Chapter 12
Frictional Damping

Elena Teidelt, Valentin L. Popov and Markus Heß

12.1 Introduction

Friction is a dissipative process, in which mechanical energy is transformed into heat. This can be both unwanted as well as purposefully taken advantage of. Even at very small amplitudes of tangential oscillations, the small slip displacements at the border of the contact area always lead to energy dissipation. This effect is the physical mechanism of damping in periodically forced frictionally engaged joints, for example, in leaf springs for commercial and transportation vehicles. Similar effects are generally exhibited in all frictionally engaged joints and are, therefore, of great interest. For the investigation of damping caused by dry friction, a dynamic tangential contact is of interest. The exact coincidence of the frictional damping in a true three-dimensional contact and its one-dimensional representation in the framework of the method of dimensionality reduction follows from general theorems concerning tangential contacts. This chapter is an illustration how the use of the MDR makes dynamic tangential problems simple without loss of exactness.

12.2 Damping by Dry Friction

In the following, a dynamic tangential contact is considered, the movement of which is damped by Coulomb friction. An elastic parabolic indenter is loaded with the normal force F_N and oscillates subsequently in the tangential direction. As discussed in Chap. 5, this problem can be mapped to an equivalent one-dimensional problem. In this equivalent problem, all elements are considered independently of one another. We can, therefore, begin with the energy dissipation of a single element, as illustrated in Fig. 12.1. Afterwards, the results will be generalized to the entire system.

V.L. Popov and M. Heß, *Method of Dimensionality Reduction in Contact Mechanics and Friction*, DOI 10.1007/978-3-642-53876-6_12

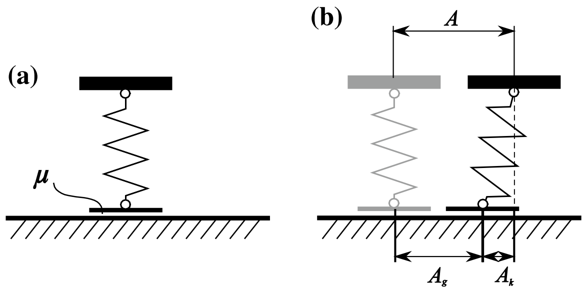

Fig. 12.1 (**a**) Contact of a spring with a rigid substrate. (**b**) The top of the spring is moved from the original position to the right by a distance of A

Let the spring be compressed in the vertical direction by u_z. It possesses a normal spring stiffness of $k_z = E^* \Delta x$ (see Eq. (3.5)) and a tangential spring stiffness of $k_x = G^* \Delta x$ (see Eq. (5.4)). If the top of the spring is moved to the side by A, then the bottom of the spring remains in a state of stick as long as the spring force in the tangential direction is smaller than the maximum force of static friction:

$$F_x = k_x A < \mu F_N = \mu k_z u_z. \tag{12.1}$$

The critical value of the displacement is

$$A_k = \mu \frac{k_z}{k_x} u_z = \mu \kappa u_z, \tag{12.2}$$

where we have introduced the constant κ:

$$\kappa := \frac{k_z}{k_x} = \frac{E^*}{G^*} = \frac{2 - \nu}{2(1 - \nu)}. \tag{12.3}$$

If A is larger than the critical value $A > A_k$, then the bottom of the spring remains in a state of stick until the critical displacement is achieved and slips for the remainder of the distance $A_g = A - A_k$. The work done by the frictional force is then

$$W = (A - A_k)\mu k_z u_z. \tag{12.4}$$

Now, we consider an oscillating spring having a peak-to-peak amplitude of $2A$. If the top of the spring is brought back by this amplitude, then the bottom of the spring sticks until the top has moved by a distance of $2A_k$ and then slips a distance of $2A - 2A_k$ (see Fig. 12.2).

In further oscillations, this distance always remains the same and is traversed two times per period. Therefore, the frictional work done per period for a cyclical movement is

$$W_{cycle} = 4(A - A_k)\mu k_z u_z. \tag{12.5}$$

Now, we consider a system of independent springs. The critical amplitude for a spring with the coordinate x is given by

$$A_k(x) = \mu u_z(x)\kappa. \tag{12.6}$$

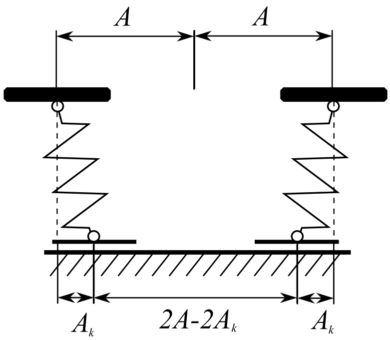

Fig. 12.2 The slip movement of a periodically oscillating frictional contact

The work of the frictional contact during one period for a cyclical movement with the amplitude $2A$ is

$$\Delta W(x) = 4(A - A_k(x))\mu u_z(x) \cdot k_z = 4\mu E^*(A - \mu\kappa u_z(x))u_z(x)\Delta x. \tag{12.7}$$

