \geq 0, qj_{ijkt} \geq 0, a^r_{jkt} \geq 0, \\ &z_{ijt} \geq 0, \textit{and all are integers} \qquad \forall i \forall j \forall k \forall t \end{aligned} \tag{13}$$

The objective function (1) expresses the total net profit over the time periods, calculated by subtracting total costs from total revenue. Revenue is the total turnover at all retailers, calculated by multiplying selling price with sales $(a^r_{jkt-1} + \sum_i qj_{ijkt} - a^r_{jkt})$. The costs include production-, inventory holding-, stock-out- and distribution costs. Constraint (2) represents the capacity limit on production at a plant. Constraint (3) forces the incurring of setup costs if items are produced. Constraint (4) makes sure that production of items is only allowed at a plant if that plant is capable of producing that item. For both these constraints, M is a sufficiently large positive number. Constraint (5) determines the amount of an item that is consumed for the production of higher level items, by summing the products of the production quantities of the higher level items with the amount of lower level items consumed for their production. Constraint (7) assures the inventory balance at a plant, with both shipments to retailers and to other plants taken into account. Constraint (8) ensures that the 'core demand' is satisfied, whilst constraint (9) ensures that no more is sold (and thus ordered from the plants at some point) than the 'forecasted demand'. Constraint (10) applies the storage capacity for inventory held by retailers. The amount of vehicles needed for transportation of items to retailers is calculated in constraint (11). Constraint (12) then defines the initial inventory levels at both plants and retailers. Note that these can be changed. The final constraint (13) enforces restrictions of non-negativity, integer and binary nature of decision variables.

The model calculates optimal production quantities x_{ikt} for all items at the different plants for all time periods and optimal amounts qj_{ijkt} to be shipped to the retailers. It will balance setup with inventory holding costs and delivery costs with stock-out costs. It can therefore occur that not all forecasted demand is satisfied, although the inventory storage capacity at retailers exists to minimise the occurrence of demand not being satisfied.

3.2 Specific Configuration of Supply Chain

The supply chain that is used for this research project is represented in Fig. 2. The model's indices, parameters and decision variables are included to show to which part of the Supply Chain they pertain. The Supply Chain consists of three

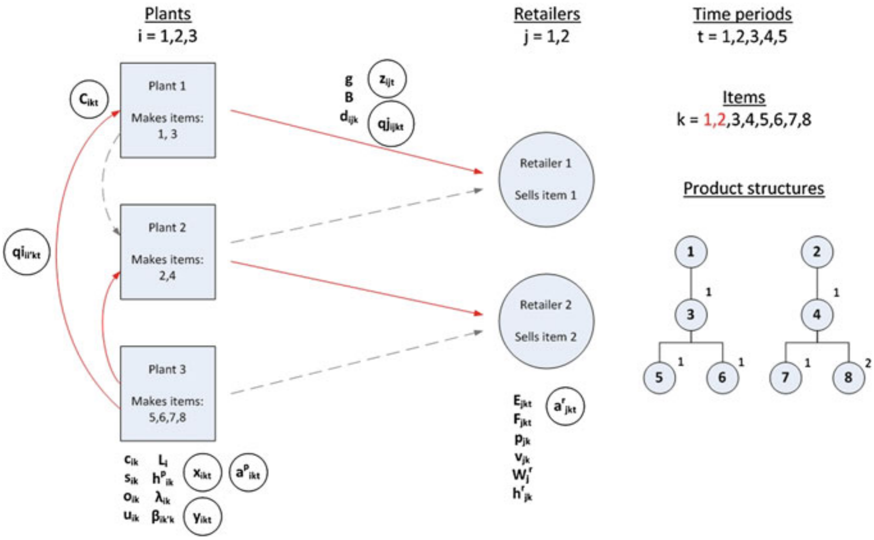


Fig. 2 Supply chain configuration used for centralised model

manufacturing plants and two retailers. There are five time periods and eight items in this problem. Of the items, only items one and two are of level one, and thus sold as end products. Their item number is indicated red for this reason. Furthermore, each item has a specific retailer, with retailer one selling item one and item two being sold by retailer two. The product structure of the final products is also given in the figure, where the required quantities of a subcomponent can be found in the top right corner of each item. Items three and four are the level two items, and items five to eight are at level three.

The item production capabilities are distributed among the plants in such a manner, that each item is produced at only one plant. Plant one makes items one and three, plant two makes items two and four, and plant three makes items five to eight. As a result, plant three supplies plants one and two with level three subcomponents. Plants one and two make their own level two subcomponents and final products. Because each retailer only sells one product, each plant only delivers to one retailer. The flow of items is represented by the red arrows in Fig. 2.

The configuration presented was determined in the assignment. However, the model has successfully been tested for other configurations. Examples are retailers selling more than one item, plants 'competing' by being able to produce the same items and common subcomponents in product structures. The flows could therefore also include the grey dashed arrows. This generality is a useful characteristic, should the model ever need to be applied to a different Supply Chain.

4 Distributed Deterministic Model

Next, the centralised deterministic model is decomposed into two separate models. These separate models each pertain to a different decision maker, one that controls the manufacturing plants and distribution of items, and one that controls the retailers. A coordination mechanism is developed to link the two models and form the distributed deterministic model. The distributed decision making process is also presented to enhance clarity. Computational results are again found in Sect. 6.

4.1 Model Manufacturer

The first decision maker has control over the production of items in the plants, and their distribution to the retailers. It is assumed that distribution of items is part of this decision maker's model because it is generally the manufacturer's responsibility to deliver a product to its customer. As done for the centralised model, first the notation is presented, then the model. Additions or changes from the centralised model are highlighted in bold.

Indices

$$i = plants,\ i \in (1, 2, 3)$$
$$j = retailers,\ j \in (1, 2)$$
$$k = items,\ k \in (1, \ldots, 8)$$
$$t = timeperiods,\ t \in (1, \ldots, 5)$$

Parameters

$$c_{ik} = unit\ processing\ cost\ of\ item\ k\ at\ plant\ i$$
$$s_{ik} = setup\ cost\ for\ item\ k\ at\ plant\ i$$
$$o_{ik} = processing\ time\ for\ item\ k\ at\ plant\ i$$
$$u_{ik} = setup\ time\ for\ item\ k\ at\ plant\ i$$
$$h^{p}_{ik} = inventory\ holding\ cost\ of\ item\ k\ atplant\ i\ per\ period\ t$$
$$\lambda_{ik} = \begin{cases} 1 & if\ plant\ i\ can\ produce\ item\ k \\ 0 & if\ plant\ i\ can\ NOT\ product\ item\ k \end{cases}$$
$$\beta_{ik'k} = required\ cuantity\ of\ item\ k\ for\ the\ production\ of\ one\ item\ k' atplanti$$
$$L_i = production\ capacity\ of\ plant\ i\ in\ time$$
$$d_{ijk} = unit\ transportation\ cost\ of\ item\ k\ between\ plant\ i\ and\ retailer\ j$$
$$g = fixed\ cost\ per\ vehicle$$
$$B = fixed\ capacity\ per\ vehicle$$
$$\boldsymbol{vi_{jk} = unit\ supply\ shortage\ penalty\ cost\ of\ retailer\ j\ for\ item\ k}$$
$$\boldsymbol{Sj_{jkt} = requested\ supply\ quantity\ for\ item\ k\ by\ retailer\ j\ in\ period\ t\ (received\ from\ j)}$$

Decision Variables

x_{ikt} = *quantity of item k produced in plant i in period t*

$$y_{ikt} = \begin{cases} 1 & \textit{if setup must be performed at plant i for item k in period t} \\ 0 & \textit{otherwise} \end{cases}$$

a^p_{ikt} = *level of inventory of item k at plant i in period t*

C_{ikt} = *quantity of item k consumed as subcomponent at plant i in period t*

$qi_{ii'kt}$ = *quantity of components k shipped from plant i to plant i′ in period t*

qj_{ijkt} = *quantity of item k transported from plant i to retailer j in period t*

z_{ijt} = *number o vehicles required for distribution from plant i to retailer j in period t*

$\boldsymbol{Z_{jkt}}$ = ***supply shortage volume of item k for retailer j in; period t***

$\boldsymbol{Si_{jkt}}$ = ***outcome variable with available supply to be sent to Retailers model***

Model

Objective function

$$\begin{aligned} & Min\left(\sum_i \sum_k \sum_t c_{ik}x_{ikt} + \sum_i \sum_k \sum_t s_{ik}y_{ikt} + \sum_i \sum_k \sum_t h^p_{ik}a^p_{ikt}\right) \\ & + \left(\sum_i \sum_j \sum_t g * z_{ijt} + \sum_i \sum_j \sum_k \sum_t d_{ijk}qj_{ijkt}\right) \\ & + \boldsymbol{\sum_j \sum_k \sum_t vi_{jk}Z_{jkt}} \end{aligned} \tag{14}$$

Subject to

$$\left(\sum_k x_{ikt} * o_{ik} + y_{ikt} * u_{ik}\right) \le L_i \quad \forall i \forall t \tag{15}$$

$$x_{ikt} \le M * y_{ikt} \quad \forall i \forall k \forall t \tag{16}$$

$$x_{ikt} \le M * \lambda_{ik} \quad \forall i \forall k \forall t \tag{17}$$

$$C_{ikt} = \sum_{k'} \beta_{ik'k} * x_{ik't} \quad \forall i \forall k \forall t \tag{18}$$

$$C_{ikt} = \sum_{i'} qi_{i'ikt} \quad \forall i \forall k \forall t \tag{19}$$

$$a^p_{ikt} = a^p_{ikt-1} + x_{ikt} - \sum_j qj_{ijkt} - \sum_{i'} qi_{ii'kt} \quad \forall i \forall k \forall t \tag{20}$$

$$\boldsymbol{\sum_i qj_{ijkt} + Z_{jkt} = Sj_{jkt} \quad \forall j \forall k \forall t} \tag{21}$$

$$\sum_k qj_{ijkt} \le B * z_{ijt} \quad \forall i \forall j \forall t \tag{22}$$

$$\boldsymbol{\sum_i qj_{ijkt} = Si_{jkt} \quad \forall i \forall j \forall k} \tag{23}$$

$$a^{p}_{ik0} = 0 \quad \forall i \forall k \tag{24}$$

$$\begin{aligned} &x_{ikt} \geq 0, a^{p}_{ikt} \geq 0, y_{ikt} \in \{0,1\}, C_{ikt} \geq 0, qi_{ii'kt} \geq 0, qj_{ijkt} \geq 0, \\ &z_{ijt} \geq 0, Z_{jkt} \geq 0, \ \textit{and all are integers} \qquad \forall i \forall j \forall k \forall t \end{aligned} \tag{25}$$

The manufacturer does not know the actual demand for final products. He only knows the requested supply quantities for each item per period as submitted by the retailers. This quantity is represented by a new parameter Sj_{jkt}. The manufacturer must endeavour to fill the requested supply quantities to the best of his ability, because it contributes to Supply Chain profitability. To make the model strive for this, a penalty will be incurred for every unit of unfilled requested supply. For this reason, a shortage penalty cost vi_{jk} and a shortage quantity decision variable Z_{jkt} have been defined.

The manufacturer has no knowledge of actual demand or of retail prices. Maximising profit is thus not a valid objective for this model. Instead, the manufacturer will try to minimise its costs while meeting supply, because that should contribute to SC profitability. The objective function (14) now only includes production, setup and inventory holding costs for the plants, distribution costs and supply shortage penalty costs. Because having shortage negatively affects the objective function, the model will try to fill all demand. The penalty cost per unit of shortage must be high enough for the manufacturer to generally prefer production and distributing to incurring the penalty.

Constraints (15–20) are the same as in the centralised model, but constraint (21) replaces the constraints that ensured filling demand. It makes sure that the amount of an item shipped from all the plants to a retailer plus any shortage equal the requested supply quantity by that retailer for that item. If the shipped amounts do not suffice, shortage is positive and the penalty will be incurred. Constraint (22) governs the amount of vehicles needed for transportation of items to retailers, like in the earlier model. Constraint (23) is also new, and calculates the supply of an item k that is available for a retailer in a period t. This decision variable, Si_{jkt}, is the connection between the manufacturer's model and the retailers' model, because it will be communicated to the retailers after the manufacturer has solved its local problem. The retailers then know the available supply quantities that they can use to satisfy demand with. It will become clear that Si_{jkt} is an input variable for the retailers' model, just like Sj_{jkt} is for the manufacturer's model.

4.2 Model Retailers

The second decision maker has control over the retailers. This is a modelling choice, as each retailer could also have its own model, in which case the index j of the retailers would be forsaken. For simplicity, this is not done in this project. The notation and model are first given, with changes or additions highlighted in bold. The explanation of the model follows hereafter.

Indices

$j = retailers, j \in (1,2)$
$k = items, \quad k \in (1,2)$
$t = timeperiods, t \in (1,\ldots,5)$

Parameters

$E_{jkt} =$ *demand for item k at retailer j in period t that must be filled*
$F_{jkt} =$ *total forecast demand for item k at retailer j in period t*, E_{jkt} *is part of* F_{jkt}
$p_{jk} =$ *unit selling price of item k at retailer j*
$h^r_{jk} =$ *inventory holding cost of item k at retailer j per period t*
$W^r_j =$ *capacity for units of inventory at retailer j*
$v_{jk} =$ *stockout cost per unit of item k at retailer j*

$$\boldsymbol{Si_{jkt}} = \textbf{\textit{offered supply quantity of item k to ret.j in period t}} \begin{cases} \textbf{\textit{first iteration it is infinite}} \\ \textbf{\textit{then, received from plants}} \end{cases}$$

Decision Variables

$\boldsymbol{q_{jkt}} =$ ***quantity of item k requested from plants by retailer j in period t***
$a^r_{jkt} =$ *level of inventory of item k at retailer j in period t*

Model

Objective function

$$Max \sum_j \sum_k p_{jk} \sum_t \left(a^r_{jkt-1} + \boldsymbol{q_{jkt}} - a^r_{jkt}\right) - \left(\sum_j \sum_k \sum_t h^r_{jk} a^r_{jkt} + \sum_j \sum_k \sum_t v_{jk} \left(F_{jkt} - \left(a^r_{jkt-1} + \boldsymbol{q_{jkt}} - a^r_{jkt}\right)\right)\right) \tag{26}$$

Subject to

$$a^r_{jkt-1} + \boldsymbol{q_{jkt}} - a^r_{jkt} \geq E_{jkt} \quad \forall j \forall k \forall t \tag{27}$$

$$a^r_{jkt-1} + \boldsymbol{q_{jkt}} - a^r_{jkt} \leq F_{jkt} \quad \forall j \forall k \forall t \tag{28}$$

$$\boldsymbol{q_{jkt}} \leq \boldsymbol{Si_{jkt}} \quad \forall \boldsymbol{j} \forall \boldsymbol{k} \forall \boldsymbol{t} \tag{29}$$

$$\sum_k a^r_{jkt} \leq W^r_j \quad \forall j \forall t \tag{30}$$

$$\boldsymbol{q_{jkt}} = \boldsymbol{Sj_{jkt}} \quad \forall \boldsymbol{j} \forall \boldsymbol{k} \forall \boldsymbol{t} \tag{31}$$

$$a^r_{jk0} = 0, \quad \forall j \forall k \tag{32}$$

$$q_{jkt} \geq 0, a^r_{jkt} \geq 0, \ \textit{and all integers} \quad \forall j \forall k \forall t \tag{33}$$

First of all, the index i for the plants is no longer present, because it does not matter for the retailers where their supply comes from, as long as it comes. The parameter Si_{jkt} is the only new parameter, and it is the available supply of an item for a retailer in period t, which is received from the manufacturer's model. Only for the first iteration of the retailers' model it is assumed to be infinite. This is because the distributed search for the optimal solution begins at the retailers, as will become apparent in the following sections. Because it does not matter from which plant the supply comes, the decision variable qj_{ijkt} is changed into q_{jkt}. The latter now only represents the item quantities requested by a retailer from the manufacturer as a whole.

The objective function (26) is programmed to maximise profits by maximising sales and minimising inventory holding costs and stock-out costs. Constraints (27) and (28) still exist to ensure 'core demand' is satisfied and 'forecasted demand' not exceeded. The small change in these constraints is that $\sum_i qj_{ijkt}$ is replaced by q_{jkt}. Constraint (29) enforces that the requested amounts of items from the manufacturer are at most what the manufacturer has indicated he can provide. Constraint (30) is copied from the centralised model. The sixth constraint is newly added to calculate the input variable for the manufacturer's model, Sj_{jkt}. It is simply equal to q_{jkt}, meaning that could also be sent to the manufacturer's model. However, for uniformity this is changed into Sj_{jkt}.

4.3 Coordination Mechanism

The characteristics of the coordination mechanism will now be discussed, drawing from the characteristics identified in Sect. 2. Information sharing in the distributed model is minimal, with only requested quantities and available quantities shared between the two decision makers. The exchange of requested and available supply quantities was inspired by the distributed local search mechanism as developed by Jung et al. (2008) Other information is kept private, accounting for a state of information asymmetry. The information exchanged however, is certain, and truthfully exchanged. The decision makers do not display opportunistic behaviour.

The distributed decision making model can also be characterised as a non-team model. Neither decision maker takes the other's interest into account, and tries to optimise its own objective function. The other's response is not anticipated either, making the grade of anticipation non-reactive. Neither decision maker has knowledge of the other's model implemented in their own model. This makes opportunism a lot more difficult too.

The requested and available quantities are exchanged between the two decision making models directly. No mediator is involved to monitor or perhaps influence the local decisions that are taken. The distributed model starts with the retailer solving his local problem of determining how much to request from the manufacturer based on customer demand. That information is then sent to the manufacturer who returns his response. If the available supply quantity is enough to at least fill ‘core demand’, i.e., the retailers’ model has a feasible solution, the iterative exchange starts. The two models exchange updated solutions back and forth until they reach a feasible solution where all requested items are delivered without shortage. The initial solution used is one that maximises sales, because it is generated by the retailers’ model.

After starting the iterative process, the number of iterations is not fixed in the model. A protocol governing coordination as such has not been programmed. Iterations will be performed manually, continuing until a feasible solution has or has not been reached. A stop criterion is therefore not formally defined. Whether this is a correct choice will become apparent from the computational results, since they will show how many iterations were performed. The expected/desired result is a best solution for the Supply Chain as a whole. Fairness is not considered in the solution, with only the retailers’ model concerned with making money. All the manufacturer’s model does is minimise costs. He obviously does not know how his decisions affect revenue, and will only find out after having the final decision is made.

Side payments are not used to distribute the benefits between the decision makers and thus make it fair. This is also not required because the goal of the DDM model is to find the SC optimum. The other goal of side payments is to ensure each decision maker’s participation. The shortage penalty cost acts as the incentive for the manufacturer to comply with requests from retailers. This could also be modelled alternatively, to let the manufacturer make a profit when he complies with demand, but the penalty method works as well.

4.4 Modifications to Guarantee Feasibility

During initial testing of the distributed models, it was found that unless exorbitant shortage penalty costs vi_{jk} (which were actually higher than the sales price) were applied, the manufacturer’s model would not supply the item with the lowest margin in the first period if capacity was tight. With margin, the difference between the penalty cost and the production-, setup-, inventory holding- and distribution cost is meant. The reason was that the model preferred to produce larger batches of one item in the first period, and then in the next period would start producing the other item. The result was infeasibility in the retailers’ model, because core demand could not be filled.

It is not realistic for a manufacturer to have complete liberty over supply quantities for his customers. A reasonable assumption is that the manufacturer and the retailers have agreed contracts, in which it is agreed that the manufacturer will endeavour to meet at least a percentage of the requested supply. If such a ‘fill rate’ is to be incorporated into the manufacturer's model, the retailers could theoretically engage in shortage gaming strategies, to ensure they always get enough. This will not happen because of the model formulation, and the corresponding assumption of no opportunism, but in reality this would be very probable. That consideration shows that only in a trusting environment, can DDM really thrive. A fill rate of 67 % of the initially requested supply quantity is reasonable, and also enough to satisfy core demand. The fill rate FR_{jk} will be added as a parameter to the model, so that it can also be changed according to any set of contractual agreements.

The additional constraint for the manufacturer's model is then:

$$\sum_i qj_{ijkt} \geq FR_{jk} * Sj_{jkt}^{initial} \quad \forall j \forall k \forall t \tag{34}$$

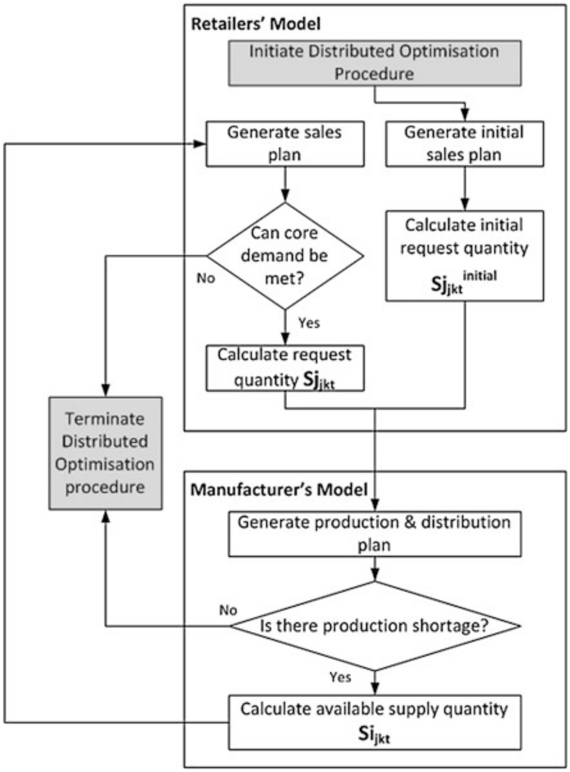
The constraint ensures that the delivered quantity of item k is at least the fill rate multiplied by the initially requested supply. $Sj_{jkt}^{initial}$ is entered into the restriction, because the regular Sj_{jkt} is updated after each iteration. However, the minimum amount to be filled is the fraction of the initially requested amount, not of the requested amount in the following iterations.

In reality, this would be easy, because the manufacturer can easily store the initially requested supply and not change it. However, for the model to function, a separate initial retailers' model must be run to ensure that it stores $Sj_{jkt}^{initial}$ somewhere where it cannot be changed. In ensuing iterations the retailers' model without the generation of $Sj_{jkt}^{initial}$ is then run.

4.5 The Decision Making Process

The flow of information and the decision making process is represented graphically in Fig. 3. The Distributed Model starts with the generation of the initial sales plan by the retailer, in which he calculates the initially requested supply quantities $Sj_{jkt}^{initial}$. These quantities are sent to the manufacturer who generates a production and distribution plan to best satisfy the requested supply quantities, at minimal cost. If there is no production shortage, then all requested supply can be delivered, which terminates the procedure. If the manufacturer cannot meet all that is requested, the available supply quantity per item, retailer and time period is calculated. This is then sent back to the retailer. He generates a new plan, checking whether he can meet his core customers' demand. If not, the problem is infeasible. If he can, then he generates a new request quantity and sends it to the manufacturer. The procedure continues until there are no production shortages.

Fig. 3 Decision making process distributed model



5 Distributed Model Under Uncertainty

In this section, the distributed deterministic model is adapted to account for uncertainty in demand. The retailers' model is the model that takes demand into account. In contrast, the manufacturer has no knowledge of demand. The adaptation to account for uncertainty will therefore be done exclusively on the retailers' model.

Peidro et al. (2009) found that several approaches exist in scientific literature for developing SC planning models under uncertainty. Most are based on analytical approaches, simulation approaches or hybrids of the former two. The models developed in these approaches generally use probability distributions based on historical data. The fuzzy set theory , pioneered by Zadeh (1965), and possibility theory are the other approaches identified. These are not based on historical data and have been applied with much success to various fields for modelling of uncertainty. Through requirement, possibility theory shall be applied to model demand uncertainty in the DDM model.

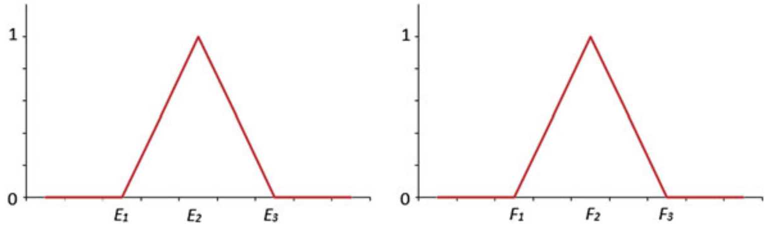


Fig. 4 Membership functions of fuzzy parameters for 'core demand' $\tilde{\boldsymbol{E}}_{jkt}$ (*left*) and 'forecasted demand' $\tilde{\boldsymbol{F}}_{jkt}$ (*right*)

Two parameters defined the demand in the retailers' model; which were 'core demand' and 'forecasted demand'. In possibility theory, these parameters are turned into diffuse coefficients. It is plausible that both parameters can turn out to be somewhat lower, or somewhat higher than initially thought. Consequently, a membership function that expresses that is required. A triangular or 'Lambda' membership function is therefore chosen to represent the fuzzy demand parameters. It has a central value with a membership degree of one, and the membership degree decreases the further the parameter moves away from the central value. Outside of two boundary values (one left and one right), the membership degree turns zero, meaning that it is not plausible that demand will take on values outside of a certain interval. Taking 'core demand' as an example, the triangular fuzzy coefficient E is defined by three parameters (E1, E2, E3). E1 is the left boundary of the fuzzy set, E2 the central value for which the membership degree equals one, and E3 is the right boundary of the set. The membership functions for 'core demand' E and 'forecasted demand' F are presented graphically in Fig. 4. One can see that for values in the interval [E1, E3] and [F1, F3], the membership degree μ is non-zero.

Furthermore, it is reasonable to assume that there is less uncertainty for the 'core demand', because it comes from a loyal customer base, than there is for the 'forecasted demand'. A smaller range of values thus belong to the fuzzy set of 'core demand than of 'forecasted demand'. This is expressed by a smaller interval (a, b) than (c, d), i.e., the range between the boundary values.

The approach used to change the deterministic model into a fuzzy model is the one used by Jiménez et al. (2007). It was developed to incorporate diffuse coefficients with trapezoidal membership functions into linear programming models. The triangular function is a simplification of the trapezoidal function, for which the two central values of the trapezoid are the same and the function is symmetrical. For the mathematical justification of the method, the reader is referred to the article by Jiménez et al. (2007).

For triangular functions, Jiménez showed that the expected interval of a diffuse coefficient $\tilde{a} = (a_1, a_2, a_3)$, can be calculated by:

$$EI(\tilde{a}) = \left[E_1^{\tilde{a}}, E_2^{\tilde{a}}\right] = \left[\frac{1}{2} * (a_1 + a_2), \frac{1}{2} * (a_2 + a_3)\right]$$

And the expected value of a diffuse coefficient can then be calculated by:

$$EV(\tilde{a}) = \left(\frac{E_1^{\tilde{a}} + E_2^{\tilde{a}}}{2}\right)$$

Objective functions and constraints with diffuse coefficients in them subsequently change, although differently. When a diffuse coefficient appears in the objective function, it is replaced by its expected value. In a symmetric triangular membership function this value corresponds with the central value, so no calculations are really required. Constraints change depending on the relationship ($\leq$, $=$ or $\geq$) defined in the constraint. The 'satisfy core demand' and 'not surpass forecasted demand' constraints affected in the retailers' model are $\geq$ and $\leq$ constraints respectively, which change as follows:

$$ax \geq b \rightarrow \left[(1-\alpha)E_2^a + \alpha E_1^a\right]x \geq \alpha E_2^b + (1-\alpha)E_1^b$$
$$ax \leq b \rightarrow \left[(1-\alpha)E_1^a + \alpha E_2^a\right]x \leq \alpha E_1^b + (1-\alpha)E_2^b$$

Where α is a parameter $\in [0, 1]$ set by the decision maker. With α he/she can vary the degree of feasibility of the fuzzy model. A higher value of α makes the fuzzy coefficients assume values that make it harder to find a feasible solution, thus covering for more of the uncertainty.

5.1 Retailers' Model Formulation Under Uncertainty

The F_{jkt} parameter in the objective function will be replaced with the expected value, so that the model is also generally valid. Because the newly defined fuzzy demand parameters only appear on the right hand sides of the constraints, only the right hand sides of the constraints are affected. The new terms are factored out to preserve linearity. The new fuzzy retailers' model is thus formulated as following, with bold highlighting the changes:

Indices

$j = retailers, \quad j \in (1, 2)$
$k = items, \quad k \in (1, 2)$
$t = timeperiods, \quad t \in (1, \ldots, 5)$

Parameters

$\vec{\alpha}$ = ***degree of feasibility parameter set by decision maker***
$\vec{\gamma} = (1 - \vec{\alpha})$, ***complement of degree of feasibility parameter set by decision maker***
$\tilde{\boldsymbol{E}}_{\boldsymbol{jkt}} = (\boldsymbol{E}_1, \boldsymbol{E}_2, \boldsymbol{E}_3)_{\boldsymbol{jkt}}$, *demand for item k at retailer j in period t that must be filled*
$\tilde{\boldsymbol{F}}_{\boldsymbol{jkt}} = (\boldsymbol{F}_1, \boldsymbol{F}_2, \boldsymbol{F}_3)_{\boldsymbol{jkt}}$, *total forecast demand for item k at retailer j in period t*
p_{jk} = *unit selling price of item k at retailer j*
h^r_{jk} = *inventory holding cost of item k at retailer j per period t*
W^r_j = *capacity for units of inventory at retailer j*
v_{jk} = *stockout cost per unit of item k at retailer j*
$Si_{jkt} = \text{offered supply quantity of item } k \text{ to ret. } j \text{ in period } t \begin{cases} \text{first iteration it is infinite} \\ \text{then, received from plants} \end{cases}$

Decision Variables

q_{jkt} = *quantity of item k requested from plants by retailer j in period t*
a^r_{jkt} = *level of inventory of item k at retailer j in period t*

Model

Objective function

$$Max \sum_j \sum_k p_{jk} \sum_t \left(a^r_{jkt-1} + q_{jkt} - a^r_{jkt}\right) - \left(\sum_j \sum_k \sum_t h^r_{jk} a^r_{jkt} + \sum_j \sum_k \sum_t v_{jk}\left(\left(\frac{1}{4}\boldsymbol{F}_1 + \frac{1}{4}\boldsymbol{F}_2 + \frac{1}{4}\boldsymbol{F}_2 + \frac{1}{4}\boldsymbol{F}_3\right) - \left(a^r_{jkt-1} + q_{jkt} - a^r_{jkt}\right)\right)\right) \tag{35}$$

Subject to

$$a^r_{jkt-1} + q_{jkt} - a^r_{jkt} \geq \frac{1}{2}\boldsymbol{\alpha}\boldsymbol{E}_2 + \frac{1}{2}\boldsymbol{\alpha}\boldsymbol{E}_3 + \frac{1}{2}\boldsymbol{\gamma}\boldsymbol{E}_1 + \frac{1}{2}\boldsymbol{\gamma}\boldsymbol{E}_2 \quad \forall j \forall k \forall t \tag{36}$$

$$a^r_{jkt-1} + q_{jkt} - a^r_{jkt} \leq \frac{1}{2}\boldsymbol{\alpha}\boldsymbol{F}_1 + \frac{1}{2}\boldsymbol{\alpha}\boldsymbol{F}_2 + \frac{1}{2}\boldsymbol{\gamma}\boldsymbol{F}_2 + \frac{1}{2}\boldsymbol{\gamma}\boldsymbol{F}_3 \quad \forall j \forall k \forall t \tag{37}$$

$$q_{jkt} \leq Si_{jkt} \quad \forall j \forall k \forall t \tag{38}$$

$$\sum_k a^r_{jkt} \leq W^r_j \quad \forall j \forall t \tag{39}$$

$$q_{jkt} = Sj_{jkt} \quad \forall j \forall k \forall t \tag{40}$$

$$a^r_{jk0} = 0, \quad \forall j \forall k \tag{41}$$

$$q_{jkt} \geq 0, a^r_{jkt} \geq 0, \text{and all are integers} \quad \forall j \forall k \forall t \tag{42}$$

Table 7 Datasets used in computations

	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8
Demand behaviour	Stable	Stable	Stable	Stable	Erratic	Erratic	Erratic	Erratic
Production capacity	Low	Low	High	High	Low	Low	High	High
Production/setup costs	Low/high	High/low	Low/high	High/low	Low/high	High/low	Low/high	High/low

6 Computational Results

This section discusses the computational results for the centralised deterministic model, distributed deterministic model and the distributed model under uncertainty. First the experimental design is explained, after which the results are presented. A discussion of the results follows to end the section.

6.1 Experimental Design

Eight different datasets were used to generate solutions with the different models. Three parameters were chosen to be varied to create the different sets. First, demand was given two different behaviours. Both had the same total demand value, but in one instance the demand was stable over the periods, whereas in the other it was very erratic, varying from near nothing to high peaks. Second, production capacity was varied. Low capacity meant that the production capacity constraints were very tight, and that it was never really possible to meet all demand. High capacity was chosen such that there should still be some slackness, meaning cost considerations would govern the decision more than capacity. These same costs were the third parameter to be varied. Combinations of low unit production costs with high setup costs, and high unit production costs with low setup costs were made to change the decisions the manufacturer would make regarding batches. Low setups obviously encouraged smaller batches. The eight combinations created the datasets found in Table 7. Due to space limitations, the details of the created datasets are not presented here, but can be made available upon request.

Another very important parameter for the distributed models is the penalty for production shortage applied to the manufacturer. Its value greatly influences the outcome of the manufacturer's decisions, as was already found by the model choosing not to serve retailers at all if it is chosen too low. Three different values for the production shortage were used; one that is only 60 % of the sales price, one of 90 % and the highest penalty is 120 % of the sales price.

The values for these parameters, and all other parameters were entered into a Microsoft Access database. An Access database was chosen because it can interact with the modelling program employed, MPL. MPL models can extract data from

Table 8 Computational results for centralised model (CM) and distributed model (DM)

Dataset	CM	DM	Absolute gap	% Gap	Iterations	Computation time (s)
vi_{jk} = 60 % of sales price						
1	116640	111244	5396	4.63	2	17.3
2	141321[a]	141691	−370	−0.26	2	10.4
3	147545	147545	0	0.00	1	1.1
4	166590	166590	0	0.00	1	0.3
5	123608	121709	1899	1.54	2	1.4
6	149340	144275	5065	3.39	2	500.6
7	148454	131895	16559	11.15	2	7.1
8	166525	165615	910	0.55	2	1.1
vi_{jk} = 90 % of sales price						
1	116640	115766	874	0.75	2	1.0
2	141321[a]	141694	−373	−0.26	2	3.4
3	147545	136164	11381	7.71	2	3.0
4	166590	166590	0	0.00	1	0.3
5	123608	121715	1893	1.53	2	0.6
6	149340	144275	5065	3.39	2	0.5
7	148454	131799	16655	11.22	2	1.6
8	166525	165572	953	0.57	2	1.5
vi_{jk} = 120 % of sales price						
1	116640	115763	877	0.75	2	1.6
2	141321[a]	141851	−530	−0.38	2	600.6
3	147545	147545	0	0.00	1	2.5
4	166590	166590	0	0.00	1	0.4
5	123608	122437	1171	0.95	2	0.7
6	149340	148460	880	0.59	2	1.1
7	148454	147544	910	0.61	2	1.5
8	166525	165615	910	0.55	2	1.4

[a] Computation was aborted after 600 s

the database and also export their solutions back to the database. This dual interaction was very useful for the exchange of the supply quantity variables Sj_{jkt} and Si_{jkt}. The MPL models were solved with the CPLEX solver on a single desktop computer with 4 GB RAM, using an academic license.

6.2 Results Centralised Deterministic Model and Distributed Deterministic Model

The results for the Centralised Deterministic Model (CM) and the Distributed Deterministic Model (DM) are presented in Table 8. Several observations were made whilst studying the data.

1. The biggest percentage gap in the objective value profit between the CM and the DM is 11.2 %. This occurs twice, whilst the second biggest gap is only 4.6 %. In general, the DM looks to be performing reasonably well compared to the CM, with many distributed solutions being close to the optimal solution.
2. The biggest percentage gaps occur when setup costs are high in relation to unit production costs. This corresponds to the odd datasets. The explanation is that high setup costs may cause the manufacturer to not want to produce a batch of a certain item, if it has enough in inventory to meet the agreed fill rate. Some of the forecasted demand can then not be met, resulting in lost sales and a suboptimal solution.
3. The DM's performance did not vary much for the production shortage penalties vi_{jk} of 60 % and 90 % of the sales price. However, for a penalty cost of 120 % of the sales price, DM performance was always equal or better than for the lower penalties, and by quite a margin. For this model therefore, a higher penalty cost seems to lead to better results.
4. Computation times were either very short, or extremely long. Two of the 24 runs of the manufacturer's model took 500 and 600 s respectively. These did not occur for the same dataset either, which seems to suggest that some combinations of data make the problem more difficult to solve optimally, because solutions are closer together.
5. One run of the CM also took a very long time, and it was aborted after 10 min (600 s) with a suboptimal solution being accepted. The DM outperformed the CM for all three shortage penalties. Only in this particular case, the DM performed better as the shortage penalty decreased. These two observations imply that for that particular dataset, not serving some of the demand was better. However, I assume that this is due to the dataset configuration and to be considered an anomaly instead of a rule.
6. The two most important observations come from the iterations column. In some cases, only one iteration is required because it is optimal for the manufacturer to deliver everything that is requested. This coincides with a stable demand behaviour and high production capacity.
7. In all the other runs, only two iterations are sufficient to generate a feasible solution to the problem. The retailers never order less than the available supply quantities. With hindsight, this is due to the decomposition choices made for the centralised model, which requires some further discussion.

Part of the assignment was to apply the same type of coordination mechanism that Jung et al. (2008) developed for their DDM model. The important difference between their model and the adaptation of Park's (2005) model developed in this chapter, is the model decomposition choices made when decomposing the central model into two distributed models. The decomposition choice to make the manufacturer responsible for delivering the items to the retailers has meant that the only consideration for the retailers is minimising inventory and stock-out costs. This means they will ask for as much as they can possibly sell every period, but no more. Keeping inventory would come into the equation if the distribution costs were also incurred by the retailers. They might then prefer smaller or bigger shipments to avoid nearly empty vehicles, resulting in inventory at the retailers. In that case, the retailers might actually change their requested supply quantities after knowing the available supply quantities, resulting in more iterations. In Jung et al.'s model, the retailer's place is taken by a third party logistics provider (3PL). He does have to take distribution costs into account, so his optimal local solution may change per received available supply quantity. I want to be clear that the decomposition choice was made on the argumentation given earlier, realism. The increased simplicity of the coordination was not taken into account.

Seeing how the distributed model would behave if the retailers' model included distribution costs instead of those being part of the manufacturer's model, would be very interesting. I expect that the amount of iterations would increase, following the above reasoning on managing distribution-, inventory and stock-out costs. The coordination mechanism would also change somewhat, although it would still function in a similar fashion.

Taking all into account, more dynamism in the coordination process would have been revealing regarding the workings of distributed decision making, but the current distributed deterministic model performs well enough to be satisfied with the result.

6.3 Results Distributed Model Under Uncertainty

The optimal solutions for the distributed model under demand uncertainty were generated with a production penalty value vi_{jk} of 120 % of the sales price, because these gave the best results in the earlier computations. Datasets one, four and five were chosen at random for the other parameters.

The parameter α was varied between 0.1 and 1. Recall that a solution for $\alpha = 0.1$ is very easily found because the uncertain demand parameters take on the most favourable values. That solution is thus the best possible outcome, but it is not very likely, and will probably leave the decision maker with unsold items. One could call it the risky solution. At the other end of the scale, $\alpha = 1$ gives the worst possible outcome. However, this solution is also certain to be possible, because the demand parameters take on the most unfavourable values that the decision makers believe they can assume. This is thus the risk-averse solution. So, the choice for α depicts the amount of risk the decision maker is willing to accept in his solutions. The computational results for different values of α are given in Table 9.

The optimal objective values for $\alpha = 0.5$ correspond with the objective values found by the deterministic DM. This is because of the symmetry in the chosen membership functions. For each dataset, the riskiest solution has the potential to perform 45 % better than the most risk-averse solution. It is therefore for the decision maker to decide how much risk he wants to take with his solutions.

The computation times are mostly quite low, with two notable exceptions. Once, for dataset 1 the entire computation takes 121 s, and for one run with dataset

Table 9 Computational results for distributed fuzzy model with demand uncertainty

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Dataset 1										
DM	129320	125433	119807	116517	115655	112638	106347	100170	94424	90563
Iterations	2	2	2	2	2	2	2	2	2	2
Computation time (s)	1.36	0.93	0.91	1.97	1.57	0.88	1.67	121	1.06	0.77
Dataset 4										
DM	183622	180739	176834	171344	166590	160362	152634	140498	135181	129627
Iterations	2	1	1	1	1	1	1	1	1	1
Computation time (s)	1.13	0.32	0.31	0.32	0.4	0.32	0.29	300	0.29	300
Dataset 5										
DM	136995	133134	131472	127204	122329	118171	112104	106683	101035	96044
Iterations	2	2	2	2	2	2	2	2	2	2
Computation time (s)	0.96	10.45	1.52	0.82	0.86	0.98	1.63	1.68	3.31	0.86

vijk—120 % of sales price

4 the solver takes a total of 300 s. These are other datasets than took long in the deterministic DM however. Consequently, this enforces the belief that long computation times result from 'unlucky' combinations of parameters that give the solver a hard time in finding the optimal solution. This time however, both longer runs did finish inside 10 min and were therefore not aborted prematurely.

7 Conclusions

An analysis of recent Distributed Decision Making related work was given in this chapter. The different works of literature were classified along differing DDM characteristics, with extra attention given to characteristics related to the coordination mechanisms used in DDM systems.

Thereafter, a centralised deterministic mixed-integer model was developed for a Supply Chain planning and distribution problem similar to that of Park (2005), but with the addition of multiple product levels. This model was further developed into a distributed deterministic model and a distributed model which accounted for demand uncertainty by applying possibility theory. The distributed model has demonstrated that it could approximate very closely the centralised model's performance, in most cases to within a per cent point. With the fuzzy distributed model, it was possible to see what the solutions could deliver in terms of objective value under various risk levels, which showed to be an insightful tool for decision makers dealing with uncertainty.

It should be noted that the objective of this chapter is not to provide a real large scale application for the proposed models. The emphasis in this chapter is on demonstrating how a proposed DDM coordination mechanism for a supply chain planning problem under uncertainty, can obtain solutions very close to those obtained by the centralized model. For this reason and for illustrative purposes, we focus on a small size case study. It is expected that when the size of the problem grows, the computational time will be greater. Further research may investigate the application of metaheuristics approaches and other soft computing techniques in order to handle large scale problems.

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A Fuzzy Linear Programming Approach for Aggregate Production Planning

Cagatay Iris and Emre Cevikcan

Abstract Aggregate Production Planning (APP) is considered as an important stage in production systems, since it links operations with strategies and plays a key role in enterprise resource planning and organizational integration. An effective APP should not only provide the minimization of production and inventory costs, but also increase the level of service available to the customers. When maintaining APP, some of cost and demand parameters cannot be frequently determined as crisp values. Fuzzy logic is utilized in many engineering applications so as to handle imprecise data. This chapter provides a mathematical programming framework for aggregate production planning problem under imprecise data environment. After providing background information about APP problem, together with fuzzy linear programming, the fuzzy linear programming model of APP is solved on an illustrative example for different α-cut values.

Keywords Aggregate production planning · Fuzzy logic · Linear programming · Holding and backorder costs

1 Introduction

Aggregate production planning (APP) is a problem of deciding how to vary production capacity, keep stock, and subcontract to satisfy a seasonal demand in the most effective way. It is medium-term planning whereby its planning horizon is usually from 6 to 18 months (Techawiboonwong and Yenradee 2003). APP

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provides a linkage between operations management and strategic management. Additionally, APP operations with strategies and plays a key role in enterprise resource planning and organizational integration. The goal of a manufacturing enterprise for making APP is to obtain the maximum profit or minimum cost by determining the product quantity, subcontracting quantity, labor level, etc., to meet the market demand.

Among costs in APP models, backorder and holding costs can be regarded as important since they affect delivery performance and stock policy of the company as the two parameters. However, in real-world APP problems, backorder and holding costs as well demand are frequently imprecise because some information is incomplete or unobtainable. In this context, fuzzy logic provides an inference morphology that enables approximate human reasoning capabilities to be applied to knowledge-based systems. The theory of fuzzy logic provides a mathematical strength to capture the uncertainties associated with human cognitive processes, such as thinking and reasoning. Fuzzy set theory has been widely applied in different disciplines, such as operations research, management science, control theory and artificial intelligence. Fuzzy mathematical programming is one of the most popular decision making approaches based on the fuzzy set theory.

In this chapter, imprecise parameters in APP are addressed. The research will commence with discussion of various types of uncertainty and sources of uncertainty in production planning. This broad perspective will be narrowed down to an elaborate study of fuzziness in demand, holding cost and backorder cost. A fuzzy linear programming model for APP is introduced. Another point is that, this chapter also provides necessary theoretical background for decision makers to develop and implement their own tool for multi item, multi period aggregate production planning.

The rest of this chapter is organized as follows. In Sect. 2, relevant literature is reviewed. In Sect. 3, background information about APP is given. Fuzzy linear programming techniques are included in Sect. 4. Application of fuzzy linear programming to APP problem is given in Sect. 5. Finally, conclusions are provided in Sect. 6.

2 Problem Structure and Relevant Literature

As mentioned above problem structure is described by means of fundamental and extension clusters. Fundamental cluster consists of basic aggregate planning aspects, while extension may be formed with detailed parameters that are improved by authors' strategies. Literature studies in APP are mostly based on analysis of problem states and solution methodologies. There is a lack of bilateral comparison of problem type and solution method that is proposed. In this study, both problem structure and solution method is included in survey analysis. As mentioned before, fundamental and extension aspects are reflected in literature analysis. Reflecting problem structure consists of aspects that are investigated by

Table 1 Coding scheme for classification of the studies

A: Fuzzy attributes	*B: Place of fuzziness*	*X: Solution strategy*
A1: Production cost	B1: Objective	X1: Fuzzy goal programming
A2: Inventory holding cost	B2: Constraints	X2: Fuzzy linear programming
A3: Backorder cost	B3: Weights	X3: Fuzzy heuristic search
A4: Overtime cost	*C: Types of fuzzy numbers and membership*	X4: Fuzzy multi-objective modeling
A5: Subcontracting cost	C1: Trapezoidal	X5: Fuzzy genetic algorithm
A6: Setup cost	C2: Linear	*Y: Defuzzification (Crispization) of fuzzy models*
A7: Purchasing cost	C3: Triangular	Y1: Max–Min operators
A8: Hiring/laying off cost	C4: L-R type	Y2: α- level sets
A9: Production capacity	*D: Number of objectives*	Y3: Interactive constraint conf.
A10: On-hand capacity	D1: Single	Y4: Solving auxiliary MOLP
A11: Overtime capacity	D2: Multiple parallel	Y5: Fuzzy simulation
A12: Workforce capacity	D3: Multiple conflicting	Y6: Subgradient algorithm
A13: Processing time		Y7: Zimmermann method
A14: Demand		Y8: Torabi-Hassani method
A15: Quality and fraction rate		

authors in given paper. The studies reviewed in this chapter are classified to observe the nature of current literature. For this aim, a coding scheme was developed as seen in Table 1.

In the literature review, there are five aspects that are analyzed to understand APP problem structure better. These parameters are fuzzy attributes, place of fuzziness, types of fuzzy numbers and membership functions, and number of objective for related fuzzy mathematical model. In this sense, there are a number of parameters which may have fuzzy attributes. These parameters may be different cost inputs. Since cost values change frequently, different APP cost types are reflected in schema to note fuzzy parameters. Capacity values may also have fuzzy nature due to uncertainty associated. Since it is hard to measure exact capacity of system components, fuzzy variables are associated to each of capacity parameters. Other sources of variability which may be represented with fuzzy numbers are the demand pattern of the items, unit processing of each, and quality rate of production systems. These parameters are selected very carefully to fully characterize components of a fundamental aggregate production planning mathematical model. Second aspect that is presented in literature table is to understand where fuzzy parameters are embedded in the mathematical model. There may be three different alternatives. Fuzziness may be characterized in the objective function coefficients or constraint coefficients/parameters. There may be fuzzy attributes about the weights of objective functions. Here in this case, objective functions could be crisp, but the weight of each objective may be characterized with a membership function. Another aspect which is tested is the type of fuzzy numbers and membership functions utilized by the mathematical model. Four of most popular fuzzy number types are listed. This clustering may help us understand whether there is correlation between specific fuzzy parameter and its fuzzy number type. The last attribute on problem structure is the number of objective functions. There may be specific solution strategies on single objective problems. If formulated problem has multiple objectives, it becomes important whether they are in the same direction or conflicting. Formulating the problem structure is the first stage to solve to optimality.

In the second phase of literature studies, solution procedures are analyzed with respect to optimization tools and defuzzification techniques of the mathematical models. In the first aspect where solution strategies are listed, five common techniques are illustrated as: fuzzy LP, fuzzy goal programming (GP), fuzzy heuristic optimization (HO), fuzzy MOLP, fuzzy GA. Having a multiobjective problem structure, problem could only be solved via fuzzy GP or MOLP. After showing general framework of solution, details of procedure should be clarified. Crispization of fuzzy models is made by using various techniques. As stated in Table 1, solution may utilize max–min operators of extension principle. There may be some α-cuts based conversion techniques. In addition to these fuzzy logic based approaches, some techniques of fuzzy simulation and subgradient analysis are also recalled. Finally, there are some fundamental techniques that are published to defuzzify a given fuzzy LP problem. Methods of Zimmerman (1978) and Torabi and Hassini (2008) methods are the common operators. Hence, they are also involved in the literature table.

Table 2 provides detailed information on the studies about the APP problem. The represented studies correspond to a valuable part of fuzzy modeling related APP literature and cover most of the fuzzy logic features. The classification reveals a number of possible areas of research that need to be addressed in the future.

When we examine the studies regarding the fuzzy parameters, we observe that unlike the common expectation where researchers have formulated a unique parameter as a fuzzy number, papers have most commonly multiple fuzzy parameters. The first eight parameter of A codes reflect the cost values, we can easily observe that most of recent papers consider fuzzy cost parameters (Torabi and Hassini 2008; Liang et al. 2011; Yaghin et al. 2012). The ones that have fuzziness in costs are also diversified in terms of fuzziness level. It is observed that as a cost parameters of production, holding, backlogging and hiring/laying-off are most common fuzzy attributes. There are also some studies which may be called as fully fuzzified in respect to cost coefficients (Wang and Liang 2005a, b; Liang 2007; Torabi et al. 2010; Yaghin et al. 2012).

Assigning fuzzy capacity levels is another popular way to embed possibility in input parameters. Table 2 reveals that production and workforce capacity are very common as fuzzy attributes (Wang and Liang 2005a, b; Omar et al. 2012; Mezghani et al. 2012). Ending inventory capacity which is related to warehouse capacity cannot be measured directly, since utilized equipment and rack storage area change frequently parameter may also be set as a fuzzy attribute (Fung et al. 2003; Sakallı et al. 2010).

Finally, the last cluster of fuzzy attributes comes out with right-hand side values and coefficient of constraints. Here we may observe the most popular fuzzy

Table 2 Classification of the studies according to the coding schema

Authors	Problem structure											
	A											
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12
Pendharkart (1997)												
Miller et al. (1997)									*			*
Hsu and Wang (2001)		*	*			*						
Fung et al. (2003)										*		*
Wang and Liang (2005a, b)	*	*	*	*	*			*	*		*	*
Wang and Liang (2005a, b)												
Yuan and Liu (2006)												
Aliev et al. (2007)	*	*					*					
Liang (2007)		*	*	*	*			*	*			*
Selim et al. (2008)												
Jamalnia and Soukhakian (2009)												
Lan et al. (2009)	*	*										
Torabi and Hassani (2009)								*				
Baykasoglu and Gocken (2010)	*			*				*				*
Torabi et al. (2010)	*	*	*	*	*	*		*			*	
Liang et al. (2011)	*	*		*				*	*		*	
Omar et al. (2012)	*	*	*		*	*			*	*		*
Mezghani et. al. (2012)									*			*
Figueroa-García et al. (2012)												
Yaghin et al. (2012)	*	*	*	*	*			*	*	*		
Peidro et al. (2012)												
	8	9	7	6	5	3	2	7	9	4	3	8

Authors	Problem structure											
	A			B			C				D	
	A13	A14	A15	B1	B2	B3	C1	C4	C5	C6	D1	D2
Pendharkart (1997)			*		*				*		*	
Miller et al. (1997)		*			*			*			*	
Hsu and Wang (2001)				*			*				*	
Fung et al. (2003)		*			*				*		*	
Wang and Liang (2005a, b)				*	*				*		*	
Wang and Liang (2005a, b)						*		*				*
Yuan and Liu 2006		*			*				*		*	
Aliev et al. (2007)				*	*		*		*		*	
Liang (2007)				*	*				*			*
Selim et al. (2008)						*		*	*			

(continued)

Table 2 (continued)

Authors	Problem structure											
	A			B			C				D	
	A13	A14	A15	B1	B2	B3	C1	C4	C5	C6	D1	D2
Jamalnia and Soukhakian (2009)						*	*	*	*			*
Lan et al. (2009)		*		*	*				*		*	
Torabi and Hassani (2009)					*				*			
Baykasoglu and Gocken (2010)				*	*		*		*			
Torabi et al. (2010)	*	*		*	*				*		*	
Liang et al. (2011)		*		*	*				*		*	
Omar et al. (2012)	*	*		*	*				*		*	
Mezghani et al. (2012)		*			*		*		*			*
Figueroa-García et al. (2012)		*			*				*		*	
Yaghin et al. (2012)			*	*	*	*	*	*				
Peidro et al. (2012)						*			*			
	3	13	2	13	21	7	6	7	22	1	17	3

Authors	Problem structure													
	D	X					Y							
	D3	X1	X2	X3	X4	X5	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8
Pendharkart (1997)			*											
Miller et al. (1997)			*				*							
Hsu and Wang (2001)			*					*						
Fung et al. (2003)			*				*	*	*					
Wang and Liang (2005a, b)					*				*	*				
Wang and Liang (2005a, b)									*	*				
Yuan and Liu 2006						*					*			
Aliev et al. (2007)				*		*					*			
Liang (2007)					*				*	*			*	
Selim et al. (2008)	*	*					*							
Jamalnia and Soukhakian (2009)		*							*	*				
Lan et al. (2009)				*							*	*		
Torabi and Hassani (2009)	*													*
Baykasoglu and Gocken (2010)	*			*								*		
Torabi et al. (2010)			*				*		*					
Liang et al. (2011)			*							*				
Omar et al. (2012)			*					*						*
Mezghani et al. (2012)		*								*	*			
Figueroa-García et al. (2012)										*				
Yaghin et al. (2012)	*	*			*					*				*
Peidro et al. (2012)	*				*									*
	7	4	12	3	5	2	6	4	8	10	5	3	2	4

attribute of the whole literature which is demand pattern of planning items (Miller et al. 1997; Yuan and Liu 2006; Mula et al. 2006a, b). Another valuable attribute which has not taken the attention of researcher is the quality rate and different utilization rates (Pendharkart 1997; Yaghin et al. 2012). There is a future research opportunity in this topic with aggregate production planning reflection. Finally, there are some interesting studies (Wang and Liang 2005a, b; Mula et al. 2006a, b; Taghizadeh et al. 2011; Peidro et al. 2012) which don't have any fuzzy attributes. These models are fully crisp. However, the weight of each objective is defined with a fuzzy membership function.

Depending on fuzzy parameters, it changes the place of fuzziness. Most commonly fuzzy parameters are enrolled in the constraint. Whenever studies have demand fuzziness, inventory balance equation becomes fuzzified (Miller et al. 1997). There are also many cases where both objective function and constraints are fuzzy (Jamalnia and Soukhakian 2009; Baykasoglu and Gocken 2010). In such a case, crisp solution should be formulated iteratively where you had both defuzzified equivalents of constraint and objective function.

Finally, there are few studies that use fuzzy parameters not only in the mathematical model, but also in the imprecise weights of each objective (Yaghin et al. 2012). In such cases, problem becomes more complicated. Another analytical component is the type of fuzzy number. Triangular and trapezoidal fuzzy numbers are extensively assigned to attributes. There is a lack of L-R type formulation of aggregate production planning problem. The last associative of problem structure is the number of objectives. As it is seen in Table 2, in most cases a single objective mathematical modeling is formulated which is a cost minimization (Hsu and Wang 2001; Yuan and Liu 2006; Aliev et al. 2007). If the formulated mathematical model contains multiple objectives, they are most commonly conflicting. Authors have observed that once the objective-1 is a cost minimization, other objectives become maximization of machine utilization (Taghizadeh et al. 2011), minimization of total defective items (Torabi and Hassini 2009), maximizing total number of production, maximizing supply chain profit (Selim et al. 2008; Baykasoglu and Gocken 2010), minimizing idle time of production plan (Peidro et al. 2012).

After analyzing fundamental problem structures where different fuzzy aggregate production problems are revealed, we have focused on how studies solve related problem with test bed, case study, industrial data that they obtained. In most cases where a single objective is formulated, authors have utilized by the basic of fuzzy LP. If the formulated problem has a multi-objective case, fuzzy goal programming (Jamalnia and Soukhakian 2009; Mezghani et al. 2012), fuzzy MOLP (Wang and Liang 2005a, b; Liang 2007) techniques are derived. There are also some novel fuzzy heuristic search techniques for APP problem (Yuan and Liu 2006; Aliev et al. 2007). It is deduced that fuzzy formulation of APP problem has a combinatorial characteristics for larger instance sets. Hence, genetic algorithms (Yuan and Liu 2006), particle swarm optimization methods (Lan et al. 2009), tabu search (Baykasoglu and Gocken 2010) techniques are engaged to solution framework.

General framework of solution strategy is not adequate to understand problem in a better way. For this reason, we have also listed different defuzzification techniques for objectives and constraints. It is deduced that extension principle based solution techniques are continuously in focus (Miller et al. 1997). However, most common strategies are using interactive constraint relaxation and formulating auxiliary MOLP models (Wang and Liang 2005a, b; Liang 2007; Jamalnia and Soukhakian 2009). In addition to direct crispization technique, some simulation techniques are also proposed (Aliev et al. 2007; Lan et al. 2009) where Under the decision maker may expect a higher or lower possibility level of meeting the market demands for a certain product type in a given period. Method of Torabi-Hassini (2008) has also taken the attention of researchers where an interactive possibilistic programming approach for multiple objective mathematical models is set (Torabi and Hassini 2009; Peidro et al. 2012).

This study tries to conduct an equivalent model that reflects fundamental aspects of aggregate production planning. Model proposed in the next section gives information about basic preliminary components of general APP. Mathematical model is formulated with inspirations coming from literature, but is not generalized to an all-common model. It is deduced from the literature review that there is an immense trade-off between cost of holding a unit inventory and backlogging demand for a further period (Iris and Yenisey 2012). For this reason, determination of these two cost parameters is vital to obtain an aggregate production plan to be applied on the shop-floor.

However, since we don't know exactly about customer approach against backorder, and changing values of product, these two cost parameters (holding, backlogging) obviously have imprecise nature. Noting that most of the papers in the literature assume that demand is also fundamental fuzzy attribute, the model proposed has fuzzy demand pattern. In the Sect. 3, we introduce the fuzzy APP model that will be solved.

3 Aggregate Production Planning Model

The original fuzzy APP model proposed in this chapter deals with minimizing the production related costs simultaneously. Production related costs include production, inventory, shortage, subcontracting costs and those associated with hiring/laying-off man-hour. Here, the objective function is fuzzy and piecewise linear membership functions are introduced to denote the DM's satisfaction degrees with obtained objective function coefficients. There should be some assumption in order to formulate a proper model that could be generalized. Following notations are used in the model formulation (Guillermo 2013):

Index attributes

t time horizon in periods where $t = 1,2,\ldots, T$

i total number of products where $i = 1,2,\ldots, N$

Model parameters:

$\widetilde{h_{it}}$	fuzzy inventory carrying cost per unit of product i from period t to $t+1$
c_{it}	unit production cost for product i in period t
$\widetilde{\pi_{it}}$	fuzzy backorder cost per unit of product i carried from period t to $t+1$
s_{it}	unit setup cost for product i in period t
r_t	cost per man-hour of regular labor in period t
ov_t	cost per man-hour of overtime labor in period t
h_t	cost of hiring one man-hour in period t
f_t	cost of firing one man-hour in period t
$\widetilde{D_{it}}$	fuzzy demand parameter for product i in period t
cp_i	man-hours required to produce one unit of product I
cs_i	man-hours required to setup of product i
p	fraction of regular hours allowed as overtime
wd_t	working days in period t

Decision Variables:

X_{it}	units of product i to be produced in period t
I_{it}^{+}	units of product i to be left over as an inventory in period t
I_{it}^{-}	units of product i backordered at the end of period t
H_t	man-hours of regular work force hired in period t
F_t	man-hours of regular work force fired in period t
R_t	man-hours of regular labor used during period t
O_t	man-hours of overtime labor used during period t
Y_{ij}	binary variable indicating setup for item i in period t (1; if item i is produced in period t, 0 otherwise).

The linear program for given index set is:

$$\text{Min} \sum_{\text{i}} \sum_{\text{t}} [\text{c}_{\text{it}} X_{it} + \widetilde{h_{it}} I_{it}^{+} + \widetilde{\pi_{it}} I_{it}^{-} + s_{it} Y_{it}] + \sum_{t} [\text{r}_{\text{t}} R_t + ov_t O_t + h_t H_t + f_t F_t]$$

s.t.

$$X_{it} + I_{it-1}^{+} - I_{it}^{+} - I_{it-1}^{-} + I_{it}^{-} = \widetilde{D_{it}} \quad \forall \text{i,t}$$

$$R_t - R_{t-1} - H_t + F_t = 0 \quad \forall \text{t}$$

$$\sum (cp_i X_{it} + cs_i Y_{it}) \le R_t + O_t \quad \forall \text{t}$$

$$O_t - pR_t \le 0 \quad \forall \text{t}$$

$$X_{it} \le M(Y_{it}) \quad \forall \text{i, t}$$

$$X_{it}, I_{it}^{+}, I_{it}^{-}, R_t, O_t, H_t, F_t \ge 0, Y_{it} \in \{0, 1\} \quad \forall \text{i, t}$$

The model covers multiple-items with single-level product structure to be planned over T periods. The objective (1) aims to minimize total cost of production activities. Constraint (1) is inventory balance equation for each product and period. Constraint (2) satisfies the balance of man-hours in the shop floor. Hiring and firing values should be subtracted and added to beforehand amount of current workforce level. Due to the fact that there is a single production capacity consumer (no setup involved), production limitations are formulated by constraint (3) consisting of production time with an upper bound of threshold of overall capacity and overtime amount. Constraint (4) ensures that whenever production takes place of each item, a setup operation is performed. Constraint (5) is a control technique to limit the maximum available overtime regarding production capacity. Constraint set (6) reflects non-negativity conditions and binary variable of setup activities.

4 Fuzzy Linear Programming Techniques

Fuzzy linear programming can be derived by using fuzzy sets as coefficient values in objective function, constraints or right-hand sides of the constraints. Providing solutions for fuzzy linear programming models, three different techniques are discussed in this section.

4.1 Zimmermann's Approach

Both fuzzy objective and constraint functions are considered in Zimmermann's approach (Zimmermann 1991).

$$\begin{aligned}
&\widetilde{\mathrm{Max}} \quad cx \\
&\text{s.t.} \\
&a_i x \leq b_i \quad i = 1, \ldots, m \\
&x \geq \tilde{0}
\end{aligned} \tag{1}$$

An aspiration level and a tolerance interval are proposed for the fuzziness of the objective function. In the fuzzy constraints, a fuzzy inequality can be considered as fuzzy right-hand sides. On the condition that an aspiration level of objective value is denoted as b_0, the fuzzy mathematical model is called a symmetric fuzzy model and can be written as follows (Shih 1999):

$$\begin{aligned}
&cx \geq b_0 \\
&a_i x \leq b_i \quad i = 1, \ldots, m \\
&x \geq 0
\end{aligned} \tag{2}$$

The following matrix notation is used for defining symmetric fuzzy model:

$$\begin{aligned} & Ax \leq b \\ & x \geq 0 \\ & A = \begin{bmatrix} -c \\ a_i \end{bmatrix} b = \begin{bmatrix} -b_0 \\ b_i \end{bmatrix} \end{aligned} \tag{3}$$

Having an interval, the fuzzy inequality violation of right-hand-side values, b_i, is equivalent to the fuzzy inequality. The membership function of the degree of violation of the fuzzy inequality is expressed as follows where p_i is a tolerance level in the fuzzy relationship:

$$\mu_i(a_i x) = \left\{ \begin{array}{ll} 1 & a_i x_i \\ 1 - \frac{a_i x - b_i}{p_i} & b_i ix \leq b_i + p_i \\ 0 & b_i + p_i \leq a_i x \end{array} \right\} \tag{4}$$

The problem is now to obtain the maximum value of the membership grade in expression (4) which can be expressed as the following model which presents the final solution.

$$\begin{aligned} & Max\ \lambda \\ & \text{s.t.} \\ & \lambda \leq 1 - \frac{a_i x - b_i}{p_i} \\ & \lambda \leq 1 \\ & x \geq 0 \end{aligned} \tag{5}$$

4.2 Chanas' Approach

The approach of Chanas can be considered as parametric programming method for fuzzy linear programming. A parameter is included to Eq. (2) as follows (Chanas 1983):

$$\begin{aligned} & cx \geq b_0 - \theta p_0 \\ & a_i x \leq b_i + \theta p_i \quad i = 1, \ldots, m \\ & x \geq 0 \end{aligned} \tag{6}$$

The optimization of the membership of constraints can be given as the following max–min operation oriented equation:

$$\mu_c(A_x(\theta)) = \min_{i=1,,,,m} [\mu_i(a_i x(\theta))] = \alpha = 1 - \theta \tag{7}$$

α the minimum membership grade of all constraints
θ the complementary term of α

In this approach, the objective value has a piecewise-linear membership function in which aspiration and tolerance levels are utilized.

$$\mu_0(x(\theta)) = \left\{ \begin{array}{cc} 1 & cx^* > b_0 \\ 1 - \frac{b_0 - cx^*(\theta)}{p_0} & b_0 - p_0 \leq cx^*(\theta) \leq b_0 \\ 0 & cx^*(\theta) < b_0 - p_0 \end{array} \right\} \tag{8}$$

$x^*(\theta_0)$ the admissible solution with a fixed parameter θ

Finally, a decision is made via max–min operation oriented membership function given as the following equation where μ_0 and μ_c are the membership functions of objective and constraints, respectively (Shih 1999).

$$\max \mu_D(\theta) = \max\{\min[\mu_0(\theta), \mu_0(\theta)]\} \tag{9}$$

4.3 Julien's Approach

Julien (1994) integrates the α-cut concept with the possibility programming of Buckley (1989) to resolve the problem including fuzzy objective and fuzzy right hand side by solving pairs of crisp linear programming problems in Eqs. (10) and (11). As previously mentioned Eq. (1) is a general form of fuzzy linear programming model (Allahviranloo and Afandizadeh 2008). The superscript represents an α-cut of the fuzzy parameters, and the subscripts L and U are the corresponding lower and upper cuts.

$$\begin{array}{l} \text{Max } c_L^\alpha x \\ \text{s.t.} \\ A_U^\alpha x \leq b_L^\alpha \quad i = 1, \ldots, m \end{array} \tag{10}$$

$$\begin{array}{l} \text{Max } c_U^\alpha x \\ \text{s.t.} \\ A_L^\alpha x \leq b_U^\alpha \quad i = 1, \ldots, m \\ x \geq 0 \end{array} \tag{11}$$

When method of Julien is applied to a minimization problem given in (12), the following formulation should be considered. Then, the objective function has the interval between the solutions of (14) and (13).

$$\begin{aligned} &\tilde{\mathrm{Min}} \quad cx \\ &\text{s.t.} \\ &a_i x \geq b_i \quad i = 1, \ldots, m \\ &x \geq \tilde{0} \end{aligned} \tag{12}$$

$$\begin{aligned} &\mathrm{Min}\ c_U^{\alpha} x \\ &\text{s.t.} \\ &A_L^{\alpha} x \geq b_U^{\alpha} \quad i = 1, \ldots, m \\ &x \geq 0 \end{aligned} \tag{13}$$

$$\begin{aligned} &\mathrm{Min}\, c_L^{\alpha} x \\ &\text{s.t.} \\ &A_U^{\alpha} x \geq b_L^{\alpha} \quad i = 1, \ldots, m \\ &x \geq 0 \end{aligned} \tag{14}$$

5 An Application: Aggregate Production Planning with Fuzzy Parameters

In order to test presented approach, some analytical experiments are applied to an aggregate production planning problem. It can be deduced from the literature studies that experimental test bed is relatively important to distinguish between different configurations of fuzzy LP solvers (Torabi et al. 2010). Test instances are obtained by using libraries of internet for given problem type (Guillermo 2013). It should be noted that the case of single product is used so as to reflect time dependent fuzziness of parameters.

The objective function is formulated as a combination of production, holding, backorder, workforce man-hours, overtime, hiring and firing of man-hours costs. And objective is formulated as minimization of overall costs. Since one type of item is produced, setup times and costs are inevitable for each period (where are months in this case). Hence, setup times are embedded in the unit processing time and setup costs are added to objective function as a constant. Let us assume that facility is planning a monthly aggregate plan for the next 6 months of the planning horizon. It is known that each item is produced in 90 min which constitutes a combination of operations. It is assumed that there is a 3-items initial inventory, zero initial backlogs prior to planning horizon and we will assume a unit production rate that overtime is at most 25 % (p) of regular labor.

The goal is to obtain the optimal production amounts, on-hand inventory of each period, backlogs, workforce man-hours needed of each month, and amounts of man-hours to be hired and fired. There is another assumption that there is 8-hours shift per day to reflect whole working time for a given item. And cost of each hour is 6$. To solve formulated fuzzy linear programming model, method of Julien[36] is utilized. Related labour parameters and unit production costs for the problem are given in Table 3.

Demand amounts and costs for unit inventory holding and backlogging are expressed as fuzzy triangular numbers in Table 3, since they cannot be determined precisely. The representation of fuzzy triangular parameters shows us that decision maker was able to formulate a membership function for given APP parameter. The membership functions are very acceptable considering steady characteristics of production planning environment.

The upper and lower values of bounds on costs and demand forecasts for different α-cuts are given in Tables 4 and 5, respectively. We should note that selection of α-cuts is an important decision to reflect problem structure better in the sense that expected impreciseness would be high. Calculations are made by using formulations proposed in Sect. 4.

The model is coded by using LINGO 13.0 optimization software. Results are obtained for the α-cut values of 0, 0.25, 0.50, 0.75, 1. Lower and upper bounds of optimum aggregate production plan and their related non-zero variables are given in Table 6.

In Appendix, the general code of LINGO for Aggregate production planning is also given. The notations in code are taken from APP libraries (Reveliotis 2013). It can be observed that assumptions of initial parameters (inventory, backlogging, and workforce level) are given in dataset.

As can be seen from Table 6, upper bound of optimal aggregate production plan increases with higher level of fuzziness (highest value: 692.639, α-cut = 0). Inversely, lower α-cut values yields decreasing lower bounds (lowest value: 372.425, α-cut = 0). In fact, α-cut value can be considered as the level of certainty.

Range between lower and upper bounds is inversely related to α-cut value. The underlying reason of this fact is that range of fuzzy parameters (demand and holding-backlogging costs) gets wider with higher level of vagueness.

Lower bound has the range of [372.425; 492.968]. Similarly, upper bounds are computed as the values between 573.296 and 692.639.

Regarding APP model, it is clear that there is a trade-off between holding an inventory and backlogging a demand pattern. What is more, there is another trade-off between hiring some additional workforce and making overtime with the availability on-hand. Models mostly hold inventory at the initial stages of planning horizon and they do not intend to change workforce frequently because of the high values of hiring and firing. Backlogging issue is solved by considering objective coefficients. In period where backlogging cost is high, plan resulted in hiring new workforce. Another aspect that should be covered is the number of working days. Especially in May and June when working days of month is low, aggregate

Table 3 Crisp cost and fuzzy cost/RHS parameters (costs in \$, $i = 1$)

Mont	Jan	Feb	Marc	April	May	June
c_{it}	7	8	8	8	7	8
wd_t	15	15	18	18	15	15
ov_t	22.5	22.5	27.0	27.0	22.5	22.5
h_t	1000	1000	1000	1000	1000	1000
f_t	1500	750	1250	1000	1500	950
$\widetilde{h_{it}}$	(2.1;3;3.9)	(2.8;4;5.2)	(2.8;4;5.2)	(2.8;4;5.2)	(2.1;3;3.9)	(1.4;2;2.6)
$\widetilde{\pi_{it}}$	(14;20; 26)	(17.5;25; 32.5)	(17.5;25; 32.5)	(17.5;25; 32.5)	(14;20; 26)	(10.5;15; 19.5)
$\widetilde{D_{it}}$	(70;100; 130)	(70;100; 130)	(105;150; 195)	(140;200; 260)	(105;150; 195)	(70;100; 130)

Table 4 Unit holding cost and unit backlogging cost (\$)

Unit holding	Upper/lower Cut	Jan	Feb	Marc	April	May	June
1	–	3	4	4	4	3	2
0.75	Lower	2.78	3.70	3.70	3.70	2.78	1.85
	Upper	3.23	4.30	4.30	4.30	3.23	2.15
0.5	Lower	2.55	3.40	3.40	3.40	2.55	1.70
	Upper	3.45	4.60	4.60	4.60	3.45	2.30
0.25	Lower	2.33	3.10	3.10	3.10	2.33	1.55
	Upper	3.68	4.90	4.90	4.90	3.68	2.45
0	Lower	2.10	2.80	2.80	2.80	2.10	1.40
	Upper	2.60	2.60	2.60	2.60	2.60	2.60
Unit backlog							
1	–	20	25	25	25	20	15
0.75	Lower	18.50	23.13	23.13	23.13	18.50	13.88
	Upper	21.50	26.88	26.88	26.88	21.50	16.13
0.5	Lower	17.00	21.25	21.25	21.25	17.00	12.75
	Upper	23.00	28.75	28.75	28.75	23.00	17.25
0.25	Lower	15.50	19.38	19.38	19.38	15.50	11.63
	Upper	24.50	30.63	30.63	30.63	24.50	18.38
0	Lower	14.00	17.50	17.50	17.50	14.00	10.50
	Upper	26.00	32.50	32.50	32.50	26.00	19.50

Table 5 Demand forecasts (product units)

α-cuts	Upper/lower cut	Jan	Feb	Marc	April	May	June
1	–	100	100	150	200	150	100
0.75	Lower	93	93	139	185	139	93
	Upper	108	108	161	215	161	108
0.5	Lower	85	85	128	170	128	85
	Upper	115	115	173	230	173	115
0.25	Lower	78	78	116	155	116	78
	Upper	123	123	184	245	184	123
0	Lower	70	70	105	140	105	70
	Upper	130	130	195	260	195	130

Table 6 Optimum production plans for different α-cuts

α-cuts	Var.	1	2	3	4	5	6							
1	D(t)	100	100	150	200	150	100							
	X(t)	124	124	149	150	125	124							
	I(t)	27	51	50	0	0	0							
	B(t)	0	0	0	0	25	0							
	W(t)	93	93	93	94	94	94							
Total	H(t)	93	0	0	1	0	0							
Cost:	F(t)	0	0	0	0	0	0							
532446$	O(t)	0	0	18	0	0	0							
0.75	D(t)	93	93	139	185	139	93	D(t)	108	108	161	215	161	108
Lower/upper	X(t)	113	116	139	139	116	116	X(t)	124	136	163	163	136	136
Total	I(t)	23	46	46	0	0	0	I(t)	19	47	49	0	0	0
Cost lower:	B(t)	0	0	0	0	23	0	B(t)	0	0	0	3	28	0
492968$	W(t)	85	87	87	87	87	87	W(t)	93	102	102	102	102	102
Total	H(t)	85	2	0	0	0	0	H(t)	93	9	0	0	0	0
Cost upper:	F(t)	0	0	0	0	0	0	F(t)	0	0	0	0	0	0
573296$	O(t)	0	0	0	0	0	0	O(t)	0	0	0	0	0	0
0.50	D(t)	85	85	128	170	128	85	D(t)	115	115	173	230	173	115
Lower/upper	X(t)	104	104	128	128	107	107	X(t)	140	144	173	173	144	144
Total	I(t)	22	41	41	0	0	0	I(t)	28	57	57	0	0	0
Cost lower:	B(t)	0	0	0	1	22	0	B(t)	0	0	0	0	29	0
453053$	W(t)	78	78	80	80	80	80	W(t)	105	108	108	108	108	108
Total	H(t)	78	0	2	0	0	0	H(t)	105	3	0	0	0	0
Cost upper:	F(t)	0	0	0	0	0	0	F(t)	0	0	0	0	0	0
612824$	O(t)	0	0	0	0	30	30	O(t)	0	0	18	18	0	0
0.25	D(t)	78	78	116	155	116	78	D(t)	123	123	184	245	184	123
Lower/upper	X(t)	96	96	115	117	97	97	X(t)	152	153	184	184	153	153
Total	I(t)	21	39	38	0	0	0	I(t)	32	62	62	1	0	0
Cost lower:	B(t)	0	0	0	0	19	0	B(t)	0	0	0	0	30	0
$412899	W(t)	72	72	72	73	73	73	W(t)	114	115	115	115	115	115
Total	H(t)	72	0	0	1	0	0	H(t)	114	1	0	0	0	0
Cost upper:	F(t)	0	0	0	0	0	0	F(t)	0	0	0	0	0	0
$653192	O(t)	0	0	0	18	0	0	O(t)	0	0	0	0	0	0
0	D(t)	70	70	105	140	105	70	D(t)	130	130	195	260	195	130
Lower/upper	X(t)	86	87	104	104	88	88	X(t)	160	162	195	195	163	162
Total	I(t)	19	36	35	0	0	0	I(t)	33	65	65	0	0	0
Cost lower:	B(t)	0	0	0	1	18	0	B(t)	0	0	0	0	32	0
$372425	W(t)	65	65	65	65	66	66	W(t)	120	122	122	122	122	122
Total	H(t)	65	0	0	0	1	0	H(t)	120	2	0	0	0	0
Cost upper:	F(t)	0	0	0	0	0	0	F(t)	0	0	0	0	0	0
$692639	O(t)	0	30	0	0	0	0	O(t)	0	0	0	0	30	0

production plan opted to make an overtime to come over the problem of slack capacity. In all scenarios, either backlogging or overtime is used as a tool to meet the exact demand. In all cases, backlogging is applied with different reasons.

However, overtime is observed in six different α-cuts. The reason for the variability in results is the high volatility in input parameters.

In addition, pattern of decisions for inventory on-hand and firing workforce are not sensitive to fuzzy parameters. As mentioned before, in early stages inventory is kept high, and zero workforce is fired in whole planning horizons.

6 Conclusion

In this chapter, fuzzy linear programming model is provided to solve the aggregate production planning problem. Then the proposed model is applied to a test problem in literature. The advantage of using fuzzy linear programming is the incorporation of uncertainty of the customer demands, and unit holding and backordering costs of production plan. The optimal production amounts, amounts to be kept on-hand at the end of each period, backlogging amounts and the workforce policy have been determined for different fuzziness levels.

According to the uncertainties influencing the aggregate production plan, fuzzy numbers are used to model the problem. Fuzzy logic helps production planners to know the value of membership degree of development plan in the optimum set. In fact, for specific uncertainty, the planner can understand the range of optimum planning costs. According to the results of this research, planners will be able to decide how to develop the production plan under imprecise demand data.

As a future research topic, different fuzzy linear programming methods can be compared for aggregate production planning problem. Another research direction would be to integrate different lot sizing rules to model in order to impose new constraints such as minimum lot sizes or warehouse capacities. What is more detailed sensitivity analyses approaches could be applied to test different fuzzy membership functions.

Appendix: LINGO code of given Aggregate Production Planning Model

```
MODEL:
  SETS:
  months/1..6/:P,W,O,H,F,I,B,WD,D,pc,hc,oc,fc,ic,bc;
  ENDSETS
min = @sum(months(t):pc(t)*P(t) + 8*WD(t)*W(t)*6
+ oc(t)*O(t) + hc(t)*H(t) + fc(t)*F(t) + ic(t)*I(t)
+ bc(t)*B(t));
  @for(months(t)| t#GT#1: P(t) + I(t - 1) + B(t) - I(t)
- B(t - 1) = D(t););
  P(1) + I0 + B(1) - I(1) - B0 = D(1);
```

```
  @for(months(t)|t#GT#1:W(t) − W(t − 1)
 − H(t) + F(t) = 0;);
  W(1) − W0 − H(1) + F(1) = 0;
  @for(months(t): 90*P(t) − 8*WD(t)*W(t) − O(t) < 0;);
  @for(months(t):O(t) <= 0.25*W(t)*WD(t)*8;);
  B(6) = 0;
  @for(months(t): @GIN(H));
  @for(months(t): @GIN(F));
  @for(months(t): @GIN(W));
  @for(months(t): @GIN(P));

  DATA:
  D = 100,100,150,200,150,100;
  WD = 15,15,18,18,15,15;
  pc = 7, 8, 8, 8, 7, 8;
  oc = 22.5,22.5,27,27,22.5,22.5;
  hc = 1000,1000,1000,1000,1000,1000;
  fc = 1500,750,1250,1000,1500,950;
  ic = 3, 4, 4, 4, 3, 2;
  bc = 20,25,25,25,20,15;
  I0 = 3;
  B0 = 0;
  W0 = 0;
  ENDDATA
END
```

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Batch Production Plan for Periodic Demands with Uncertain Recycling Rate in a Closed-Loop Supply System

Hsiao-Fan Wang and Chung-Yuan Fu

Abstract Environmental issues and legislation pressures have forced the manufacturers to exert more effort in product recovery. This necessitates a production plan to take the product recovery and greenhouse gas emission into account. However, before doing so, a myth that re-using the recycled products would increase the total production cost or decrease the profit needs to be clarified. Therefore, in this study, we shall first show that a closed-loop production plan to consider both manufacture and remanufacture would be more economic and beneficial than a single activity of either manufacture or remanufacture. Second, when we conduct recycling activity in reality, how to estimate the amount of the recycled products to be re-utilized is another issue. In this study, the concept of the expected value transformed from a fuzzy recycling rate is adopted with intervals to describe its degree of uncertainty. Then, based on the periodic demands, a production plan for batch manufacture and remanufacture is proposed and analyzed in the form of a fuzzy mixed integer programming model (FMIP), such that the total costs of production cost, holding cost, emergency procurement cost, backlogging cost and the penalty for excessive carbon emission can be minimized with different degrees of satisfaction. A numerical example is presented to illustrate the validity of the model and the impact of recycling rate on the cost of such a close-loop production system for flexible applications.

Keywords Batch manufacture and remanufacture · Periodic demands · Fuzzy recycling rate · Green supply chain · Closed-loop production system

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1 Introduction

Given the limited energy and resources, sustainability has become an important goal for environmental protection in recent years. Anticipated consequences have compelled the implementation of measures to protect the environment. Environmental concerns extend far beyond local issues to encompass the entire globe. As a result, many countries have formed global organizations and signed agreements to achieve balance between development and the environment. In 1993, the United Nations Framework Convention on Climate Change (UNFCCC) was signed with the intention of reducing global warming. It is possible for governments and business leaders to use accounting tools to quantify, and manage greenhouse gas emissions according to the Greenhouse Gas Protocol (GHG Protocol). The European Union has imposed strict restrictions on manufacturers, such as (1) restrictions on the use of certain hazardous substances in electrical and electronic equipment (RoHS), (2) wasted electrical and electronic equipment (WEEE), imposing the responsibility for the disposal of used electrical and electronic equipment on the manufacturers, and (3) eco-design requirements for energy- use products (EUP).

Today, under the pressure from international treaties and environmental legislation, manufacturers must learn to produce products that meet customer needs and do so in an environmentally responsible manner. In other words, manufacturers are responsible for the entire life cycle of products. Therefore, the manufacturers need to deal with returned products by repairing, refurbishing, remanufacturing, cannibalizing, and disposing. In this scenario, product recovery and greenhouse gas emission must be considered in the traditional production planning system. The Green Supply Chain (GSC) management is now suggested as an efficient tactic to achieve this goal. On the basis of the 3Rs, recycling, recovery, and remanufacture in GSC management, a company has to be aware of environmental issues and put effort to prevent carbon emitted from any production activities. Therefore, closed-loop logistics are required to facilitate the 3R processes while greenhouse gas emission has to be controlled. The closed-loop logistics for a green company consists of two parts: the forward logistics and the reverse logistics. Apart from the conventional logistics, GSC has an additional role to allow green logistics to operate with the additional functions of recovery and remanufacture.

The uncertainty embedded in reverse logistics has been a challenge for GSC managers. For the conventional supply chain, the uncertain demand has affected the inventory level, production amounts, and the logistics. The uncertain factors of the reverse supply chain are more complex than those of the forward supply chain. In particular, the value of recovery rate is difficulty in estimation, yet it is the major factor of the reverse logistics management and we shall tackle this issue in this chapter based on fuzzy logic approach.

Production planning involves the overall operations of an organization over a specified period of time. To achieve the mission set by the organization, planners must determine efficient strategies in response to market conditions and allocate resources within a given time frame. This is thus our study which considers a single item, the capacitated dynamic lot sizing problem with batch manufacturers and remanufacturers in a closed-loop supply chain when the issues of carbon emissions produced during manufacture and re- manufacture at period are considered.

The rest of this chapter is organized as follows. In Sect. 2, we review the literature of the production plans especially in the framework of green supply chain management. In Sect. 3, the uncertain returned rate is considered and a fuzzy mixed integer programming model is developed. In Sect. 4, a numerical example is provided for illustration. Finally, the discussion and conclusion are provided in Sect. 5.

2 Literature Review

In this section, we shall focus on the uncertain issues of GSC management raised in literature with a particular production mode of lot-size models. After the basic knowledge of uncertain environment, the resolution methods will be discussed.

2.1 Uncertain Issues in Green Supply Chain Management

Ever since Fleischmann et al. (2000) pointed out that product recovery has reversed the product stream, its design remains complicated because of the high uncertainty in many aspects. These factors of uncertainty on the demand, landfilling, and recovery rate attribute to the differences between the conventional and green supply chains (e.g. Biehl et al. 2007; Kongar 2004). As regard to the logistics, a detailed comparison between the forward and reverse supply chains have been summarized by Kongar (2004), which have been extended by many researchers (e.g. Ovidiu and Dekker 2005; Ovidiu 2007; Salema et al. 2007) with different resolution approaches.

To cope with the uncertainties in a close-loop logistics, Wang and Hsu (2010, 2012) have applies fuzzy programming to cope with the three major uncertain factors. However, how to integrate the manufacture and re-manufacture in an optimal production mix plan remains an issue.

2.2 The Characteristics of Lot Sizing Model

According to different production environment, there are two kinds of the production planning: stochastic and deterministic. By different demand types, deterministic production planning and inventory control models are subdivided into static and dynamic models (Li et al. 2007). Static models correspond to economic order quantity (EOQ), seeking an optimal tradeoff between fixed setup and variable holding costs. Dynamic models correspond to the lot sizing problem, which deals with the determination of the proper timeframe and quantities to minimize setup and production and holding costs. The characteristics of lot sizing decisions can be referred to the following:

2.2.1 Planning Horizon

Planning horizon specifies how long the output of planning is valid. In general, for an aggregate planning, the planning horizon ranges between 2 and 12 months. In other words, planning horizons assume a finite-horizon in the lot sizing problem.

2.2.2 Capacity

Managers must understand the limits in manpower, equipment, and budgets in production systems according to long-term plans. In this manner, the upper bound of manufacturing and remanufacturing for each period can be inferred. Lia et al. and Pan et al. considered the concept of capacity for single items and assumed that manufacturing and remanufacturing are independent in the process of production. However, their outputs are the same products and they may overlap to a certain degree in the production process. Therefore, the relationship of capacity utilization between manufacturing and remanufacturing should be taken into consideration in the lot size problem.

2.2.3 Periodic Demand

Karimi et al. pointed out that periodic demand is important as an item of the input data of the lot size problem and described how demand changes over time. Managers can obtain this input by forecasting, using one of two methods: (1) qualitative methods based on soft information such as expert opinions, consumer market surveys, and a jury of executive opinions; (2) quantitative methods based on historical sales data or the data retrieved from test markets, to forecast the market demand such as time-series forecasts and associative forecasts.

2.2.4 Inventory

To satisfy anticipated demand or reduce fluctuations in production, decision-makers must decide (according to forecast), how much the finished goods or work-in-process should be produced in advance and stored as inventory. However, due to the limitation of capacity, the situation that the supply is unable to meet the demand in time may still occur, resulting in shortages. Lia et al. mentioned the option of emergency procurement to prevent shortages, and occasionally the demand of the current period can be satisfied in the following periods. In other words, backlogging can be a strategy to manage the shortage.

2.3 The Current Developments of Lot Sizing Model in the GSC Management

Since Wagner et al. proposed the Wagner/Whitin's dynamic production planning and inventory control model, development of the production planning models to minimize the total cost in a finite discrete-time horizon has drawn intensive research interest, especially, in dynamic lot sizing problem. To address environmental concerns, a lot sizing problem has been integrated within the process of product recovery. One important aspect of product recovery is remanufacturing, involving activities that render remanufactured products or major modules marketable again, as good as new. Richter et al. (2000) proposed the reversed Wagner/Whitin's dynamic production planning and inventory control model, in which a single item related to production planning is considered within a discrete-time finite horizon with minimal total cost. In this model, apart from no difference between manufacturing and remanufacturing, relevant information about the demand and the number of returned products over the entire planning horizon must be clearly stated and shortages are not permitted.

Due to the fact that returned products may exceed current demand, not all returned products require complete remanufacturing. Golany et al. (2001) provided three options for returned products: (1) disposal of the returned products, (2) storing the returned products for following periods, and (3) remanufacturing the returned products. Besides, Golany et al. (2001) pointed that the careful selection of the correlation between the demand and the returned product is crucial for applying the production planning model. Due to capacity constraints, Li et al. (2007) proposed a capacitated dynamic lot-sizing model with substitutions and products returned for both manufacturing and remanufacturing, in a model concerned primarily with batch production. Then, Tang et al. formulated a general model for a capacitated dynamic lot sizing model including manufacturing and remanufacturing capacity, disposal, and the impermissibility of shortages in a closed-loop supply chain.

These researchers have already provided sound frameworks for single item production planning in a closed-loop supply chain with regard to product recovery. As regard the costs, returned products can be remanufactured as new products, stored as inventory, or discarded in a closed-loop supply chain. While the previous researchers have emphasized on the improvement of the refurbishment of products, few studies dealt with the issue of carbon emissions resulting from manufacturing and remanufacturing.

2.4 Fuzzy Approach to the Uncertain Models

There have been many models developed to cope with uncertain parameters. Among them, fuzzy approach has been commonly adopted to deal with the imprecise coefficients which are defined by a generalized membership function as shown in (1) where is the fuzzy coefficient.

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & \forall x \in (-\infty, a_1], \\ f_a(x) & increasing\ on\ [a_1, a_2] \\ 1 & \forall x \in [a_2, a_3], \\ g_a(x) & decreasing\ on\ [a_3, a_4], \\ 0 & \forall x \in [a_4, \infty). \end{cases} \tag{1}$$

where and are continuous functions, is increasing from 0 to 1, is decreasing from 1 to 0.

To cope with such functions, Jimenez et al. has based on Heilpern's concept (1992) of expected interval and expected value defined in Definition 2.1 to resolve the functions.

Definition 2.1 (Heilpern 1992) The expected interval (EI) and expected value (EV) of an interval random set generated by a fuzzy number $\tilde{a}$ are defined as follow:

$$EI(\tilde{a}) = [E_1^a, E_2^a] = \left[\int_0^1 f_a^{-1}(x)dx, \int_0^1 g_a^{-1}(x)dx \right] \tag{2}$$

$$EV(\tilde{a}) = \frac{E_1^a + E_2^a}{2} \tag{3}$$

If the fuzzy number is linear trapezoidal defined by $\tilde{a} = (a_1, a_2, a_3, a_4)$ as shown in Fig. 1, its expected interval and its expected value are shown in (4) and (5) respectively:

$$EI(\tilde{a}) = [\frac{1}{2}(a_1 + a_2), \frac{1}{2}(a_3 + a_4)] \tag{4}$$

$$EV(\tilde{a}) = \frac{1}{4}(a_1 + a_2 + a_3 + a_4) \tag{5}$$

Then, based on the definitions (4) and (5), proposed a ranking method for each pair of fuzzy numbers $\tilde{a}$ and $\tilde{b}$, and the degree of which $\tilde{a}$ is bigger than $\tilde{b}$ is defined as follows:

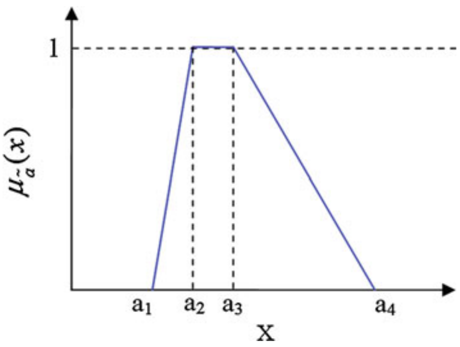


Fig. 1 Trapezoidal fuzzy number

Definition 2.2 For two linear trapezoidal fuzzy numbers, $\widetilde{a}$ and $\widetilde{b}$, the degree of $\widetilde{a}$ is bigger than $\widetilde{b}$ is defined by

$$\mu_M(\widetilde{a}, \widetilde{b}) = \begin{cases} 0 & \text{if } E_2^a - E_1^b < 0, \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & \text{if } 0 \in [E_1^a - E_2^b, E_2^a - E_1^b], \\ 1 & \text{if } E_1^a - E_2^b > 0, \end{cases} \tag{6}$$

where $[E_1^a, E_2^a]$ and $[E_1^b, E_2^b]$ are the expected intervals of $\widetilde{a}$ and $\widetilde{b}$

When $\mu_M(\widetilde{a}, \widetilde{b}) = 0.5$ we say that $\widetilde{a}$and $\widetilde{b}$ are indifferent; whereas when $\mu_M(\widetilde{a}, \widetilde{b}) \geq \alpha$, it represents that $\widetilde{a}$ is bigger than, or equal to $\widetilde{b}$ at least to the degree of α and will be denoted by $\widetilde{a} \geq_\alpha \widetilde{b}$.

Then Parra et al. (2005) pointed out that although Jimenez et al. used $\mu_M(\widetilde{a}, \widetilde{b}) = 0.5$ for the indifference of $\widetilde{a}$ and $\widetilde{b}$, it is inflexible to handle indifference. In order to established flexible indifference, Parra et al. (2005) defined α-indifference between $\widetilde{a}$ and $\widetilde{b}$ in terms of "is approximately 1/2".

Definition 2.3 For two linear trapezoidal fuzzy numbers, $\widetilde{a}$ and $\widetilde{b}$, $\widetilde{a}$is indifferent from $\widetilde{b}$ to a degree α, $0 \leqq \alpha \leqq 1$, denoted by $\widetilde{a} \approx_\alpha \widetilde{b}$ if the following relationships hold simultaneously: $\widetilde{a} \leq_{\alpha/2} \widetilde{b}$ and $\widetilde{b} \leq_{\alpha/2} \widetilde{a}$, i.e., $\widetilde{a}$is indifferent from $\widetilde{b}$ to a degree α if $\frac{\alpha}{2} \leq \mu_M(\widetilde{a}, \widetilde{b}) \leq 1 - \frac{\alpha}{2}$.

Now, we consider the following fuzzy mathematical programming model in which the constraints involve fuzzy numbers.

$$\begin{gathered} min\ z = cx \\ \text{s.t. } \widetilde{a}_i x \geq_\alpha \widetilde{b}_i,\ i = 1, \ldots, l \\ \widetilde{a}_i x =_\alpha \widetilde{\widetilde{b}}_i,\ i = l + 1, \ldots, m \\ x \geq 0 \end{gathered} \tag{7}$$

Based on Definition 2.2 we know $\widetilde{a}_i x \geq_\alpha \widetilde{b}_i \; i = 1, \ldots, l$ are equivalent to:

$$\frac{E_2^{a_i x} - E_2^{b_i}}{E_2^{a_i x} - E_1^{a_i x} + E_2^{b_i} - E_1^{b_i}} \geq \alpha, \; i = 1, \ldots, l \tag{8}$$

or

$$[(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i}, \; i = 1, \ldots, l \tag{9}$$

And based on Definition 2.3, $\widetilde{a}_i x =_\alpha \widetilde{b}_i$, $i = l+1, \ldots, m$ are equivalent to:

$$[(1-\frac{\alpha}{2})E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}]x \geq \frac{\alpha}{2}E_2^{b_i} + (1-\frac{\alpha}{2})E_1^{b_i}, \; i = l+1, \ldots, m \tag{10-1}$$

$$[\frac{\alpha}{2}E_2^{a_i} + (1-\frac{\alpha}{2})E_1^{a_i}]x \leq (1-\frac{\alpha}{2})E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i}, \; i = l+1, \ldots, m \tag{10-2}$$

Definition 2.4 Given a decision vector $\boldsymbol{x}$, it is feasible at degree α if $\min\limits_{i=1,\ldots,m}\left\{\mu_M\left(\widetilde{a}_i x, \widetilde{b}_{\partial i}\right)\right\} = \alpha$ where $\widetilde{a}_i = (\widetilde{a}_{i1}, \widetilde{a}_{i2}, \ldots, \widetilde{a}_{in})$.

Based on Definition 2.4, the equivalent auxiliary crisp α-parametric model of the model (7) can be written as follows:

$$\begin{aligned} &Min\, Z = cx \\ &Subject\ to \\ &\quad [(1-\alpha)E_2^{a_i} + \alpha E_1^{a_i}]x \geq \alpha E_2^{b_i} + (1-\alpha)E_1^{b_i}, \; \mathrm{i = 1,\ldots,m} \\ &[(1-\frac{\alpha}{2})E_2^{a_i} + \frac{\alpha}{2}E_1^{a_i}]x \geq \frac{\alpha}{2}E_2^{b_i} + (1-\frac{\alpha}{2})E_1^{b_i}, \; i = l+1, \ldots, m \\ &[\frac{\alpha}{2}E_2^{a_i} + (1-\frac{\alpha}{2})E_1^{a_i}]x \leq (1-\frac{\alpha}{2})E_2^{b_i} + \frac{\alpha}{2}E_1^{b_i}, \; i = l+1, \ldots, m \\ &\qquad\qquad x \geq 0 \end{aligned} \tag{11}$$

2.5 Conclusion and Discussion

The above discussion has largely clarified the issues related to the capacitated dynamic lot sizing problem. However, from the previous researches there are some critical issues which have not been considered: (1) the relationship of capacity utilization between manufacturing and remanufacturing, (2) the strategies of shortage management, (3) the carbon emissions resulting from manufacturing and remanufacturing, and (4) the correlation between the demand and the returned product. For that reasons, the aim of this study is to develop a dynamic lot sizing model that can address four crucial factors.

First, by assuming the remanufactured products are as new ones, the process of manufacturing and remanufacturing can largely overlap in the utilization of system resources. Therefore, the variance in output resulting from the tug-of-war between manufacturing and remanufacturing will be considered.

Second, production capacity is limited. Therefore, current output and inventory may fail to meet the current demand, and lead to the shortages. To prevent shortages, two strategies of: emergency procurement to meet current demand; and backlogging, in which unsatisfied demand is provided in following periods will be considered.

Third, by considering the minimum total cost including excessive carbon emissions, the trading off production between manufacturing and remanufacturing production will be considered.

Fourth, a number of parameters can be estimated using historical data, but most are difficult to assign due to unavailable or incomplete data in real-world situations, particularly the quantity of returned products in reverse supply chains. For this reason, the return rate is often provided by experts based on experience, and tends to be expressed in linguistic terms. Fuzzy set theory will be adopted to cope with this issue.

To sum up, this study proposes a single item production plan for batch manufacture and remanufacture with uncertain recycling rate as a fuzzy number to minimize total costs, both financial and environmental, so that four major issues of: the overlap of manufacturing and remanufacturing in utilizing system resources; the options for preventing shortages; and tradeoff between manufacturing and remanufacturing to reduce carbon emissions will be resolved with the minimum total cost.

3 The Proposed Model

When a factory produces a certain kind of product, for which demand is deterministic but time-varying during a finite planning horizon, this section will propose a model to cope with several issues stated in the previous section.

Since periodic demands have to be satisfied, strategies of the emergency procurement or backlogging are considered of which the cost of emergency procurement is assumed to be higher than the production cost including batch manufacturing and remanufacturing, otherwise the production system will become meaningless.

Since the factory is responsible for the used products, yet the amount returned from customers is not sure. Let the returned rate be assumed to be a fuzzy number $\widetilde{R} = (r_1, r_2, r_3, r_4)$ with linear trapezoidal membership function as defined below:

$$\mu_{\tilde{R}}(x) = \begin{cases} 0 & \forall x \in (-\infty, r_1] \\ \frac{x-r_1}{r_2-r_1} & \forall x \in [r_1, r_2] \\ 1 & \forall x \in [r_2, r_3] \\ \frac{x-r_3}{r_4-r_3} & \forall x \in [r_3, r_4] \\ 0 & \forall x \in [r_4, \infty) \end{cases} \tag{12}$$

3.1 The Structure of the Closed-Loop Supply Chain System and Assumption

Assumed the lifespan of new products in a market is U periods. Initially, there is no inventory of returned or new products. After U periods, a portion of products at the end of their lifespan are recycled by the factory. Two options are available to be considered for these returned products: remanufacturing and disposal. Remanufactured products can be sold as new, but the capacities for manufacturing and remanufacturing are limited, such that these two parallel processes will compete for utilizing system resources.

Different production processes for manufacture and remanufacture produce different quantities of carbon emissions. If a manufacturer produces exceed the limit of carbon emissions in a given period, a penalty must be paid. The structure of the closed-loop supply chain is illustrated in Fig. 2.

3.2 Mathematical Formulation

In this section, we shall develop a mathematical model to analyze a capacitated lot sizing problem for a single-item demand satisfied by batch manufacturing and remanufacturing within a finite planning horizon. While the total costs need to be minimized, the embedded items of production costs (batch manufacturing, batch remanufacturing, and disposal), holding costs (new products, and returned products), emergency procurement costs, backlogging costs, and penalties for excessive carbon emissions are considered. In addition, due to uncertainty of the recycled rate for each period, a trapezoidal fuzzy number defined by (12) is assumed for ease of computation and its merit of obeying Convolution Law.

We first define the notations used in our model:

Parameters

T	Planning horizon
t	Index for periods in the planning horizon, $t = 1, 2...T$
K	Resources for manufacturing or remanufacturing
k	Index for resources $k = 1, 2...K$
U	The lifespan period of the new product
D_t	The demand of new products in period t
p	The ratio of returned product collected from each period could be remanufactured
MC	The cost of manufacturing per batch
RC	The cost of remanufacturing per batch
DC	The cost of disposal per unit
MS	The set up cost of manufacturing per period

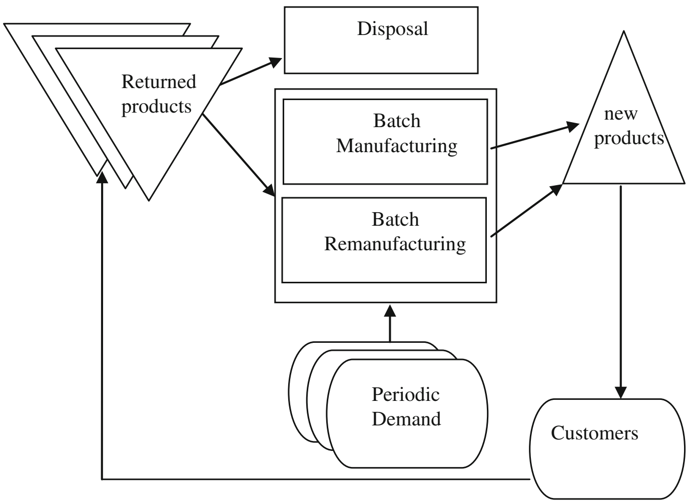


Fig. 2 The closed-loop supply chain with batch manufacturing, batch remanufacturing, and disposal

RS	The set up cost of remanufacturing per period
MN	The number of products manufactured per batch
RN	The number of returned products remanufactured per batch
C_k	The amount of resource k available for each period
CM_k	The quantity of resource k required per batch of manufacturing
CR_k	The quantity of resource k required per batch of remanufacturing
SC	The emergent procurement cost of new product per unit
BC_t	The cost of backlogging after t period per unit
HS	The holding cost of new product per unit
HR	The holding cost of returned product per unit
CE	The limit of carbon emission per period
CC	The penalty of excessive carbon emission per unit
ME	The carbon emission of manufacturing per batch
RE	The carbon emission of remanufacturing per batch
M	Large number
$\tilde{R}$	The recycled rate for each period with a trapezoidal fuzzy number ($\tilde{R} = (r_2, r_3, r_2 - r_1, r_4 - r_3)$) where the membership function is defined in (12)

Variables

x_t	The quantity of batches manufactured in period t
y_t	The quantity of batches remanufactured in period t
z_t	The disposal quantity of returned products in period t
r_t	The quantity of returned products in period t

is_t Inventory of new products at the end of period t
ir_t Inventory of returned products at the end of period t
s_t The shortage of new products
b_{ij} Demand of the ith period supplied by jth period $1 \le i < T$ and $i < j \le T$
m_t 0–1 binary variable for manufacturing set-up in period t

$$m_t = \begin{cases} 1 & \text{if the product is manufactured in period } t \\ 0 & \text{otherwise} \end{cases}$$

rm_t 0–1 binary variable for remanufacturing set-up in period t

$$rm_t = \begin{cases} 1 & \text{if the returned product is remanufactured in period } t \\ 0 & \text{otherwise} \end{cases}$$

o_t The excessive carbon emission in period t

Based on the structure of the closed-loop supply chain system and the stated problem, a fuzzy mixed integer programming model is proposed as below:

The Fuzzy Mixed Integer Programming Model

Minimize

$$\sum_{t=1}^{T} [(MSm_t + MCx_t) + (RCy_t + RSy_t) + (DCz_t) + (HSis_t + HRir_t) + SCs_t + CCo_t] + \sum_{i=1}^{T-1} \sum_{j=i+1}^{T} BC_t b_{ij} \tag{13}$$

Subject to

$$is_0 = 0 \tag{14 – 1}$$

$$ir_0 = 0 \tag{14 – 2}$$

$$r_t = 0 \; t \in T \textit{ and } t \le U \tag{15 – 1}$$

$$r_t = \tilde{R}[D_{t-U} + \sum_{i=1}^{t-U-1} b_{i,t-U} - \sum_{i=t+1}^{T} b_{t-U,i} - s_{t-U}] \quad t \in T \textit{ and } t > U \tag{15 – 2}$$

$$MNx_t + \tilde{R}Ny_t + is_{t-1} + s_t + \sum_{i=t+1}^{T} b_{ti} = D_t + \sum_{i=1}^{t-1} b_{it} + is_t \quad t \in T \tag{16}$$

$$ir_t - ir_{t-1} + \tilde{R}Ny_t + z_t = pr_t \quad t \in T \tag{17}$$

$$CM_k x_t + CR_k y_t \le C_k \quad t \in T, k \in K \tag{18}$$

$$x_t \leq Mm_t \quad t \in T \tag{19}$$

$$y_t \leq Mrm_t \quad t \in T \tag{20}$$

$$MEMNx_t + RERNy_t - CE \leq o_t \quad t \in T \tag{21}$$

$$o_t, r_t, z_t, ir_t, \geq 0 \tag{22}$$

$$x_t, y_t, is_t, s_t \geq 0 \ and \ integer \, t \in T \tag{23}$$

$$m_t, rm_t \, binary \, variable \tag{24}$$

$$b_{ij} \geq 0 \ and \ integer \, 1 \leq i < T \ and \ i < j \leq T \tag{25}$$

The objective function (13) minimizes the total setup cost, manufacturing cost, remanufacturing cost, disposal cost, holding cost, emergency procurement cost, backlogging cost and the penalty for excessive the limit of carbon emission. Constraint (14-1) is the inventory of new product which is zero in the initial period. Constraint (14-2) is the inventory of returned product which is zero in the initial period. Constraints (15)–(17) are the inventory flow conservation equations for new products and returned product. Constraint (18) is the limit of capacity for batch manufacturing and remanufacturing activities. Constraints (19) and (20) represent the setup costs of manufacturing and remanufacturing if new product is manufactured and remanufactured in period t. Constraint (21) represent the carbon emission for each period. Constraints (22)–(23) are decision variable constraints. While m_t and rm_t are binary variable, o_t, r_t, z_t, and ir_t are non-negative real number. The rest of variables are non-negative and integer.

According to the constraints (14-1) and (14-2), the constraint (17) can be rewritten into the following:

$$ir_t - ir_{t-1} + \tilde{R}Ny_t + z_t = 0 \quad t \in T \ \ and \ t \leq U \tag{26 – 1}$$

$$ir_t - ir_{t-1} + \tilde{R}Ny_t + z_t - p\tilde{R} \sum_{i=1}^{t-U-1} b_{i,t-U} + p\tilde{R} \sum_{i=t+1}^{T} b_{t-U,i} + p\tilde{R}s_{t-U} = p\tilde{R}D_{t-U}$$
$$t \in T, \ and \ t > U \tag{26 – 2}$$

3.3 Properties of the Model

According to (11), the FMIPM can be transformed into the equivalent auxiliary crisp model (ACM) as follows:

Minimize

$$\sum_{t=1}^{T}[(MSm_t + MCx_t) + (RCy_t + RSy_t) + (DCz_t) + (HSis_t + HRir_t) + SCs_t + CCo_t] \\ + \sum_{i=1}^{T-1}\sum_{j=i+1}^{T} BC_t b_{ij} \tag{27}$$

Subject to

$$is_0 = 0 \tag{28 – 1}$$

$$ir_0 = 0 \tag{28 – 2}$$

$$MNx_t + RNy_t + is_{t-1} + s_t + \sum_{i=t+1}^{T} b_{ti} = D_t + \sum_{i=1}^{t-1} b_{it} + is_t \quad t \in T \tag{29}$$

$$ir_t - ir_{t-1} + RNy_t + z_t = 0 \quad t \in T\,,\ and\ t \le U \tag{30}$$

$$ir_t - ir_{t-1} + RNy_t + z_t - p[(1-\frac{\alpha}{2})E_2^R + \frac{\alpha}{2}E_1^R]\sum_{i=1}^{t-U-1} b_{i,t-U} + p[(1-\frac{\alpha}{2})E_2^R + \frac{\alpha}{2}E_1^R]\sum_{i=t+1}^{T} b_{t-U,i} \\ + p[(1-\frac{\alpha}{2})E_2^R + \frac{\alpha}{2}E_1^R]s_{t-U} \ge p[\frac{\alpha}{2}E_2^R + (1-\frac{\alpha}{2})E_1^R]D_{t-U} \quad t \in T,\ and\ t > U \tag{31 – 1}$$

$$ir_t - ir_{t-1} + RNy_t + z_t - p[(\frac{\alpha}{2}E_2^R + (1-\frac{\alpha}{2})E_1^R]\sum_{i=1}^{t-U-1} b_{i,t-U} + p[(\frac{\alpha}{2}E_2^R + (1-\frac{\alpha}{2})E_1^R]\sum_{i=t+1}^{T} b_{t-U,i} \\ + p[\frac{\alpha}{2}E_2^R + (1-\frac{\alpha}{2})E_1^R]s_{t-U} \le p[(1-\frac{\alpha}{2})E_2^R + \frac{\alpha}{2}E_1^R]D_{t-U} \quad t \in T,\ and\ t > U \tag{31 – 2}$$

$$CM_k x_t + CR_k y_t \le C_k \quad t \in T\,, k \in K \tag{32}$$

$$x_t \le Mm_t \quad t \in T \tag{33}$$

$$y_t \le Mrm_t \quad t \in T \tag{34}$$

$$MEMNx_t + RERNy_t - CE \le o_t \quad t \in T \tag{35}$$

$$o_t, r_t \ge 0 \tag{36}$$

$$o_t, r_t, z_t, ir_t, \ge 0 \tag{37}$$

Table 1 Input data

(a) Periodic demands								
t	1	2	3	4	5	6	7	8
Demand (unit)	450	850	531	348	500	700	350	670
(b) Resource								
k (unit)	1	2	3					
C_k	350	450	100					
CM_k	20	50	10					
CR_k	35	5	10					
(c) The cost of backlogging after t period per unit								
t	1	2	3	4	5	6	7	
BC_t($)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
(d) Other parameters								
Parameter	Value	Parameter	Value					
T	8 periods	*RN*	70 unit					
R	(0.7,0.7,0.3,0.3)	*SC*	$2.3					
U	2 periods	*HS*	$0.3					
MC	$90	*HR*	$0.1					
RC	$60	*CE*	400 kg					
DC	$0.15	*CC*	$2.5					
MS	$100	*ME*	50 kg					
RS	$60	*RE*	10 kg					
MN	80 unit	*p*	0.9					

$$x_t, y_t, is_t, s_t \geq 0 \ and \ integer \, t \in T \tag{38}$$

$$b_{ij} \geq 0 \ and \ integer \, 1 \leq i < T \ and \ i < j \leq T \tag{39}$$

Then, there are $T(6 + K)$-U constraints and $(T^2 + 19T)/2$ variables in this model.

4 Numerical Illustration

In this section, we use an example to illustrate this fuzzy model. The input data are assumed as following:

D_t : the demand of new products in period $t = 1\ldots8$ (Table 1).

There are 108 variables and 64 constraints in total.

This study employed the optimization package ILOG OPL Studio 3.5 on a Pentium IV 1.6 GHz PC.

First, from Fig. 3, a comparison of manufacture only, re-manufacture only and both is shown to specify the cost down with close-loop manufacture system. Regarding to the carbon emission, Fig. 4 shows the carbon emission of portions of manufacture and remanufacture in the total amount for each periods. It also shows the carbon emission of remanufacture is much less than those from manufacture.

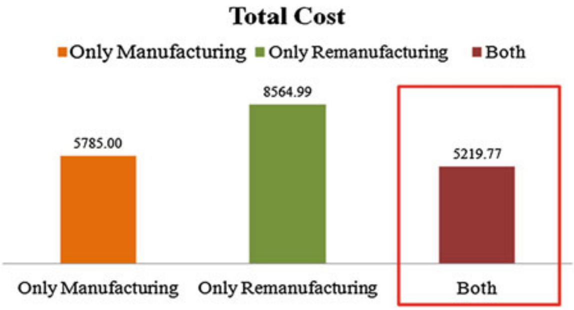


Fig. 3 Comparison of costs between one-way manufacturing and close-loop system

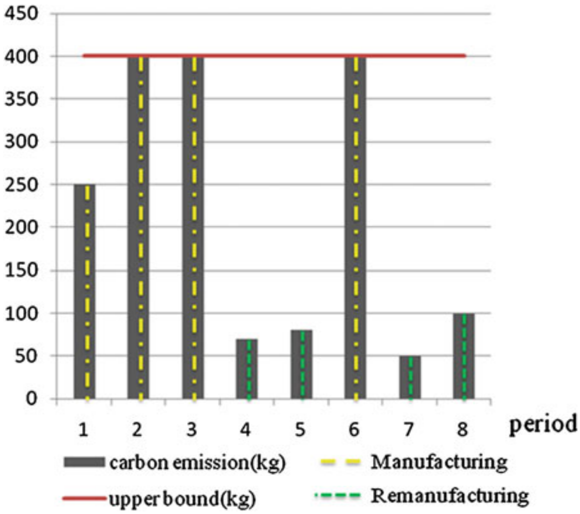


Fig. 4 The Carbon emission of each period

When uncertain demand is estimated, Table 2 illustrates the optimal production patterns with respective to the different feasibility degrees of α. When production processes remain the same with different degrees of feasibility α, we can infer that most of the cost items are equal, except for the inventory cost of returned products. When α increase, the constraints become stricter and the region of variation for returned product is reduced. As a result, satisfying the demand for remanufacturing requires higher storage of the returned products for the following periods, which increases the inventory cost of returned products.

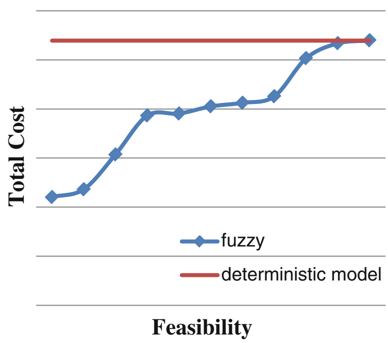
Figure 5 summarizes the total cost changes with respective the degree of feasibility α in the fuzzy model. The positive relation is induced from the fact that the greater the degree to which the constraints are satisfied (feasibility degree α), the smaller the feasible region becomes; and the worse the optimal value. In other words, we found that the total cost using the fuzzy model were lower than or equal to those using the deterministic model for all α values. When α approaches 1, the fuzzy model becomes crisper and the total cost also increases. With this

Table 2 Output analysis: The values of embedded costs

Feasibility degree α	Manuf. cost	Re-manuf. cost	Disposal cost	Shortage	
				Emergent procurement	Backlogging
0.0 ∼ 0.1	2370	2460	0	89.7	59
0.2	2460	2400	0	66.7	42
0.3 ∼ 0.4	2830	2160	0	0	40.8
0.5	2830	2220	0	0	42.8
0.6 ∼ 0.7	2830	2220	0	0	29.8
0.8	2920	2160	0	0	38.8
0.9	3010	2040	2.2293	0	48.8
1	3010	2040	3.465	0	48.8
Deterministic model	3010	2040	3.465	0	48.8

Feasibility degree α	Inventory cost		Total cost
	New item	Returned item	
0	33.9	47.633	5060.233
0.1	33.9	55.5301	5068.1301
0.2	66.9	68.0232	5103.6232
0.3	60.6	51.8868	5143.2868
0.4	60.6	53.8524	5145.2524
0.5	9.6	50.1878	5152.5878
0.6	18.6	57.8076	5156.2076
0.7	18.6	64.425	5162.825
0.8	15.6	67.2189	5201.6189
0.9	21.6	94.2509	5216.8802
1	21.6	95.907	5219.772
Deterministic model	21.6	95.907	5219.772

Fig. 5 The total cost under different feasibility degree



information, a decision maker is able to trade off between the degree to which constraints are satisfied and the value of the objective to determine the best production plan.

5 Summary and Conclusion with Further Study

In this study, a production plan is developed to cope with four issues: (1) periodic demand, which can be fulfilled by emergency procurement and backlogging; (2) recovery, in which the used products can be returned to the primary market by remanufacturing or leave the primary market through disposal; (3) environmental protection, in which the carbon emission from manufacturing and remanufacturing are considered to reduce the environment impact; (4) the correlation between the demand and the returned product, in which the Fuzzy set theory are considered to cope with the uncertainty of the recycled rate.

We presented a fuzzy mixed integer programming model for a single item lot sizing problem. The fuzzy mixed integer programming model provided a total solution for (1) the quantity of new products to be produced in each period, (2) the assignment of the production mix (manufacturing and remanufacturing) in each period, (3) the level of inventory held in each period, and (4) the strategy employed to meet periodic demands to minimize total cost. Besides, according to the preference, a decision maker can choose the various degrees of feasibility, and determine his optimal production option. The result also suggests that if the decision maker wants to have the higher satisfaction on the resource utilization, he/she will pay more cost in the production plan.

Further research could be considered from two directions. First, when the problem size is large, this approach will not necessarily derive the optimal solution rapidly. Therefore, a heuristic or meta-heuristic algorithm could be developed for the proposed lot sizing model to search for the optimal solution more efficiently. Secondly, the proposed model only considers single item in the production plan. Future study could be extended to multiple products in the production system by taking the characteristics of each product into consideration.

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Part V
Optimization in Supply Chain Under Fuzziness

Optimization Models for Supply Chain Production Planning Under Fuzziness

Josefa Mula, David Peidro and Raúl Poler

Abstract The aim of this chapter is to propose diverse fuzzy mathematical programming models based on the fuzzy set theory for supply chain production planning in a multi-product, multi-plant environment with fuzziness and capacity constraints. The proposed models consider fuzziness in demand and at the aspiration level of total costs. The main contribution of this chapter to the field of fuzzy sets is a practical application of flexible programming approaches (or fuzzy constraints) in linear programming with diverse aggregation schemes to the model originally proposed by McDonald and Karimi (1997)for supply chain production planning.

Keywords Supply chain · Production planning · Optimization · Uncertainty · Fuzziness

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1 Introduction

Production planning implies simultaneously determining a firm's production, inventory and capacity levels in a finite planning horizon for the purpose of minimizing the total costs generated by the production plan.

Real situations in production planning problems are often inaccurate or uncertain. The scientific and research community has shown interest in dealing with uncertainty modeling in production planning problems (Mula et al. 2006b; Peidro et al. 2009). The modeling approach employed in most of the works proposed in the scientific literature is of the analytic type, specifically stochastic programming models. These models have been employed for all the production planning levels (strategical, tactical and operational) and they have also been applied to real cases. For dynamic programming, very few models have been found, and they are mainly theoretical. Analytical models are followed by simulation-based models. As regards artificial intelligence models, the fuzzy set theory-based ones are the most widely used, although more have been employed at the operational level in planning. Finally, conceptual models with different approaches complete the list.

The supply chain (SC) takes on all the activities required to meet customer demand, from acquiring raw materials in the supply network and production in the production network, to packaging and delivery in the distribution network (Hahn et al. 2000). There is considerable literature which provides many models and algorithms to tackle SC management problems (information management, supply contracts, international aspects, etc.). For extensive reviews, see Tayur et al. (1999) and Peidro et al. (2009).

The SCs of industrial firms have to face not only continuous changes in the environment, but also the presence of uncertainty in their production processes. Therefore, developing models and algorithms for SC production planning, which can recognize and model the many uncertainties that emerge in the various planning phases, is necessary. In this context, the research community has detected that more and more attention is being paid to the effects of demand variability through the SC (see Mula et al. 2006b; Peidro et al. 2009).

This work aims to propose several fuzzy mathematical programming models based on the fuzzy set theory (Bellman and Zadeh 1970) which allow production planning in a multi-product, multi-plant SC environment with uncertainty and capacity constraints. The main contribution of this work is a practical application of flexible programming approaches or fuzzy constraints in linear programming, with several aggregation schemes to the model originally proposed by McDonald and Karimi (1997) for mid-term SC production planning. Other flexible programming applications can be found in Miller et al. (1997); Pendharkar (1997); Dubois et al. (2003); Itoh et al. (2003) and Mula et al. (2006a, 2007a, b; Mula and Poler 2010), among others. For an extensive literature review of the topic we refer readers to Peidro et al. (2009) and Mula et al. (2010a, b).

The rest of the chapter is organized as follows. Section 2 describes the deterministic model adopted as the basis of this work. Section 3 formulates the fuzzy mathematical programming models for SC production planning. Section 4 presents the computational results. Finally, Sect. 5 provides the conclusions and further research.

2 Deterministic Formulation of the Problem

The deterministic model of mid-term SC production planning proposed originally by McDonald and Karimi (1997) has been adopted as the representative formulation of the present work. This model has also been considered the basis of the work by Mula and Poler (2010). The objective of this model is to establish the sources of limited resources in a firm and to optimally allocate its manufacture resources to meet market demand at a minimum cost. The contemplated SC consists of many globally localized production plants that produce numerous products. The demand of these products lies in a series of customers. The mid-term planning horizon spans from 1 to 2 years. Each production plant is characterized by one or more semi-continuous production resource(s) with limited capacity. The various products, which are grouped into product families, compete for the limited capacity of these resources. This decision-making process can be divided into two different phases: the production phase and the distribution or logistic phase. The production phase centers on efficiently allocating the production capacity in each production plant in order to adopt optimum operational policies. In the distribution phase, post-production activities are considered, such as satisfying demand and inventories management to satisfy demand.

This model has a deterministic structure in which demand uncertainty is dealt with by using safety stocks.

2.1 Mathematical Nomenclature

The generic mid-term SC production planning model proposed by McDonald and Karimi (1997) consists in a mixed-integer linear programming model, which we call SCM&K, with the following mathematical notation:

The set of products in the system is denoted by the set of indices $I \equiv \{i\}$. This set can be classified into three categories: (i) raw materials, denoted by set I^{RM}; (ii) intermediate or half-finished products, I^{IP}; and (iii) end products, I^{FP}. A half-finished product can also belong to the end products set. The set of machines or resources is referred to as $J \equiv \{j\}$, and the set of production plants where these resources are localized is denoted $S \equiv \{s\}$. The set of product families is referred to as $F \equiv \{f\}$. The set of customers is represented by $C \equiv \{c\}$, whereas the set of time periods is known as $T \equiv \{t\}$.

The decision variables are defined as:

P_{ijst} Production quantity of product $i \in I \backslash I^{\mathrm{RM}}$ in resource j in plant s during period t.

RL_{ijst} Production time of product $i \in I \backslash I^{\mathrm{RM}}$ in resource j in plant s during period t.

FRL_{fjst} Production time of family f in resource j in plant s during period t.

C_{ist} Consumption of raw material or half-finished $i \in I \backslash I^{\mathrm{FP}}$ in plant s during period t.

I_{ist} Inventory level of product $i \in I \backslash I^{\mathrm{RM}}$ in plant s at the end of period t.

S_{isct} Supply of end product $i \in I \backslash I^{\mathrm{FP}}$ of resource s to customer c during period t.

$\sigma_{iss\prime t}$ Flow of intermediate product $i \in I \backslash I^{\mathrm{IP}}$from plant s to s' during period t.

I^{-}_{ict} Quantity of delayed demand of end product $i \in I \backslash I^{\mathrm{FP}}$ for customer c during period t. It is assumed that delayed demand can be satisfied during a future period, although delayed deliveries are penalized.

$I^{\wedge}_{ist}$ Deviation below the target safety stock for product $i \in I \backslash I$ in plant s during period t.

Y_{ijst} Binary variable indicating whether product i is produced in resource j in s during period t.

The cost coefficients are defined as follows:

h_{ist} Cost of maintaining the inventory of one unit of product i in plant s during period t.

μ_{ic} Profit per unit of product $i \in I \backslash I^{\mathrm{FP}}$ sold to customer c.

P_{is} Price of raw material $i \in I \backslash I^{\mathrm{RM}}$ in plant s.

ζ_{is} Penalization for maintaining safety stock below the target of product i in plant s.

V_{ijs} Variable cost of producing one unit of product $i \in I \backslash I^{\mathrm{RM}}$ in resource j in plant s.

$t_{ss'}|t_{sc}$ Transport cost of moving one product unit from plant s to plant s' or to customer c.

f_{fjs} Set production cost for family f in resource j in plant s.

Other general data are required:

R_{ijst} Efficient ratio for product i in resource j in plant s during period t.

$\beta_{i'is}$ Quantity of raw material or half-finished product $i \in I \backslash I^{\mathrm{FP}}$ that must be consumed to produce one unit of $i' \in I \backslash I^{\mathrm{RM}}$ in plant s.

k_{if} Parameter 0–1, which indicates if product i belongs to the family of products f.

H_{jst} Quantity of production time available in resource j in plant s during time period t.

MRL_{fjs} Minimum production time required for family f in resource j in plant s. This parameter is considered for operational efficiency reasons, and is usual in semicontinuous production processes, as in chemical firms, or in batch-based production processes with considerable times in changing entries. This parameter can be omitted in those batch-based production processes where entry change costs are insignificant.

d_{ict} Demand of end product i for customer c at the end of period t.

I^{L}_{ist} Target safety stock for product i in plant s during period t.

I_{is0} Inventory of product i in plant s at the beginning of the planning horizon.

The objective function (1) minimizes the total cost that combines production and distribution phase costs. Production phase costs include fixed and variable production costs, costs of purchasing raw materials and transport costs of delivering intermediate products between plants. Distribution phase costs include the costs to transport end products, costs of maintaining inventories, costs involved in delayed demand and penalization costs for neglecting the target safety stocks level.

Min

$$\begin{aligned} C = \sum_{i,j,s,t} v_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,c,t} t_{sc} S_{isct} \\ + \sum_{i,s,s',t} t_{ss'} \sigma_{iss't} + \sum_{i,s,t} \zeta_{is} I^{\wedge}_{ist} + \sum_{i,c,t} \mu_{ic} I^{-}_{ict} + \sum_{f,j,s,t} f_{fjs} Y_{fjst} \end{aligned} \tag{1}$$

Subject to

Equation (2) relates production quantity to production time by means of the production ratio.

$$P_{ijst} = R_{ijst} RL_{ijst}, \forall i \in I \backslash I^{RM}, \forall j \in J, \forall s \in S, \forall t \in T \tag{2}$$

Equations (3) and (4) provide the upper and lower limits for the production times of each product family.

$$FRL_{fjst} \le H_{jst} Y_{fjst}, \forall f \in F, \forall j \in J, \forall s \in S, \forall t \in T \tag{3}$$

$$FRL_{fjst} \ge MRL_{fjs} Y_{fjst}, \forall f \in F, \forall j \in J, \forall s \in S, \forall t \in T \tag{4}$$

Allocating products to product families is done by Eq. (5).

$$FRL_{fjst} = \sum_{k_{if}=1} RL_{ijst}, \forall f \in F, \forall j \in J, \forall s \in S, \forall t \in T \tag{5}$$

Equation (6) models the production capacity constraints. This model assumes that the minimum length required for the production times of a product family is much shorter than the length of a time period.

$$\sum_{f} FRL_{fjst} \le H_{jst}, \forall j \in J, \forall s \in S, \forall t \in T \tag{6}$$

Equation (7) models the consumption of raw materials or half-finished products based on the list of materials. Raw materials arrive from an external supplier and it is assumed that they will be available whenever required, although they can be easily amended by including the constraints of the upper and lower limits. The half-finished products that are consumed in plant s may be supplied from the same plant or sent from plant s′.

$$C_{ist} = \sum_{i' \ni \beta i' is \neq 0} \beta_{i'is} \sum_{j} P_{i'jst}, \forall i \in I \backslash I^{FP}, \forall j \in J, \forall s \in S, \forall t \in T \tag{7}$$

Equation (8) implies that all the materials sent to plant s are consumed during the same period of time. This guarantees that the inventory is maintained where the products are produced, thus avoiding redundant materials flowing in the network.

$$C_{ist} = \sum_{s'} \sigma_{is'st}, \forall i \in I \backslash I^{IP}, \forall s \in S, \forall t \in T \tag{8}$$

Equation (9) represents the inventory balance constraint. The inventory available at the end of time period t is equal to the inventory available at the end of the former period (t – 1) plus the production during period t, less the outgoing flow of half-finished products to other plants and deliveries of end products to customers. End products are sent only between plants if they are also half-finished products at the same time.

$$I_{ist} = I_{is(t-1)} + \sum_{j} P_{ijst} - \sum_{s'} \sigma_{iss't} - \sum_{c} S_{isct}, \forall i \in I \backslash I^{RM}, \forall t \in T \tag{9}$$

Delays in customer demand are the differences accumulated between demand and supply. Equation (10) indicates that delays in supply accumulate from one period to the following period:

$$I^{-}_{ict} \geq I^{-}_{ic(t-1)} + d_{ict} - \sum_{c} S_{isct}, \forall i \in I \backslash I^{FP}, \forall t \in T \tag{10}$$

Equation (11) helps satisfy former demands based on current period supply, which is always subject to the upper limit of the total demand accumulated until this period.

$$\sum_{s,t' \leq t} S_{isct'} \leq \sum_{t' \leq t} d_{ict'}, \forall i \in I \tag{11}$$

Equation (12) ensures that delayed demand is always lower than the upper limit of the total demand accumulated until this period.

$$I^{-}_{ict} \leq \sum_{t' \leq t} d_{ict'}, \forall i \in I, \forall c \in C \tag{12}$$

Equation (13) determines the excesses and deviations in the inventory in relation to the level of the set target safety stock.

$$I_{ist}^{\wedge} \geq I_{ist}^{L} - I_{ist}, \forall i \in I, \forall s \in S, \forall t \in T \tag{13}$$

Equations (14) and (15) establish the upper and lower limits set for the different decision variables.

$$I_{ist}^{\wedge} \leq I_{ist}^{L}, \forall i \in I, \forall s \in S, \forall t \in T \tag{14}$$

$$\begin{gathered} P_{ijst,} RL_{ijst}, C_{ist,} I_{ist,} I_{ict}^{-}, \sigma_{iss't}, I_{ist}^{\wedge} \geq 0, Y_{ffst} \in \{0,1\} \\ \forall i \in I, \forall j \in J, \forall c \in C, \forall f \in F, \forall s \in S, \forall t \in T \end{gathered} \tag{15}$$

3 The Fuzzy Models Approach

One of the key sources of uncertainty in any production system is end product demand (Gupta and Maranas 2003). Not considering fluctuations in demand could lead to customer demand not being met, which could imply loss of market share or excessively high inventory maintenance costs. A firm can adopt two strategic positions to face uncertainty in demand; being a firm that (a) *outlines* demand or (b) *adapts* to demand. In the first strategy, the firm's objective is to restructure the distribution of demand. This it achieves through contracts and agreements with the customer. For example, the firm could offer a supply contract which promises a minimum/maximum quantity in exchange for a discount in price (Anupindi and Bassok 1999). In the second strategy, the firm does not attempt to influence the level of market uncertainty, but aims to control the risk of exposing its assets, such as inventory levels and profit margins, by constantly adapting its operations to satisfy demand. An adaptation to demand viewpoint is considered in this research work.

In this section, the formulation of the deterministic SCM&K model presented in the former section is reformulated by assuming the uncertainty that is inherent to the demand in the SC's tactical planning problems and by formalizing it using the approaches based on the fuzzy set theory. The possible uncertainty related with the SC's production planning process costs is also considered.

The proposed models are called SCM&KFuzzy, followed by the following notation: Firstly, the objective function is fuzzy, OF. Secondly, how uncertainty is formalized is specified by the fuzzy constraints modeled by fuzzy sets, RF.

Next, the type of linear membership used to represent fuzzy sets or fuzzy numbers is indicated: linear, which monotonously increases or decreases in a triangular (T), trapezoidal (Tr) interval (L), or a combination of them. Next, the operator used is specified: min-operator (MIN), the convex combination of min-operator and max-operator (COM), or the fuzzy and (FA) operator. Finally, the model provides a deterministic solution, SD.

3.1 Models with the Fuzzy Objective Function

Most of the costs indicated in (2) cannot be easily measured because this greatly implies human perceptions to evaluate them. For instance, in some manufacturing environments, planners employ a perceived mean production cost per hour when calculating production unit costs, which means that these costs are fuzzy. Furthermore, the cost of delayed demand is fuzzy as far as its composition is concerned. This cost consists in both the administrative cost to manage delayed orders and the cost incurred from losing customers. Such a cost is frequently estimated when using human criteria. Basically in these environments, the objective function should be considered fuzzy because of these possible fuzzy costs.

3.1.1 The SCM&KFuzzy.OF/RF/L/MIN/SD_1 Model

This section presents a fuzzy linear programming model to solve the SC's midterm production planning problem where the aspiration levels of the total costs of the objective function and market demand are considered fuzzy. Market demand comprises customers' firm orders and demand forecasts. Firm orders are known at the beginning of each planning horizon. The demand forecast is based on factors like sales from previous years, supplies from other sources, competitors, etc. These factors make Constraints (10), (11) and (12) of the SCM&K model fuzzy in nature. The model also includes other constraints with crisp data.

The solution intended to be obtained with this model is deterministic. Negoita and Sularia (1976) demonstrate that the problem of making a maximization/minimization determination can be reduced to a mathematical programming problem. Particularly in the fuzzy linear programming case, the problem boils down to a linear programming one (Hamacher et al. 1978). The main advantage of a maximization/minimization decision is that it is easier for the planner to interpret it.

In this model, it is assumed that the planner can determine a level of aspiration, z, for the objective function value that he/she wishes to accomplish. Besides, the market demand value is considered inaccurate.

In this section, the strict SCM&K problem requirements ease and the problem is reformulated as a fuzzy model where objective function (1) of the SCM&K model to be minimized is transformed into Constraint (16) of the fuzzy model. Thus, this model is completely symmetrical to the objective function and the constraints.

$$\sum_{i,j,s,t} v_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,s',t} t_{ss'} \sigma_{iss't} + \sum_{i,s,t} \zeta_{is} I^{\wedge}_{ist} + \sum_{i,c,t} \mu_{ic} I^{-}_{ict} + \sum_{f,j,s,t} f_{fjs} Y_{fjst} \in z1 \tag{16}$$

Inequality constraints (10), (11) and (12) are transformed into fuzzy constraints (17), (18) and (19). Symbol $\tilde{\leq}$ represents the fuzzy version of $\leq$ whose linguistic interpretation is "essentially lower than or equal to". These constraints show that the planner wishes to make the left-hand side of the constraints smaller than or equal to the right-hand side, "if possible".

$$-I^{-}_{ict} + I^{-}_{ic(t-1)} - \sum_{c} S_{isct} \in -d_{ict}, \forall i \in I \backslash I^{FP}, \forall t \in T \tag{17}$$

$$\sum_{s,t' \leq t} S_{isct'} \in \sum_{t' \leq t} d_{ict'}, \forall i \in I \tag{18}$$

$$I^{-}_{ict} \in \sum_{t' \leq t} d_{ict'}, \forall i \in I, \forall c \in C \tag{19}$$

The remaining non fuzzy constraints are incorporated into the model and they remain unalterable if compared with the SCM&K model.

Within the flexible programming framework, various fuzzy modeling approaches have been put forward (see Bellman and Zadeh (1970); Zimmermann 1978 and Luhandjula 1982, among others). In this model, the approach proposed by Bellman and Zadeh (1970) is adopted where the objective function and some constraints are described as fuzzy inequalities, which are explicitly specified by the corresponding membership function.

The fuzzy set's membership function of the model's fuzzy "decision", which employs the min-operator, is defined by:

$$\mu_{\tilde{D}}(x) = \min_{i}\{\mu_i(x)\} \tag{20}$$

The fuzzy set of decision D consists in all the x values that fulfill the fuzzy constraints. Let us assume that we wish to obtain an accurate "optimum" or a non fuzzy solution, then we can consider "maximizing the solution" of Eq. (20):

$$\max_{x \geq 0} \min_{i}\{\mu_i(x)\} = \max_{x \geq 0} \mu_{\tilde{D}}(x) \tag{21}$$

All the fuzzy constraints of the model, these being (17), (18) and (19), are to be represented by a fuzzy set whose membership function is defined by $\mu_i(x)$.

$\mu_i(x)$ can be interpreted as the degree to which x (decision variable) fulfills the fuzzy inequality. It is considered that $\mu_i(x)$ is 0 if the constraints (including the objective function (16)) are considerably neglected, and 1 if they are properly fulfilled (in the deterministic sense); besides, $\mu_i(x)$ is considered to increase monotonously from 0 to 1.

$$\mu_i(x) = \begin{cases} 1 & si \quad B_i x \leq d_i \\ \in [0,1] & si \quad d_i < B_i x \leq d_i + p_i \\ 0 & si \quad B_i x > d_i + p_i \end{cases} \qquad i = 1, \ldots, m+1 \tag{22}$$

or

$$\mu_i(x) = \begin{cases} 1 & si \quad B_i x \geq d_i \\ \in [0,1] & si \quad d_i - p_i < B_i x \leq d_i \\ 0 & si \quad B_i x < d_i - p_i \end{cases} \qquad i = 1, \ldots, m+1 \tag{23}$$

where $B_i x$ represents the left-hand side of the fuzzy constraints, d_i denotes the independent term of the fuzzy constraints, and p_i refers to the extension of the "tolerance interval" of neglecting the constraints or, more specifically, they are subjectively selected constants that define the admissible neglected constraints and the objective function.

If the simplest membership function of the fuzzy sets is used, and a linear increase over the tolerance interval is assumed $[d_i, d_i + p_i]$, then Eqs. (22) and (23) become (24) and (25), respectively:

$$\mu_i(x) = \begin{cases} 1 & si \quad B_i x \leq d_i \\ 1 - \frac{B_i x - d_i}{p_i} & si \quad d_i < B_i x \leq d_i + p_i \\ 0 & si \quad B_i x > d_i + p_i \end{cases} \qquad i = 1, \ldots, m+1 \tag{24}$$

and

$$\mu_i(x) = \begin{cases} 1 & si \quad B_i x \geq d_i \\ 1 - \frac{d_i - B_i x}{p_i} & si \quad d_i - p_i < B_i x \leq d_i \\ 0 & si \quad B_i x < d_i - p_i \end{cases} \qquad i = 1, \ldots, m+1 \tag{25}$$

These linear membership functions offer one clear advantage, which is that the planner has to specify only two values to define each membership function: the higher and lower aspiration levels or the two tolerance interval limits. These functions are computationally efficient and provide equivalent linear models in combination with many operators (Zimmermann 1996). By substituting Eq. (24) in the problem (21), and after making a few changes (Zimmermann 1976) and some further assumptions, we obtain:

$$\max_{x \geq 0} \min_i \left(1 - \frac{B_i x - d_i}{\mu_i} \right) \tag{26}$$

By introducing a new variable, $\lambda \in [0, 1]$, which corresponds essentially to Eq. (20) and which represents the degree of fulfillment of the least fulfilled constraint (in the deterministic sense), the fuzzy model is transformed into the equivalent deterministic model known as SCM&KFuzzy.OF/RF/L/MIN/SD_1:

$$\text{Maximize } \lambda \tag{27}$$

Subject to

$$\begin{aligned} &\sum_{i,j,s,t} v_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,s',t} t_{ss'} \sigma_{iss't} \\ &\quad + \sum_{i,s,t} \zeta_{is} I^{\wedge}_{ist} + \sum_{i,c,t} \mu_{ic} I^{-}_{ict} + \sum_{f,j,s,t} f_{fjs} Y_{fjst} + \lambda p1_t \leq z1_t + p1_t \end{aligned} \tag{28}$$

$$-I_{ict}^{-} + I_{ic(t-1)}^{-} - \sum_{c} S_{isct} + \lambda p2_{ict} \leq -d_{ict} + p2_{ict}, \forall i \in I \backslash I^{FP}, \forall t \in T \tag{29}$$

$$\sum_{s,t' \leq t} S_{isct'} \leq \sum_{t' \leq t} (d_{ict'} + p2_{ict'} - \lambda p2_{ict'}), \forall i \in I \tag{30}$$

$$I_{ict}^{-} \leq \sum_{t' \leq t} (d_{ict'} + p2_{ict'} - \lambda p2_{ict'}), \forall i \in I, \forall c \in C \tag{31}$$

$$0 \leq \lambda \leq 1 \tag{32}$$

In Eq. (28), $z1_t$ is an estimated value corresponding to the lower tolerance interval limit for the level of total costs desired, which is generated by the production plan during period t. Moreover, $p1_t$ represents the maximum extension of $z1_t$ in the tolerance interval of the total costs desired during period t. If the values of $z1_t$ and $p1_t$, cannot be easily estimated, they can be estimated from the original deterministic model solution.

In Eqs. (29), (30) and (31), d_{ict} is an estimated value corresponding to the lower limit of the tolerance interval for customer's c demand of product i during period t. Hence, $p2_{ict}$ represents the maximum extension of d_{ict} during the tolerance interval of demand.

Those constraints that are not considered fuzzy are added deterministically to the fuzzy model; that is, they can remain the same as in the base model SCM&K.

The λ value must be somewhere between 0 and 1 (32).

The optimum solution to the SCM&KFuzzy.OF/RF/L/MIN/SD_1 model is $\lambda, P_{ijst}, RL_{ijst}, C_{ist}, I_{ist}, I_{ict}^{-}, \sigma_{iss't}, I_{ist}^{\wedge}, Y_{fjst}$ where $P_{ijst}, RL_{ijst}, C_{ist}, I_{ist}, I_{ict}^{-}, \sigma_{iss't}, I_{ist}^{\wedge}, Y_{fjst}$ is the maximization solution (21) of the SCM&KFuzzy.OF/RF/L/MIN/SD_1 model by assuming the membership function specified in (24). This solution may be obtained by solving a standard linear programming model with an additional variable (λ) and with another constraint (that corresponding to the initial objective function) than the deterministic SCM&K model. Therefore in computational terms, this approach proves highly efficient.

The main advantage that the SCM&KFuzzy.OF/RF/L/MIN/SD_1 model offers, if compared with formulating the deterministic problem, SCM&K, is that the planner is not obliged to make an exact estimation of market demand for mathematical reasons should he or she be capable of only describing these values in inaccurate terms. Evidently, the membership functions of the linear fuzzy sets are a rough approach but, in contrast, membership functions can be easily managed (Zimmermann 1996).

The data structure for the SCM&KFuzzy.OF/RF/L/MIN/SD_1 model is similar to that presented by the SCM&K model, despite the following data having been included: $z1_t$, the lower tolerance interval level of the total costs during period t; $p1_t$, the maximum extension in the tolerance interval of the total costs during period t; and $p2_{ict}$, the maximum extension of the tolerance interval corresponding to the demand of product i for customer c during period t. For the lower tolerance interval limit of demand, parameter d_{ict} is still employed.

3.1.2 The SCM&KFuzzy.OF/RF/L/MIN/SD_2 Model

The model proposed in this section considers the inaccuracy of the aspiration level of the total costs of the objective function and market demand, just as the previous model does. The solution for this model is deterministic.

The SCM&KFuzzy.OF/RF/L/MIN/SD_1 model can be slightly amended if a new variable t_i, is defined, where i is the number of constraints and $0 \leq t_i \leq p_i$, t_i measures the degree of neglecting the fuzzy constraints (Zimmermann 1996). The equation corresponding to the fuzzy set's (24) membership function becomes:

$$\mu_i(x) = 1 - \frac{t_i}{p_i} \tag{33}$$

The model named SCM&KFuzzy.OF/RF/L/MIN/SD_2 becomes:

$$\text{Maximize } \lambda \tag{34}$$

Subject to

$$\begin{aligned} &\sum_{i,j,s,t} v_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,s',t} t_{ss'} \sigma_{iss't} + \sum_{i,s,t} \zeta_{is} I^{\wedge}_{ist} \\ &\quad + \sum_{i,c,t} \mu_{ic} I^{-}_{ict} + \sum_{f,j,s,t} f_{fjs} Y_{fjst} - t1_t \leq z1_t \end{aligned} \tag{35}$$

$$\lambda p1_t + t1_t \leq p1_t, \forall t \in T \tag{36}$$

$$t1_t \leq p1_t \tag{37}$$

$$-I^{-}_{ict} + I^{-}_{ic(t-1)} - \sum_{c} S_{isct} - t2_{ict} \leq -d_{ict}, \forall i \in I \backslash I^{FP}, \forall t \in T \tag{38}$$

$$\sum_{s,t' \leq t} S_{isct'} \leq \sum_{t' \leq t} (d_{ict'} + t2_{ict'}), \forall i \in I \tag{39}$$

$$I^{-}_{ict} \leq \sum_{t' \leq t} (d_{ict'} + t2_{ict'}), \forall i \in I, \forall c \in C \tag{40}$$

$$\lambda p2_{ict} + t2_{ict} \leq p2_{ict}, \forall i \in I, \forall c \in C, \forall t \in T \tag{41}$$

$$t2_{ict} \leq p2_{ict}, \forall i \in I, \forall c \in C, \forall t \in T \tag{42}$$

$$0 \leq \lambda \leq 1 \tag{43}$$

$$t1_t, t2_{ict} \geq 0, \forall i \in I, \forall c \in C, \forall t \in T \tag{44}$$

In addition, those constraints that are not considered fuzzy are added deterministically to the fuzzy model; that is to say, they remain the same as in the base model SCM&K.

The SCM&KFuzzy.OF/RF/L/MIN/SD_2 model employs a larger number of constraints than the SCM&KFuzzy.OF/RF/L/MIN/SD_1 model. The set of constraints $t_i \leq p_i$, is redundant, but it is useful to interpret shadow prices. The SCM&KFuzzy.OF/RF/L/MIN/SD_2 model offers some advantages as opposed to the SCM&KFuzzy.OF/RF/L/MIN/SD_1, model, which are basically to do with performing a sensitivity analysis. The data structure for the SCM&KFuzzy.OF/RF/L/MIN/SD_2 model is identical to that presented by the SCM&KFuzzy.OF/RF/L/MIN/SD_1 model.

3.1.3 The SCM&KFuzzy.OF/RF/L/MIN/SD_3 Model

The chief difference between this model and the previous two lies in the planner indicating inaccurate information. Rather than having tolerance intervals of types $[z1_t, z1_t + p1_t]$ and $[d_{\text{ict}}, d_{\text{ict}} + p2_{\text{ict}}]$, they are of types $[z1_t, k1z1_t]$ and $[d_{\text{ict}}, ld_{ict}]$, where $k1$ and l are constant values for the various planning horizon periods. This can be useful when any feasible inaccuracies in information cannot be defined in detail for each planning period, although the level of inaccuracy throughout the planning horizon is generally known. For example, the inexact demand of an SC in a fashion footwear sector at the beginning of the Fall-Winter season can be the interval formed between the firm orders obtained at the trade fair during this season and the value of these orders multiplied by 1.5, which could be the weight of the replenishment orders. The intention is to obtain a deterministic solution for this problem.

In this model, the approach put forward by Liu and Shi (1994) to model linear programming problems with multiple fuzzy constraints is adopted. The main contribution that these authors make is the development of procedures to solve problems with multiple criteria and multiple fuzzy constraints (Shi and Liu 1993). The authors name this type of problems MC^2 (Multiple criteria and multiple constraints). This section adopts the approach set forward by Liu and Shi (1994), but employs models with a single optimization criterion.

Pendharkar (1997) applies this approach to solve a production planning problem in the coal industry. The ever growing importance of ensuring a quality production planning process in the coal industry led this author to develop a fuzzy linear programming model to evaluate the different production alternatives by incorporating measures of fuzzy quality.

Based on the fuzzy model (16)–(19) utilized in the two previous sections, the membership functions associated with fuzzy variables z and d_{ict} can be defined as:

$$\mu_i(x) = \begin{cases} 1 & si \quad B_i x \leq d_i \\ \frac{ld_i - B_i x}{ld_i - d_i} & si \quad d_i < B_i x \leq ld_i \\ 0 & si \quad B_i x > ld_i \end{cases} \quad i = 1, \ldots, m+1 \tag{45}$$

where $l \geq 1$. When l approaches 1, the inaccuracy of variable d_i diminishes. If λ represents the degree at which fuzzy parameters $z1$ and d_{ict} are acceptable, then the equivalent deterministic model, SCM&KFuzzy.OF/RF/L/MIN/SD_3, is formulated as follows:

$$\text{Maximize } \lambda \tag{46}$$

Subject to

$$\lambda \leq \frac{k1z1 - \left(\sum_{i,j,s,t} v_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,s\prime,t} t_{ss'} \sigma_{iss't}\right.}{k1z1 - z1}$$
$$\frac{\left. + \sum_{i,s,t} \zeta_{is} I^{\wedge}_{ist} + \sum_{i,c,t} \mu_{ic} I^{-}_{ict} + \sum_{f,j,s,t} f_{fjs} Y_{fjst}\right)}{k1z1 - z1} \tag{47}$$

$$\lambda \leq \frac{ld_{ict} - 2d_{ict} - \left(I^{-}_{ict} - I^{-}_{ic(t-1)} + \sum_{c} S_{isct}\right)}{ld_{ict} - d_{ict}}, \forall i \in I \backslash I^{FP}, \forall t \in T \tag{48}$$

$$\lambda \leq \frac{l \sum_{t' \leq t} d_{ict'} - \sum_{s,t' \leq t} S_{isct'}}{l \sum_{t' \leq t} d_{ict'} - \sum_{t' \leq t} d_{ict'}}, \forall i \in I \tag{49}$$

$$\lambda \leq \frac{l \sum_{t' \leq t} d_{ict'} - \left(I^{-}_{ict}\right)}{l \sum_{t' \leq t} d_{ict'} - \sum_{t' \leq t} d_{ict'}}, \forall i \in I, \forall c \in C \tag{50}$$

$$0 \leq \lambda \leq 1 \tag{51}$$

The data structure for the SCM&KFuzzy.OF/RF/L/MIN/SD_3 model is similar to that presented by the SCM&K model, although the following data have been incorporated: *z1*, the lower tolerance interval level of the total costs for all the periods; *k1*, the constant value that provides the maximum extension in the tolerance interval of the total costs for all the periods; *l*, the constant value that provides the maximum extension of the demand tolerance interval for all products, customers and periods. Parameter d_{ict} continues to be employed for the lower demand tolerance interval limit.

3.1.4 SCM&KFuzzy.OF/RF/L/COM/SD

The main difference between this model and the three former ones lies in the use of the convex linear combination of the min-operator and max-operator operators as the operator for the aggregation of the objective function and the fuzzy constraints (Zimmermann and Zysno 1980). The intention is to obtain a deterministic solution for this problem.

Miller et al. (1997) use this approach to solve a production planning problem in a company that packs fresh tomatoes. In packing fresh tomatoes, uncertain elements attributed to human perceptions are quite commonplace, such as harvest, the packed tomato ratio, demand and stockout cost. The authors develop a linear programming model to determine the firm's production program. Then they develop a fuzzy programming model that contemplates the elements' fuzziness. When real data are used, the costs obtained by both models are compared, resulting in the cost obtained by the linear programming model being considerably higher. This higher cost is the result of the rigidness in the limits of the linear programming model's constraints.

In this model, the min-operator as the model for the "Y logic" and the max-operator as the model of the "O logic" are combined (Zimmermann and Zysno 1980).

$$\mu_D(x) = \gamma \min_{i=1}^{m} \mu_i(x) + (1-\gamma) \max_{i=1}^{m} \mu_i(x), \gamma \in [0,1] \tag{52}$$

or

$$\mu_D(x) = (1-\gamma) \min_{i=1}^{m} \mu_i(x) + \gamma \max_{i=1}^{m} \mu_i(x), \gamma \in [0,1] \tag{53}$$

Parameter γ indicates the degree of closeness to the strict logic meaning of "Y" and "O".

Based on the fuzzy model (16)–(19), and after introducing the two new auxiliary variables, we obtain:

$$\lambda_1 = \min_{i=1}^{m} \mu_i(x) \tag{54}$$

and

$$\lambda_2 = \max_{i=1}^{m} \mu_i(x) \tag{55}$$

where λ_1 is the degree of fulfilment of the least fulfilled constraint, $0 \le \lambda_1 \le 1$, and λ_2 is the degree of fulfilment of the most fulfilled constraint, $0 \le \lambda_2 \le 1$. Using membership function (24) for the fuzzy constraints, the SCM&KFuzzy.OF/RF/L/COM/SD model becomes:

$$\text{Maximize z} = \gamma\lambda_1 + (1-\gamma)\lambda_2 \tag{56}$$

Subject to

$$\begin{aligned}\sum_{i,j,s,t} v_{ijs}P_{ijst} + \sum_{i,s,t} p_{is}C_{ist} + \sum_{i,s,t} h_{ist}I_{ist} + \sum_{i,s,c,t} t_{sc}S_{isct} + \sum_{i,s,s',t} t_{ss'}\sigma_{iss't} + \sum_{i,s,t} \zeta_{is}I^{\wedge}_{ist} \\ + \sum_{i,c,t} \mu_{ic}I^{-}_{ict} + \sum_{f,j,s,t} f_{fjs}Y_{fjst} + \lambda_1 p1 \le z1_t + p1_t\end{aligned} \tag{57}$$

$$\sum_{i,j,s,t} v_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,s\prime,t} t_{ss\prime} \sigma_{iss\prime t} + \sum_{i,s,t} \zeta_{is} I^{\wedge}_{ist}$$
$$+ \sum_{i,c,t} \mu_{ic} I^{-}_{ict} + \sum_{f,j,s,t} f_{fjs} Y_{fjst} + \lambda_2 p1_t \leq z1_t + p1_t + M\pi_1 \tag{58}$$

$$-I^{-}_{ict} + I^{-}_{ic(t-1)} - \sum_{c} S_{isct} + \lambda_1 p2_{ict} \leq -d_{ict} + p2_{ict}, \forall i \in I \backslash I^{FP}, \forall t \in T \tag{59}$$

$$-I^{-}_{ict} + I^{-}_{ic(t-1)} - \sum_{c} S_{isct} + \lambda_2 p2_{ict} \leq -d_{ict} + p2_{ict} + M\pi_2, \forall i \in I \backslash I^{FP}, \forall t \in T \tag{60}$$

$$\sum_{s,t\prime \leq t} S_{isct\prime} \leq \sum_{t\prime \leq t} (d_{ict\prime} + p2_{ict\prime} - \lambda_1 p2_{ict\prime}), \forall i \in I \tag{61}$$

$$\sum_{s,t\prime \leq t} S_{isct\prime} \leq \sum_{t\prime \leq t} (d_{ict\prime} + p2_{ict\prime} + M\pi_2 - \lambda_2 p2_{ict\prime}), \forall i \in I \tag{62}$$

$$I^{-}_{ict} \leq \sum_{t\prime \leq t} (d_{ict\prime} + p2_{ict'} - \lambda_1 p2_{ict'}), \forall i \in I, \forall c \in C \tag{63}$$

$$I^{-}_{ict} \leq \sum_{t' \leq t} (d_{ict'} + p2_{ict'} + M\pi_2 - \lambda_2 p2_{ict'}), \forall i \in I, \forall c \in C \tag{64}$$

$$\sum_{i=1}^{m} \pi_i \leq m - 1 \tag{65}$$

$$\begin{gathered} 0 \leq \lambda_1, \lambda_2 \leq 1 \\ \pi_1, \pi_2 \in \{0, 1\} \end{gathered} \tag{66}$$

where M represents a very high value and m is the number of constraints with fuzzy parameters, in this case $m = 2$.

Objective function (55) maximizes the linear combination of the degree of fulfilment of the least fulfilled constraint (minimum operator) and the degree of fulfilment of the most fulfilled constraint (maximum operator).

The objective function and the fuzzy constraints are represented by two sets of constraints: one for the min-operator and the other for the max-operator. Each constraint has a tolerance parameter indicating the level of tolerated deviation. These parameters are linear (linear membership functions).

The degrees of fulfilling the constraints are reflected in fulfillment indicators λ_1 and λ_2. The objective is to strike a balance between them in order to maximize total fulfilment. For example, if λ_1 and λ_2 are both 1, then all the constraints are fulfilled (in the deterministic sense). If their values are between 0 and 1, then they are inside the fuzzy region.

For the min-operator, the structure of constraint $\leq$ and the maximization criterion are such that the maximum is selected from the minimum of all the possible λ values.

For the max-operator, the logic of binary variable, π, Constraint (64), and the maximization criterion select the level of the fulfillment of the most fulfilled constraint.

Similarly to the previous fuzzy models, the deterministic constraints do not require amendments and can be added to the model.

To solve the problem, the compensatory value, γ, can be set at 0.6, and this value is considered effective for most circumstances (Zimmermann 1996).

The data structure for the SCM&KFuzzy.OF/RF/L/COM/SD model is identical to that presented by the SCM&KFuzzy.OF/RF/L/MIN/SD_1 and SCM&KFuzzy.OF/RF/L/MIN/SD_2 models.

3.1.5 The SCM&KFuzzy.OF/RF/L/FA/SD Model

In this model, the *fuzzy and* operator (Werners 1987) is to be employed as an operator for the aggregation of the objective function and the fuzzy constraints. The intention is to obtain a deterministic solution to the problem.

By using the same fuzzy model (16)–(19) as in previous sections, the fuzzy set of the decision described in (53) and membership function (24), the SCM&KFuzzy.OF/RF/L/FA/SD model becomes:

$$\text{Maximize } z = \lambda + (1-\gamma)\frac{1}{2}\sum\nolimits_{i=1}^{2} \lambda_i \tag{67}$$

Subject to

$$\begin{aligned}\sum_{i,j,s,t} v_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,s\prime,t} t_{ss\prime} \sigma_{iss\prime t} + \sum_{i,s,t} \zeta_{is} I^{\wedge}_{ist} \\ + \sum_{i,c,t} \mu_{ic} I^{-}_{ict} + \sum_{f,j,s,t} f_{fjs} Y_{fjst} + (\lambda + \lambda_1) p1 \leq z1_t + p1_t\end{aligned} \tag{68}$$

$$\begin{aligned}-I^{-}_{ict} + I^{-}_{ic(t-1)} - \sum_{c} S_{isct} + (\lambda + \lambda_2) p2_{ict} \leq -d_{ict} + p2_{ict} \\ \forall i \in I \backslash I^{FP}, \forall t \in T\end{aligned} \tag{69}$$

$$\sum_{s,t\prime \leq t} S_{isct\prime} \leq \sum_{t\prime \leq t} (d_{ict\prime} + p2_{ict\prime} - (\lambda + \lambda_2) p2_{ict\prime}), \forall i \in I \tag{70}$$

$$I^{-}_{ict} \leq \sum_{t\prime \leq t} (d_{ict\prime} + p2_{ict\prime} - (\lambda + \lambda_2) p2_{ict\prime}), \forall i \in I, \forall c \in C \tag{71}$$

$$0 \leq \lambda, \lambda_1, \lambda_2 \leq 1 \tag{72}$$

Parameter γ in the objective function indicates the degree of closeness to the strictest sense of the "Y logic". For $\gamma = 1$, the *fuzzy and* operator becomes the min-operator, and for a value of $\gamma = 0$, it acts as the arithmetic mean of the fuzzy constraints.

Each constraint has a tolerance parameter indicating the level of tolerated deviation. These parameters are linear (linear membership functions).

The fulfilment levels of the constraints are reflected in the fulfilment indicators, λ, λ_1 and λ_2. The objective is to strike a balance among them in order to maximize total fulfilment. For example, if they are all 1, then all the constraints are fulfilled (in the deterministic sense). If values are between 0 and 1, then they are inside the fuzzy region.

Similarly to the previous fuzzy models, deterministic constraints do not need to be amended and they can be added to the model.

To solve the problem, the compensatory value, γ, can be set at 0.6, and this value is considered effective for most circumstances (Zimmermann 1996).

The data structure for the SCM&KPFuzzy.OF/RF/L/FA/SD model is identical to that presented by the SCM&KFuzzy.OF/RF/L/MIN/SD_1 and SCM&KFuzzy.OF/RF/L/MIN/SD_2 models.

4 Computational Experiment

In order to evaluate the way the proposed models function, Example 1, provided by the work of McDonald and Karimi (1997), is employed. This example is considered very characteristic of the chemical sector, where a couple of production plants produce 34 products. Each production plant contains a single processor, thus set J is superfluous. Production plant 1 produces 23 products and there are 11 product families whose ratios, minimum production lots and fixed costs are supplied. Production plant 2 depends on Production plant 1 and produces 11 products, which require 1 unit of the first product of each family from Production plant 1. Twelve monthly planning periods are considered in the planning horizon with the demand required at the end of each period. It is assumed that Production plant 2, which depends on Production plant 1, has no capacity constraints. The demand of the 11 products from Production plant 2 is derived as 50 % of the demand of the products that consume the supply from the first plant. The costs and production speeds for Production plant 2 are also provided. The target safety stock levels are assumed to equal the mean monthly demand.

Furthermore, studying the impact on the results of those models by considering, or not, safety stocks and grouping products into product families were believed relevant.

To help demonstrate the information in the tables, a short code was assigned to each evaluated model (Table 1).

The models have been implemented with the MPL V4.2 modeling language (Maximal Software Incorporation 2004). The solution was performed with

Table 1 Code assigned to the models

Model name	Code
SCM&K	M0
SCM&K without safety stocks (NoSS)	M0_NoSS
SCM&K with product families (F)	M0_F
SCM&K with F and NoSS	M0_F_NoSS
SCM&KFuzzy.OF/RF/L/MIN/SD_1	M1
SCM&KFuzzy.OF/RF/L/MIN/SD_1 NoSS	M1_NoSS
SCM&KFuzzy.OF/RF/L/MIN/SD_1 with F	M1_F
SCM&KFuzzy.OF/RF/L/MIN/SD_1 with F and NoSS	M1_F_NoSS
SCM&KFuzzy.OF/RF/L/MIN/SD_2	M2
SCM&KFuzzy.OF/RF/L/MIN/SD_2 NoSS	M2_NoSS
SCM&KFuzzy.OF/RF/L/MIN/SD_2 with F	M2_F
SCM&KFuzzy.OF/RF/L/MIN/SD_2 with F and NoSS	M2_F_NoSS
SCM&KFuzzy.OF/RF/L/MIN/SD_3	M3
SCM&KFuzzy.OF/RF/L/MIN/SD_3 NoSS	M3_NoSS
SCM&KFuzzy.OF/RF/L/MIN/SD_3 with F	M3_F
SCM&KFuzzy.OF/RF/L/MIN/SD_3 with F and NoSS	M3_F_NoSS
SCM&KFuzzy.OF/RF/L/COM/SD	M4
SCM&KFuzzy.OF/RF/L/COM/SD NoSS	M4_NoSS
SCM&KFuzzy.OF/RF/L/COM/SD with F	M4_F
SCM&KFuzzy.OF/RF/L/COM/SD with F and NoSS	M4_F_NoSS
SCM&KFuzzy.OF/RF/L/FA/SD	M5
SCM&KFuzzy.OF/RF/L/FA/SD NoSS	M5_NoSS
SCM&KFuzzy.OF/RF/L/FA/SD with F	M5_F
SCM&KFuzzy.OF/RF/L/FA/SD with F and NoSS	M5_F_NoSS

Optimization Solver CPLEX 6.6 (CPLEX Optimization Inc. 1994). Finally, the model's input and output data were managed by a Microsoft Access 2000 database.

The experiment was carried out in a PC with an Intel Pentium M processor, 1400 MHz and 504 MB of RAM.

To carry out the experiment, each planning model was run for all the planning periods (1,…,12) by updating the demand values, the initial existing inventory and the delay in existing demand, which originate from the launches planned from previously calculated periods.

The proposed evaluation method consists in evaluating the way the models function for a set of measurable proposals originally described by Mula et al. (2006b): the models' computational efficiency, service level, inventory levels, total costs of planning and planning nervousness.

4.1 Summary of the Results

The models proposed in this chapter have been evaluated using data from an SC in the chemical sector. However, the final objective of this evaluation was not to select the best model to solve a sporadic situation, but to validate the way the developed models function.

Table 2 summarizes the computational effort of all the models when launching the first planning horizon. As observed, models M4 and M4_NoSS (continuous linear programming problems), which employ the operator of the linear convex min- and max-operators combination, obtains the optimum solution with zero iterations. Evidently, the number of iterations can change in the remaining launches of the planning horizon, depending on the problem input data. Nevertheless, the level of difficulty presented (number of iterations) during the first launch to obtain the solution to the problem may be representative.

It is also important to stress the minimum differences between computational effort, measured from the information storage requirements between fuzzy models and deterministic models:

Model M2_F has the largest number of variables, 5539, as opposed to deterministic model M0_F, 5118, which implies 421 additional variables.

As for the number of integer variables, all the integer programming models present the same number, 528, except models M4_F and M4_F_NoSS, which employ 530. This is because these models use the operator from the linear convex min- and max-operators combination and, therefore, they require another integer variable for each fuzzy constraint, the objective function and the constraints where demand is considered inaccurate.

The number of the deterministic model constraints with product families is 9474, whereas the fuzzy model with a larger number of constraints is once again the model that employs the combined operator, M4_F, with 12559 constraints, which implies 3085 more constraints.

Regarding the number of nonzero elements of the constraints matrix, model M2_F has a larger number. Finally, models M2_NoSS and M2_F_NoSS present greater constraints matrix density, 0.4, which implies more information storage requirements.

Another important aspect to evaluate the proposed models' computational efficiency is the measure of the time required by the CPU to solve the model for the first planning horizon launch. To the mixed-integer programming models, a CPU time limit of 100 s has been added, which they all used up.

Models M1, M2, M3, M4 and M5 and their variants are computationally more intensive. This is because these models have the additional requirement of reaching the desired level of total costs. It is interesting to highlight that the model

Table 2 Computational efficiency

Model	Iterations	Variables	Integer	Constraints	Nonzero elements	Matriz density (%)	CPU time (s)
M0	462	4854	0	8946	7534	0.17	0.16
M0_F	276428	5118	528	9474	77742	0.16	100
M0_F_NoSS	317976	4302	528	8658	76110	0.2	100
M0_NoSS	1114	4038	0	8130	73722	0.2	0.24
M1	1473	4855	0	8958	98123	0.2	0.3
M1_F	82534	5119	528	9486	102095	0.2	100
M1_F_NoSS	306134	4303	528	8670	95567	0.3	100
M1_NoSS	909	4039	0	8142	91595	0.3	0.97
M2	1883	5275	0	9378	159647	0.3	0.51
M2_F	46262	5539	528	9906	163619	0.3	100
M2_F_NoSS	1059	4723	528	9090	157091	0.4	100
M2_NoSS	1170	4459	0	8562	153119	0.4	0.28
M3	1704	4855	0	8947	82350	0.19	0.39
M3_F	241978	5119	528	9475	84870	0.17	100
M3_F_NoSS	137380	4303	528	8659	82830	0.2	100
M3_NoSS	1071	4039	0	8131	80310	0.2	0.25
M4	0	4858	2	12031	126734	0.2	1.37
M4_F	250411	5122	530	12559	132290	0.2	100
M4_F_NoSS	352340	4306	530	11743	120866	0.2	100
M4_NoSS	0	4042	2	11215	115310	0.3	1.16
M5	1456	4857	0	8958	98123	0.2	0.29
M5_F	109065	5121	528	9486	102095	0.2	100
M5_F_NoSS	139748	4305	528	8670	95567	0.3	100
M5_NoSS	585	4041	0	8142	91595	0.3	0.13

which employs the combined operator, and is one of the models with greater information storage requirements, needs one of the longest CPU times.

In relation to the other evaluation criteria considered (Mula et al. 2006b), service level, inventory levels, total costs of planning and planning nervousness, if we look for a model that performs as best as possible in all the evaluation criteria, or even if the evaluation criteria are weighted, the following procedure is used:

A score of 100 is assigned to the best value obtained in each evaluation criterion.

If the best values for each criterion are ranked in decreasing order, which occurs with computational time, total costs and planning nervousness, the following formula is applied to the remaining values in this ranking to obtain their corresponding score:

$$Score_i = 100 * \left(\frac{MaximumValue - Value_i + MinimumValue}{MaximumValue} \right) \tag{73}$$

If the best values of each criterion are ranked in increasing order, which occurs with service level and minimum inventory levels, the following formula is applied to the rest of the values in this ranking to obtain their corresponding score:

$$Score_i = 100 * \frac{Value_i}{MaximumValue} \tag{74}$$

The whole score obtained is summed by weighing (as required) each evaluation criteria.

Table 3 offers the results obtained by applying this procedure should all the evaluation criteria have the same weight.

As seen in the table above, assigning the same weight to all the evaluation criteria, model M3_F_NoSS will be selected, which performs better than deterministic model M0. It is worth stressing in general that the models which function without safety stocks obtained a better score than those which contemplate these stocks. To the quantitative comparison made, the qualitative comparison must be added, which derives from the model characteristics. These characteristics are presented in Sect. 3 of this chapter, and will be taken into account when deciding on a specific model.