Within the framework of the method of dimensionality reduction, a parabolic indenter with the profile $\tilde{z} = r^2/(2R)$ is replaced with the profile $\tilde{z} = x^2/R$. If the indentation depth of the indenter is equal to d, then the spring with the coordinate x is indented by

$$u_z(x) = d - \frac{x^2}{R}. \tag{12.8}$$

Therefore, the work of the frictional contact of one spring is

$$\Delta W(x) = 4\mu E^* \left(A - \mu\kappa\left(d - \frac{x^2}{R}\right)\right)\left(d - \frac{x^2}{R}\right)\Delta x. \tag{12.9}$$

The entire energy dissipated during one period is then the integral over the slip domain in the contact:

$$W = 2\int_c^a 4\mu E^* \left(A - \mu\kappa\left(d - \frac{x^2}{R}\right)\right)\left(d - \frac{x^2}{R}\right) \mathrm{d}x. \tag{12.10}$$

The geometric relation $a = \sqrt{Rd}$ is valid for the outer radius of the contact. The lower boundary of the integral is determined by the springs that are in the stick state exactly at their maximum displacement: $\mu k_z u_z(c) = Ak_x$. Therefore, by using Eq. (12.8), we obtain $c = \sqrt{Rd - AR/(\mu\kappa)}$. The entire work done is then calculated as

$$W = 8E^* R^{1/2}\kappa^{-3/2}\mu^{-1/2} \begin{pmatrix} A_{k0}(A - A_{k0})\left((A_{k0})^{1/2} - (A_{k0} - A)^{1/2}\right) \\ + \frac{1}{3}(2A_{k0} - A)\left((A_{k0})^{3/2} - (A_{k0} - A)^{3/2}\right) \\ - \frac{1}{5}\left((A_{k0})^{5/2} - (A_{k0} - A)^{5/2}\right) \end{pmatrix}, \tag{12.11}$$

with $A_{k0} = \mu\kappa d$. If the oscillation amplitudes are small, then the equation can be converted to a Taylor series:

$$W = 8E^* R^{1/2} \kappa^{-3/2} \mu^{-1/2} A_{k0}^{5/2} \left(\frac{1}{12} \left(\frac{A}{A_{k0}} \right)^3 + \frac{1}{48} \left(\frac{A}{A_{k0}} \right)^4 + \frac{3}{320} \left(\frac{A}{A_{k0}} \right)^5 \right). \tag{12.12}$$

The leading term of this series is

$$W \approx \frac{2}{3} \kappa^{-2} E^* R^{1/2} \mu^{-1} d^{-1/2} A^3, \tag{12.13}$$

which corresponds *exactly* to the results from Mindlin et al. [1].

12.3 Damping of Elastomers for Normal Oscillations

In elastomers, energy is also dissipated for the vertical oscillation of contact partners. We consider an axially-symmetric indenter that is pressed into an elastomer to a depth of d by an average normal force of F_N so that the (static) contact radius a is formed. If the indenter is now moved according to a harmonic law

$$d = d_0 + A \cos \omega t = d_0 + \frac{A}{2} \left(e^{i\omega t} + e^{-i\omega t} \right) \tag{12.14}$$

with a small amplitude A, then this movement leads to energy dissipation. Within the framework of the reduction method, a contact with a diameter of $2a$ is replaced by a contact with a viscoelastic foundation having a length of $L = 2a$ using Eq. (7.29):

$$f_N(t) = 4\Delta x \int_0^t G(t - t') \dot{u}_z(t') \mathrm{d}t'. \tag{12.15}$$

The oscillation of an element of the form $u_1 = (A/2)e^{i\omega t}$ leads to the force $f_1 = 4G(\omega) \cdot \Delta x \cdot u_1$ and an oscillation of the form $u_2 = (A/2)e^{-i\omega t}$ leads to the force $f_2 = 4G(-\omega) \cdot \Delta x \cdot u_2$. Due to linearity, an oscillation of

$$\Delta d(t) = \frac{A}{2} \left(e^{i\omega t} + e^{-i\omega t} \right) = u_1 + u_2 \tag{12.16}$$

leads to the force

$$f_N = f_1 + f_2 = 2A \cdot \Delta x \cdot \left(G(\omega) e^{i\omega t} + G(-\omega) e^{-i\omega t} \right). \tag{12.17}$$