Table 3 Quantitative evaluation of the models

Model	Service levels	Inventory levels	Planned period nervousness	Planned quantity nervousness	Total costs	TOTAL score
M3_F_NoSS	98.84	100.00	79.31	95.83	81.99	455.98
M0	100.00	95.00	96.55	64.58	74.55	430.69
M5_F_NoSS	99.51	95.00	75.86	66.67	82.58	419.62
M0_NoSS	100.00	50.00	100.00	75.00	83.43	408.43
M0_F_NoSS	99.38	95.00	51.72	77.08	78.61	401.80
M4	100.00	95.00	89.66	37.50	78.23	400.39
M4_NoSS	100.00	50.00	100.00	54.17	93.32	397.49
M1_F_NoSS	100.00	95.00	48.28	70.83	80.06	394.16
M4_F_NoSS	99.39	95.00	51.72	70.83	76.67	393.62
M2_F_NoSS	99.01	95.00	48.28	60.42	89.01	391.72
M2	100.00	95.00	75.86	33.33	81.77	385.97
M2_NoSS	100.00	55.00	93.10	50.00	86.74	384.84
M1	100.00	95.00	65.52	35.42	83.58	379.51
M2_F	99.99	55.00	72.41	77.08	73.82	378.31
M1_NoSS	100.00	60.00	79.31	47.92	87.67	374.90
M1_F	99.71	60.00	68.97	68.75	71.70	369.12
M5	99.96	95.00	55.17	33.33	85.55	369.02
M3	99.98	90.00	44.83	33.33	97.41	365.55
M5_F	99.95	65.00	44.83	66.67	88.23	364.68
M3_F	99.44	85.00	34.48	58.33	83.83	361.09
M5_NoSS	100.00	55.00	72.41	39.58	89.94	356.93
M3_NoSS	99.96	55.00	51.72	31.25	98.18	336.11
M4_F	99.34	20.00	51.72	64.58	79.57	315.21
M0_F	100.00	20.00	44.83	72.92	77.14	314.88

5 Conclusions

This chapter addresses the mid-term production planning problem of a supply chain (SC) under fuzziness in market demand and at the aspiration level of the total costs to be generated. The deterministic model, originally put forward by McDonald and Karimi (1997), has been adopted as a representative formulation to incorporate fuzziness into the decision-making process. The considered SC is multi-product, multi-plant and multi-period. Other key considerations include the capacity constraints of the manufacturing resources and delayed demand.

By taking the deterministic model as a basis, five fuzzy mathematical models have been developed. One noteworthy development is that the objective function of the SCM&K model is the minimization of costs, while it is the maximization of the fulfilment of all the constraints that the fuzzy models contemplate. Thus, total fulfilment aggregates individual satisfaction measures which are associated with the operational cost range and with variations in the technological coefficients or in the independent terms of the constraints.

The min-operator version (Bellman and Zadeh 1970) is employed in the majority of the proposed fuzzy models, and other operators are also utilized: the linear convex combination of the min-operator and the max-operator (Zimmermann and Zysno 1980) and the fuzzy and operator (Werners 1987). The membership functions used were assumed linear. Yet the existing literature suggests other operators, concepts and operations. Moreover, different research lines remain open, of which some intend to take on new research projects.

In general, the fuzzy models provide "freedom of action" as far as inaccurate constraints are concerned, with a slight increase in information storage requirements and a moderate increase in the CPU time needed. They also provide medium service levels and minimum inventory levels, which are similar to those obtained by deterministic models. All the fuzzy models proposed, M1, M2, M3, M4 and M5, generate lower total costs than the basic M0 deterministic model. These differences in total costs are mainly due to considering feasible future fluctuations in demand, which lead to greater production and/or inventories in order to avoid heavily penalized delayed demand. It is worth stressing that those models operating without safety stocks generate lower total costs. In general however, the models operating with product families generate lower total costs. Likewise, the models operating with product families are those which obtain greater planning period nervousness. Besides, those models that do not consider using safety stocks obtain lower planning period nervousness. The fuzzy models do not present greater planning period nervousness if compared to the planned quantity than the M0 deterministic model. Yet the models operating with product families provide less planning nervousness as compared with planned quantity. Something similar occurs with the models working without safety stocks. The changes in the quantity of the planned order are more marked than during the planned time period. This is logical in the example applied, where product demand is found in virtually all the planning periods, and where minimizing the costs of maintaining inventories is the objective, among other costs.

Some circumstances which can represent future research lines have been presented throughout this research work: (i) compare these modeling approaches and their results with those based on representing epistemic uncertainty with fuzzy coefficients, which produce fuzzy solutions; (ii) introduce soft computing techniques to optimize the computational efficiency of those models operating with product families. As the models operating with product families generally display good performance for all the evaluation criteria, except CPU time, it is considered important to improve these computational times by soft computing techniques, which combine the fuzzy sets theory and meta-heuristic methods (neuronal networks and genetic algorithms) to solve real combinational optimization problems, such as production planning in SCs; and (iii) model uncertainty due to delivery times and its implication on capacity planning.

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Recent Models and Solution Methodologies for Optimization Problems in Supply Chain Management Under Fuzziness

Seda Yanık Uğurlu and Ayca Altay

Abstract Supply chain (SC) involves collaborating with business partners which uniquely specialize on only a few key strategic activities. The network structures formed in SC's have emerged in the last decade with the accelerated developments in globalization, outsourcing and information technology. The complex network structures have introduced novel problems to both industry and academia while traditional complications are yet investigated. The intensification points of SC problems are mainly configuration of distribution networks, forming distribution strategies, trade-off analyses, managing inventory and cash-flow. One of the main challenges in modeling and solving these problems is to deal with the uncertainties involved in the complex nature of SC. Demand has been the main uncertain aspect of the problems of the related literature followed by internal parameters, supplier related parameters, environmental parameters and price. The uncertainty issues have been commonly dealt with fuzzy approaches in the literature. Fuzzy approaches become beneficial under uncertainties such as the absence of data, use of qualitative data or the need for subjective judgments. Hence, fuzzy techniques in SC optimization problems are vastly implemented in the literature. The purpose of this study is basically to summarize the fuzzy techniques employed for SC optimization models, their past applications, solutions algorithms and offer directions for future research.

Keywords Supply chain management · Fuzzy set theory · Fuzzy optimization · Fuzzy mathematical programming · Chance-constrained programming

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1 Introduction

The evolution of markets to strong competition on quality and time, uncertainties related to environment and complexities due to globalization have necessitated firms to change and evolve into more reliable and robust entities (Mentzer et al. 2001). This challenge has led the firms to develop close relations and collaborations with their suppliers and buyers in order to operate in a more efficient and quick way. Consequently, the concept which is named as "Supply Chain" (SC) has been defined as the alignment of upstream (i.e. supply) and downstream (i.e. distribution) firms that bring products or services to market (Lambert et al. 1998). Supply chain emphasizes on the close coordination among independent firms which take part in the production and delivery to the point-of-consumption such as producers, product assemblers, distributors, retailers and transportation companies.

A set of management activities is required in order to achieve the coordination and integration of the activities of the partners of a supply chain. Supply Chain Management (SCM) is defined as the process of planning, implementing and controlling the operations of the supply chain in an efficient way (Melo et al. 2009). A systematic approach is adopted through supply chain management to view the supply chain as a whole and manage all movements and storage of raw materials, work-in-process inventory, and finished goods as well as information and finances between the point-of-origin and the point-of-consumption (Mentzer et al. 2001. Together with the challenge of coordination and integration of the SC partners, the aforementioned management activities of SC needs to cope with the high uncertainty related to the pace of change in technology, high competition in the marketplace and the globalization.

Main processes of SCM include demand management, capacity planning, inventory planning, order fulfillment, manufacturing flow management, procurement, and product development, customer service management and customer relationship management (Mentzer et al. 2001). Due to above mentioned reasons, uncertainty which is defined as the difference between the amount of information required to perform a task and the amount of information already possessed (Mula et al. 2006) applies to almost all of the SCM processes. For example, uncertainty of demand contributes a significant challenge in planning capacities, inventory levels and order fulfillment along the SC. Consequently, a common approach, optimization, is used to make decisions on SCM processes is redefined to deal with the uncertainty and to give more flexible results.

Uncertainty in SCM optimization problems is typically incorporated into mathematical programming models through a parameter (i.e. quantity demanded or a cost factor related to the problem). In literature, uncertainty in SCM is dealt under the following analytical models: robust optimization, stochastic programming, fuzzy mathematical programming (possibilistic programming) and hybrid models. In robust optimization, uncertain parameters are defined by determining specific bounds. Whereas, stochastic programming employs the probability theory and the uncertain parameters of the problem are specified using probability distributions. In some cases, uncertainty exists not due to randomness but fuzziness where doubt arises about the exactness of concepts, correctness of statements and judgments having little to do with occurrence of events which is the back-bone of probability theory; thus, stochastic programming (Luhandjula 2007). This type of uncertainty is handled using fuzzy set theory which was developed by Zadeh (1965). In fuzzy mathematical programming the uncertain parameters are defined using fuzzy numbers and fuzzy relationships in order to soften mathematical programming models. There also exist hybrid models where stochastic fuzzy programming or robust fuzzy programming models are combined together. In all of them, the goal of the mathematical model is to find a feasible solution for the defined data and somewhat seek optimality.

In this chapter, it is aimed to summarize fuzzy techniques employed for SC optimization models, their past applications, solutions algorithms and offer directions for future research. For this purpose we have reviewed database libraries of "Science Direct", "Web of Science" and "IEEE" for bibliographic references of the last decade (Chen and Lee 2004; Xie et al. 2006; Yugang and Guang 2006; Selim and Ozkarahan 2006, 2008; Wang and Shu 2007; Aliev et al. 2007; Chen et al. 2007; Xu et al. 2008, 2009; Jing-min et al. 2008; Petrovic et al. 2008; Xu and Zhai 2008, 2010; Liu et al. 2008; Yuansheng et al. 2008; Handfield et al. 2009; Gong et al. 2009; Lau et al. 2009; Ghatee and Hashemi 2009; Peidro et al. 2009; Mahnam et al. 2009; Gumus et al. 2009; Mitra et al. 2009; Pishvaee and Torabi 2010; Miller and John 2010; Bilgen 2010; Qin and Ji 2010; Wong 2010; Mula et al. 2010; Pinto-Varela et al. 2011; Jazemi et al. 2011; Kabak and Ülengin 2011; Chandran and Kandaswamy 2012; Paksoy et al. 2012; Kubat and Yuce 2012; Björk 2012; Paksoy and Yapici-Pehlivan 2012; Zheng and Ling 2013; Pishvaee and Razmi 2012; Liang 2012; Nepal et al. 2012; Vahdani et al. 2012; Makkar et al. 2011; Arikan 2013; Özkır and Başlıgil 2013). The journals which are corresponded in this search and could be identified to be related to the particular area of "Optimization Problems in Supply Chain Management under Fuzziness" have been presented in Table 1.

The keywords which are used in this search are "supply chain", "fuzzy", "optimization" and sought to be included in the title, keywords or abstract. Approximately 220 articles (some articles duplicated in the three databases searched) have been obtained. After an exhaustive examination, the number of articles which are related to optimization in SCM under fuzziness has been found to be 46. Then, the final 46 articles have been investigated for the taxonomy developed in this study. The distribution of 46 articles examined with respect to their published year is presented in Fig. 1. It is clearly seen that number of articles has increased in the last half of this decade more than 5 times than the articles published in the first half of the decade.

The chapter is arranged as follows: The SC problems dealt with fuzziness have been classified with respect to decision types and fuzzy components in Sect. 2. The model types of SC are set out in Sect. 3. Then, the approaches and the solution methodologies for optimization problems in SC under fuzziness are described and classified in Sect. 4. Section 5 summarizes the application areas of SC optimization under fuzziness and reviews the validation approaches of the collection of works studied. Finally, conclusions are drawn and future lines of research are discussed in Sect. 6.

Table 1 Journals presenting articles of "Optimization Problems in SCM under Fuzziness"

Journal name	Number of articles
European Journal of Operational Research	6
Expert Systems with Applications	5
International Journal of Production Economics	3
Fuzzy Sets and Systems	3
Information Sciences	3
Applied Mathematical Modelling	3
Computers and Industrial Engineering	2
Computers and Chemical Engineering	2
Others	19

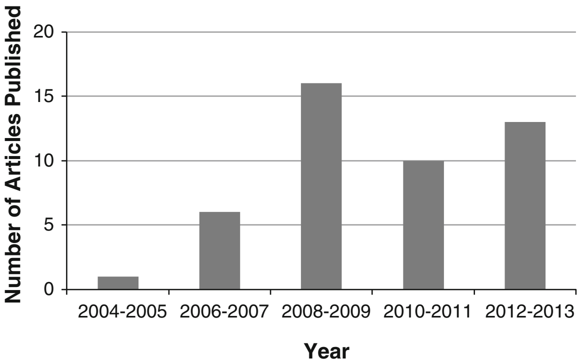

Fig. 1 The distribution of articles of "optimization in SCM under fuzziness" with respect to year

2 SCM Optimization Problems and Uncertainty as Fuzziness

In this section, the dimensions of defining SCM optimization problems are presented first. These dimensions have been used to classify different types of SCM problems. Then, the uncertainty issues which lead to more realistic SCM models have been analyzed in detail. Finally, the uncertainty parameters which are included in the SCM models are investigated.

2.1 SCM Problems Classification

The articles with respect to the decision type of the presented problem have been reviewed. The most commonly sought decisions within the SCM efforts are in the domain of location-allocation, production, inventory, procurement and capacity planning problems.

Location-allocation problems are commonly encountered to be dealt together. In location-allocation problems, the aim is to decide the facilities to be used (opened) and the assignment of customers to be served from these facilities taking into account a set of spatially distributed customers and facilities in order to minimize factors such as cost, distance or maximize coverage, profit, etc.

There also exists a vast effort to combine allocation decisions with one or more decision types such as: production, inventory, procurement and capacity decisions. In these types of problems, the aim is to determine (i) the types of the processes/production capabilities to be provided by the facilities; (ii) the amount of the production in the facilities; (iii) the storage amounts along the SC; (iv) which supplier to be used; (v) the economic order quantities from suppliers and (vi) the capacities of the processes and the facilities under various constraints related to demand, operations, capacity and transportation.

SCM optimization problems include some other decisions such as routing, sales amount, transportation mode and product development either combined with other decision types or handled as the single decision. In routing problems, the sequence of the stops are searched considering a set of geographically dispersed points in order to achieve an optimal criteria such as cost or distance minimization. In transport mode decisions, the best mode for the distribution is searched taking into account constraints such as connections, facilities. In product development problems, the aim is to find the product configuration which is best compatible with SCM capabilities such as production capabilities, material procurement.

In Table 2, we present the frequency count and percentage of the decision types without regarding combinations with other decision types simultaneously in the optimization model. We observe that allocation decisions have been referred as the most common decision type in studies of SCM optimization under fuzziness with 32 %. Then, inventory, location, procurement and production decisions have been studied with 17 and 12 %. Decisions regarding the capacity, product development, routing, sales amount and transport mode are noticed as open areas that are least studied as decision types of SCM under fuzziness research.

In Table 3, we keep the track of the counts of both individual and combination of the decision types handled in the articles reviewed. It is seen that location-allocation decision are frequently studied together.

Allocation decision has also been combined with other decisions such as production, inventory, procurement and capacity commonly. Besides, procurement-location and inventory-procurement decisions have been combined and studied together. On the other side, it is observed that inventory decisions are dealt also commonly as single decisions in the literature while combined with other decisions.

The SCM problems with the above mentioned decisions have been extended in order to represent the real-life problems. A well-studied extension is to define multi-period problems to correspond to parameters changing by time. By this means, decisions can be adapted to the changes over the planning time. In these types of problems, the planning horizon is divided into more than one period. Table 4, shows that among 46 articles reviewed in total, 30 % consider a multi-period planning horizon. The distribution of the count of articles for single and

Table 2 Frequency of articles for single-decision type

Decision type	Frequency	Percent (%)
Allocation	30	31.9
Capacity	4	4.3
Inventory	16	17.0
Location	16	17.0
Procurement	11	11.7
Product development	1	1.1
Production	11	11.7
Routing	2	2.1
Sales amount	2	2.1
Transport mode	1	1.1
Total	94	100

Table 3 Frequency analysis for multi-decision types

Decision type	Frequency	Percent (%)
Allocation	4	9
Capacity-allocation	1	2
Capacity-inventory-allocation	1	2
Capacity-procurement-allocation-transport mode	1	2
Capacity-production-location-Allocation	1	2
Inventory	7	15
Inventory-allocation	1	2
Inventory-location-allocation	1	2
Inventory-procurement-location-Allocation	1	2
Inventory-procurement-production-Allocation	2	4
Inventory-production	1	2
Inventory-production-allocation	1	2
Inventory -production-allocation-SalesAmount	1	2
Location	1	2
Location-allocation	9	20
Procurement	4	9
Procurement-location-allocation	1	2
Procurement-production-location-allocation	1	2
Procurement-location-allocation-sales amount	1	2
Product development	1	2
Production	1	2
Production-allocation	2	4
Production-allocation-routing	1	2
Routing	1	2
	46	100

Table 4 Frequency analysis of number of periods, commodities, echelons w.r.t. number of decisions

Number of decisions	Period		Commodity		Echelon	
	Single	Multiple	Single	Multiple	Single	Multiple
1	20	8	16	11	14	14
2	5	2	2	5	3	4
3	5	2	2	5	1	6
4	2	2	1	3	0	4
Count	32	14	21	24	18	28
Percent (%)	70	30	46	52	39	61

multiple periods are given with respect to the number of decisions searched in the problem.

Similar to extending the number of periods, the number of commodities dealt in the problem can be defined as single or multiple commodities with different parameters such as demand, cycle times, material and others. The literature review shows that multi-commodity problems are usually defined with more than one decision type.

Additional to the number of period and commodity defined in the SCM problems, the number of layers and the assumption of interrelations among the layers of SC is an important dimension for SCM optimization problems. More than half of the reviewed SCM studies (61 %) deal with multi-echelon planning problems. As the number of decisions related to different layers of the SCM (i.e. inventory, production and distribution) increase, the problem has been defined as multi-echelon leading to SC layers interacting with each other.

The SCM under fuzziness studies have also been reviewed whether the reverse logistics planning is considered. 7 articles (15 %) of 46 have been defined as either closed-loop or reverse SC planning problems. These studies have been mostly specified as single-decision, single-period, single-commodity and multi-echelon problems.

2.2 SCM Uncertainties as Fuzziness

Besides the above mentioned dimensions and related extensions of SCM problems, uncertainty components of SC have been commonly included in the SCM optimization problems. In SCM, the firms are not considered as independent entities but interacting entities which need to coordinate and integrate their process along the SC. Thus, the uncertainties related to external and internal processes constitute a challenge for the coordinated and integrated processes of SCM.

In literature, the sources of uncertainty have been mainly classified into three groups: demand, process/manufacturing and supply (Peidro et al. 2009). For a firm's point of view, demand and supply uncertainties represent the external uncertainties, where variation in the suppliers' operations/outputs (e.g. quality of the supplied material, delivery times) are mentioned with supply uncertainties on one side and on the other side, the variation in the demand (e.g. volatility of sales amounts, unpredictable requests of new product specifications, etc.) are specified as the demand (or customer related) uncertainties. Finally, the third source of process/ manufacturing uncertainties is mainly related to the internal production processes such as unpredictable machine breakdowns, variability in cycle times, etc.

Uncertainties related to above mentioned sources are faced in almost all of the real life cases. The modeling efforts for optimizing SC commonly disregard the uncertainties in order to simplify complex real case problems and seek for solutions for deterministic cases. On the other side, efforts in the area of robust optimization, stochastic programming, fuzzy mathematical programming (possibilistic programming) and hybrid models include uncertain parameters in the model to end up with more realistic models and solutions (Melo et al. 2009). In SC optimization studies, we reviewed which parameters are most commonly considered to be uncertain and represented using fuzzy set theory. Table 5 presents the frequency analysis of the classification of fuzzy parameters used in SC optimization problems.

Each fuzzy parameter has been classified based on its relation to supply, process or demand operations of the SC. Supply and demand related parameters have been defined to be external to a firm and process related parameters internal parameters of a firm within the SC.

Fuzzy parameters are employed to deal with situations where data is lacking. Thus, quantitative parameters such as demand, costs, quantities, capacity have been frequently modeled as fuzzy parameters as observed in Table 4. Fuzzy set theory is also an effective way to deal with uncertainty existing due to doubt about the exactness of concepts, correctness of statements and judgments as well as qualitative data. As a result of the review, it may be concluded that this type of parameters which may be named as quality, compatibility, risk, failure, delay, objective weight has not been studied well.

We have also clustered the reviewed SC optimization studies using k-means clustering algorithm based on the types of fuzzy parameters. 20 different types of parameters (e.g. demand, costs, etc.) have been used as binary input variables in the cluster analysis and the most efficient cluster number has been found to be three in the analysis. In Fig. 2, the three clusters have been represented as three circles. In each cluster, the fuzzy parameters which have been observed frequently among the reviewed articles are shown. More frequent types of fuzzy parameters are highlighted by bold and capital letters. For example, the fuzzy parameter "demand" is written with bold and capital letters in the intersection of cluster 1 and 2. This shows that the reviewed articles in cluster 1 and 2 commonly and frequently use demand as a fuzzy parameter.

The decision types of the studies in each of the three clusters are investigated and it is observed that studies in cluster 1 are mainly related to demand/sales and supply/inventory types of problems. This analysis shows which types of fuzzy parameters are used in demand/sales and supply/inventory types of SC problems. Cluster 2 has been detected to be related to process related decisions such as

Table 5 Frequency analysis of the fuzzy parameters used in sc optimization problems

Parameter description	Supply-process-demand	External–internal	Count	Percent (%)
Demand	Demand	External	29	26
Transport cost, travel time	Process	External	13	12
Capacity of facility/labor	Process	Internal	11	10
Quantity to be transported, sold, stored, procured	Supply-demand	External	11	10
Inventory holding cost, shortage cost, penalty cost	Supply-demand	External	6	5
Lead time	Supply-process	Both	6	5
Fixed cost of facilities	Process	Internal	5	5
Production costs	Process	Internal	5	5
Quality, compatibility, service level	Process	Internal	5	5
Objective weights	–	–	4	4
Risk, failure, delay, rate of return	All	Both	3	3
Price	Demand	External	2	2
Raw material cost	Supply	External	2	2
Inventory cycle length	Process-demand	Both	2	2
Annual total cost	Supply-process	Both	2	2
Budget	Process	Both	1	1
Number of networks	All	Both	1	1
Profit	All	Both	1	1
Revenue	All	Both	1	1
Machine idle time, setup time	Process	Internal	1	1

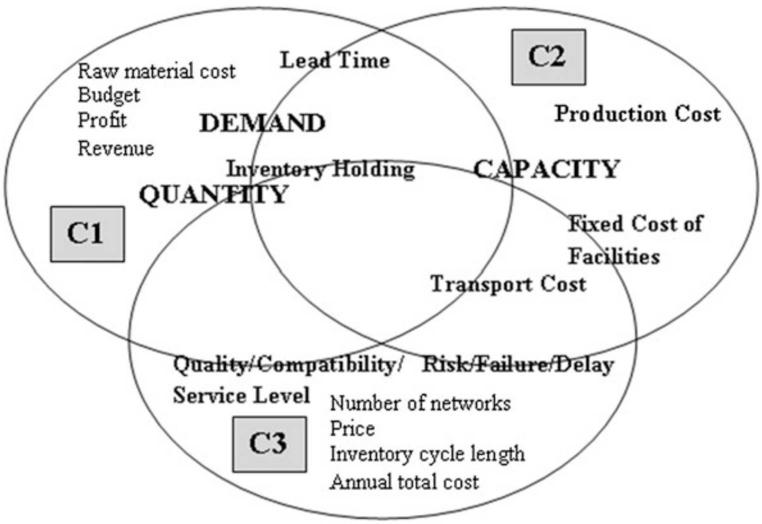

Fig. 2 Cluster analysis of observations and positions of fuzzy parameters in the clusters

production, location, allocation. The fuzzy parameters used in these types of problems are shown in circle "C2". The third cluster contains articles mainly related to product development and risk. The types of fuzzy parameters used in this group are shown in circle C3. The number of observations in each cluster respectively has been found to be 24, 6 and 10.

3 SCM Models and Solution Approaches Under Fuzziness

3.1 Fuzzy Measures and Fuzzy Sets

According to Ross et al. (2002) and Ross (2002), the main distinction between fuzzy sets and measures are the concept of ambiguity. In fuzzy sets, the boundaries are vague, e.g. let A be a crisp set composed of numbers from 1 to 10, then the boundaries fuzzy set of "small numbers in A" are not certainly determined. However, in fuzzy measures, the ambiguity lies in making the proper assignment given a number of evidences. For example, jury members decide on the innocence of a criminal by either both the existence and the lack of evidences. The sets of "innocent" or "guilty" are crisp, that is, their boundaries are known. However, the decision is ambiguous and depends on the evidence (Klir and Yuan 1995). Moreover, real world data is mostly incomplete and vague. Therefore, the credibility of information is to be measured and tested. Fuzzy measures, which are also called Sugeno measures, measure the degree of existence of an evidence for a given set of variables (Friedman and Halpern 1995). Fuzzy measures that are defined on a set W, fuzzy measures are functions that are defined $2^W \rightarrow [0, 1]$, which means each subset of W is mapped to a degree between 0 and 1 (Friedman and Halpern 1995), where 2^W is defined as $\wp(x)$. Fuzzy measures also hold the following properties

1. $h(\varnothing) = 0$ and $\mathrm{h(X)} = 1$
2. For every $A_1, A_2 \in \wp(X)$ *if* $A_1 \subseteq A_2$, then $h(A_1) \leq h(A_2)$.
3. For $A_i \in \wp(X)$, $i = \{1, 2, \ldots\}$ of subsets of X, if either $A_1 \subseteq A_2 \subseteq \ldots$or $A_1 \supseteq A_2 \supseteq \ldots$ holds then $\lim_{i\to\infty} h(A_i) = h(\lim_{i\to\infty} A_i)$

The first property is known as the boundary condition (Türkşen 2004), stating for the null set, no evidence holds, and for the universal set, there is complete evidence (Ross 2002). The second property states the monotonousness of the measures (Ross 2002) and implies that the evidence of a set should be as large as the evidence that are of any of the subset of the related set (Türkşen 2004). Lastly, the third property is known as continuity and it is based on the monotonousness property (Türkşen 2004).

3.1.1 Belief and Plausibility

Belief and plausibility are two main fuzzy measures. Belief measure is derived from “preconceived notions” and plausibility measure is derived from “information that is plausible” (Ross 2002). Belief measure states “the degree of belief” that an element X belongs to the set A. This measure is known to be an upper semi-continuous function (Wang and Klir 1992) and has the property that:

$$\mathrm{bel}(A_1 \cup A_2 \cup \ldots \cup A_n) \geq \sum\nolimits_i \mathrm{bel}(A_\mathrm{i}) - \sum\nolimits_{i<j} \mathrm{bel}\left(A_i \cap A_j\right) + \cdots \\ + (-1)^{n+1}\mathrm{bel}(A_i \cap A_2 \cap \ldots \cap A_n) \tag{1}$$

The dual of the belief measure is the plausibility measure and defined by (Ross 2002):

$$\mathrm{pl}(A) = 1 - \mathrm{bel}(\bar{A}) \tag{2}$$

Plausibility is a lower semi-continuous function (Wang and Klir 1992) and has the property that:

$$\mathrm{pl}(A_1 \cap A_2 \cap \ldots \cap A_n) \leq \sum\nolimits_i \mathrm{pl}(A_i) - \sum\nolimits_{i<j} \mathrm{pl}\left(A_i \cup A_j\right) + \cdots \\ + (-1)^{n+1}\mathrm{pl}(A_i \cup A_2 \cup \ldots \cup A_n) \tag{3}$$

Belief and plausibility measures are interpreted as a lower bound and an upper bound for probability measures (Wang and Klir 1992). Hence, if bel (A) = pl (A), then this value equals to the probability of set A. The belief and plausibility measures also hold the following properties:

1. $\mathrm{bel}(A \cap B) = \min(\mathrm{bel}(A), \mathrm{bel}(B))$
2. $\mathrm{pl}(A \cup B) = \max(\mathrm{pl}(A), \mathrm{pl}(B))$

3.1.2 Necessity and Possibility

If the sets $A_1, A_2, \ldots, A_n$ are nested or consonant, that is, if $A_1 \subset A_2 \subset \ldots \subset A_n$, the belief and possibility measures are called necessity and possibility measures (Ross 2002; Wang and Klir 1992) and hold the following properties:

$$\mathrm{Nec}\left(\bigcap\nolimits_{k \in K} A_k\right) = \inf \mathrm{Nec}(A_k) \tag{4}$$

$$\mathrm{Pos}\left(\bigcup\nolimits_{k \in K} A_k\right) = \sup \mathrm{Pos}(A_k) \tag{5}$$

3.1.3 Credibility

The necessity and possibility measures are duals of each other, however, they are not self dual, that is, the following properties do not hold.

$$\mathrm{Nec}(A) + \mathrm{Nec}(\bar{A}) = 1 \tag{6}$$

$$\mathrm{Pos}(A) + \mathrm{Pos}(\bar{A}) = 1 \tag{7}$$

Credibility differs from Sugeno fuzzy measures in a way that fuzzy measures are not self dual. Credibility is proposed by Liu and Liu as a self-dual possibilistic indicator (Georgescu and Kinnunen 2011). The credibility measure is calculated as (Li and Ralescu 2009):

$$\mathrm{Cr}(A) = \frac{1}{2}(\mathrm{Nec}(A) + \mathrm{Pos}(A)) \tag{8}$$

The main difference between necessity, possibility and credibility measures is that necessity measure assesses truth degree of a set whereas the possibility measure assesses the impossibility of the complementary set. On the other hand, credibility measure acts as a corrective measure between necessity and possibility (Zhao and Tang 2007). The credibility measure holds for axioms (Li and Ralescu 2009).

- Axiom 1. $\mathrm{Cr}(\varnothing) = 0$ and $\mathrm{Cr}(X) = 1$
- Axiom 2. For every $A_1, A_2 \in \wp(X)$ if $A_1 \subseteq A_2$, then $\mathrm{Cr}(A_1) \leq \mathrm{Cr}(A_2)$.
- Axiom 3. $\mathrm{Cr}(A) + \mathrm{Cr}(\bar{A}) = 1$
- Axiom 4. $\mathrm{Cr}\left(\bigcup_{k \in k} A_k\right) = \sup \mathrm{Cr}(A_k)$ for any event Ak where $\sup \mathrm{Cr}(A_k) \leq 0.5$

Moreover, the theorems below are proven for the credibility measure:

- Theorem 1a. $\mathrm{Cr}(A \cup B) = \mathrm{Cr}(A) \vee \mathrm{Cr}(B)$, if $\mathrm{Cr}(A \cup B) \leq 0.5$
- Theorem 1b. $\mathrm{Cr}(A \cap B) = \mathrm{Cr}(A) \wedge \mathrm{Cr}(B)$, if $\mathrm{Cr}(A \cap B) \geq 0.5$
- Theorem 2. $\mathrm{Cr}(A \cup B) \leq \mathrm{Cr}(A) + \mathrm{Cr}(B)$
- Theorem 3. If $\{B_i\}_{i \in N} \subseteq \in \wp(\mathrm{X})$ and $\lim\limits_{\mathrm{i} \to \infty} \mathrm{Cr}\{\mathrm{B_i}\} = 0$, then for any event A $\lim_{i \to \infty} \mathrm{Cr}(A \cup B_i) = \lim_{i \to \infty} \mathrm{Cr}(A \backslash B_i) = \mathrm{Cr}(A)$
- Theorem 4. $\lim\limits_{i \to \infty} \mathrm{Cr}(A_i) = \mathrm{Cr}\left(\lim\limits_{i \to \infty} A_i\right)$

The second theorem is known as the credibility sub-additivity theorem and the fourth theorem is known as the credibility semi-continuity law.

3.2 *Fuzzy Mathematical Programming*

A linear mathematical program has the form:

$$\min(x) \tag{9}$$

subject to

$$g_i(x) \leq b_i \quad i = 1, \ldots, m \tag{10}$$

$$x \in X = \{x \in \mathbb{R}^n | x \geq 0\} \tag{11}$$

The constraints can be made flexible with the introduction of soft constraints to allow a level of the violation in the satisfaction of the objective or constraints. These violations may be represented using fuzzy sets of the membership functions of which are $\mu_{\mathrm{i}};\ \ i = 0,\ 1, \ldots, m$ defined as follows (Luhandjula 2007):

$$\mu_i(x) = 0\, if\ g_i(x) > b_i + d_i \tag{12}$$

$$\mu_i(x) \in (0,\ 1) \quad if\ b_i < g_i(x) \leq b_i + d_i \tag{13}$$

$$\mu_i(x) = 1 \quad if\ g_i(x) \leq b_i \tag{14}$$

where $d_{\mathrm{i}}(i = 0,\ 1, \ldots, m)$ are subjectively chosen constants for admissible violation. Then, a membership function for $\mu_i(x)$ can be defined with a piecewise function:

$$\mu_i(x) = \begin{cases} 1; & if g_i(x) \leq b_i \\ 1 - \frac{g_i(x) - b_i}{d_i}; & if\ if\ b_i < g_i(x) \leq b_i + d_i \\ 0; & g_i(x) > b_i + d_i \end{cases} \tag{15}$$

The solution under fuzziness may or may not fulfill the goal and the constraints. The optimality seeks for $x^* \in X$ with the highest membership degree in the fuzzy set intersection of fuzzy sets representing the objective function and the constraints. This problem is defined as (Luhandjula 2007):

$$\max \min \mu_i(x) \tag{16}$$

subject to

$$x \in \bigcap\nolimits_{i=0}^{m} Supp\ \mu_i \tag{17}$$

which has the below mathematical program form:

$$\max \lambda \tag{18}$$

$$\lambda \leq 1 - \frac{g_i(x) - b_i}{d_i}; \quad i = 1, \ldots, m \tag{19}$$

$$x \geq 0 \tag{20}$$

This linear programming model may be solved by many exact general solvers.

Other than introducing a violation level of the satisfaction of the constraints, fuzzy parameters may be directly included in the mathematical program. One of the most common parameters referred as uncertain in the SC literature is the demand and commonly introduced as fuzzy parameters in the SCM fuzzy optimization literature. In this case, the mathematical program has the form:

$$\min f(x, \tilde{a}) \tag{21}$$

subject to

$$g_i\left(x, \tilde{b}_i\right) \le \tilde{c}_i \quad i = 1, \ldots, m \tag{22}$$

$$x \in \mathbb{R}^n \tag{23}$$

where $\tilde{a}$ is a fuzzy vector and $\tilde{b}_i$ and $\tilde{c}_i$ (i = 0, 1,…, m) are fuzzy numbers. A common way to solve this model is to defuzzy the fuzzy parameters and obtain a deterministic version of the model. Most commonly used defuzzification methods are α-level sets, center of area and weighted average. This approach does not guarantee a minimum certainty level. Thus, it is only appropriate when the fuzzy quantities are not too large (Luhandjula 2007).

Another approach which ensures minimum uncertainty principle is uncertainty-constrained approach which is similar to stochastic and chance constrained programming explained in Sect. 4.2. The mathematical program of uncertainty-constrained approach has the form together with the constraint given in Eq. (23):

$$\max_{\mathrm{x}} \max \lambda \tag{24}$$

subject to

$$F_M\{f(x, \tilde{a}) \ge \lambda\} \ge \alpha \tag{25}$$

$$F_M\left\{g_i\left(x, \tilde{b}_i\right) \le \tilde{c}_i; i = 1, \ldots, m\right\} \ge \beta \tag{26}$$

where α and β are thresholds and F_M are some uncertainty measure like possibility, necessity or credibility. The solution approaches used for the uncertainty-constrained approach is commonly metaheuristics like genetic algorithms, particle swarm optimization.

3.3 Stochastic and Chance Constrained Programming

A stochastic programming model has the form

$$\min f(x) = c^T x = \sum\nolimits_{j=1}^{n} c_j x_j \tag{27}$$

subject to

$$A_i^T X = \sum\nolimits_{j=1}^{n} a_{ij} x_j \le b_i \quad i = 1, \ldots, m \tag{28}$$

$$x_j \ge 0 \; j = 1, \ldots, n \tag{29}$$

where c_j, a_{ij} and b_i are random variables (Rao 2009). Since the technology constraint coefficients, right hand side values and objective function coefficients are random variables, deterministic techniques such as the Simplex Method, cannot be applied for solving such models. Chance Constrained Programming is one of the special forms for solving stochastic models. Chance Constrained Programming is introduced by Charnes and Cooper (1959) and formed by rewriting the stochastic linear model as given below, in addition to the constraint given in Eq. (29):

$$\min f(x) = c^T x = \sum_{j=1}^{n} c_j x_j \tag{30}$$

subject to

$$P\left[\sum_{j=1}^{n} a_{ij}x_j \leq b_i\right] \geq p_i \; i = 1, \ldots, m \tag{31}$$

where p_i are specified probabilities.

For a chance constrained model, if technology constraint coefficients and right hand side values are composed of fuzzy numbers, and the objective function value is determined by possibilities, then the model becomes a fuzzy chance constrained programming and the model becomes (Gong et al. 2009) as given below (Eq. 29 should also be added as a constraint)

$$\min \overline{f(x)} \tag{32}$$

subject to

$$Pos\left\{f(x) \leq \bar{f}\right\} \geq \alpha \tag{33}$$

$$\sum_{j=1}^{n} \tilde{a}_{ij}x_j \leq \tilde{b}_i \tag{34}$$

which signifies that the objective function should be satisfied for a given confidence interval. Maximizing credibility can also be an objective function for fuzzy chance constrained programming (Huang 2006). Solving the chance constrained programming models, metaheuristics, especially genetic algorithms, are commonly used in the studies.

3.4 Fuzzy Components of SCM Models

Place of fuzziness in an SCM optimization model can be at any combination of any parameter: objective value, objective coefficients, right-hand side and constraint coefficients, or even decision variables. However, especially in stochastic and goal programming models, the target service level remains uncertain and the service level is indicated with a dummy fuzzy variable, whose membership value signifies the service level, that is, a higher membership value indicates a higher service

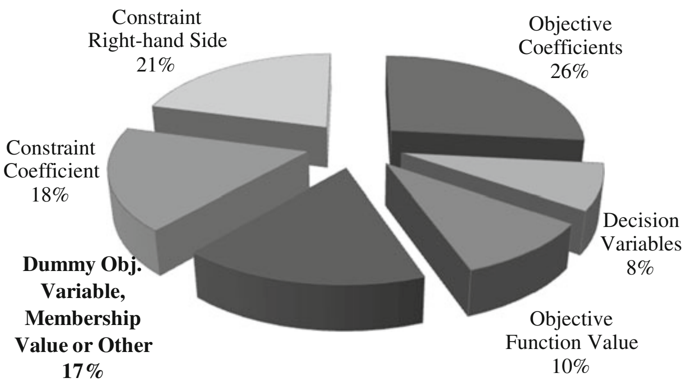

Fig. 3 Position of fuzziness in studies

level which is not present in the main problem or model. The percentages of the different types of fuzzy components of the models are given in Fig. 3.

Analyzing the relation among the models and the methods with respect to fuzzy components of the mathematical model (e.g. objective function coefficients, etc.…) and the fuzzy parameters of the problem (e.g. demand, unit transportation cost, etc.…), it can be seen that demand is the most fuzzified aspect of SC optimization, leading numerous combination of methods and models depending on the SC environment and problem. Moreover, if the fixed opening and operating costs such as operating a distribution center or opening a warehouse are fuzzy, then they are unsurprisingly represented as part of the coefficients of the objective function that aims to minimize total cost (Ghatee and Hashemi 2009). However, in further research when service level is a dummy fuzzy variable and the main variable of the objective function, the total cost objective is converted into a constraint with a target level which makes fixed cost as a constraint coefficient parameter in mathematical models involving credibility (Qin and Ji 2010). Likely, fuzzy unit shortage and holding costs are part of the objective function when the objective function aims to minimize costs. However, fuzzy unit production cost is also observed to be a constraint coefficient, since the models involving unit production cost are credibility based and they are added as a constraint to the models. Mathematical models involving fuzzy lead time and optimum quantity to be transported/stocked/procured/sold/stock also involve fuzzy demand and they are included in the objective function. Fuzzy lead time appears in the objective function as a cost of tardiness and also appears as a constraint coefficient as to provide strict due dates. The most complicated fuzzy assets in terms of interactions are unit transportation cost/travel time and capacities of facility and/or labor. In literature, when those two parameters are both included in the mathematical model, the models also involve demand, unit shortage and/or holding cost, fixed costs and unit production cost as fuzzy numbers and the mathematical model becomes fully fuzzified where objective function coefficients, constraint coefficients and right-hand sides are all fuzzy. Moreover, with the insertion of service

Table 6 Frequencies of model types

Model type	Frequency	Percentage
Mixed Integer Linear Programming (MILP)	11	22
Linear Programming (LP)	7	14
Mixed Integer Programming (MIP)	7	14
Nonlinear Programming (NP)	7	14
Multi-Objective Linear Programming(MOLP)	5	10
Mixed Integer Nonlinear Programming (MINLP)	4	8
Combinatorial Optimization Models (CO)	4	8
Fuzzy Mathematical Programming (FMP)	4	8
Chance Constraint Programming (CCP)	4	8
Stochastic Programming (SP)	3	6
Grey Programming (GP)	3	6
Fuzzy Grey Programming (FGP)	1	2

level to the model signifies a dummy membership indicating fuzzy variable in the objective function, thus, making the objective function value a fuzzy variable as well. In cases when unit transportation cost/travel time is fuzzy but capacities of facility and/or labor is not included in the model, the transportation cost is included in the objective function as a part of cost minimization. Likely, when capacities of facility and/or labor is fuzzy but unit transportation cost/travel time is not included in the model, the capacity serves as a constraint and the constraint right-hand side become fuzzy.

4 Models and Methodology in SCM Optimization Problems

4.1 Models in SCM Optimization

In literature, the optimization of SCM problems is dealt with mathematical models. The most exploited model is detected as Mixed Integer Linear Programming. The frequencies of models are given in Table 6. Analyzing models according to the years do not propose deterioration, that is, former and recent studies do not differ in terms of models. One exception is that fuzzy grey programming has become a popular method in the recent years (Table 7).

Analyzing decision types with respect to the models built produces several correlations. Being the most exploited model, MILP models are only built for allocation/distribution problems whereas, all routing problems are modeled with NLP. On the other hand, when allocation/distribution problems are accompanied by location problems such as facility, warehouse, distribution center location problems, these problems are dealt with using either MIP or NLP models. As for the commodity-period-echelon combinations, it is observed that common modeling

Table 7 Model types with respect to years

Model type	2004	2006	2007	2008	2009	2010	2011	2012	2013	Total
MILP	0	1	0	0	1	1	0	1	1	5
LP	0	0	1	0	2	0	0	4	0	7
MIP	0	0	0	1	2	2	0	2	0	7
NLP	0	1	0	2	2	1	0	1	0	7
MOLP	0	0	2	0	2	2	2	2	1	11
MINLP	1	0	0	1	1	0	1	0	0	4
CO	0	0	1	1	1	1	0	0	0	4
FMP	0	0	0	2	1	0	0	0	0	3
CCP	0	0	0	1	2	0	1	0	0	4
SP	0	0	0	1	2	1	0	0	0	4
GP	0	0	0	1	0	0	0	0	0	1
FGP	0	1	0	1	0	0	0	1	0	3

applications include MIP for single commodity—single period—multi echelon problems, and MILP for multi commodity—multi period—multi echelon problems. In LP and MIP models, the fuzziness is mostly generated in the objective function coefficients; and in MINLP models a dummy fuzzy variable is generated that measures a satisfaction level as an objective function value.

In cases that the problem involves fuzzy unit transportation costs and fuzzy unit productions costs, the problems are dealt mostly with MILP models. However, LP and NLP models are observed as minorities.

4.2 Methodology in SCM Optimization

The methodologies dealing with fuzzy SCM optimization problems can be classified under four groups. The first methodology group involves methods that lead to exact optimized results and that are not problem specific but generalizable. By being problem specific, it is implied that the solutions method is proposed for solving merely a specific type of problem, such as the Cheapest Insertion Heuristic solving Traveling Salesman Problems. On the contrary, general solution methods can be applied to any or at least a variety of problem types such as branch and bound being applied to all Integer Programming problems regardless of the context. In this class of methods, authors tend to construct a mathematical model for optimization and solve it with a general solver.

The second class offers problem specific approaches that provide exact results. This class involve author proposed, iterative solution approaches with involvement of exact general methods in either stage of recursions. For example, Chandran and Kandaswamy (2012) offer and iterative approach for solving a general multi-item problem that minimizes transportation costs between specific sellers and buyers. In other cases, a large problem can be divided into smaller problems which are easier to be dealt with. Their approach shortens the execution time of the algorithm

Table 8 Methods with respect to years

Model	2004	2006	2007	2008	2009	2010	2011	2012	2013	Total
General exact solver	1	2	2	3	4	3	2	4	2	23
Problem specific exact solver	0	0	0	1	0	0	0	4	0	5
Problem based heuristics	0	2	0	0	3	1	1	1	0	8
Metaheuristics	0	0	2	3	5	2	0	2	0	14

by checking for non-degeneracy of solutions. Petrovic et al. (2008) divide their transportation problem by echelons of retail and warehouse, and optimize the distribution cost minimization problem as a whole after adjusting each echelon. Yet, these methods are not classified under heuristic solutions since they guarantee the exact optimum solution.

The third class of methods involves problem based heuristic solutions which do not guarantee an exact solution, yet provide a satisfactory one. Solving continuous and stochastic inventory problems such as (Q, r) problems require such approaches; since, solving large problems is computationally either expensive or impossible to solve them using exact methods. These approaches either are iterative and single, or involve hybridizations of heuristics with other heuristics or exact methods.

The last class of methodologies involves metaheuristics. The most exploited metaheuritic is Genetic Algorithms (Iris and Serdar-Asan 2012) followed by Particle Swarm Optimization. Yanxue et al. (Gong et al. 2009) use an Adaptive Genetic Algorithm for minimizing costs in a closed-loop supply chain. The adaptation is achieved by recalculation of crossover and mutation parameters at each iteration in order to avoid premature convergence and failing into a local optimum. For solving SCM problems, metaheuristics are both fuzzified and redesigned in order to fit the problem. Aliev et al. (2007) use fuzzified demand forecasts as an input at an aggregative production–distribution problem solved with Genetic Algorithms.

The distribution of methodology selections over the years is shown in Table 8. Over the years, the most exploited method is general and exact solving methods, followed by metaheuristics, whereas problem specific exact methods appear as the least exploited. A yearly analysis do not yield any distinction between method classes, yet, there is a sudden intensification in problem specific and exact solving methods in year 2012 and the exploitation of metaheuristics is continued in recent years.

Analyzing methods versus decisions, it is observed that problem specific exact methods are mostly utilized for single item, single period and single echelon, forward networks with fuzzy demand; whereas general exact methods are applied mostly to multiple item and multiple echelon models. In capacity decisions, heuristics have not been exploited at all; the methods exploited are exact methods. In supplier selection, 80 % of the studies involve exact methods, only Particle Swarm Optimization and Genetic Algorithms are utilized for once. In facility location problems, any problem specific exact methods have not been developed yet.

4.3 Relations of Problem Classes and Models and Methods

The observations over the relations with models and methods show that problems that involve decisions of both allocation/distribution and sales amount determination with uncertain demand in closed loop networks are modeled with Mixed Integer Linear Programming and solved using general exact solvers. Allocation/distribution decisions together with supplier selection or production decisions, which have multi products and multi echelons in forward loop networks are also dealt with Mixed Integer Linear Programming and solved using general exact solvers. Decisions that involve both sales and production amount determination in multi item and multi echelon forward networks are dealt with general algorithms; problems specific approaches (either exact solution or heuristic) are not applied. Sales amount distribution decisions in single period and multi echelon systems are optimized using metaheuristics, namely Genetic Algorithms. General exact solvers are also widely exploited in multi product, single period, multi-echelon forward networks with conflicting multiple objectives.

5 Application of SCM Optimization Under Fuzziness and Validation of Methods

In literature, different optimization schemes are applied to different industries; yet, a vast majority of studies do not specify the industry of their applications (See Fig. 4). As for the studies which have identified their industries, the most common ones are automotive and food & beverage.

SCM optimization studies in automotive industry are achieved on a single period, multiple echelon and forward networks, all aiming to minimize costs. However, one study also tries to minimize the environmental impacts of production. The problems are modeled as either MILP or MIP. The most important character of these studies are that they try to optimize the flow from procurement and supplier selection to allocation to customers. The studies are also inventory-focused. The common fuzzy parameters in this industry include demand and capacity of facility or labor. All models are modeled with triangular fuzzy numbers and for defuzzification, most of them use α-cut method. The results are validated by illustrative examples, real world cases and sensitivity calculations.

For food and beverage industry, all studies focus on a single period, forward network, where the main decisions include distribution of goods and locations of distribution centers. Both single and multi objective methods are built where the main objective is minimizing total costs. However, since food and beverage are perishable goods minimizing distribution time and customer satisfaction maximization are also considered as main objectives. Food and beverage applications involve numerous fuzzy parameters other than demand. One of the most common fuzzy parameters are the lead time, travel time, quantities to be transported and

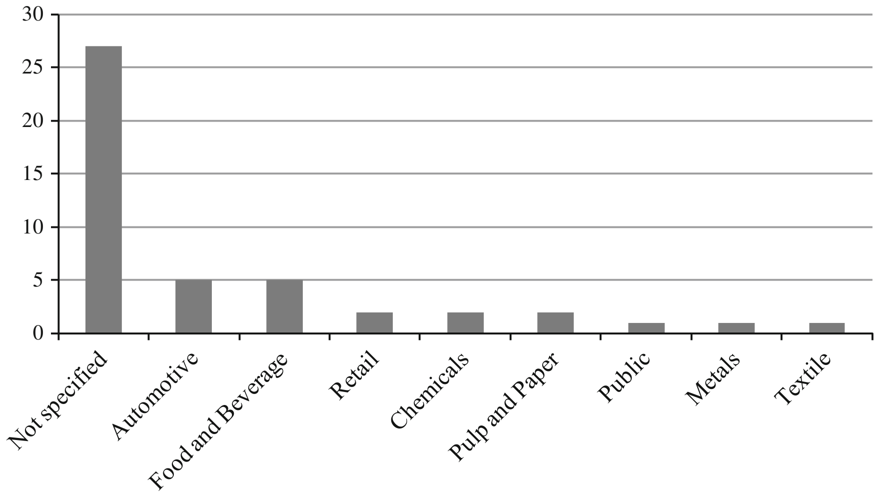

Fig. 4 The frequencies of industries among studies

customer service level due to the perishability of goods. Budget and capacity of facilities are other fuzzy attributes analyzed in such studies. The main validity of proposed models and methods is achieved through comparisons with other techniques, since new combinatorial and multi objective models are developed and problem specific methods and metaheuristics are used.

Although the number of studies that analyze chemicals and, pulp and paper industry, there are many similarities in these papers. Both chemical industry related studies involve multi item and multi echelon forward networks involving a fuzzy demand that are modeled with MILP. As for the pulp and paper industry, the main decision is facility layout in a multi item, single period environment of a forward network. Profit maximization, cost and environmental impact minimization are the main objectives, where inventory cycle length is the main fuzzy parameter.

A common way to validate the model and the proposed solution methodology is to solve an illustrative example or a benchmark problem -if any exists. Then, a sensitivity analysis follows to show the effect of various parameters on the solution. In the sensitivity analysis, a common parameter to change is the relative importance of the goals if the problem is a multi-objective problem. The effect of the level of the fuzziness of the problem is also analyzed by comparing the solution for various uncertainty levels. In some studies, types of fuzzy membership functions and their interval width have been compared to obtain more realistic solutions. Other types of sensitivity analyses in the studies compare the effect of parameters related to the SC such as demand, capacity and cost.

In some studies, the solutions obtained from other solution methods have been compared to validate the proposed methodology. As well, real case studies have been presented for the validation of the models and solutions techniques.

6 Conclusions

The literature review of the studies of optimization problems in SC under fuzziness has revealed some areas which are not studied well in the literature. The review shows that uncertainties related to the demand and sales have been extensively studied. However, process and product related uncertainties of the SC are not yet investigated in the fuzzy optimization literature. The fuzziness of process related to the costs, quality, risk, delay may be included and examined further in the SC problems. Moreover, another dimension of the problem types, the multi-period SC problems have not also been studied well in the fuzzy optimization literature compared to single-period problems. The fuzzy mathematical programming and chance-constrained programming models are commonly used in the literature. On the other side, the fuzzy grey programming models are newly explored in the recent years. It can be concluded that the model and the solution methodology are directly related to the type of the SC problem and the network structure of the network. The fuzzy component or types of fuzziness do not directly affect the solution methodology. The review shows that studies including fuzzy decision variables are relatively less than other fuzzy components such as right-hand side or coefficients of the fuzzy mathematical program models in the collection of works studied.

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A Multiple Means Transportation Model with Type-2 Fuzzy Uncertainty

Juan Carlos Figueroa-García and Germán Hernández

Abstract Uncertainty in transportation problems is commonly handled by probabilistic tools. When statistical information is not available, the role of the experts takes an important role in decision making, so the use of fuzzy sets allows us to apply their knowledge to optimization models. To do so, we propose an optimization strategy using Interval Type-2 fuzzy sets, which is applied to a transportation model.

Keywords Type-2 fuzzy sets · Fuzzy optimization · Fuzzy transportation model

1 Introduction and Motivation

Supply chain optimization is a challenge for many decision makers who manage different organizations. Many decision making problems contain uncertainty, so its solution requires more complex methods and models. To do so, the analyst should keep in mind that uncertainty affects their decisions and the way to solve problem, so uncertainty needs to be treated using specialized methods and models.

Transportation problems should be managed in order to integrate supply chain decisions with multiple echelons. One of its main problems includes multiple transportation means, where any decision affects the behavior of the echelons of the supply chain. Sometimes fuzzy uncertainty appears in the supplies and demands of the chain, defined by multiple experts, each one defining a fuzzy set using their own criterion, so there is no agree about which opinion and/or perception is adequate.

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This chapter is intended to present a model for a transportation problem with multiple transportation means and fuzzy uncertainty in their supplies and demands using Interval Type-2 Fuzzy Sets (IT2FS) and mathematical programming methods.

In this way, the problem is modeled and solved using both a single-expert and a multiple-experts fuzzy model. This leads us to use IT2FSs to deal with their opinions and perceptions. Some basics about Type-2 fuzzy sets are shown, and a method for finding optimal solutions is applied.

1.1 Fuzzy Transportation Approaches

Different approaches to transportation problems are based on the idea of having different fuzzy parameters (coefficients, costs or constraints). Tada (1996), Basirzadeh (2011), Gani and Razak (2004), Pandian and Natarajan (2010), Chanas and Kuchta (1996), Chanas et al. (1984), Buckley and Jowers (1992), Gani et al. (2011) and Liu and Kao (2006) proposed fuzzy models for transportation problems using classical fuzzy sets, and based on the idea of a single membership function per set.

The use of IT2FSs in optimization is not wide, and its application to transportation problems is a challenge for many decision makers. Figueroa and Hernández (2012) have provided the first report about the use of IT2FSs in transportation problems, so its results can be extended to related problems.

2 Non-Fuzzy (Crisp) Multiple Means Transportation Model

Transportation decisions regard to the problem of sending products from suppliers to customers using different means e.g. trucks, train, trailers, airplane, etc. This situation can be summarized as the problem of sending a specific quantity x of product from the i_{th} supplier to the j_{th} customer using an amount v of the k_{th} vehicle. A network representation of the problem is shown in Fig. 1.

This is a combinatorial problem where decision making should be focused in how to satisfy the demands of the customers and what kind of transportation means should be used to carry out them, at an optimum cost. As usual, the amount of and vehicles should be an integer variable, this is $x \in \mathbb{R}, v \in \mathbb{Z}$, so the problem is defined as a combinatorial one.

Then, the decision maker to solve two main goals: a first one regards to the quantities to be sent and a second one regarding the transportation means used for. At a first glance, there are two separated goals, but they are intimately related to each other, so any strategy used to solve the problem should include both goals at the same time. Now, a mathematical definition of the problem is:

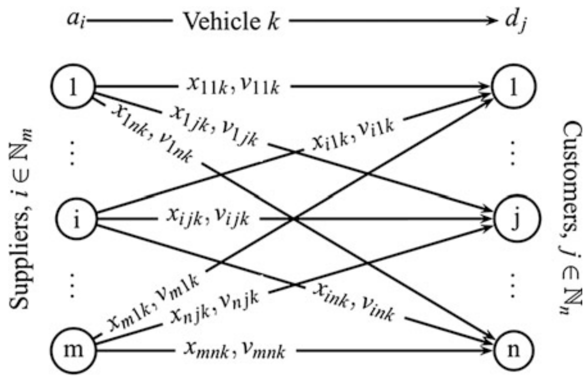

Fig. 1 Transportation network

$$\min_{(i,j)} z = \sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} c_{ijk}x_{ijk} + s_k v_{ijk} \tag{1}$$

s.t

$$\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} x_{ijk} \le a_i \quad \forall\, i \in \mathbb{N}_m \tag{2}$$

$$\sum_{i\in\mathbb{N}_m}\sum_{k\in\mathbb{N}_K} x_{ijk} \ge d_j \quad \forall\, j \in \mathbb{N}_n \tag{3}$$

$$x_{ijk} \le p_k v_{ijk}\; \forall\, i \in \mathbb{N}_m,\; j \in \mathbb{N}_n,\; k \in \mathbb{N}_K \tag{4}$$

$$\sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n} v_{ijk} \le q_k \quad \forall\, k \in \mathbb{N}_K \tag{5}$$

where $x_{ijk}, c_{ij}, s_k, d_j, a_i, p_k, q_k \in \mathbb{R},\; v_{ijk} \in \mathbb{Z}^+$.

Index Sets:

$\mathbb{N}_m$ is the set of all i suppliers, $i \in \mathbb{N}_m$
$\mathbb{N}_n$ is the set of all j customers, $j \in \mathbb{N}_n$
$\mathbb{N}_K$ is the set of all k vehicle types, $k \in \mathbb{N}_K$

Decision Variables:

x_{ijk} Quantity of product to be sent from the i_{th} supplier to the j_{th} customer using the vehicle type k
v_{ijk} Quantity of vehicles type k used to carry products from the i_{th} supplier to the j_{th} customer

Parameters:

c_{ij} Unitary cost of carrying a product from the i_{th} supplier to the j_{th} customer
a_i Availability of product of the i_{th} supplier
d_j Demand of product of the j_{th} customer

s_k Unitary cost of using a vehicle type k
p_k Capacity (in units of product) of the k_{th} vehicle
q_k Availability of the k_{th} vehicle

In this model, $\leqslant$ and $\geqslant$ are crisp partial orders, and all its parameters are defined as constants, so we refer this model as the *Crisp multiple means transportation model*.

Constraints (2) and (3) are equilibrium constraints of the transportation model where $\leqslant a_i$ is the availability limit per supplier, $\geqslant d_j$ is the minimum demand to be supplied per customer, constraint (4) refers to the availability $(\leqslant)$ of the vehicles that can be used to carry out products in the $i \rightarrow j$ route, and finally constraint (5) defines the maximum amount $(\leqslant)$ of vehicles of type k that can be used for transportation.

Usually, this kind of models can be solved using optimization techniques including branch and bound, branch and cut and integer programming algorithms. This kind of algorithms works only with crisp parameters, so its scope precludes the use of other kind of parameters as probabilistic, possibilistic, fuzzy, etc.

3 Fuzzy Uncertainty and Fuzzy Sets

In many applications, the supplies and demands of the system are not crisp parameters i.e. they are not constants. When those parameters are defined as random variables, the models proposed by Kall and Mayer (2010) and Mora (2001) are useful tools to find a solution of the model.

When statistical information is not available, then a way to model this kind of problems is by using information coming from experts of the system. A way to measure this information is through fuzzy sets, which are mathematical representations of uncertainty and natural language. Fuzzy sets refer to the knowledge of an expert about a variable regarding a concept (or a word). In next sections, we provide some basic concepts about fuzzy sets and its use in optimization problems.

3.1 Basics on Fuzzy Sets

A fuzzy set is a generalization of a *crisp* or Boolean set. It is defined on an universe of discourse X and is characterized by a *Membership Function* namely $\mu_A(x)$ that takes values in the interval [0, 1]. A fuzzy set A may be represented as a set of ordered pairs of a generic element x and its grade of membership function, $\mu_A(x)$, i.e.,

$$A = (x, \mu_A(x)) \mid x \in X \tag{6}$$

In the fuzzy approach, x can be defined on multiple fuzzy sets $\{A_1, A_2, \cdots, A_m\}$, each one is defined by a membership function $\left\{\mu_{A_1}(x), \mu_{A_2}(x), \cdots, \mu_{A_m}(x)\right\}$ and $\mu_A(x)$ is a measure of belongings of x regarding any fuzzy set F.

Here, A is a *Linguistic Label* which defines the sense of the fuzzy set through the word A. This word defines how an expert perceives the variable X and the shape of each set, in a single representation (membership function) per label.

3.2 Multiple Experts and Interval Type-2 Fuzzy Sets

Type-1 fuzzy sets refer to the perception of a single expert, since its knowledge can be represented by a single membership function. When multiple experts provide their own perception about the same variable, then the problem becomes more complex. To do so, we propose the use of IT2FSs to involve those opinions.

IT2FSs are useful measures to represent the knowledge of multiple experts (or alternatively, ambiguity about the definition of a fuzzy set). These kind of fuzzy sets involve an infinite amount of Type-1 fuzzy sets into a single set, which is a representation of linguistic uncertainty itself.

In Fig. 2, the opinion of the n_{th} expert about A can be represented through μ_A^n, and the opinion of all experts can be summarized by $\mu_{\tilde{A}}$. This measure represents the differences among all perceptions about A, so those differences can be seen as a linguistic uncertainty source, leading to define $\tilde{A}$ and the use of IT2FS.

IT2FS allows to model linguistic uncertainty, Mendel (2001), Mendel and John (2002), Mendel et al. (2006) and Melgarejo (2007a, b) provided formal definitions of IT2FS, and Figueroa (2008, 2013, 2012, 2012) used IT2FSs in LP problems. Some basic definitions are shown next.

3.3 Basics on Interval Type-2 Fuzzy Sets

A Type-2 fuzzy set is a collection of infinite Type-1 fuzzy sets into a single fuzzy set. It is defined by two membership functions: a one defining the membership of the universe of discourse Ω and a second one weighting each Type-1 fuzzy set. Karnik and Mendel (2001), Karnik et al. (1999), Liang and Mendel (2000), Melgarejo (2007a, b), Mendel (2001, 2003a, b), Mendel and John (2002), Mendel et al. (2006) and Mendel and Liu (2007) provided the following basic definitions of Type-2 fuzzy sets:

Definition 3.1 (*Interval Type-2 Fuzzy Set*) An Interval Type-2 fuzzy set $\tilde{A}$, is:

$$\tilde{A} = \left(x, \mu_{\tilde{A}}(x)\right) \mid x \in X \tag{7}$$

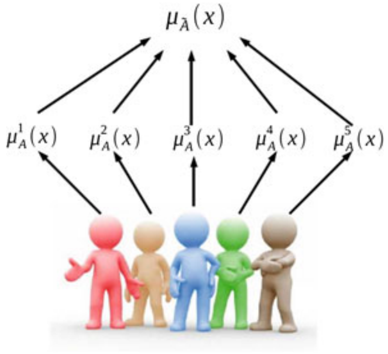

Fig. 2 Multiple experts' opinion

where $\mu_{\tilde{A}}(x)$ is a Type-2 membership function which is composed by an infinite amount of Type-1 fuzzy sets, in two ways: primary fuzzy sets $\mathrm{J_x}$ weighted by secondary fuzzy sets $\mathrm{f_x(u)} = 1$. In other words

$$\tilde{A} = ((x, u), J_x, 1) \mid x \in X; u \in [0, 1] \tag{8}$$

and its extended representation is:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x, u) = \int_{x \in X} \left[\int_{u \in J_x} 1/u \right] \Big/ x,$$

A graphical representation of an IT2FS is shown in Fig. 3.

Here, $\tilde{A}$ is an Interval Type-2 fuzzy set defined over an universe of discourse $a \in \Omega$, its support supp $(\tilde{a})$ is enclosed into the interval $a \in \left[\bar{a}^{\nabla}, \bar{a}^{\Delta}\right]$. $\mu_{\tilde{a}}$ is a linear Type-2 fuzzy set with parameters $\bar{a}^{\nabla}, \bar{a}^{\Delta}, \underline{a}^{\nabla}, \underline{a}^{\Delta}$ and $\bar{a}$. *FOU* is the *Footprint of Uncertainty* of the Type-2 fuzzy set and A_e is a Type-1 fuzzy set embedded in the FOU.

Uncertainty about A is conveyed by the union of all of the primary memberships into the FOU of $\tilde{A}$, $\mathrm{FOU}(\tilde{A})$, i.e.

$$\mathrm{FOU}(\tilde{A}) = \bigcup_{x \in X} J_x \tag{9}$$

Therefore, the FOU involves all the embedded J_x characterized by a secondary membership function $f_x(u)/u$, in this case $1/u$. An FOU is bounded by two membership functions: an *Upper* membership function (UMF) $\bar{\mu}_{\tilde{A}}$ and a *Lower* membership function (LMF) $\underline{\mu}_{\tilde{A}}$ having e embedded sets (A_e).

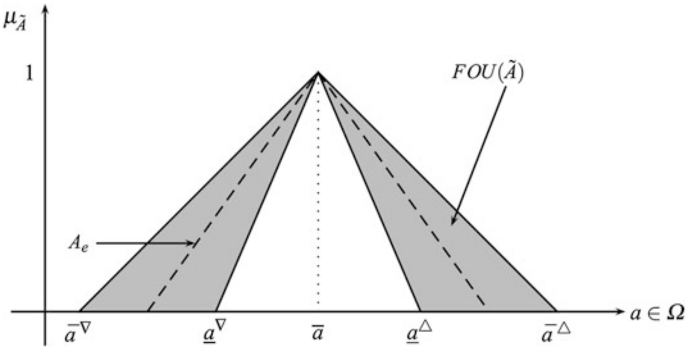

Fig. 3 Interval type-2 fuzzy set $\tilde{A}$

3.4 Uncertain Constraints

Now we have defined what a Type-2 fuzzy set is, then an uncertain constraint should be defined as an IT2FS in order to be used in LP problems, as shown next.

Definition 3.2 (*IT2FS Constraint*—Figueroa 2013) Consider a set of constraints of an LP problem defined as an IT2FS called $\tilde{b}$ defined on the closed interval $\tilde{b}_i \in [\underline{b}_i, \bar{b}_i]$, $\{\underline{b}_i, \bar{b}_i\} \in \mathbb{R}$ and $i \in \mathbb{N}_m$. The membership function which represents $\tilde{b}_i$ is:

$$\tilde{b}_i = \int_{b_i \in \mathbb{R}} \left[\int_{u \in J_{b_i}} 1/u \right] \Big/ b_i, \quad i \in \mathbb{N}_m, J_{b_i} \subseteq [0, 1] \tag{10}$$

Note that $\tilde{b}$ is bounded by both LMF and UMF, namely $\underline{\mu}_{\tilde{b}}(x)$ with parameters $\underline{b}^{\nabla}$ and $\underline{b}^{\Delta}$, and $\bar{\mu}_{\tilde{b}}(x)$ with parameters $\bar{b}^{\nabla}$ and $\bar{b}^{\Delta}$. Now, its FOU is bounded by two distances called Δ and ∇, defined as follows.

Definition 3.3 Consider an Interval FLP problem (IFLP) with constraints in the form $\geqslant$. Then ∇ is defined as the distance between $\underline{b}^{\nabla}$ and $\bar{b}^{\nabla}$, $\nabla = \bar{b}^{\nabla} - \underline{b}^{\nabla}$ and Δ is defined as the distance between $\underline{b}^{\Delta}$ and $\bar{b}^{\Delta}$, $\Delta = \bar{b}^{\Delta} - \underline{b}^{\Delta}$.

A graphical representation of $\tilde{b}_i$ is shown in Fig. 4.

In Fig. 4, $\tilde{b}$ is an IT2FS with linear membership functions $\underline{\mu}_{\tilde{b}}(x)$ and $\bar{\mu}_{\tilde{b}}(x)$. A particular value b projects an interval of membership degrees $u \in J_b$, as follows

$$J_b \in [{}^{\alpha}\bar{b}, {}^{\alpha}\underline{b}] \forall b \in \mathbb{R} \tag{11}$$

where J_b is the set of all possible membership degrees associated to $b \in \mathbb{R}$. Now, the FOU of $\tilde{b}$ can be composed by the union of all values of u, i.e.

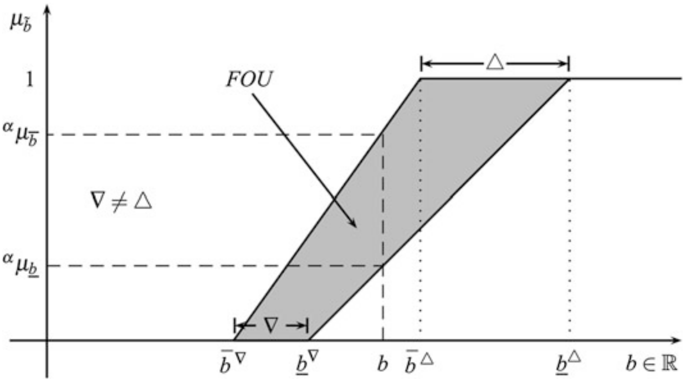

Fig. 4 IT2FS constraint with joint uncertain Δ and ∇

Definition 3.4 (*FOU of* $\tilde{b}$) Using (11) it is possible to compose the footprint of uncertainty of $\tilde{b}, u \in J_b$ as follows:

$$FOU(\tilde{b}) = \bigcup_{b \in \mathbb{R}} [{}^{\alpha}\overline{b}, {}^{\alpha}\underline{b}] \; \forall b \in \tilde{b}, \; u \in J_b, \; \alpha \in [0,1] \tag{12}$$

Remark 3.1 Definition 3.1 presents an L-R Type-2 fuzzy set defined as the union of all possible L-R Type-1 fuzzy sets into its FOU, all of them seen as fuzzy numbers (See Klir and Yuan 1995). Definition (3.2) defines an uncertain constraint as a monotonic decreasing Type-2 fuzzy set which represents the words "Approximately less or equal than b_i". In this way, we refer to an uncertain constraint as defined in Definition (3.2) with membership functions and parameters as displayed in Fig. 4.

An optimal solution in terms of the decision variables $x \in \mathbb{R}$ given uncertain constraints $\tilde{b}$ lead to define an optimization strategy. Next section shows the transportation model to be solved alongside with an optimization strategy.

4 A Fuzzy Multiple Means Transportation Model

Given the concept of an uncertain constraint and the crisp multiple means transportation problem, its fuzzy version is:

$$
\begin{aligned}
&\min_{(i,j)} \; z = \sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} c_{ijk}x_{ijk} + s_k v_{ijk} \\
&\qquad\qquad\qquad s.t. \\
&-\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} x_{ijk} \succsim -\tilde{a}_i \qquad\qquad \forall\, i\in\mathbb{N}_m \\
&\sum_{i\in\mathbb{N}_m}\sum_{k\in\mathbb{N}_K} x_{ijk} \succsim \tilde{d}_j \qquad\qquad \forall\, j\in\mathbb{N}_n \\
&\quad x_{ijk} \leqslant p_k v_{ijk} \qquad \forall\, i\in\mathbb{N}_m,\; j\in\mathbb{N}_n,\; k\in\mathbb{N}_K \\
&\sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n} v_{ijk} \leqslant q_k \qquad\qquad \forall\, k\in\mathbb{N}_K
\end{aligned}
\tag{13}
$$

where $x_{ijk}, c_{ij}, s_k, p_k, q_k \in \mathbb{R}$, $v_{ijk} \in \mathbb{Z}^+$. $\tilde{d}_j, \tilde{a}_i$ are IT2FS vectors, each one defined by four parameters: $\bar{a}_i^{\nabla}, \bar{a}_i^{\Delta}, \underline{a}_i^{\nabla}, \underline{a}_i^{\Delta}, \bar{d}_j^{\nabla}, \bar{d}_j^{\Delta}, \underline{d}_j^{\nabla}, \underline{d}_j^{\Delta} \in \mathbb{R}$, and $\succsim$ is a Type-2 fuzzy partial order. Note that we have defined $\tilde{a}_i$ in (13) as negative to change its direction from $\precsim$ to $\succsim$. The (LMF) of $\tilde{d}_j$ is:

$$
\underline{\mu}_{\tilde{d}_j}\left(x; \underline{d}_j^{\nabla}, \underline{d}_j^{\Delta}\right) = \begin{cases} 0, & x \leqslant \underline{d}_j^{\nabla} \\ \frac{x-\underline{d}_j^{\nabla}}{\underline{d}_j^{\Delta}-\underline{d}_j^{\nabla}}, & \underline{d}_j^{\nabla} \leqslant x \leqslant \underline{d}_j^{\Delta} \\ 1, & x \geqslant \underline{d}_j^{\Delta} \end{cases} \tag{14}
$$

and its (UMF) is:

$$
\underline{\mu}_{\tilde{d}_j}\left(x; \bar{d}_j^{\nabla}, \bar{d}_j^{\Delta}\right) = \begin{cases} 0, & x \leqslant \bar{d}_j^{\nabla} \\ \frac{x-\bar{d}_j^{\nabla}}{\bar{d}_j^{\Delta}-\bar{d}_j^{\nabla}}, & \bar{d}_j^{\nabla} \leqslant x \leqslant \bar{d}_j^{\Delta} \\ 1, & x \geqslant \bar{d}_j^{\Delta} \end{cases} \tag{15}
$$

5 Optimization Strategy

There are different ways to solve crisp transportation problems, including combinatorial optimization, heuristics, etc. In this chapter we focus on a transportation model which involves fuzzy uncertainty coming from experts of the system instead of an optimization routine. This kind of uncertainty has to be modeled using special techniques, so the problem turns into a more complex than the classical approach.

A first approach for solving IT2FS problems is by reducing its complexity into a simpler form, allowing use well known algorithms. In this case, we propose two strategies: a first one that uses a single fuzzy set embedded into the FOU of each IT2FS constraint, and a second one where its first step is to compute a fuzzy set of optimal solutions namely $\tilde{z}$ and afterwards, a Type-reduction strategy to find an embedded Type-reduced fuzzy set Z. In general, the above is currently the problem of finding a vector of solutions $x \in \mathbb{R}^m$ such that:

$$\max_{x \in \mathbb{R}^n} {}^{\alpha} \left\{ \bigcap_{i=1}^{m} \{\tilde{b}_i, b_i\} \bigcap \tilde{z} \right\} \tag{16}$$

where, α is an α-cut made over all fuzzy constraints. Now, $\tilde{z}$ is defined as follows

$$\mu_{\tilde{z}}(x^*) = \sup_{z=c'x^*+c_0} \min_i \left\{ \mu_{\tilde{b}_i}(x^*) \mid x^* \in \mathbb{R}^m \right\} \tag{17}$$

Given $\mu_{\tilde{z}}$, the problem becomes in how to find the maximal intersection between $\tilde{z}$ and $\tilde{b}$, for which α is defined as an auxiliary variable. In practice, the problem is solved by $x*$, so α allows us to find $x*$, according to (16).

5.1 Single Expert Approach

When having single fuzzy sets for $\tilde{a}_i$ and $\tilde{d}_j$, then the Zimmermann soft constraints method (See Zimmermann 1978 and Zimmermann and Fullèr 1993) can be applied. This method imposes a restriction on $\tilde{a}_i$ and $\tilde{d}_j$: they shall be linear shaped L-R fuzzy numbers. As example, $\tilde{d}_j$ is described in Fig. 5.

Zimmermann proposed a method for solving this fuzzy constrained problem, described as follows:

Algorithm 5.1

1. Compute the inferior boundary of optimal solutions $\min\{z^*\} = \underline{z}$ by using $(\underline{a}_i, \underline{d}_j)$ as a right hand side of the model.
2. Compute the superior boundary of optimal solutions $\max\{z^*\} = \bar{z}$ by using $(\bar{a}_i, \bar{d}_j)$ as a right hand side of the model.
3. Define an L-R fuzzy set $Z(x*)$ with parameters $\underline{z}$ and $\bar{z}$. This set is the set of all feasible solutions regarding a goal, i.e. a thick solution of the fuzzy problem (See Kall and Mayer 2010 and Mora 2001). If the goal is to maximize, then μ_Z is:

$$\mathrm{p}\mu_Z(z; \underline{z}, \bar{z}) = \begin{cases} 0, & c'x \leqslant \underline{z} \\ \frac{\bar{z}-c'x}{\bar{z}-\underline{z}}, & \underline{z} \leqslant c'x \leqslant \bar{z} \\ 1, & \bar{z} \end{cases} \tag{18}$$

Its graphical representation is displayed in Fig. 6

4. Create an auxiliary variable α and solve the following model:

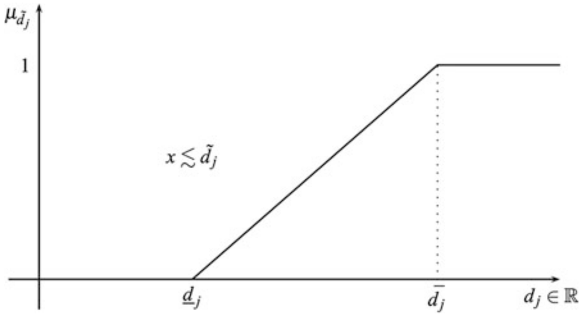

Fig. 5 Fuzzy set $\tilde{d}_j$

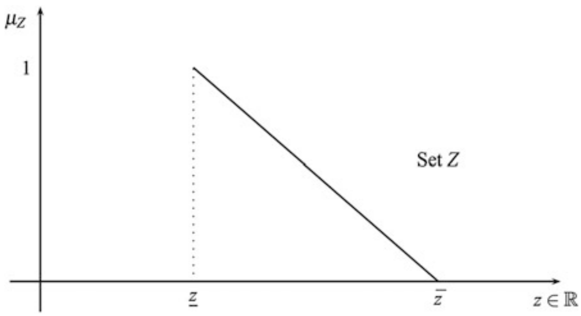

Fig. 6 Fuzzy set Z

$$
\begin{aligned}
&\max \{\alpha\} \\
&s.t. \\
&\sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} c_{ijk}x_{ijk} + s_k v_{ijk} + \alpha(\bar{z} - \underline{z}) = \hat{z} \\
&-\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} x_{ijk} - \alpha(\bar{a}_i - \underline{a}_i) \geq -\underline{a}_i \qquad \forall\, i \in \mathbb{N}_m \\
&\sum_{i\in\mathbb{N}_m}\sum_{k\in\mathbb{N}_K} x_{ijk} - \alpha(\bar{d}_j - \underline{d}_j) \geq \underline{d}_j \qquad \forall\, j \in \mathbb{N}_n \\
&x_{ijk} \leq p_k v_{ijk} \qquad \forall\, i \in \mathbb{N}_m,\ j \in \mathbb{N}_n,\ k \in \mathbb{N}_K \\
&\sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n} v_{ijk} \leq q_k \qquad \forall\, k \in \mathbb{N}_K
\end{aligned}
\tag{19}
$$

where $x_{ijk}, c_{ij}, s_k, p_k, q_k, \underline{d}_j, \bar{d}_j \in \mathbb{R}$, $v_{ijk} \in \mathbb{Z}^+$.

This method uses α as a global satisfaction degree of all constraints regarding to a fuzzy set of optimal solutions Z. In fact, α operates as a balance point between the use of the resources (supplies and demands) and the desired costs (denoted by z), since the use of more supplies to satisfy more demands lead to high costs, at different uncertainty degrees.

Then, the main idea of this method is to find an overall satisfaction degree of both goals (costs vs. supplies-demands usage) that maximizes the global satisfaction of all constraints and its goal, i.e. minimizing the global uncertainty.

5.2 Multiple Experts Approach

Figueroa (2008), Figueroa and Hernández (2013) and Figueroa et al. (2012) proposed a method that uses Δ and ∇ as auxiliary variables with weights c^{Δ} and c^{∇} respectively, in order to find an optimal fuzzy set embedded into the FOU of the problem and then solve it by using the Zimmermann's method. Its fuzzified version is presented next.

Algorithm 5.2

1. Calculate an optimal upper boundary called *Z minimum* $(\bar{z})$ by using $\underline{a}_i^{\Delta} - \Delta_i$ and $\underline{d}_j^{\Delta} - \Delta_j$, where $\Delta_i, \Delta_j$are auxiliary variables weighted by c^{Δ}which represents the lower uncertainty interval subject to the following statement:

$$\Delta_i \leq \underline{a}_i^{\Delta} - \bar{a}_i^{\Delta} \tag{20}$$

$$\Delta_j \leq \underline{d}_j^{\Delta} - \bar{d}_j^{\Delta} \tag{21}$$

To do so, Δ_i^*, Δ_j^* are obtained solving the following LP problem:

$$\begin{aligned}
&\min_{(i,j)} z = \sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} c_{ijk}x_{ijk} + s_k v_{ijk} + c_i^{\Delta}\Delta_i + c_j^{\Delta}\Delta_j \\
&\qquad\qquad s.t. \\
&-\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} x_{ijk} + \Delta_i \geq -\underline{a}_i^{\Delta} \qquad \forall\, i\in\mathbb{N}_m \\
&\sum_{i\in\mathbb{N}_m}\sum_{k\in\mathbb{N}_K} x_{ijk} + \Delta_j \geq \underline{d}_j^{\Delta} \qquad \forall\, j\in\mathbb{N}_n \\
&x_{ijk} \leq p_k v_{ijk} \qquad \forall\, i\in\mathbb{N}_m,\ j\in\mathbb{N}_n,\ k\in\mathbb{N}_K \\
&\sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n} v_{ijk} \leq q_k \qquad \forall\, k\in\mathbb{N}_K
\end{aligned} \tag{22}$$

2. Calculate an optimal lower boundary called *Z maximum* $(\underline{z})$ by using $\underline{a}_i^{\nabla} - \nabla_i$ and $\underline{d}_j^{\nabla} - \nabla_j$, where ∇_i, ∇_j are auxiliary variables weighted by c^{∇}which represents the lower uncertainty interval subject to the following statement:

$$\nabla_i \leq \underline{a}_i^{\nabla} - \bar{a}_i^{\nabla} \tag{23}$$

$$\nabla_j \leq \underline{d}_j^{\nabla} - \bar{d}_j^{\nabla} \tag{24}$$

To do so, ∇_i^*, ∇_j^* are obtained solving the following LP problem

$$\begin{aligned}
\min_{(i,j)} z = \sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} c_{ijk}x_{ijk} + s_k v_{ijk} + c_i^{\nabla}\nabla_i + c_j^{\nabla}\nabla_j \\
s.t. \\
-\sum_{j\mathbb{N}_n}\sum_{k\in\mathbb{N}_K} x_{ijk} + \nabla_i \geq -\underline{a}_i^{\nabla} \qquad & \forall\, i\in\mathbb{N}_m \\
\sum_{i\in\mathbb{N}_m}\sum_{k\in\mathbb{N}_K} x_{ijk} + \nabla_j \geq \underline{d}_j^{\nabla} \qquad & \forall\, j\in\mathbb{N}_n \\
x_{ijk} \leq p_k v_{ijk} \qquad & \forall\, i\in\mathbb{N}_m,\ j\in\mathbb{N}_n,\ k\in\mathbb{N}_K \\
\sum_{i\in\mathbb{N}_m}\sum_{j\in\mathbb{N}_n} v_{ijk} \leq q_k \qquad & \forall\, k\in\mathbb{N}_K
\end{aligned} \tag{25}$$

3. Find the final solution using the third and subsequent steps of the Algorithm 5.1 using the following values of $\underline{b}$ and $\bar{b}$:

$$\bar{a}_i = \underline{a}_i^{\Delta} - \Delta_i^* \tag{26}$$

$$\bar{d}_j = \underline{d}_j^{\Delta} - \Delta_j^* \tag{27}$$

$$\underline{a}_i = \underline{a}_i^{\nabla} - \nabla_i^* \tag{28}$$

$$\underline{d}_j = \underline{d}_j^{\nabla} - \nabla_j^* \tag{29}$$

Remark 5.1 (*About c^{Δ} and c^{∇}*) In Algorithm 5.1, we have defined c^{Δ} and c^{∇} as weights of Δ and ∇. In this approach, we define $c_i^{\Delta}, c_i^{\nabla}, c_j^{\Delta}, c_j^{\nabla}$ as the unitary costs associated to decrease the use of available products and demands, namely $\underline{a}_i^{\Delta}, \underline{a}_i^{\nabla}\underline{d}_j^{\Delta}$ and $\underline{d}_j^{\nabla}$. Note that the main goal of this transportation model is to get minimum transportation costs, not to send more products increasing costs.

Therefore, Δ and ∇ are auxiliary variables that operate as a Type-reducers,[1] this means that for each uncertain $\tilde{a}_i, \tilde{d}_j$, we obtain a fuzzy set embedded on its FOU, where $\Delta_i^*, \Delta_j^*, \nabla_i^*$ and ∇_j^* become $\bar{a}_i, \underline{a}_i, \bar{d}_j$ and $\underline{d}_j$ for the Algorithm 5.1.

[1] A Type-reduction method finds a fuzzy set embedded into the FOU of a Type-2 fuzzy set.

6 Application Example

To illustrate our proposal, we present an example of three suppliers, three customers and three vehicle types. The perception of the experts of the system is provided in two fronts: experts of the behavior of the customer and experts of the suppliers' capability.

Therefore, if different experts provide an opinion based on their previous knowledge, the problem is about how to use the information they have provided. Sometimes, the experts provide an opinion using words instead of numbers using sentences as "*I think that the demand of X should be between* d_1 *and* d_2", where d_1 and d_2 become $\underline{d}_j$ and $\bar{d}_j$, as presented in Sect. 5.1.

The multiple opinions of the experts about the supplies and demands of the system are summarized using IT2FS, where the main idea is to maximize the profits of the system. In this way, we need to compute $\tilde{z}$ and $z^* = c(x^*)$ using (17), $\tilde{a}$ (in units of product), $\tilde{d}$ (in units of product) $c_{ijk}, c^{\nabla}, c^{\Delta}$ (in monetary units), s_k (in monetary units) p_k (in units of product) and q_k (amount of available vehicles), as follows.

$$\bar{d}_j^{\nabla} = \begin{bmatrix} 12 \\ 7 \\ 20 \end{bmatrix} \bar{d}_j^{\Delta} = \begin{bmatrix} 20 \\ 11 \\ 25 \end{bmatrix} \underline{d}_j^{\nabla} = \begin{bmatrix} 18 \\ 10 \\ 24 \end{bmatrix} \underline{d}_j^{\Delta} = \begin{bmatrix} 27 \\ 15 \\ 40 \end{bmatrix} \underline{a}_i^{\Delta} = \begin{bmatrix} 25 \\ 22 \\ 35 \end{bmatrix}$$

$$\underline{a}_i^{\nabla} = \begin{bmatrix} 32 \\ 25 \\ 42 \end{bmatrix} \bar{a}_i^{\Delta} = \begin{bmatrix} 31 \\ 26 \\ 40 \end{bmatrix} \bar{a}_i^{\nabla} = \begin{bmatrix} 36 \\ 30 \\ 45 \end{bmatrix}$$

$$c_i^{\Delta} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} c_j^{\Delta} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \Delta_i = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} \Delta_j = \begin{bmatrix} 5 \\ 5 \\ 7 \end{bmatrix}$$

$$c_i^{\nabla} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} c_j^{\nabla} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \nabla_i = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix} \nabla_j = \begin{bmatrix} 6 \\ 3 \\ 4 \end{bmatrix}$$

And the unitary costs are described in Tables 1 and 2

6.1 *Single Expert Results*

This approach only uses a fuzzy set embedded into the FOU of $\tilde{a}_i$ and $\tilde{d}_j$. Now, there is an infinite amount of possible choices that the analyst can use to solve the problem. In this way, we have divided $\Delta_i, \Delta_j, \nabla_i$ and ∇_j into 21 proportional segments using an auxiliary variable called $\delta \in [0, 1]$, for which we have applied

Table 1 unitary transportation costs c_{ijk}

	$i = 1$			$i = 2$			$i = 3$		
	$j = 1$	$j = 2$	$j = 3$	$j = 1$	$j = 2$	$j = 3$	$j = 1$	$j = 2$	$j = 3$
$k = 1$	2	4	2	3	4	2	3	2	4
$k = 2$	3	4	2	4	3	5	4	2	5
$k = 3$	3	2	4	2	4	3	3	2	2

Table 2 Transportation fleet parameters

Vehicle type	Availability (q_k)	Capacity (in units) p_k	Unitary cost (s_k)
1	4	10	100
2	3	12	140
3	2	15	170

Table 3 Single experts results for from 0 to 1

	Integer MLP			Relaxed LP		
δ	$\underline{z}$	$\bar{z}$	α^*	$\underline{z}$	$\bar{z}$	α^*
0	1272	1238	0.5	1272	1238	0.5
0.05	1274.6	1239.3	0.5	1274.6	1239.3	0.5
0.1	1277.2	1240.6	0.5	1277.2	1240.6	0.5
0.15	1279.8	1241.9	0.5	1279.8	1241.9	0.5
0.2	1282.4	1243.2	0.5	1282.4	1243.2	0.5
0.25	1285	1244.5	0.5	1285	1244.5	0.5
0.3	1287.6	1245.8	0.5	1287.6	1245.8	0.5
0.35	1290.2	1247.1	0.5	1290.2	1247.1	0.5
0.4	1292.8	1248.4	0.5	1292.8	1248.4	0.5
0.45	1295.4	1249.7	0.5	1295.4	1249.7	0.5
0.5	1298	1251	0.5	1298	1251	0.5
0.55	1300.6	1252.3	0.5	1300.6	1252.3	0.5
0.6	1303.2	1253.6	0.5	1303.2	1253.6	0.5
0.65	1305.8	1254.9	0.5	1305.8	1254.9	0.5
0.7	1308.4	1256.2	0.5	1308.4	1256.2	0.5
0.75	1311	1257.5	0.5	1311	1257.5	0.5
0.8	1313.6	1258.8	0.5	1313.6	1258.8	0.5
0.85	1317.15	1260.1	0.5042	1316.2	1260.1	0.5
0.9	1320.6	1261.4	0.5077	1318.8	1261.4	0.5
0.95	1324.8	1262.7	0.5141	1321.4	1262.7	0.5
1	1329	1264	0.52	1324	1264	0.5

the Algorithm 5.1. This leads to have 21 fuzzy sets and therefore 63 LP optimization problems including its fuzzy solution. The obtained results are shown in Table 3.

In Table 3, we have divided the results into two parts: a first one which contains the results of the fuzzy model using a branch-and-bound algorithm and a second one using a relaxed LP model.

Note that all results are similar since the fuzzy constraints operate as a relaxation of the problem, which means that the integer problem has closer results to the LP ones, so we can see that the fuzzy approach allows to find appropriate solutions.

Now, the analyst needs a solution in terms of x_{ijk}, so we have selected $\delta = 0.5$ as a medium point to get a solution. The optimal solution is summarized as follows:

$$\begin{array}{ccc} \Delta^*_{i=1}=3 & \nabla^*_{i=1}=2 & x^*_{111}=10 \\ \Delta^*_{i=2}=2 & \nabla^*_{i=2}=2.5 & x^*_{131}=20 \\ \Delta^*_{i=3}=2.5 & \nabla^*_{i=3}=1.5 & x^*_{231}=7.25 \\ \Delta^*_{j=1}=3.5 & \nabla^*_{j=1}=3 & x^*_{213}=9.25 \\ \Delta^*_{j=2}=2 & \nabla^*_{j=2}=1.5 & x^*_{322}=10.75 \\ \Delta^*_{j=3}=7.5 & \nabla^*_{j=3}=2 & \end{array}$$

$$v^*_{111} = 1 v^*_{131} = 2 v^*_{122} = 2$$
$$v^*_{113} = 1 \quad v^*_{213} = 1 \quad v^*_{231} = 1 \quad v^*_{322} = 1$$

The total cost of this selection is $z^* = 1274.5$, so the analyst can implement the obtained results in practice with an uncertainty degree of $\alpha = 0.5$ as proposed by the Algorithm 5.1. Note that in this approach; the analyst should increase all the demands to be satisfied, demanding more products from all suppliers as well.

6.2 Multiple Experts Results

Now, all the opinions of the experts are combined to compose uncertain constraints, and therefore IT2FS. The consensus of all experts is defined in the sense of involving their individual perceptions into the FOU of each constraint, which finally is a way to involve their knowledge into a single representation.

As mentioned in previous sections, our main idea is to handle uncertainty while solving an optimization LP problem using convex optimization methods, in this case, the Zimmermman's soft constraints method and LP models. First we solve two interval LP models for finding a single Type-1 fuzzy set into the FOU of each constraint and then use the Zimmermman's method for finding a crisp solution of the problem.

Using the Algorithm 5.2, the obtained fuzzy set $\tilde{Z}$ has the following boundaries (See Figueroa and Hernandez 2012):

$$\bar{z}^{\nabla} = 1{,}264,\ \bar{z}^{\Delta} = 1{,}329,\ \underline{z}^{\nabla} = 1{,}238,\ \underline{z}^{\Delta} = 1{,}272$$

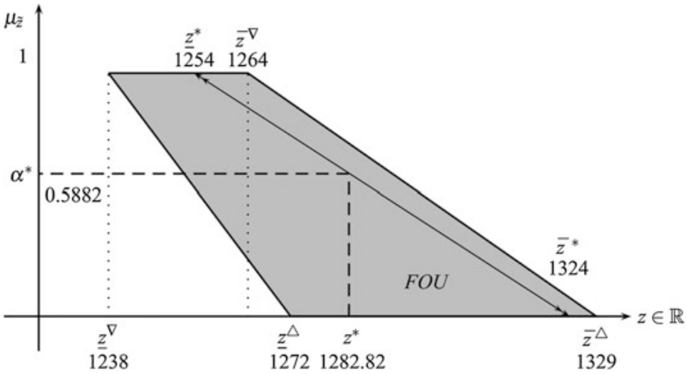

Fig. 7 Interval Type-2 fuzzy set of solutions $\tilde{Z}$

After solving the LP models shown in (22) and (25), the values of $\underline{z}^*$ and $\bar{z}^*$ are 1,254 and 1,324, respectively. Using the Zimmermann's method, we have obtained a crisp solution of $\alpha^* = 0.5882$ and $z^* = 1282, 82$. A detailed report is shown next.

$$
\begin{array}{lll}
\Delta^*_{i=1} = 0 & \nabla^*_{i=1} = 0 & x^*_{131} = 27.88 \\
\Delta^*_{i=2} = 0 & \nabla^*_{i=2} = 0 & x^*_{231} = 3.88 \\
\Delta^*_{i=3} = 0 & \nabla^*_{i=3} = 0 & x^*_{213} = 16.70 \\
\Delta^*_{j=1} = 7 & \nabla^*_{j=1} = 6 & x^*_{322} = 12.94 \\
\Delta^*_{j=2} = 0 & \nabla^*_{j=2} = 0 & \\
\Delta^*_{j=3} = 0 & \nabla^*_{j=3} = 4 &
\end{array}
$$

$$
v^*_{131} = 3 \quad v^*_{122} = 1 \quad v^*_{231} = 1
$$
$$
v^*_{213} = 2 \quad v^*_{322} = 2
$$

The obtained results show that x^*_{ijk} units should be sent from suppliers to customers using v^*_{ijk} as shown above. Figure 7 shows the Type-reduced fuzzy set of optimal solutions $\tilde{z}$ which is embedded into the FOU of $\tilde{Z}$, where the global satisfaction degree of $\alpha^* = 0.5882$ allows us to find an optimal solution of the problem, which leads to the above values of x^*_{ijk} and v^*_{ijk}.

Remark 6.1 (*About the boundaries of* Z) The boundaries of the set $\tilde{Z}$ i.e. $\bar{z}^\Delta, \bar{z}^\nabla, \underline{z}^\Delta$and $\bar{z}^\Delta$ were computed using the results of Figueroa (2012). In other words, each boundary comes from an LP problem using one of the following combinations:

$$\begin{gathered}\left(\bar{d}^{\Delta}, -\bar{a}^{\Delta}\right) \rightarrow \underline{z}^{\nabla}\\ \left(\underline{d}^{\Delta}, -\underline{a}^{\Delta}\right) \rightarrow \bar{z}^{\nabla}\\ \left(\bar{d}^{\nabla}, -\bar{a}^{\nabla}\right) \rightarrow \underline{z}^{\Delta}\\ \left(\underline{d}^{\nabla}, -\underline{a}^{\nabla}\right) \rightarrow \bar{z}^{\Delta}\end{gathered}$$

6.3 Discussion of the Results

The multiple experts' solution using IT2FSs outperforms the single expert approach in the sense that it selects only the critical constraints of the problem, so the total cost is minimized at higher α-cut values. Note that even when the proposed method has additional costs (Δ and ∇), its final solution is better in terms of α, for $\delta > 0.71$.

The values of $c^{\Delta}$and c^{∇} affect the solution, so in a first glance, the method should not increase delivering costs, which does not happen as shown before. Moreover, the method selects some auxiliary variables which increase delivering costs. This happens since the method selects the constraints that improve the objective function, accomplishing (19) instead of giving the same weight to all constraints.

Although there is an infinite amount of possible choices of x_{ijk}, we point out that our proposal is based on a deffuzification process, so its results are based on a Type-reduction strategy which selects only one of the possible choices of α^*.

It is important to note that our method only selects critical constraints using Δ and ∇. Although the results of the single expert approach can be similar, the proposed one obtains a higher deffuzification degree (i.e. smaller uncertainty degree) using less resources which means better costs.

7 Concluding Remarks

The proposed method can deal with Type-2 fuzzy supplies and demands using well known fuzzy optimization techniques, achieving appropriate results.

A solution in terms of $x^*$and α^*, is found. The method obtains a fuzzy set embedded into the FOU of $\tilde{a}_i, \tilde{d}_j$ and $\tilde{Z}$; this set is used to find an optimal solution using the Zimmermann's method which returns the values of $x^*$and α^*.

The presented approach is based in a transportation problem, which is a basic problem in supply chain decisions. Related problems including TSP, VRP and its extensions can be solved using our proposal due to its flexibility and interpretability.

Finally, the proposed method is intended as a guide about how to address a problem where some of its constraints are considered as Type-2 fuzzy sets, involving the opinions and perceptions of different experts and using their previous knowledge and non-probabilistic uncertainty. Other methods can be used for, but our proposal is only an approach for modeling and solving this kind of problems.

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Part VI
Warehouse Management Under Fuzziness

A Fuzzy Set Theoretic Approach to Warehouse Storage Decisions in Supply Chains

Avninder Gill

Abstract Warehouse facilities in a supply chain provide the necessary product storage before consumption. When the shipments are received in a warehouse, the first decision encountered by a logistic manager is where to store the product. The product can be sent to long-term reserve storage, short-term primary storage or it can be directly cross-docked. This decision calls for an expert judgment and knowledge of certain decision rules. However, it would be impossible for a human being to comprehend these rules and process the information to take real-time decisions. The present chapter demonstrates how the fuzzy linguistic modeling concept and fuzzy set theory can be effectively used to capture, present, organize and synthesize the expert knowledge in terms of fuzzy decision rules to provide a powerful tool to the decision maker. The approach has been illustrated with the help of an example and computation experience provided.

Keywords Warehousing · Fuzzy decision making · Fuzzy logics · Fuzzy expert systems · Supply chain management

1 Introduction

The main objective of a warehouse facility is to provide a temporary shelter to the product. Warehouses also facilitate the movement of goods from the suppliers to the customers encompassing the entire supply chain. Looking at the types and activities undertaken in a modern warehouse, the role of a warehouse has certainly evolved into a logistical switching facility rather than a mere storage facility. The need for warehouses mainly arises due to the time gaps between production and

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consumption of goods. For example, most goods would be produced ahead of their consumption times. Therefore, we would need a warehousing facility to store the goods from the time they are produced to the time they are sold. Another reason why we end up storing goods is the imbalance between the demand and supply of seasonal products. For some products, there is a uniform demand but a seasonal supply. Most agricultural, produce and processed food supplies would fall into this category. There is abundant supply of these products during a short growing season but the demand for these products is consistent throughout the year. For other products, there is a seasonal demand but they can be manufactured throughout the year to take advantage of the limited and expensive capacity. In order to manage this supply and demand gap, we need to store these products till the time of consumption. Besides, a storage facility acts as a safeguard against many supply chain related risks and disruptions such as floods, fire, employee strikes etc. The recent emphasis on just-in-time principles and adoption of sophisticated computer-based information systems that provide real-time information on stock locations, have certainly reduced the need for holding a large stock of inventory and hence the need for keeping warehouse facilities. Nevertheless, achieving a perfect match and integration between production and logistics planning, and to forecast the demand with a high degree of accuracy remains a distant dream for many supply chain authorities. Most supply chains will have to consolidate the supply of goods from multiple sources and therefore, they will experience a mismatch in demand and supply. Hence there are valid reasons to have warehouses and distribution centers. Some of the benefits of having a warehouse and the functions performed by a warehouse facility are listed below.

- Providing temporary storage for the product till the product is sold;
- Providing a buffer to match supply and demand due to lack of poor planning as well as due the seasonal nature of demand and supply;
- Consolidating various products and enabling economies of scale in shipments to lower the line-haul costs;
- Receiving better prices through large purchases;
- To cover for planned or unplanned disruptions in production by ensuring continuity and regulating the flow of goods;
- Performing packaging and re-packaging on the product;
- Performing light assembly, product assortment and kit assembly operations;
- Minimizing risk through postponement strategies by delaying the final product decision until the customer order;
- Providing protection to goods from loss, damage, dust, moisture, heat and minimize wastage and spoilage;
- Grading the product through inspection techniques;
- Show market presence of a company through brick and wall warehouses;
- Reaping the benefits of real-estate appreciation through investment in warehouses;
- Creating employment opportunities through warehousing jobs.

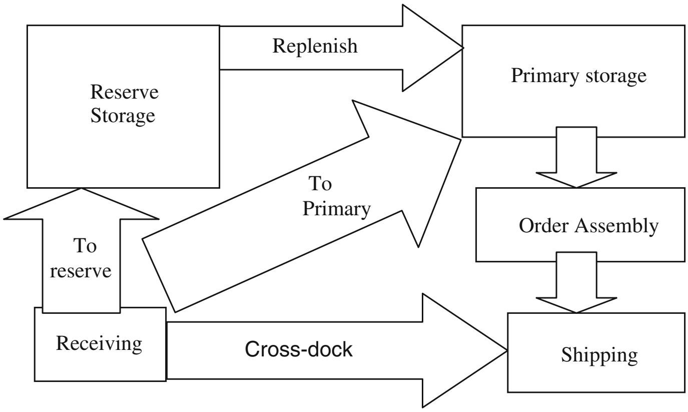

Fig. 1 Material flow in a warehouse

1.1 Warehouses Operations

The objective of warehouse operations is to fulfill the customer orders and to have an effective utilization of space, equipment and labor. In order to achieve these objectives, a warehouse has to perform various activities or operations and follow a basic flow of material to get the product in and out of a warehouse. Although these operations would be common for most warehouses, but a deviation from this flow is possible depending on the nature of a warehouse. Figure 1 shows a basic material flow and layout for a warehouse. With an aim to minimize the travel distance for the order pickers and to have an efficient storage policy, most warehouses would separate their long-term reserve storage from the primary storage and order picking areas. Such a separation not only results in a better utilization of expensive warehouse space but it also increases the ability to conveniently and quickly access the stock. Furthermore, this separation helps to reduce the interference between order picking and bulk material movement. However, if the total inventory for a stock item is small, it will not be beneficial to hold it at separate locations. The main operations in a warehouse are receiving, storage, replenishment, order-picking, order-assembly and shipping.

1.1.1 Receiving

The receiving function is the starting point for an inventory control system in a warehouse. It is a collection of activities that involve the orderly receipt of all materials in the warehouse. It is also the function for collecting information needed for keeping product details accurate. The receiving function begins with an advance notification of arrival of the goods to the warehouse and includes assigning bay numbers to incoming vehicles, verifying product quantity and quality, preparing receiving reports, and sending those reports to designated departments.

1.1.2 Storage

While merchandise is waiting for demand, the physical shelter of that merchandise is called storage. Storage functions also refers to the movement of products from the receiving docks to a storage location, the recording of the storage location and quantity, and updating of storage records so that the product can be easily found when it is needed. Storage function starts with a put-away operation which is the process of transporting and placing merchandise in storage. Before product can be put away, an appropriate storage area must be determined which essentially is the objective of this chapter. The main storage options are in terms of reserve or back-up storage which is the largest space in a warehouse, primary storage for easy pick-up or directly cross-docking a product to the shipping area. Eventually, this decision determines how quickly and at what cost the item is later retrieved to meet the customer order.

1.1.3 Replenishment

This is the movement of goods in larger-than-order-quantities from the reserve area to the primary storage area to ensure that that order picking locations maintain their stock availability and fill-rates. For example, moving a whole pallet from reserve storage to order picking would be an act of replenishment.

1.1.4 Order Picking and Retrieval

Order picking is the process of removing merchandise from either reserve or primary storage locations as per the requirements given in a specific customer order. Typically, order pickers will print a list of items on the customer order and move to the primary or reserve area with a material handling equipment to look for the items and desired quantities of those items to fill an order. It is an important and highly cost consuming activity in a warehouse and most warehouse facilities are designed keeping in view the convenience of the order pickers.

1.1.5 Order-Assembly and Shipping

Order assembly and shipping operations is where the final steps are taken to prepare orders for shipment via the requested mode of transit. The tasks performed usually include weighing cartons, consolidating amounts collected by different order pickers, checking for completeness, and packaging in appropriate containers, applying address labels, recording shipment information and preparing shipment documents, sorting the packages and moving them to assigned shipping docks.

2 Warehouse Storage Decisions

As discussed in the earlier section, when the shipments are received in a warehouse, a number of decisions need to be made by the warehouse managers. One such decision and probably the first decision encountered by a warehouse manager is where to store the product. Warehouse managers have a number of options available. For example, the product can be sent to long-term reserve storage area. The product can be stored in a primary or short-term storage if it is needed in the near future. Furthermore, the incoming product can also be staged and sent to the vehicles waiting for outbound shipments if it is needed immediately, a concept commonly known as cross-docking. This decision has important implications for retrieval time and retrieval cost at later stages. These decisions are taken for all the shipments several times in a day. The repetitive nature of this decision acts as a multiplier for cost and time savings. The final decision as to where to send the received shipment depends on the expert judgment and knowledge of the decision maker and it is based on certain decision rules. However, it would be impossible for a human being to comprehend these decision rules and process the information in real time to take on the spot decisions. Furthermore, a decision maker is expected to take crisp decisions while the decision situations are vague and the data provided is ambiguous. Under such scenarios, the fuzzy set theory and its associated tools provides an ideal approach to model the decision situation using vague linguistic terms and process those terms to arrive at crisp decisions. In the present chapter, we will demonstrate how the linguistic variable concept and fuzzy set theory can be effectively used to process information for repetitive activities such as the ones described above, to arrive at sound business decisions for product storage in a warehouse. The chapter will propose an approach to present, organize and synthesize the expert knowledge in terms of fuzzy decision rules. In the ensuing sections, we will first introduce the important aspects of fuzzy set theory and demonstrate how these concepts can be utilized to effectively model the decision situation described above. The approach has been illustrated with the help of an example.

3 Literature Review

First we briefly review the general warehousing literature and then discuss specific applications of fuzzy set theory in warehousing research. Some introductory material on warehousing facility layout and planning can be found in most standard logistics and facility planning texts such as Ballou (1999), Daganzo (1999) and Tompkins et al. (2003). Extensive surveys of applications of operations research models to warehouse facility design in general and warehouse layout and design in particular, are available in Cormier and Gunn (1992), (1996), Montulet et al. (1995), Vandenberg and Zihm (1999), Rouwenhorst et al. (2000). White and

Francis (1971) after specifying a probability mass function for space demand, derive network flow formulations for storage. Ashayeri and Gelders (1985) discussed the warehouse design optimization from a layout perspective. Park and Webster (1989) considered factors such as control procedures, storage assignment rules, patterns for product flow etc. for product storage in a warehouse. Gray et al. (1992) discussed the design and operational optimization for a warehouse using the order consolidation concepts and a top-down approach. Cormier and Gunn (1996) tackled a warehouse sizing problem under the assumption of arbitrarily varying product demands over a finite planning horizon. Applications of fuzzy set theory in product distribution are relatively fewer. Gonzalez and Fernadez (2000) proposed the use of fuzzy sets to represent the provisional information related to costs, demands and other variables in a product distribution model. Govindaraj et al. (2000) provided an object oriented expert system for a top-down warehouse and distribution facility design and conclude that fewer orders picked simultaneously results in fewer dock doors. Lau et al. (2002) presented a fuzzy logic methodology to monitor supply chain performance. Chan and QI (2003) employ a fuzzy evaluation process to measure performance of complex supply chains. Shore and Venkatachalam (2003) combined fuzzy logic with analytical hierarchy process to model the information sharing capabilities of a supply chain. Yang and Sun (2004) developed an expected value model to minimize transportation cost in a fuzzy warehouse layout problem. Gill (2009) proposed a fuzzy set theoretic based approach to decide the number of loading docks in a warehouse. Jassbi et al. (2010) suggested a new approach based on adaptive neuro-fuzzy inference system for evaluating agility in supply chains. Arif (2010) provided the linguistic fuzzy logic and the computations using words to analyze the decision making processes in social sciences. Ganga and Carpinetti (2011) proposed a supply chain performance model based on fuzzy logic to predict performance based on causal relationships in the supply chain operations reference model. Maleki et al. (2011) considered a multi-objective class based storage model by treating demand rates and efficiency as fuzzy linguistic variables. The concept of hesitant fuzzy sets was introduced in Torra (2010) and further expanded in Rodriguez et al. (2012). Shoar et al. (2012) modeled the efficiency of supply chain by measuring the bullwhip effect using a fuzzy logic approach. Lee and Chen (2013) presented a new fuzzy decision making method based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets.

It is evident from the past literature that there is hardly any application of fuzzy logics to evaluate the storage decisions described in the earlier section. The contribution of the present work lies in providing a fuzzy logic based methodology to represent the implicit knowledge about warehousing decisions in terms of fuzzy rules which can be processed quantitatively to make intelligent decisions.

The following section presents some basic concepts of fuzzy logics that have been used to develop the decision methodology. Having defined the fuzzy sets for the warehousing problem, we present the decision methodology with the help of an illustrative example.

4 Basic Fuzzy Logic Concepts

Zadeh (1965) developed the concept of fuzzy set theory to deal with the issue of uncertainty in systems modeling. The concept of fuzzy set theory challenged the conventional binary logic of traditional set theory as fuzzy sets were defined as sets with imprecise boundaries. The applications of fuzzy set theory in mathematical programming have been discussed by many authors (Kaufmann and Gupta 1988; Zimmermann 1991). Since its inception by Zadeh (1965), fuzzy logic has created a silent revolution to bridge the gap between human reasoning that is often imprecise and computers that require precise definitions. For example, terms such as 'high', 'low', 'substantially more' and 'reasonably large' are commonly used and understood by decision makers to express knowledge. However a computer requires a rigid definition to process these terms. That's where fuzzy logic plays an important role by allowing us to define these terms as fuzzy sets that could further be input into a computerized system. Here, we present some basic concepts in fuzzy set theory. For details on fuzzy set theory, the reader is referred to Zadeh (1965) and Zimmermann (1991).

4.1 Fuzzy Set and Membership Function

A fuzzy set A in X is characterized by a membership function (MF), $\mu_A(x)$ which associates with each element in X, a real number in the interval [0,1] with the value of $\mu_A(x)$ at x representing the "degree of membership" of x in A. Mathematically, if $X = \{x\}$ is a collection of elements denoted generically by x i.e. $x \, \varepsilon \, X$ with $X \subset R$, then a fuzzy set in X is denoted by a set of ordered pairs, $A = \{x, \mu_A(x) \,/\, \forall x \, \varepsilon \, X\}$ such that $\mu_A(x) \, \varepsilon \, [0,1]$ i.e. $\mu_A(x)$ maps X to membership space $[0,1] \; \forall x \, \varepsilon \, X$. Therefore, a membership function assigns to each element x of X a number in the closed unit interval [0, 1] that characterizes the degree of membership of x. The closer the value of membership function to one, the greater the membership of x in fuzzy set A. Hence a fuzzy set is a generalization of classical set and the membership function is a generalization of the characteristic function.

4.2 Union of Fuzzy Sets (OR Operator)

Union of two fuzzy sets A and B with respective MF's $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set "A OR B" whose MF is $\mu_C(x) = \mathrm{Max}\{\mu_A(x), \mu_B(x)\}$, $x \, \varepsilon \, X$.

4.3 Intersection of Fuzzy Sets (AND Operator)

Intersection of two fuzzy sets A and B with respective MF's $\mu_A(x)$ and $\mu_B(x)$ is a fuzzy set "A AND B" whose MF is $\mu_C(x) = \text{Min}\{\mu_A(x), \mu_B(x)\}$, x ε X.

4.4 Linguistic Variables

The concept of fuzzy linguistic variable helps in modeling several applications in decision-making, approximate reasoning, optimization, and statistics. Variables whose values are words or linguistic terms in a natural or artificial language are called linguistic variables. In fuzzy linguistic theory, the fuzzy numbers represent linguistic concepts like "very small", "small", "medium", and "high" and these values are set as per the user's discretion. To illustrate the concept of linguistic variable, consider the word weather in a natural language. It cannot be characterized precisely. However, we can use fuzzy sets that can describe weather approximately in terms of linguistic values like very hot, hot, mild, cold and very cold. Hence weather is a linguistic variable whose values are words like very hot, hot, mild, cold and very cold. These values are also called labels of the linguistic variable and are expressed by fuzzy sets that relate these terms to a range of temperatures on a universal. Each such set is expressed by an appropriate membership function.

4.5 Fuzzy Linguistic Relations

Fuzzy linguistic modeling which is based on certain fuzzy rules helps in knowledge acquisition by extracting knowledge either from the experts or from the data. These approaches are built around rule-based models and if-then type of statements on fuzzy linguistic variables. Natural language terms such as small, medium, more or less equal, roughly the same weight are commonly used by managers in their day to day life. Furthermore, they use these terms to make important decisions at work. Although these terms are used in all walks of life, but their use is rampant in fuzzy control theory, fuzzy decision making, and several other areas. Most of these terms are utilized in a framework of fuzzy IF–THEN statements called linguistic expressions. When these fuzzy expressions are equated to a generic fuzzy set, then we get a fuzzy linguistic equation or a fuzzy relation. Suppose X is an input fuzzy linguistic variable which takes linguistic values xi and Y is an output linguistic variable that takes values yi. A fuzzy if-then relation or equation can be generically represented as: Ri = If X is xi then Y is yi. Some examples of this fuzzy equation are:

R1 = IF X is large THEN Y is small.
R_2 = IF X is somewhat small THEN Y is medium.

Often in decision making situations, it becomes necessary to combine different fuzzy expressions using ‘AND’ or ‘OR’ type of logic depending on whether both the conditions need to be met or the satisfaction of either condition is good enough to lead to the decision. As an example, consider relationship 3 (R_3) as:

R_3 = IF (X is large AND Z is medium) THEN Y is small.

Therefore, using the logical AND-OR operators defined in Sects. 4.2, 4.3, a fuzzy decision model can be developed. The approach is fairly flexible and can be applied to model a wide variety of complex situations. The output results from these fuzzy equations are also fuzzy sets. Most of the time, the decision situations, their data and modeling approaches are fuzzy in nature but the decision makes would prefer to take crisp and dichotomous decisions. Therefore, it would often be necessary to de-fuzzy the output results to arrive at a dichotomous decision.

5 Storage Decision Rules and Example

As mentioned earlier, the main storage options available in a warehouse to a decision maker are: (a) put away to reserve for efficient low cost storage in bigger unit loads such as pallets, (b) put away to primary if the product is needed in the near future and the inventory in the order picking area is sufficiently depleted and (c) cross-dock the product from receiving to the shipping area if it is needed immediately. Reserve storage is mainly meant for buffer stocks and low cost efficient storage. Having space reserved for extra inventory reduces the risk of running out of stock during critical times. On the other hand, the primary storage area minimizes the travel for order pickers and provides an efficient product retrieval and order picking mechanism. The third option, cross docking reduces inventory carrying costs, transportation costs, order fulfillment costs and material handling costs. While cross docking may not be viable solution for many situations, it can lead to significant benefits. A company can achieve a significant volume of cross-docked product if it has an appropriate order processing and material handling system, and it maintains a warehouse management system that can match incoming goods to the existing orders.

This section discusses some of the key factors that are normally considered by the decision makers in order to choose a storage option. While making decisions, expert’s judgment may consist of a few rules of thumbs that are applied to the decision problem. It should be noted that the objective of this chapter is not to provide an exhaustive list of decision rules. Depending on the management philosophy or problem situation, we may have an entirely different set of decision rules. The emphasis in this chapter is on demonstrating a modeling approach on how a particular piece of knowledge can be presented as a decision rule and then how fuzzy logics can be used to process those decision rules. No doubt, the benefits of this approach are more obvious for a large size problem where it becomes impossible for a decision maker to comprehend, process and interpret a large set of rules and take on the spot decisions. For illustrative purposes, we focus on a small set of decision rules. However, the approach is general enough to incorporate more rules. Often, a manager who deals with these repetitive decisions situations on a day-to-day basis would assign the modeling parameters, linguistic terms and the expert decision rules. However, there may be situations involving multiple authorities (e.g. more than managers on various shifts) where the assignment of parameters should be based on group decision making in a team environment.

Suppose, an experienced logistic manager decides to cross dock the product if the product is needed immediately. Then a decision rule can be designed as "If product is needed immediately, then cross-dock". Similarly, if the manager feels that the inventory in primary storage is low, again it would be appropriate to cross-dock. Therefore, the second statement would be "if the primary storage inventory is low, then cross-dock". Furthermore, the decision maker may feel that only one of these conditions need to be met to cross-dock the product, then it becomes necessary to further combine these statements in order to have a decision rule. An "OR" operator discussed earlier would be appropriate to combine these statement and the resulting decision rule 1 will be as follows. Decision rule 1 and 4 is an example of an "OR" operator. If the decision maker feels that all the conditional statements need to be satisfied in order to choose a storage option, then an "AND" operator would be more appropriate. Decision rules 2, 3 and 5 depict such a situation using "AND" operators.

Decision Rule 1. If (product is needed immediately or primary storage inventory is low), then cross-dock.
Decision Rule 2. If (reserve storage inventory is high and primary storage inventory is low), then put away to primary.
Decision Rule 3. If (product is needed on a short-term basis and primary storage inventory is low), then put away to primary.
Decision Rule 4. If (product is needed on a long-term basis or primary storage inventory is high), then put away to reserve.
Decision Rule 5. If (reserve storage inventory is low and primary storage inventory is medium), then put away to reserve.

We focus on these five decision rules. As discussed above, the decision rules can be situation specific and can capture the manager's knowledge differently but the approach is flexible enough to model various situations. These five decision rules are summarized in the following Table 1.

6 Fuzzy Linguistic Variables for the Example

In this section, we define the fuzzy linguistic terms used in the above fuzzy decision rules and their associated fuzzy sets. We may define the fuzzy variable "product shipment due date" that will assume values or labels "immediately",

Table 1 Expert knowledge about the decision situation

Rule number	Shipment due date	Primary inventory	Reserve inventory	Connector operator	Decision
Rule 1	Immediate	Low	–	OR	Cross Dock
Rule 2	–	Low	High	AND	Put away to primary
Rule 3	Short-term	Low	–	AND	Put away to primary
Rule 4	Long-term	High	–	OR	Put away to reserve
Rule 5	–	Medium	Low	AND	Put away to reserve

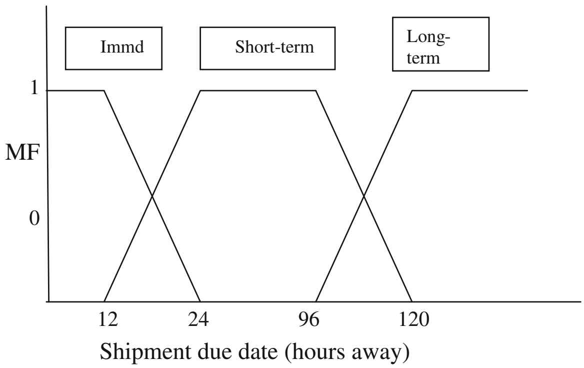

Fig. 2 Fuzzy sets for the linguistic variable shipment due date

"short-term" or "long-term" from a natural language. These values in turn will be defined as fuzzy sets. Suppose we define the due date "immediately" as something up to 12 h but it can go up to 24 h with a varying degree of membership. Similarly, the label "short-term" due date refers to a shipment schedule that is certainly between 24 to 96 h, however it could be as low as 12 h or as high as 120 h with varying degrees of membership in these ranges. Finally, the label "long-term" refers to a shipment schedule that is certainly more than 120 h away but it can be as low as 96 h with a gradual degree of memberships. Such a linguistic variable and its associated fuzzy sets are defined in Eqs. (1)–(3) and shown in Fig. 2.

Similarly, we define the fuzzy sets for inventory situation in the primary storage area. Suppose that the inventory situation is described by three linguistic terms or labels: Low, medium, and high. These values are defined as fuzzy sets. The low inventory situation in primary storage is worth 2 days of supply or less but it can go up to 4 days of supply. A "medium" inventory in this area refers to between 4 to 6 days worth of stock, however it could be as low as 2 days or as high as 8 days with varying degrees of membership in these ranges. A high primary area inventory is 8 days or more worth of stock but it can be as low as 6 days. These fuzzy sets are defined in Eqs. (4)–(6) and shown in Fig. 3.

Finally, we defined the fuzzy labels for inventory situation in the reserve storage area using two fuzzy labels or linguistic terms: low reserve inventory and high reserve inventory. A low reserve inventory in this situation is defined as less than 30 days of supply but it could go up to 60 days of supply with various degrees

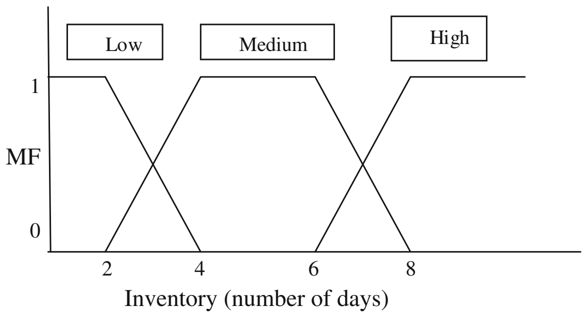

Fig. 3 Primary storage inventory (number of days' worth of supply)

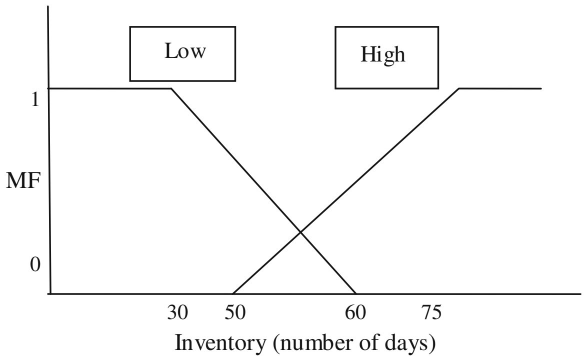

Fig. 4 Reserve storage inventory (number of days' worth of supply)

of membership. A high reserve inventory situation is more than 75 days of stock on hand but it could be as low as 50 days, again depending on the membership function. The fuzzy sets and their membership functions for reserve inventory situation are given in Eqs. (7) and (8) and shown in Fig. 4. The fuzzy sets used in the current chapter are trapezoidal in shape but other fuzzy set shapes such as triangular are also common in the fuzzy set theory literature (Kaufmann and Gupta 1988). The trapezoidal shape is generic enough to include the triangular fuzzy sets as well.

Immediate Shipment.

$$\mu_A = \begin{cases} 1, & x < 12 \\ \frac{x-24}{12-24}, & 12 \le x \le 24 \\ 0, & x > 24 \end{cases} \tag{1}$$

Short-term Shipment.

$$\mu_A = \begin{cases} 0, & x < 12 \\ \frac{x-12}{24-12}, & 12 \le x \le 24 \\ 1, & 24 < x < 96 \\ \frac{x-120}{96-120}, & 96 \le x \le 120 \\ 0, & x > 120 \end{cases} \tag{2}$$

Long-term Shipment.

$$\mu_A = \begin{cases} 0, & x < 96 \\ \frac{x-96}{120-96}, & 96 \le x \le 120 \\ 1, & x > 120 \end{cases} \tag{3}$$

Low Primary Inventory.

$$\mu_A = \begin{cases} 1, & x < 2 \\ \frac{x-4}{2-4}, & 2 \le x \le 4 \\ 0, & x > 4 \end{cases} \tag{4}$$

Medium Primary Inventory.

$$\mu_A = \begin{cases} 0, & x < 2 \\ \frac{x-2}{4-2}, & 2 \le x \le 4 \\ 1, & 4 < x < 6 \\ \frac{x-8}{6-8}, & 6 \le x \le 8 \\ 0, & x > 8 \end{cases} \tag{5}$$

High Primary Inventory.

$$\mu_A = \begin{cases} 0, & x < 6 \\ \frac{x-6}{8-6}, & 6 \le x \le 8 \\ 1, & x > 8 \end{cases} \tag{6}$$

Low Reserve Inventory.

$$\mu_A = \begin{cases} 1, & x < 30 \\ \frac{x-60}{30-60}, & 30 \le x \le 60 \\ 0, & x > 60 \end{cases} \tag{7}$$

High Reserve Inventory.

$$\mu_A = \begin{cases} 0, & x < 50 \\ \frac{x-50}{75-50}, & 50 \le x \le 75 \\ 1, & x > 75 \end{cases} \tag{8}$$

7 Decision Methodology

Suppose that the logistic manager in the above situation receives a new shipment and has to decide where to store the product. At the time of the receipt, the shipment due date for that product was 12 h away, there was 5 days' worth of stock left-over in primary area and 30 days' worth of stock in reserve area. At first sight, the answer is not so obvious. It will be even less obvious if the decision rules

Table 2 Summary of calculations

Decision rule	Shipment date	Connector	Primary inventory	Connector	Reserve inventory	Decision
Rule 1	Immediately	OR	Low			Cross dock
	1	OR	0			1
Rule 2			Low	AND	High	Primary
			0	AND	0	0
Rule 3	Short-term	AND	Low			Primary
	0	AND	0			0
Rule 4	Long-term	OR	High			Reserve
	0	OR	0			0
Rule 5			Medium	AND	Low	Reserve
			1	AND	1	1

to be processed are large in number and complex in nature. Next we demonstrate how fuzzy logic and expert system can be used to make an intelligent decision.

Equation (1) evaluates the term immediate shipment to a degree of satisfaction of ‘1’ and Eq. (4) evaluates the term low primary inventory to a degree of ‘0’. Decision rule 1 combine these membership functions using an ‘OR’ operator. Therefore decision rule 1 evaluates the decision “cross dock” to a degree of ‘1’. Similarly, low primary inventory and high reserve inventory are the two premises of decision rule 2 which are connected using an ‘AND’ operator giving it a rigid intersection meaning as defined in Sect. 4.3. In a strict intersection sense, the ‘AND’ operator chooses the lesser of the degree of membership. Therefore, decision rule 2 will evaluate this situation for a degree of ‘0’ regarding the primary storage decision. Following the same logic and calculations, decision rule 3 will evaluate the primary storage to degree for ‘0’; decision rule 4 will evaluate the reserve storage decision to a degree of ‘0’; and finally, decision rule 5 will evaluate reserve storage decision to a degree of ‘1’. A summary of these calculations has been provided in Table 2.

- It may be noted that, decision rules 2 and 3 both refer to a primary storage decision with a degree of fulfillment of ‘0’. There is no ambiguity here. However, decision rules 4 and 5 suggest the reserve storage decision to a degree of ‘0’ and ‘1’ respectively. At this stage, it might be important for the decision maker to assign a relative importance for decisions rules 4 and 5. The decision maker has lot of flexibility to interpret these rules.
- If decision rules 4 and 5 are both to be satisfied, then they must be treated in a strict ‘AND’ (i.e. intersection) sense and the overall satisfaction level for decision reserve storage will be minimum (0, 1).
- If satisfaction of either of decision rules 4 and 5 is fine with the decision maker, then they must be treated in an ‘OR’ (union) sense and the overall satisfaction level for decision reserve storage will be maximum (0, 1).
- The third option the decision maker has is to assign relative importance to decision rules 4 and 5. For simplicity, we assume that decision rule 4 and

decision rule 5 are equally important and therefore, assume the same weights for them. Here, the final decision that emerges from decision rules 4 and 5 is reserve storage to a degree of belief of '0.5'.

There is another layer of complexity in a decision making approach that basically relates to the nature of decision options available. The decision options available may be completely disjoint or they may fall across a uniform scale or spectrum. For example, if a logistic manager has to take a decision choosing between trucking, railways or air mode of transportation, the decision options are somewhat disjoint. In such cases, a ranking system for decision options will be more appropriate. In other cases, the decision options available will be related to each other on some graded scale and they can be conveniently put across a spectrum. For example, in an automatic control system, a sensor may sense the temperature in a fuzzy sense and turn on the air-conditioner to three decision options: cold, very cold and extremely cold. It may be noted that the decision options in this situation can be conveniently put across a decision spectrum or graded scale and the final decision would depend on to which decision option the final evaluation is closet. The objective of this work is not to suggest if the decisions of cross-docking, primary storage and secondary storage are disjoint or they can fall on a spectrum. The main emphasis is on the modeling approach. Therefore, we will explain both the approaches here and leave it to the reader to interpret the decision options in a graded sense or disjointed sense. The benefits of graded decision making include their flexibility to vary policy variables along the spectrum, appropriateness for short-to-medium term operational and tactical decisions. On the other hand, disjoint decisions will be more rigid, not-easy-to-change and more suitable for strategic level decisions. Furthermore, graded decisions will provide more options on a continuous spectrum whereas the disjoint decisions represent very few well-defined discrete options.

For the disjointed decision options case, as evident from the Table 2 that decision rule 1 suggests a cross-docking decision with a degree of belief of '1'. Decision rules 2-3 suggest a primary storage decision with a degree of belief of '0'. Decision rules 4-5 suggest a reserve storage decision with a degree of belief of '0.5' while assuming equal weights for decisions 4 and 5. Therefore, there emerges a clear ranking for the decisions based on the degree of beliefs as cross-docking (1), reserve storage (0.5) and primary storage (0). In such a case, the decision makes will prefer a cross-docking decision.

For the graded decision options case, the three decision options can be put on a spectrum (scale 1–10) with cross-docking positioned at 1, primary storage positioned at 5 and reserve storage decision positioned at number 10. The degrees of belief of these decisions as calculated above i.e. 1 for cross-dock, 0 for primary storage and 0.5 for reserve storage are treated as their weights and a weighted average is calculated for the decision situation. The final decision will tend to balance at its weighted average which for the current example is 4. This weighted average is 3 units apart from cross-docking decision, 1 unit apart from primary storage and 6 units apart from the reserve storage. Hence the final situation

Table 3 Computation experience and model validation

Problem #	Input parameters: due date (h); primary stock (days); reserve stock (days)	Evaluation of decision options (membership functions)	Disjointed decision option
1	24;10,20	Cross dock(0); primary(0); reserve(0.5)	Reserve
2	150;3,30	Cross dock(0.5); primary(0); reserve(0.75)	Reserve
3	20;4,10	Cross dock(0.33); primary(0); reserve(0.5)	Reserve
4	12;1;60	Cross dock(1); primary(0.2); reserve(0)	Cross-dock
5	36;3;25	Cross dock(0.5); primary(0.25); reserve(0.25)	Cross-dock
6	96;10;25	Cross dock(0); primary(0); reserve(0.5)	Reserve
7	15;5;30	Cross dock(0.75); Primary(0); Reserve(0.5)	Cross-dock
8	*20;6;30*	Cross dock(0.33); primary(0); reserve(0.5)	Reserve
9	*22;2;10*	Cross dock(1); primary(0.42); reserve(0)	Cross-dock
10	*24;15;10*	Cross dock(0); primary(0); reserve(0.5)	Reserve
11	*15;3;70*	Cross dock(0.75); primary(0.375); reserve(0.5)	Cross-dock
12	*30;3;10*	Cross dock(0.5); primary(0.25); reserve(0.25)	Cross-dock

evaluates to a weighted average which is closest to the primary storage decision and a decision maker will be inclined to make primary storage decision.

The approach suggested in this chapter are modeled using MS excel spreadsheet and twelve problems have been tested. The results are summarized in Table 3 for the disjointed decision case. A graded decision case would often require the decision maker to assign weights and those results requiring subjective weights are not listed in the computation experience. In the 12 problem listed below, the results mainly led to the cross-dock or reserve decision and none of the results led to the primary storage decision because that decision has been too restricted through 'AND' statements. However, in a graded decision making scenario where we compute the weighted average, several results will led to the primary storage decision.

8 Conclusions and Further Work

Storage activities form an integral part of supply chain planning and execution process. These activities are repetitive, time consuming and absorb a significant portion the logistic dollar. Therefore, these activities need to be planned and carried out using sound logistical decisions that will result in substantial cost savings. The repetitive nature of these decisions act as a multiplier for cost savings. When the shipments are received in a warehouse, a number of decisions need to be made by the warehouse managers. One such decision and probably the first decision encountered by a warehouse manager is where to store the product. Warehouse managers have a number of options available, for example, long-term storage, and short-term primary storage or cross-docking. The final decision depends on the expert judgment and knowledge of the decision maker and it based on certain decision rules. However, it would be impossible for a human being to comprehend these decision rules and process the information involved in real time to take on the spot decisions. In the present chapter, we have demonstrated how the linguistic variable concept and fuzzy set theory can be effectively used to capture, present, organize and synthesize the expert knowledge in terms of fuzzy decision rules to provide a powerful tool to the decision maker. Fuzzy logic can put these rules to work on your data in order to make rational decisions. The chapter is not intended to provide a mathematically rigid decision approach or to provide an exhaustive list of decision rules but how fuzzy decision rules can be used to process information for repetitive activities such as the ones described above to arrive at sound business decisions.

Further work on the model will involve testing the approach on data sets obtained from various real-life scenarios or conducting some simulation experiments on the approach. Another development of this approach may consider the economic impact of various decision options in a fuzzy linguistic sense. The author also endeavors to develop a decision making software tool based on the present approach that would help managers to make such repetitive decisions. Further enhancement in voice recognition technology may make it possible to use linguistic approach as a framework for data entry and voice communication with computers using linguistic terms.

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Fuzzy C-Means Algorithm with Fixed Cluster Centers for Uncapacitated Facility Location Problems: Turkish Case Study

Şakir Esnaf, Tarık Küçükdeniz and Nükhet Tunçbilek

Abstract In this study, a new algorithm to solve uncapacitated facility location problems is proposed. The algorithm is a special version of original fuzzy c-means (FCM) algorithm. In FCM algorithm, unlabeled data are clustered and the cluster centers are determined according to priori known stopping criterion iteratively. Unlike the original FCM, the proposed algorithm allows the unlabeled data are to be assigned with single iteration to related clusters centers, which are assumed to be fixed and known a priori like location of facilities according to their degrees of membership. First, the proposed algorithm is applied to various benchmark problems from literature and compared with integer programming. Second, the proposed algorithm is tested and compared with particle swarm optimization (PSO) and artificial bee colony optimization (ABC) algorithms based uncapacitated facility location method on alternative versions such as discrete, continuous, discrete with local search and continuous with local search in literature for a Turkish fertilizer producer's real data. Numerical results obtained from real life application show that the proposed algorithm outperforms the PSO-based and ABC-based algorithms.

Keywords Uncapacitated facility location problems · Fuzzy C-means · Fixed cluster centers

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1 Introduction

The facility location problem is the classical, combinatorial problem of finding the number and locations of a set of facilities (warehouses, plants, machines etc.) and assigning customers to these in such a way that the total cost is minimized. If an arbitrary number of customers can be connected to a facility, the problem is called an uncapacitated facility location problem (Wu et al. 2006). The uncapacitated facility location (UFL) problem assumes the cost of satisfying the customer requirements has two components: a fixed cost of setting up a facility in a given site and a transportation cost of satisfying the customer requirements from a facility (Ghosh 2003). Many researchers developed and proposed exact methods to solve uncapacitated facility location problems [Khumawala (1972), Erlenkotter (1978), Van Roy (1986), Barcelo et al. (1990), Klose (1998)]. Exact algorithms for this problem do exist, but its NP-hard nature makes heuristics the natural choice for larger instances (Resende and Werneck 2006). There are also several papers deal with solving UFL problem with heuristics and metaheuristics methods [Ghosh (2003), Hsieh and Tien (2004), Levin and Ben-Israel (2004), Xu and Xu (2005), Greistorfer and Rego (2006), Sevkli and Guner (2006), Dohn et al. (2007), Guner and Sevkli (2008), Kashan et al. (2012)]. Hence the performance of meta-heuristics methods depends on initial parameter values, these methods calculate the optimal or near-optimal solution of an UFL problem as an interval instead of a single value. Also this interval can change for each run. However, solution assignments can be determined exactly when clustering is applied. The crisp (classical hard) clustering methods are proposed to solve uncapacitated discrete and continuous multi-facility location problems (mflps) (Levin and Ben-Israel 2004). In contrast with crisp clustering tools, fuzzy clustering allows gradual memberships of data points to clusters in [0, 1] [Hu and Sheu (2003), Döring et al. (2006)]. Unlike exact and metaheuristics methods, this feature allows customers to split their demand among facilities. The fuzzy c-means clustering algorithm (Bezdek 1981) is the most popular and its extensions have many successful applications in finance, medicine, MR imaging, inventory planning, cellular manufacturing, acoustics, upwelling prediction in oceans and face recognition. However, the number of studies that employ fuzzy c-means for solving continuous uncapacitated mflps is very limited [Chepoi and Dimitrescu (1999), Zalik (2006), Ayoub et al. (2007), Esnaf and Kucukdeniz (2009, 2013)], single-iterated version of fuzzy c-means algorithm for solving very large UFL problems like exact methods could not be found in the literature. This study is proposed to contribute to fill the above-mentioned gaps.

In this study, an algorithm, which is a special version of FCM algorithm, is proposed for uncapacitated facility location problem, by using the fuzzy clustering analysis in a different way. Fuzzy clustering based new model that gives near-optimal solutions is explained in this study. Model cannot guarantee the optimal solution like other heuristics and metaheuristics; however, they can be an alternative for solving especially extremely large discrete optimization problems which could not be solved by exact methods. The proposed algorithm is a single iterated version of the fuzzy c-means algorithm, and it is adopted to solve discrete uncapacitated facility location problems with fixed cluster centers. Unlike the original fuzzy c-means algorithm, this algorithm assigns points (customers) to known cluster centers (supply centers) with a single iteration according to membership degrees. In the real-life problems, companies may want to add new facilities to existing ones. With this method, this need is satisfied, and logistics operations of a Turkish fertilizer producer are improved. The proposed unique method outperforms recently developed PSO-based and ABC-based alternative models. There is no study that can be found in literature to solve UFL, which has discrete nature by using modified fuzzy c-means.

The remainder of this chapter is organized as follows. Procedures of methodology and the proposed method are presented in Sect. 2. Experimental results are reported in Sect. 3. A case study, with numerical results generated using the proposed method and comparisons are summarized in Sect. 4. Finally, the concluding remarks are presented in Sect. 5.

2 Fuzzy C-Means Algorithm with Fixed Cluster Centers for Uncapacitated Facility Location Problem

The algorithm presented in this chapter is a specific version of Bezdek's (1981) classical fuzzy c-means clustering algorithm, which is adopted for solving location and allocation problems. In the fuzzy c-means algorithm, known data are clustered for a desired number of cluster centers. The centers of gravities of these clusters are determined iteratively according to a specific termination criterion. In the proposed algorithm, cluster centers are assumed to be known beforehand. Data belonging to these clusters are determined with one iteration according to their degree of membership; and allocation is thus made. Here in this study cluster center denotes facility location, unlabeled data denotes unassigned demand points. If geographical coordinates are not known, transportation costs between demand points and facilities could be used. The total transportation cost is a function of transported quantities and distances that result from matching at each cluster.

2.1 The Extended Uncapacitated Facility Location Problem

A UFL problem with m customers and n candidate facility sites can be represented by a network with $m + n$ nodes and mn arcs. In the UFL model, f_j is used to represent the cost of opening facility j and c_{ij} is used to represent the cost of serving customer i from facility j or assigning customer i to facility j. We assume that $c_{ij} \geq 0$ for all $i = 1,\ldots, m$ and $j = 1,\ldots, n$ and $f_j > 0$ for all $j = 1,\ldots, n$. A binary variable y_j is used to represent the status of facility j in the model. Facility j will be opened only if $y_j = 1$ in the solution. A binary variable x_{ij} is used for the road from customer i to facility j in the model. Customer i will be served by facility j only if $x_{ij} = 1$ in the solution. However, each x_{ij} can be treated as a continuous variable and will have a binary value in the solution. The solution process of the UFL problem is to find an optimal solution that satisfies all customer demand and minimizes the total cost. The UFL problem can be formally stated as Sun (2006).

$$Min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{j=1}^{n} f_j y_j \tag{1}$$

$$\text{s.t.} \sum_{j=1}^{n} x_{ij} = 1 \qquad for\ i = 1, \ldots\ldots, m \tag{2}$$

$$x_{ij} \leq y_j \qquad for\ i = 1, \ldots\ldots, m\ and\ j = 1, \ldots\ldots, n \tag{3}$$

$$x_{ij} \geq 0 \qquad for\ i = 1, \ldots\ldots, m\ and\ j = 1, \ldots\ldots .n \tag{4}$$

$$y_j = \{0, 1\} \qquad for\ j = 1, \ldots\ldots, n \tag{5}$$

There is no limit of capacities for any candidate facility and whole demand of each customer has to be assigned to one of the facilities.

In the case of extending the existing locations in the logistics networks with new facilities, which is often occurred in real life problems, the UFL problem can be reformulated as follows.

$$Min \sum_{i=1}^{m} \sum_{k=1}^{o} c_{ik} x_{ik} + \sum_{i=1}^{m} \sum_{j=o+1}^{n+o} c_{ij} x_{ij} + \sum_{k=1}^{o} f_k + \sum_{j=o+1}^{n+o} f_i y_j, j \in S_c, k \in S_e \tag{6}$$

$$\text{s.t.} \sum_{k=1}^{o} x_{ik} + \sum_{j=o+1}^{n+o} x_{ij} = 1 \qquad for\ i = 1, \ldots\ldots, m \tag{7}$$

$$x_{ij} \leq y_j \qquad for\ i = 1, \ldots\ldots, m\ and\ j = 1, \ldots\ldots, n \tag{8}$$

$$\sum_{j=o+1}^{n+o} y_j = n_{max} \qquad for\ j = 1, \ldots\ldots, n \tag{9}$$

$$x_{ij}, x_{ik} \geq 0 \qquad for\ i = 1, \ldots\ldots, m,\ j = 1, \ldots\ldots, n\ and\ k = 1, \ldots\ldots, o \tag{10}$$

$$y_j = \{0, 1\} \qquad for\ j = 1, \ldots\ldots, n \tag{11}$$

where f_k is the fixed cost of existing facility k, c_{ik} is the cost of serving customer i from existing facility k, x_{ik} is the road from customer i to existing facility k (Customer i will be served by existing facility k only if $x_{ik} = 1$ in the solution),

o is the number of existing facilities, n is the number of candidate facilities, n_{max} is the maximum number of opening facilities, S_c is set of opening facilities and S_e is the set of existing facilities.

2.1.1 Illustrative Example of the Extended UFL Problem

Following simple example is developed to illustrate extended UFL problem, which considers existing and opening facilities. In this example first and second facilities are the existing facilities and we want to open three new facilities among five alternatives (facilities 3–7). There are eight customers in the system and transportations cost for each customer to each facility are given.

$$\begin{aligned}\text{Total cost } = & \text{ sum of fixed costs for all existing facilities} \\ & + \text{ sum of fixed costs for open new facilities} \\ & + \text{ sum of transportation cost for each customer from a facility.}\end{aligned}$$

(The facility that will serve to each customer is chosen so that it has minimum cost among all open facilities.)

Considering the 7-facility to 8-customer example shown in Table 1, the total cost of open facility vector (y_j) can be calculated as follows:

$$\begin{aligned}\text{Total Cost } = & 12 + 4 \\ & + 7 + 9 + 10 \\ & + \min(2, 8, 7, 1, 11) \\ & + \min(-^{**}, 11, 4, 12, 6) \\ & + \min(11, 6, 5, 8, 4) \\ & + \min(19, 5, 16, 13, 8) \\ & + \min(3, 12, 7, 10, 5) \\ & + \min(4, 18, 6, -^{**}, 9) \\ & + \min(6, 9, 11, 3, 20) \\ & + \min(7, 10, 12, 4, 22) \\ = & 16 + 26 + 1 + 4 + 4 + 5 + 3 + 4 + 3 + 4 = 70\end{aligned}$$

If the number of open new facilities is desired to be limited by a maximum number, then "maximum number of new locations" parameter is used. In this case, a feasible solution should have a number of open new facility locations less than or equal to desired maximum number. Algorithm assigns infinite total cost, when the solution contains more than desired number of open facility locations. Hence, other feasible solutions are preferred. In other words, assigning extremely large cost penalizes infeasible solutions.

Table 1 An example of 7-facility to 8-customer

Facility locations		1	2	3	4	5	6	7
Fixed cost (existing facility)		12	4	–	–	–	–	–
Fixed cost (new facility)		–	–	5	3	7	9	10
Existing facility vector (always equals to one)		1	1	–	–	–	–	–
Open new facility vector (y_j)		–[a]	–[a]	0	0	1	1	1
Max. number of new facilities		3						
Customers	1	2	8	3	6	7	1	11
	2	–[b]	11	5	8	4	12	6
	3	11	6	6	14	5	8	4
	4	19	5	18	21	16	13	8
	5	3	12	9	8	7	10	5
	6	4	18	7	9	6	–[b]	9
	7	6	9	10	7	11	3	20
	8	7	10	11	10	12	4	22

[a] At each iteration, solution algorithm used offers several binary "open new facility vectors" as alternative solutions for the UFL problem. Each one of these vectors is used to calculate the total cost. For existing open facilities, each facility's position value is taken as 1, disregarding the value for that facility at open new facility vector

[b] If it is not possible to deliver from a facility to a customer, there is no transportation cost given in this matrix

2.2 *Fuzzy C-Means Algorithm with Fixed Cluster Centers*

The FCM can be seen as the fuzzified version of the k-means algorithm and is based on the minimization of an objective function called c-means functional (Kenesei et al. 2006):

$$J(X;U;V) = \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{ik})^m \|x_k - v_i\|_A^2 \tag{12}$$

where A_i is a set of objects (data points) in the i-th cluster, unlike original FCM v_i is the known mean for that points over cluster i here, $V = [v_1, v_2, \ldots.., v_c]$, $v_i \in R^n$ is a vector of cluster prototypes (centers), which have to be known a priori, $D_{ikA}^2 = \|x_k - v_i\|_A^2 = (x_k - v_i)^T A(x_k - v_i)$ is a squared inner product distance norm, and the Nxc matrix U $= [\mu_{ik}]$ represents the fuzzy partitions, where μ_{ik} denotes the membership degree that the i-th data point belongs to the k-th cluster. Its conditions are given by:

$$\mu_{ij} \in [0,1], \forall_i, k, \sum_{k=1}^{c}\mu_{ik} = 1, \forall_i, 0 < \sum_{i=1}^{N}\mu_{ik} < N, \forall_k \tag{13}$$

Unlike original FCM algorithm, proposed algorithm creates clusters around known cluster centers.

Given the dataset X which includes geographical X and Y coordinates, the number of clusters $1 < c < N$, the weighting exponent $m > 1$, and the norm-inducing matrix A, the algorithm tracks the following steps with Balasko et al. notation (2005). It should be noted that the termination tolerance $\varepsilon > 0$ is not required.

Step 1: Include the known cluster prototypes:

$$V_i, 1 \leq i \leq c \tag{14}$$

Step 2: Compute the distances:

$$D_{ikA}^2 = (x_k - v_i)^T A(x_k - v_i), 1 \leq i \leq c, 1 \leq k \leq N. \tag{15}$$

Step 3: Compute the partition matrix:

$$\mu_{i,k} = \frac{1}{\sum_{j=1}^{c} \left(D_{ikA}/D_{jkA}\right)^{2/(m-1)}} \tag{16}$$

The computer that was used during test runs in both Sects. 3 and 4 has the following configuration; Intel CPU at 2.26 GHz with 8 GB of RAM. Codes of the FCM with fixed cluster centers and integer programming were developed and executed by MATLAB R2012b. The main inputs for FCM clustering algorithm are x and y coordinates of demand points. In this study FCM is applied to uncapacitated facility location problem using weighting exponent, m, is taken as 2 and termination tolerance is determined as 10^{-6}.

Therefore customers are grouped by FCM in respect to their geographical locations. Then plants are located at the proposed cluster centers.

The pseudo code of the proposed algorithm is presented as follows:

Initialize

if n is the number of additional facilities to open,

then S' is all combinations of the additional n facilities to existing ones: $S' = S_e \cup \binom{S_c}{n}$

repeat for each $s \in S'$

```
{
  Run FCM algorithm with fixed cluster centers on s
  Assign customers to the facilities by using membership
     values
  Calculate the cost of the assignments
}
Choose the solution with minimum cost among s
```

3 Experimental Study

The objective of this section is to evaluate FCM with fixed cluster center algorithm's performance against to integer programming. Also it is tried to be shown that the proposed algorithm solves the original UFL problem, which is given with formulas (6–11) in Sect. 2 with a same cost and lower CPU time then the other benchmark algorithms. Different data sets from different sources are used in order to prove this claim. The data set (pmedcap1-15) is the combination of the pmedcap 1-15 datasets, totally containing 1000 data points. Pmedcap16-20 are also combined into a separate dataset (pmedcap16-20). Also all pmedcap datasets are combined into a single dataset (pmedcap All). These pmedcap datasets are taken from Osman and Christofides (1994). Another two datasets are Taillard (2003)'s dataset containing 2863 data points and Bongartz et al. (1994)'s dataset containing 287 data points. Also three randomly generated big datasets, containing 10000 and 50000 data points are employed for benchmarking. The last data set is taken from the real life example, which is explained in Sect. 4.

The results of the FCM with fixed cluster center algorithm and integer programming are given in Table 2, and computational costs of each algorithm are shown in Table 3.

4 Case Study

The algorithm proposed in the second section of this study with formulas (6–11) is used in developing a model for solving the location-allocation problem of a fertilizer production firm and in proposing new port locations for the models. The supply chain and logistics activities of the firm are examined. Thus, it is intended to deliver the products on time, at the best level of customer services and with the lowest possible costs.

The distributed fertilizers are grouped into two categories as imported and domestic. Domestic fertilizers are produced in the facilities of the firm located in Yarimca and are distributed to the demand points from here. Imported products are transported from abroad by ships and are unloaded onto one of the four ports used at present. They are distributed to the relevant demand points from the depots at these ports.

Logistics activities for the distribution of the imported and domestic products of the firm are analyzed for the current situation and the proposed alternative ports. The influences of the usage of alternative ports on costs are then presented. The four ports used at present for the imported products are Yarimca, Izmir, Iskenderun and Samsun. The proposed seven ports are Tekirdag, Bandirma, Canakkale, Antalya, Mersin, Giresun and Rize. The aim is to determine the ports which should be added to the existing four ports with different combinations such as four plus one; four plus two; and four plus three. Special software which is developed for this

Table 2 Transportation cost performances of the FCM with fixed cluster center algorithm and integer programming

Multi-facility location allocation data	Number of demand points	Number of candidate facilities	Number of facilities/clusters to be opened	Number of combinations	Proposed FCM algorithm	Integer programming
Bongartz et al. (1994)	287	10	5	252	38,381	38,381
Bongartz et al. (1994)	287	15	5	3,003	45,528	45,528
Bongartz et al. (1994)	287	20	10	184,756	61,475	n/a*
Bongartz et al. (1994)	287	25	5	53,130	28,414	n/a*
Pmedcap1-15	1,000	10	5	252	193,403	193,403
Pmedcap1-15	1,000	15	5	3,003	199,858	199,858
Pmedcap1-15	1,000	20	10	184,756	140,393	n/a*
Pmedcap1-15	1,000	25	5	53,130	190,344	n/a*
Pmedcap16-20	498	10	5	252	116,228	116,228
Pmedcap16-20	498	15	5	3,003	105,549	105,549
Pmedcap16-20	498	20	10	184,756	72,385	n/a*
Pmedcap16-20	498	25	5	53,130	97,020	n/a*
Pmedcap All	1,481	10	5	252	289,726	289,726
Pmedcap All	1,481	15	5	3,003	301,250	301,250
Pmedcap All	1,481	20	10	184,756	210,274	n/a*
Pmedcap All	1,481	25	5	53,130	288,593	n/a*
Taillard (2003)	2,863	10	5	252	3,247,444,620	n/a*
Taillard (2003)	2,863	15	5	3,003	2,706,513,315	n/a*
Taillard (2003)	2,863	20	10	184,756	2,063,109,134	n/a*
Taillard (2003)	2,863	25	5	53,130	2,323,267,984	n/a*
Random dataset 1	10,000	10	5	252	102,017,525	n/a*
Random dataset 1	10,000	15	5	3,003	91,850,319	n/a*
Random dataset 1	10,000	20	10	184,756	69,679,103	n/a*
Random dataset 1	10,000	25	5	53,130	93,685,819	n/a*
Random dataset 2	10,000	10	5	252	98,448,492	n/a*

(continued)

Table 2 (continued)

Multi-facility location allocation data	Number of demand points	Number of candidate facilities	Number of facilities/clusters to be opened	Number of combinations	Proposed FCM algorithm	Integer programming
Random dataset 2	10,000	15	5	3,003	99,346,475	n/a*
Random dataset 2	10,000	20	10	184,756	68,599,727	n/a*
Random dataset 2	10,000	25	5	53,130	95,635,940	n/a*
Random dataset 3	50,000	10	5	252	27,951,357,511	n/a*
Random dataset 3	50,000	15	5	3,003	23,001,342,620	n/a*
Random dataset 3	50,000	20	10	184,756	17,906,208,189	n/a*
Random dataset 3	50,000	25	5	53,130	23,156,742,063	n/a*

*n/a** Incalculable due to the out of memory error

Table 3 CPU times of the FCM with fixed cluster center algorithm and integer programming in s

Multi-facility location allocation data	Number of demand points	Number of candidate facilities	Number of facilities/clusters to be opened	Number of combinations	Proposed FCM algorithm (s)	Integer programming (s)
Bongartz et al. (1994)	287	10	5	252	0.55	3.86
Bongartz et al. (1994)	287	15	5	3,003	6.19	5.25
Bongartz et al. (1994)	287	20	10	184,756	447.41	–
Bongartz et al. (1994)	287	25	5	53,130	108.70	–
Pmedcap1-15	1,000	10	5	252	0.89	59.89
Pmedcap1-15	1,000	15	5	3,003	10.55	672.55
Pmedcap1-15	1,000	20	10	184,756	745.64	–
Pmedcap1-15	1,000	25	5	53,130	183.80	–
Pmedcap16-20	498	10	5	252	0.78	15.57
Pmedcap16-20	498	15	5	3,003	6.98	24.49
Pmedcap16-20	498	20	10	184,756	500.36	–
Pmedcap16-20	498	25	5	53,130	123.19	–
Pmedcap All	1,481	10	5	252	1.05	560.32
Pmedcap All	1,481	15	5	3,003	10.35	3,959.74
Pmedcap All	1,481	20	10	184,756	788.97	–
Pmedcap All	1,481	25	5	53,130	175.11	–
Taillard (2003)	2,863	10	5	252	1.22	–
Taillard (2003)	2,863	15	5	3,003	13.18	–
Taillard (2003)	2,863	20	10	184,756	1,169.81	–
Taillard (2003)	2,863	25	5	53,130	225.24	–
Random dataset 1	10,000	10	5	252	3.07	–
Random dataset 1	10,000	15	5	3,003	32.37	–
Random dataset 1	10,000	20	10	184,756	3,422.76	–
Random dataset 1	10,000	25	5	53,130	521.68	–
Random dataset 2	10,000	10	5	252	2.65	–

(continued)

Table 3 (continued)

Multi-facility location allocation data	Number of demand points	Number of candidate facilities	Number of facilities/clusters to be opened	Number of combinations	Proposed FCM algorithm (s)	Integer programming (s)
Random dataset 2	10,000	15	5	3,003	31.19	–
Random dataset 2	10,000	20	10	184,756	3,309.87	–
Random dataset 2	10,000	25	5	53,130	498.78	–
Random dataset 3	50,000	10	5	252	17.14	–
Random dataset 3	50,000	15	5	3,003	194.39	–
Random dataset 3	50,000	20	10	184,756	22,404.28	–
Random dataset 3	50,000	25	5	53,130	3,248.29	–

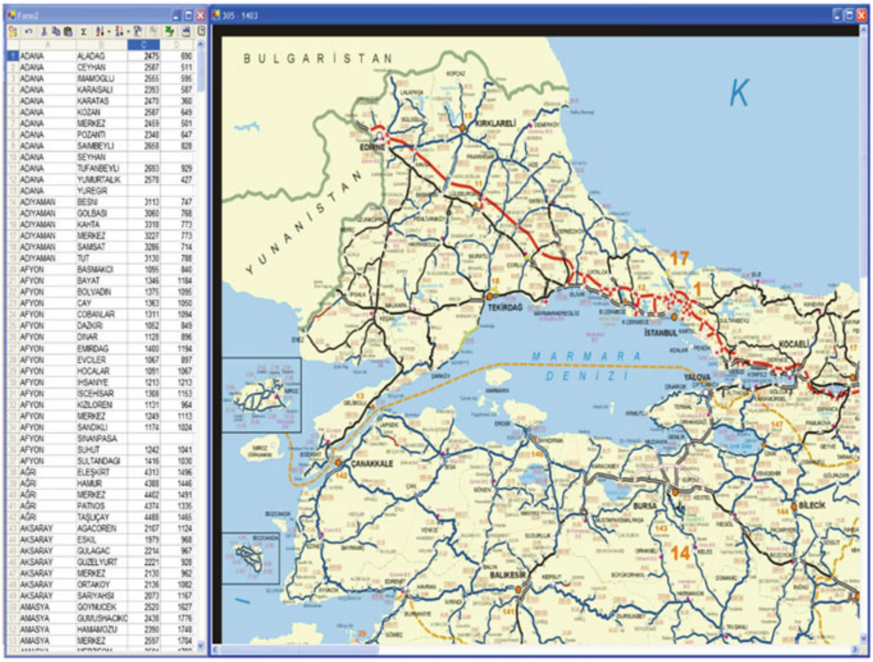

Fig. 1 Interface of the software developed for calculating the point coordinates

Table 4 Facility coordinates calculated by special software developed for this study

Facility	X	Y
Bandirma	626	1686
Iskenderun (existing)	2684	365
Mersin	2277	437
Rize	3738	1865
Samsun (existing)	2713	1920
Yarimca (existing)	1079	1789
Antalya	1250	501
Tekirdag	242	1649
Canakkale	3216	1801
Giresun	537	1901
Izmir (existing)	386	1072

study calculates the X and Y coordinates of the demand points plotted on a map of Turkey, by taking the lower left corner of the map as a reference. In all the analysis, this coordinate system is taken into account. The interface of this software is depicted in Fig. 1. Calculated facility coordinates by special software developed for this study are given in Table 4.

Table 5 Transportation cost performances of the FCM with fixed cluster center algorithm and integer programming

Multi-facility location allocation data	Number of demand points	Number of candidate facilities	Number of facilities/clusters to be opened	Proposed FCM algorithm	Integer programming
Real world dataset	768	11	4	56,088,182	n/a*
Real world dataset	768	11	5	55,264,095	n/a*
Real world dataset	768	11	6	56,055,000	n/a*
Real world dataset	768	11	7	56,970,040	n/a*

*n/a** Incalculable due to the out of memory error

Table 6 CPU times of the FCM with fixed cluster center algorithm

Multi-facility location allocation data	Number of demand points	Number of candidate facilities	Number of facilities/clusters to be opened	Number of combinations	Proposed FCM algorithm (s)
Real world dataset	768	11	4	–	3.02
Real world dataset	768	11	5	462	4.00
Real world dataset	768	11	6	462	3.90
Real world dataset	768	11	7	330	2.74

For this purpose the necessary data are determined and they are tried to be sorted out of the firm's existing enterprise resource planning (ERP) system. Transport and delivery data obtained from the system are carried onto an SQL server database. At the data collection stage, domestic and import transport prices determined for transport from the present facilities/depots to the provincial centers, coordinates of the 768 demand points, information of the 74823 transports made in a year, 40561 orders belonging to a specific year, unit costs per ton of the activities in the ports and real distances between the demand points and the facilities are also obtained.

The results of the FCM with fixed cluster center algorithm and integer programming are given in Table 5, and computational costs of the proposed algorithm is shown in Table 6.

4.1 Benchmark Algorithms

In order to make a comparison, the same problem was solved with Continuous PSO (CPSO) proposed by Sevkli and Guner (2006) and Discrete PSO (DPSO) proposed by Guner and Sevkli (2008) and discrete ABC algorithm which is described in Sect. 4.1.1 in Kashan et al. (2012). These PSO-based and ABC-based algorithms were applied to the problem with the development of software. The results of the algorithms are compared to each other.

It should be noted that all allocation-based solution methods for facility location problems given in the literature, including PSO-based algorithms and ABC-based algorithms couldn't find a solution according to the X and Y coordinates. Moreover, the algorithms given in the literature allow for allocation from a source to only one demand point; on the other hand, the proposed FCM-based method overcomes this obstacle by allocation according to the degree of membership. However, in order to make an objective comparison with a PSO and ABC-based algorithms, this allocation characteristic of the FCM-based method was removed. A demand point is assumed to meet its demand from a facility with which it has the highest degree of membership. This modified algorithm is referred to as winner-takes-all allocation algorithm.

4.1.1 Artificial Bee Colony Optimization Algorithm

Artificial Bee Colony (ABC) algorithm is one of the most recently introduced swarm intelligence based meta-heuristic method by Karaboga (2005). The algorithm has been motivated by the intelligent behavior of honeybees. This algorithm is as simple as Particle Swarm Optimization (PSO) and Differential Evolution (DE) algorithms.

Karaboga proposed the artificial bee colony (ABC) algorithm, which is inspired by foraging behavior of honeybees. In the ABC algorithm, a problem is solved by exploring good solutions, which are represented as food sources. The quality of the solution is represented by the nectar amount of that food source. In this algorithm, the first half of the bee colony are employed bees, the second half are the onlooker bees. The number of food sources is same as the number of employed bees. It is assumed that there is only one employed bee for every food source. Each employed bee is placed on a food source, and starts extracting nectar. The employed bee becomes a scout when the food source has no more nectar and moves away to look for another food source. As soon as a scout bee finds a new food source it again becomes an employed bee. The ABC algorithm initially places all employed bees on randomly generated food sources (solutions). Then iteratively, every employed bee determines a food source nearby their currently associated food source and evaluates its nectar amount (fitness). If it has more nectar than that of its current food source, then that employed bee moves to this new food source, otherwise it stays on its current food source.

The search can be materialized with following steps:

- Employed bees locate a food source close to current food source, which is in their memory.
- The other half of the colony, onlooker bees, wait in the hive and get information about rich food sources from employed bees, which returned into the hive. Then the onlookers decide to go to one of the food sources and locate to a food source, which is close to this food source.

- After some period, food source may be exhausted. An employed bee on such a food source becomes a scout and starts to search a new food source randomly.

The pseudo code of the ABC algorithm is presented as follows:

```
Initialize food sources at random positions and locate
  employed bees
repeat
  Move the employed bees around their food sources and
     determine their nectar amounts.
  Move the onlookers towards rich food sources and
     determine their nectar amounts.
  Determine exhausted food sources and assign employed
     bees as scout bees for searching new food sources.
  Memorize the best food source found so far.
until requirements are met
```

This cycle is repeated up to a predefined number of iterations or predefined limit on CPU time.

A food source can be interpreted as a possible solution to the optimization problem. The nectar amount of a food source represents the quality of the solution represented by that food source.

Scout bees move to new directions so that colony can explore new food sources. While onlookers and employed bees exploiting good solutions in the search space, the scouts explore new unknown solutions (Karaboga and Basturk 2008).

Onlooker bees move according to the information taken from an employed bee that is returned back to the hive. When employed bees have finished collecting nectar, they come back to their hive and share information with the onlooker bees by dancing longer or shorter, according to nectar amount of the last visited food source. Onlooker bees select a food source according to a probability, which is proportional to the nectar amount of that food source. The probability p_i of selecting a food source i is determined using the following expression:

$$p_i = \frac{fit_i}{\sum_{n=1}^{SN} fit_n} \tag{17}$$

fit_i fitness value of i-th solution which represents nectar amount at the food source at i-th position

SN number of employed bees (also number of food sources).

Since the objective is minimizing $f(x)$ values, fitness function is calculated as shown in (18), so that smaller $f(x)$ values get higher fitness. In the meantime, $f(x)$ values that are close to zero will get fitness close to 1. If $f(x)$ has greater values, fitness value gets closer to zero. Hence fitness values can be used as weights for probability of selection in (17).

$$fitness(i) = \begin{cases} 1 + |f(i)|, & f(i) < 0 \\ 1/(f(i) + 1), & f(i) \geq 0 \end{cases} \tag{18}$$

This weighted probability calculation shows that good food sources attract more onlookers than the bad ones. After all onlookers have decided which food source to move to, each of them determines a food source in the neighborhood of their current food source and computes its fitness. Onlookers try to determine the best food source among all the neighboring locations nearby a particular food source i and it will be the new location of the food source i. After a predetermined number of iterations, if a solution represented by a particular food source does not improve, then that food source is abandoned by its associated employed bee, it becomes a scout and starts searching for a new food source randomly. In other words, this scout is assigned to a randomly generated food source (solution) and its status is changed from scout to employed bee. This process is repeated until the termination condition is satisfied.

Movement of an employed bee around its current position is probabilistically formulated. Scanning food sources in the neighborhood of a particular food source is done by altering the value of one randomly chosen solution parameter (dimension) and keeping other parameters unchanged. The value of the chosen parameter is changed by using the following formula:

$$v_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}) \tag{19}$$

where

j 1,.., D randomly chosen dimension

v_{ij} new candidate location

x_{ij} current location

Φ_{ij} random factor between −1 and +1 which is generated by uniform probability distribution

k a randomly chosen neighbor, where $i \neq k$

x_{kj} location of a randomly chosen neighbor (k) at chosen dimension j, where $i \neq k$ and $k \in 1, .., SN$.

If the calculated value v_{ij} exceeds the acceptable range for dimension j, it is set to the corresponding extreme value in that range. (x_{minj} and x_{maxj})

Fitness values for x_{ij} and v_{ij} are compared and the one with better fitness value is chosen as the new position for i-th employed bee. This is greedy selection process. If nectar amount of the candidate location is better than the present one, the bee forgets the present location and memorizes the candidate location produced by (19). Otherwise, the bee keeps its present location in the memory.

Onlooker bees also set their location according to formula (19), same as employed bees.

Behavior of scouts is also probabilistically defined as follows:

$$x_i^j = x_{min}^j + rand\,[0,1]\left(x_{max}^j - x_{min}^j\right) \tag{20}$$

If a particular employed bee does not improve the solution in a predefined number of iterations called "limit", then that employed bee becomes a scout by leaving current position and looking for a new food source at randomly set new position, generated by formula (20).

In formulations explained above, it is shown that basic ABC has three control parameters: The number of food sources, which is equal to the number of employed, or onlooker bees (SN), the value of food limit and the maximum cycle number (MCN) (Karaboga and Akay 2009).

4.1.2 Artificial Bee Colony Optimization Algorithm for Uncapacitated Facility Location Problem

Parameters of ABC algorithm are number of employed/onlooker bees, number of scout bees, food limit and number of iterations. Throughout test runs in this study, these parameters are taken as 10, 1, 250 and 1000 respectively.

In UFL problem, there are n facilities under question. Some of these facilities will be opened and other will not, so that the total cost of serving customers from these opened facilities is optimized. Search space of the artificial bee colony is n-dimensional, where n is the total number of facilities.

n-dimensional position vector of each employed bee is mapped to a binary valued vector to determine whether each facility will be opened or not. Position values are converted to binary variables as follows:

$$y_i = \lfloor |x_i| (mod\ 2) \rfloor \tag{21}$$

The absolute value of a position value is first divided by 2 and then the remainder is floored to the nearest integer, which may be 0 or 1 (Guner and Sevkli 2008).

These values construct open facility vector (Yi) for n-facilities under question. Yi represents the opening or closing facilities based on the position vector Xi, $Yi = [y_{i1}, y_{i2}, y_{i3}, \ldots, y_{ik} \ldots, y_{in}]$, where y_{ik} represents opening or closing k-th facility of the i-th particle. For an n-facility problem, each particle contains n number of dimensions.

Pseudo code for ABC algorithm for UFL problem is as follows:

```
Initialize food source positions randomly
for all food sources do
  Calculate open facilities vector (21)
  Calculate and memorize fitness value
end for
while maximum iteration is not reached do
  for all employed bees do
    Randomly choose a dimension
    Move the position in chosen dimension by a random
      value between (-1;1) (12)
    Update position (13)
    Find open facility vectors (21)
    Calculate fitness value (total cost) using open
      facility vector
    if new fitness value is better than the previous one
then
      Move to the new position
    else
      Increment the counter by 1
      if counter > food limit then
        Employed bee becomes a scout bee
        A scout bee becomes an employed be at a randomly
          set position (9)
      end if
    end if
    Memorize best solution and position vector
  end for
  for all onlooker bees do
    Randomly choose a food source pointed by employed
      bees that return to hive (17)
    Evaluate the chosen position, the same way described
      for employed bees
    if fitness value is better then
      Move food source to evaluated position
    end if
  end for
  (ABC_LS) Apply local search to the best solution
end while
```

Similar to CPSO or DPSO, fitness value for each employed bee is calculated by adding up delivery costs from opened facilities to nearest customers.

4.1.3 Local Search for CPSO, DPSO and ABC Algorithms

It is observed that CPSO and DPSO could not reach optimal solutions for large problems. In order to improve solutions that are found by CPSO and DPSO, Guner and Sevkli (2008) employed a local search algorithm to CPSO and DPSO. The local search method looks for better solutions in the neighborhood of the global best particle in every generation.

The way of how neighbor solutions are produced is extremely essential to get better results. Local search algorithm takes the global best solution at the end of

```
Set globalbest open facility vector (Y_g) to s_0 (for DPSO_LS)
Set globalbest position vector (X_g) to s_0 (for CPSO_LS)
Set globalbest food source (X_g) to s_0 (for ABC_LS)
Modify s_0 based on η, κ and set to s
Set 0 to loop
repeat
  Apply Flip to s and get s_1
  if (f(s_1) ≤ f(s))
    Replace s with s_1
  else
    loop = loop + 1
until loop = n
if (f(s) ≤ f(s_0))
  Replace Y_g with s (for DPSO_LS)
  Replace X_g with s (for CPSO_LS)
  Replace X_g with s (for ABC_LS)
end if
```

Fig. 2 Pseudo code for local search algorithm

each iteration, and two randomly selected position values of the position vector are modified. In CPSO_{LS}, this is done by adding 1 to x_i (22). In DPSO_{LS}, value of y_i is flipped between 0 and 1, by subtracting y_i from 1 (23).

$$x_i \leftarrow x_i + 1, \tag{22}$$

$$y_i \leftarrow 1 - y_i. \tag{23}$$

This operation is repeated as far as new neighbor generates better solution. The local search algorithm is shown in Fig. 2. At the end of each iteration, global best result of CPSO and DPSO is taken as input by the local search algorithm. In order to generate diverse alternatives, two facilities (η and κ) are picked and their values are flipped.

The global best found at the end of each iteration of CPSO and DPSO is adopted as the initial solution by the local search algorithm. In order not to lose the best found and to diversify the solution, the global best is modified with two facilities, which are randomly chosen. Then, flip operator is applied to existing solution, as long as it gets a better solution. If the produced alternative does not have a better solution, loop counter is incremented by 1. Local search algorithm allows maximum of n unsuccessful trials in order to guarantee reasonable run time (Guner and Sevkli 2008).