The average power of this force averaged over one period is equal to

$$\Delta P = \langle f_N \cdot \Delta \dot{d} \rangle = \left\langle 2A \cdot \Delta x \cdot \left(G(\omega) e^{i\omega t} + G(-\omega) e^{-i\omega t} \right) \frac{A}{2} \left(i\omega e^{i\omega t} - i\omega e^{-i\omega t} \right) \right\rangle, \tag{12.18}$$

and yields

$$\Delta P = i\omega A^2 \cdot \Delta x \cdot (-G(\omega) + G(-\omega)). \tag{12.19}$$

By writing the complex shear modulus in the form

$$G(\omega) = G'(\omega) + iG''(\omega) \tag{12.20}$$

and taking into account that $G'(-\omega) = G'(\omega)$ and $G''(-\omega) = -G''(\omega)$, we obtain the average energy dissipation power of one spring:

$$\Delta P = 2\omega A^2 G''(\omega) \cdot \Delta x. \tag{12.21}$$

The dissipation power in the entire contact area is then

$$P = 2\omega A^2 G''(\omega) L = 4\omega A^2 G''(\omega) a. \tag{12.22}$$

12.4 Problems

Problem 1 Determine the attenuation behavior of the horizontal oscillation of a mass, the movement of which is impeded by a frictionally engaged joint with a sphere (see Fig. 12.3). The initial displacement of the mass is $A(0) = A_0$.

Solution In principle, we are dealing with an oscillator with frictional damping. For a harmonic oscillation, the total energy is equal to the maximum potential energy:

$$U = \frac{k_x A^2}{2}, \tag{12.23}$$

where A is the amplitude of the oscillation. The change in this energy during one period $T = 2\pi/\omega$ is equal to the work done by the frictional dissipation (see Eq. (12.13)):

$$\Delta U = -\frac{2}{3}\kappa^{-2} E^* R^{1/2} \mu^{-1} d^{-1/2} A^3. \tag{12.24}$$

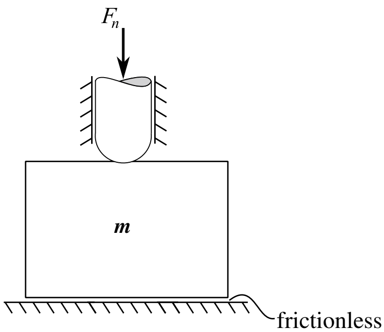

Fig. 12.3 Spherical contact between an elastic sphere and a rigid mass m

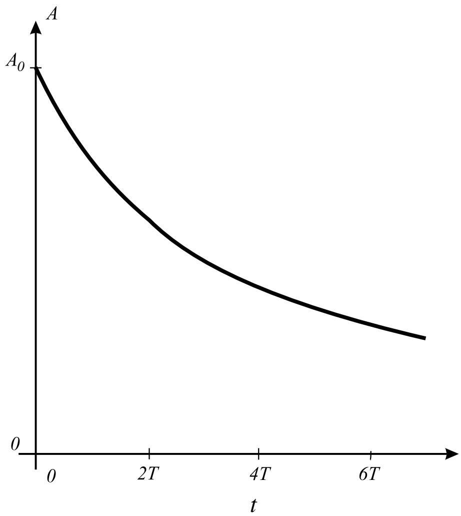

Fig. 12.4 The dependence of amplitude with respect to time

The change in potential energy per unit time is then

$$\frac{dU}{dt} = \frac{\Delta U}{T} = -\frac{1}{T}\frac{2}{3}\kappa^{-2} \cdot E^* R^{1/2} \mu^{-1} d^{-1/2} A^3. \tag{12.25}$$

Rearranged with respect to the amplitude and assuming that $k_x = G^* 2\sqrt{Rd}$ (see Eq. (5.1)), the differential equation reads

$$\frac{dA}{dt} = -\frac{1}{3}\frac{A^2}{\kappa T \mu d}. \tag{12.26}$$

The solution to this differential equation with the initial condition $A(0) = A_0$ is

$$A = \frac{A_0}{1 + \frac{1}{3}\frac{A_0}{\kappa T \mu d} t}. \tag{12.27}$$

For an amplitude at which complete slip is first exhibited ($A_0 \approx A_{k0} = \mu \kappa d$), the following is valid:

$$A = \frac{A_0}{1 + \frac{1}{3T} t}. \tag{12.28}$$

The attenuation behavior of the amplitude per period is presented schematically in Fig. 12.4. It is clear that the oscillations are only slowly (according to a power law) damped by dry friction. This means that it is recommended to integrate a further damping mechanism into vibration sensitive systems.

Reference

1. R.D. Mindlin, W.P. Mason, J.F. Osmer, H. Deresiewicz, Effects of an oscillation tangential force on the contact surfaces of elastic spheres, in *Prof. 1st US National Congress of Applied Mechanics*, vol. 227 (ASME, New York, 1952), pp. 203–208