The same local search is applied to ABC, since bees in ABC algorithm have n-dimensional position vectors similar to CPSO. Application of ABC algorithm to UFL problem was described in the Sect. 4.1.2.

4.2 Transportation Cost

The case study solved with the proposed model with the formulas (6–11) given in Sect. 2. Transportation cost of this problem is calculated by using the formula (24).

$$TC = \sum_{j=1}^{n}\sum_{i=1}^{c} w_{ij}^{import} c_{ij}^{import} + \sum_{i=1}^{c} w_i^{domestic} c_i^{domestic} + \sum_{j=1}^{n}\sum_{i=1}^{c} w_{ij}^{import} c_j^{handling} \quad (24)$$

where;

TC	Total Transportation cost
w_{ij}^{import}	Demand quantity of imported products of demand point i assigned to port in cluster j
$w_i^{domestic}$	Demand quantity of domestic products of demand point i
c_{ij}^{import}	Transportation cost of imported products between the port in cluster j and demand point i
$c_i^{domestic}$	Transportation cost of domestic products between production facility of domestic products and demand point i
$c_j^{handling}$	Handling cost of the port in cluster j.

4.3 Comparison of the Proposed Algorithm with CPSO, DPSO, $CPSO_{LS}$, $DPSO_{LS}$, ABC and ABC_{LS} Algorithms

FCM with known cluster centers algorithm is compared with Sevkli and Guner's (2006) CPSO; Guner and Sevkli's (2008) DPSO, $CPSO_{LS}$ and $DPSO_{LS}$; ABC and ABC_{LS} algorithms. Comparison is based on the total cost of offered solutions for facility location problem.

In order to make a fair comparison, CPSO, DPSO, $CPSO_{LS}$, $DPSO_{LS}$, ABC, and ABC_{LS} algorithms are programmed in the same platform, in conformance with the definition of Karaboga (2005), Sevkli and Guner (2006), and Guner and Sevkli (2008). Algorithms were developed as a Windows application with Microsoft Visual C++ 2008 Express Edition. The computer that was used during test runs has the following configuration; Intel Core i5-430 M processor at 2.26 GHz with 4 GB RAM. A sample form of this application can be seen in Fig. 3. On this form, for six different algorithms, PSO and ABC parameters are set to some default values. The user may change these parameters. It is possible to load problem data sets from TXT files, which have the format of any test data such as OR-library files or from Microsoft Excel files with a similar format.

In the top middle section, it is possible to see the input values that are loaded from the selected file. After running the algorithm with desired number of repeat

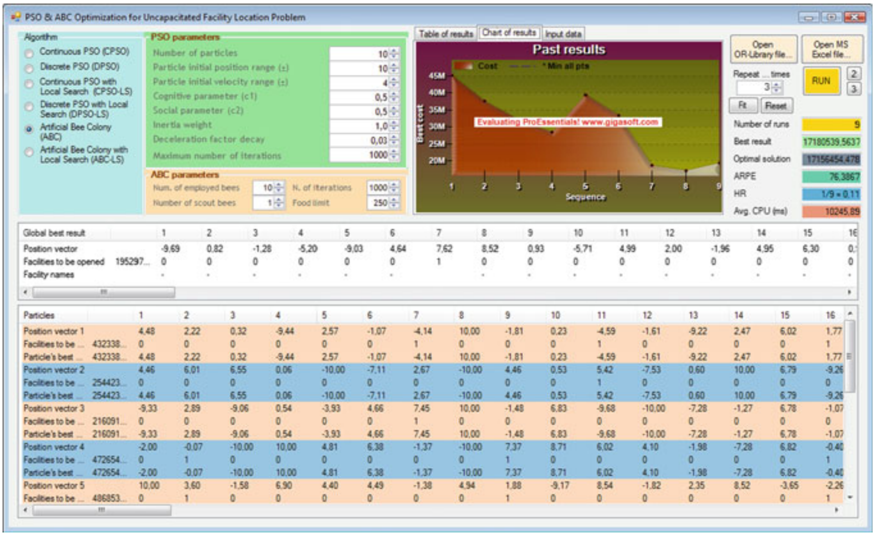

Fig. 3 Interface of the software developed for six algorithms

count, best result of each run is shown on a table and also on a plot chart, which is a component named ProEssentials by Gigasoft Company.

On the table in the bottom section, it is possible to see actual values of each particle. These are current position vector, particle's best location's position vector, and facility open vector depending on position vector. On the middle part, position vector and facility open vector for global best result is shown. On upper right section, results of all iterations are summarized.

CPSO, DPSO, $CPSO_{LS}$, $DPSO_{LS}$ algorithms were executed 3 times up to 1000 iterations with 10 particles, where for all dimensions, initial particle positions were randomly set to a value between −10 and 10, initial particle velocities were randomly set to a value between −4 and 4. Cognitive parameter (c_1) was taken as 0.5 and social parameter (c_2) was taken as 0.5. Inertia weight was 0.9 and deceleration factor decay was 0.03.

ABC and ABC_{LS} algorithms were also executed same number of times, 3 times up to 1000 iterations with 10 employed bees, 10 onlooker bees and 1 scout bee. Food limit was taken as 250, which is one fourth of the total number of iterations.

Number of iterations per run: 1000
Number of runs: 3
Number of particles/bees: 10

Termination criteria are completion of given number of iterations. This comparison does not include values for CPU time. It is observed that the algorithm can return the result in a reasonable time.

As explained above, the fertilizer company works with four ports at present, which are Yarimca, Samsun, Izmir and Iskenderun. In addition to the existing four

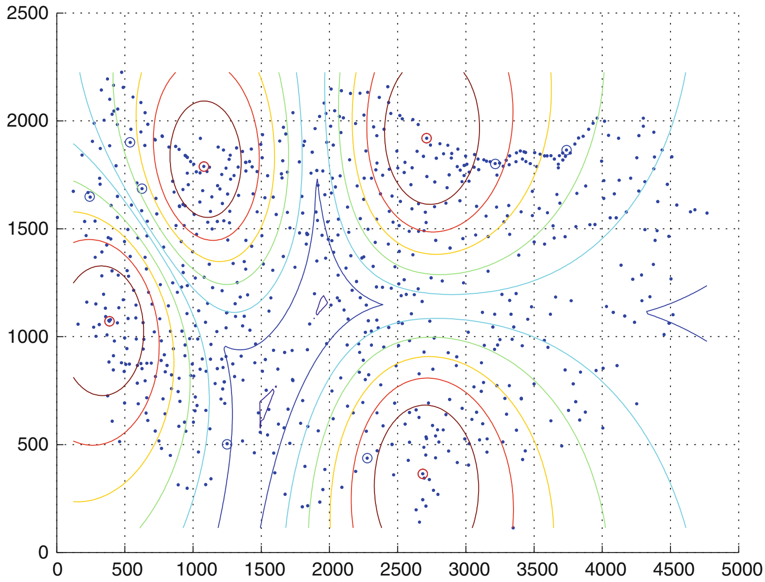

Fig. 4 FCM with known cluster centers algorithm applied to the presently used ports. (4 clusters)

ports, the ports of Tekirdag, Canakkale, Bandirma, Giresun, Rize, Mersin and Antalya are also evaluated as alternative ports. These alternative ports are considered with 4, 5, 6 and 7 cluster models having one port at each cluster center.

The CPSO, DPSO, $CPSO_{LS}$, $DPSO_{LS}$, ABC and ABC_{LS} algorithms are tested with four facilities to be opened by default and 0, 1, 2, 3 more new facilities to be opened additionally, which makes totally 4, 5, 6, 7 new facilities, respectively. Default open facilities are marked as open in the input file and number of allowed new facilities is also given as a parameter.

The four ports that are used by the firm at present are analyzed for the known cluster centers by using the fuzzy c-means clustering algorithm. In the five, six and seven port models, the four current ports are being continued to be used. These ports are included in the model with fuzzy c-means algorithm for fixed cluster centers. The resulting clusters are depicted in Figs. 4, 5, 6 and 7.

Results of the proposed and CPSO, DPSO, $CPSO_{LS}$, $DPSO_{LS}$, ABC, and ABC_{LS} algorithms are represented in Table 7. Cost values in this study have been changed for commercial confidentiality reasons.

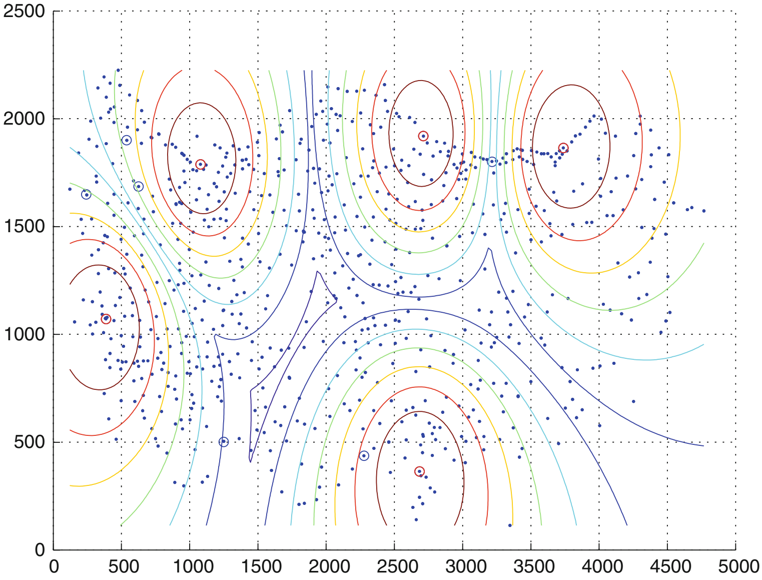

Fig. 5 FCM with known cluster centers algorithm applied to the presently used ports. (5 clusters)

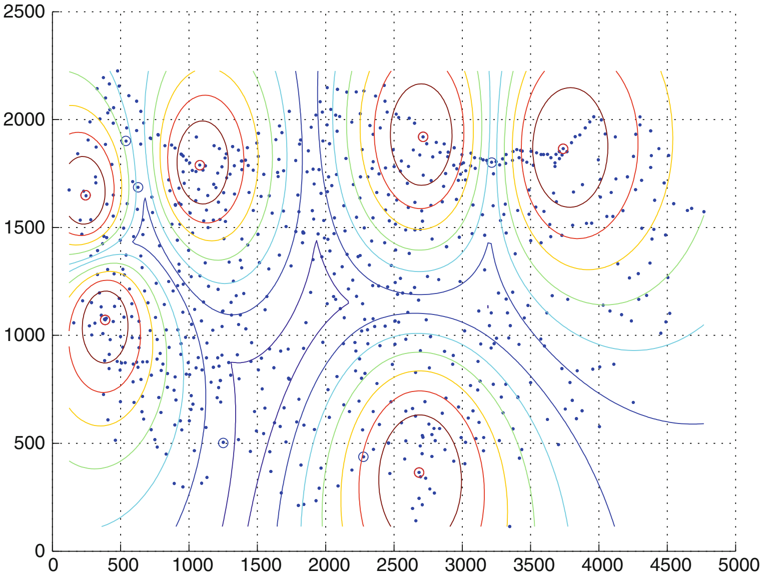

Fig. 6 FCM with known cluster centers algorithm applied to the presently used ports. (6 clusters)

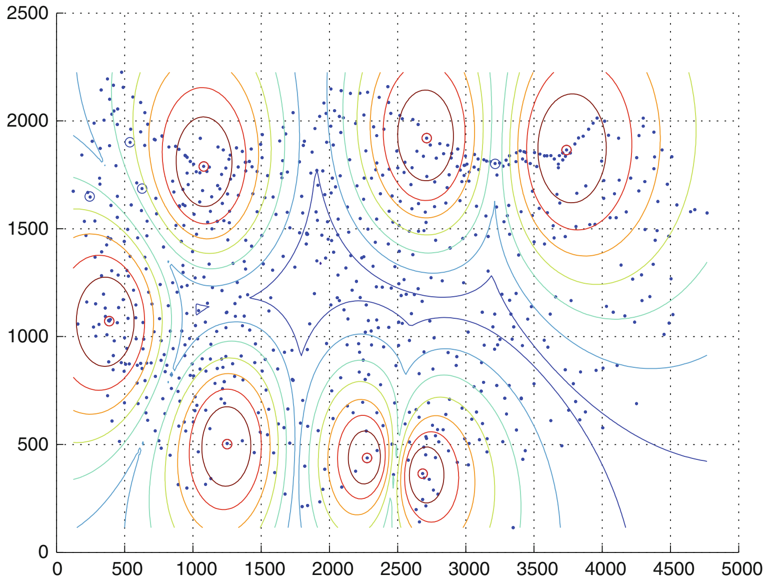

Fig. 7 FCM with known cluster centers algorithm applied to the presently used ports. (7 clusters)

Table 7 Transportation cost performances of FCM with fixed cluster center, CPSO, DPSO, $CPSO_{LS}$, $DPSO_{LS}$, ABC, and ABC_{LS} algorithms for uncapacitated facility location problem of the fertilizer company. (winner-takes-all allocations)

Model Code	Solution of the CPSO, DPSO, ABC, $CPSO_{LS}$, $DPSO_{LS}$, ABC_{LS} algorithms (TRY)	Selected ports by $CPSO_{LS}$; $DPSO_{LS}$ based algorithms	Solution of the FCM with fixed cluster center algorithm (TRY)	Selected ports by FCM with fixed cluster centers
4 Cluster	64,521,252	Iskenderun	56,088,182	Iskenderun
		Izmir		Izmir
		Samsun		Samsun
		Yarimca		Yarimca
5 Cluster	60,199,046	Iskenderun	55,264,094	Iskenderun
		Izmir		Izmir
		Samsun		Samsun
		Yarimca		Yarimca
		Canakkale		Rize
6 Cluster	58,558,971	Iskenderun	56,055,000	Iskenderun
		Izmir		Izmir
		Samsun		Samsun
		Yarimca		Yarimca
		Tekirdag		Rize
		Canakkale		Tekirdag
7 Cluster	57,806,059	Iskenderun	56,970,039	Iskenderun
		Izmir		Izmir
		Samsun		Samsun
		Yarimca		Yarimca
		Bandırma		Antalya
		Tekirdag		Rize
		Canakkale		Mersin

5 Results and Conclusions

In this study, a new solution approach is developed for uncapacitated facility location and allocation problem by using the FCM algorithm in a different way. Fuzzy clustering analysis is seldom used in facility location problems. This new approach gives nearly optimal solutions, i.e. it cannot guarantee the optimal solution like other heuristics and metaheuristics; however, it can be an alternative to PSO and ABC algorithms for discrete facility location-allocation problems. In the proposed method, degrees of membership for facilities are calculated by fuzzy c-means with single iteration according to the fixed cluster centers, which are the location of existing facilities already known. Then transportation costs are calculated by assuming that there is matching between demand points within each cluster and facilities at cluster centers.

The performance of the proposed algorithm is benchmarked against integer programing approach on several datasets. But due to the high computational needs of the integer programming; only a small set of benchmark scenarios could be completed by integer programming. This experimental study shows that the proposed algorithm gives the same results as Integer Programming models with lower CPU times. Also proposed algorithm was able to complete its calculations on all of the cases where integer programing fails.

In the case study, the appropriate port group for transporting the fertilizer products to the 768 demand points from 11 different ports, with four, five, six and seven port alternatives, is tried to be found. Proposed FCM algorithm with fixed cluster centers gives better results when compared with CPSO, DPSO, CPSO_{LS}, DPSO_{LS}, ABC, and ABC_{LS} algorithms. In four, five, six and seven clustered models, FCM algorithm with fixed cluster centers resulted in 10.15, 4.42, 6.14 and 5.42 % smaller costs respectively. The results indicate that in every scenario the proposed method is performed better than CPSO, DPSO, CPSO_{LS}, DPSO_{LS}, ABC and ABC_{LS} algorithms.

As a future research, another version of the fuzzy c-means algorithm, which is adopted to solve uncapacitated facility location problems, which have both discrete and continuous features, will be developed. The difference from the original fuzzy c-means algorithm, this algorithm will concurrently assigns points (customers) to known and unknown cluster centers (supply centers).

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A Supply-Chain Production Inventory Model with Warehouse Facilities Under Fuzzy Environment

K. Maity

Abstract In reality, there are different types of supply-chain system for production. One such type may be that a producer purchases raw materials from several vendors and the finished products are sold to a retailer. The retailer may plan to procure in large quantity to avail the price discount, transportation advantage, etc., and adopt for warehouse facilities system-one warehouse at the market place from where sale is conducted and the other (if necessary) at a distance away from the market place from which the units are transported to the market warehouse (MW) continuously to keep MW full. This motivated us to take up the following three supply-chain production inventory models. In the first model, the above mentioned type two warehouse supply chain model (SCM) is considered with imprecise stock dependent demand and in this model the objective goal is assumed to be fuzzy. There are budget and space constraints which are also in fuzzy nature. The fuzziness are defuzzified following possibility, necessity and credibility measures. In the second model (i) nature of collection of raw-material is different; (ii) demand is increasing with time in a decreasing rate, (iii) selling price of the partial backlogging units depends on the waiting time of the customers. The model is formulated with defective production system and learning effect which is fuzzy in nature. Learning effect i.e., experience is introduced in reducing the defective rate in production. In last model, an integrated production-inventory model is presented for a supplier, manufacturer, and retailer supply chain under conditionally permissible delay in payments in uncertain environments. The supplier produces the item at a certain rate, which is a decision variable, and purchases the item to the manufacturer. The manufacturer has also purchased and produced the item in a finite rate. The manufacturer sells the product to the retailer and also gives the delay in payment to the retailer. The retailer purchases the item from the manufacture to sell it to the customers. Ideal costs of supplier, manufacturer, and retailer have been taken into account. The SCMs have been

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developed and solved analytically fuzzy environments, and finally, corresponding individual profits are calculated numerically and graphically.

Keywords Delay in payment · Possibility/Necessity/Credibility · Production inventory system · Supply chain · Warehouses

1 Introduction

A supply chain model (SCM) is a network of supplier, producer, distributor and customer which synchronizes a series of inter-related business process in order to have: (i) optimal procurement of raw materials from nature; (ii) transportation of raw-materials into warehouse; (iii) production of the goods in the production center and (iv) distribution of these finished goods to retailer for sale to the customers. With a recent paradigm shift to the supply chain (SC), the ultimate success of a firm may depend on its ability to link supply chain members seamlessly.

One of the earliest efforts to create an integrated SCM dates back to Bookbinder, Oliver and Webber (1982), Cohen and Baghanan (1998) and Cachon and Zipkin (1999). They developed a production, distribution and inventory (PDI) planning system that integrated three supply chain segments comprised of supply, storage/location and customer demand planning. The core of the PDI system was a network model and diagram that increased the decision maker's insights into supply chain connectivity. The model however was confined to a single-period and single-objective problem. Viswanathan and Piplani (2001) concerned an integrated inventory model through common replenishment in the SC. Agarwal et al. (2004) have developed a dynamic balancing of inventory model in supply chains management. Rau et al. (2004) developed an integrated SCM of a deteriorating item with shortages. All the above SCMs are considered with constant, known demand and production rates in crisp environment.

By decision-making in a fuzzy environment is meant a decision process in which the goals and/or the constraints, but not necessarily the system under control, are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined. Many decision making processes in supply chain management are under vague and uncertain environment. For instance supplier selection process is made under an environment which selection criteria and alternatives are evaluated imprecisely.

Different types of uncertainty such as fuzziness, randomness, roughness are common factors in SCM. In many cases, it is found that some inventory parameters involve fuzzy uncertainty. For example, the inventory related costs holding cost, set-up cost, demand and selling price depend on several factors such as bank interest, stock amount, market situation, etc. which are uncertain in fuzzy sense. To be more specific, inventory holding cost is sometimes represented by a fuzzy number and it depends on the storage amount which may be imprecise and range within an interval due to several factors such as scarcity of storage space, market fluctuation, human estimation/thought process. Following papers have been developed in this environment.

Wang and Shu (2005) developed a fuzzy decision methodology that provides an alternative framework to handle SC uncertainties and to determine SC inventory strategies, while there is lack of certainty in data or even lack of available historical data. Fuzzy set theory is used to model SC uncertainty. A fuzzy SC model based on possibility theory is developed to evaluate SC performances. Based on the proposed fuzzy SC model, a genetic algorithm approach is developed to determine the order-up-to levels of stock-keeping units in the SC to minimize the SC inventory cost subject to the restriction of fulfilling the target fill rate of the finished product. The proposed model allows decision makers to express their risk attitudes and to analyze the trade-off between customer service level and inventory investment in the SC and better SC inventory strategies can be made. A simulation approach is used to validate the concept developed.

Das et al. (2007) have presented a joint performance of a supply chain (SC) with two warehouse facilities under fuzzy environment. A realistic two warehouse multi collection production-inventory model with constant/stock dependent demand, defective production system and fuzzy budget constraint has been formulated and solved in an SC context. Later Chen et al. (2007) developed multi-criteria fuzzy optimization for locating warehouses and distribution centers in a supply chain network.

Peidro et al. (2010) develops a fuzzy linear programming model for tactical supply chain planning in a multi-echelon, multi-product, multi-level, multi-period supply chain network in fuzzy environment. In this approach, the demand, process and supply uncertainties are jointly considered. The aim is to centralize multi-node decisions simultaneously to achieve the best use of the available resources along the time horizon so that customer demands are met at a minimum cost. This proposal is tested by using data from a real automobile SC. The fuzzy model provides the decision maker (DM) with alternative decision plans with different degrees of satisfaction.

Chu (2011) developed the supply chain flexibility that has become increasingly important. This study thus builds a group decision-making structure model of flexibility in supply chain management development. This study presents a framework for evaluating supply chain flexibility comprising two parts, an evaluation hierarchy with flexibility dimensions and related metrics, and an evaluation scheme that uses a three-stage process to evaluate supply chain flexibility. This study then proposes an algorithm for determining the degree of supply chain flexibility using a fuzzy linguistic approach. Evaluations of the degree of supply chain flexibility can identify the need to improve supply chain flexibility, and identify specific dimensions of supply chain flexibility as the best directions for improvement. The results of this study are more objective and unbiased for two reasons. First, the results are generated by group decision-making with interactive consensus analysis. Second, the fuzzy linguistic approach used in this study has more advantage to preserve no loss of information than other methods. Additionally, this study presents an example using a case study to illustrate the availability of the proposed methods and compare it with other methods.

Kristianto et al. (2012) developed an adaptive fuzzy control application to support a vendor managed inventory (VMI). This paper also guides management in allocating inventory by coordinating suppliers and buyers to ensure minimum inventory levels across a supply chain. Adaptive fuzzy VMI control is the main contribution of this paper.

In the literature many journal papers on fuzziness in supply chain management have been published but not in a single book. In this chapter of the book, three models under fuzziness are planned to be included systematically.

In first model, a single period two warehouse optimal collection-production-inventory-retailer problem has been formulated for multi raw-materials, defective and fresh product units which are screened continuously at the time of production and then defective units are reworked. The problem is solved under an imprecise budget constraint. The total cost is expressed as an integral and minimization of expected total cost is formulated as a imprecise with fuzzy demand and converted into crisp one by using possibility or necessity approach and weighted average of optimistic and pessimistic levels. The optimum collection, production and stock levels are determined for known demand function using GRG technique (cf. Gabriel and Ragsdell (1977)). The model is illustrated through numerical examples and results are presented for different types of demand.

The second model consists of supplier, producer, retailer and customers for a defective production item under fuzzy environment. The shortages to the retailer are partially backlogged and selling price of these units depend on waiting time of the customers.

In the last (three) model, a three layer supply chain production inventory model (SCPIM) under conditionally permissible delay in payments formulated under fuzzy environment. Using expectation of fuzzy variable, the fuzzy model is converted the equivalent crisp model and the finally corresponding individual profits are calculated through numerically and graphically.

The rest of this chapter is organized as follows. Section 2 for literature review, Sect. 3 for solution methodologies, Sect. 4 for first model, Sect. 5 for second model, Sect. 6 for last (third) model and Sect. 7 for conclusion part.

2 Literature Review

One of the earliest efforts to create an integrated supply chain model dates back to Karabakal et al. (2000). They developed a production, distribution and inventory (PDI) planning system that integrated three supply chain segments comprised of supply, storage/location and customer demand planning. The core of the PDI system was a network model and diagram that increased the decision maker's insights into supply chain connectivity. The model, however was confined to a single-period and single-objective problem. Agarwal et al. (2004), Viswanathan and Piplani (2001) concerned an integrated inventory model through common replenishment in the SC. Rau et al. (2004) developed an integrated SCM of a deteriorating item with shortages. All the above SCMs are considered with constant, known demand and production rates.

Gradually the time varying demand over a finite planning horizon has attracted the attention of researchers (cf. Bhunia and Maiti (1997), Maity and Maiti (2005) and others). This type of demand is observed in the case of fashionable goods, seasonable products, etc. Moreover, there are a lot of items which deteriorate continuously. Articles (cf. Zhou et al. (2003), Maity and Maiti (2005) and others) on inventory model of deteriorating items are available in the literature.

Peidro et al. (2010) have developed a fuzzy linear programming based approach for tactical supply chain planning in an uncertainty environment. Kabak and Ulengin (2011) have developed a possibilistic linear-programming approach for supply chain networking decisions.

Also, Monthatipkul and Yenradee (2008) have developed an inventory/distribution control system in a one-warehouse/multi-retailer supply chain model. Later, due to large stock and limited capacity of exiting storage (market warehouse, MW), an additional storage of infinite capacity (with sufficient space) (rented warehouse, RW) which is located away from MW is rented to store the excess items. Several authors (e.g. Pakkala and Achary (1992), Bhunia and Maiti (1997), Maiti and Maiti (2006), Das et al. (2007), Maity (2011) and others) have considered these type of inventory models for defective/deteriorating items under crisp/fuzzy environment. Wang et al. (2012) have developed a two-stage fuzzy-AHP model for risk assessment of implementing green initiatives in the fashion supply chain model. Paksoy and Pehlivan (2012) have developed a fuzzy linear programming model for the optimization of multi-stage supply chain networks with triangular and trapezoidal membership functions.

In the traditional economic order quantity (EOQ) model, it often assumed that the retailer must pay off as soon as the items are received. In fact, the supplier offers the retailer a delay period, known as trade credit period, in paying for purchasing cost, which is a very common business practice. In this research field, Goyal (1985) was the first to establish an EOQ model with a constant demand rate under the condition of permissible delay in payments. Khanra et al. (2011) have developed an EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. Also, Maihami and Abadi (2012) have established joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging.

3 Solution Methodologies

3.1 Possibility/Necessity/Credibility/Expectation Measures Under Fuzzy Environment

Any fuzzy subset $\tilde{a}$ of $\Re$ (where $\Re$ represents a set of real numbers) with membership function $\mu_{\tilde{a}}(x) : \Re \rightarrow [0, 1]$ is called a fuzzy number. Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy quantities with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$ respectively. Then according to Dubois and Prade (1983), Liu and Iwamura (1998) and others, the measure of $\widetilde{a} * \widetilde{b}$ in optimistic and pessimistic sense are

$$Pos(\tilde{a} * \tilde{b}) = sup\{(min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \Re, x * y\} \tag{1}$$

$$Nes(\tilde{a} * \tilde{b}) = inf\{(max(1 - \mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \Re, x * y\} \tag{2}$$

where the abbreviation Pos and Nes stand for possibility and necessity respectively, and * is any of the the relations >, <, =, ≤, >.

Let $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be a triangular fuzzy numbers. Then for these fuzzy numbers, following Das et al. (2007), Maity and Maiti (2007) and Maity (2011), we have used Lemmas1–3.

Lemma 1

$$Nes\,(\tilde{a} > \tilde{b}) > \eta \quad iff\, \frac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2} < 1 - \eta, (a_2 > b_2, b_3 > a_1)$$

Lemma 2

$$pos\,(\tilde{a} \geq \tilde{b}) > \eta \quad iff\, \frac{a_3 - b_1}{b_2 - b_1 + a_3 - a_2} > \eta, (a_2 < b_2, a_3 > b_1)$$

Lemma 3

$$pos\,(a = \tilde{b}) > \eta \quad iff\, \frac{a - b_1}{b_2 - b_1} > \eta,\ (b_1 < a < b_2)\, or\, \frac{b_3 - a}{b_3 - b_2} > \eta,\ (b_2 < a < b_3)$$

If the attitude of the DM is toward optimistic, Eq. (1) is the measure of best case and in pessimistic sense Eq. (2) gives the measure of worst case of that event. Now if we consider ρ the optimistic and pessimistic index determine the combined attitude of DM, then the measure of Weighted Possibility and Necessity (WPN) of $\widetilde{a} * \widetilde{b}$ is

$$WPN(\widetilde{a} * \widetilde{b}) = \rho Pos(\widetilde{a} * \widetilde{b}) + (1 - \rho)Nes(\widetilde{a} * \widetilde{b}) \tag{3}$$

Note: In particular when $\rho = \frac{1}{2}$, WPN is known as credibility of that event, i.e.,

$$Cr(\widetilde{a} * \widetilde{b}) = \frac{1}{2}(Pos(\widetilde{a} * \widetilde{b}) + Nes(\widetilde{a} * \widetilde{b})) \tag{4}$$

Based on the credibility measure, Liu and Liu (2002) presented the expected value operator of a fuzzy variable as follows.

The expected value of a normalized fuzzy variable $\tilde{a}$ is defined by

$$E[\tilde{a}] = \int_{0}^{\infty} Cr(\tilde{a} \geq r)dr - \int_{-\infty}^{0} Cr(\tilde{a} \leq r)dr \tag{5}$$

When the right hand side of (5) is of form $-\infty$ to ∞, the expected value is not defined.

Lemma 4 *The expected value of triangular fuzzy variable $\tilde{a}$ is defined as*

$$E[\tilde{a}] = \frac{1}{2}[(1-\rho)a_1 + a_2 + \rho a_3] \tag{6}$$

$$= \frac{1}{2}[a_1 + 2a_2 + a_3],\ for\ \rho = \frac{1}{2} \tag{7}$$

Lemma 5 *The expected value operation has been proved to be linear for bounded fuzzy variables, i.e., for any two bounded fuzzy variables $\tilde{a}$ and $\tilde{b}$, we have $E[c\tilde{a} + d\tilde{b}] = cE[\tilde{a}] + dE[\tilde{b}]$ for any real numbers c and d.*

3.2 Single Objective Problem Under Necessity/Possibility/ Credibility Measures

A general single-objective mathematical programming problem with fuzzy parameters should have the following form:

$$\begin{array}{cc} \text{Min} & f(u,\xi) \\ \text{subject to} & g_j(u,\xi) \leq m_j,\ j = 1,2,\ldots,k \end{array} \tag{8}$$

where u is the decision vector, ξ is a vector of fuzzy parameters, f(u, ξ) is an imprecise objective function, $g_j(u,\ \xi)$s are constraint functions, j=1,2,…, k. To convert the fuzzy objective and constraints to their crisp equivalents, Liu and Iwamura (1998) proposed a method to convert the problem (8) into an equivalent fuzzy programming problem under possibility constraints. Similarly we can convert (8) to the following fuzzy programming problem under necessity/possibility/credibility constraints.

$$\text{Min} \quad \alpha \tag{9}$$

$$\begin{array}{ll} \text{subject to} & \text{Pos}\,(\alpha = f(u,\xi)) > \eta \\ \text{and} & \text{Nes}\{\xi | g_j(u,\xi) \le m\} > \eta_{1j}\,\text{or}\,\text{Pos}\,\{\xi | g_j(u,\xi) \le m\} > \eta_{2j} \end{array} \tag{10}$$

$$\text{or} \quad \text{Cr}\{\xi | g_j(u,\xi) \le m\} > \eta_j \tag{11}$$

where η_{1j}, η_{2j} and η_j, $j = 1, 2\ldots k$. are predetermined confidence levels for fuzzy constraints. Nes {.} denotes the necessity of the event in {.}. So a point ξ is feasible if and only if necessity of the set $\{\xi/g_j(u,\xi) < m\}$ is at least η_{1j}. Similarly, Pos {.} and Cr {.} denotes the possibility and credibility of the event in {.}. So a point ξ is feasible if and only if possibility of the set $\{\xi/g_j(u,\xi) \le m\}$ is at least η_{2j} and η_j respectively. $j = 1, 2,..,k$.

4 A Two Warehouse Supply Chain Inventory Model Under Fuzzy Environment

Now-a-days, with the advent of multi-nationals, specially in developing countries, there is an acute scarcity of marketing space in important market places like municipality market, super market, corporation market,etc. Normally, due to large stock and limited capacity of exiting storage (market warehouse, MW), an additional storage of infinite capacity (with sufficient space) (rented warehouse, RW) which is located away from MW is rented to store the excess items. Several authors (e.g. Pakkala and Achary (1992), Bhunia and Maiti (1997), Maiti and Maiti (2006), Das et al. (2007), Maity (2011) and others) have considered these type of inventory models for defective/deteriorating items in crisp/fuzzy environment.

In this model, a single period two warehouse optimal collection-production-inventory-retailer problem has been formulated for multi raw-materials, defective and fresh product units which are screened continuously at the time of production and then defective units are reworked. The problem is solved under an imprecise budget constraint which is of possibility/necessity or their combination type. Usually, the holding cost is more in MW than that in RW and this realistic scenario has been ignored by many earlier workers. The actual service to the customer is done at MW only. In practice, although the holding cost at MW is higher than the holding cost in RW so an attempt is made by the management to keep MW full of stock as the demand is stock dependent. It also protects from the loss of goodwill. So, in order to start the business and to maintain the steady demand, the produced units are first transported from production centre to MW and after full filling the MW, the units are stocked in RW and the units are continuously transported to MW from RW at the time of sale (cf. Figs. 1 and 2).

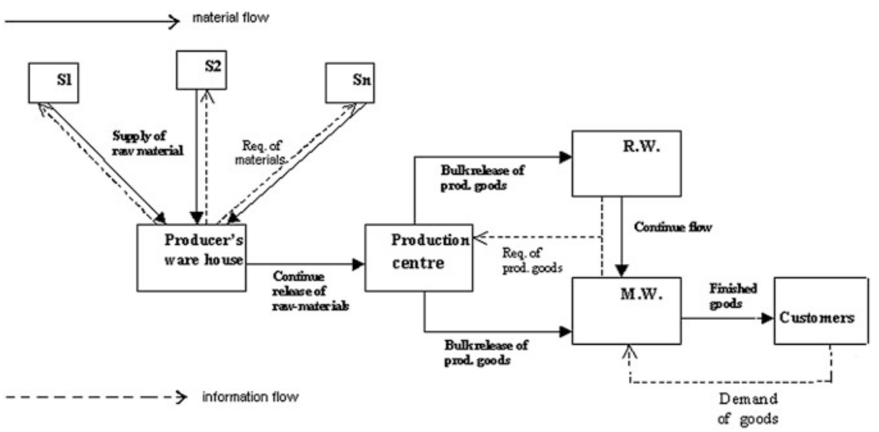
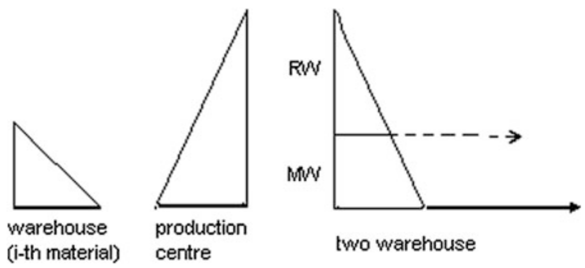


Fig. 1 Supply chain network

Fig. 2 Supply chain inventory level



The total cost is expressed as an integral and minimization of expected total cost is formulated as a fuzzy demand with possibility or necessity approach and weighted average of optimistic and pessimistic levels. The optimum collection, production and stock levels are determined for known demand function using GRG technique (cf. Gabriel and Ragsdell (1977)). The model is illustrated through numerical examples and results are presented for different types of demand.

4.1 Assumption and Notation

The proposed model is based on the following assumptions and notations.

(i) Demand of the product item is stock dependent.
(ii) Shortages are not allowed.
(iii) Defective units are screened continuously in the time of production and the defective items are reworked instantaneously.
(iv) Holding cost is more in MW than that in RW.
(v) Unit production cost is known and constant.

(vi) Inventory level, demand and production are assumed to be continuous function of time with appropriate units.
(vii) The units at production centre or RW are continuously transferred to MW.
(viii) This is a single period inventory model with infinite time horizon.
(ix) Transportation is made from the production centre to MW and RW simultaneously and the transportation cost is distance dependent. MW is filled up with its maximum capacity W and $Q_p - W$ is kept in RW.

T	time length of the cycle (to be determined)
$\tilde{M} = [M1, M2, M3]$	available total budgetary capital, which is fuzzy in nature
C_i	constant collection rate of i-th raw material (to be determined)
P	constant production rate (to be determined)
Q_{si}	collected amount of i-th raw material for supplier
Q_p	total produced amount
$X(t)$	stock level at time t in warehouses
$TM(= tm_0 + tm_1 d(P, M))$	total transportation cost from production centre to MW, where tm_0, tm_1 are positive constants and $d(P, M)$ is distance between production centre to MW
$TR(= tm_0 + tm_1 d(P, R))$	total transportation cost from production centre to RW, where tr_0, tr_1 are positive constants and $d(P, M)$ is distance between production centre to RW
$TRW(= trm_0 + trm_1 d(R, M))$	total transportation cost from RW to MW, where trm_0, trm_1 are positive constants and $d(R, M)$ is distance between RW to MW
P	probability of defectiveness with expected value $\widehat{p}$
$D(X(t))$	stock dependent demand
$c_{si}, /c_p$	collection cost (for ith item)/production cost per unit item
s_p	unit screening cost of defective quantity
r_p	reworked cost of defective quantity per unit item
h_m, h_r	holding cost per unit item for MW and RW respectively ($h_m > h_r$)
CS	total collection cost for supplier
HS, HP	total holding cost for supplier and producer respectively for raw material
PP, SP, RP, IP	total production, screening, reworked, inventory cost for producer respectively for product item
HW	total holding cost of product item for warehouse
ETC	expected total cost.

4.2 Model Formulation in Crisp Environment

Supplier's collection: The supplier's collection rate is C_i for the ith raw material, and the collection cost per unit is c_{si}. Therefore, the supplier's inventory holding cost is: $HS = h_s \sum_{i=1}^{n} \frac{Q_{ci}^2}{2C_i}$, and collection cost $CS = \sum_{i=1}^{n} c_{si} Q_{ci}$.

Manufacturer's set up: The manufacturer purchases the collects raw-materials $\sum_{i=1}^{n} Q_{ci}$ from supplier, from which goods are produced with a production rate P of which probability of defective production is p. Therefore manufacturer inventory holding cost for raw materials is: $HP = h_p \sum_{i=1}^{n} \frac{Q_{ci}^2}{2P}$.

Manufacturer's production: The manufacturer produces both good and defective units. Therefore manufacturer production cost is: $PP = c_p Q_p$.

Cost of screening = $SP = s_p.Q_p$, mean cost of rework = $RP = r_p.Q_p.\widehat{p}$, and inventory cost for produced goods = $IP = h_p \left\{ \frac{Q_p^2}{2P} \right\}$.

Warehouse's set up: The quantity Q_p is stored in a two warehouse system, in which market warehouse has a fixed W capacity of storing and stock dependent demand is considered in the market warehouse.

The differential equation to MW during the time-period, T (having the stock in RW + after complete the exhaustion of stock in RW)is:

$$\dot{X}_M(t) = D_M(X(t)) - D_M(X(t)) = 0 \quad \text{in } (0, T') \tag{12}$$

$$= -D_M(X(t)) \qquad \text{in } (T', T) \tag{13}$$

After production, the stock equation in RW is:

$$\dot{X}_R(t) = -D_R(X(t)) \quad \text{in } (0, T') \tag{14}$$

with boundary conditions $X_M(T') = W, X_M(T) = 0, X_R(0) = Q_p - W, X_R(T') = 0$.

Here, the time scale is shifted from (2T, 3T) to (0, T) as it does not effect the results. Assuming the market warehouse is of finite capacity W and total cost consisting of holding costs, and transportation cost (including transportation from production centre to rented warehouse and from rented warehouse to market warehouse). Stock period at RW and MW for stock dependent demand are $T' = \int_0^{Q_p - W} \frac{dX}{D_M(X(t))}$ and $T = T' + \int_0^{W} \frac{dX}{D_M(X(t))}$

Inventory holding cost of two warehouse are $HW = h_r(Q_p - W)\frac{T'}{2} + h_m(T + T')\frac{W}{2}$

Hence, Expected Total Cost of the SC is:

$$ETC = E\{HS + CS + HP + PP + SP + RP + IP + HW + TR + TM + TRM\} \tag{15}$$

4.2.1 Equivalent Crisp Model for Fuzzy Resources

Let the budget resources be uncertain in in fuzzy sense. Then with the imprecise budget $\tilde{M}$ the crisp objective model in terms of possibility or necessity constraints is:

$$Z = \quad Min(ETC) \tag{16}$$

$$\text{subject to} \quad Nes\{s_c Q_c + c_p Q_p + s_p.Q_p + r_p.\tfrac{Q_p.P}{K} < \tilde{M}\} > \eta_1 \tag{17}$$

$$or \quad Pos\{s_c Q_c + c_p Q_p + s_p.Q_p + re.\tfrac{Q_p.P}{K} < \tilde{M}\} > \eta_2 \tag{18}$$

$$or \quad Cr\{s_c Q_c + c_p Q_p + s_p.Q_p + r_p.\tfrac{Q_p.P}{K} < \tilde{M}\} > \eta_3 \tag{19}$$

4.2.2 Equivalent Crisp Model for Fuzzy Demand and Fuzzy Resources

In this case, the coefficients of the demands are assumed to be imprecise. So due to the fuzzy demand function $\widetilde{D}(X(t))$, the expected total cost ETC in (16) become $E\tilde{T}C = [ETC1, ETC2, ETC3]$ which is triangular fuzzy number. Using the possibily theory, the problem represented by (17, 18 and 19) can be expressed as:

$$\text{Min} \quad Z \tag{20}$$

$$\begin{array}{ll} \text{subject to} & Pos(Z = E\tilde{T}C) > \eta(for(16)) \\ \text{and} & (17), (18)\ (19) \end{array} \tag{21}$$

4.3 De-Fuzzyfication of Fuzzy Constraints

The fuzzy constraints, presents as crisp ones through possibility and necessity in (17, 18, and 19) and the objective in terms of possibility present in (21) can be normalized as:

$$\frac{(TC - M1)}{(M2 - M1)} < (1 - \eta_1);\ \text{when}\ TC > M1\ and\ TC < M2\ (for\ (17)) \tag{22}$$

$$\frac{(M3 - TC)}{(M3 - M2)} < \eta_2;\ \text{when}\ TC > M2;\ and\ TC < M3;\ (for(18)) \tag{23}$$

$$\rho \cdot \frac{(TC - M1)}{(M2 - M1)} + (1 - \rho) \cdot \frac{(M3 - TC)}{(M3 - M2)} < \rho \cdot (1 - \eta_3) + (1 - \rho)\eta_3;\ (for(19)) \tag{24}$$

Table 1 Result of Sect. 4.2.1 for stock dependent crisp demand and fuzzy resource constraint

	T	T'	(C_1, C_2)	P	(Q_{s1}, Q_{s2})	Q_p (Fresh,defective)	ETC
w.r.t Nes	1.52	0.81	(4.24, 16.07)	19.11	(6.47, 24.53)	(20.67, 8.78)	1332.80
w.r.t Pos	1.73	0.87	(8.13, 13.48)	20.93	(14.04, 23.32)	(25.35, 10.86)	1881.09
w.r.t Cr	1.58	0.81	(7.59, 10.31)	19.98	(11.99, 16.29)	(22.11, 9.47)	1678.14

Table 2 Result of Sect. 4.2.2 for stock dependent fuzzy demand and fuzzy resource constraint

	T	T'	(C_1, C_2)	u	(Q_{s1}, Q_{s2})	Q_p (Fresh,defective)	ETC
w.r.t Nes	1.62	0.81	(4.24, 16.07)	19.11	(7.43, 19.19)	(27.51, 1.45)	1332.73
w.r.t Pos	1.58	0.87	(8.13, 13.48)	20.93	(12.82, 21.25)	(31.35, 1.65)	1881.08
w.r.t Cr	1.49	0.89	(6.56, 10.74)	19.54	(12.92, 16.73)	(30.13, 1.30)	1638.10

where $TC = \left\{s_c Q_c + c_p Q_p + s_p \cdot Q_p + r_p \cdot \frac{Q_p \cdot P}{K}\right\}$ $and\ \widetilde{M} = (M1, M2, M3)$ and

$$\frac{(ETC3 - Z)}{(ETC3 - ETC2)} < \eta;\ \text{when}\, Z > ETC2;\ and\, Z < ETC3;\ (for\,(21)) \tag{25}$$

Then the reduced crisp objective is optimized for the given constraints via GRG technique (Gabriel and Ragsdell 1977).

4.4 A Numerical Example

To illustrate the above inventory model numerically, defectiveness is considered as uniform distribution, i.e.,

$$f(p) = \begin{cases} \frac{1}{Q_p - 0} & for\, 0 < p < Q_p \\ 0 & elsewhere \end{cases}$$

and the other relevant input data are:

(M_1, M_2, M_3 = (900\$, 1000\$, 1100\$) and η_1 = 0.01, η_2 = 0.65 η = 0.60, h_m = \$15, h_r = \$12, p = 0.05*unit*, c_s \$50*per unit*, c_p \$15, s_p = \$0.5, r_p = \$1.5, $d(P,R)$ = 150 km, $d(P,M)$ = 200 km, $d(R,M)$ = 50 km, W = 12 *units*.

Now, we consider the result for stock dependent exponential demand $D(X(t)) = ae^{bX(t)}$ units with a = 10, b = 0.1 in crisp environment, the results are shown in Table 1 and stock dependent exponential fuzzy demand $D(\widetilde{X(t)}) = \tilde{a}e^{\tilde{b}X(t)}$ units with $(\tilde{a} = (9.8, 10, 10.2))$ and $(\tilde{b} = (0.08, 0.1, 0.12))$ under fuzzy environment, the results are shown in Table 2.

5 A Supply-Chain Model with Waiting Time Dependent Revenue for Partial Backlogging and Fuzzy Learning

5.1 Assumption and Notations

The following assumptions are used for the proposed SCM.

5.1.1 Assumptions

(i) The model is developed for a finite time horizon.
(ii) There are a single supplier, producer and retailer in the system.
(iii) Only one type of raw materials and finished product are considered.
(iv) Producer possesses a warehouse and a production centre.
(v) Shortages of goods are allowed and partially backlogged.

The supply chain inventory network is shown in Fig. 3.

5.2 Model Description and Formulation

This model consists of supplier, producer, retailer and customers for a defective production item under fuzzy environment. The shortages to the retailer are partially backlogged and selling price of these units depend on waiting time of the customers.

5.2.1 Inventory Model for the Retailer with Finished Goods

If $q_{R_i}(t)$ be the inventory of the finished goods at any time t for the retailer with demand $d(t)$ and if W_{i+1} be the time of zero inventory for the retailer in the ith cycle $\left[\frac{iT}{n}, \frac{(i+1)T}{n}\right]$, the governing differential equations are:

$$\frac{dq_{R_i}}{dt} = -d(t), \quad \text{in } \left[\frac{iT}{n}, W_{i+1}\right] \tag{26}$$

$$= -\frac{s_0 - s_1\left((i+1)\frac{T}{n} - t\right)}{s_0} k_0 d(t), \quad \text{in } \left[W_{i+1}, \frac{(i+1)T}{n}\right] \tag{27}$$

The inventory conditions for the model are: $q_{R_i}(t) = 0, \text{ at } t = W_{i+1}$

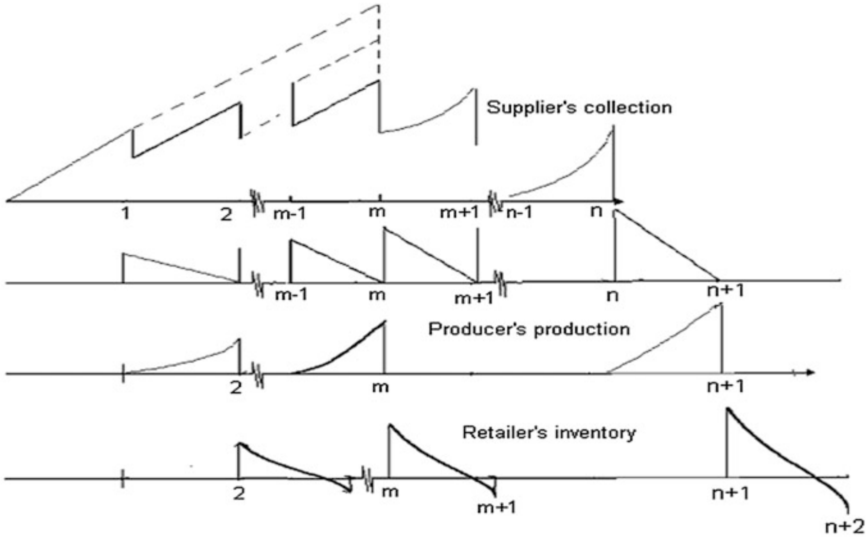


Fig. 3 Supply chain network

Therefore, the inventories during the holding and shortage period at any time t within the ith cycle are given by:

$$
\begin{aligned}
q_{R_i}(t) &= \int_t^{W_{i+1}} d(u)du \text{ in}\left[\frac{iT}{n}, W_{i+1}\right] \\
&= -k_0\left(1-\frac{(i+1)Ts_1}{ns_0}\right)\int_{W_{i+1}}^t d(u)du - k_0\frac{s_1}{s_0}\int_{W_{i+1}}^t u.d(u)du, \text{ in}\left[W_{i+1}, \frac{(i+1)T}{n}\right]
\end{aligned}
\tag{28}
$$

The total inventory during the ith cycle is:

$$
\begin{aligned}
H_{R_i} &= \int_{\frac{iT}{n}}^{W_{i+1}}\left(u-\frac{iT}{n}\right)d(u)du \\
&= \frac{D_0}{D_1^2}e^{-D_1\frac{iT}{n}} - \frac{D_0}{D_1}\left(W_{i+1}-\frac{iT}{n}+\frac{1}{D_1}\right)e^{-D_1W_{i+1}}
\end{aligned}
\tag{29}
$$

The ordered quantity of the retailer is:

$$Q_{R_0} = q_{R_0}(0) \tag{30}$$

and for $1 \le i \le n$

$$Q_{R_i} = q_{R_i}\left(\frac{iT}{n}\right) + \int_{W_{i+1}}^{\frac{(i+1)T}{n}}\int_{W_{i+1}}^t q_{Ri}(u)dudt \tag{31}$$

For partial backlogging, shortage amount is waiting time dependent price $s_{R_i}(t) = s_0 - s_1\left(\frac{(i+1)T}{n} - t\right)$. Total sales revenue during the shortage with partial backlogging is:

$$\begin{aligned}
TS_{1_i} =& s_0[(1 - \frac{(i+1)Ts_1}{ns_0})^2 \int_{W_{i+1}}^{\frac{(i+1)T}{n}} (u - W_{i+1})d(u)du \\
&+ \frac{s_1}{s_0}(1 - \frac{(i+1)Ts_1}{ns_0})\{\int_{W_{i+1}}^{\frac{(i+1)T}{n}} (u - W_{i+1})u.d(u)du \\
&+ 0.5\int_{W_{i+1}}^{\frac{(i+1)T}{n}} (u^2 - (W_{i+1})^2)d(u)du\} + (\frac{s_1}{s_0})^2 \int_{W_{i+1}}^{\frac{(i+1)T}{n}} (u^2 - (W_{i+1})^2)u.d(u)du] \\
=& s_0[(1 - \frac{(i+1)Ts_1}{ns_0})^2(\frac{D_0}{D_1}e^{-d_1W_{i+1}} - (((i+1)T/n - W_{i+1})(1 + \frac{D_0}{D_1}))e^{-D_1(i+1)T/n}) \\
&+ \frac{s_1}{s_0}(1 - \frac{(i+1)Ts_1}{ns_0})\{(W_{i+1}\frac{D_0}{D_1^2} + 2\frac{D_0}{D_1^3})e^{-D_1W_{i+1}} - (((\frac{(i+1)T}{n})^2 - W_{i+1}\frac{(i+1)T}{n})\frac{D_0}{D_1} \\
&+ 2(\frac{(i+1)T}{n} - W_{i+1})(D_0/D_1^2) + 2(D_0/D_1^3))e^{-D_1(i+1)T/n} \\
&+ \frac{1}{2}((2W_{i+1}\frac{D_0}{D_1^2} + 2\frac{D_0}{D_1^3})e^{-D_1W_{i+1}} - 0.5(((\frac{(i+1)T}{n})^2 - W_{i+1}\frac{(i+1)T}{n})(D_0/D_1) \\
&+ 2(\frac{(i+1)T}{n})(D_0/D_1^2) + 2(D_0/D_1^3))e^{-D_1(i+1)T/n}\} \\
&+ (\frac{s_1}{s_0})^2\{(2W_{i+1}^2\frac{d_0}{D_1^2} + 6W_{i+1}\frac{d_0}{D_1^3} + 6W_{i+1}\frac{D_0}{D_1^4})e^{-D_1W_{i+1}} \\
&- 0.5(((\frac{(i+1)T}{n})^3 - \frac{(i+1)T}{n})W_{i+1}^2)(D_0/D_1) + 3((\frac{(i+1)T}{n})^2 - W_{i+1}^2)(D_0/D_1^2) \\
&- 6(\frac{(i+1)T}{n}D_0/D_1^3) + 6(D_0/D_1^4))e^{-D_1(i+1)T/n}\}]
\end{aligned}$$

Total profit of the retailer individually for finite time period is given by:

$$\begin{aligned}
PF_R =& s_0\frac{D_0}{D_1}\sum_{i=2}^{n+2}(e^{-D_1\frac{iT}{n}} - e^{-D_1W_{i+1}}) \\
&+ \sum_{i=2}^{n+2} TS_{1i} - h_R\sum_{i=2}^{n+2} H_{Ri} - p_R\sum_{i=2}^{n+2} Q_{Ri}
\end{aligned} \tag{32}$$

5.2.2 Producer's Production Model

The finished goods inventory level for the producer with (demand dependent)controllable production function $u_i(t) = p_0d_i(t)$ is described by the following differential equation:

$$\frac{dq_{P_{i-1}}}{dt} = (1 - \delta_{i-1})u_{i-1}(t), \qquad (i-1)T/n \le t \le iT/n \tag{33}$$

The initial condition is $q_{P_{i-1}}((i-1)T/n) = 0$.Therefore the non-defective product amount up to time t in the $(i-1)$th period is given by:

$$I_{P_{i-1}}(t) = (1-\delta_{i-1}) \int_{\frac{(i-1)T}{n}}^{t} u_{i-1}(\xi) d\xi \tag{34}$$

Total quantity of non-defective units amount for each cycle is

$$\begin{aligned} Q_{P_{i-1}} &= q_{P_{i-1}}(\frac{iT}{n}) \\ &= (1-\delta_{i-1}) P_0 \frac{D_0}{D_1} e^{-D_1 \frac{iT}{n}} \left(e^{D_1 \frac{T}{n}} - 1 \right) \end{aligned} \tag{35}$$

The holding amount of good products for each cycle is:

$$\begin{aligned} H_{P_{i-1}} &= (1-\delta_{i-1}) \int_{\frac{(i-1)T}{n}}^{\frac{iT}{n}} \left(\xi - \frac{(i-1)T}{n} \right) u_{i-1}(\xi) d\xi \\ &= P_0 \frac{D_0}{D_1^2} (1-\delta_{i-1}) \left[e^{-D_1 \frac{iT}{n}} \left(D_1 \frac{T}{n} - 1 + \frac{D_1}{n} e^{-D_1 T} \right) \right] \end{aligned} \tag{36}$$

Note that δ_{i-1} is the defective rate of the production which decreases with imprecise learning rate $\alpha = \frac{\ln r}{\ln 2}$, where learning slope/co-efficient is $r \varepsilon$ (0, 1].

$$\text{So}, \delta_{i-1} = \delta_1 i^{-\alpha}, \text{and} \sum_{i=1}^{n} \delta_{i-1} \text{ is approximated to } (i > 2)\ \delta_1 \int_0^n x^{-\alpha} dx.$$

Introducing learning effect concept,

$$H_{P_{i-1}} = P_0 D_0 \left(1 - \delta_1 \frac{n^{-\alpha-1}}{\alpha+1} \right) \left[e^{-D_1 \frac{iT}{n}} (D_1 \frac{T}{n} - 1 + \frac{D_1}{n} e^{-D_1 T}) \right] \tag{37}$$

5.2.3 Inventory for Producer's Storage of Raw Materials

The inventory level of raw materials at the producer's warehouse at time t, $q_{PW_{i-1}}$ is determined by the linear differential equation

$$\frac{dq_{PW(i-1)}}{dt} = -u_{(i-1)}(t), \qquad (i-1)T/n \le t \le iT/n \tag{38}$$

Therefore the inventory at any time t in the $(i-1)$th period is given by:

$$q_{PW_{(i-1)}}(t) = -\int_{t}^{iT/n} u_{i-1}(\xi) d\xi \tag{39}$$

Therefore the ordered quantity of raw materials in the warehouse for the $(i-1)$th period is:

$$\begin{aligned} Q_{PW_{i-1}} &= \int_{(i-1)T/n}^{iT/n} u_{i-1}(\xi)d\xi \\ &= P_0 \frac{D_0}{D_1} e^{-D_1 \frac{iT}{n}} \left(e^{D_1 \frac{T}{n}} - 1 \right) \end{aligned} \tag{40}$$

The stock of raw materials within the time period is:

$$\begin{aligned} H_{PW_{i-1}} &= \int_{(i-1)T/n}^{iT/n} (u - (i-1)T/n) u_{i-1}(\xi) d\xi \\ &= P_0 D_0 \left[e^{-D_1 \frac{iT}{n}} \left(D_1 \frac{T}{n} - 1 + \frac{D_1}{n} e^{-D_1 T} \right) \right] \end{aligned} \tag{41}$$

Total cost only to the producer (sum of the costs in production centre and producer's warehouse) in n cycles due to the linear production function $u_i(t) = p_0 d(t)$can be expressed as the sum of the setup cost, purchasing cost of raw materials, production cost and holding cost (raw material + finished goods), i.e.,

$$\begin{aligned} TC_P + TC_{PW} = &\sum_{i=1}^{n+1} (p_P Q_{Pi} + h_P H_{Pi} + (h_{PW} + sc_P \delta_1 \frac{n^{-\alpha-1}}{\alpha+1}) H_{PWi} \\ &+ p_{PW} Q_{PWi}) + nA_P \end{aligned} \tag{42}$$

$$PF_P = p_R \sum_{1}^{n+1} Q_{Ri} - (TC_P + TC_{PW}) \tag{43}$$

5.2.4 Inventory Model of Supplier's Raw Materials

In this model supplier collects raw materials from open market/nature for the producer at a constant collection rate C_0 up to m-cycle(decision variable). And after that the collection rate decreases with a rate C_1. Therefore supplier's raw materials inventory quantity $q_{S_{i-2}}(t)$ at any time $t\varepsilon[(i-2)T, (i-1)T]$ in the ith cycle, with initial condition $q_{S_n} = 0$, at $t = T$ is given by.

$$\begin{aligned} \frac{dq_{S_{(i-2)}}}{dt} &= C_0, \qquad \text{in}[iT, (i+1)T], \qquad \text{for } i = 0, 1, \cdots, (m-1). \\ &= C_0 - C_1 t, \quad \text{in } [iT, (i+1)T], \qquad \text{for } i = m, m+1, \cdots, n. \end{aligned} \tag{44}$$

Total collected amount in the time horizon is:

$$Q_S = C_0 T - \frac{C_1}{2n^2}(n^2 - m^2)T^2 \tag{45}$$

Stock of raw materials up to the mth cycle is

$$H_{S0} = \frac{1}{2}C_0\frac{m^2T^2}{n^2} - \frac{T}{n}\left(\sum_{j=1}^{m-1}(m-j)Q_{PW_j}\right) \tag{46}$$

Stock of raw materials during (m + 1)th cycle up to the end of time horizon is

$$H_{S1} = \sum_{j=m}^{n}\left\{\left(\frac{C_1}{2}T^2 - C_0T\right)\frac{T}{n} + \frac{C_0}{2}(2j+1)(T/n)^2 - \frac{C_1}{6}(3i^2+3i+1)(T/n)^3\right\} \tag{47}$$

The total cost for raw materials of the supplier during whole period is the sum of the set up cost, collection cost, holding cost. Hence,

$$TC_S = nA_S + c_SQ_S + h_S(H_{S0} + H_{S1}) \tag{48}$$

$$PF_S = p_{PW}\sum_{i=0}^{n}Q_{PQi} - TC_S \tag{49}$$

Crisp Model:

Model-1. (Integrated Formulation): Assuming the whole SC system is owned and managed by a single concern/management house, the problem reduces to a single objective to maximize the profit (PF) (exclude the purchasing costs (Producer's and Retailer's)).Also the holding and setup costs are higher with respect to non-integrated model as:

$$\begin{aligned}\operatorname{Max} PF = &\left\{s_0\frac{d_0}{d_1}\sum_{i=2}^{n+2}(e^{-d_1\frac{iT}{n}} - e^{-d_1W_{i+1}}) + \sum_{i=2}^{n+2}TS_{1i} - h_{R'}\sum_{i=2}^{n+2}H_{Ri}\right\}\\ &- \left\{\sum_{1}^{n+1}\left(p_PQ_{Pi} + h_{P'}H_{Pi} + (h_{PW'} + sc_P\delta_1\frac{n^{-\alpha-1}}{\alpha+1})H_{PWi})\right) + nA_{P'}\right\}\\ &- \{nA_{S'} + c_{S'}Q_S + h_{S'}(H_{S0} + H_{S1})\}\end{aligned} \tag{50}$$

Model-2. (Non-integrated Formulation): In this formulation, members of the chain are assumed to be different from each other but they operate/work together in collective/collaborative manner. Hence, the problem is to find the no of cycles n, and the production rate P_0 and collection rate C_0, C_1, such that these parameters/ quantities minimize their profits individually.i.e.,

$$\begin{cases}\text{Max} & PF_S\\ \text{Max} & PF_P\\ \text{Max} & PF_R\end{cases} \tag{51}$$

Fuzzy Model:

Assuming the fuzzy learning coefficient $\tilde{r}$ is converted into an interval number $[r_L, r_R]$. $\Rightarrow$ learning rate, $\widetilde{\alpha} = [\alpha_L, \alpha_R]$. Using Grzegorzewski (2002) and Moore (1979), the total cost to the producer $TC_P + TC_{PW}$ becomes

$$[TC_{PL} + TC_{PWL}, TC_{PR} + TC_{PWR}] = \sum_{i=1}^{n+1}(p_P Q_{Pi} + h_P[H_{PLi}, H_{PRi}] + (h_{PW} + \delta_1 sc_P \frac{n^{[-\alpha_R - 1, -\alpha_L - 1]}}{[\alpha_R + 1, \alpha_L + 1]})H_{PWi} + p_{PW} Q_{PWi}) + nA_P \tag{52}$$

and the other expressions remain same as these are in the model of crisp model and the model is solved by Genetic algorithm (cf. Goldberg 1989).

5.3 A Numerical Example

To illustrate the above SCM, the following input data are considered.

H $= 500; A_R = 130\$; A'_R = 140\$; p_R = 90\$; h_R = 4\$; h_{R'} = 4.5\$; s_0 = 110\$; A_P = 120\$;\ \ A'_P = 125\$; p_P = 30\$, h_P = 4\$; h_{P'} = 4.5\$; s_c = 5\$, p_{PW} = 25\$; h_{PW} = 2\$; h_{PW'} = 2.5\$; A_S = 100\$;\ \ A_{S'} = 120\$; h_S = 2\$; h_{S'} = 2.4\$; c_S = 22\$; \delta_1 = 0.1\%, \alpha = 0.5, \alpha_L = 0.25, \alpha_R = 0.65; D_0 = 250$ unit, $D_1 = 0.90$;

Results for all the models for different cases have been presented in the following tables. (Table 3)

5.4 Discussion

In this formulation, reduction in selling price depends the waiting time during shortage period and backlogging increases as waiting time for customer decreases. Here, four cases have been considered under the fuzzy environments.

The case-1 ($s_1 = 0$, $k_0 = 0.8$) represents the procurement-production-inventory model with partial backlogging, not reducing the unit selling price during the shortage period. For the cases-II & III, both partial backlogging and selling price change are allowed. Fully backlogging with constant selling price is considered in case-IV. For all cases, integrated and non-integrated models are formulated and results are obtained. From the Tables 1 and 2, the following observation are made:

(i) As the fuzzy parameter has been replaced by an interval number, the crisp model results lies between the profit interval under fuzzy environment. (ii) As expected, the backlogging amount is maximum in case-IV and profit is highest for this model under both environment. Similarly, the opposite observations are observed for case-I. (iii) Moreover, in all cases, integrated model gives more profit than the non-integrated models under the fuzzy environment.

Table 3 Optimal solutions for fuzzy model

N	m	C_0, C_1	P_0	BL	PROFS	$PROFP_L$	$PROFP_R$	PROFR
Case-I: s_1=0.0, k_0=0.8;								
16	12	37.46, 0.67	25.82	48.78	4301.62	12072.84	12673.77	7814.46
18	12	49.48, 0.56	79.64	49.54	5215.81	11996.58	12304.37	7740.03
19	10	73.48, 0.25	32.58	51.34	5041.06	12580.73	12582.17	7880.45
19	17	17.74,0.78	76.62	50.74	4823.43	13015.87	13087.68	7878.16
21	18	19.37, 0.76	81.45	52.39	5105.85	12407.45	12711.60	7762.81
23	12	51.39, 0.35	30.00	53.75	5536.56	12135.83	12445.98	7670.92
22	18	52.53, 0.45	29.30	52.85	–	–	[25944.95, 26517.25]	–
Case-II: s_1=0.7, k_0=0.8;								
17	10	42.38, 0.34	78.12	54.38	4799.09	12686.58	13648.50	8051.98
19	17	17.74, 0.75	76.62	52.97	4928.87	12963.81	13464.84	7821.85
19	9	60.60, 0.38	73.90	52.78	5148.68	13130.54	13834.99	7870.39
20	11	44.33, 0.54	10.00	51.95	4927.71	11941.83	13455.83	7833.17
22	19	47.94, 0.46	36.61	53.05	5192.76	13010.09	13012.89	7879.00
21	16	45.25, 0.75	32.25	53.95	–	–	[26137.70, 27534.45]	–
Case-III: s_1=0.7, k_0=1;								
15	12	33.10, 0.60	88.88	61.27	4983.31	15386.87	15387.60	8113.16
19	17	17.74, 0.80	76.62	62.90	5746.10	14215.08	14919.68	7052.02
21	15	54.38, 0.60	87.03	63.18	5747.43	14597.81	15205.92	7979.31
22	14	15.20, 0.91	26.96	64.39	5767.84	13876.33	14178.37	7872.54
23	17	57.43, 0.56	92.17	64.80	5030.25	15254.12	15961.06	8061.59
23	17	57.55, 0.45	35.55	64.73	–	–	[29052.49,29361.25]	–
Case-IV: s_1=0, k_0=1;								
21	9	19.37, 0.45	81.45	64.13	5726.56	15318.50	15722.39	7923.79
23	17	27.53, 0.65	55.75	67.71	5651.59	14793.84	15984.21	7835.41
24	16	21.94, 0.81	47.17	71.29	5757.10	15492.92	15801.76	8018.40
25	18	25.05, 0.65	93.56	72.59	5578.65	14845.89	15652.93	7829.06
24	18	32.23, 0.75	72.90	72.05	–	–	[28359.44, 29460.12]	–

5.5 Practical Implementation

A production industry starts with the purchase of raw materials and ends with the sale of finished product. In this, several operational decisions are taken to effectively and efficiency manage of the material flow in previous model $\oint$ 4. In India there are several small and large scale industries which necessarily are involved in such planned supply chain system. Even the multi-nationals also follow the supply chain policies.

To illustrate, in developing countries like India, chin and other countries, production of fruit products like jelly, jam, etc., are produced under small scale industry sector. The fruits (these deteriorate with time) are supplied by some suppliers who collect these from villagers or village markets. The product centre purchases and stores these fruits. Fruit products (which also deteriorate) are produced and sold to retailers who later sell these in the market. In this process, there may or may not be some resource constraints like limited capital for purchase, limitation on transportation cost, etc. Similar process is also followed in rice mills. Here some middle men collect paddy from villages and supply to a rice-mill owner. The rice-mill owner makes a temporary stock of paddy and produces rice out of it. This rice is sold to retailers for sale. Here both paddy (raw materials) and rice (product) deteriorate due to dryness, vaporization, etc.

The proposed study of these real-life supply chain problems will help the industry sector to take effective and efficient planned supply chain decisions.

6 Three Layers Supply Chain Production Inventory Model Under Permissible Delay in Payments Under Fuzzy Environment

In the traditional economic order quantity (EOQ) model, it often assumed that the retailer must pay off as soon as the items are received. In fact, the supplier offers the retailer a delay period, known as trade credit period, in paying for purchasing cost, which is a very common business practice. Suppliers often offer trade credit as a marketing strategy to increase sales and reduce on hand stock level. Once a trade credit has been offered, the amount of period that the retailer's capital tied up in stock is reduced, and that leads to a reduction in the retailer's holding cost of finance. In addition, during trade credit period, the retailer can accumulate revenues by selling items and by earning interests. As a matter of fact, retailers, especially small businesses which tend to have a limited number of financing opportunities, rely on trade credit as a source of short-term funds. In this research field, Goyal (1985) was the first to establish an EOQ model with a constant demand rate under the condition of permissible delay in payments. Khanra et al. (2011) have developed an EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment. Also, Maihami and Abadi (2012) have established joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging.

In this model, a three layer supply chain production inventory model (SCPIM) under conditionally permissible delay in payments formulated under fuzzy environment. Supplier produced the item at a finite rate and purchase the item to the manufacturer. Manufacturer has also purchased and produced the item in a certain rate which is the decision variable. Manufacturer sale his product to the retailer and also give the delay in payment to the retailer. Retailer purchase the item from manufacture and to sale the customers. Ideal costs of supplier, manufacturer and retailer have been taken into account. Integrated model has been developed and solved analytically under fuzzy environments and finally corresponding individual profits are calculated through numerically and graphically.

6.1 Assumptions and Notations

The following assumption and notation are consider to develop the model:

Assumptions:

(i) Model is developed for single item product.
(ii) Lead time is negligible.
(iii) Joint effect of supplier, manufacturer, retailer is consider in a supply chain management.
(iv) Supplier produced the item with constant rate p_s unit per unit time.
(v) Total production rate of manufacturer is equal to the demand rate of manufacturer which is decision variable .
(vi) The manufacturer give the opportunity to the retailer conditionally permissible delay in payment.
(vii) Idle cost of suppliers, manufacturer and retailer are taken into account.

Notations:

p_s	constant production rate for the suppliers
p_m	demand rate or production rate for the manufacturer(decision variable)
D_R	constant demand rate for the retailer
D_c	constant demand rate of customer
C_s	purchase cost of unit item for suppliers
C_m	selling price of unit item for suppliers which is also purchase cost for manufacturer
C_r	selling price of unit item for manufacturer which is also purchase cost for retailers
C_{r1}	selling price for retailers
t_s	production time for supplers
T_s	cycle length for the suppliers
T_R	length of each time period of retailer
T'	last cycle length of the retailer
T	total time for the integrated model
h_s	holding cost per unit per unit time for suppliers
h_m	holding cost per unit per unit time for manufacturer
h_r	holding cost per unit per unit time for retailers
A_s	ordering cost for suppliers
A_m	ordering cost for manufacturer
h_r	ordering cost for retailers
id_s	idle cost per unit time for suppliers
id_m	idle cost per unit time for manufacturer
id_r	idle cost per unit time for retailers
n	number of cycle for retailers
r	number of cycle where manufacturer stop the production
M	retailers trade credit period offered by the manufacturer to the retailers in years

I_p	interest payable to the manufacturer by the retailers
$I_{re}, I_{re}^{\sim}$	interest earned by the retailers in crisp and fuzzy environment respectively
$q_s(t)$	inventory level of suppliers in time [0,T]
$q_m(t)$	inventory level of manufacturer in time [0,T]
$q_r(t)$	inventory level of retailers in time [0,T]
ATP	average total profit for the integrated models
p_{m^*}	optimum value of p_m for integrated models
ATP^*	optimum value of average total profit for the integrated models

6.2 Mathematical Formulation of the Model

6.2.1 Formulation of Suppliers Individual Average Profit

Differential equation for the supplier in Fig. 4 in [0, T] is given by

$$\frac{dq_s}{dt} = \begin{cases} p_s - p_m, & 0 \le t \le t_s \\ -p_m, & t_s \le t \le T_s \end{cases}$$

with boundary condition $q_s(t) = 0$, $t = 0$, T_s. Solving the differential equation with boundary condition, we have

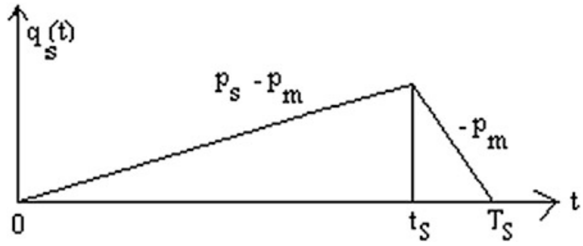
$$q_s(t) = \begin{cases} (p_s - p_m)t, & 0 \le t \le t_s \\ p_m(T_s - t) & t_s < t \le T_s \end{cases} \tag{53}$$

By continuity at $t = t_s$, we get $p_m T_s = p_s t_s$ and total unit produced by the supplier in [0, t_s] $Q_s = p_s t_s = p_m T_s$(= Total demand during [0, T_s])

$$\begin{aligned} H_s &= \text{Holding cost of supplier} \\ &= h_s\left[\int_0^{t_s}(p_s - p_m)t\,dt + \int_{t_s}^{T_s} p_m(T_s - t)dt\right] \\ &= h_s\left[\frac{p_s t_s^2}{p_m} - p_s t_s^2\right] \end{aligned}$$

The idle cost of supplier $= id_s\left[T_R + p_s t_s\left(\frac{1}{D_c} - \frac{1}{p_m}\right)\right]$
Total purchase cost = $c_s\ p_m T_s$
Total selling price = $c_m\ p_m T_s$
and ordering cost = A_s

Fig 4 Inventory level of supplier



$$\begin{aligned}\text{APS} &= \text{Average profit of supplier.}\\ &= \frac{1}{T}\,[\text{revenue from sale} - (\text{purchase} + \text{holding} + \text{idle} + \text{ordering})\text{cost.}]\\ &= \frac{1}{T}[(c_m - c_s)p_s t_s - h_s(\frac{p_s^2 t_s^2}{p_m} - p_s t_s^2) - id_s(T_R + p_s t_s\left(\frac{1}{D_c} - \frac{1}{p_m}\right)) - A_s]\end{aligned} \tag{54}$$

6.2.2 Formulation of Manufacturer Individual Average Profit

Inventory level of manufacturer in Fig. 5 in [0, T] is given by

$$q_m(t) = \begin{cases} p_m t, & 0 \le T_R \\ p_m t - iD_R, & iT_R < t \le (i+1)T_R, i = 1, 2, \ldots, (r-1) \\ p_m t - rD_R, & rT_R < t_s \\ p_m T_s - rD_R, & T_s < t \le (r+1)T_R \\ p_m T_s - iD_R, & iT_R < t \le (i+1)T_R, i = r+1, r+2, \ldots, n-1 \\ p_m T_s - nD_R, & nT_R < t \le (n+1)T_R \end{cases} \tag{55}$$

with boundary condition $q_m(0) = 0$, and $q_m(iT_R + 0) = q_m(iT_R) - D_R$

$$\begin{aligned} H_m &= \text{Holding cost for manufacturer.}\\ &= h_m[\int_0^{T_R} p_m t dt + \sum_1^{r-1} \int_{iT_R}^{(i+1)T_R} (p_m t - iD_R)dt + \int_{rT_R}^{(T_s} (p_m t - rD_R)dt \\ &\quad + \int_{T_s}^{(r+1)T_R} (p_m T_s - rD_R)dt + \sum_{r+1}^{n-1} \int_{iT_R}^{(i+1)T_R} (p_m T_s - iD_R)dt + \int_{nT_R}^{(n+1)T_R} (p_m T_s - nD_R)dt] \\ &= h_m\left[np_m T_s T_R - \frac{n^2 + n - 2r - 2}{2} T_R D_R - \frac{p_s^2 t_s^2}{2p_m}\right]\end{aligned}$$

The idle cost of manufacturer $= id_m\left[\frac{p_m T_m - nD_R}{D_c}\right]$
Total purchase cost = $c_m p_m T_s$
Total selling price = $c_r p_m T_s$
and ordering cost = A_m

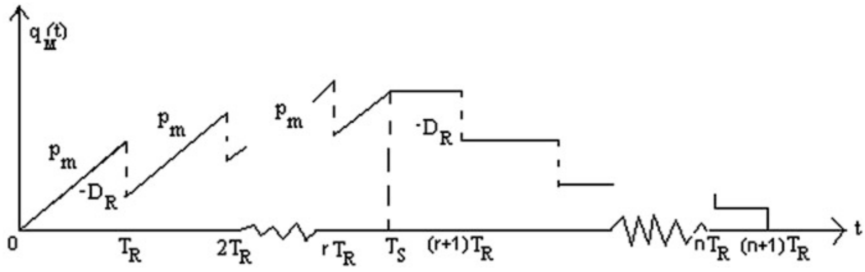


Fig. 5 Inventory level of manufacturer

6.2.3 Case-I (When $M \leq T^{'} \leq T_R$)

$$\begin{aligned}
I_{em} = I_{pr} &= \text{Amount of interest earned by the manufacturer in } [0, \mathrm{T}] \text{ from retailer.}\\
&= \text{Amount of interest paid by the retailer to the manufacturer in } [0, T].\\
&= c_r i_p [n \int_M^{T_R} (D_R - D_c t)dt + \int_M^{T'} (p_m T_s - nD_R - D_c t)dt]\\
&= \frac{nc_r i_p}{2}[T_R D_R + D_c M^2 - 2MD_R] + c_r i_p [(\frac{(p_m T_s - nD_R)^2}{2D_c}\\
&\quad + (p_m T_s - nD_R)M + \frac{D_c M^2}{2}]
\end{aligned}$$

$$\begin{aligned}
APM_1 &= \text{Average profit of Manufacturer.}\\
&= \frac{1}{T}\ [\text{revenue from sale} - \text{purchase cost} - \text{holding cost} - \text{idle cost}\\
&\quad + \text{earned interest} - \text{ordering cost.}]\\
&= \frac{1}{T}[(c_r - c_m)p_m T_s - h_m(np_m T_s T_R - \frac{n^2 + n - 2r - 2}{2} T_R D_R - \frac{p_s^2 t_s^2}{2p_m})\\
&\quad - id_m(\frac{p_m T_m - nD_R}{D_c}) + \frac{nc_r i_p}{2}[T_R D_R + D_c M^2 - 2MD_R]\\
&\quad + c_r i_p(\frac{(p_m T_s - nD_R)^2}{2D_c} + (p_m T_s - nD_R)M + \frac{D_c M^2}{2}) - A_m]
\end{aligned} \tag{56}$$

6.2.4 Case-II (When $T' \le M \le T_R$)

$$\begin{aligned} I_{em} = I_{pr} &= \text{Amount of interest earned by the manufacturer in } [0, \mathrm{T}] \text{ from retailer.} \\ &= \text{Amount of interest paid by the retailer to the manufacturer in } [0, \mathrm{T}]. \\ &= c_r i_p [n \int_M^{T_R} (D_R - D_c t) dt] \\ &= \frac{n c_r i_p}{2} [T_R D_R + D_c M^2 - 2 M D_R] \end{aligned}$$

$$\begin{aligned} APM_2 &= \text{Average profit of Manufacturer.} \\ &= \frac{1}{T} [\text{revenue from sale } - \text{purchase cost} - \text{holding cost} - \text{idle cost} \\ &\quad + \text{ earned interest} - \text{ordering cost.}] \\ &= \frac{1}{T} [(c_r - c_m) p_m T_s - h_m \left(n p_m T_s T_R - \frac{n^2 + n - 2r - 2}{2} T_R D_R - \frac{p_s^2 t_s^2}{2 p_m} \right) \\ &\quad id_m \left(\frac{p_m T_m - n D_R}{D_c} \right) + \frac{n c_r i_p}{2} [T_R D_R + D_c M^2 - 2 M D_R] - A_m] \end{aligned} \tag{57}$$

6.2.5 Formulation of Retailer Individual Average Profit

Inventory level of retailer in Fig. 6 in [0, T] is given by

$$q_r(t) = \begin{cases} D_c t, & i T_R \le t \le (i+1) T_R \\ p_m T_s - n D_R - D_c t, & (n+1) T_R \le t \le T \end{cases} \tag{58}$$

with boundary condition $q_r((n+1)T_R) = 0$, and $q_r(T) = 0$

$$\begin{aligned} H_r &= \text{Holding cost of retailer.} \\ &= n h_r \left[\int_0^{T_R} (D_R - D_c t) dt + \int_0^{T'} (p_m T_s - n D_R - D_c t) dt \right] \\ &= \frac{h_r}{2} \left[\frac{p_m^2 T_s^2}{D_c} - 2 n p_m T_s T_R - (2n+1) T_R D_R \right] \end{aligned}$$

The idle cost of retailer = $id_r T_R$
Total purchase cost = $c_r p_m T_s$
Total selling price = $c_{r1} p_m T_s$
and ordering cost = A_r,

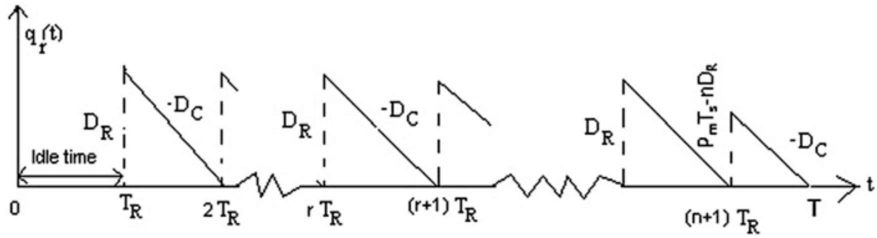


Fig. 6 Inventory level of retailer

6.2.6 Case-I (When $M \le T' \le T_R$)

Interest earned by the retailers for $(n + 1)$ cycle is given by

$$
\begin{aligned}
I_{er} &= \text{Amount of interest earned by the retailer from Bank in } (\text{n}+1) \text{ cycle.}\\
&= (n+1)c_{r_1} i_e \left[\int_0^M (M-t)D_c dt\right]\\
&= \frac{(n+1)c_{r_1} i_e D_c M^2}{2},\\
I_{pr} &= \text{Amount of interest paid by the retailer to the manufacturer in } [0,\text{T}].\\
&= c_r i_p \left[n\int_M^{T_R} (D_R - D_c t)dt + \int_M^{T'} (p_m T_s - nD_R - D_c t)dt\right]\\
&= nc_r i_p \left[\frac{T_R D_R + D_c M^2 - 2MD_R}{2}\right] + c_r i_p \Big[\frac{(p_m T_s - nD_R)^2}{2D_c}\\
&\quad + (p_m T_s - nD_R)M + \frac{D_c M^2}{2}\Big],\\
APR_1 &= \text{Average profit of retailer.}\\
&= \frac{1}{T}[\text{revenue from sale } - \text{purchase cost} - \text{holding cost}\\
&\quad + \text{earned interest} - \text{payable interest} - \text{idle cost} - \text{ordering cost}]\\
&= \frac{1}{T}\Big[c_{r_1} p_m T_s - c_r p_m T_s - \frac{h_r}{2}\Big(\frac{p_m^2 T_s^2}{D_c} - 2np_m T_s T_R - (2n+1)T_R D_R\Big)\\
&\quad + \frac{(n+1)c_{r_1} i_e D_c M^2}{2} - \frac{nc_r i_p}{2}[T_R D_R + D_c M^2 - 2MD_R] - id_r T_R - A_r\\
&\quad + (p_m T_s - nD_R)M + \frac{D_c M^2}{2}\Big) - c_r i_p\Big(\frac{(p_m T_s - nD_R)^2}{2D_c}\Big]
\end{aligned} \tag{59}
$$

6.2.7 Case-II (When $T' \leq M \leq T_R$)

Interest earned by the retailer for (n + 1) cycle

$$\begin{aligned} I_{er} &= \text{Amount of interest earned by the retailer from Bank in n cycle.} \\ &= c_{r_1} i_e \left[n \int_0^M (M-t) D_c dt + \int_0^{T'} (T'-t) D_c dt + (M-T')(p_m T_s - nD_R) \right] \\ &= \frac{nc_{r_1} i_e D_c M^2}{2} + \frac{c_{r_1} i_e}{2} (p_m T_s - nD_R)(2M - T') \end{aligned}$$

Interest payable by the retailers for 1st n cycle is given by

$$\begin{aligned} I_{pr} &= \text{Amount of interest paid by the retailer to the manufacturer in } [0, T]. \\ &= c_r i_p [n \int_M^{T_R} (D_R - D_c t) dt] \\ &= \frac{nc_r i_p}{2} [T_R D_R + D_c M^2 - 2MD_R] \end{aligned}$$

$$\begin{aligned} APR_2 &= \text{Average profit for retailer.} \\ &= \frac{1}{T} \text{ [revenue from sale − purchase cost − holding cost} \\ &\quad + \text{ earned interest − payable interest − idle cost − ordering cost].} \\ &= \frac{1}{T} [(c_{r_1} - c_r) p_m T_s - \frac{h_r}{2} \left(\frac{p_m^2 T_s^2}{D_c} - 2np_m T_s T_R - (2n+1) T_R D_R \right) \\ &\quad + \frac{nc_{r_1} i_e D_c M^2}{2} + \frac{c_{r_1} i_e}{2} (p_m T_s - nD_R)(2M - T') \\ &\quad - \frac{nc_r i_p}{2} [T_R D_R + D_c M^2 - 2MD_R] - id_r T_R - A_r] \end{aligned} \tag{60}$$

6.3 Integrated Model Under Fuzzy Environment

We consider $I_{re}^{\sim} = (I_{re_1}, I_{re_2}, I_{re_3})$ be a triangular fuzzy number. Then the objective reduce to

6.3.1 For case-I $M \leq T^{'} \leq T_R$

$$
\begin{aligned}
A\tilde{T}P_1 = {} & \frac{D_c}{p_m T_s + D_R}[(c_m - c_s)p_m T_s - h_s\left(\frac{p_s t_s^2}{p_m} - p_s t_s^2\right) - id_s\left(T_R + p_s t_s(\frac{1}{D_c} - \frac{1}{p_m})\right) \\
& - A_s + (c_r - c_m)p_m T_s - h_m\left(np_m T_s T_R - \frac{n^2 + n - 2r - 2}{2}T_R D_R - \frac{p_s^2 t_s^2}{2p_m}\right) \\
& - A_m + (c_{r_1} - c_r)p_m T_s - \frac{h_r}{2}\left(\frac{p_m^2 T_s^2}{D_c} - 2np_m T_s T_R - (2n+1)T_R D_R\right) \\
& - id_m\left(\frac{p_m T_m - nD_R}{D_c}\right) + \frac{(n+1)c_{r_1}\tilde{I}_{re}]D_c M^2}{2} - id_r T_R - A_r] \\
= {} & \frac{D_c}{p_m T_s + D_R}\left[\frac{h_m - h_s}{2}p_m T_s^2 + (\frac{h_s}{2p_s} - \frac{h_r}{2D_c})p_m^2 T_s^2 + Ap_m T_s + B_1\right]
\end{aligned}
\tag{61}
$$

where $p_m\ T_s = p_s\ t_s$,

$$
\begin{aligned}
A & = (c_{r_1} - c_s) + n(h_r - h_m)T_R - \frac{id_m + id_s}{D_c}, \\
B_1 & = \left[h_m\frac{n^2 + n - 2r - 2}{2} + h_r\frac{2n+1}{2}\right]T_R D_R + \frac{(n+1)c_{r_1}E[I_{re}^{\sim}]D_c M^2}{2} \\
& \quad + (nid_m - id_s - id_r)T_R + id_s T_s - (A_s + A_m + A_r)
\end{aligned}
\tag{62}
$$

6.3.2 For case-II ($T^{'} \leq M \leq T_R$)

$$
\begin{aligned}
A\tilde{T}P_2 = {} & \frac{D_c}{p_m T_s + D_R}[(c_m - c_s)p_m T_s - h_s\left(\frac{p_s t_s^2}{p_m} - p_s t_s^2\right) - id_s\left(T_R + p_s t_s\left(\frac{1}{D_c} - \frac{1}{p_m}\right)\right) \\
& - (A_m + A_s) + (c_{r1} - c_m)p_m T_s - h_m\left(np_m T_s T_R - \frac{n^2 + n - 2r - 2}{2}T_R D_R - \frac{p_s^2 t_s^2}{2p_m}\right) \\
& - id_m\left(\frac{p_m T_m - nD_R}{D_c}\right) - \frac{h_r}{2}\left(\frac{p_m^2 T_s^2}{D_c} - 2np_m T_s T_R - (2n+1)T_R D_R\right) \\
& + \frac{nc_{r_1}I_{re}^{\sim}D_c M^2}{2} + \frac{c_{r_1}I_{re}^{\sim}]}{2}(p_m T_s - nD_R)(2M - T') - id_r T_R - A_r] \\
= {} & \frac{D_c}{p_m T_s + D_R}\left[\frac{h_m - h_s}{2}p_m T_s^2 + \left(\frac{h_s}{2p_s} - \frac{h_r}{2D_c}\right)p_m^2 T_s^2 + Ap_m T_s + C_1\right]
\end{aligned}
\tag{63}
$$

where $p_m T_s = p_s t_s$,

$$A = (c_{r_1} - c_s) + n(h_r - h_m)T_R - \frac{id_m + id_s}{D_c},$$

$$\begin{aligned}C_1 = &(h_m \frac{n^2 + n - 2r - 2}{2} + h_r \frac{2n+1}{2})T_R D_R + (nid_m - id_s - id_r)T_R + id_s T_s \\ &+ \frac{nc_{r1} I_{re}^{\sim}]D_c M^2}{2} + \frac{c_{r_1} I_{re}^{\sim}}{2}(p_m T_s - nD_R)(2M - T') - (A_s + A_m + A_r)\end{aligned}$$

6.3.3 Equivalent Crisp Model

Using lemma 4, We put $E[\tilde{\tilde{I}}_e] = \frac{1}{2}[(1-\rho)I_{re_1} + I_{re_2} + \rho I_{re_3}]$ where $0 < \rho < 1$. The expected total average profit,

6.3.4 For case-I ($M \le T' \le T_R$)

$$\begin{aligned}E[ATP_1] = &\frac{D_c}{p_m T_s + D_R}[(c_m - c_s)p_m T_s - h_s\left(\frac{p_s t_s^2}{p_m} - p_s t_s^2\right) - id_s\left(T_R + p_s t_s\left(\frac{1}{D_c} - \frac{1}{p_m}\right)\right) \\ &- h_m\left(np_m T_s T_R - \frac{n^2 + n - 2r - 2}{2} T_R D_R - \frac{p_s^2 t_s^2}{2p_m}\right) - id_m\left(\frac{p_m T_m - nD_R}{D_c}\right) \\ &- A_m - A_s + (c_{r_1} - c_r)p_m T_s - \frac{h_r}{2}\left(\frac{p_m^2 T_s^2}{D_c} - 2np_m T_s T_R - (2n+1)T_R D_R\right) \\ &+ (c_r - c_m)p_m T_s + \frac{(n+1)c_{r_1}[(1-\rho)I_{re_1} + I_{re_2} + \rho I_{re_3}]D_c M^2}{4} - id_r T_R - A_r]\end{aligned}$$

6.3.5 For Case-II ($T' \le M \le T_R$)

$$\begin{aligned}E[ATP_2] = &\frac{D_c}{p_m T_s + D_R}[(c_m - c_s)p_m T_s - h_s\left(\frac{p_s t_s^2}{p_m} - p_s t_s^2\right) - id_s\left(T_R + p_s t_s\left(\frac{1}{D_c} - \frac{1}{p_m}\right)\right) \\ &- h_m[\left(np_m T_s T_R - \frac{n^2 + n - 2r - 2}{2} T_R D_R - \frac{p_s^2 t_s^2}{2p_m}\right) - id_m[\left(\frac{p_m T_m - nD_R}{D_c}\right) \\ &- A_m + (c_{r1} - c_r)p_m T_s - \frac{h_r}{2}\left(\frac{p_m^2 T_s^2}{D_c} - 2np_m T_s T_R - (2n+1)T_R D_R\right) \\ &- A_s + (c_r - c_m)p_m T_s + \frac{nc_{r_1}[(1-\rho)I_{re_1} + I_{re_2} + \rho I_{re_3}]D_c M^2}{4} \\ &+ \frac{c_{r_1}[(1-\rho)I_{re_1} + I_{re_2} + \rho I_{re_3}]}{4}(p_m T_s - nD_R)(2M - T') - id_r T_R - A_r]\end{aligned} \tag{64}$$

Table 4 Input data of different parameter for case-I and case-II

Parameter	Case-I	Case-II	Parameter	Case-I	Case-II
c_s	8	8	D_R	120	120
c_m	14	14	D_C	50	50
c_r	25	25	id_s	1	1
c_{r_1}	30	30	id_m	2	2
h_s	0.05	0.05	id_r	3	3
h_m	0.1	0.1	A_s	20	20
h_r	0.2	0.2	A_m	30	30
T_s	10	10	A_r	40	40
p_s	150	150	$\tilde{\tilde{I}}_e$	(0.08, 0.09, 0.11)	(0.08, 0.09, 0.11)
n	5	6	M	1.6	2
r	4	4	i_p	0.1	0.1
ρ	0.5	0.5			

Table 5 Optimal values of objective and decision variable in equivalent crisp environment for case-I & case-II

Parameter	Case-I	Case-II	Parameter	Case-I	Case-II
ATP^*	1041.43	1110.75	APS^*	248.71	253.91
p_m^*	69.86	79.31	APM^*	467.76	463.61
T'	1.97	1.46	APR^*	324.96	393.23

6.4 A Numerical Example

Tables 4 and 5.

6.5 Discussion

From Table 5, It is observed that under fuzzy environment optimal profits for supplier and retailers are better in case-II (i.e. when $T' \leq M \leq T_R$) than case-I (i.e. when $T' \leq M \leq T_R$). But manufacturer individual profit is less in case-II then case-I. This is due to large delay in payment, retailer earned interest is greater for greater value of delay in payment. Concave nature of the integrated model is shown analytically and graphically in Fig. 7 for the both cases.

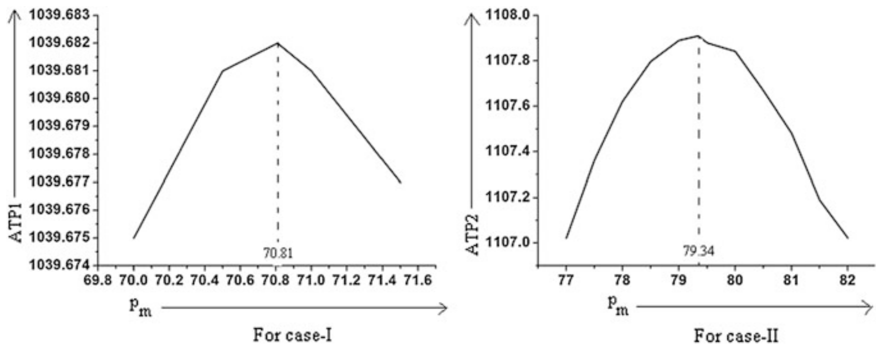


Fig. 7 Average profit versus supply rate of supplier's in case-I and case-II

7 Conclusion

Supply chain management spans all movement and storage of raw materials, work-in-process inventory, and finished goods from point of origin to point of consumption. By decision-making in a fuzzy environment is meant a decision process in which the goals and/or the constraints, but not necessarily the system under control, are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined. Many decision making processes in supply chain management are under vague and uncertain environment. For instance supplier selection process is made under an environment which selection criteria and alternatives are evaluated imprecisely.

In the first model a real-life necessity, possibility and credibility constraints and possibility objective in the context of a finite number of raw material collection, manufacturing, extra stock in rented warehouse and sale from market warehouse is defined and defuzified. This model has showned an optimal policy for necessity/possibility/credibility based constraints on fuzzy resources. For the first time, a collection-production-inventory system in a SC including two warehouse with fuzzy resource constraints/objective has been formulated and solved as an optimal control problem through GRG technique.

The second model addresses optimal order placement and delivery rate policies for a three stage SCS. The system consists of a supplier, a producer and a retailer and work-in-process inventory and production of finished products (by producer) and sale of good products (by retailer) for a finite time period. Here, demand of the item is time-dependent. The production has a defective rate, which decreases cycle to cycle due to the learning effect. Shortages, if any are allowed and partially backlogged. The system has been formulated as integrated (the whole system under a single management-single objective model) and non-integrated (multi-objective) model. An appropriate GA is developed and applied to solve the models.

In the third model, a production inventory three layers supply chain model under fuzzy environment has been developed. Here suppliers is also manufacturer, collect the row material(ore) and produce the row material of actual manufacture. For example, In petrochemical industries suppliers collect the ore and produced the naphthalene, which is the row material of manufacturer. Then manufacturer produce the usable product to sale the retailer.

The formulation and analysis of the above models are quite general in nature and can be extended for future research work to other collection-production-inventory problems with fuzzy defectiveness, price discount, etc. Deterioration can be allowed for produced items of manufacturer and also in case of retailer. The models also can be extended for future research work to other collection-production-inventory problems with uncertain environments like fuzzy, bi-fuzzy and fuzzy rough environments.

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Selection and Assignment of Material Handling Devices Under Uncertainty

Alp Ustundag

Abstract Material handling devices (MHDs) transfer the materials among the machines, work stations and support services in manufacturing and distribution facilities. It is very important to select and assign the required number and types of MHDs to minimize the operating and investment costs. Generally, the parameters have uncertain values by solving the MHD selection and assignment problem. Especially, the number of batches required to be transported cannot be determined as crisp values. This chapter provides a mathematical programming basis for MHD selection and assignment problem under fuzzy environment. In this study, background information about MHD selection and assignment as well as fuzzy linear programming is given. Additionally, a related hypothetical problem is solved for a specific case.

Keywords Material handling device · Selection · Assignment · Fuzzy linear programming

1 Introduction

Material handling is an activity that uses the right method to provide the right amount of the right material at the right place, at the right time, in the right sequence, in the right position and at the right cost (Tompkins et al. 2003). Without a well-designed material handling system, manufacturing and logistics operations could encounter delays, production times could increase, products could get damaged or contaminated, and cost of movement within a facility could increase thereby increasing operating cost (Hassan 2010).

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Material handling device (MHD) selection is an important function in the design of a material handling system, and thus a crucial step for facilities planning. MHDs transfer the materials among the machines, work stations and support services in manufacturing and distribution facilities Using proper MHD can enhance the production process, provide effective utilization of manpower, increase production, and improve system flexibility (Rao 2007). In a typical manufacturing company, multiple MHDs are used in combination to form a material handling system. MHDs have been classified into the following main groups of industrial trucks, conveyors, automated guided vehicles (AGVs), cranes, storage/retrieval systems and industrial robots (Kulak 2005).

It is very important to select and assign the required number and types of MHDs to minimize the operating and investment costs. Generally, the parameters have uncertain values in this problem. Especially, the number of batches required to be transported cannot be determined as crisp values. Therefore MHD selection and assignment problem can be solved using fuzzy linear programming model under uncertainty.

Fuzzy set theory has been widely applied in different disciplines, such as operations research, management science, control theory and artificial intelligence. Fuzzy mathematical programming is one of the most popular decision making approaches based on the fuzzy set theory (Shih 1999). In this chapter, imprecise parameters and fuzzy situation in MHD selection and assignment is addressed. The inspiration of this chapter is to provide a mathematical programming basis for this problem under imprecise data.

The rest of this chapter is organized as follows. In Sect. 2, relevant literature is reviewed. In Sect. 3, MHD selection and assignment model is described. Fuzzy linear programming techniques are included in Sect. 4. Application of fuzzy linear programming to MHD selection and assignment problem is given in Sect. 5. Analysis of the results is indicated in Sect. 6. Finally, discussion and conclusions are provided in Sect. 7.

2 Literature Study

In the literature, there are various studies focusing on the solution of the complicated problem of material handling device selection and assignment. These studies generally propose analytical models and decision support systems to solve the problem (Hassan 2010). Malmborg et al. (1987) developed a prototype expert system considering 17 equipment attributes and 47 devices for industrial truck type selection. Swaminathan et al. (1992) developed EXCITE, the expert consultant for in-plant transportation equipment, addressing 35 equipment types, and 28 material, move, and method attributes. Chu et al. (1995) developed a computer-aided material handling equipment selection system called ADVISOR. Chan et al. (2001) described the development of an intelligent material handling equipment selection system called material handling equipment selection advisor (MHESA).

Fonseca et al. (2004) developed a knowledge-based system for conveyor equipment selection. Lashkari et al. (2004) presented an integrated approach to operation allocation (OA) and material handling systems selection (MHSS) in cellular manufacturing systems. Kulak (2005) developed a decision support system called FUMAHES—fuzzy multi-attribute material handling equipment selection. Chakraborthy and Banik (2006) focused on the application of the AHP technique in selecting the optimal material handling equipment for a specific material handling equipment type. Sujono and Lashkari (2007) proposed a method for simultaneously determining operation allocation and material handling system selection in an FMS environment with multiple performance objectives. The 0–1 integer programming model was developed to select machines, assign operations of part types to the selected machines, and assign material handling equipment to transport the parts from machine to machine, as well as to handle the part in a given machine. Rao (2007) proposed graph theory and matrix approach (GTMA) for a conveyor selection problem. Onut et al. (2009) proposed a combined multi-criteria decision making (MCDM) methodology for evaluation and selection of MHD types for a company in the steel construction industry using fuzzy analytic network process (FANP) and fuzzy technique for order preference by similarity to ideal solution (TOPSIS).

When considering the previous studies in the literature, it is revealed that there is insufficient number of papers handling the MHD selection and assignment problem under uncertainty. Additionally, there has not been published any study using fuzzy linear programming model to solve this problem in uncertain conditions. In fuzzy mathematical programming, models have been proposed to incorporate fuzziness of objective and constraint functions. In this chapter, imprecise parameters and fuzzy situation in MHD selection and assignment is addressed. Furthermore, a hypothetical problem is solved with the consideration of vagueness in number of batches of parts transported between machines in a factory environment.

3 MHD Selection and Assignment Model

In this chapter, a MHD selection and assignment model is described considering various types of machines, MHDs and parts in a manufacturing environment. The objective of the model is to select the required number and types of MHDs and assign them to material-handling moves by minimizing the operating and annualized investment costs of the MHDs (Heragu 2008). A material-handling move is the physical move performed by a MHD to transport a unit load between a pair of machines. The volume and transfer batch size of each part type to be manufactured and the number of machines to be visited are the decisive factors for the number of material-handling moves. The main goal is here to evaluate all possible moves that can be executed by the potential MHD types and to determine the optimal selection and assignment. While the numbers of batches of different part types to be transported have fuzzy values, the other parameters have crisp values in the model. The indices, parameters and decision variables of the model are given below (Heragu 2008):

Indices:

i Part type index, $i = 1, 2,\ldots, p$

j Machine type index, $j = 1, 2,\ldots, m$

l MHD type index, $l = 1, 2,.., n$

Parameters:

L_i Set of MHDs that can be used to transport part type i

PP Length of planning period

$\widetilde{Q}_i$ Fuzzy number of batches of part type i required to be transported

K_{ij} Set of machines to which a batch of part type i can be sent from machine j for the next processing step

M_{ij} Set of machines from which a batch of part type i can be sent to machine j for the next processing step

E_i Set of machine types required for the first operation on part type i

F_i Set of machine types required for the last operation on part type i

S_l Purchase cost of MHD PP_l

T_{ijkl} Time required to move one batch of part type i from machine type j to k using MHD l

C_{ijkl} Unit transportation cost to move one batch of part type i from machine j to k using MHD l

Decision variables:

X_{jjkl} Number of batches of part type i to be transported from machine j to k using MHD l

Y_l Number of units of MHD type l selected

The problem is formulated as a linear optimization model:

$$Min \sum_{l=1}^{n} S_l Y_l + \sum_{i=1}^{p} \sum_{j=1}^{m} \sum_{k \in K_{ij}} \sum_{l \in L_i} C_{ijkl} X_{ijkl} \tag{1}$$

s.t.

$$\sum_{j \in E_i} \sum_{k \in K_{ij}} \sum_{l \in L_i} X_{ijkl} = \tilde{Q}_i \quad \text{for } i = 1, 2, \ldots, p \tag{2}$$

$$\sum_{k \in M_{ij}} \sum_{l \in L_i} X_{ijkl} - \sum_{k \in K_{ij}} \sum_{l \in L_i} X_{ijkl} = 0 \quad \text{for } i = 1, 2, \ldots, p \quad j : j \notin E_i \cup F_i \tag{3}$$

$$\sum_{j \in F_i} \sum_{k \in M_{ij}} \sum_{l \in L_i} X_{ijkl} = \tilde{Q}_i \quad \text{for } i = 1, 2, \ldots, p \tag{4}$$

$$\sum_{i=1}^{p}\sum_{j=1}^{m}\sum_{k\in K_{ij}} T_{ijkl}X_{ijkl} \le PPY_l \quad \text{for } l = 1, 2, \ldots, n \tag{5}$$

$$X_{ijkl} \ge 0 \text{ and integer} \quad i = 1, 2, \ldots, p \quad j = 1, 2, \ldots, m \quad k = 1, 2, \ldots, m \quad l = 1, 2, \ldots, n \tag{6}$$

$$Y_l \ge 0 \text{ and integer} \quad l = 1, 2, \ldots, n \tag{7}$$

In the model above, the objective function minimizes the purchase and transportation costs in Eq. (1). In Eqs. (2–4), the constraints ensure the balanced flow of parts to be processed between the machines. Equation (5) is the MHD capacity constraint and finally the Eqs. (6–7) are the integer constraints for the number of batches of parts and number of units of MHD, respectively.

4 Fuzzy Linear Programming Methods

Fuzzy linear programming can be derived by using fuzzy sets as coefficient values in objective function, constraints or right hand sides of the constraints. There are several methods in the literature to solve fuzzy linear programming models (Verdegay 1982; Chanas 1983; Zimmermann 1991; Julien 1994; Negoita and Sularia 1976; Carlsson and Korhonen 1986; Werners 1987; Buckley 1989). In this section, the methods of Verdegay (1982), Zimmerman (1991) and Julien (1994) are discussed respectively.

In Verdegay's method (1982), right-hand side constant in Eq. (8) is considered as fuzzy value with toleration level of pi in the interval of $[b_i, b_i + p_i]$.

$$\begin{aligned} &\text{Max } cx \\ &\text{s.t.} \\ &a_i x \le b_i \quad i = 1, \ldots, m \\ &x \ge 0 \end{aligned} \tag{8}$$

The membership function of the fuzzy constraints is defined in Eq. (9).

$$\mu_i(a_i x) = \left\{ \begin{array}{cc} 1 & a_i x \le b_i \\ 1 - \frac{a_i x - b_i}{p_i} & b_i \le a_i x \le b_i + p_i \\ 0 & b_i + p_i \le a_i x \end{array} \right\} \tag{9}$$

Verdegay transformed Eq. (8) to a parametric programming problem in Eq. (10). The parametric solution is obtained when $(1-\alpha)$ is replaced with the parameter $\theta \in [0,1]$. So, as the parameter θ moves from 0 to 1, the satisfaction level decreases from 100 % to 0.

$$
\begin{aligned}
& \text{Max}\ cx \\
& \text{s.t.} \\
& a_i x \le b_i + p_i(1 - \alpha) \quad i = 1, \ldots, m \\
& \alpha \in [0,\ 1] \\
& x \ge 0
\end{aligned} \tag{10}
$$

In Zimmermann's approach (1991), both objective and constraint functions are considered as fuzzy values in the following equation.

$$
\begin{aligned}
& \widetilde{\text{Max}}\ cx \\
& \text{s.t.} \\
& a_i x \lesssim b_i \quad i = 1, \ldots, m \\
& x \ge 0
\end{aligned} \tag{11}
$$

An aspiration level and a tolerance interval are proposed for the fuzziness of the objective function. In the fuzzy constraints, a fuzzy inequality can be considered as fuzzy right-hand sides. On the condition that an aspiration level of objective value is denoted as b_0, the fuzzy mathematical model is called a symmetric fuzzy model and can be written as follows (Rao 2007):

$$
\begin{aligned}
& cx \gtrsim b_0 \\
& a_i x \lesssim b_i \quad i = 1, \ldots, m \\
& x \ge 0
\end{aligned} \tag{12}
$$

The following matrix notation is used for defining symmetric fuzzy model:

$$
\begin{aligned}
& Ax \lesssim b \\
& x \ge 0 \\
& A = \begin{bmatrix} -c \\ a_i \end{bmatrix} \quad b = \begin{bmatrix} -b_0 \\ b_i \end{bmatrix}
\end{aligned} \tag{13}
$$

Having an interval, the fuzzy inequality violation of right-hand-side values, b_i, is equivalent to the fuzzy inequality. The membership function of the degree of violation of the fuzzy inequality is expressed as follows where p_i is a tolerance level in the fuzzy relationship as in Eq. (9). The problem is now to obtain the maximum value of the membership degree which can be expressed as the following model which presents the final solution.

$$\begin{aligned}
&\text{Max } \lambda \\
&\text{s.t.} \\
&\lambda \leq 1 - \frac{a_i x - b_i}{p_i} \\
&\lambda \leq 1 \\
&x \geq 0
\end{aligned} \tag{14}$$

Julien (1994) transformed the fuzzy linear programming problem with the best and the worst linear programming problem at different α-cut levels, and got the possibility distribution of the optimal objective value. Julien associated the α-cut concept with Buckley's (1989) possibility programming to resolve the maximization problem in Eq. (11) including fuzzy objective and fuzzy right hand side. Therefore, the crisp linear programming problems in Eqs. (15) and (16) are solved where the superscript represents an α-cut of the fuzzy parameters, and the subscripts L and U are the corresponding lower and upper cuts (Allahviranloo and Afandizadeh 2008).

$$\begin{aligned}
&\text{Max } c_L^{\alpha} x \\
&\text{s.t.} \\
&A_U^{\alpha} x \leq b_L^{\alpha} \quad i = 1, \ldots, m \\
&x \geq 0
\end{aligned} \tag{15}$$

$$\begin{aligned}
&\text{Max } c_U^{\alpha} x \\
&\text{s.t.} \\
&A_L^{\alpha} x \leq b_U^{\alpha} \quad i = 1, \ldots, m \\
&x \geq 0
\end{aligned} \tag{16}$$

When Julien's method is applied to a minimization problem given in Eq. (17), the Eqs. (18) and (19) should be considered to determine the interval of the objective value.

$$\begin{aligned}
&\widetilde{\text{Min}}\ cx \\
&\text{s.t.} \\
&a_i x \underset{\sim}{\geq} b_i \quad i = 1, \ldots, m \\
&x \geq 0
\end{aligned} \tag{17}$$

$$\begin{aligned}
&\text{Min } c_U^{\alpha} x \\
&\text{s.t.} \\
&A_L^{\alpha} x \geq b_U^{\alpha} \quad i = 1, \ldots, m \\
&x \geq 0
\end{aligned} \tag{18}$$

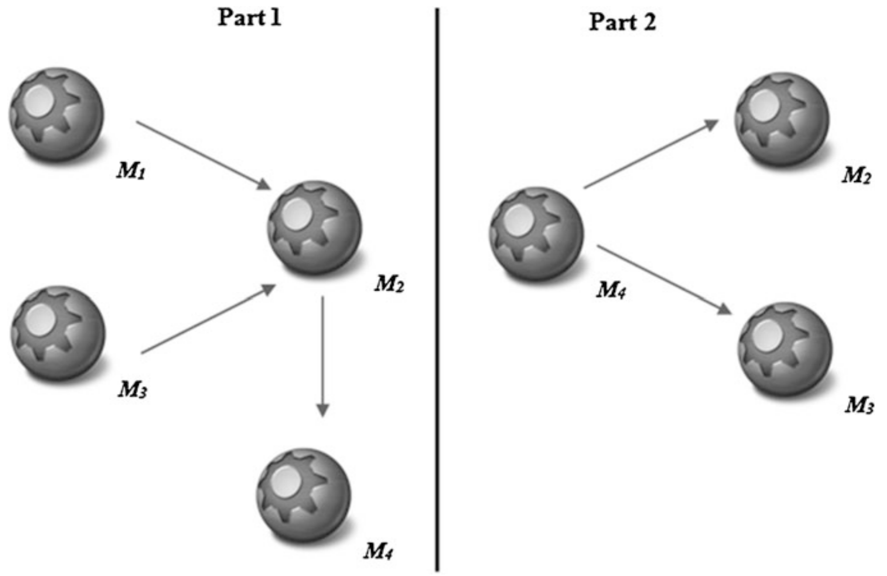

Fig. 1 Processing steps for Part 1 and Part 2

$$\begin{aligned}&\text{Min}\, c_L^\alpha x\\&\text{s.t.}\\&A_U^\alpha x \geq b_L^\alpha \quad i = 1,\ldots,m\\&x \geq 0\end{aligned} \tag{19}$$

5 MHD Selection and Assignment Application with Fuzzy Parameters

In the hypothetical case (Heragu 2008), the company plans to select and assign MHDs for a manufacturing system with the consideration of vagueness in the number of batches of parts to be processed. The company produces two types of parts called as P_1 and P_2, so Fig. 1 displays the processing steps of each part. According to this, 50 % of P_1 are processed on machine 1 and the rest are processed on machine 3. Then, the parts are sent to machine 2. After being processed on machine 2, the parts of P_1 are processed on machine 4. In addition to part type P_1, the facility also processes part type P_2. So, the parts of P_2 are first processed on machine 4 then 50 % of these are sent to machine 2 and the rest to machine 3 to be further processed.

Table 1 The upper and lower bounds of the number of parts

α-cut	Upper/lower bounds	P−1	P−2
1	–	60	80
0.75	Lower	56	74
	Upper	64	86
0.5	Lower	52	68
	Upper	70	92
0.25	Lower	46	62
	Upper	74	98
0	Lower	42	56
	Upper	78	104

Table 2 Unit transportation cost and times for part types P_1 and P_2

Part type	MHD			Machine type			
	From	Type	To	M_1	M_2	M_3	M_4
P_1	M_1	1		–	6(10)	–	–
P_1	M_1	2		–	4(8)	–	–
P_1	M_3	1		–	9(5)	–	–
P_1	M_3	2		–	4(2)	–	–
P_1	M_2	1		–	–	–	6(6)
P_1	M_2	2		–	–	–	3(5)
P_2	M_4	1		–	5(4)	10(6)	–
P_2	M_4	2		–	2(2)	12(2)	–

The number of batches of parts is expressed as fuzzy triangular numbers, since they cannot be determined precisely. Their upper and lower values for different α-cuts are given in Table 1.

The goal is here to obtain the optimal number and types of MHDs and assign them to material-handling moves by minimizing the transportation and purchase costs of the MHDs using fuzzy linear programming. Two candidates MHD_1 and MHD_2 with purchase costs of \$60,000 and \$80,000 respectively, are being considered. The unit cost for transporting P_1 and P_2 on each of the MHDs between the machines and is given in Table 2. Additionally in this table, the transportation time per unit in seconds are also provided in parentheses. In the application, there are 1250 s in the planning period and each handling device is expected to make empty trips 40 % of the time.

In this study, Julien's (1994) method is used to solve the problem since it provides computation convenience for the decision maker comparing to other methods to reach a crisp decision considering the fuzzy parameter values.

Table 3 Lower and upper bounds of optimum values for MHD selection and assignment

0		$\alpha = 0.75$		$\alpha = 0.50$	
		L	U	L	U
Cost ($)	140,900	80,910	140,964	80,840	141,053
Y_1	1	0	1	0	1
Y_2	1	1	1	1	1
X_{1121}	0	0	0	0	5
X_{1122}	30	28	32	26	30
X_{1321}	0	0	0	0	0
X_{1322}	30	28	32	26	35
X_{1241}	0	0	0	0	0
X_{1242}	60	56	64	52	70
X_{2421}	0	0	0	0	0
X_{2422}	40	37	43	34	46
X_{2431}	40	0	43	0	46
X_{2432}	0	37	0	34	0

Table 4 Lower and upper bounds of optimum values for MHD selection and assignment

	$\alpha = 0.25$		$\alpha = 0$	
	L	U	L	U
Cost ($)	80,756	141,128	80,686	141,204
Y_1	0	1	0	1
Y_2	1	1	1	1
X_{1121}	0	11	0	17
X_{1122}	23	26	21	22
X_{1321}	0	0	0	0
X_{1322}	23	37	21	39
X_{1241}	0	0	0	0
X_{1242}	46	74	42	78
X_{2421}	0	0	0	0
X_{2422}	31	49	28	52
X_{2431}	0	49	0	52
X_{2432}	31	0	28	0

6 Analysis of the Results

Using the parameters in Sect. 5, the model is constructed by using LINDO 6.1 optimization software. Results are obtained for the α-cut values of 0, 0.25, 0.50, 0.75 and 1. Lower and upper bounds of optimum values for MHD selection and assignment are given in Tables 3 and 4.

According to Tables 3 and 4, upper bound of optimum cost value increases with higher level of fuzziness where the highest value is $141,204 for α-cut level is 0. Inversely, lower α-cut values yields decreasing lower bounds where the lowest value is $80,686 with α-cut level is 0. In fact, α-cut value can be considered as the level of certainty. Range between lower and upper bounds is inversely related to α-cut value. The underlying reason of this fact is that range of fuzzy parameters of number of batches of parts gets wider with higher level of vagueness. While the lower bound has the range of [80,686; 80,910], upper bound has the range of [140,964; 141,204]. At upper bound levels, all MHDs are selected for each situation, so that the costs at upper bounds are higher than at lower bounds since only MHD_2 is selected at lower bound levels. Additionally, for part type 2, MHD_1 is used for transportation between machine 4 and 3 at upper bounds, however only MHD_2 is used at lower bound levels.

7 Discussion and Conclusion

MHDs are the most important parts of today's manufacturing systems and increasingly playing an important role in the productivity of the plant (Onut et al. 2009). Due to the huge capital investment requirement, MHD selection problem should be examined carefully. Especially, the rising uncertainty in manufacturing and logistics environment increases the complexity of this problem. In this chapter, a fuzzy linear programming model is provided to obtain the optimal number and types of MHDs and assign them to material-handling moves by minimizing the transportation and purchase costs of the MHDs. The proposed model is applied to a hypothetical problem in a manufacturing facility. The advantage of using fuzzy linear programming is here that the uncertainty of number of batches of different part types is considered in the problem. So, the optimal assignment and selection of MHDs have been determined for different fuzziness levels.

The difference between the fuzzy mathematical programming approach and conventional mathematical programming approach is in the point where a fuzzy model exists between a real world optimization problem and usual mathematical model (Inuiguchi and Ramik 2000). In real world problems, there are usually uncertainties in knowledge on parameters. Additionally, qualitative constraints and objectives are almost difficult to represent in mathematical forms. In such conditions, a fuzzy solution satisfying the given mathematically represented requirements are very useful in a sense of weak focus in the feasible area. The decision maker can select the final solution from the fuzzy solution considering implicit and mathematically weak requirements (Inuiguchi and Ramik 2000). In the MHD selection and assignment problem, the decision makers can consider fuzzy values for system parameters using the fuzzy linear programming method, thus they can understand the range of optimum costs for different uncertainty levels. Furthermore, they will be able to decide how to select and assign MHDs under fuzziness. However, the complexity of the computations is increased due to the requirements for the transformation of the fuzzy model into crisp one.

As a future study, different fuzzy linear programming methods can be compared for MHD selection and assignment problem. Another research direction would be to analyze the problem with multi-objectives.

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Part VII
Green and Reverse Logistics Under Fuzziness

Government Green Procurement: A Fuzzy-DEMATEL Analysis of Barriers

Yijie Dou, Joseph Sarkis and Chunguang Bai

Abstract Government green procurement has been a major driver of demand for environmental products and services in developing countries. This reality is not lost in China. However, government green procurement in China has still seen limited utilization. To address the obstacles facing government green procurement adoption in China, various barrier factors that may contribute to limited adoption of this practice are investigated. A fuzzy-based DEMATEL and fuzzy clustering approach is applied to identify the relative relationship and significance of various factors that negatively influence Chinese government green procurement at a municipal level. Further, managerial and research implications, and future research directions are presented.

Keywords Government green procurement · Fuzzy set · DEMATEL · Fuzzy c-means clustering

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1 Introduction

Governments in many developed countries have adopted government green procurement (GGP) which encourages and emphasizes the environmental characteristics of products and services to varying degrees (Ho et al. 2010; Correia et al. 2013; Fisher and Corbalán 2013; Nijaki and Worrel 2012; Snider et al. 2013; Walker and Brammer 2012; Zhu et al. 2013). Since the 1990s, GGP has become a promising government-led strategy to foster green markets and promote sustainable development, especially in developing countries such as China (Geng and Doberstein 2008; Zhu et al. 2013).

In the mid-1990s, GGP was diffused amongst developing countries, especially newly industrialized Asian economies such as Singapore, South Korea, Hong Kong and Taiwan. Subsequently, GGP has been a major driver of demand for environmental products and services in developing countries.

This GGP practices diffusion is important in China. China's thirty years' rapid industrial and economic development has caused severe issues of resource shortages and environmental pollution. Green markets could undoubtedly contribute to addressing these concerns. However, consumers' demand for green products in China is still weak due to their relatively minimal environmental awareness. This situation and China's centralized planned economy provides government with an even larger role. Thus, compared with the developed countries, GGP plays a more significant role in building green markets in China.

Unfortunately, GGP has seen limited utilization within China (Geng and Doberstein 2008; Guo 2012). This raises an imperative issue of what barriers hinder the implementation of GGP in China. Some authors have presented a descriptive analysis to investigate the barriers of Chinese GGP (e.g., Geng and Doberstein 2008). Nevertheless, few studies have applied formal models and tools to investigate this important issue. Formal models and tools are helpful in providing prescriptive and descriptive insights that qualitative investigations may overlook. Some of these new insights are developed here.

To address some of the barriers facing GGP implementation in China, this paper extensively investigates various factors that may contribute to limited adoption of this practice. The paper then completes a quantitative analysis utilizing a fuzzy-based DEMATEL approach to identify the relative relationship and importance of various barriers, especially at the municipal level. By incorporating fuzzy c-means (FCM) into the approach, the barriers are also classified into different clusters.

Overall, the goal of the chapter is to explicate how a fuzzy-based DEMATEL approach could be served as a managerial tool to evaluate Chinese GGP barriers and their interrelationships to each other and significance for Chinese policy makers. Using a literature review this paper identifies the major barriers facing GGP implementation, especially at the municipal level. Then a fuzzy-based DEMATEL and fuzzy clustering approach to evaluate the relationships of the barriers is introduced. To evaluate the barriers, this article initially gets input from an actual government official in charge of procurement. The case application illustrates the efficacy of the technique. Methods of fuzzy clustering analysis will also be integrated into the methodological analysis, in order to capture more information and insights of the relationships among the barriers. Finally the conclusion of the chapter is proposed.

2 Background

2.1 Government Green Procurement

The genesis of government green procurement can be easily traced to the early 1990s. Green purchasing was incorporated into the principles of sustainable production and consumption in the Rio Declaration at the 1992 Earth Summit. Consistent with the 1992 Earth Summit is a series of Action Plans initiated by the United Nations Commission on Sustainable Development (UNCSD) and the Organization for Economic Cooperation and Development (OECD). One of significant strategies presented by the Action Plans is to promote GGP. Subsequently, GGP practices were initially and effectively developed in OECD countries (Günther and Scheibe 2006).

OECD countries have made a variety of GGP efforts. Each European Union (EU) country's public procurement policies follow the rules formulated by the EU procurement directives. The US has promoted GGP practices at municipal, state and federal levels since their first initiative in 1993. Canada's 1995 "Directions on Greening Government Operations" paid special attention to the potential environmental benefit of GGP. It is reported that 40 % of total government purchases in Sweden and Denmark are focused on green products and services (Day 2005). Currently in the US, the Federal Government spent about $500 billion and state/local governments spent an additional $400 billion on green procurement (Ho et al. 2010). To date, GGP in developed regions has played a significant role in enhancing green markets and supporting national sustainable development policy goals (Geng and Doberstein 2008; Brammer and Walker 2011).

In the mid-1990s, GGP started to diffuse to developing countries. Much less progress has been made in these developing regions when compared to developed countries. Yet, notable exceptions do exist. GGP practices are increasingly prevalent within newly industrialized regions in East and Southeast Asia, including South Korea, Taiwan and Hong Kong (Mol 2006). In 2005 South Korea introduced a regulation for promoting the purchase of environmental products by government agencies. The Ministry of Environment was authorized to develop the "Purchasing Guidelines for Environmentally-friendly Products" and to aid related agencies to establish proper green purchasing strategy plans and specific initiatives. The government agencies are required to procure listed green products labeled with the South Korean Eco-label, Energy Saving Mark or Good Recycled Mark (Ho et al. 2010). A significant increase in GGP in South Korea was generated.

In Taiwan, the Action Plan for Implementing Green Procurement by Government Agencies was put into effect in 2002. Central government agencies and first level government agencies, government-owned firms, public schools and hospitals were obliged to obey the Action Plan. Increased designated green product categories were included in the Action Plan, and the scope of the products involved has been increasingly expanded. The Taiwanese government has initiated a Green Mark Scheme to identify green products meriting "green purchasing" preference (Ho et al. 2010). In 2008, 876 Green Mark labeled products were introduced and the amount of government green procurement value in Taiwan has soared to 6.77 billion NTD (approximately $210 billion) (Ho et al. 2010).

In Hong Kong, the Store Procurement Regulation (SPR) was amended in 2000 to require the Hong Kong government to pay special attention to the purchase of green products, with comprehensive consideration of economical rational and environmental performance. Afterwards, the Government Logistics Department produced green specifications for a series of commonly purchased products including recycled paper, environmentally preferable cleaning materials and clean fuel. The total amount of green purchasing for products with green specifications was HK$1.76 billion (approximately $220 million), which represented approximately 45 % of Government Logistics Department's total purchasing value in 2008 (Ho et al. 2010).

Since the start of this century, GGP has seen increased interest in China. A number of GGP policy initiatives were successively introduced. The law on government procurement approved in 2002 mentioned that China's government procurement should be supportive of environmental protection. In 2004, a new regulation of "suggestions on promoting government procurement of energy-saving products" was jointly released by the Ministry of Finance and the National Development and Reform Commission. This regulation requires all levels of Chinese governmental agencies, including national, provincial and municipal levels, to pay attention to purchasing of energy-saving products and to implement GGP starting in 2005. In 2005, a detailed government procurement list on energy-saving products was first publicized. The original list covered 6 categories and includes more than 1,000 types of products such as TV sets, lighting, printer and air-conditioners.

As of October 2012, China's government list of energy-saving products was enlarged to 27 categories and more than 5,000 types of products. Further, a government procurement list for environmentally labeled products was enacted in 2007. 14 categories and more than 800 types of products, including low pollution light weight vehicles, fax machines, printers, furniture and photocopiers, were involved included the original list for environmentally labeled products. The list for environmentally labeled products was expanded to 41 categories with more than 2,000 products. Though the amount of green purchasing has been increasing, China's green product purchases are still very limited (Guo 2012). There is still a long way to go for China to promote GGP and further to foster the development of green markets. Identifying and evaluating these barriers is important for such a large influential institution to address environmental issues.

2.2 Barriers to Implementing Government Green Procurement

Following the above outline of various national GGP regulations and practices, this section now depicts barriers of GGP, from the literature, which are summarized in Table 1.

A quick inspection of Table 1 shows that the barriers unearthed from the literature are categorized into internal and external levels. Within these levels individual and organizational sub-categories are also identified.

2.2.1 Internal Factors

(1) *Individual related factors.*

Lack of purchasing staffs' environmental awareness: Daily and Huang (2001) state that employees are often the initiators and recipients of a proactive environmental practice. Proactive environmental programs often face failure due to the unwillingness of employee involvement (Murillo-Luna et al. 2011) and lack of knowledge by these employees (Zhu et al. 2013). Thus, lack of purchasing staffs' environmental awareness is justified as a barrier for implementing GGP.

Lack of knowledge on how to implement GGP: Governmental procurement officials may lack knowledge on how to implement GGP (Bala et al. 2008; Walker and Brammer 2009).

This lack of knowledge represents a lack of expertise and a reliance on external capabilities if management truly wishes to invest in GGP. Lack of knowledge also indicates an inability for promoting green procurement (Günther and Scheibe 2005). The know-how for promoting GGP may constitute knowledge of identifying green products/services and of understanding the procedures for procuring green products. Typically the responsibility for sustainability-related policies often is given to environmental departments and hence other departments such as procurement agencies may lack knowledge of how to promote green programs (Hall and Purchase 2006).

Difficulty of evaluating products' total cost over the life cycle: Accounting for the total product and services cost over the whole life cycle is important (Geng and Doberstein 2008). Products or services with low prices can have high disposal costs. A thoughtful total cost analysis should be conducted by procurement agencies. Given that many green products may be more expensive, a 'green premium', the difficulty of evaluating products' total cost over the life cycle could be a barrier for agencies seeking to implement GGP.

Difficulty of tracking environmental impact of large-scale construction projects: This factor is especially unique to China. In recent years, numerous large-scale projects such as energy, transport, housing and waste treatment have been constructed, particularly in newly industrialized countries like China. Many of these

Table 1 Barriers to implementing GGP

Categories	Factors	Sub-factors	Source
Internal factors	Individual–related	Lack of purchasing staffs' environmental awareness	Daily and Huang 2001, Murillo-Luna et al. 2011
		Lack of knowledge of how to implement GGP	Günther and Scheibe 2005, Bala et al. 2008, Walker and Brammer 2009
		Difficulty of evaluating products' life cycle environmental impact	Geng and Doberstein 2008
		Difficulty of tracking environmental impact of some big projects	Hall and Purchase 2006
		Perception of poor quality of green products	Warner and Ryall 2001, Bala et al. 2008
		Perception of too redundant operations for GGP	Coggburn 2003
	Organization-level	Lack of top management commitment	van Hemel and Cramer 2002
		Lack of middle management support	Carter et al. 1998
		Focus on supporting local economies regardless of products' environmental metric	Rimmington et al. 2006
		Organization culture of resisting changes	Walker and Brammer 2009, Preuss and Walker 2011
		Incapability of aligning GGP into other broader agency objectives	New et al. 2002, Coggburn 2004
		Lack of intra-communication	New et al. 2002, Preuss and Walker 2011
		Corrupt behaviors sacrifice potential opportunities of some green suppliers	Coggburn 2004, Wong 2009
External factors	Regulation-related	National regulation sets conflicting priorities	Dawson and Probert 2007
		Provincial regulation sets conflicting priorities	Dawson and Probert 2007
		Lack of practical guidelines of how to undertake GGP	Bouwer et al. 2006, Geng and Doberstein 2008
	User-related	Lack of environmental awareness of end users	Preuss 2007, Preuss and Walker 2011
	Supplier-related	Higher prices of green products	Li and Geiser 2005, Geng and Doberstein 2008
		Difficulty of finding suppliers for products or services	Swanson et al. 2005, Testa et al. 2012

projects in China are government funded. Good environmental practices are not widely applied in the social housing sector (Hall and Purchase 2006). One critical reason is the difficulty of assessing and tracking the environmental impact of large-scale construction projects. Procurement officials acknowledge that such practices of tracking environmental impact frequently involve too much effort and are sometimes not feasible due to data unavailability (Hall and Purchase 2006).

Perception of poor quality of green products: Poor performance and quality, actual or perceived, of green products is one of the main constraints to GGP (Warner and Ryall 2001). Perceived poorer performance of some green products when compared to regular products has been empirically shown to be a barrier (Bala et al. 2008). If purchasing staff perceive green products as poor quality ones, the implementation of GGP could easily be hindered, as purchasing staff can use these subjective biases to support lessened green purchasing.

Perception of redundant operations for GGP: The cumbersome and bureaucratic process of government procurement has long been criticized by the literature (e.g., Coggburn 2003). Given the involvement of additional environmental criteria, GGP makes it more complex and burdensome for purchasing staffs. If purchasing officials have the perception of redundant or voluntary characteristics of GGP, intentional ignorance of GGP may be likely to occur.

(2) *Organization-level factors.*

Lack of top management commitment: Top management support is critical for the success of any environmental initiative (Zhu et al. 2011; Zhu et al. 2008). With strong top management commitment, purchasing staff would have less fear of promoting GGP, and probably receive greater financial and social support for their initiatives. Conversely, reluctant top management can easily cause elimination or starving of environmental behaviors such as GPP (van Hemel and Cramer 2002).

Lack of middle management support: The literature has emphasized that middle management may play an important role in promoting proactive management practices (e.g., Carter et al. 1998). If middle managers do not perceive GGP as a necessity, barriers may appear for such proactive environmental behaviors.

Focus on supporting local economies regardless of products' environmental metric: Local economies are the typical focus of local governments' procurement policies (Rimmington et al. 2006). Supporting the local economy through government procurement is an important goal of local governments, but the availability of green products may not be very high. Of course, much of this issue depends on the level of local economic performance. However, focusing only on supporting local economies regardless of products' environmental metrics can hamper the implementation of GGP. In some regions of China, economic development is the major issue and environmental performance metrics are put less weight.

Organizational culture of resisting change: Governmental agencies with an innovative culture could have greater potential for implementing proactive environmental initiatives such as GGP. Alternatively, organizational cultures that resist changes may cause purchasing staffs to hesitate initiation of GGP practices (Walker and Brammer 2009; Preuss and Walker 2011). Governmental bureaucracies are noted for this perceived inflexibility (Thompson 1965; Fernandez and Moldogaziev 2011).

Incapability of aligning GGP into other broader agency objectives: The success of green supply initiatives lies heavily in an organization's ability to align activity with dominant corporate objectives (New et al. 2002; Sarkis et al. 2011). If purchasing agencies could viably integrate GGP into governments' overall missions with regard to economic development and social welfare, the probability of GGP implementation success could be amplified. Conversely, if GGP is perceived to be only an environmental initiative, and implemented in the absence of authentic stakeholder participation and strategic integration, then it will likely fail to live up to its potential (Coggburn 2004). Subsequently, it can be concluded that incapability of aligning GGP into other broader agency objectives may act as a barrier for implementing GGP.

Lack of intra-organizational communication: One barrier of implementing GGP is the difficulty of establishing links between appropriate members of staff and functions within an organization (New et al. 2002). If procurement officials could have viable and proactive interaction with other proper personnel such as material and service designers and users, it would be easier to incorporate environmental issues into purchasing decisions. Barriers may arise in situations where effective channels of communication of purchasing departments may be very competent within themselves but not the involvement of other departments (Preuss and Walker 2011).

Corrupt behaviors sacrifice potential opportunities of some green suppliers: Literature has recognized corruption as a barrier to an effective government purchasing decision (e.g., Coggburn 2004; Wong 2009). Corrupt behaviors that procurement officials favor rent seekers producing brown products do sacrifice potential opportunities for suppliers producing green products. Thus corruption is posited as a key barrier for implementing GGP.

2.2.2 External Factors

(1) *Regulation-related.*

National or provincial regulations have conflicting priorities: Conflict between sustainable and economic/social decisions could be attributed to slow adoption of GGP. Normally one key requirement of government procurement guidelines is to achieve the "best value for money", which implies purchasing goods and services that are of good quality and are environmentally friendly. But in many cases combining these criteria is difficult (Dawson and Probert 2007). Barriers could occur when national or provincial regulations have conflicting priorities in China.

Lack of practical guidelines of how to undertake GGP: A swath of loopholes existing within the legal system and the sometimes voluntary nature of these regulations could strongly hinder the implementation of GGP. Without clear practical guidelines of how to undertake GGP, it is difficult for purchasing staffs to promote GGP (Geng and Doberstein 2008). The study of Bouwer et al. (2006)

reveals that the successes of "Green 7" in EU with good performance of GGP partly derive from the strong political push through national guidelines or action plans.

(2) *User-related.*

Lack of environmental awareness of end users: Governmental end users' demand for green products could act as a driving force for implementing proactive environmental behaviors. Generally, the government purchasing department would integrate the environmental requirements of end users into purchasing decisions. However, lack of environmental awareness of end users could be a major barrier of embedding GGP into procurement decisions (Preuss 2007; Preuss and Walker 2011).

(3) *Supplier-related.*

Higher prices of green products: Green products often require higher prices due to higher costs from the adoption of advanced innovative technologies or materials, particularly during the early market introduction stages (Li and Geiser 2005; Geng and Doberstein 2008). Cost-cutting is a priority of government procurement decisions, especially in developing countries. As a result, higher prices of green products often serve as a key barrier of implementing GGP.

Difficulty of finding suppliers: The practices of the State of California's Procurement Division demonstrate that to determine the availability of suitable products is a key procedure in implementing GGP (Swanson et al. 2005). Hence, difficulty of finding proper environmentally preferable products could be another supplier related barrier. Indeed, 27 % among 249 Italian public administrations have declared that difficulty in finding green suppliers is a main obstacle of carrying out GGP (Testa et al. 2012). This obstacle necessitates the significance of providing a variety of incentives to stimulate the building of a significant green supply base.

2.3 Evaluating Green Government Practices Implementation

Various methods can be applied to evaluate interrelationships among factors restricting GGP. These tools may incorporate econometric methodologies and regression models which require significant sample sizes, but cannot fully evaluate the various interactions and causal relationships that exist amongst these barriers. To help address the complex relationships, especially in situations where little understanding on the relationships of barriers exists, causal mapping and modeling may be useful. These tools such as interpretative structural modeling (ISM), systems analysis and design techniques, and general cognitive mapping, are all possible techniques for evaluating the complexity of relationships amongst factors. Each of these has various limitations including the inability to provide joint qualitative and quantitative relationships that exist when evaluating organizational and managerial barriers. The tool presented in this paper is more capable of addressing the complex situations in a clear structural set of relationships.

In order to evaluate the identified GGP implementation barriers, this paper applies the Decision-Making Trial and Evaluation Laboratory (DEMATEL) methodology. DEMATEL has been applied in a variety of fields ranging from human resource management (Wu and Lee 2007), knowledge management (Wu 2008) to risk management (Fan et al. 2012). The technique has also been proven to a viable tool for managerial decision making support in environmental issues (Zhu et al. 2011; Fu, et al. 2012). To advance the methodology, fuzzy c-means (FCM) combined with DEMATEL to evaluate GGP implementation barriers is further introduced.

DEMATEL allows for determination of a final prominence-causal relationship diagram visually showing the interrelationships among factors. DEMATEL has some additional value over other multi-criteria decision making methods by aiding the evaluation of indirect relationships into a cause-effect diagram, effectively examining the overall structure, analyzing causal relationships among criteria (Tzeng et al. 2007). The disadvantages of DEMATEL may occur when too many factors are involved since it may require geometrically increased effort by analysts. Hence, a filtering process of decreasing the number of barriers, before conducting DEMATEL, is firstly introduced.

In order to help address the problem of incomplete information and epistemic uncertainty, fuzzy numbers are incorporated into DEMATEL. Therefore, the formal modeling approach proposed in the paper uses a fuzzy-based DEMATEL integrating FCM, can provide valuable insights into GGP implementation and policy analysis.

3 The Fuzzy-Based DEMATEL Methodology

Fuzzy set theory, Fuzzy c-means clustering, DEMATEL steps are shown in this section.

3.1 Fuzzy Set Theory

Fuzzy set theory was introduced by Zadeh (1965) for dealing with uncertainty and incomplete information. Different from "ordinary" numbers, a fuzzy number is a quantity whose value is imprecise. Any fuzzy number can be regarded as a function whose domain is a specific set of real numbers, and whose range is the span of non-negative real numbers between (and including) 0 and 1. This article uses triangular fuzzy numbers. A triangular fuzzy number $\tilde{z} = (l, m, r)$

$$\mu_{\tilde{z}}(x) = \begin{cases} 0, x \leq l \\ \dfrac{x-l}{m-l}, l < x \leq m \\ \dfrac{r-x}{r-m}, m < x \leq r \\ 0, x > u \end{cases} \quad (1)$$

Let $\tilde{z}_1$ and $\tilde{z}_2$ be two triangular fuzzy numbers. The primary operations of the two fuzzy numbers are as follows:

$$\tilde{z}_1 \oplus \tilde{z}_2 = (l_1 + l_2, m_1 + m_2, r_1 + r_2) \quad (2)$$

$$\tilde{z}_1 \, \Theta \, \tilde{z}_2 = (l_1 - l_2,\ m_1 - m_2,\ r_1 - r_2) \quad (3)$$

$$\tilde{z}_1 \otimes \tilde{z}_2 = (l_1 \times l_2, m_1 \times m_2, r_1 \times r_2) \quad (4)$$

$$\frac{\tilde{z}_1}{\tilde{z}_2} = (\frac{l_1}{r_2}, \frac{m_1}{m_2}, \frac{r_1}{l_2}) \quad (5)$$

$$\tilde{z}_1 \otimes k = (l_1 \times k, m_1 \times k, r_1 \times k), \text{ for each } k \in R \quad (6)$$

To deal with the issues in a fuzzy environment, an effective fuzzy aggregation method is needed. This necessitates the application of a defuzzification method, in order to convert fuzzy numbers into crisp values. In this paper the CFCS (Converting Fuzzy data into Crisp Scores) defuzzification method (Opricovic and Tzeng 2003) is adopted.

The CFCS method, introduced by Opricovic and Tzeng (2003), includes a five-step algorithm. Let $\tilde{z}_{ij}^{k} = (l_{ij}^{k}, m_{ij}^{k}, r_{ij}^{k})$ indicate the fuzzy assessment of evaluator k ($k = 1, 2, 3, \ldots, n$) about the degree to which the item i affects the item j. The CFCS method completes the following steps:

(1) *Normalization.*

$$xl_{ij}^{k} = \frac{(l_{ij}^{k} - \min l_{ij}^{k})}{\Delta_{\min}^{\max}} \quad (7)$$

$$xm_{ij}^{k} = \frac{(m_{ij}^{k} - \min l_{ij}^{k})}{\Delta_{\min}^{\max}} \quad (8)$$

$$xr_{ij}^{k} = \frac{(r_{ij}^{k} - \min l_{ij}^{k})}{\Delta_{\min}^{\max}} \quad (9)$$

where $\Delta_{\min}^{\max} = \max r_{ij}^k - \min l_{ij}^k$

(2) *Compute left* (*ls*) *and right* (*rs*) *normalized values.*

$$xls_{ij}^k = \frac{xm_{ij}^k}{1 + xm_{ij}^k - xl_{ij}^k} \tag{10}$$

$$xrs_{ij}^k = \frac{xr_{ij}^k}{1 + xr_{ij}^k - xm_{ij}^k} \tag{11}$$

(3) *Compute the total normalized crisp value.*

$$x_{ij}^k = \frac{xls_{ij}^k(1 - xls_{ij}^k) + xrs_{ij}^k xrs_{ij}^k}{1 - xls_{ij}^k + xrs_{ij}^k} \tag{12}$$

(4) *Compute crisp value.*

$$z_{ij}^k = \min l_{ij}^k + x_{ij}^k \Delta_{\min}^{\max} \tag{13}$$

(5) *Integrate crisp values.*

$$z_{ij} = \frac{1}{n}(z_{ij}^1 + z_{ij}^2 + \cdots + z_{ij}^k + \cdots + z_{ij}^n) \tag{14}$$

3.2 Fuzzy C-means Clustering

Fuzzy c-means (FCM) clustering is a technique that has been widely applied to feature analysis, clustering, and classifier designs in fields such as astronomy, geology, medical imaging, target recognition, and image segmentation (Izakian and Abraham 2011). As one of the commonly used clustering methods, FCM differs from crisp or hard segmentation methods by introducing fuzziness for the 'belongingness' of each factor and hence retaining more information from the original data (Zhang et al. 2009). Therefore, this paper here uses the FCM as our clustering method.

It partitions a set of n objects $X = \{x_1, x_2, \ldots, x_n\} \subset R_P$ in R_P dimensional space into c $(1 < c < n)$ fuzzy clusters with $V = \{v_1, v_2, \ldots, v_c\}$ cluster centers or centroids. The fuzzy clustering of objects is described by a fuzzy matrix U with n rows and c columns in which n is the number of data objects and c is the number of clusters. u_{ik}, the element in the ith row and kth column in U, indicates the degree of association or membership function of the ith object with the kth cluster. The characters of U are as follows:

$$u_{ik} \in [0,1] \quad \forall i = 1,2,\ldots,n; \quad \forall k = 1,2,\ldots,c; \tag{15}$$

$$\sum_{k=1}^{c} u_{ik} = 1, \quad \forall i = 1,2,\ldots,n; \tag{16}$$

$$0 \le \sum_{i=1}^{n} u_{ik} \le n \quad \forall k = 1,2,\ldots,c; \tag{17}$$

The objective function of FCM algorithm is to minimize the expression (18)

$$\min J(U,V) = \sum_{i=1}^{n}\sum_{k=1}^{c} u_{ik}^{m}(\|x_i - v_k\|)_A, \tag{18}$$

where m ($m > 1$) is a scalar value termed the weighting exponent and controls the fuzziness of the resulting clusters and $\|\bullet\|_A$ $A = I$ is the Euclidian distance from object x_i to the cluster center v_k, when $A = C^{-1}$ (where C is the covariance matrix) it would be the Mahalanobis norm. The cluster centers and related membership functions, the solutions of constrained optimization problem in expression (18), are given in expression (19) and expression (20), respectively.

$$v_{k,t} = \frac{\sum_{i=1}^{n} (u_{ik})^m x_i}{\sum_{i=1}^{n} (u_{ik})^m}, \quad k = 1,2,\cdots,c; \tag{19}$$

$$u_{ik,t} = \left[\sum_{j=1}^{c} \left(\frac{\|x_i - v_{k,t-1}\|_A}{\|x_i - v_{j,t-1}\|_A}\right)^{2/(m-1)}\right]^{-1}, k \ne j \tag{20}$$

3.3 DEMATEL

The DEMATEL method was developed by the Geneva Research Centre of the Battelle Memorial Institute (Gabus and Fontela 1973; Fontela and Gabus 1976). By means of matrices or digraphs, the method is used to visualize the structure of obscure causal relationships. The relationships between system components with the quantitatively described strengths of the relationships are portrayed by the matrices or graphs. The DEMATEL method assumes a system contains a set of components C = {C1, C2,…, Cn}, with pairwise relations that can be evaluated.

DEMATEL can be decomposed into four generic stages:

Step 1 *Develop a pairwise direct-relation matrix between system components through decision maker input.*

Step 2 *Determine the initial influence matrix by normalizing the direct-relation matrix.*

On the basis of the overall crisp direct-relation matrix *Z*, the normalized direct-relation matrix M can be obtained through expressions (21) and (22):

$$M = s \bullet Z \tag{21}$$

$$s = \frac{1}{\max\limits_{1 \le i \le n} \sum\limits_{j=1}^{n} z_{ij}}, \quad i,j = 1,2,\ldots,n \tag{22}$$

Step 3 *Determine a total relation (influence) matrix.*

The total relation matrix (T) is determined by expression (23) where I represents an $n \times n$ identity matrix.

$$T = M + M^2 + M^3 + \ldots = \sum_{i=1}^{\infty} M^i = M(I - M)^{-1} \tag{23}$$

Step 4 *Determine the cause/effect relationships (prominence-causal diagram) amongst the components and relative strengths.*

Firstly, row (R_i) and column (D_j) sums for each row i and column j from the total relation matrix T are acquired, through the following formulas (24) and (25):

$$R_i = \sum_{j=1}^{n} t_{ij}\ \forall i \tag{24}$$

$$D_j = \sum_{i=1}^{n} t_{ij}\ \forall j \tag{25}$$

The row values R_i stand for the overall direct and indirect effect of a barrier i on other barriers, while the column values D_j represent the overall direct and indirect effects of all the barriers on barrier j.

Moreover, the overall importance or prominence (P_i) of a barrier i and net effect (E_i) of barrier i are determined, through expressions (26) and (27).

$$P_i = \{R_i + D_j | i = j\} \tag{26}$$

$$E_i = \{R_i - D_j | i = j\} \tag{27}$$

P_i describes the overall prominence or importance of barrier i regarding the overall relationships with other barriers. The larger is the value of P_i, then the greater the overall prominence of barrier i. If $E_i > 0$ then barrier i is a net cause for other green supplier development programs. If $E_i < 0$, barrier i relies on (net effect of) operation of other barriers (Tzeng et al. 2007). The values may then be plotted onto a two-dimensional axis for each barrier.

Subsequently, the digraph relationship for each barrier in relation with other barriers can be determined. The causal diagraph can visualize the complicated causal relationships among barriers.

3.4 A Fuzzy-Based DEMATEL and Clustering Approach

The analytical procedure of the fuzzy-based DEMATEL approach with incorporation of FCM will be completed as follows:

Step 1 *Define a fuzzy pairwise influence comparison scale for the assessments.*

Step 2 *Develop the fuzzy direct-relation matrix X by introducing the fuzzy pairwise influence relationships between the barriers in a matrix.* Note that all the principal diagonal elements are initially set to a crisp value of zero ("N" = no influence).

Step 3 *The fuzzy direct relation matrix is transformed into a crisp matrix Z, by using the modified-CFCS method.*

Step 4 *Establishing and analyzing the structural model.* Based on the initial direct relation crisp matrix Z, the normalized direct relation matrix M is acquired. Further, the total-relation matrix T is accrued. Then, the cause/effect relationships (prominence-causal diagram) amongst the components and relative strengths can be determined.

Step 5 *Clustering the similar barriers into various groups.* By using FCM, all similar barriers with the respective values of prominence (P_{i}) and net effect (E_{i}) come into a cluster. Hence, with the aid of the causal diagram including different clusters, decision makers may achieve appropriate decisions by recognizing the similarities and difference between and within the cause and effect clusters.

4 Case Study

4.1 Application of the Fuzzy-Based DEMATEL

Under the pressure of sustainable development, many local Chinese governments have attempted to initiate GGP. However, many barriers impede the effective implementation of GGP. This section explains how a local government of City A can use our proposed approach to analyze the barriers and their interrelationships.

A filtering process to reduce the number of alternative barriers is completed to arrive at a consensus set. This process is first conducted by purchasing officials to help keep the number of comparisons in latter steps to a manageable level. Only those factors that the respondents felt really hinder the implementation of GGP in practice were considered and kept. The officials felt that remaining factors shown in Table 2 represented the barriers they have consistently encountered. A total of

Table 2 Final barriers used in case study

No.	Barriers
a1	Lack of knowledge of how to implement GGP
a2	Lack of purchasing staffs' environmental awareness
a3	Lack of top management commitment
a4	Lack of practical guidelines of how to undertake GGP
a5	Corrupt behaviors sacrifice potential opportunities of some green suppliers
a6	Focus on supporting local economies regardless of products' environmental metric
a7	Higher prices of green products
a8	Perception of too redundant operations for GGP
a9	Incapability of aligning GGP into other broader agency objectives
a10	Difficulty of evaluating products' life cycle environmental impact
a11	Perception of poor quality of green products
a12	Difficulty of tracking environmental impact of some big projects

Table 3 The fuzzy linguistic scale

Linguistic terms	Triangular fuzzy numbers
No influence (N)	[0, 0, 0.25]
Very low influence (VL)	[0, 0.25, 0.5]
Low influence (L)	[0.25, 0.5, 0.75]
High influence (H)	[0.5, 0.75, 1.0]
Very high influence (VH)	[0.75, 1.0, 1.0]

twelve barrier factors existed in the final set. Then, the twelve barriers are given to the officials to complete the pairwise comparative analysis for DEMATEL. The following case exemplifies how one respondent applied our proposed fuzzy DEMATEL method to evaluate the barriers of GGP and their interrelationship.

Step 1 *Define a fuzzy pairwise influence comparison scale for the assessments.* A five level scale with the items of N (no influence), VL (very low influence), L (low influence), H (high influence), and VH (very high influence) is used in the paper. The fuzzy linguistic scale is shown in Table 3.

Step 2 *Develop the fuzzy direct-relation matrix X by introducing the fuzzy pairwise influence relationships between the barriers in a matrix.* Note that all the principal diagonal elements are initially set to a crisp value of zero ("N" = no influence). Table 4 shows the linguistic scale direct-relation matrix completed by one respondent. Then, the linguistic scales in Table 4 can be translated into triangular numbers by using Table 3.

Step 3 The fuzzy direct relation matrix is transformed into a crisp matrix Z, by using the modified-CFCS method. Table 5 is the crisp direct relation matrix Z.

Step 4 *Establishing and analyzing the structural model.* Based on the initial direct relation crisp matrix Z, the normalized direct relation matrix M is acquired. Further, the total-relation matrix T is resultant. Then, the cause/effect relationships (prominence-causal diagram) amongst the components and relative strengths can

Table 4 The linguistic scale direct-relation matrix completed by the respondent

GGP	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12
a1	N	L	N	N	N	N	N	N	H	H	VH	N
a2	L	N	N	N	N	N	N	N	L	N	VL	N
a3	L	H	N	N	H	N	N	N	L	N	N	VH
a4	N	VH	VH	N	H	N	N	N	VH	H	H	VH
a5	H	H	H	N	N	N	N	N	H	N	H	VH
a6	N	N	N	N	N	N	L	N	N	N	L	N
a7	N	N	VL	N	N	VL	N	N	N	N	N	N
a8	N	N	N	N	N	N	N	N	N	N	L	H
a9	N	N	N	N	VL	N	N	N	N	VH	H	VH
a10	N	N	N	N	L	N	N	N	N	N	H	H
a11	N	N	N	N	L	N	N	N	VH	H	N	H
a12	N	N	N	N	H	N	N	N	N	N	VH	N

Table 5 The crisp direct relation matrix Z

GGP	a1	a2	a3	a4	a5	a6	a7	a8	a9	a10	a11	a12
a1	0.03	0.50	0.03	0.04	0.03	0.04	0.04	0.04	0.73	0.73	0.97	0.03
a2	0.50	0.03	0.03	0.04	0.03	0.04	0.04	0.04	0.50	0.03	0.27	0.03
a3	0.50	0.73	0.03	0.04	0.73	0.04	0.04	0.04	0.50	0.03	0.03	0.97
a4	0.03	0.97	0.97	0.04	0.73	0.04	0.04	0.04	0.97	0.73	0.73	0.97
a5	0.73	0.73	0.73	0.04	0.03	0.04	0.04	0.04	0.73	0.03	0.73	0.97
a6	0.03	0.03	0.03	0.04	0.03	0.04	0.49	0.04	0.03	0.03	0.50	0.03
a7	0.03	0.03	0.27	0.04	0.03	0.25	0.04	0.04	0.03	0.03	0.03	0.03
a8	0.03	0.03	0.03	0.04	0.03	0.04	0.04	0.04	0.03	0.03	0.30	0.73
a9	0.03	0.03	0.03	0.04	0.27	0.04	0.04	0.04	0.03	0.97	0.73	0.97
a10	0.03	0.03	0.03	0.04	0.50	0.04	0.04	0.04	0.03	0.03	0.73	0.73
a11	0.03	0.03	0.03	0.04	0.50	0.04	0.04	0.04	0.03	0.97	0.03	0.73
a12	0.03	0.03	0.03	0.04	0.73	0.04	0.04	0.04	0.03	0.03	0.97	0.03

be determined. Table 6 demonstrates the prominence and net cause/effect values of the twelve barriers. Figure 1 is the prominence-causal DEMATEL graph.

Step 5 *Clustering the similar barriers into various groups.* FCM is used to classify the various barriers into groups. Three groups are identified, as shown in Fig. 2. The barriers are classified into three groups because the classification of three groups can maximize the distance among the clusters and minimize the distance among the barriers in a cluster. Group 1 includes seven barriers: a1, a2, a3, a4, a5, a9, a10. Barriers in Group 1 are characterized by having relatively high R and D values, while they play more important role of influencing other barriers. In brief, Group I is dominated by causal barriers. Conversely, Group III containing a11 and a12 is more of a group of consequential barriers. Additionally, three barriers of a6, a7 and a8, having relatively medium R and D values, are classified into Group II.

Table 6 The degree of prominence and net cause/effect values

	R sum	D sum	R + D(P)	R−D(E)
a1	0.883	0.605	1.49	0.28
a2	0.466	0.817	1.28	−0.35
a3	1.072	0.569	1.64	0.50
a4	1.812	0.142	1.95	1.24
a5	1.371	1.163	2.53	0.21
a6	0.339	0.182	0.52	0.16
a7	0.243	0.212	0.46	0.03
a8	0.378	0.142	0.52	0.24
a9	0.893	0.953	1.85	−0.06
a10	0.672	1.123	1.79	−0.45
a11	0.728	1.791	2.52	−1.06
a12	0.642	1.799	2.44	−1.16

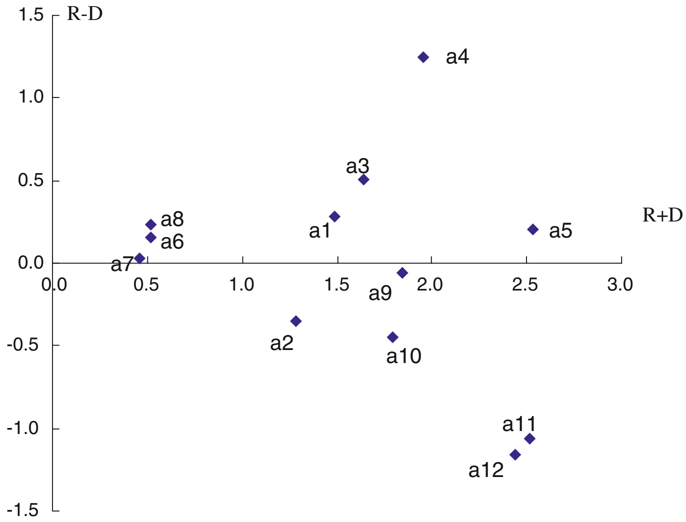

Fig. 1 The prominence-causal DEMATEL graph

4.2 Discussion

In the exemplified case, the local government attempted to understand the barriers to GGP by using the proposed methodology. And the evaluation results provide useful analytical information for the governmental officials. Interpreting the results could effectively provide officials practical insights.

Firstly, a5 (corrupt behaviors sacrifice potential opportunities of some green suppliers) and a11 (perception of poor quality of green products) are recognized as the two most important barriers that require initial and special attention. According

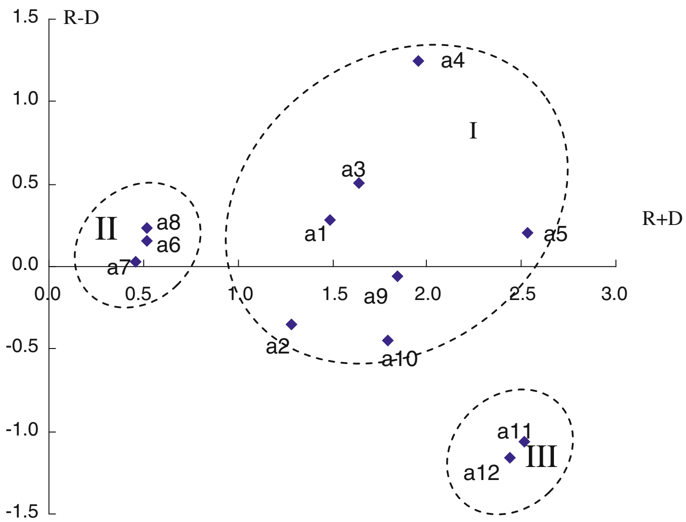

Fig. 2 The clustering result of prominence-causal DEMATEL graph

to Table 4 and Fig. 1, a5 (2.53) and a11 (2.52), with high R + D scores, are the two barriers having the strongest propensity of relationships with other barriers. This implies that corruption may implicitly hamper numerous opportunities for green suppliers and consequently limit the effective implementation of GGP. Significant attention needs to be given to addressing corruption in order to promote the effectiveness of GGP. Local governments may establish an information platform for GGP to facilitate purchasing information exchange and promote public participation.

Further, if the purchasing staff perceives green products as normal poor quality ones, the implementation of GGP could be greatly hindered. Hence, green suppliers may need to show credible evidence of good quality to purchasing officials. Acquiring certification of quality management systems or guarantees might be something for consideration by green suppliers. Quality management system certified products of green suppliers authorized by third-party organizations, along with easily identified labels, may draw consumers' attention and to some extent create confidence of quality products. Also, green suppliers may try to invite purchasing officials to visit their plants to verify good quality characteristics of their products.

Secondly, a4 (lack of practical guidelines of how to undertake GGP) is regarded as the most influencing barrier and a12 (difficulty of tracking environmental impact of some big projects) is regarded as the least influencing barrier. Table 4 identifies factor a4 is the cause barrier with the highest R-D score. This result indicates that a4 may be quite difficult to alter. But, if a4 can be successfully addressed, it is likely that many other barriers can subsequently be addressed. Chinese local governments may learn lessons from the specific practical GGP guidelines originating from developed countries/regions. However, considering the different financial and industrial capabilities existing among various Chinese local governments, the foreign experiences of undertaking GGP will require alterations to fit with Chinese specific situations. Also, a12, with the lowest R-D score, is the effect barrier that may be influenced by other barriers. That means, many other barriers can cause the occurrence of a12.

Thirdly, the clustering analysis shows that the barriers in Group I are more cause oriented barriers. Hence, the local government should focus more attention on the barriers in Group I (a4, a3, a1, a5, a9, a2, a10). Many other barriers will be influenced when these barriers in Group I are initially addressed.

The barriers of a4 and a5 have been discussed. With the second highest R-D score, factor a3 (lack of top management commitment) should also be evaluated and considered. Numerous environmental programs fade due to poor top management support. Chinese higher officials generally pay greater attention to their general performance requirements. The current performance evaluation system in China stresses economic indicators such as GDP growth proportion, reducing emphasis on greening indicators and practices. Hence, a practical aspect may be that purchasing officials may try to initially purchase environmental products from local green suppliers in order to gain support from top management (local economic benefits from local purchases may be seen as a good indicator). Also, purchasing officials may cooperate with local investment promotion department to attract green enterprises by promising that the green products from the enterprises could be prioritized purchased. In so doing, top management in local governments would more likely support win–win GPP programs.

5 Conclusions

In developing countries, government green procurement may act as an important driving force to achieve sustainable development. This reality is not lost in China. Yet, government green procurement in China is still facing limited utilization. To address the issues facing GGP adoption in China, various factors that may contribute to limited adoption of this practice are investigated. Thus, this paper does a comparative analysis utilizing a fuzzy-based DEMATEL approach to identify the relative relationship and importance of various barriers, especially at the municipal level. By incorporating FCM into the approach, the barriers are also classified into different clusters.

Our proposed method successfully extends the DEMATEL method by integrating both a fuzzy set theory and fuzzy clustering method. Hence, it can effectively deal with the issues of vague and incomplete information. Also, the clustering analysis allows a more systematic observation of the interrelationships among GGP barriers.

By the application of the proposed methodology in a Chinese city, this chapter gets input from actual government officials. The evaluation results show the cause-effect relationship among the barriers and many useful insights into effectively implementing GGP have been provided.

Even with the advantages of this proposed approach and its application, there are some limitations and room for further research. For example, this paper does the analysis at the municipal level. A further analysis in a provincial or national level may also produce edified results.

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Facility Location Selection in Reverse Logistics Using a Type-2 Fuzzy Decision Aid Method

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Abstract Location selection for the process of moving goods from their final destination to ensure proper value creation is a multi-faceted issue which requires consideration of social, economic, environmental and technical factors. The fuzzy sets theory is a good tool for dealing with complex and subjective problems which make use of implicit human judgments. Type-2 fuzzy sets provide more degrees of freedom to reflect the uncertainty and the ambiguity of real cases. The aim of this study is to suggest a multi criteria approach for the selection of the most appropriate reverse logistics facility location using a type-2 fuzzy TOPSIS methodology. Using proposed methodology, a case study from an e-waste recycling industry is conducted. In the evaluations, criteria like social acceptability, environmental risks, biodiversity conservation, operation and investment costs, energy and transportation infrastructure, legal/political environment, and growth potentials of the region are considered.

Keywords Facility location selection · Multi criteria · Reverse logistics · TOPSIS · Type-2 fuzzy sets

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1 Introduction

Reverse logistics (RL) can be represented as the process of planning and controlling of backward moves of products or materials. Issues related with RL and considerations associated with sustainability have become more apparent in both academic and business environment. Parallel to these, regulatory initiatives which impose strict environmental obligations have become to be enforced on actors in the chain. These developments force actors select appropriate logistics structures and prefer to benefit from systematic facility location selection tools in order to deal with difficulties of RL activities.

Optimizing the allocation decisions considering the material flows and ensuring the maximum value recovery under defined constraints, facility location models aim to provide optimal logistic structure for the actors which take place in the RL system. Due to its major impacts on the long term financial structure of the organizations, facility location selection is considered one of the most critical strategic decisions for an institution at the level of establishment (Queiruga et al. 2008).

The process of evaluating existing and alternative locations of facilities is a multi dimensional problem which requires considering quantitative and qualitative factors. Moreover, as actors in a chain (original equipment manufacturers, distributors, and logistic providers) may have different expectations, choosing the best location requires considering multiple and usually conflicting perspectives. In contrast to forward logistics, there are a limited number of studies which focuses on facility location in RL research area. Emerging studies show that multiple criteria decision making techniques are useful tools for handling RL location selection issues.

Fuzzy methods are good tools for dealing with the uncertainty resulting from subjective human judgments (Zadeh 1965). In order to transform the linguistic terms into fuzzy numbers, conversion scales are used. In the classical set theory, it is believed that an element can not be in and out of a set simultaneously. Conversely, fractional membership can be accepted in the fuzzy set theory. By using fuzzy systems, favorable dimensions have been added into the existing domain of engineering and location planning problems.

There is a growing literature on multi criteria decision making using Type-2 fuzzy sets. Type-2 fuzzy sets can perform better in defining uncertain parameters than Type-1 fuzzy sets (Chen and Lee 2010). Although Type-2 fuzzy sets are found to be more difficult to use and understand in contrast to Type-1 fuzzy sets (Mendel and John 2002), researchers can prefer to use Type-2 fuzzy methods in order to handle more uncertainty. If uncertainty associated with information or data is relatively high, Type-2 sets which are used for perceptions and subjective judgments are preferred to Type-1 fuzzy sets (Dereli et al. 2011).

The purpose of this study is to suggest a Type-2 fuzzy TOPSIS facility location selection methodology within RL context. To do this, a case study from Turkish e-waste recycling industry is conducted. Using social, economic, environmental, and technical attributes; experts from academy and industry have evaluated five different districts of Turkey in order to determine the best reverse logistics facility location for e-waste products. Conducting a sensitivity analysis, the robustness of the results is also tested.

2 Reverse Logistics and Facility Location Selection

In the view of increasing attraction to environmental protection, RL which includes logistic activities for collecting used or scrapped products from users and recovering to make them usable (Fleischmann et al. 1997), becomes an important business strategy to achieve sustainability in various industries. RL process is commonly forced by legislative, commercial and economic drivers (Fleischmann et al. 2001) in order to manage the backward flow of goods and materials caused from several reasons such as quality control returns, product recalls, warranty returns etc. (De Brito and Dekker 2004). In the logistics context, product recovery activities begin with consolidation of used products in collection centers and then, if product recovery activities are performed, they are sent to disassembly facilities. The last point for returned products is disposal area or remanufacturing facility where products will be recovered (Aras et al. 2008).

In the scope of supply chain, RL is an essential subcategory for facility location problems (Farahani et al. 2010). There are many structural options in a RL network converging from consumers to a few demand nodes (Ko and Evans 2007). Therefore, the establishment of RL networks by deciding location of centers and allocation of products is a critical issue to ensure product recovery activities efficiently and to increase the performance of supply chain management.

A large number of optimization models which aim to select best locations at the least cost, maximum profit or at optimum level of other objectives have been presented in the RL literature. In one of the first studies on RL network design, Gottinger (1988) proposed a facility site model for solid waste management considering fixed costs for defined locations and decrease investment costs for others. Spengler et al. (1997) dealt with a multi level warehouse location problem in a German–French region by proposing a mixed integer linear programming model. Louwers et al. (1999) presented a facility location-allocation model for recovery activities of carpet wastes by carrying out two different applications. Krikke et al. (1999) studied an uncapacitated warehouse location and transportation model by conducting three different scenarios in the view of remanufacturing context. In the study of Beamon and Fernandes (2004), a multi period integer programming model utilizing the present worth method is used for defining the number and location of warehouse and collection centers and the flow of materials between centers. Ahluwalia and Nema (2006) proposed a multi objective, integer linear programming model which selects optimum locations of facilities and allocation of goods or materials by minimizing cost and environmental risk. Aras et al. (2008) proposed a mixed integer nonlinear facility location-allocation model that also includes the decision of the best satisfactory incentives for various types of returned products. Demirel and Gökçen (2008) used a mixed integer mathematical model to solve a facility location problem in a multi-product and multi-phase distribution system. Similarly, Cruz-Rivera and Ertel dealt with an uncapacitated facility location problem for end-of- life vehicles by considering three scenarios regarding different collection quantities (Cruz-Rivera and Ertel 2009). In order to determine the optimum locations for disposal, landfill, and remanufacturing centers, Chu et al. (2010) formulated a multi echelon RL network model which addresses the uncertainty in demand and supply by using fuzzy-chance constrained programming.

On the other hand, there are a number of studies using multi-criteria decision making techniques. Queiruga et al. (2008) used the discrete multi-criteria decision method PROMETHEE (Preference Ranking Organisation Method for Enrichment Evaluations) for evaluation of Spanish municipalities in order to see which locations are appropriate for installation of recycling plants. They considered economic objectives (land costs, personnel costs, energy prices), infrastructural objectives (facility access, agglomeration effects, proximity to inhabited areas, absence of other e-waste recycling plants, availability of labor), and legal objectives (availability of a local waste processing programs, environmental grants). Yüksel (2009) used AHP decision model to evaluate predetermined locations under selected criteria such as cost, common effect, access, facilities, legal arrangements, environment, and area. Gan (2010) used an integrated AHP and data envelopment analysis (DEA) approach and considered social benefit, economic benefit, and technical performance criteria to evaluate the recycle center performances. Barker and Zabinsky (2011) proposed a multi criteria decision making model by benefiting from AHP method in order to assess and prioritize eight candidate locations in RL. In the study, a sensitivity analysis is also performed to indicate the dependencies of network structure decisions on many factors.

Literature review on RL network design reveals that there is a vast literature on facility location selection. A summary of the tangible and intangible criteria which are used in facility location selection problems can be found in Table 1.

3 Steps of the Type-2 Fuzzy TOPSIS Method

It is assumed that there are X alternatives, where $X = \{x_1, x_2, \ldots, x_n\}$ and Y criteria, where $Y = \{y_1, y_2, \ldots, y_n\}$. There are k experts $D_1, D_2, \ldots,$ and D_k. The set Y of criteria can be separated into two groups Y_1 and Y_2 where Y_1 represents the set of benefit criteria and Y_2 represents the set of cost criteria. The details of arithmetic operations in type-2 fuzzy sets can be found in Chen and Lee (2008, 2010).

Table 1 Criteria considered in different facility location problems

References	Criteria
Alberto (2000)	Cost, quality of living, local incentives, environmental aspects, logistic service criteria
Awasthi (2011)	Accessibility, security, connectivity to multimodal transport, costs, environmental impact, proximity to customers/suppliers, resource availability, conformance to sustainable freight regulations, possibility of expansion, quality of service
Barker and Zabinsky (2011)	Costs, business relations
Boran (2011)	Political environment, proximity to markets and customers, supplier networks, expansion potential, availability of transportation systems and utility, quality-of-life issues, culture issues
Demirel et al. (2010)	Costs, labor characteristics, infrastructure, markets, macro environment
Gan (2010)	Social benefit, economic benefit, and technical performance
Kahraman et al. (2003)	Environmental regulation, hostcommunity, competitive advantage, political risk
Kaya and Cinar (2007)	Environmental aspects, costs, quality of living, local incentives, time reliability provided to customers, response flexibility to customers' demands, proximity to suppliers, other company's, complementary facilities, and customers, integration with customers
Kuo (2011)	Port rate, import/export volume, location resistance, extension transportation convenience, transshipment time, one stop service, information abilities, port & warehouse facilities, port operation system, density of shipping line
Queiruga et al. (2008)	Economic objectives (land costs, personnel costs, energy prices), infrastructural objectives (facility access, agglomeration effects, proximity to inhabited areas, absence of other E-WASTE recycling plants, availability of labor), and legal objectives (availability of a local waste processing programs, environmental grants)
Thai and Grewal (2005)	Proximity to customers' bases, availability and quality of labour workforce, availability of utilities, local tax environment, in land transport infrastructures, expansion capability, customs administration and regulations, local standards of living
Turgut et al. (2011)	Cost, transportation, infrastructure, geographic location, suitability of climate
Yang and Lee (1997)	Access to markets/distribution centers and suppliers/resources, community/government access, competitive considerations, environmental factors, labour, taxes and financing, utilities services, transportation
Yüksel (2009)	Cost, common effect, access, facilities, legal arrangements, environment, and area

The method can be presented as follows:

- Step 1: Using linguistic terms and interval type-2 fuzzy sets (Table 2), obtain the decision matrix Y_p of the pth expert and the average decision matrix $\bar{Y}$, respectively, shown as follows (Chen and Lee 2010):

Table 2 Linguistic terms and their corresponding interval type-2 fuzzy sets

Linguistic terms	Interval type-2 fuzzy sets
Very low (VL)	((0, 0, 0, 0.1;1, 1), (0, 0, 0, 0.05; 0.9, 0.9))
Low (L)	((0, 0.1, 0.1, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.9, 0.9))
Medium low (ML)	((0.1, 0.3, 0.3, 0.5; 1, 1), (0.2, 0.3, 0.3, 0.4; 0.9, 0.9))
Medium (M)	((0.3, 0.5, 0.5, 0.7; 1, 1), (0.4, 0.5, 0.5, 0.6; 0.9, 0.9))
Medium high (MH)	((0.5, 0.7, 0.7, 0.9; 1, 1), (0.6, 0.7, 0.7, 0.8; 0.9, 0.9))
High (H)	((0.7, 0.9, 0.9, 1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9))
Very high (VH)	((0.9, 1, 1, 1; 1, 1), (0.95, 1, 1, 1; 0.9, 0.9))

$$Y_p = \left(\tilde{\tilde{y}}_{ij}^p\right)_{mxn} = \begin{array}{c} \\ y_1 \\ y_2 \\ \vdots \\ y_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{y}}_{11}^p & \tilde{\tilde{y}}_{12}^p & \cdots & \tilde{\tilde{y}}_{1n}^p \\ \tilde{\tilde{y}}_{21}^p & \tilde{\tilde{y}}_{22}^p & \cdots & \tilde{\tilde{y}}_{2n}^p \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{y}}_{m1}^p & \tilde{\tilde{y}}_{m2}^p & \cdots & \tilde{\tilde{y}}_{mn}^p \end{bmatrix} \end{array} \tag{1}$$

$$\bar{Y} = (\tilde{\tilde{y}}_{ij})_{mxn}, \tag{2}$$

where $\tilde{\tilde{y}}_{ij} = \left(\frac{\tilde{\tilde{y}}_{ij}^1 \oplus \tilde{\tilde{y}}_{ij}^2 \oplus \tilde{\tilde{y}}_{ij}^3 \oplus \tilde{\tilde{y}}_{ij}^4}{k}\right)$, $\tilde{\tilde{y}}_{ij}$ is an interval type-2 fuzzy set, $1 \le \mathrm{i} \le \mathrm{m}$, $1 \le \mathrm{j} \le \mathrm{n}$, $1 \le \mathrm{p} \le k$ and k denotes the number of experts (Chen and Lee 2010):

- Step 2: Obtain the weighting matrix W_p of the attributes of the pth expert and find the average weighting matrix $\bar{W}$ (Lee and Chen 2008):

$$W_p = (\tilde{\tilde{w}}_i^p)_{1xn} = \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & \cdots & y_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{w}}_i^p & \tilde{\tilde{w}}_2^p & \cdots & \tilde{\tilde{w}}_m^p \end{bmatrix} \end{array} \tag{3}$$

$$\bar{W} = (\tilde{\tilde{w}}_i)_{1xm}, \tag{4}$$

where $\tilde{\tilde{w}} = \left(\frac{\tilde{\tilde{w}}_i^1 \oplus \tilde{\tilde{w}}_i^2 \oplus \tilde{\tilde{w}}_i^3 \oplus \tilde{\tilde{w}}_i^4}{k}\right)$, $\tilde{\tilde{w}}_i$ is an interval type-2 fuzzy set, $1 \le \mathrm{i} \le \mathrm{m}$, $1 \le \mathrm{p} \le \mathrm{k}$ and k denotes the number of decision-makers.

- Step 3: Compute the (weighted) decision matrix $\bar{Y}_w$,

$$\bar{Y}_w = (\tilde{\tilde{v}}_{ij})_{mxn} = \begin{array}{c} \\ y_1 \\ y_2 \\ \vdots \\ y_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{\tilde{v}}_{11} & \tilde{\tilde{v}}_{12} & \cdots & \tilde{\tilde{v}}_{1n} \\ \tilde{\tilde{v}}_{21} & \tilde{\tilde{v}}_{22} & \cdots & \tilde{\tilde{v}}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{v}}_{m1} & \tilde{\tilde{v}}_{m2} & \cdots & \tilde{\tilde{v}}_{mn} \end{bmatrix} \end{array}, \tag{5}$$

where $\tilde{\tilde{v}}_{ij} = \tilde{\tilde{w}}_i \oplus \tilde{\tilde{y}}_{ij}, 1 \le \mathrm{i} \le \mathrm{m}, 1 \le \mathrm{j} \le \mathrm{n}$.

- Step 4: Using Eq. (10), compute the ranking value $Rank\left(\tilde{\tilde{v}}_{ij}\right)$ of the interval type-2 fuzzy set $\tilde{\tilde{v}}_{ij}$, where $1 \le \mathrm{j} \le \mathrm{n}$. Obtain the ranking weighted decision matrix $\bar{Y}_w^*$ (Chen and Lee 2010):

$$Rank\left(\tilde{\tilde{A}}_i = M_1\left(\tilde{A}_1^u\right) + M_1\left(\tilde{A}_i^L\right) + M_2\left(\tilde{A}_i^u\right) + M_2\left(\tilde{A}_i^L\right) + M_3\left(\tilde{A}_i^u\right)\right.$$
$$+ M_3\left(\tilde{A}_i^L\right) - \frac{1}{4}\left(S_1\left(\tilde{A}_i^u\right) + S_1\left(\tilde{A}_i^L\right) + S_2\left(\tilde{A}_i^u\right) + S_2\left(\tilde{A}_i^L\right) + S_3\left(\tilde{A}_i^u\right) + S_3\left(\tilde{A}_i^L\right)\right.$$
$$\left. + S_4\left(\tilde{A}_i^u\right) + S_4\left(\tilde{A}_i^L\right)\right) + H_1\left(\tilde{A}_i^u\right) + H_1\left(\tilde{A}_i^L\right) + H_2\left(\tilde{A}_i^u\right) + H_2\left(\tilde{A}_i^L\right) \tag{6}$$

$$\bar{Y}_w^* = \left(Rank(\tilde{\tilde{v}}_{ij})\right)_{m \times n}, \tag{7}$$

where $1 \le i \le m$, $1 \le j \le n$.

- Step 5: Find the positive ideal solution (PIS) $x^+ = \left(v_1^+, v_2^+, \ldots, v_m^+\right)$ and the negative-ideal solution (NIS) $x^- = \left(v_1^-, v_2^-, \ldots, v_m^-\right)$, where

$$v_i^+ = \begin{cases} \max\left\{\left\{Rank(\tilde{\tilde{v}}_{ij})\right\}, \text{ if } y_i \in Y_1 \right. \\ \min\left\{\left\{Rank(\tilde{\tilde{v}}_{ij})\right\}, \text{ if } y_i \in Y_2 \right. \end{cases} \quad 1 \le j \le n \tag{8}$$

and

$$v_i^- = \begin{cases} \min\left\{\left\{Rank(\tilde{\tilde{v}}_{ij})\right\}, \text{ if } y_i \in Y_1 \right. \\ \max\left\{\left\{Rank(\tilde{\tilde{v}}_{ij})\right\}, \text{ if } y_i \in Y_2 \right. \end{cases} \quad 1 \le j \le n \tag{9}$$

where Y_1 denotes the set of benefit attributes, Y_2 denotes the set of cost attributes, and $1 \le i \le m$ (Lee and Chen 2008).

- Step 6: Compute the distances PIS and the NIS and find the relative degree of closeness $C(x_j)$ using the equations below:

$$d^+\left(x_j\right) = \sqrt{\sum_{i=1}^{m}\left(Rank\left(\tilde{\tilde{v}}_{ij}\right) - v_i^+\right)^2}, \tag{10}$$

$$d^-\left(x_j\right) = \sqrt{\sum_{i=1}^{m}\left(Rank\left(\tilde{\tilde{v}}_{ij}\right) - v_i^-\right)^2}, \tag{11}$$

$$C(x_j) = \frac{d^-(x_j)}{d^+\left(x_j\right) + d^-(x_j)} \tag{12}$$

- Step 7: Finally, rank the closeness scores $C(x_j)$ in a descending order. Choose the alternative with the highest $C(x_j)$ (Chen and Lee 2008, 2010).

4 A Case Study in E-waste Recycling Industry

In Turkey, the procedures of collection and recycling of e-waste are defined by Environment and Forestry Ministry. Although the directive has not practically come into force yet, actors in the chain have begun to make arrangements in their

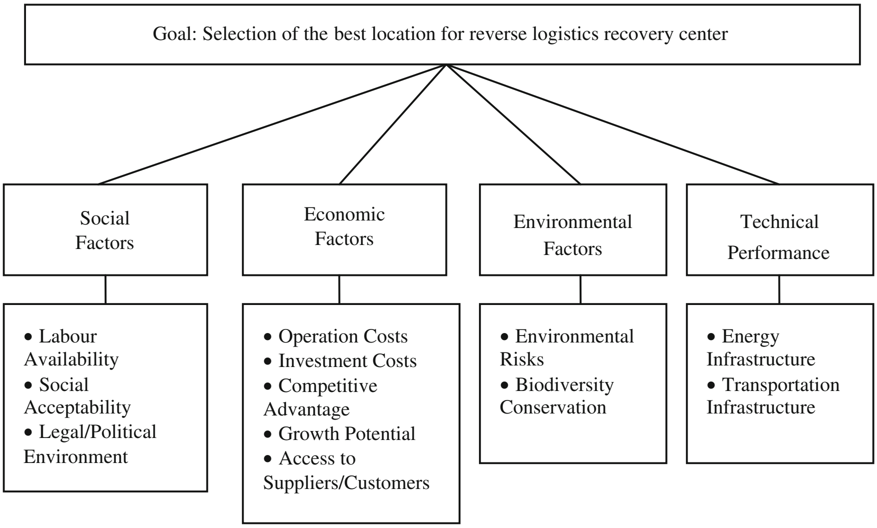

Fig. 1 Criteria structure for the e-waste reverse logistics location selection problem

strategies and operations. It is expected that the directive will go into expect in the near future. The directive sets out target of minimum 4 kg (per inhabitant) e-waste collection per year. If the collection of 4 kg e-wastes per inhabitant and per year is to be achieved, a total of 300,000 tons e-wastes will be recovered per year. In Turkey, the rate of e-wastes was 2.5 kg/year in 2008. However, there are only 15 recycling companies that collect and recycle e-wastes. Therefore, the role of recycling centers is significant for fulfillment of e-waste directive. The treatment of 300,000 tons e-wastes is insufficient until more facilities with appropriate and new technologies will be installed. With this background, it is clear that one of the main issues in e-waste directive is to define the best places for e-waste recycling centers (Queiruga et al. 2008).

In this study, the criteria structure seen in Fig. 1 is proposed for the e-waste reverse logistics location selection problem. The studies of Barker and Zabinsky (2011), Gan (2010), Queiruga et al. (2008) and Yüksel (2009) are used as a base for the suggested value hierarchy. The selection is made among five location alternatives (as seen at Fig. 2) which represent the most populated and industrialized regions of Turkey: İzmit (a_1), Ankara (a_2), İzmir (a_3), Adana (a_4), Samsun (a_5).

The definitions of sub criteria are given as the following:

- Social Factors:
  - Labor Availability (C_1): This criterion considers the potential of alternative locations in providing skilled labor force. Labor availability differs based on development level of locations and educational structure.
  - Social Acceptability (C_2): Social acceptability indicates the environmental consciousness level of the society. It differs according to environmental

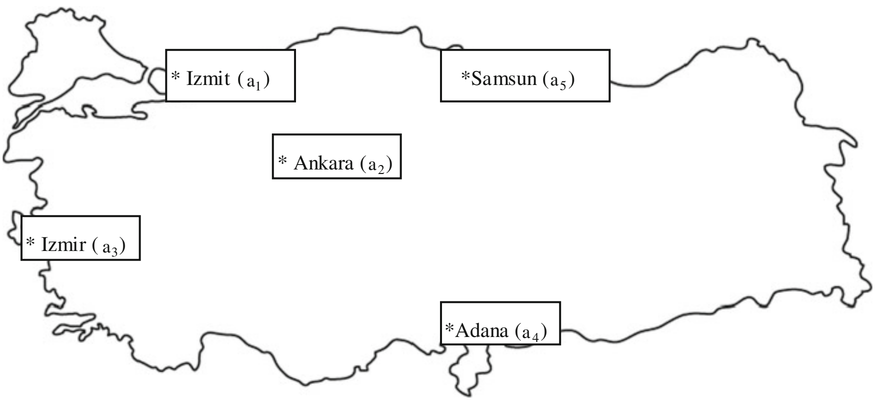

Fig. 2 Five location alternatives

responsibilities enforced on inhabitants of alternative locations, environmental investments made by municipalities of alternative locations, and economic and educational structure of alternative locations.
– Legal/Political Environment (C_3) : This criterion refers to the legal sanctions and arrangements that are imposed on OEMs, distributors, recycling companies, and consumers. It varies in respect to how effectively environmental directives come into force by municipalities of alternative locations.

- Economic Factors:

  – Operation Costs (C_4): Operation costs include labor costs, collection (take back) costs, recycling costs, and transportation costs. Labor costs can change with respect to the quality of living in alternative locations. Collection (take back) costs can vary according to the price paid to distributors for each collection at alternative locations. Recycling costs can vary in respect to labor skill and availability, and economic infrastructure of alternative locations. Transportation costs can change according to transportation types, economic and geographical structure of alternative locations.
  – Investment Costs (C_5): This criterion considers costs born at the beginning of establishing the logistics structure. It changes with respect to economic infrastructure, transportation decisions and land prices of alternative locations.
  – Competitive Advantage (C_6): This criterion refers to the potential of alternative locations in responding new collection potentials quickly. It changes according to number and location of competitors at alternative locations.
  – Growth Potential (C_7): This criterion considers the potential of alternative locations in expanding their existing capacities in the future. It changes in respect to geographical conditions and product return potentials of alternative locations.
  – Access to Material Suppliers, Collection Centers, Consumers (C_8): This criterion takes into account the distance of alternative locations to the

material suppliers who buy the recycled e-products; collection centers where the returned products are collected; consumers who are the end users of e-products. It shows the ability of alternative locations to collect the returned products or sell the recycled products in expected time.

- Environmental Factors:
  - Environmental Risks (C_9): This criterion ensures the ability of alternative locations to have minimum negative impact on environment and inhabitants. It changes in respect to disposal operations and legal sanctions imposed at alternative locations.
  - Biodiversity Conservation (C_{10}): This criterion considers the ability of alternative locations in conserving natural living standards of all livings. It changes according to legal sanctions imposed at alternative locations, geographical conditions of alternative locations, and disposal operations.
- Technical Performance:
  - Energy Infrastructure (C_{11}): This criterion considers the potential of alternative locations in utilizing cheap energy resources. It varies according to type of energy used in alternative locations.
  - Transportation Infrastructure (C_{12}): This criterion refers to the potential of alternative locations in providing easy transportations and different transportation modes. It changes according to the geographical conditions of alternative locations.

After determining the evaluation criteria, the steps of the type-2 fuzzy TOPSIS algorithm are implemented. In order to obtain the importance of the criteria, the experts made use of a seven point scale given in Table 2.

In the next step, using Tables 2 and 3, the aggregated type-2 fuzzy weights $(\tilde{\tilde{w}}_i)$ for the evaluation criteria (C_i) are obtained as in Table 4.

Next step is the determination of the most appropriate location for the establishment of an e-waste reverse logistic facility with the proposed type-2 fuzzy TOPSIS procedure. To do this, four experts evaluated five alternative locations (İzmit (a_1), Ankara (a_2), İzmir (a_3), Adana (a_4), Samsun (a_5)) in Turkey with respect to each criterion using Table 2. Evaluation results can be found in Table 5.

In the next step, first, using Eqs. (3–5) type-2 fuzzy weighted evaluation matrix is obtained. Then using Eqs. (11, 12) the ranks $\left(rank(\tilde{\tilde{v}}_{ij})\right)$ for the alternatives are obtained as in Table 6.

Next, making use of Table 6 and Eqs. (8 and 9) the rank coordinates of the positive and negative ideal solutions are determined as in Table 7. Then, using Eqs. (10 and 11), the distances from the positive and negative ideal solutions are obtained as in Table 8.

Finally, using Eq. (12), the closeness index (C*) figures and the rankings among the location alternatives are obtained.

According to Table 9, a_1 (İzmit) is the most convenient location for the settlement of an e-waste reverse logistics centre. The second and third best

Table 3 Experts' evaluations on the importance level of the criteria (C*i*)

	E_1	E_2	E_3	E_4
C_1	VH	H	MH	M
C_2	MH	H	M	MH
C_3	VL	MH	M	H
C_4	H	H	MH	M
C_5	H	VH	MH	H
C_6	ML	MH	ML	M
C_7	VH	VH	VH	M
C_8	VH	VH	H	VH
C_9	MH	L	MH	H
C_{10}	ML	L	M	MH
C_{11}	H	H	M	MH
C_{12}	VH	VH	M	H

Table 4 Type-2 fuzzy weights ($\tilde{\tilde{w}}_i$) for the evaluation criteria (C_i)

$\tilde{\tilde{w}}_1 =$	((0.6, 0.775, 0.775, 0.9; 1, 1), (0.6875, 0.775, 0.775, 0.8375; 0.9, 0.9))
$\tilde{\tilde{w}}_2 =$	((0.5, 0.7, 0.7, 0.875; 1, 1), (0.6, 0.7, 0.7, 0.7875; 0.9, 0.9))
$\tilde{\tilde{w}}_3 =$	((0.375, 0.525, 0.525, 0.675; 1, 1), (0.45, 0.525, 0.525, 0.6; 0.9, 0.9))
$\tilde{\tilde{w}}_4 =$	((0.55, 0.75, 0.75, 0.9; 1, 1), (0.65, 0.75, 0.75, 0.825; 0.9, 0.9))
$\tilde{\tilde{w}}_5 =$	((0.7, 0.875, 0.875, 0.975; 1, 1), (0.7875, 0.875, 0.875, 0.925; 0.9, 0.9))
$\tilde{\tilde{w}}_6 =$	((0.25, 0.45, 0.45, 0.65; 1, 1), (0.35, 0.45, 0.45, 0.55; 0.9, 0.9))
$\tilde{\tilde{w}}_7 =$	((0.75, 0.875, 0.875, 0.925; 1, 1), (0.8125, 0.875, 0.875, 0.9; 0.9, 0.9))
$\tilde{\tilde{w}}_8 =$	((0.85, 0.975, 0.975, 1; 1, 1), (0.9125, 0.975, 0.975, 0.9875; 0.9, 0.9))
$\tilde{\tilde{w}}_9 =$	((0.425, 0.6, 0.6, 0.775; 1, 1), (0.5125, 0.6, 0.6, 0.6875; 0.9, 0.9))
$\tilde{\tilde{w}}_{10} =$	((0.225, 0.4, 0.4, 0.6; 1, 1), (0.3125, 0.4, 0.4, 0.5; 0.9, 0.9))
$\tilde{\tilde{w}}_{11} =$	((0.55, 0.75, 0.75, 0.9; 1, 1), (0.65, 0.75, 0.75, 0.825; 0.9, 0.9))
$\tilde{\tilde{w}}_{12} =$	((0.7, 0.85, 0.85, 0.925; 1, 1), (0.775, 0.85, 0.85, 0.8875; 0.9, 0.9))

alternatives are a_2 (Ankara) and a_3 (İzmir). The rank order of the rest is a_4 (Adana) and a_5 (Samsun).

5 Sensitivity Analysis

In order to track the sensitivity of the closeness figures to changes in the weight configurations, a sensitivity analysis is conducted. Table 10 summarizes the weight configurations with respect to different cases. In the computation procedure, first, the linguistic importance levels given in Table 10 are converted into type-2 fuzzy weights using Table 3. Then, using Eqs. (3–12) the closeness indexes

Table 5 Evaluation scores of the alternatives

		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
DM1	a_1	VH	VH	VH	ML	ML	M	VH	MH	MH	M	H	H
	a_2	VH	VH	VH	ML	ML	MH	VH	VH	H	H	H	VH
	a_3	VH	VH	VH	L	L	H	VH	H	H	H	MH	H
	a_4	VH	VH	MH	L	L	M	VH	H	H	MH	H	H
	a_5	VH	VH	H	L	L	M	VH	H	H	H	H	H
DM2	a_1	H	H	MH	ML	M	M	M	H	H	H	H	MH
	a_2	H	H	VH	ML	L	M	M	MH	MH	MH	MH	MH
	a_3	VH	VH	M	ML	ML	ML	ML	ML	MH	MH	MH	M
	a_4	ML	ML	ML	ML	L	L	M	MH	MH	MH	M	M
	a_5	ML	ML	L	L	VL	L	L	L	MH	MH	MH	M
DM3	a_1	H	H	H	L	ML	MH	H	H	MH	H	H	H
	a_2	H	H	H	L	L	H	H	H	MH	MH	H	H
	a_3	H	H	H	L	L	H	H	H	MH	MH	H	H
	a_4	MH	MH	H	ML	ML	MH	H	MH	MH	MH	H	MH
	a_5	M	M	M	ML	ML	H	H	H	H	H	H	H
DM4	a_1	H	H	H	ML	ML	H	H	VH	MH	H	H	VH
	a_2	MH	M	MH	M	M	ML	M	M	ML	ML	MH	MH
	a_3	M	ML	MH	M	M	ML	ML	M	ML	ML	MH	MH
	a_4	ML	MH	MH	L	L	L	ML	ML	M	M	MH	MH
	a_5	ML	ML	MH	MH	MH	ML	ML	M	ML	ML	M	M

Table 6 The ranks $(rank(\tilde{\tilde{v}}_{ij}))$ for the alternatives

	a_1	a_2	a_3	a_4	a_5
C_1	7.888	7.672	7.566	6.336	6.11
C_2	7.484	7.087	6.991	6.496	5.887
C_3	6.425	6.505	6.127	5.748	5.453
C_4	4.853	5.071	4.869	4.651	5.086
C_5	5.513	5.041	5.041	4.55	5.178
C_6	5.457	5.329	5.322	4.707	4.95
C_7	7.933	7.429	6.916	7.172	6.68
C_8	8.689	8.128	7.4	7.411	7.137
C_9	6.375	6.026	6.026	6.2	6.192
C_{10}	5.615	5.282	5.282	5.288	5.391
C_{11}	7.628	7.212	7.004	6.994	6.994
C_{12}	8.053	7.815	7.432	7.194	7.184

Table 7 The ranks for the positive ideal and negative ideal solutions

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
$\boldsymbol{x}^+$	7.888	7.484	6.505	4.651	4.55	5.457	7.93	8.689	6.375	5.615	7.628	8.053
$\boldsymbol{x}^-$	6.11	5.887	5.453	5.086	5.513	4.707	6.68	7.137	6.026	5.282	6.994	7.184

Table 8 Distances from the positive and negative ideal solutions

	a_1	a_2	a_3	a_4	a_5
$d^+(a_i)$	0.988	1.292	2.122	2.83	3.589
$d^-(a_i)$	3.555	2.754	2.151	1.405	0.459

Table 9 The closeness index (C^*) and the rankings

	a_1	a_2	a_3	a_4	a_5
$C^*(a_i)$	0.783	0.681	0.503	0.332	0.113
Ranking	1	2	3	4	5

Table 10 Importance levels of attributes with respect to different cases

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
Case 1	M	M	M	M	M	M	M	M	M	M	M	M
Case 2	M	M	VH	M	VH	M	M	M	M	M	M	M
Case 3	M	M	VH	M	VH	M	M	M	VL	VL	M	M
Case 4	M	M	VH	M	VH	M	M	M	VL	VL	VL	VL

VL very low, *L* low, *ML* medium low, *M* medium, *MH* medium high, *H* high, *VH* very high

Table 11 The closeness index (C^*) figures with respect to different cases

	İzmit (a_1)	Ankara (a_2)	İzmir (a_3)	Adana (a_4)	Samsun (a_5)
CS	0.783	0.681	0.503	0.332	0.113
Case 1	0.810	0.688	0.547	0.311	0.140
Case 2	0.722	0.727	0.566	0.373	0.142
Case 3	0.719	0.754	0.580	0.375	0.134
Case 4	0.714	0.764	0.597	0.385	0.136

$C^*(a_i)$ are calculated for each case. Table 11 demonstrates the computed closeness index (C*) figures with respect to different cases.

In the current situation (CS), as stated earlier, a_1 (İzmit) is the best alternative, followed by a_2 (Ankara) and a_3 (İzmir) respectively. In Case 1, the situation where all the criteria weights are equal (at a level of "medium" importance) is considered. It can be seen that, the ranking among alternatives in Case 1 is the same with that of CS. In Case 2, the importance levels of C_3 (legal/political environment) and C_5 (investment costs) are increased significantly with respect to Case 1. It is seen in Fig. 3 that this change made the closeness index figures of a_1 (İzmit) and a_2 (Ankara) almost equal while it had no impact on the ranking among the rest of the alternatives. In the third case, the weights of C_9 (energy infrastructure) and C_{10} (biodiversity conservation) are decreased significantly. When the weights of these environmental attributes decrease, it can be seen in Fig. 3 that a_2 (Ankara) becomes the alternative with the highest preference level where a_1 (İzmit) falls to the second place. Finally, in Case 4, when the weights of C_{11} (energy

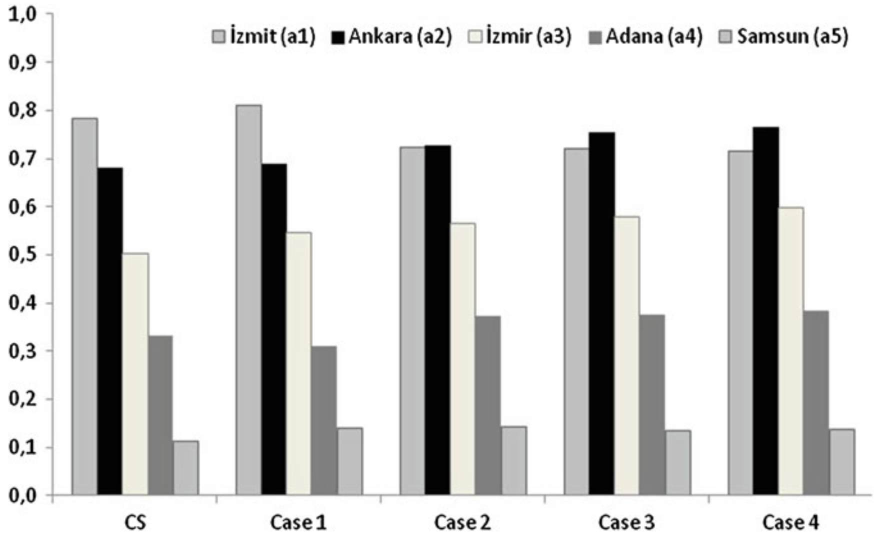

Fig. 3 Sensitivity analysis

infrastructure) and C_{12} (transportation infrastructure) are decreased significantly with respect to Case 3, a_2 (Ankara) becomes the best location (as in Case 3) where a_1 (İzmit) keeps its second place. It should be noted that the ranking among a_3 (İzmir), a_4 (Adana), and a_5 (Samsun) does not change in either of the cases.

Sensitivity analysis show that the ranking among the last three alternatives are robust to changes in weight configurations. On the other hand, while robust to small to medium changes, the ranking among a_1 (İzmit) and a_2 (Ankara) can be considered sensitive to significant changes in the weights of certain attributes.

6 Conclusion

The scope of reverse logistics covers all activities associated with the reuse of products and materials. The purpose of this study was to propose a type-2 fuzzy multiple attribute approach for ranking the facility location alternatives within e-waste reverse logistics context. Using a type-2 fuzzy TOPSIS methodology, an application from the Turkish e-waste recycling industry is conducted. Considering economic, social, environmental, and technical aspects, four experts have evaluated five alternatives (İzmit, Ankara, İzmir, Adana, Samsun) located in the most industrialized regions of Turkey. The results of the type-2 fuzzy decision aid procedure showed that, followed by Ankara and İzmir, İzmit region is the most appropriate location for the establishment of an e-waste reverse logistics facility in Turkey. Sensitivity analyses demonstrated that the results are robust to changes in weight configurations unless significant changes are made in the weights of certain attributes.

In the future studies, using the proposed structure, findings of this study can be compared with that of type-2 fuzzy PROMETHEE, ELECTRE, AHP, and evidential reasoning methodologies. Moreover, with some modifications, proposed reverse logistics facility location selection approach could be applied to different sectors and/or in other countries.

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Green and Reverse Logistics Management Under Fuzziness

Mohammad Mousazadeh, S. Ali Torabi and Mir Saman Pishvaee

Abstract Green supply chain management (GrSCM) has its roots in supply chain management (SCM) and environmental management. In fact, adding "green" concept into traditional SCM leads to studying environmental impact of SCM-related processes. Logistics activities which form the main part of SCM-related processes belong to the most influential sources of environmental pollution and greenhouse emissions which may cause harmful impacts both on human health and ecosystem quality. In order to reduce hazardous environmental impacts of logistics activities, the concept of green logistics (GrLog) and reverse logistics (RL) was introduced. Similar to traditional supply chain, uncertainty plays an important role in GrSCM; however, considering the environmental factors beside the quantity and quality of end-of-life products elevates the degree of uncertainty in GrLog and RL problems. In this chapter, designing and planning problems in GrLog and RL are investigated in a fuzzy environment via a systematic review and analysis of recent literature. Three selected fuzzy mathematical models from the recent literature are elaborated. A real industrial green logistics case study is described and investigated and a number of avenues for further research are finally suggested.

Keywords Green logistics · Green supply chain management · Reverse logistics · Fuzzy mathematical programming

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1 Introduction to Green and Reverse Logistics Management

The design and operation of supply chains has traditionally been upon economical and technological objectives such as maximizing revenue/minimizing cost, maximizing responsiveness, increasing flexibility, etc. For example, companies take into account various factors such as price, quality and flexibility when selecting their suppliers or just consider economical aspects when choosing their production technologies or selecting their transportation modes.

Since 1990s, green issues are increasingly considered by governments, people, industries and scientists in design and planning problems in both micro and macro levels. For example, governments force manufacturers to include green aspects into their products and production processes and taking into account green considerations in their logistics-related processes such as supplier selection and material movements. People prefer to buy products from those companies with higher reputation in environmental protection. As a result, including green aspects in products gradually becomes as a competitive advantage for manufacturers. Establishing international standards (such as ISO 14000 series) and international conventions (such as Kyoto Protocol in 1997) could also be considered as important drivers for environmental protection.

Among the logistics activities, manufacturing and transportation activities are the main sources of waste generation, ecosystem disruption, and depletion of natural resources (Fiksel 1996). As such, governments force the firms to decrease the environmental impact of their activities and all of these urge the manufacturers to consider environmental issues through their supply chains (Büyüközkan 2012). Paying more attention to GrLog not only can decrease the ecological impact of industrial activities but also can maintain or even increase quality, reliability, performance, energy efficiency or decrease cost (Srivastava 2007).

1.1 Importance and Drivers

The growing importance of GrSCM/GrLog is driven mainly by the escalating deterioration of the environment. Nevertheless, it is not only environmental issues that matters; it is good business sense and higher profits too (Srivastava 2007). In fact, the perspective of "greening as a burden" gradually changes toward "greening as a potential source of competitive advantage" (Van Hoek 1999).

According to the study of De Brito and Dekker (2004), companies involve in green practices either because they can profit from it (competitive advantages); or/ and because they have to doing so due to environmental legislations; or/and because they "feel" socially motivated to do it (social responsibility).

By reviewing a great number of papers in the relevant literature, the following drivers of GrSCM/GrLog could be realized:

- Deterioration of the environment involving:
  - limited natural resources;
  - diminishing raw material resources;
  - increase in solid and hazardous wastes (Fiksel 1996);
  - increasing level of pollution (water and air);
- economic advantages and savings (Porter and Van der Linde 1995a, b) by saving resources, eliminating wastes and productivity improvement;
- environmental legislations and regulatory requirement like:
  - Montreal Protocol in 1987 that limit the production of substances harmful to the stratospheric ozone layer, such as CFCs;
  - the Kyoto Protocol in 1997 that limits the emissions of greenhouse gases from industrialized countries;
- environmental management standards and guidelines (e.g., ISO 14000 series);
- consumer pressures (Lamming and Hampson 2005; Elkington 1994).

In addition to abovementioned drivers, benefits acquired by managing used product for further utility, adding customer's value, etc., are some other drivers enforcing manufacturers to address RL in their production activities (Wang and Sun 2005).

1.2 Definition and Scope

Zhu and Sarkis (2004) mentioned that the scope of GrSCM can range from a simple act of green purchasing to implementing an integrated green supply chain flowing from suppliers to customers, and even reverse flows of logistics. On the other hand, Srivastava (2007) defined the range of GrSCM as "the flow of material from the final customers back to retailers, collection points, manufacturers, and/or disposal sites". According to this definition, the scope of GrSCM includes reactive monitoring of the general environmental management programs and/or proactive practices implemented through reduce, re-use, rework, refurbish, reclaim, recycle, remanufacture, or as a whole, reverse logistics activities. Particularly, in the area of reverse logistics, researchers have explored various topics and issues, including reusing, recycling, remanufacturing, etc. (see Kroon and Vrijens 1995; Barros et al. 1998; Jayaraman et al. 1999).

RL was defined by Council of Logistics Management as "The role of logistics in recycling, waste disposal, and management of hazardous materials; a broader perspective included all relating to logistics activities carried out in source reduction, recycling, substitution, reuse of materials and disposal".

Also, Rogers et al. (1999) have defined RL as "the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal".

Srivastava (2007) defined GrSCM as "integrating environmental thinking into supply chain management, including product design, material sourcing and selection, manufacturing processes, delivery of the final product to the consumers as well as end-of-life management of the product after its useful life".

Sarkis et al. (2011) reviewed different concepts and definitions related to GrSCM including "sustainable supply network management (Young and Kielkiewicz-Young 2001; Cruz and Matsypura 2009), Supply and demand sustainability in corporate socially responsible networks (Kovacs 2004; Cruz and Matsypura 2009), supply chain environmental management (Sharfman et al. 2009), green purchasing (Min and Galle 1997) and procurement (Günther and Scheibe 2006), environmental purchasing (Carter et al. 2000; Zsidisin and Siferd 2001), green logistics (Murphy and Poist 2000) and environmental logistics (Gonzalez-Benito and Gonzalez-Benito 2006) and sustainable supply chains (Linton et al. 2007; Bai and Sarkis 2010)".

According to the above-mentioned descriptions, here we define GrSCM as "integrating environmental and economical aspects into all decisions of supply chain management through all stages of product life cycle (cradle-to-grave) in order to create (more) sustainable value for broad range of stakeholders".

1.3 Classification of Planning Problems in Green and Reverse Logistics Management

Different classifications on green supply chain have ever been proposed in the literature. Among them, Srivastava (2007) introduced a classification based on problem context in which GrSCM is classified into (1) green design and (2) green operations. In this classification, subjects such as life cycle assessment (LCA) and ergonomic comfort design (ECD) are related to green design while green manufacturing and remanufacturing, reverse logistics, network design and waste management are subfields of green operations.

Recently, Ilgin and Gupta (2010) classified environmentally conscious manufacturing and product recovery into four main categories including (1) product design, (2) reverse and closed-loop supply chain, (3) remanufacturing and (4) disassembly.

Similar to the traditional SCM, GrSCM can also be classified according to the length of decision horizon, i.e., strategic (STRG), tactical (TCTL) and operational (OPRL) decisions. Issues such as green supply chain network design and integrated forward-reverse logistics network design are considered as strategic decisions; problems concerning with the amount of material flows between each pair of network's facilities at each medium-term time period (e.g., monthly) with respect to their environmental concerns as well as cost objectives are known as tactical decisions and finally decisions such as green daily production scheduling and material transportations are operational level ones.

In the literature, there are several multi-attribute decision making (MADM) techniques used to evaluate the performance of whole GrSCM/GrLog/RL, suppliers and third-party logistics providers (see Shen et al. 2013; Ravi 2012; Lin 2013; Kannan et al. 2009; Kannan et al. 2013; Govindan et al. 2013; Dhouib 2013; Akman and Pışkın 2013); however, in this chapter we have focused on GrLog and RL designing and planning problems.

From the operations research (OR) perspective, different modeling approaches including mixed-integer linear programming (MILP), multi-objective integer linear programming (MOILP), mixed-integer goal programming (MIGP), multi-objective mixed integer programming (MOMIP), fuzzy goal programming (FGP), credibility-based fuzzy mathematical programming (CFMP), multi-objective possibilistic mixed integer linear programming (MOPMILP) have been used to formulate planning problems in the context of GrSCM/GrLog. In addition, in order to solve the developed mathematical models, different approaches are often applied in the literature which include: commercial optimization solvers (like CPLEX) to find optimal solutions in small to medium-scaled problems, decomposition-based exact/approximation methods (like Benders decomposition/Lagrangean relaxation) and heuristic or metaheuristic methods to yield near-optimal or optimal solutions in large-scaled instances.

As discussed before, uncertainty plays an important role in GrSCM/GrLog and RL contexts. Three main approaches including: (1) fuzzy programming, (2) stochastic programming and (3) robust programming are used to cope with uncertainty. Uncertainty is usually considered in the model parameters involving: Demands (D), Transportation Costs (TC), Handling Costs (HC), Quantity of Returns (QnR), Quality of Returns (QlR), Fixed Opening Costs (FOC), Manufacturing Costs (MC), Processing Costs (PC), Operations Costs (OC), Remanufacturing Costs (RC), Capacity levels (Cap), Recovery Percentages (RPer), Landfill Percentages (LPer), Number of Created Jobs (NCJ), Emission Factors (EF), Production Rates (PR), Collection Costs (CC), Distribution Costs (DC) and Recovery Fractions (RF) or is incorporated into the objective function(s) such as Flexibility of Goals (FG) and Preference of DM's over objective function (POF) in multi-objective models.

A detailed review of selected papers from the literature related to GrSCM/GrLog, RL and closed-loop supply chain (CLSC) based on abovementioned classifications is provided in Table 1.

For more comprehensive and detailed review of GrSCM/GrLog and RL, interested readers can consult with (Srivastava 2007; Sbihi and Eglese 2007; Ilgin and Gupta 2010; Sarkis et al. 2011), and (Fleischmann et al. 1997; Beamon 1999; De Brito and Dekker 2004; Wang and Sun 2005; Pishvaee et al. 2010a; Souza 2013), respectively.

The rest of the chapter is organized as follows. In Sect. 2, the concept of GrLog and RL management under uncertainty is discussed. A classification for different types of uncertainty, main programming approaches to cope with uncertainties, advantages of fuzzy mathematical programming approach over other competing approaches and a classification for fuzzy mathematical models are also given in this section. Afterwards, in Sect. 3, three selected fuzzy mathematical models

Table 1 Classification of some selected papers

Author	Scope	Modeling approach	Solution approach	Type of uncertainty	Uncertain parameters
Pati et al. (2008)	STRG, TCTL	MIGP	Solver	–	–
Mele et al. (2009)	STRG	MOILP	Solver	–	–
Pishvaee et al. (2009)	STRG	MILP	Solver	Stochastic	D, TC, QnR, QIR
Tsai and Hung (2009)	TCTL, OPRL	FGP		Fuzzy	FG, POF
Kannan et al. (2010)	TCTL, OPRL	MILP	Metaheuristic	–	–
Pishvaee and Torabi (2010)	STRG, TCTL	MOPMIL	Solver	Fuzzy Possibilistic	D, TC, QnR, FOC, MC, PC, RC, Cap
Pishvaee et al. (2010a)	STRG	MOMIP	Metaheuristic	–	–
Pishvaee et al. (2010b)	STRG	MILP	Metaheuristic	–	–
Qin and Ji (2010)	STRG	MILP	Hybrid fuzzy and GA	Fuzzy	QnR, FOC, HC
Wang and Hsu (2010)	TCTL	MILP		Fuzzy	D, RPer, LPer
Abdallah et al. (2011)	STRG, TCTL	MILP	Solver	–	–
Pinto-Varela et al. 2011	STRG, TCTL	MILP	Solver	Fuzzy	FG
Pishvaee et al. (2011)	STRG	MILP	Solver	Robust	D, QnR, TC
Pishvaee and Razmi (2012)	STRG	MOPMIL	Solver	Fuzzy	D, QnR, FOC, TC, PC, Cap
Ubeda et al. (2011)	OPRL	MILP	Heuristic	–	–
Wang et al. (2011)	STRG, TCTL	MOMILP	Solver	–	–
Zhang et al. (2011)	TCTL	MILP	Piecewise Interval Programming	–	–
Cardoso et al. (2013)	TCTL	MILP	Solver	Stochastic	D
Chaabane et al. (2012)	STRG, TCTL	MOMILP	Solver	–	–
Jamshidi et al. (2012)	STRG	MOMILP	Hybrid Memetic and Taguchi	–	–

(continued)

Table 1 (continued)

Author	Scope	Modeling approach	Solution approach	Type of uncertainty	Uncertain parameters
Pishvaee et al. (2012a)	STRG	MOMILP	Solver	Robust Possibilistic	D, FOC, NCJ, Cap
Pishvaee et al. (2012b)	STRG, TCTL	MILP	Interactive fuzzy	Credibility-based fuzzy	D, Cap, FOP, TC, PC, EF
Vahdani et al. (2013a)	STRG	MOMILP	Solver	Fuzzy Possibilistic Queuing	FOP, TC, CC, DC, MC, RC, D, PR, QnR, RF
Vahdani et al. (2012)	STRG	MOMILP	Solver	Fuzzy Robust Queuing	FOP, TC, PR, OC, Cap

addressing GrLog and RL planning problems are presented and discussed. In Sect. 4, an industrial case study is provided and finally, some possible future directions for further research are presented in Sect. 5.

2 Green and Reverse Logistics Management Under Uncertainty

The complex nature and structure of commercial supply chains and working in a dynamic and chaotic business environment, imposes a high degree of uncertainty in supply chain planning decisions and significantly affects their overall performance (Klibi et al. 2010). The degree of complexity in green and reverse logistics is even greater than traditional supply chains, since highly imprecise parameters such as quantity and quality of returned products and environmental factors should also be taken into account (Erol et al. 2011; Pishvaee et al. 2012b).

As it could be seen in Table 1, most of the published papers are related to strategic level decisions rather than tactical or/and operational decisions. Decisions regarding locations and number of required manufacturing, remanufacturing, and collection centers as well as aggregated material flows between these centers and consumers in forward and reverse directions are some of main decisions made in the strategic level. It is quite clear that the degree of uncertainty in strategic decisions is significantly higher than mid-term and short-term decisions. The reason goes back to difficulty of forecasting and providing confident values for input parameters in a longer time horizon.

In the light of above-mentioned points, accounting for uncertainty in GrLog and RL is inevitable. Therefore, different approaches to cope with uncertainty are used in the literature including stochastic programming (e.g., Pishvaee et al. 2009; Cardoso et al. 2013), fuzzy programming (e.g., Tsai and Hung 2009; Qin and Ji 2010; Wang and Hsu 2010; Pishvaee and Torabi 2010; Pishvaee and Razmi 2012; Pishvaee et al. 2012a; Pishvaee et al. 2012b; Pinto-Varela et al. 2011; Vahdani et al. 2013a; Vahdani et al. 2012) and robust programming (e.g., Pishvaee et al. 2011; Pishvaee et al. 2012a; Vahdani et al. 2012) approaches. Among these approaches, fuzzy programming methods are mostly utilized in recent years due to their capability in handling both epistemic and vague uncertainties.

In this section, a useful taxonomy is provided to classify different kinds of uncertainty in green and reverse logistics planning problems. Then, various types of fuzzy programming methods which have already been applied in the context of GrLog and RL along with their characteristics are studied and analyzed.

2.1 Classification of Uncertainties

Different general and SCM-related classifications for uncertainty have ever been proposed in the literature from different points of view. Among them, according to

Tang (2006) and Klibi et al. (2010), uncertainty in supply chains can be classified into two groups: (1) business-as-usual (or operational) uncertainty, such as usual fluctuations in demand and supply data which mostly includes events with low to medium impact, medium to high likelihood; (2) disaster uncertainty, that covers rare events with high business impacts but low likelihood such as uncertainty in supply disruptions due to occurrence of a natural disaster (e.g., flood or earthquake) in supplier location. Terms such as "hazard" and "disruption" can also be used instead of the term "disaster" here. This type of uncertainty can be originated generally from natural sources i.e., earthquake, flood, Tsunami or man-made sources such as war, terrorist attacks, labor strikes, sanctions, etc.

From a general view, Dubois et al. (2003) classified uncertainty as: (1) uncertainty in input data, and (2) flexibility in constraints and goals. The first type is the most common uncertainty faced in supply chains which is usually referred to epistemic uncertainty and possibilistic programming methods are used to handle such kind of uncertainty. The second type of uncertainty deals with flexibility in target value of fuzzy goals and/or right hand side (RHS) of soft constraints for which flexible mathematical programming models are utilized to cope with such flexible values (Bellman and Zadeh 1970; Mula et al. 2006).

Uncertainty in data can be classified into two categories (Mula et al. 2006; Mula et al. 2007): (1) randomness, that stem from the random nature of parameters and stochastic programming methods are the most applied approaches to cope with this sort of uncertainty; (2) Epistemic uncertainty, that deals with ill-known and imprecise parameters arising from lack of knowledge regarding the exact value of these parameters for which possibilistic programming approaches are usually applied (Pishvaee and Torabi 2010; Mula et al. 2006).

From a different point of view, Davis (1993) classified the potential sources of uncertainty in supply chains in three main categories, i.e., (1) supply uncertainty, (2) process uncertainty and (3) demand uncertainty. In general, changes in supplier's performance such as lateness in delivery of raw materials or delivery of defective materials by suppliers leads to supply uncertainty. On the other hand, faults occurring in production and/or distribution processes are the main sources of process uncertainty. Finally, imprecise estimation of future demands for special products, changes in market, changes in customers attitude, changes in fashion, etc. are the main sources of demand uncertainty which is the most frequent uncertainty in real-life situations.

Another classification of uncertainty in the context of production systems is provided by Ho (1989) as: (1) environmental uncertainty and (2) system uncertainty. Similar to afore-mentioned classifications in the context of supply chain, environmental uncertainty is related to demand side uncertainties derived from customer behavior and market trends as well as supply side uncertainties stemmed from the performance of suppliers. Furthermore, system uncertainty refers to those uncertainties within the production, distribution, collection and recovery processes for example uncertainties pertaining to production costs/times and actual capacity of different processes.

It should be mentioned that all of the reviewed classifications are meaningful in the context of GrSCM/GrLog, RL and CLSC but the main point is that how should we cope with these uncertainties in mathematical models?

2.2 *Overview of Different Approaches to Cope with Uncertainty*

As the body of literature shows, three main approaches are mostly employed to deal with uncertainty in the context of mathematical programming, i.e., (1) stochastic programming, (2) fuzzy programming and (3) robust optimization. Based on the structure and context of the concerned problem, type of uncertainty and the level of incompleteness in the model's parameters, one or a combination of these approaches can be applied. Nevertheless, each method has its unique characteristics which differentiate it from the others. Hence, one should delicately study and analyze the type(s) of uncertainty involved in the concerned problem and then choose the most appropriate method(s) to cope with recognized uncertainty or uncertainties.

2.2.1 Stochastic Programming

Stochastic programming methods can be used whenever randomness is the main source of uncertainty in input data for which random variables with known probability distributions are often utilized.

Sahinidis (2004) classified stochastic programming into two main categories: programming with recourse (i.e., two-stage stochastic programming) and probabilistic (chance constrained) programming. In the former, the decision variables are partitioned into two sets. The first stage decisions are those that have to be made before the actual realization of the uncertain parameters and the second stage decisions are those that must be made after realization of uncertain parameters. This method is mostly suggested when infeasibility is allowed with charging penalty costs. Traditionally, the second-stage variables are interpreted as corrective measures or recourse against any infeasibilities arising due to a particular realization of uncertainty. From a different point of view, one can refer to first-stage decisions as strategic decisions and the second-stage decisions as tactical or operational decisions following the first-stage plan that has been made in an uncertain environment. The objective is usually to determine the first-stage decisions in such a way that minimizes total first-stage costs and the expected value of second-stage costs. On the other hand, the former focus on the reliability of the system, i.e., the ability of system to meet feasibility in an uncertain environment. This reliability could be translated as a minimum requirement on the probability of satisfying constraints (i.e., the confidence level of satisfaction).

For detailed classification on stochastic programming approaches and their mathematical challenges, the reader may consult with Sahinidis (2004) and Birge and Louveaux (1997).

2.2.2 Robust Optimization

Robust programming/optimization provides risk-averse methods to cope with uncertainty in optimization problems. According to Pishvaee et al. (2012a), "a solution to an optimization problem is said to be robust if it has both feasibility and optimality robustness. Feasibility robustness means that the solution should remain feasible for (almost) all possible values of uncertain parameters and optimality robustness means that the value of objective function should remain close to optimal value or have minimum (undesirable) deviation from the optimal value for (almost) all possible values of uncertain parameters".

Robust programming approaches can be classified into three groups (Pishvaee et al. 2012a): (1) hard worst case robust programming (Soyster 1973; Ben-Tal and Nemirovski 1998; Ben-Tal et al. 2009), (2) soft worst case robust programming (Inuiguchi and Sakawa 1998; Bertsimas and Sim 2004) and (3) realistic robust programming (Mulvey et al. 1995).

The hard worst case approach is the most pessimistic approach since in this approach it is assumed that all parameters could get their worst case value simultaneously. Although this approach gives maximum safety against uncertainty by giving feasible solution for all realization of uncertain parameters, the matter of highly conservatism made by this approach found itself confronted by intense criticisms (Bertsimas and Sim 2004). That is, they believe that it is highly unrealistic or over pessimistic approach. However, Ben-Tal et al. (2009) supports this approach because it does not need any information about the possibility or probability distribution of uncertain parameters. Also, Pishvaee et al. (2012a) expressed that hard worst case is appropriate for risk averse DMs and it is especially applicable in the cases that a small perturbation from the expected performance of the system causes catastrophic outcomes (e.g., in military and emergency cases).

The second approach is more flexible than hard worst case approach. By this approach, like the hard worst case, one tries to minimize the worst case value of objective function but the difference is that it does not satisfy (all) the constraints in their extreme worst case.

Finally, the realistic robust programming approach aims to seek trade-off between the robustness of achieved solution and the cost of robustness (a cost–benefit logic). This approach is appropriate for profit-seeking and flexible DMs and could be applicable in most of business cases (Pishvaee et al. 2012a).

For more information about the RP theory, the interested readers are referred to Beyer and Sendhoff (2007), Ben-Tal et al. (2009) and Pishvaee et al. (2012a).

2.2.3 Fuzzy Programming

Fuzzy programming can handle both epistemic uncertainty in data as well as flexibility in goals and/or elasticity in constraints. Using this approach, imprecise parameters are modeled by appropriate possibilistic distributions in the form of fuzzy numbers. Moreover, flexible target values and vague (soft) inequalities/equalities are formulated through fully subjective preference-based fuzzy membership functions.

Accordingly, fuzzy mathematical programming can be classified into two main classes (Inuiguchi and Ramík 2000; Mula et al. 2006; Torabi and Hassini 2008): (1) possibilistic programming and (2) flexible programming. Possibilistic programming is used when there is lack of knowledge (epistemic uncertainty) about exact values of input data (parameters) due to unavailability or insufficiency of required data. Accordingly, suitable possibilistic distributions based upon both available objective data and subjective opinions of DMs are introduced for modeling imprecise data in the form of fuzzy numbers. On the other hand, flexible programming is used to cope with flexibility in target value of goals and/or elasticity in soft constraints. The latter refers to those constraints tainted with soft inequalities/equalities in the form of $\tilde{\leq}$, $\tilde{\geq}$ and $\cong$ in which tilde sign shows the softness of respective constraints. For example, $x_1 \tilde{\leq} 20$ means that x_1 should be less than or equal to 20 but small deviations could be accepted subject to less constraint's satisfaction degrees. In flexible programming, a subjective, i.e., preference-based fuzzy membership function is usually adopted for each vague target value or soft constraint. It is quite clear that both possibilistic and flexible programming approaches could be simultaneously applied in a mathematical model when there is a mixture of aforementioned types of uncertainties.

2.3 *Advantages of Fuzzy Approaches*

In many cases, due to lack of historical data, it is hard or even impossible to fit a probability distribution for some objective-natured parameters such as products' demands or unit processing times of manufacturing operations. Furthermore, some other input data have a fully subjective nature like those of judgmental data quoted by expert(s) in most of decision making situations. In the former case, it is a reasonable option to fit a suitable possibilistic distribution for each parameter based upon the available (but often insufficient) objective data as well as subjective opinions of DMs, but in the latter, a fully subjective (preference-based) fuzzy set is adopted for each judgmental data based upon expert's subjective knowledge, experience and feelings. However, in both cases, fuzzy numbers can be used to formulate the incomplete, vague and ambiguous data and fuzzy programming approaches are the most suitable tools for coping with such uncertainties (Qin and Ji 2010; Wang and Hsu 2010).

In the context of GrLog and RL, there is not only greater lack of historical data but also existence of more complex relationships between some data, makes the estimation of related parameters even more impossible. To overcome this deficiency, the fuzzy mathematical programming approaches are being more employed in the context of GrLog, RL and CLSC (Pishvaee and Torabi 2010; Qin and Ji 2010).

In brief, the major advantages of fuzzy programming can be summarized as follows (Mula et al. 2006; Pishvaee and Torabi 2010): (1) it can appropriately handle both the imprecise and vague data; (2) it can integrate subjective and objective data (i.e., using of both available historical data and human subjective knowledge) to formulate business decision problems in practical situations; (3) it can resolve the issue of infeasibility in some decision making situations such as applications of hierarchical planning (Torabi et al. 2010); (4) problems formulated as fuzzy programming models can be easily reformulated to their equivalent crisp counterparts for which commercial optimization solvers could be used to obtain optimal solutions; (5) fuzzy programming can offer enough flexibility for obtaining various solutions by taking into account the tolerances provided by fuzzy data which can then be evaluated by DM to find a most preferred final solution based on her/his preferences; (6) compared to the stochastic programming approach that its deterministic counterpart increases numerical complexity of the problem in a great degree, by using a fuzzy programming approach, a final solution could be obtained with much fewer computation. In the next subsection, a comprehensive review of fuzzy programming approaches is provided in the context of green and reverse logistics.

2.4 Review of Relevant Papers

As mentioned in Sect. 2.2, the fuzzy programming approaches can be classified into two groups: flexible programming and possibilistic programming.

Literature review demonstrates that the most of published works in the context of reverse and green logistics addressing the fuzziness, use either one of the possibilistic programming approaches (see for example: Pishvaee and Torabi 2010; Qin and Ji 2010; Pishvaee and Razmi 2012; Pishvaee et al. 2012a; Pishvaee et al. 2012b; Vahdani et al. 2013b) or a mixture of possibilistic and flexible programming approaches (see for example: Tsai and Hung 2009; Wang and Hsu 2010; Özceylan and Paksoy 2013) when different type of fuzziness (i.e., imprecise coefficients in objective functions and/or constraints as well as flexible target values for objectives and/or soft inequalities) are introduced in the formulated problem. In this subsection, the related papers are reviewed in more details.

2.4.1 Possibilistic Programming

Pishvaee and Torabi (2010) propose a possibilistic programming approach for a closed-loop supply chain network design problem in which some parameters are imprecise. A bi-objective possibilistic mixed-integer programming model is proposed which integrates the strategic network design for both forward and reverse flows with material flows tactical decisions. An efficient interactive fuzzy solution approach is developed by combining Jimenez et al. (2007), Parra et al. (2005), TH (see Torabi and Hassini 2008) and SO (see Selim and Ozkarahan 2008) methods, that is capable of generating both balanced and unbalanced efficient solutions based on decision maker's preferences.

Qin and Ji (2010) propose three credibility measure based fuzzy programming approaches, i.e., expected value (see Liu and Liu 2002), chance constrained programming (see Liu and Iwamura 1998) and dependent-chance constrained programming (see Liu 1999) to design a product recovery network. In order to solve the proposed MILP models, a hybrid intelligent algorithm is used that integrates fuzzy simulation and genetic algorithm.

Pishvaee and Razmi (2012) propose a multi-objective fuzzy mathematical programming model for designing an environmental supply chain. In the proposed model, a life-cycle assessment (LCA) based method is applied in order to quantify the environmental impact of different options. The main decisions of the proposed model are the location of production and collection centers as well as flow quantities between different facilities under two different objectives, i.e., minimization of total costs and total environmental impacts. In order to solve the proposed model, an interactive fuzzy solution approach based on the ε-constraint method is developed and finally a real industrial case study is provided to show the usefulness of the proposed model as well as the solution approach.

Pishvaee et al. (2012a) propose a novel robust possibilistic programming (RPP) approach and use it for design of a socially responsible supply chain network. This approach involves six variants of RPP which are elaborated in the next section. The model aims to select a set of locations for plants and distribution centers among candidate locations, an appropriate production technology for each opened plant and estimate material flows between different facilities while 1) minimizing the total costs including fixed opening costs, variable production costs and transportation costs, and 2) maximizing the social responsibility of the concerned network including: maximization of job opportunities, minimization of total produced wastes, lost days caused from work's damages and the number of potentially hazardous products. Finally, a real industrial case is provided to illustrate the efficiency and applicability of this novel approach.

Pishvaee et al. (2012b) propose a bi-objective credibility-based fuzzy mathematical programming model for designing supply chain network design in which green issues are also taken into account. The model aims to make a trade-off between two conflicting objectives, i.e., minimization of total costs and minimization of the environmental impacts by defining CO_2 equivalent index in order to quantify the environmental burden of logistics activities. Also, an interactive fuzzy solution approach by mixing two credibility measure based approaches (i.e., expected value and chance constrained programming) is developed to solve the original bi-objective fuzzy model. A real industrial case study is also provided that supports the applicability of the proposed model.

Finally, Vahdani et al. (2013a) propose a possibilistic-queuing model for designing a reliable closed-loop supply chain network. The model aims to minimize the total costs and the expected transportation costs after failure of bi-directional facilities of the concerned network. A new probabilistic queuing constraint is introduced in order to overcome capacity limitations and an efficient hybrid solution method by combining the queuing theory, possibilistic programming and fuzzy multi objective programming approaches is developed to solve the model.

2.4.2 Flexible Programming

Among the relevant papers, Tsai and Hung (2009) introduce a fuzzy goal programming approach for green supply chain optimization. In the proposed approach, the well-known activity-based costing (ABC) and performance evaluation in value-chain structure are integrated aiming to find the optimal supplier selection and flow allocation. Also, analytical hierarchy process (AHP) is utilized to determine the final objective structure and as an illustrative case example, the green supply chain of mobile phone is studied.

Also, Wadhwa et al. (2009) propose a flexible multi criteria decision-making (MCDM) model based on fuzzy-set theory for reverse logistics systems. Their model collect required information from DMs in order to select the most suitable alternative(s) for product reprocessing concerning five different criteria, i.e., cost/time, environmental impacts, market factors, quality factors and legislative factors. To assess the rating of the criteria, they use verbal values collected from product return experts instead of crisp values due to this fact that the crisp evaluation of the criteria is quite impossible.

2.4.3 Mixed Possibilistic and Flexible Programming

Wang and Hsu (2010) study a closed-loop supply chain network design in which some imprecise parameters and soft constraints are introduced. The decisions to be made involve: the location of production, distribution and dismantler centers and amount of material flows between these centers. An interval programming method is applied in order to reformulate the crisp counterpart of the original fuzzy model.

Özceylan and Paksoy (2013) propose a multi-objective mixed-integer fuzzy mathematical model for optimizing an integrated forward and reverse closed-loop supply chin network with multiple period and multiple items. The concerned decisions consist of: opening of potential plants and retailers alongside with amount of shipment between different set of facilities while minimization of total transportation, purchasing, refurbishing and fixed costs simultaneously. In the proposed model, capacity and reverse rates as model parameters and also objective and demand constraints are considered as fuzzy data. In order to build the crisp counterpart, the linear membership functions are defined for all fuzzy objective functions and α-value and weighted average methods are used to convert the fuzzy inequality constraints into crisp ones.

3 Selected Fuzzy Mathematical Models

In this section, three different fuzzy mathematical models are elaborated in the context of green and reverse logistics in which different fuzzy programming approaches are employed to capture inherent fuzziness in the data. For the sake of simplicity, the notations used in this section are the same as those represented in the original papers.

3.1 A GrLog Model with Mixed Expected Value and Chance Constrained Programming Approach

In this subsection, a brief discussion of fuzzy mathematical model introduced by Pishvaee et al. (2012b) along with the respective defuzzification process to formulate the crisp counterpart are provided as a sample in the current GrLog literature under fuzziness. The problem is a single product, three-echelon supply chain which includes multiple production and distribution centers and customer zones. Products are produced in production centers and are then transported to the distribution centers through which are finally delivered to the customer zones. The locations of the customers are fixed and each customer has its own demand which must be completely fulfilled. There are a number of potential sites for establishing production and distribution centers at different capacity levels. Furthermore, multiple options of production technologies are available for each established production center and different transportation modes can be used for transporting products between each pair of nodes in the network. The model aims to determine the number, location and required capacity of production and distribution centers alongside the preferred production technology at each production center as well as transportation mode between each pair of nodes. The model has two different objectives i.e., minimization of overall opening, production and transportation cost and minimization of overall environmental effects. In order to assess and quantify burden of logistics activities including production and transportation activities on environment, the CO2 equivalent index based on the Eco-indicator 99 database (Goedkoop and Spriensma 2000) is used. The structure of the problem is depicted in Fig. 1 and notations are described thereafter.

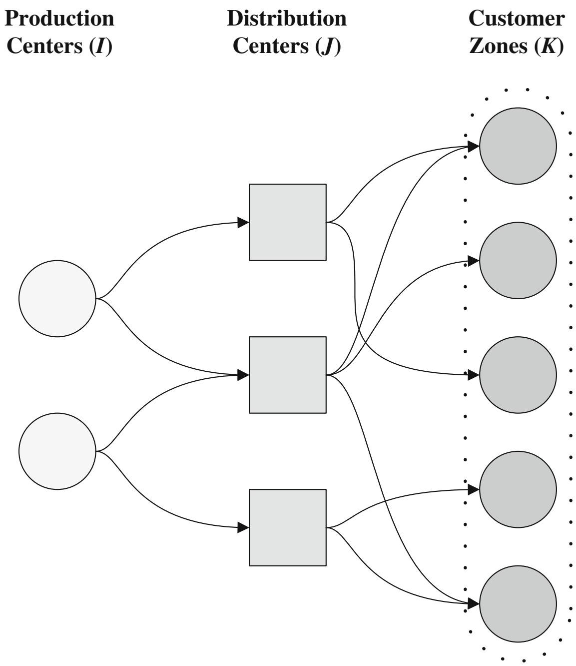

Fig. 1 Structure of the discussed green logistics network (adopted from Pishvaee et al. 2012b)

Indices

i index of candidate production centers $i \in \{1, 2, \ldots, I\}$
j index of candidate distribution centers $j \in \{1, 2, \ldots, J\}$
k index of fixed customer zones $k \in \{1, 2, \ldots, K\}$
m index of capacity levels available for production centers $m \in \{1, 2, \ldots, M\}$
n index of capacity levels available for distribution centers $n \in \{1, 2, \ldots, N\}$
l index of potential production technologies $l \in \{1, 2, \ldots, L\}$
p index of potential transportation modes $p \in \{1, 2, \ldots, P\}$.

Parameters

d_k demand of customer zone k
f_i^{ml} fixed cost of opening production center i with capacity level m and production technology l
g_j^n fixed cost of opening distribution center j with capacity level n
c_{ij}^p unit transportation cost from production center i to distribution center j via transportation mode p
a_{jk}^p unit transportation cost from distribution center j to customer zone k via transportation mode p
ρ_i^l unit manufacturing cost at production center i with production technology l
τ_i^m capacity of production center i with capacity level m
φ_j^n capacity of distribution center j with capacity level n

l	CO_2 equivalent emission per unit product produced using technology l
t_{ij}^{p}	CO_2 equivalent emission per unit product shipped from production center i to distribution center j using transportation mode p.
s_{jk}^{p}	CO_2 equivalent emission per unit product shipped from distribution center j to customer zone k using transportation mode p.

Variables

u_{ij}^{lp}	quantity of product manufactured at production center i using technology l and shipped to distribution center j using transportation model p
q_{jk}^{p}	quantity of products shipped from distribution j to customer zone k using transportation mode p
x_{i}^{ml}	1, if potential production center i with capacity level m and technology l is opened; 0, otherwise
y_{j}^{n}	1, if potential distribution center j with capacity level n is opened; 0, otherwise

Using abovementioned notation, the proposed mathematical model is as follows:

$$\min w_1 = \sum\nolimits_{i,m,l} f_i^{ml} x_i^{ml} + \sum\nolimits_{j,n} g_j^n y_j^n + \sum\nolimits_{i,j,l,p} \left(\rho_i^l + c_{ij}^p\right) u_{ij}^{lp} + \sum\nolimits_{j,k,p} a_{jk}^p q_{jk}^p \quad (1)$$

$$\min w_2 = \sum\nolimits_{i,j,l,p} \left(l + t_{ij}^p\right) u_{ij}^{lp} + \sum\nolimits_{j,k,p} s_{jk}^p q_{jk}^p \quad (2)$$

$$s.t: \sum\nolimits_{j,p} q_{jk}^p \geq d_k \quad \forall k \quad (3)$$

$$\sum\nolimits_{i,l,p} u_{ij}^{lp} = \sum\nolimits_{k,p} q_{jk}^p \quad \forall j \quad (4)$$

$$\sum\nolimits_{j,p} u_{ij}^{lp} \leq \sum\nolimits_{m} x_i^{ml} \tau_i^m \quad \forall i, l \quad (5)$$

$$\sum\nolimits_{k,p} q_{jk}^p \leq \sum\nolimits_{n} y_j^n \varphi_j^n \quad \forall j \quad (6)$$

$$\sum\nolimits_{m,l} x_i^{ml} \leq 1 \quad \forall i \quad (7)$$

$$\sum\nolimits_{n} y_j^n \leq 1 \quad \forall j \quad (8)$$

$$x_i^{ml}, y_j^n \in \{0, 1\} \quad \forall i, j, l, m, n \quad (9)$$

$$u_{ij}^{lp}, q_{jk}^p \geq 0 \quad \forall i, j, k, p \quad (10)$$

Objective function (1) minimizes the total fixed opening costs, production costs and transportation costs while objective function (2) minimizes the total CO_2 equivalent emission. Demand fulfillment of each customer zone is guaranteed by constraints (3). Constraints (4) ensure that all of the manufactured products must be transported to distribution centers. Equations (5) and (6) are the capacity constraint for production and distribution centers, respectively. Equation (7) ensure that at most one capacity level and one technology can be assigned to each production center at each candidate location. Similarly, assigning at most one capacity level to each distribution center at each candidate location is guaranteed via (8). Finally, the binary and non-negativity restrictions on the corresponding decision variables are indicated in (9) and (10).

As mentioned earlier, most of the parameters in logistics network design are tainted with epistemic uncertainty. To cope with this uncertainty, a new credibility-based chance constrained programming model is proposed in this paper. Modeling all of the imprecise parameters in the model as trapezoidal possibility distributions, and substituting Eqs. (1)–(3), (5) and (6) with Eqs. (11)–(15), the possibilistic programming counterpart of the discussed problem could be formulated as below:

$$\begin{aligned} \min E[w_1] = {} & \sum\nolimits_{i,m,l} E\left[\tilde{f}_i^{ml}\right] x_i^{ml} + \sum\nolimits_{j,n} E\left[\tilde{g}_j^n\right] y_j^n \\ & + \sum\nolimits_{i,j,l,p} \left(E\left[\tilde{\rho}_i^l\right] + E\left[\tilde{c}_{ij}^p\right]\right) u_{ij}^{lp} + \sum\nolimits_{j,k,p} E\left[\tilde{a}_{jk}^p\right] q_{jk}^p \end{aligned} \tag{11}$$

$$\min E[w_2] = \sum\nolimits_{i,j,l,p} \left(E[l] + E\left[\tilde{t}_{ij}^p\right]\right) u_{ij}^{lp} + \sum\nolimits_{j,k,p} E\left[\tilde{s}_{jk}^p\right] q_{jk}^p \tag{12}$$

$$Cr\left\{\sum\nolimits_{j,p} q_{jk}^p \geq d_k\right\} \geq \beta_k \quad \forall k \tag{13}$$

$$Cr\left\{\sum\nolimits_{j,p} u_{ij}^{lp} \leq \sum\nolimits_{m} x_i^{ml} \tau_i^m\right\} \geq \lambda_i \quad \forall i, l \tag{14}$$

$$Cr\left\{\sum\nolimits_{k,p} q_{jk}^p \leq \sum\nolimits_{n} y_j^n \varphi_j^n\right\} \geq \theta_j \quad \forall j \tag{15}$$

In this model, the expected value method is used to convert the possibilistic objective functions into their crisp ones. To do so, according to Liu and Liu (2002), the expected value of a trapezoidal fuzzy number $\tilde{\Psi}$ with four prominent points $\tilde{\Psi} = \left(\Psi_{(1)}, \Psi_{(2)}, \Psi_{(3)}, \Psi_{(4)}\right)$ will be equal to $\left(\Psi_{(1)} + \Psi_{(2)} + \Psi_{(3)} + \Psi_{(4)}/4\right)$. Meanwhile, by adopting a chance-constrained programming approach, a minimum confidence level is set to ensure satisfaction of each possibilistic constraint pertaining to most critical constraints (i.e., demand and capacity restrictions) at some acceptable level. A, based on (Zhu and Zhang 2009), for α-critical values greater than 0.5, the following substitutions could be used:

$$Cr\{\Psi \leq r\} \geq \alpha \leftrightarrow r \geq (2 - 2\alpha)\Psi_{(3)} + (2\alpha - 1)\Psi_{(4)} \tag{16}$$

$$Cr\{\Psi \geq r\} \geq \alpha \leftrightarrow r \leq (2\alpha - 1)\Psi_{(1)} - (2 - 2\alpha)\Psi_{(2)} \tag{17}$$

Consequently, after converting abovementioned possibilistic terms into their crisp equivalents, the crisp counterpart of (11)–(15) is reformulated as below:

$$\begin{aligned}\min E[w_1] = &\sum\nolimits_{i,m,l}\left(\frac{f^{ml}_{i(1)} + f^{ml}_{i(2)} + f^{ml}_{i(3)} + f^{ml}_{i(4)}}{4}\right)x^{ml}_i + \sum\nolimits_{j,n}\left(\frac{g^{n}_{j(1)} + f^{n}_{j(2)} + f^{n}_{j(3)} + f^{n}_{j(4)}}{4}\right)y^n_j \\ &+ \sum\nolimits_{i,j,l,p}\left(\frac{\rho^{l}_{i(1)} + \rho^{l}_{i(2)} + \rho^{l}_{i(3)} + \rho^{l}_{i(4)} + c^{p}_{ij(1)} + c^{p}_{ij(2)} + c^{p}_{ij(3)} + c^{p}_{ij(4)}}{4}\right)u^{lp}_{ij} \\ &+ \sum\nolimits_{j,k,p}\left(\frac{a^{p}_{jk(1)} + a^{p}_{jk(2)} + a^{p}_{jk(3)} + a^{p}_{jk(4)}}{4}\right)q^p_{jk}\end{aligned} \tag{18}$$

$$\begin{aligned}\min E[w_2] = &\sum\nolimits_{i,j,l,p}\left(\frac{{}^{l}_{(1)} + {}^{l}_{(2)} + {}^{l}_{(3)} + {}^{l}_{(4)} + t^{p}_{ij(1)} + t^{p}_{ij(2)} + t^{p}_{ij(3)} + t^{p}_{ij(4)}}{4}\right)u^{lp}_{ij} \\ &+ \sum\nolimits_{j,k,p}\left(\frac{s^{p}_{jk(1)} + s^{p}_{jk(2)} + s^{p}_{jk(3)} + s^{p}_{jk(4)}}{4}\right)q^p_{jk}\end{aligned} \tag{19}$$

$$\sum\nolimits_{j,p} q^p_{jk} \geq (2 - 2\beta_k)d_{k(3)} + (2\beta_k - 1)d_{k(4)} \quad \forall k \tag{20}$$

$$\sum\nolimits_{j,p} u^{lp}_{ij} \leq \sum\nolimits_{m} x^{ml}_i\left[(2\lambda_i - 1)\tau^m_{i(1)} + (2 - 2\lambda_i)\tau^m_{i(2)}\right] \quad \forall i, l \tag{21}$$

$$\sum\nolimits_{k,p} q^p_{jk} \leq \sum\nolimits_{n} y^n_j\left[\left(2\theta_j - 1\right)\varphi^n_{j(1)} + \left(2 - 2\theta_j\right)\varphi^n_{j(2)}\right] \quad \forall j \tag{22}$$

3.2 RL Using Dependent-Chance Constrained Programming

In this part, the model proposed by Qin and Ji (2010) is presented as a sample model for reverse logistic in which three different credibility measure based possibilistic programming methods, i.e., expected value, chance constrained programming and dependent-chance constrained programming are implemented independently on the original model.

The problem is of reverse logistics network design type that includes multiple consumers, collection centers and manufacturing centers. Suppose that there is a set of potential sites for collection centers and the DM must make decision about the number and location of collection centers as well as the quantity of returned products from each customer zones to each collection center. In the proposed model, minimization of total setup costs, penalty costs, handling and transportation costs are considered as the objective function. The following notations are used for model formulation.

Indices

i index of consumer zones $i \in \{1, 2, \ldots, I\}$
j index of candidate collection centers $j \in \{1, 2, \ldots, J\}$.

Parameters

ξ_i quantity of returned product from consumer zone i
η_j cost of opening collection center j
ζ_j unit handling cost in collection center j
c_i penalty cost per unit of uncollected returned product from consumer i
p_{ij} unit transportation cost from consumer zone i to collection center j
q_j unit transportation cost from collection center j to manufacturing center
V_j maximum capacity of collection center j
M maximum number of opened collection centers
γ minimum service level.

Variables

x_{ij} quantity of returned products from consumer zone i to collection center j
y_j equal 1, if collection center j is opened and 0 otherwise.

Using the abovementioned notations, the proposed mathematical model is as follows:

$$\mathrm{Min}C(x, y) = \sum\nolimits_j \eta_j y_j + \sum\nolimits_{i,j} p_{ij} x_{ij} + \sum\nolimits_i c_i \left(\xi_i - \sum\nolimits_j x_{ij} \right) + \sum\nolimits_{i,j} (\zeta_j + q_j) x_{ij} \tag{23}$$

$$s.t : \gamma\xi_i \leq \sum\nolimits_j x_{ij} \quad \forall i \tag{24}$$

$$\sum\nolimits_i x_{ij} \leq y_j V_j \quad \forall j \tag{25}$$

$$1 \leq \sum\nolimits_j y_j \leq M \tag{26}$$

$$x_{ij} \geq 0 \quad \forall i, j \tag{27}$$

$$y_j \in \{0, 1\} \quad \forall j \tag{28}$$

The objective function (23) is to minimize total opening costs, transportation costs, handling costs and penalty costs of not collected returned products from consumer zones. Constraints (24) ensure that minimum service level must be fulfilled for each consumer zone. Capacity constraint for each collection center is proposed via (25). Constraint (26) ensures that at most M collection centers from all candidate sites could be opened and finally decision variables types are assured via (27) and (28).

Since it is difficult or even impossible to predict the quantity of returned products as well as opening and transportation costs exactly, these parameters, i.e., ξ_i, η_j and ζ_j are then considered as independent possibilistic variables modeled by fuzzy numbers and three different possibilistic programming approaches, i.e., expected value, chance constrained programming and dependent-chance constrained programming are applied independently on the original mathematical model. Also, the imprecise parameters might have triangular, trapezoidal or normal membership functions. Since the first two approaches are employed in the previous model, in this subsection, we only elaborate the dependent-chance constrained programming for the concerned model.

Dependent-chance constrained programming was first introduced by Liu (1999) and then became one the most commonly used possibilistic programming approaches. In this approach, the decision maker tries to maximize the credibility degree of a possibilistic term not exceeding from a given value (here the total costs not exceeding from the capital limit (C_0)) subject to some credible constraints (here the demand fulfillment constraints). Accordingly, for the discussed model, we would have:

$$\max \quad Cr\{C(x,y) \le C_0\} \tag{29}$$

$$s.t: Cr\left\{\gamma\xi_i \le \sum\nolimits_j x_{ij}\right\} \ge \beta_i \quad \forall i \tag{30}$$

$$(25)\text{-}(28)$$

Now, suppose that ξ_i, η_j and ζ_j are independent fuzzy numbers with normal membership functions $v\left(e_i^1, \sigma_i^1\right), v\left(e_i^2, \sigma_i^2\right)$ and $v\left(e_i^3, \sigma_i^3\right)$, respectively. Hence, the linear crisp counterpart of the above dependent-chance programming model is as follows:

$$\max \quad \left(1 + exp\left(\frac{\pi(e^* - C_0)}{\sqrt{6}\sigma^*}\right)\right)^{-1} \tag{31}$$

$$s.t: \sum\nolimits_j x_{ij} \ge \gamma e_i^1 + \frac{\sqrt{6}\gamma\sigma_i^1}{\pi} ln\left(\frac{1-\beta_i}{\beta_i}\right) \forall i \tag{32}$$

$$(25)\text{-}(28)$$

in which e* and σ* are as follows:

$$e^* = \sum\nolimits_i c_i e_i^1 + \sum\nolimits_j \left[y_j e_j^2 + \left(e_j^3 + q_j\right)\sum\nolimits_i x_{ij}\right] + \sum\nolimits_{i,j} p_{ij} x_{ij} \tag{33}$$

$$\sigma^* = \sum\nolimits_i c_i \sigma_i^1 + \sum\nolimits_j \left(y_j \sigma_j^2 + \sigma_j^3 \sum\nolimits_j x_{ij}\right) \tag{34}$$

The interested reader may refer to Qin and Ji (2010) for more details.

3.3 RL Using a Robust Possibilistic Programming Approach

To benefit from the advantages and capabilities of both robust programming and possibilistic programming, a novel approach entitled "robust possibilistic programming" was introduced by Pishvaee et al. (2012a) for the first time in the literature.

In that chapter, five different robust possibilistic programming (RPP) approaches covering hard worst case, soft worst case and realistic robust programming approaches are proposed and efficiency of each one is tested by using an industrial case study. The results show that each of the proposed approaches has its strengths, weaknesses and are useful to be applied in some specific situations. For example, the hard worst case is useful for risk-averse decision makers (DM) while soft worst case is suitable for risk-neutral or benefit seeking DMs. In the studied case study, it is proved that among the developed RPPs, the RPP-II model is more effective than other introduced approaches. This model is useful when DM is only sensitive about over deviation from expected optimal value like situations where achieving lower total cost is more desirable. Also, one of the main advantages of this method is that the model optimizes the minimum confidence level since it is defined as decision variable in the model.

Since the structure of the problem discussed in Qin and Ji (2010) is similar to that of presented by Pishvaee et al. (2012a), here we modify the model developed by Qin and Ji (2010) as an application for RPP-II model.

In this new version of model developed by Qin and Ji (2010), the capacity of collection centers (V_j) in addition to previously mentioned parameters are considered as imprecise ones whose their possibilistic distributions are of trapezoidal type, i.e., $\tilde{\eta}_j = \left(\eta_{j(1)}, \eta_{j(2)}, \eta_{j(3)}, \eta_{j(4)}\right)$, $\tilde{\xi}_i = \left(\xi_{i(1)}, \xi_{i(2)}, \xi_{i(3)}, \xi_{i(4)}\right)$, $\tilde{\zeta}_j = \left(\zeta_{j(1)}, \zeta_{j(2)}, \zeta_{j(3)}, \zeta_{j(4)}\right)$ and $\tilde{V}_j = \left(V_{j(1)}, V_{j(2)}, V_{j(3)}, V_{j(4)}\right)$. Accordingly, the RPP-II version of this model is as follows:

$$\begin{aligned} \text{Min } & E[C(x,y)] + \tau\left(C(x,y)_{max} - C(x,y)_{min}\right) + \sum\nolimits_i \delta\left[\xi_{i(4)} - (1-\beta_i)\xi_{i(3)} - \beta_i \xi_{i(4)}\right] \\ & + \sum\nolimits_j \pi\left[\alpha_j V_{(1)j} + \left(1-\alpha_j\right)V_{(2)j} - V_{(1)j}\right] y_j \end{aligned} \tag{35}$$

$$s.t: \gamma\left[(1-\beta_i)\xi_{i(3)} + \beta_i \xi_{i(4)}\right] \le \sum\nolimits_j x_{ij} \quad \forall i \tag{36}$$

$$\sum\nolimits_i x_{ij} \le \left[\alpha_j V_{1(j)} + \left(1-\alpha_j\right)V_{(2)j}\right] y_j \quad \forall j \tag{37}$$

$$0.5 \le \alpha_j, \beta_i \le 1 \quad \forall i,j \tag{38}$$

$$(26)\text{-}(28)$$

where parameters $\delta$and π are the penalty rate of violating the demand and capacity constraints. In practice, these parameters could be considered as penalty cost of not collecting each unit of returned products and cost of each unit of extra capacity needed in collection centers to handle all collected returned products. Also, in abovementioned model we have:

$$E[C(x,y)] = \sum_j E(\eta_j)y_j \sum_{i,j} p_{ij}x_{ij} + \sum_i c_i\Big(E(\xi_i) - \sum_j x_{ij}\Big) \\ + \sum_{i,j}\big(E(\zeta_j) + q_j\big)x_{ij} \tag{39}$$

$$C(x,y)_{max} = \sum_j \eta_{(4)j}y_j + \sum_{i,j} p_{ij}x_{ij} + \sum_i c_i\Big(\xi_{(4)i} - \sum_j x_{ij}\Big) \\ + \sum_{i,j}\big(\zeta_{(4)j} + q_j\big)x_{ij} \tag{40}$$

$$C(x,y)_{min} = \sum_j \eta_{(1)j}y_j + \sum_{i,j} p_{ij}x_{ij} + \sum_i c_i\Big(\xi_{(1)i} - \sum_j x_{ij}\Big) \\ + \sum_{i,j}\big(\zeta_{(1)j} + q_j\big)x_{ij} \tag{41}$$

In fact, the first term of objective function is the expected value function while the second and third terms refer to optimality and feasibility robustness, respectively. Also, equations (36) and (37) are crisp counterpart of possibilistic form.

As could be seen, the last term of objective function is non-linear. Therefore, by introducing new variables $\mu_j = \alpha_j . y_j$, the linear counterpart of the model can be written as below.

$$\text{Min } E[C(x,y)] + \tau\big(C(x,y)_{max} - C(x,y)\big) + \sum_i \delta\big[\xi_{i(4)} - (1-\beta)\xi_{i(3)} \\ -\beta\xi_{i(4)}\big] + \sum_j \pi\big[\mu_j V_{(1)j} + \big(y_j - \mu\big)V_{(2)j} - y_j \cdot V_{(1)j}\big] \tag{42}$$

$$\sum_i x_{ij} \le \big[\mu_j V_{1(j)} + \big(y_j - \mu_j\big)V_{(2)j}\big] \quad \forall j \tag{43}$$

$$\mu_j \le L \times y_j \quad \forall j \tag{44}$$

$$\mu_j \ge L \times \big(y_j - 1\big) + \alpha_j \quad \forall j \tag{45}$$

$$\mu_j \le \alpha \quad \forall j \tag{46}$$

$$(26), (28), (36), (38) \tag{47}$$

It should be noted that the parameter L in the model is a large number.

4 Case Study

In this section a real green supply chain case study, presented in Pishvaee and Razmi (2012) is reviewed. The case study is related to an Iranian single-use medical needle and syringe manufacturer that has one production plant with capacity of producing about 600 million products per year. The firm feeds both domestic and overseas customers. Reviewing the World Health Organization (WHO) report (2005) demonstrates that around 16 billion injections are carried out per year while reusing unsterilized needles and syringes leads to 8-16 million hepatitis B, 2.3-4.7 million hepatitis C and 80000-160000 human immunodeficiency virus (HIV) infections around the globe. These data shows that the end-of-life (EOL) management of this medical product is very critical from the environmental viewpoint. In order to decrease infection risks, needles and syringes are put into safety boxes and one of available EOL options such as following ones are used:

- Incineration methods like cement incinerator and rotary kiln incinerator which can be used conveniently with low cost, and are capable of energy recovery but at the same time are considered as a major source of emissions with considerable amount of negative impact on environment;
- non-incineration methods, such as steam autoclave with sanitary landfill and microwave disinfection;
- recycling that can be used by considering solutions for disinfecting the used products.

The respective supply chain structure is depicted in Fig. 2 in which new products that are produced in manufacturing centers are transported to the customer zones in forward network and after being used, the EOL products are transported to the collection centers by reverse flows. After that, the EOL products can be delivered to incineration and/or recycling centers. It is assumed that all the customer demands must be fulfilled and also all of the returned products (a predefined percent of customer's demand) must be collected.

The manufacturer serves 13 domestic and two foreign customer zones from two neighbor countries but the firm is just in charge of collecting the EOL products from domestic customer zones. The firm has already opened one plant with about 600 million production capacity per year but seven other potential locations are available for increasing the production capacity of needles and syringes. At the reverse side, there are 11 candidate locations which can be selected for establishing collection centers. Furthermore, four steel and plastic recycling centers and three incineration centers are also available for handling used products. The aim of model is to find the number and location of opened production/collection centers as well as quantity of the material flows between different facilities with respect to two conflicting objective functions, i.e., minimization of total cost and minimization of total environmental impact in which Eco-indicator 99 (see Goedkoop and Spriensma 2000) is used to quantify the second objective.

Due to lack of sufficient historical data and also dynamic nature of the problem which does not guarantee that behavior of uncertain parameters comply with historical data, the uncertain parameters are presented by fuzzy numbers and possibilistic programming approach is used to handle these uncertain parameters in the model. In order to solve the problem, an interactive fuzzy solution method based on ε-constraint method is used in which for each value of minimum

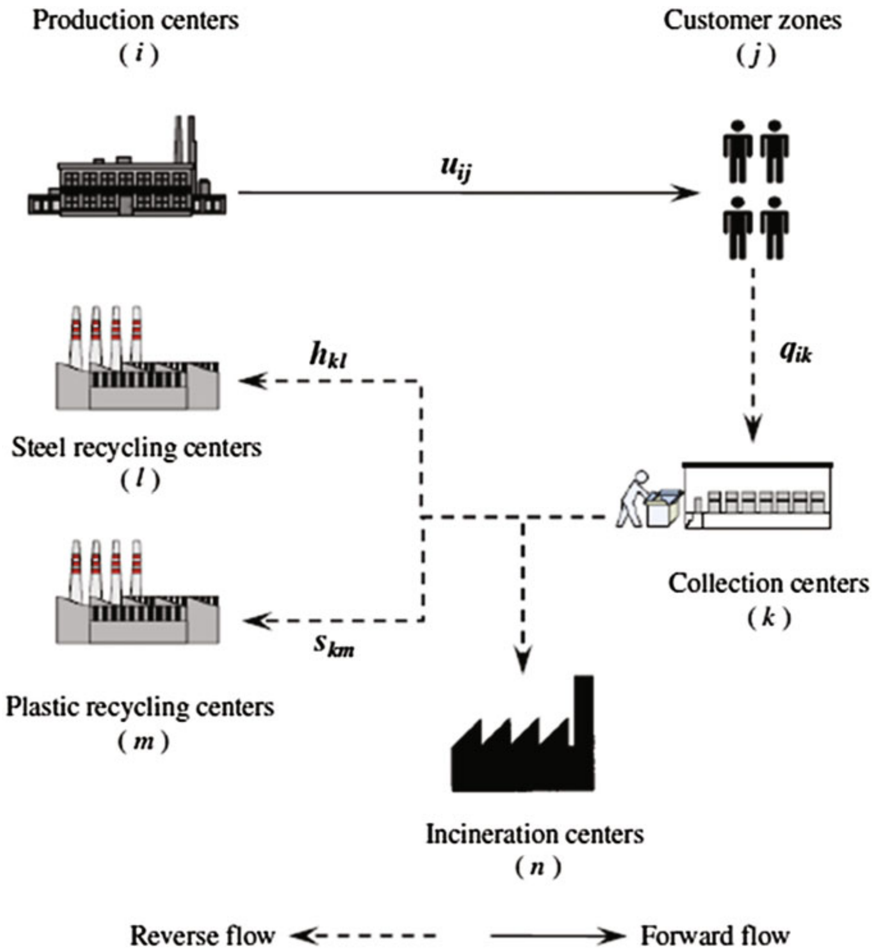

Fig. 2 The structure of the concerned supply chain (adopted from Pishvaee and Razmi 2012)

acceptable feasibility degree (α) ranged from 0.6 to 1, six Pareto-optimal solutions are generated. It should be mentioned that in the proposed method, the satisfaction degree of environmental objective (μ_1) is kept as the objective function of the ε-constraint method and satisfaction degree of cost objective (μ_2) is used as a side constraint.

Solving the discussed model using abovementioned method, one can see that when α-level value increases (in response to uncertainty with higher confidence level), it will lead to increase in values of both objective functions because more resources (raw material, products, transportations, etc.) must be used to fulfill the demand and collection of returned products.

In addition, as it was expected, the two objective functions are in conflict. In fact, the cost-based objective function has a tendency towards designing a centralized network with less total cost while the environmental-based objective function offers a more decentralized network since this structure decreases transportation distances between centers that has less negative environmental impacts. Finally, based on the firm's preference, the decision maker sets minimum acceptable feasible degree (α) equal to 0.9 by which the satisfaction degree for both objectives were selected as $\mu_1 = 0.85$ and $\mu_2 = 0.694$. In this preferred solution, two production centers and five collection centers should be opened.

5 Future Research Directions

Given the current state-of-the-art literature in GrLog and RL areas, there are various avenues for further research among them we refer to the following ones:

- Considering social aspects when designing commercial supply chains is so limited in the current literature. Therefore, to move towards more sustainable supply chain networks, it is necessary to include the social aspects beside the environmental and economical dimensions,
- Integrating tactical and operational planning issues into the current strategic models to broaden the scope of developed models could be another interesting research direction with significant practical relevance,
- It can be realized that some lessons from best practices in commercial supply chains (such as applying Milk-run systems when collecting used products) could be learnt and might be beneficial for reverse logistics,
- Accounting for flexibility in objectives' target values and/or elasticity in soft constraints along with imprecise input data and accordingly developing new mixed flexible-possibilistic approaches to cope with this kind of mixed uncertainty can fill a major methodological gap in this research stream,
- Since most of real life problems are large, and the exact methods can solve only small to moderate sized problem instances, devising tailored solution approaches including heuristics, meta-heuristics or Mat-heuristics (the interoperation of meta-heuristics and mathematical programming techniques) would be of particular interest.

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An Axiomatic Design Approach to the Classification of Reverse Logistics Network Design Studies Under Fuzziness

Didem Cinar, Gül Tekin Temur and Y. Ilker Topcu

Abstract Reverse logistics (RL) is recently receiving much attention because of growing environmental problems. Reusing, recycling and remanufacturing are considered as environmentally and financially effective processes for various drivers—government, corporations and customers. Because many parameters have to be defined by decision makers based on their experiences in RL, a significant number of articles that apply fuzzy set theory have been written in the relevant literature. This paper proposes a model with an axiomatic design (AD) approach on the subject of RL and analyzes the fuzzy set theory used models in relation to steps defined in AD model. AD is applied to generate a conceptual framework for RL network design by distinguishing objectives and means of RL at different levels. The model proposed in this study can be used as a road map for both organizations needing to enhance an existing network, organizations intending to design a network from the beginning and researchers who want to advance in their RL studies by using fuzzy based models.

Keywords Reverse logistics network design · Axiomatic design · Fuzzy logic

1 Introduction

Due to the increasing environmental and economical concerns of government, customers and manufacturers, reverse logistics (RL) is receiving growing attention. Optimization and development of entire supply chain (SC) management is

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starting to be considered in the scope of sustainability. A complete SC comprises product returns during recycling, remanufacturing, or disposal besides forward logistics (FL) activities. Fleischmann et al. (1997) define RL as "a process which encompasses the logistics activities all the way from used products no longer required by the user to products again usable in a market". RL has grown to be seen as crucial service management activities. A well designed and well managed RL can satisfy both customer retention and environmental protection by recycling, remanufacturing, recovery, and procurement (Srivastava 2008). With an appropriate RL network, efficient implementation can be set up for the goods flowing from users to producers (Fleischmann et al. 1997). In the RL literature, a significant number of articles have been written about specific aspects of RL. However, not so many articles consider RL network design holistically (Fleischmann et al. 2000). In this study, an axiomatic design (AD) approach is used to construct a conceptual map for the construction of a RL network. RL processes include many subjective judgments and uncertainty in supply volume and input quality. Fuzzy methods are supportive tools in order to deal with the uncertainty caused as a result of subjective judgments in decision making approach development phases. By using the fuzzy set theory that is firstly introduced by Zadeh (1965), the linguistic terms can be expressed and vagueness and subjectivity that are caused in uncertain nature of RL can be resolved. Since there is a growing RL literature using fuzzy set, it is reasonable to reveal general features of RL studies under fuzziness. The contributions of this study can be mentioned in twofold: (1) to develop a framework to design a RL network, (2) to define usability of fuzzy set theory in RL literature. RL literature review under fuzziness is conducted to highlight the characteristics of the fuzzy set theory using studies for researchers who want to advance in RL research area.

The remainder of this study has been organized as follows: Sect. 2 comprises a brief introduction to define AD. The proposed AD for RL is presented in Sect. 3. Studies on RL under fuzziness are classified using proposed AD model in Sect. 4. Finally, the study is summarized by some concluding remarks in Sect. 5.

2 Axiomatic Design

AD is a way of developing systems, products, software, machines, etc. in a rational and efficient manner while satisfying the functional requirements and constraints. It utilizes the experience and creativity of designers through underlying principles, theories and methodologies (Suh 2001). Developing a scientific basis for design and improving design processes by theoretical foundations is the fundamental aim of AD. The goal of AD is stated by Suh (2001): "to make human designers more creative, to reduce the random search process, to minimize the iterative trial-and-error process, to determine the best design among those proposed, and to endow the computer with creative power through the creation of a scientific base for the design field".

According to Suh (2001) design arises from the relationship between “what we want to achieve” and “how we want to achieve”. The first step of design is to determine the customer needs or attributes. Then, the functional requirements and constraints are determined. Functional requirements (FRs) are the answer to “what”. Once the FRs are decided, the design parameters (DPs) of FRs are chosen. DPs correspond to “how” and should also be selected while considering the constraints. After the DPs are chosen, process variables (PVs) should be identified.

An important advantage of AD is the differentiation between requirements (what) and solutions (how). Another advantage is the decomposition of the problem (Marasco 2008). In order to develop a feasible design, abstract FRs and DPs at the top level should be decomposed to a more detailed level. FRs and DPs can be displayed in the same hierarchy and this representation will be used in this study because of the easily understandable visualization. According to this method, FR at the top level is defined (FR1). Then a DP corresponding to the FR at the top level is found (DP1). Both FR and DPs are decomposed to the lower level. Decomposition continues until it reaches a convenient level of detail. The result of the decomposition is a hierarchy of requirements (or objectives) and solutions (or means) (Marasco 2008).

AD uses axioms which cannot be derived by any theoretical foundation and there are no exceptions or counter-examples. Two axioms are defined to create good design and determine the best design among proposed alternatives. These axioms can be denoted as follows (Schnetzler et al. 2007):

Independence Axiom: FRs must be independent from each other. This means that a DP regarding a FR cannot belong to another FR. This axiom is used to generate an acceptable design.

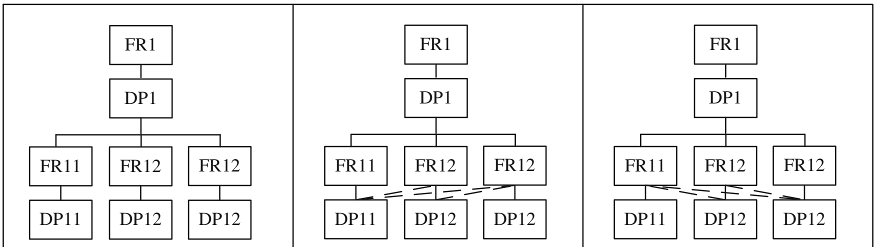
Information Axiom: This axiom is used to choose the best design among the proposed solution alternatives. According to this axiom, the design which has the least information is the best one. This can be described simply as ‘simple is better’.

The relationship between FRs and DPs can be indicated by a design matrix A. In this matrix, an entry with X measures the effect of DP on FR, and an entry with 0 represents the lack of any connection between DP and FR. The representation of this relation can be stated as follows:

$$\mathrm{FR} = \mathrm{A\ DP}$$

DP and FR are defined as vectors while A is a matrix called a design matrix. The design matrix, A, displays both DPs affecting an FR and FRs affected by a DP. An equal number of FRs and DPs and the independence axiom are necessary for an ideal design. The design matrix, A, can indicate the structure of the design and can also show whether the design satisfies the independence axiom or not.

There are three types of designs; uncoupled design, decoupled design and coupled design. An uncoupled design satisfies the independence axiom. In an uncoupled design, the design matrix is diagonal. In other words, there is only one DP corresponding to each FR. In a decoupled design, called partially coupled, the design matrix is upper triangular. Independence of FRs is satisfied by arrangement



$$\begin{bmatrix} FR1 \\ FR2 \\ FR3 \end{bmatrix} = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{bmatrix} \begin{bmatrix} DP1 \\ DP2 \\ DP3 \end{bmatrix} \quad \begin{bmatrix} FR1 \\ FR2 \\ FR3 \end{bmatrix} = \begin{bmatrix} X & 0 & 0 \\ X & X & 0 \\ X & X & X \end{bmatrix} \begin{bmatrix} DP1 \\ DP2 \\ DP3 \end{bmatrix} \quad \begin{bmatrix} FR1 \\ FR2 \\ FR3 \end{bmatrix} = \begin{bmatrix} X & X & X \\ 0 & X & X \\ 0 & 0 & X \end{bmatrix} \begin{bmatrix} DP1 \\ DP2 \\ DP3 \end{bmatrix}$$

Fig. 1 Example of uncoupled, decoupled and coupled design

of DPs in a suitable sequence. In a coupled design, the design matrix is neither diagonal nor upper triangular. Coupled design does not satisfy the independence axiom. Simple design matrices for each type of design structure are given in Fig. 1. Uncoupled design is considered to be an ideal design while coupled design is a poor design.

As mentioned before, AD can be applied to various areas: design of products, machines, software, organizations, systems, materials, manufacturing, etc. (Suh 2001). In this study we contribute to the literature by developing a model that can be utilized to develop a reverse supply chain strategy.

3 Axiomatic Design for Reverse Logistics

In this study we AD is applied to RL in order to construct a road map for network design. The proposed AD model is developed based on the RL literature and judgments of SCM professionals. The proposed methodology can be used for enhancing an existing recovery network or as a guide for companies entering RL because of cost or legislative factors. In AD development, first of all, we need to know the requirements of customers. Both customers paying for produced products and producers paying for recovering products are customers for a RL. Logistics is one of the main cost drivers for producers. Besides cost, environmental factors are also considered for logistic activities. Thus, producers are not only concerned with the economic impact of logistics policies, but also with the wider effects on society. The effect of pollution on the environment is one of the factors that concern producers (Sbihi and Eglese 2007). So, environmentally sensitive logistic design is the highest functional requirement. The design parameter of this functional requirement is “Assure environmentally sensitive logistics system”. This means that in order to get an environmental logistics design, we need to design a RL network. In this structure, the functional requirement is the answer to "what" while the design parameter is the answer to "how". The top level functional requirement and its design parameter are as follows:

FR1: Assure environmentally sensitive logistics system
DP1: Design reverse logistics network

If DP is not clear enough to understand or apply, the designer should continue the decomposition. For this reason, the first decomposition is made and RL network is divided into two phases: "Deliver Return" (FR11) and "Source Return" (FR12) activities as outlined in SCOR (2008). Deliver return contains the returns from customers to distributors or collectors while source return includes the transportations from distributors and collectors to producers, recyclers or suppliers. This partition is used to make the design process easier: "Design deliver return network" (DP11) and "Design the network with respect to product feature" (DP12).

As mentioned before, in the AD approach, there are some axioms that a designer should perform. The independence axiom requires that the independency of functional requirements be satisfied. This means that the design matrix should be uncoupled or decoupled. As seen in Fig. 2, the proposed design is a decoupled design and it satisfies the independence axiom. Decomposition should continue to get a sufficiently detailed model that can be applied and understood. So we go on to decompose the functional requirements (FR11 and FR12). The whole AD model is seen in Fig. 3.

There are two components in the deliver return network: collection and inspection activity. Returned products can be gathered in collection centers, then an inspection activity is undertaken in order to classify the returns and send them as appropriate for reprocessing or disposal. Returned product can be delivered to collection centers by customers. Another alternative is direct collection by distributors or producers. After a decision is made for the collection process, an inspection decision should be made. This means that a decision process is needed about whether the inspection of returned products is made by either collection centers or producers. Based on the decomposition of the deliver return activity, we will have "an appropriate collection process" (FR111), "classification of the used products" (FR112) with an inspection procedure.

The need to state an appropriate collection process is met by choosing "a reliable logistics service" (FR1111). Third-party logistics are widely used alternatives in return product logistics (Marasco 2008). After a proper logistics service is selected in a decision procedure, "a proper collection method" (FR1112) should be determined. The decision procedure for logistics service selection requires "analysis of the system" (FR11111) and "a selection procedure" (FR11112). With a sufficiently detailed system analysis, the objective of the system and the criteria used for selection of an appropriate logistics service are determined. After system analysis, the best logistics service is chosen among alternatives according to the criteria (Jharkharia and Shankar 2007; Meade and Sarkis 2002; Savaskan et al.

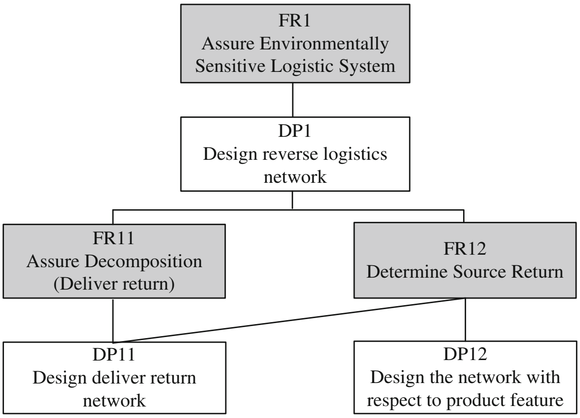


Fig. 2 First decomposition of design

2004). After deciding on a reliable logistics service, a proper collection method is defined. In this phase, "the customers' profile (location, quantity of return products, etc.) is analyzed" (FR11121). According to the customers' features; "location, capacities of collection centers, and routings of vehicles are determined" (FR11122) and "the requirements (labor, vehicle, etc.) are planned" (FR11123) (Alshamrania et al. 2007; Demirel and Gökçen 2008; Lee and Chan 2008; Min and Ko 2008).

As aforementioned, inspection is the second requirement of the return activity. After gathering at collection centers, some types of product (e.g. cellular phones, cameras, etc.) need to be classified because of their various levels of quality. In other words, all products may not be reprocessed in the same way. For example, some components of a cellular phone need refurbishing while other parts are sent for disposal. The first requirement is "the decision of which components or products to inspect" (FR1121). Criteria (quality level, age, etc.) are determined for classification of products. In order to decide where the inspection activity is undertaken, the decision of whether a collection center(s) should be built or not should be made. If a collection center is not established or used, the inspection activity can be made at the retailer or manufacturer. After deciding what products to inspect and where, the required resources are planned. "A developed resource planning system is stated as a design parameter to meet the requirement" (FR1122).

So far, the return activity (collection and inspection) network design has been modeled. In further analysis, the deliver return activity is considered. After collection, reused products are delivered to convenient facilities according to the inspection. So the model is divided into three parts: recycled (FR121), remanufactured (FR122), and reused (FR123) product networks. These network types have different features. The degree of centralization, number of levels, links with other networks, loop structure (open-closed), and degree of branch cooperation are features that are different among these network types (Fleischmann et al. 1997).

Fig. 3 Holistic design of RL network

Recycled products are sent to recycling facilities which may be corporate or private facilities. Recycling network types have uncertainty in supply volume and input quality. Since the investment and technology cost is high, cooperation is the preferred way for recycling. Cooperation also provides high volume of returned products to decrease the operation cost. So the first requirement for designing a recycling network is “to make the design feasible” (FR1211) in terms of cost. A developed procedure for cooperation is held to find the alternative with least cost. Then “the location and the capacity of the facility are determined” (FR1212). The last step for the recycling network is “to plan the requirements” (FR1213) in terms of labor, vehicle, technology etc.

If the return products have components that can be included to produce new products, the current logistics and production systems can be extended to incorporate remanufacturing (Fluente et al. 2008). In assembly product remanufacturing networks, supply uncertainty is a major cost determinant. To incorporate product recovery, the factors that impact production planning and control for closed-loop SCs have to be determined (Guide et al. 2003). “Determination of remanufacturing components” (FR1221), “economic implementation of disassembly” (FR1222), “effective usage of distribution channels” (FR1223) and arrangement of an inventory strategy are requirements of remanufacturing networks. To determine the remanufacturing components, it is necessary to explore the characteristics of remanufactured products such as returns volume, timing, quality, product complexity, test and evaluation complexity, and remanufacturing complexity. A procedure for the location of the disassembly process is applied to carry out the disassembly economically. In closed loop SCs, integration of the FL and RL is the main concern and companies need to determine the elements and functions of a logistics network (Minner 2001). By combining forward and reverse transportation and inventory, “distribution and inventory management will be less costly and more effective” (FR1224). After the design of the entire remanufacturing network, “resource planning” (FR1225) is done according to production, distribution and inventory planning.

The last part of the model is for reusable products such as packaging, containers, etc. The reused products do not need an inspection activity. So the main cost driver in this type of network is the transportation cost. “Integration with the existing transportation system” (FR1231) is the crucial problem which is a decision based on the quantity and routings of the original and reusable products. “To decrease the transportation cost” (FR1232), a lot sizing procedure is applied. Because in a convenient strategy, not only is the demand satisfied in the desired time but also vehicles can be used effectively for transportation of both original and reusable products (Gonzalez-Torre et al. 2004). In order to determine the appropriate vehicle routing and lot size, the demand for the original products and reusable products should be known. Uncertainty in timing and number of returns is the most important problem. Some stochastic approaches can be used to estimate timing and quantity of returns and these estimates can be included in the quantitative analyzing procedure for determining a suitable lot size.

As seen in Fig. 3, in accordance with the design of deliver return activity and product features, three alternative source return network designs are possible to choose. In some sectors, more than one recovery network can be used. The proposed methodology can be used as a road map for organizations needing to enhance an existing recovery network and a guide for companies entering RL because of cost or legislative factors.

4 Reverse Logistics Under Fuzziness

RL studies have been evolving through the addressing the risk and uncertainty in the amount of products and assessment of decision makers. Fuzzy programming tools are utilized in many research papers to deal with uncertain and complex nature of RL processes. As fuzzy programming tools allow considering linguistic terms, it is reasonable to utilize them in RL problems which have inputs that are forecasted based on subjective ideas of experts. Studies using fuzzy set theory in RL network design are seen at Table 1. The studies can be basically classified into three groups according to the product return: (1) source return, (2) deliver return and (3) both deliver and source return.

4.1 Studies on Source Return

The RL activities can be performed by manufacturer, retailer or third-party reverse logistics providers (3PRLP) (Senthil et al. 2012). The organization of return flow needs special infrastructures with considerably high investment cost. In order to increase the efficiency and decrease the total logistics cost, the companies usually outsource their RL activities to the 3PRLP which are the most desirable options to integrate the reverse flow into the forward logistics. The best 3PRLP selection (which can be considered as FR1111) is the most widely studied subject in source return literature. Development of multi criteria decision making (MCDM) process that helps decision makers choose the best alternative is necessary for firms (Senthil et al. 2012). Since there is uncertainty in the objectives of experts, fuzzy modeling is preferred to be used in decision making procedure.

Kannan (2009) adopted fuzzy AHP in order to develop a decision making model that select the best 3PRLP. Efendigil et al. (2008) developed an integrated model based on fuzzy analytic hierarchy process (AHP) and artificial neural network (ANN) for the selection of the best 3PRLP. Fuzzy AHP was used for rating the criteria to obtain the input for ANN which was used for the selection of 3PRLP. Kannan et al. (2009) combined interpretive structural modeling (ISM) and fuzzy technique for order of preference by similarity to ideal solution (TOPSIS) to select the best 3PRLP for a battery manufacturing industry in India. ISM was used to analyze the interaction among criteria while fuzzy TOPSIS is implemented for selection of the most appropriate 3PRLP. Zhang et al. (2012) proposed an evaluation tool for RL servicer selection. The model includes three main steps: index system

Table 1 Studies on RL network design under fuzziness

Study	FR-1111	FR-1112	FR-112	FR-12	FR-121	FR-122	FR-123
Dhouib (2014)				X			
Amin and Zhang (2013)						X	
Datta et al. (2013)	X						
Amin and Zhang (2012)						X	
Mehrbod et al. (2012)					X	X	
Paksoy et al. (2012)						X	
Pishvaee and Razmi (2012)		X			X		
Pochampally and Gupta (2012)		X		X			
Lee et al. (2012)							X
Senthil et al. (2012)	X						
Zhang et al. (2012)	X						
Vahdani et al. (2012a)					X		
Vahdani et al. (2012b)					X	X	
Zarandi et al. (2011)					X		
Karimi et al. (2011)					X		
Sasikumar and Haq (2011)	X				X	X	
Su (2011)		X		X			
Tuzkaya et al. (2011)		X			X	X	
Trappey et al. (2010)	X	X			X	X	
Pishvaee and Torabi (2010)					X	X	
Chu et al. (2010)		X				X	
Qin and Ji (2010)		X					
Sasikumar et al. (2010)						X	
Kannan et al. (2009)	X						
Wadhwa et al. (2009)				X			
Kannan (2009)	X						
Efendigil et al. (2008)	X						
Fernández et al. (2008)				X			
Tuzkaya and Gulsun (2008)					X		

design for servicer selection, determination of index weights by factor analysis and a fuzzy comprehensive evaluation method design for selection of servicers.

Design of collection centers (FR1112) is another subject studied in source return network design. Qin and Ji (2010) proposed a fuzzy programming which is generated under the expected value model, chance constrained programming and dependent-chance programming.

4.2 *Studies on Deliver Return*

One of the important considerations in RL systems is to select the best recovery option (FR12). According to the AD model developed in this study, an appropriate recovery network (recycling, remanufacturing or reusing) should be selected to perform the RL activities efficiently. Fernández et al. (2008) proposed a fuzzy decision support system to analyze recovery policies suggested by different researchers for different products. Pochampally and Gupta (2012) addressed the best recovery policy determination problem with the selection of collection centers. A linear physical programming model was used for the selection and ranking of collection center while Fuzzy Logic and Bayesian Updating were applied to determine whether the repairing an end-of-use product is more reasonable than remanufacturing or recycling. Linguistic rating was utilized to evaluate the components of end-of-use products whether they are more proper to sale after a repair process than remanufacturing or recycling. Dhouib (2014) developed a MCDM tool for evaluating RL options by utilizing a fuzzy version of MACBETH (measuring attractiveness by a categorical based evaluation technique) methodology that considers linguistic parameters. Wadhwa et al. (2009) proposed a fuzzy based flexible MCDM model that considers quantitative and qualitative criteria simultaneously.

In a logistic system, design of the network for determination of the quantity and location of centers/facilities is an essential issue. In RL literature, it is observed that recycling network design (FR121) is one of the most studied subjects. Fuzzy logic has been used to evaluate recycling processes or the facility location problems. Karimi et al. (2011) used fuzzy AHP to determine the best wastewater treatment process and to rank the alternative processes in a real life case. Tuzkaya et al. (2011) developed a decision making system on location evaluation that uses an integrated analytic network process-fuzzy technique. It was shown that this hybrid system gives satisfactory results on decision making systems that have multiple conflicting criteria. Vahdani et al. (2012a) proposed a model for recycling location allocation problem of closed loop logistics systems. In the model a fuzzy mixed nonlinear programming is used in order to have more realistic results under the uncertain and hard nature of closed loop systems. Similarly, Zarandi et al. (2011) used a fuzzy goal programming model which has a dynamic structure between decision maker and model for recycling location allocation problem. Minimization of cost and offering service in satisfactory time were the main goals of the proposed fuzzy programming model.

Remanufacturing network design (FR122) is another popular subject in RL literature. Since the quantity and quality of returned products are always uncertain and difficult to estimate, some parameters have been considered as fuzzy variables for the design of product recovery network. Amin and Zhang (2012) developed an integrated approach optimization of a close loop SC network. They proposed a fuzzy model for the evaluation of external suppliers and a mixed-integer linear programming model, which includes the objective of maximizing profit and weights of suppliers and minimizing the defect rates, to select the suppliers and refurbishing sites and determine the number of parts and products. Combining the supplier selection and order allocation is the main contribution of the paper. Amin and Zhang (2013) extended this study with the consideration of remanufacturing subcontractors and stochastic demand. Fuzzy set theory was used for the assessment of suppliers, remanufacturing subcontractors and refurbishing sites.

Pishvaee and Torabi (2010) developed a possibilistic mixed-integer programming model to determine the location of distribution, collection, recycling and recovery centers and the number of items among plants, retailers and centers for single-product multi-period closed-loop logistics under uncertainty in the demand. The objectives of the model were the minimization of total cost and the minimization of the total tardiness of the delivered products. They proposed a fuzzy approach by combining several approaches from the literature. Mehrbod et al. (2012) addressed the similar problem by considering multi-product multi-period closed-loop logistics. They developed a multi-objective mixed-integer nonlinear programming formulation with the objective of minimization of total cost and the minimization of total delivery and collection time. The model was solved by fuzzy goal programming approach for handling multiple and conflicting objectives.

Paksoy et al. (2012) developed a fuzzy programming model to determine an effective distribution strategy for green closed-loop SC network with the objective of minimizing total cost of the forward and reverse transportation, total CO2 emissions produced by trucks and purchasing costs minus the total opportunity profits. AHP, the fuzzy AHP, and the fuzzy TOPSIS are used and compared for weighting each objective. Sasikumar et al. (2010) proposed a mixed integer nonlinear programming model that maximizes the profit and has the decision variables such as location opening, amount of opening centers and flow amounts of the materials. Vahdani et al. (2012b) introduced an integrated hybrid methodology that incorporates robust optimization approach, queuing theory and fuzzy multi-objective programming in order to design the closed loop logistics systems effectively.

Lee et al. (2012) optimized the reusable RL network by considering time factor besides cost. This method addressed integration of just-in-time system in RL. The developed optimization methodology incorporates priority-based encoding/decoding and the hybrid genetic algorithm (GA) with a fuzzy logic controller and identifies suboptimal delivery routes.

4.3 Studies on Both Source Return and Deliver Return

Various studies have handled the network design problem comprehensively by considering both the source return and deliver return network. Sasikumar and Haq (2011) integrated the deliver return network design with 3PRLP selection process to achieve cost efficiency. A fuzzy MCDM model was proposed for the selection of best 3PRLP. A mixed integer linear programming model was used to determine the amount of raw material, produced, distributed and recycled/remanufactured products for the case of battery industry.

Su (2011) proposed a hybrid fuzzy model to get solution for fuzzy multi-attribute group decision making problems. A modified VIKOR method and a modified gray relational analysis (GRA) method were integrated in the hybrid model and two main RL problems (return center location selection and RL operation selection) from relevant RL literature were considered for implementation. Chu et al. (2010) used a fuzzy-chance constrained programming approach to design of both deliver return and source return. The capacity of a disposal centre and maximum tolerable travelling distance by customers were handled as the fuzzy variables. Customer satisfaction levels were included in the model to analyze the relationship between the total cost and customer satisfaction levels. Tuzkaya et al. (2011) developed RL network design model including two steps: evaluation of return centers and RL network design. A hybrid study incorporating ANP and fuzzy-TOPSIS methodology was used in return center evaluation and GA that uses weights obtained at first step is proposed. Pishvaee and Razmi (2012) proposed a multi-objective fuzzy mathematical programming model to determine the numbers and locations of production and collection centers, the end-of-life options and the material flow quantities between different facilities. The efficiency and applicability of the developed model was demonstrated with a real industrial case with the aim of minimizing the total cost and environmental impact.

Technology is one of significant requirements in RL system development. RFID is used for gathering actual data in operations. Trappey et al. (2010) aimed to evaluate changing in system performance when RFID is activated into the RL system. A hybrid model including both of qualitative and quantitative approach was developed by utilizing fuzzy cognitive maps and GA. Because the causal relationships between parameters are linguistic, fuzzy cognitive maps was used in the decision making model.

5 Conclusions and Further Suggestions

Especially with global warming, countries are more and more concerned about climate and tend to produce cleaner energy and consume less energy. Since logistics activities are important cost drivers for companies and the environmental sensitivity of consumers is increasing, green logistics are becoming popular. This study focuses on one of the main subjects of green logistics, called reverse logistics, because of its positive effect on the environment and customer expectations. This study contributes to the literature by modeling a RL network design in a logical and conceptual methodology and addresses the main characteristics of fuzzy theory set used studies in RL literature.

Proposed model is a system of objectives and means that is designed in a systematic manner according to the principles of AD. The model can be used for RL network design with consideration of strategy, situation, and context of a company. In this way, objectives and measures of operational RL can be defined and positioned to satisfy optimal support to the strategy. In addition, the methodology provides a procedure for the development and improvement of a RL network. The proposed model satisfies both general and systematic views to construct a new network or evaluate an existing network.

RL systems have some decision mechanisms that are developed with subjective expressions. Because of this reason, fuzzy set theory is one of commonly used tool in RL problem area. In this study, RL literature under fuzziness is reviewed in relation to proposed holistic road map defined by AD. Although it cannot be claimed that this study is the most exhaustive analysis on this field, it helps to stress some important characteristics of RL studies under fuzziness. In accordance with the results, we can highlight that the most popular RL problems that are solved under fuzziness are (1) to decide reliable logistics service (FR1111), (2) to determine the proper collection method (FR1112), (3) to manage recycling network (FR121) and (4) to manage remanufacturing network (FR122). This result is caused from the reality that the structure of service provider and collection method selection and network design models can not be known with certainty and all these four decisions include subjective opinion based solutions. Therefore, preference of fuzzy programming tools in these problems is compatible with our expectations. Secondly, it is observed that product classification problems (FR-112) have not considered in fuzzy studies yet. In product classification studies, it can be assumed that the structure of the product classification problem is known with certainty or another method is used in the methodology.

As a further study, it can be analyzed that what kind of solution methods are proposed in that kind of problem area and a solution method under fuzziness can be developed and the results can be compared in order to reveal the performance of the methods. The proposed AD structure can also be validated with quantitative results obtained by computational tests in further studies. Through the applications, the gaps and concepts of the model can be defined and improved. For specific applications, the focus can be placed on a particular area of the model, such as recycling, vehicle routing, inventory management, etc., and the model can be improved in detail.

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Green Supply Chain Technology: A Comprehensive Evaluation and Justification Multiattribute Decision Modeling Approach

Chunguang Bai and Joseph Sarkis

Abstract Green and sustainable supply chain activities are implemented as organizations and their partners respond to environmental pressures. Many of these activities require supporting technologies to help meet their goals. These technologies may include product, process, or organizational technologies that can help in planning and managing the activities. The evaluation of these technologies is not always a trivial approach that is based only on cost. Multiple factors and attributes come are considered in such situations. Thus, tools and models to help in green supply chain management technology evaluation and justification decisions can prove valuable. Using regular, grey, and fuzzy numbers within a TOPSIS methodology we seek to address this issue. Data is evaluated with an illustrative illustration to exemplify the utility of this approach. Insights for the reader are also presented in this Chapter.

Keywords Green supply chain technology · Grey number · Fuzzy number · TOPSIS

1 Introduction

Decisions in sustainable supply chains and reverse logistics are necessarily complex because they incorporate the management of multiple dimensions for strategic decision making. Complexities increase in these decisions because additional

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dimensions such as multiple decision makers, functions, and sometimes organizations are involved (Sarkis 2012). This strategic complexity includes variations in factors and attributes. Some of these factors and attributes are tangible and easily quantifiable with ease of aggregation. There are also factors and attributes that have intangible characteristics requiring subjective and perceptual input (Presley and Sarkis 1994). Integrating these factors into a logical decision modeling framework that is effectively implementable by managers, analysts, decision makers and analysts is far from a trivial exercise. This type of contextual environment is what motivates the modeling effort in this chapter. The importance of decision modeling for organizations revolves around making the business case for green supply chain technologies and developments (Presley et al. 2007).

Using a green supply chain management technology selection decision, we model this environment using some recent developments in multiple attribute decision making. Many of these developments require the utilization and application of fuzzy and fuzzy-like (grey) theoretic approaches. We integrate these mathematical logic approaches with a generalized framework based on the TOPSIS decision modeling approach.

Thus, we provide some details on the integration of these solution dimensions to a practical greening and sustainability issue around green technology development. Green technological developments, especially those that focus on greening of supply chains and reverse logistics networks are important from a social, environmental, and economic perspective. The dimensions of influence will include operational and strategic elements. We will consider various new performance developments and metrics that would include various performance metrics that have typically been ignored by organizational business cases seeking to justify these technologies.

The important theoretical driver behind justifying these green technologies is based on ecological modernization theory (Zhu et al. 2011; Sarkis and Cordeiro 2012). Ecological modernization theory states that organizations and societies can garner 'win–win' outcomes improving simultaneously on economic and environmental dimensions. An addition of social improvements makes ecological modernization an even more powerful ideological stance. Thus, technology can be seen to advance competition on numerous dimensions. Although our goal is not to advance this theoretical philosophical ideology, we do think there is merit in including some of its win–win competitive ideals into organizational decision making and strategic planning activities.

In order to solve the complex hybrid multiple attribute decision-making problems of combining the various dimensions of definite number, grey number and fuzzy numbers with the increasing, decreasing and constant numerical values, a model for hybrid multiple attribute decision-making will be established based on TOPSIS method, grey system theory, fuzzy number and membership functions. The calculation examples for the justification of green supply chain technologies will be provided.

Both validity and feasibility of the model will be evaluated with some discussion of actual practical implications. An illustrative example will be the vehicle for development within this chapter. We expect the results to at least provide a ranking/quantitative ordering of candidate technological choices.

2 Background

2.1 Green Supply Chain Processes and Technology

Green and sustainable supply chain management (GSCM) has gained increasing attention by organizations and their supply chains for many reasons. GSCM can provide competitive advantages, reduce organizational risk, improve corporate image, and allow for organizations with the basic right to operate in a market (Sarkis 2009).

GSCM as a strategic and operational organizational construct has been accepted in the literature for a number of years (see Sarkis 1998; Sarkis 1999; Sarkis 2003). The elements of GSCM have varied over the years and are dependent on where the boundaries of the supply chain are drawn. They can include upstream, downstream external activities, and internal operational activities. GSCM has also included reverse logistics activities that can 'close-the-loop' (Bai et al. 2012).

The processes and activities in GSCM have been well developed and delineated and could include green purchasing, supplier development, ecodesign, cleaner production, green marketing, green distribution, investment recovery and reverse logistics processes (Sarkis 1995; Zhu and Sarkis 2004). Each of these process and activity dimensions of GSCM can be substantially influenced, usually in a positive way, through technology investments (Klassen and Vachon 2009). But, usually these technology investments are strategic in nature because of their influence on major portions of the supply chain which would include intra- and inter-organizational functions.

The linkage between technologies in general, and environmental technologies in particular, between supply chain practices (both upstream and downstream) has been established (Vachon 2007). Environmental technologies can include pollution prevention, pollution control, and environmental management systems (Vachon 2007), where some of these technologies have been viewed as complements to GSCM practices, as well (Darnall et al. 2008). The argument has even been made that investment by plants in environmental technologies cannot be made independently of other organisations in the supply chain (Vachon and Klassen 2007).

Example technologies could be categorized in many ways. The pollution prevention, pollution control, and environmental management systems technology can be a good generalized set of categories. But, there are also categorizations in terms of the type of environmental media that is influenced such as air, water, and/or solid waste technologies. Material, product, process, and information technology categorizations can also be utilized. Sectoral technologies may also be used for categorization such as pulp and paper, electronics, automotive, chemical, or construction environmental technologies. Another set may focus on the function within the supply chain such as distribution technologies (e.g. green vehicles); design technologies (e.g. material, life cycle analysis, and information support tools technology); clean production (e.g. closed loop production systems, energy efficiencies); purchasing technologies (databases of suppliers and decision tools); reverse logistics technologies (e.g. sorting and recycling), are all examples. The groupings are not a major concern for us at this time, since there are significant overlaps in the technology categorizations and characteristics. But, the evaluation of each of these types of technology is not necessarily a uni-dimensional (e.g. cost) decision, but can incorporate many intangible and other tangible factors.

Given this initial background, we now evaluate how technology investments and decisions are made in general, and how these techniques and newer techniques can play a role in more effective evaluation of GSCM and environmental technologies.

2.2 *Strategic Technological Investment Justification and Needs from a Greening and Sustainability Perspective*

Payback, return on investment and discounted cash flow techniques are traditional techniques for investment justification for organizations. Their limitations are that they are short sighted and are poor instruments for taking intangible and strategic factors into consideration (Lefley and Sarkis 1997). Further developments in capital appraisal and justification approaches for strategic technologies, such as those associated with clean and sustainability technologies are needed (Presley et al. 2007).

Strategic decision-making requires comprehensive evaluations, and the capability of integrating and aggregating multiple dimensions of complex decisions. The strategic justification literature has supported the application of multi-criteria (multi-dimensional) evaluations (the tool de préférence) (Sarkis and Sundarraj 2000).

Models and tools developed for strategic and sustainable projects evaluation include utilization of multiattribute utility theory including TOPSIS, AHP and ANP (Kalbar et al. 2012); (Sarkis 1998; Sarkis et al. 2012), activity based costing (Presley and Sarkis 1994), data envelopment analysis (Sarkis 1999; Bai and Sarkis 2012), (Sarkis and Weinrach 2001), outranking (Khalili and Duecker 2012), and real options analysis (Sarkis and Tamarkin 2005), and rough set theory (Bai and Sarkis 2010).

Few of these tools, however, explicitly introduce additional sustainability concerns such as social sustainability, into their analyses, with some recent advances in this direction (e.g. Bai and Sarkis 2011) at the organizational level. At the individual corporate (or supply chain) level, study of strategic sustainability justification and evaluation tools is, at best, limited (Presley et al. 2007).

Tools to aid in strategic evaluation and justification from a sustainability perspective are still limited, especially flexible and practical tools. Many of these may use more than one methodology. Focus on green supply chain technologies and their justification, strategically or otherwise, are virtually non-existent (see Seuring 2013, for a recent review).

We expand on the research in this field by introducing a multi-stage approach that uniquely integrates some of the broadest characteristics of input data to help decision makers arrive at a strategic decision focusing on sustainability. We now define the various analytical elements and the methodology.

3 Development and General Background of Methodology and Approaches

3.1 Grey System Theory

Grey system theory can be used to solve uncertainty problems in cases with discrete data and incomplete information (Deng 1989). The major advantage is that it can generate satisfactory outcomes using a relatively small amount of data or with great variability in factors (Li et al. 1997). Grey system theory is an approach for analysis and modeling of systems with limited and incomplete information, and which may exhibit random uncertainty. Grey system theory has many successful applications, in areas such as supply chain management, economics, agriculture, medicine, geography, and disasters.

We will now introduce some general notation and operations within grey system that will be applied in our methodology.

Definition 1: Let x denote a closed and bounded set of real numbers. A grey number, $\otimes x$, is defined as an interval with known upper and lower bounds but unknown distribution information for x (Deng 1982; Deng 1988; Huang et al. 1995). That is, $\otimes x = [\underline{\otimes} x, \bar{\otimes} x] = [x' \in x | \underline{\otimes} x \leq x' \leq \bar{\otimes} x]$ where $\underline{\otimes} x$ and $\bar{\otimes} x$ are the lower and upper bounds of $\otimes x$, respectively.

Definition 2: Let $\otimes x_1 = [\underline{x}_1, \bar{x}_1]$ and $\otimes x_2 = [\underline{x}_2, \bar{x}_2]$ be two grey numbers. Generally, some basic grey number mathematical operations are represented by the following relationships (expressions 1–4):

$$\otimes x_1 + \otimes x_2 = [\underline{x}_1 + \underline{x}_2, \bar{x}_1 + \bar{x}_2] \tag{1}$$

$$\otimes x_1 - \otimes x_2 = [\underline{x}_1 - \bar{x}_2, \bar{x}_1 - \underline{x}_2] \tag{2}$$

$$\otimes x_1 \times \otimes x_2 = [\min(\underline{x}_1\underline{x}_2, \underline{x}_1\bar{x}_2, \bar{x}_1\underline{x}_2, \bar{x}_1\bar{x}_2),\ \max(\underline{x}_1\underline{x}_2, \underline{x}_1\bar{x}_2, \bar{x}_1\underline{x}_2, \bar{x}_1\bar{x}_2)] \tag{3}$$

$$\otimes x_1 \div \otimes x_2 = [\underline{x}_1, \bar{x}_1] \times \left[\frac{1}{\underline{x}_2}, \frac{1}{\bar{x}_2}\right] \tag{4}$$

Definition 3: Let the distance measure of two grey numbers be Minkowski space distance which is represented in expression (5).

$$L(\otimes x_1, \otimes x_2) = [\frac{1}{2}((\underline{x}_1 - \underline{x}_2)^p + (\bar{x}_1 - \bar{x}_2)^p)]^{\frac{1}{p}} \tag{5}$$

where p is some exponential power, in our application $p = 2$ (quadratic power).

These distance and basic operations will be necessary in the specific relationship evaluations for the TOPSIS methodology which is further defined later in the case illustration.

3.2 Triangular Fuzzy Number

A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. The most commonly used fuzzy numbers are triangular fuzzy numbers. We now briefly introduce some basic definitions of the triangular fuzzy number function. Definition 4 is the definition of triangular fuzzy number. Definition 5 describes the basic operational laws of triangular fuzzy numbers. Definition 6 is the triangular fuzzy number distance measure.

These basic definitions will be used in next section, especially the distance measure will be applied in our TOPSIS method.

Definition 4: A triangular fuzzy number $\tilde{x}$ can be defined by a triplet (x_l, x_m, x_u). The membership function is defined as (Dubois and Prade 1980), depicted as in Fig. 1.

$$\mu_{\tilde{x}}(x) = \begin{cases} (x - x_l)/(x_m - x_l), & x_l \leq x < x_m \\ 1, & x = x_m \\ (x_u - x)/(x_u - x_m), & x_m < x \leq x_u \\ 0, & otherwise \end{cases} \tag{6}$$

where $x_l \leq x_m \leq x_u$, and x_l and x_u are the lower and upper bounds of $\tilde{x}$, respectively. x_m is the mean of $\tilde{x}$.

Obviously, if $x_l = x_m = x_u$ then the triangular fuzzy number $\tilde{x}$ is reduced to a real number. Conversely, real numbers are easily rewritten as triangular fuzzy numbers. Thus, the triangular fuzzy number can be flexible to represent various semantics of uncertainty (Li 2012). The triangular fuzzy number is based on a three-value judgment: the minimum possible value x_l, the most possible value x_m and the maximum possible value x_u.

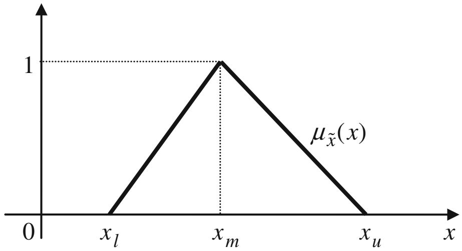

Fig. 1 A triangular fuzzy number$\tilde{x}$

Definition 5: Let $\tilde{x}^1 = (x_l^1, x_m^1, x_u^1)$ and $\tilde{x}^2 = (x_l^2, x_m^2, x_u^2)$be two triangular fuzzy numbers. The triangular fuzzy number mathematical operations are defined as (Yu and Hu 2010).

$$\tilde{x}^1 \oplus \tilde{x}^2 = (x_l^1 + x_l^2, x_m^1 + x_m^2, x_u^1 + x_u^2) \tag{7}$$

$$\tilde{x}^1 \otimes \tilde{x}^2 = (x_l^1 x_l^2, x_m^1 x_m^2, x_u^1 x_u^2) \tag{8}$$

$$\frac{\tilde{x}^1}{\tilde{x}^2} = (\frac{x_l^1}{x_l^2}, \frac{x_m^1}{x_m^2}, \frac{x_u^1}{x_u^2}) \tag{9}$$

$$\lambda \times \tilde{x} = (\lambda \times x_l, \lambda \times x_m, \lambda \times x_u), \lambda \geq 0, \lambda \in R \tag{10}$$

Definition 6: Let the distance measure of two triangular fuzzy numbers be Euclid space distance which is represented in expression (11) (Chen 2000).

$$L(\tilde{x}^1, \tilde{x}^2) = \sqrt{1/3((x_l^1 - x_l^2)^2 + (x_m^1 - x_m^2)^2 + (x_u^1 - x_u^2)^2)} \tag{11}$$

3.3 *TOPSIS Method*

The TOPSIS (technique for order preference by similarity to an ideal solution) method takes into consideration how an object performs on the basis of multiple critieria. TOPSIS is a multiple criteria method to identify alternative that it should have the shortest distance from the ideal solution and the farthest distance from the negative-ideal solution (Hwang and Yoon 1981; Chen and Hwang 1992). This method has been widely applied in the literature (Chen and Tzeng 2004; Opricovica and Tzeng 2004; Krohling and Campanharo 2011).

The ideal solution is a solution that maximizes the "benefit" criteria (criteria which improve as they increase in value) and minimizes the cost criteria (criteria which improve as they decrease in value), whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria. We now provide some definitions to help us set additional foundation for the methodology.

Definition 7: Let $S = (U, C, V, f)$ be an "information system" where U is the universe, and C is decision factor sets for U; $V = \bigcup_{a \in C} V_a$ indicates the factor range of factor a; $f : U \times C \rightarrow V$ is an information function, that is for $\forall x \in U$ if $a \in C$ then $f(x, a) \in V_a$.

The TOPSIS can be expressed using the following steps:

(1) Normalize the decision matrix $U = (x_{ij})_{n \times m}$ using expression (12):

$$v_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{n} x_{kj}^2}}, \quad i = 1, \ldots, n; j = 1, \ldots, m \tag{12}$$

But this expression is not applicable for grey numbers and the triangular fuzzy numbers. So in our model, we will develop a membership function transformation (see expressions 18–26) to map all data types over the same range of "0–1" and complete this process.

(2) Determine the ideal and negative-ideal solution.

$$\begin{aligned} S^+ &= \{v_1^+, \ldots, v_m^+\} \\ &= \{(\max_i v_{ij} | j \in I), (\min_i v_{ij} | j \in J)\} \end{aligned} \tag{13}$$

$$\begin{aligned} S^- &= \{v_1^-, \ldots, v_m^-\} \\ &= \{(\min_i v_{ij} | j \in I), (\max_i v_{ij} | j \in J)\} \end{aligned} \tag{14}$$

where I is associated with benefit criteria, and J is associated with cost criteria.

(3) Calculate the separation measures using the n-dimensional Euclidian space distance. The separation of each alternative from the ideal solution is given as

$$\mu_i^+ = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^+)^2}, i = 1, \ldots, n. \tag{15}$$

Similarly, the separation from the negative-ideal solution is given as

$$\mu_i^- = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j^-)^2}, i = 1, \ldots, n. \tag{16}$$

(4) Calculate the relative closeness to the ideal solution. The relative closeness of the alternative S_i with respect to S^+ is defined as

$$T_i = \frac{\mu_i^-}{\mu_i^+ + \mu_i^-} \tag{17}$$

(5) Rank the preference order. The larger the value of T_i, the better the alternative S_i. The best alternative is the one with the greatest relative closeness to the ideal solution. Alternatives can be ranked in decreasing order using this index (Opricovica and Tzeng 2004).

4 Methodological Exposition and Illustrative Application

A hybrid multiple attribute decision making model based on TOPSIS, classical grey system theory and fuzzy triangular scores is now presented. This model is constructed using regular, fuzzy, and grey number indicators. The indicators have increasing, decreasing and peaked (target) number characterizations. This numerical set and methodology will be applied to a green supply chain technology decision.

The methodology is composed of seven steps. The seven steps include (1) Populating the Information Decision System Table; (2) Normalizing the Information Decision System; (3) Determining the overall factor weight level; (4) Determining the overall final factor value; (5) Determining the ideal and negative-ideal solution; (6) Calculating the separation measures; and (7) Ranking by calculating the relative closeness to the ideal solution.

We shall go through each of these steps in detail with methodological operations and selected results explicitly presented with the case situation and data.

4.1 The Illustrative Case Situation

Let us assume that a company is seeking to integrate a new green supply chain technology, specifically green transportation technology, and requiring this of itself and/or its suppliers. This green transportation technology (e.g. hybrid vehicle/electric vehicle) requires that various aspects of environmental, economic/business and social factors be taken into consideration. For the purposes of minimizing the exposition only a listing of potential factors for this decision are shown in Table 1, with the value type of the indicator measure identified. One quick note, we put a 'peaked' on the recycling measure instead of 'increasing' given the sometimes negative connotations associated with recycled material or because of safety reasons. Most of the other factors are self-explanatory. Many other criteria can be included and are left off for the example case. In each grouping we have provided a different type of criterion valuation.

Table 1 Decision factors, measures, and measure types for the illustrative case example

Abbreviation	Description	Measure type
Economic/business measures		
NPC	Net present cost (dollars)	Decreasing(D)
FlexPerf	Flexibility of vehicle (VL-VH)	Increasing(I)
SCCT	Maintenance (reliability) rates (visits per year)	Decreasing(D)
ComDate	Date of delivery (scheduled date)	Peaked(P)
Environmental measures		
WstGen	Amount of fuel used (miles per gallon/charge)	Increasing(I)
Emiss	Carbon emissions levels (emissions/mile)	Decreasing(D)
ProdRec	Recycled material in vehicles (60 %)	Peaked(P)
Social measures		
Safe	Safety of vehicles (VL-VH)	Increasing(I)
ExPop	Community complaints about vehicles (number)	Decreasing(D)
Aesthetics	Perceived aesthetics (nice, but not too nice)	Peaked(P)

In our scenario a number of identified green transportation alternatives will be evaluated and ranked using this methodology. In this situation, we assume that there are three vendors each with three potential models from which one is to be selected. That is, nine green transportation vehicle alternatives exist. The steps with some details illustrated are used to describe the methodology.

Step 1: Populate the Information System Table.

First, let us define the information system table for the illustrative application. This table is defined by $T = (U, C, V, f)$, where $U = \{S_i, i = 1, 2, \ldots, n\}$ is a set of n alternative objects called the universe. $C = \{c_j, j = 1, 2, 3, \ldots, m\}$ is a set of m decision factors, and usually contain three types of data: the incremental value (Increasing), regressive value (Decreasing), the target value (Peaked). Where $f : U \times C \rightarrow V$ are the functions used to define the set of values V for each decision factor. For this illustrative case $U = \{S_i, i = 1, 2, \ldots, 9\}$ with nine green transportation vehicle alternatives, and ten decision factors $C = \{c_j, j = 1, 2, 3, \ldots, 10\}$ each. The decision factors represent the three various aspects of environmental, economic/business and social factors for sustainability. There are four economic/business decision factors, NPC, FlexPerf, SCCT, and ComDate; three environmental factors WstGen, Emiss, and ProdRec; and three social factors Safe, ExPop, and Aesthetics.

The value ranges and illustrative data for each factor are shown in Table 2. Note that some of these raw values are in crisp (regular) form, some in range (grey) form, and some are based on qualitative (to be changed to fuzzy) judgments. Each of these types of valuations will be normalized.

Step 2: Normalize the Information Decision System.

For consistency in the evaluations we introduce a normalization procedure so that metric values for each decision factor and all the later calculations, e.g.

Table 2 Evaluation of vehicle alternatives on factors by decision makers

Vehicle alternatives	Economic/business measures				Environmental measures			Social measures		
	NPC (D)	FlexPerf (I)	SCCT (D)	ComDate (P)	WstGen (I)	Emiss (D)	ProdRec (P)	Safe (I)	ExPop (D)	Aesthetics (P)
M1V1	[12,15]	VH	[6,7]	[45 days,60 days]	5	[11,14]	[4,8]	H	[10,15]	M
M1V2	[18,21]	H	[1,3]	[31 days,36 days]	7	[22,29]	[23,28]	VH	[4,8]	VH
M1V3	[34,35]	VH	[4,5]	[17 days,20 days]	19	[13,16]	[20,23]	MH	[11,15]	H
M2V1	[26,29]	L	[9,10]	[52 days,55 days]	13	[15,20]	[16,19]	VH	[9,17]	MH
M2V2	[22,27]	M	[3,4]	[39 days,39 days]	27	[31,35]	[10,12]	M	[23,28]	M
M2V3	[13,15]	MH	[3,4]	[50 days,50 days]	12	[14,20]	[11,13]	VL	[37,44]	L
M3V1	[9,11]	ML	[2,3]	[37 days,39 days]	15	[12,18]	[10,11]	L	[13,17]	M
M3V2	[21,27]	M	[5,8]	[39 days,45 days]	10	[9,15]	[16,19]	ML	[16,19]	ML
M3V3	[15,19]	VL	[8,9]	[45 days,49 days]	25	[18,27]	[20,23]	H	[30,35]	VH
Range	5–40	VL-VH	1–10	10 days-60 days	1–30	1–40	1–30	VL-VH	1–50	VL-VH
(Goal)	5	VH	1	30 days	30	1	15	VH	1	MH

Table 3 The linguistic variables and their corresponding triangle fuzzy numbers

Linguistic variables	Triangular fuzzy numbers
Very low (0)	(0,0,0.1)
Low (1)	(0,0.1,0.3)
Medium low (2)	(0.1,0.3,0.5)
Medium (3)	(0.3,0.5,0.7)
Medium high (4)	(0.5,0.7,0.9)
High (5)	(0.7,0.9,1.0)
Very high (6)	(0.9,1.0,1.0)

distance measures, are using similar scales. This normalization will adjust all the decision factor values for each alternative (x_{ij}) to be $0 \le x_{ij} \le 1$.

SubStep1: Transform linguistic (qualitative) variables into fuzzy numbers.

For intangible, textual or qualitative, evaluations, we introduce a fuzzy numerical scale table that would correspond to the qualitative values that would be chosen by decision makers. In this illustrative study seven linguistic variables, namely "very low", "low", "medium low", "medium", "medium high", "high" and "very high", are used to assess the level of performance criteria and which will be expressed in triangular fuzzy numbers. The fuzzy based transformation values for these qualitative ranges are shown in Table 3. Using Table 3, we transform the qualitative variables and natural language variables into triangular fuzzy numbers. Note that all these values are already normalized. For the illustrative example it is assumed that the fuzzy base scales are the same for all the decision factors. FlexPerf is an example of a qualitatively valued factor thus the transformation to a triangular fuzzy number for this factor is given as: VH = (0.9,1.0,1.0).

The next sub-step is to normalize all the numeric values. To complete this sub-step we rely on a membership function.

SubStep2: Normalize Numeric variables by membership function.

Each decision factor has a different maximum and minimum value which can be determined from a historical perspective. If the traditional normalization procedure (such as expression (12)) is used grey and triangular fuzzy numbers cannot be normalized. To solve this problem, we introduce a membership function (expressions 18–26). This normalization allows the decision factor values (x_{ij}) to be transformed to regular (crisp) numbers on a scale of $0 \le x_{ij} \le 1$, for grey numbers based with $[0,0] \le \otimes x_{ij} \le [1,1]$ and fuzzy numbers based with $(0,0,0) \le \tilde{x}_{ij} \le (1,1,1)$. The incremental value, regressive value and target value are each transformed into an incremental value.

1. Normalization of the incremental (increasing) value of regular (crisp) number (expression (18)), grey number (expression (19)) and fuzzy number (expression (20)) membership function:

$$U(x) = \begin{cases} 0 & \textit{if } x \leq Lower, \\ y & \textit{if } Lower \leq x \leq Upper, \\ & \quad where\ y = \frac{x - Lower}{Upper - Lower} \\ 1 & \textit{if } x \geq Upper, \end{cases} \tag{18}$$

where x is the specific evaluation value, *Lower* is the minimum historical value, and *Upper* is the maximum historical value for a factor.

$$U(x) = \begin{cases} [0,0] & \textit{if } x \leq Lower, \\ [\underline{y}, \bar{y}] & \textit{if } Lower \leq x \leq Upper, \\ & where\ \underline{y} = \frac{\underline{x} - Lower}{Upper - Lower}, \bar{y} = \frac{\bar{x} - Lower}{Upper - Lower}, \\ [1,1] & \textit{if } x \geq Upper, \end{cases} \tag{19}$$

$$U(x) = \begin{cases} (0,0,0) & \textit{if } x \leq Lower, \\ (y_l, y_m, y_u) & \textit{if } Lower \leq x \leq Upper, \\ & where\ y_l = \frac{x_l - Lower}{Upper - Lower}, y_m = \frac{x_m - Lower}{Upper - Lower}, \\ & y_u = \frac{x_u - Lower}{Upper - Lower} \\ (1,1,1) & \textit{if } x \geq Upper, \end{cases} \tag{20}$$

2. The regressive (decreasing) value of regular number (expression (21)), grey number (expression (22)) and fuzzy number (expression (23)) membership function:

$$U(x) = \begin{cases} 1 & \textit{if } x \leq Lower, \\ z & \textit{if } Lower \leq x \leq Upper, \\ & where\ z = \frac{Upper - x}{Upper - Lower} \\ 0 & \textit{if } x \geq Upper, \end{cases} \tag{21}$$

$$U(x) = \begin{cases} [1,1] & \textit{if } x \leq Lower, \\ [\underline{z}, \bar{z}] & \textit{if } Lower \leq x \leq Upper, \\ & where\ \underline{z} = \frac{Upper - \bar{x}}{Upper - Lower}, \bar{z} = \frac{Upper - \underline{x}}{Upper - Lower}, \\ [0,0] & \textit{if } x \geq Upper, \end{cases} \tag{22}$$

$$U(x) = \begin{cases} (1,1,1) & \textit{if } x \leq Lower, \\ (z_l, z_m, z_u) & \textit{if } Lower \leq x \leq Upper, \\ & where z_l = \frac{Upper - x_u}{Upper - Lower}, z_m = \frac{Upper - x_m}{Upper - Lower}, \\ & z_u = \frac{Upper - x_l}{Upper - Lower} \\ (0,0,0) & \textit{if } x \geq Upper, \end{cases} \tag{23}$$

3. The target (peaked) value for regular number (expression (24)), grey number (expression (25)) and fuzzy number (expression (26)) membership functions:

$$U(x) = \begin{cases} 0 & \text{if } x \le Lower, \\ y & \text{if } Lower \le x \le Target, \\ 1 & \text{if } x = Target, \\ z & \text{if } Target \le x \le Upper, \\ 0 & \text{if } x \ge Upper, \end{cases} \tag{24}$$

$$U(x) = \begin{cases} [0,0] & \text{if } x \le Lower, \\ [\underline{y},\bar{y}] & \text{if } Lower \le x \le Target, \\ [1,1] & \text{if } x = Target, \\ [\underline{z},\bar{z}] & \text{if } Target \le x \le Upper, \\ [0,0] & \text{if } x \ge Upper, \end{cases} \tag{25}$$

$$U(x) = \begin{cases} (0,0,0) & \text{if } x \le Lower, \\ (y_l, y_m, y_u) & \text{if } Lower \le x \le Target, \\ (1,1,1) & \text{if } x = Target, \\ (z_l, z_m, z_u) & \text{if } Target \le x \le Upper, \\ (0,0,0) & \text{if } x \ge Upper, \end{cases} \tag{26}$$

where y, z,$[\underline{y},\bar{y}]$, $[\underline{z},\bar{z}]$, (y_l, y_m, y_u), (z_l, z_m, z_u) is the membership function calculated by reference expressions (18–23).

For each of the green transportation vehicle alternatives and decision factor metric types (increasing, decreasing, target) that were exemplified in the above sub-step an example calculation and results using expressions (18–26) is shown.

$$M1V1_{\text{NPC}} = [12, 15],$$
$$\underline{v}_{11} = \frac{|x_1^{\max} - \bar{x}_{11}|}{|x_1^{\max} - x_1^{\min}|} = \frac{|40-15|}{|40-5|} = 0.71;$$
$$\bar{v}_{11} = \frac{|x_1^{\max} - \underline{x}_{11}|}{|x_1^{\max} - x_1^{\min}|} = \frac{|40-12|}{|40-5|} = 0.80.$$

Thus, the normalized value for grey number $M1V1_{\text{NPC}} = [0.71, 0.80]$.

$$M1V1_{\text{WstGen}} = 5,$$
$$v_{51} = \frac{|x_{51} - x_5^{\min}|}{|x_5^{\max} - x_5^{\min}|} = \frac{|5-1|}{|30-1|} = 0.14.$$

Thus, the normalized value for regular number $M1V1_{\text{WstGen}} = 0.14$.

$$M1V1_{\text{Aesthetics}} = \text{ M } = (0.3,\ 0.5,\ 0.7)\,,$$
$$v^{u}_{10,1} = \frac{|x^{u}_{10,1} - x^{\min}_{10}|}{|x^{\text{target}}_{10} - x^{\min}_{10}|} = \frac{|0.3-0|}{|0.7-0|} = 0.43;$$
$$v^{m}_{10,1} = \frac{|x^{m}_{10,1} - x^{\min}_{10}|}{|x^{\max}_{10} - x^{\min}_{10}|} = \frac{|0.5-0|}{|0.7-0|} = 0.71;$$
$$v^{l}_{10,1} = \frac{|x^{l}_{10,1} - x^{\min}_{10}|}{|x^{\max}_{10} - x^{\min}_{10}|} = \frac{|0.7-0|}{|0.7-0|} = 1.$$

Thus, the normalized value for fuzzy number $M1V1_{\text{Aesthetics}} = (0.43, 0.71, 1.0)$.

We can arrive at a normalized matrix v_{ij} from the original matrix x_{ij} with the expressions identified in this substep. The normalization process also transforms all the normalized decision factors to have increasing values representing better performance of factors. The resulting normalized values are shown in Table 4.

Step 3: Determine the overall factor weight level w_j.

In this step we determine the importance weight for each of the major sustainability factors. Assuming that a decision evaluation group has K evaluators, we let w_j^k represent the weight of jth decision factor assessed by the kth evaluator. To integrate the different opinions of evaluators, this study adopted a simple averaging aggregation for the subjective judgment from the K evaluators, as shown in expression (27).

$$w_j = \frac{1}{K}\left[w_j^1 + w_j^2 + \cdots + w_j^k\right] \tag{27}$$

The aggregated weight value meets the condition:

$$\sum_{j=1}^{n} w_j = 1 \tag{28}$$

where w_j is the decision factor importance weight for each factor j.

For example, the adjusted factor importance weight for NPC ($j = 1$) and decision maker 1 ($k = 1$) is $w_1^1 = 0.20$.

The adjusted overall weight for factor NPC (w_1) is:

$$\begin{aligned} w_1 &= \frac{1}{4}\left[w_1^1 + w_1^2 + w_1^2 + w_1^4\right] \\ &= \frac{1}{4}[0.20 + 0.12 + 0.11 + 0.17] \\ &= 0.15 \end{aligned}$$

The final adjusted factor importance weight values are shown in Table 5.

Table 4 Adjusted (normalized) scores of vehicle alternatives on factors

Vehicle alternatives	Economic/business measures				Environmental measures			Social measures		
	NPC (D)	FlexPerf (I)	SCCT (D)	ComDate (P)	WstGen (I)	Emiss (D)	ProdRec (P)	Safe (I)	ExPop (D)	Aesthetics (P)
M1V1	[0.71,0.8]	(0.9,1,1)	[0.33,0.44]	[0,0.5]	0.14	[0.67,0.74]	[0.21,0.5]	(0.7,0.9,1)	[0.71,0.82]	(0.43,0.71,1)
M1V2	[0.54,0.63]	(0.7,0.9,1)	[0.78,1]	[0.8,0.97]	0.21	[0.28,0.46]	[0.13,0.47]	(0.9,1,1)	[0.86,0.94]	(0,0,0.33)
M1V3	[0.14,0.17]	(0.9,1,1)	[0.56,0.67]	[0.35,0.5]	0.62	[0.62,0.69]	[0.47,0.67]	(0.5,0.7,0.9)	[0.71,0.8]	(0,0.33,1)
M2V1	[0.31,0.4]	(0,0.1,0.3)	[0,0.11]	[0.17,0.27]	0.41	[0.51,0.64]	[0.73,0.93]	(0.9,1,1)	[0.67,0.84]	(0.71,0.71,1)
M2V2	[0.37,0.51]	(0.3,0.5,0.7)	[0.67,0.78]	[0.7,0.7]	0.9	[0.13,0.23]	[0.64,0.79]	(0.3,0.5,0.7)	[0.45,0.55]	(0.43,0.71,1)
M2V3	[0.71,0.77]	(0.5,0.7,0.9)	[0.67,0.78]	[0.33,0.33]	0.38	[0.51,0.67]	[0.71,0.86]	(0,0,0.1)	[0.12,0.27]	(0,0.14,0.43)
M3V1	[0.83,0.89]	(0.1,0.3,0.5)	[0.78,0.89]	[0.7,0.77]	0.48	[0.56,0.72]	[0.64,0.71]	(0,0.1,0.3)	[0.67,0.76]	(0.43,0.71,1)
M3V2	[0.37,0.54]	(0.3,0.5,0.7)	[0.22,0.56]	[0.5,0.7]	0.97	[0.64,0.79]	[0.73,0.93]	(0.1,0.3,0.5)	[0.63,0.69]	(0.14,0.43,0.71)
M3V3	[0.6,0.71]	(0,0,0.1)	[0.11,0.22]	[0.37,0.5]	0.83	[0.33,0.56]	[0.47,0.67]	(0.7,0.9,1)	[0.31,0.41]	(0,0,0.33)

Table 5 The adjusted factor importance values w_j

Sustainability factor	Average adjusted importance weight (w_j)
NPC	0.15
FlexPerf	0.13
SCCT	0.08
ComDate	0.09
WstGen	0.15
Emiss	0.09
ProdRec	0.12
Safe	0.06
ExPop	0.07
Aesthetics	0.06

Step 4: Determine the overall final decision factor value by adjusting with the factor weight.

Considering the average decision maker weights of each factor, the weighted normalized decision matrix can be computed by multiplying the importance weights of evaluation criteria and the values in the normalized decision matrix. This step is completed with expression (29) for regular numbers, expression (30) for grey numbers and expression (31) for fuzzy numbers:

$$wv_{ij} = w_j \times v_{ij} \, \forall i \in n \tag{29}$$

$$\otimes wv_{ij} = w_j \times \otimes v_{ij} = [\min(w_j \underline{v}_{ij}, w_j \bar{v}_{ij}), \max(w_j \underline{v}_{ij}, w_j \bar{v}_{ij})] \, \forall i \in n \tag{30}$$

$$w\tilde{v}_{ij} = w_j \times \tilde{v}_{ij} = (w_j \times v_l^{ij}, w_j \times v_m^{ij}, w_j \times v_u^{ij}) \, \forall i \in n \tag{31}$$

For the green transportation vehicle alternative 1, factor 1 (NPC) the adjusted grey value is: $\otimes wv_{11} = w_1 \times \otimes v_{11} =$ [min (0.15 × 0.71, 0.15 × 0.80), max (0.15 × 0.71, 0.15 × 0.80)] = [0.107, 0.120]. For the green transportation vehicle alternative 1, factor 10 (Aesthetics) the adjusted triangular fuzzy value is: $w\tilde{v}_{12} = w_2 \times \tilde{v}_{12}$ = (0.06 × 0.43, 0.06 × 0.71, 0.06 × 1.00) = (0.026, 0.043, 0.060).

The overall adjusted aggregate factor scores results for each green transportation vehicle alternative is presented in Table 6.

Step 5: Determine the ideal and negative-ideal solution.

First, the most 'ideal' reference solution $S^+(wv)$ is determined by selecting the maximum value from amongst each of the factors using expression (32).

$$S^+(wv) = \{\max(wv_{i1}),\ \max(wv_{i2}), \ldots, \max(wv_{in})\} \tag{32}$$

Second, the most 'negative-ideal' reference solution $S^-(wv)$ is determined by selecting the minimum value from amongst each of the factors using expression

$$S^-(wv) = \{\min(wv_{i1}),\ \min(wv_{i2}),\ \ldots, \min(wv_{in})\} \tag{33}$$

Table 6 Combined weight scores of vehicle alternatives on factors

Vehicle alternatives	Economic/business measures				Environmental measures			Social measures		
	NPC (D)	FlexPerf (I)	SCCT (D)	ComDate (P)	WstGen (I)	Emiss (D)	ProdRec (P)	Safe (I)	ExPop (D)	Aesthetics (P)
M1V1	[.107,.12]	(.117,.13,.13)	[.027,.036]	[0,.045]	.021	[.06,.067]	[.026,.06]	(.042,.054,.06)	[.05,.057]	(.026,.043,.06)
M1V2	[.081,.094]	(.091,.117,.13)	[.062,.08]	[.072,.087]	.031	[.025,.042]	[.016,.056]	(.054,.06,.06)	[.06,.066]	(0,0,.02)
M1V3	[.021,.026]	(.117,.13,.13)	[.044,.053]	[.032,.045]	.093	[.055,.062]	[.056,.08]	(.03,.042,.054)	[.05,.056]	(0,.02,.06)
M2V1	[.047,.06]	(0,.013,.039)	[0,.009]	[.015,.024]	.062	[.046,.058]	[.088,.112]	(.054,.06,.06)	[.047,.059]	(.043,.043,.06)
M2V2	[.056,.077]	(.039,.065,.091)	[.053,.062]	[.063,.063]	.134	[.012,.021]	[.077,.094]	(.018,.03,.042)	[.031,.039]	(.026,.043,.06)
M2V3	[.107,.116]	(.065,.091,.117)	[.053,.062]	[.03,.03]	.057	[.046,.06]	[.086,.103]	(0,0,.006)	[.009,.019]	(0,.009,.026)
M3V1	[.124,.133]	(.013,.039,.065)	[.062,.071]	[.063,.069]	.072	[.051,.065]	[.077,.086]	(0,.006,.018)	[.047,.053]	(.026,.043,.06)
M3V2	[.056,.081]	(.039,.065,.091)	[.018,.044]	[.045,.063]	.145	[.058,.072]	[.088,.112]	(.006,.018,.03)	[.044,.049]	(.009,.026,.043)
M3V3	[.09,.107]	(0,0,.013)	[.009,.018]	[.033,.045]	.124	[.03,.051]	[.056,.08]	(.042,.054,.06)	[.021,.029]	(0,0,.02)

Using expressions (32) and (33) for this illustrative problem, we will complete 2 sub-steps in this step. First, the most ‘ideal’ reference green transportation vehicle alternative S^+ is determined to be:

$$\begin{aligned} S^+ = \{&[0.124, 0.133],\ (0.117, 0.13, 0.13),\ [0.062, 0.08],\\ &[0.072, 0.087], 0.145,\ [0.06, 0.072],\ [0.088, 0.112],\\ &(0.054, 0.06, 0.06),\ [0.06, 0.066],\ (0.043, 0.043, 0.06)\} \end{aligned}$$

Second, the most ‘negative-ideal’ reference green transportation vehicle alternative S^- is determined as:

$$\begin{aligned} S^- = \{&\ [0.021, 0.026],\ (0, 0, 0.013),\ [0, 0.009],\ [0, 0.024],\ 0.021,\\ &[0.012, 0.021],\ [0.016, 0.056],\ (0, 0, 0.006),\ [0.009, 0.019],\ (0, 0, 0.02)\} \end{aligned}$$

Step 6: Calculate the n-dimensional distance for separation distance.

Based on the grey numbers distance expression (5), fuzzy numbers distance expression(11) and the TOPSIS separation measure expressions (15) and (16), we define new separation measures for an alternative object and ‘ideal’ (expression 34) and ‘negative-ideal’ (expression 35) alternative for a given decision factor.

$$\mu_{ij}^+(S^+(j), S_i(j)) = L(S^+(j), S_i(j)) \tag{34}$$

$$\mu_{ij}^-(S^-(j), S_i(j)) = L(S^-(j), S_i(j)) \tag{35}$$

where $S^+(j)$ is a generic number type value (either regular, grey, or fuzzy) for a factor C_j for the ‘ideal’ reference solution S^+, $S^-(j)$ is a general number value for a factor C_j for the ‘negative-ideal’ reference solution S^-, $S_i(j)$, is general number value of factor C_j of candidate alternative S_i.

The final total positive and negative scores are evaluated by aggregating positive μ_i^+ and negative μ_i^- values (see expressions 36 and 37).

$$\mu_i^+ = \sum_{j=1}^{n} \mu_{ij}^+(S^+(j), S_i(j)) \tag{36}$$

$$\mu_i^- = \sum_{j=1}^{n} \mu_{ij}^-(S^-(j), S_i(j)) \tag{37}$$

For the illustrative example, we show an example calculation for μ_{11}^+ from expression (34) as part of expression (36).

$$
\begin{aligned}
\mu_1^+ &= \sum_{j=1}^{n} \mu_{1j}^+(S^+(j), S_1(j)) \\
&= \sqrt{1/2((0.124 - 0.107)^2 + (0.133 - 0.120)^2)} + \sum_{j=2}^{n} \mu_{1j}^+(S^+(j), S_1(j)) \\
&= 0.015 + \sum_{j=2}^{n} \mu_{1j}^+(S^+(j), S_1(j)) \\
&= 0.326
\end{aligned}
$$

Respectively the solutions for the alternatives' separation distances from the ideal point are:

$$
\begin{aligned}
&\mu_1^+ = 0.326,\ \mu_2^+ = 0.310,\ \mu_3^+ = 0.316,\ \mu_4^+ = 0.418,\ \mu_5^+ = 0.299, \\
&\mu_6^+ = 0.370,\ \mu_7^+ = 0.282,\ \mu_8^+ = 0.272,\ \mu_9^+ = 0.416
\end{aligned}
$$

For the separation distance for each alternative from the nadir point (negative ideal) for the illustrative example, we now show an example calculation for μ_{11}^- from expression (35) as part of expression (37):

$$
\begin{aligned}
\mu_i^- &= \sum_{j=1}^{n} \mu_{1j}^-(S^-(j), S_1(j)) \\
&= \sqrt{\frac{1}{2}((0.021 - 0.107)^2 + (0.026 - 0.120)^2)} \\
&\quad + \sum_{j=2}^{n} \mu_{1j}^-(S^-(j), S_1(j)) \\
&= 0.090 + \sum_{j=2}^{n} \mu_{1j}^-(S^-(j), S_1(j)) \\
&= 0.435
\end{aligned}
$$

The final values for the alternatives' separation distances from the ideal point for the illustrative example are:

$$
\begin{aligned}
&\mu_1^- = 0.435,\ \mu_2^- = 0.441,\ \mu_3^- = 0.447,\ \mu_4^- = 0.337,\ \mu_5^- = 0.464,\ \mu_6^- = 0.390, \\
&\mu_7^- = 0.483,\ \mu_8^- = 0.488,\ \mu_9^- = 0.335
\end{aligned}
$$

Step 7: Calculate the relative closeness to the ideal solution.

The relative closeness of the alternative S_i with respect to S^+ is calculated using expression (17). The relative closeness coefficient helps for rank ordering of all alternatives, allowing the decision-makers to select the most feasible alternative. A larger for T_i value represents a more superior alternative.

Using expression (17), the final comparative distances T_i are shown in Table 7. An example calculation for the first alternative is presented here:

Table 7 The relative closeness of vehicle alternatives

Vehicle alternatives	$\bar{\mu}_i$	$\underline{\mu}_i$	T_i
M1V1(1)	0.326	0.435	0.572
M1V2(2)	0.310	0.441	0.587
M1V3(3)	0.316	0.447	0.586
M2V1(4)	0.418	0.337	0.446
M2V2(5)	0.299	0.464	0.608
M2V3(6)	0.370	0.390	0.513
M3V1(7)	0.282	0.483	0.631
M3V2(8)	0.272	0.488	0.642
M3V3(9)	0.416	0.335	0.446

$$T_1 = \frac{\mu_1^-}{\mu_1^+ + \mu_1^-} = \frac{0.435}{0.326 + 0.435} = 0.572$$

Thus, with a score of 0.642, green transportation vehicle alternative M3V2 is the most preferred alternative from among the nine alternatives in the original set.

4.2 Results Analysis

As can be seen the results show that the eighth alternative (M3V2) has the highest TOPSIS value. It happens to be the closest alternative whose aggregate value is closest to the positive ideal value. It also happens to be the alternative whose aggregate value is the furthest from the nadir (lowest, negative ideal) value. This situation of having the highest nadir distance doesn't always guarantee the best ranking since another alternative may have a better positive distance (smaller) distance value. Although in this case, the rank ordering is in the same exact order as the $\bar{\mu}_i$ ordering. But, the overall rank ordering does not follow the same rank ordering of the nadir values.

The final ranking of the solutions, using the T_i values from Table 7, from best to worst are: M3V2, M3V1, M2V2, M1V2, M1V3, M1V1, M2V3, M2V1 and M3V3 (the last two are tied). Thus, based on the criteria used and the final evaluations, we see that the scores may not be that different. In this situation, a sensitivity analysis may be helpful to determine the robustness of the final results. That is, if some parameters change slightly, what would happen to the final ordering. Another approach would be the possibility to consider other, secondary, evaluation data that might make a difference.

The technique had a number of steps and aggregated three types of numbers into the analysis. The normalizations and the distance calculations (types of calculations) completed may each have influenced the results. For example, even the definition of the linguistic variables for the fuzzy triangular numbers can be arbitrary in terms of the values. Although in the case of the illustrative example the simple range (0.2) value differences with some overlap were used. These ranges were essentially linear values as they increased. In a real world situation, the definitions of these and other intangible values may require significant thought and acceptance by the users of the model.

This last point of input from real decision makers can make a substantial difference since perceptual information is critical to the analysis of intangible valued factors. Even some of the more tangible numbers, such as net present cost of vehicles may only be an averaged value, which may require ranges to be established. In these circumstances the need for sensitivity and parametric perturbation analysis is advisable. The sensitivity analysis for this type of problem can be completed in a number of ways. Two additional ways include variations in the values of the relative importance of the decision makers and also through the relative importance determination of the factors used in the model.

Determining the relative importance of the decision makers inputs in a real world setting may not be so easily completed. These weights can be determined by asking decision makers what level of experience or level of certainty about the decision environment, which in this case is green vehicle technology. Level of expertise and knowledge are good proxies for weighting. The other factor that comes into play in relative importance of decision makers is their managerial or organizational position. Since we are considering environmental measures, some decision makers may be very aware and involved in the environmental aspects of the decision, but not as well versed in costing and financial, or even social data. How to balance this level of knowledge across the metrics and their weighting scheme are important issues for effective and accurate results.

The relative importance of each of the factors may also vary greatly. Who determines and when the relative importance of the factors are issues. The “who” aspect goes back to the relative focus of the decision maker (e.g. an environmental, social, or economics based job). The when aspect is based on whether it should be completed as an aggregated weighting by all the decision makers before weighting of decision maker importance or after that stage.

How the relative importance rankings for factors and decision makers are completed can be based on the utilization of multiattribute utility and valuation tools such as the analytical hierarchy process. These are some possible directions for future research.

We did not spend much time on the acquisition of data for these models. The data is assumed to be accrued and available. Typically in many of these situations, missing data plays a role, getting a more complete and accurate data set is important in this situation, but estimates may need be made in many circumstances.

The results had a clear-cut winner, if there were closer values in the rankings or ties, the situation would require consideration of additional metrics (secondary metrics) to help in discriminating between the alternatives. There are many more that could be considered in each dimension.

5 Summary and Conclusions

5.1 Summary

In this chapter we introduced a TOPSIS-based decision system that can utilize fuzzy, grey, and regular (crisp) numbers to help in ranking alternatives. The specific situation that was faced is the determination of a green vehicle as part of a green logistics (and supply chain) effort by a company. Since we are focusing on general sustainability issues, we showed how the various elements of sustainability and the triple bottom line can be integrated into this multiple dimension decision making problem.

In sustainability decisions, especially those decisions that focus on the supply chain, the decisions can become quite complex as multiple functions, multiple organizations, and the diversity in metrics comes into play for a decision. Thus, the decision tools can greatly benefit managerial decision making, if developed and used appropriately. Given these scenarios and the importance of thoughtful decision making for a sustainable world, the inclusion of the broad variety of sustainability factors into a relatively straightforward methodology is important. Topsis is a very intuitive approach that seeks to balance how far from an ideal solution an object is separated. This intuitive appeal is important for managerial acceptance of any technique. Highly complex and opaque approaches may not be valuable from a practicing manager perspective and thus not garner as much support.

We provided a detailed exposition of the application of the variety of numerical values and the technique as applied to a green vehicle decision. Interestingly, little research has been completed in green vehicle and transportation decisions by corporations (Bae et al. 2011). This lack of publication and focus may mean that organizations are still relying on myopic and traditional financial appraisal approaches for this problem.

The case was illustrative and that, in itself, is a limitation of this research. Actual practical application is necessary to help address some of the concerns brought forth in our results discussion in the previous section. Practical, or ‘face’, validity of the technique is necessary. The best way to achieve the face validity goal is through a practical application and follow-up of a decision. Given that sustainable supply chain management is still in its relative infancy, there may be substantial opportunities in the near future for application of such models.

5.2 Conclusion

From a developmental perspective, linking and incorporating this modeling approach with a broader decision support system for capital and sustainable project appraisal approaches is needed. Whether the solutions from a TOPSIS approach are as valid, accurate, or easily completed as other approaches that can utilize similar data is needed.

Although we incorporated decision makers form differing perspectives, technological decisions that require collaborations and decisions made by two organizations which may diverse strategic and operational objectives could be modeled with this technique. Finding unique and interesting environmental decisions such as this in sustainable supply chain management is expected to also increase. Having tools such as this available for these type of emergent decision situations is important for all involved.

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