Chapter 4
The Hadronic String

> *[T]he string model originated as a model for the S-matrix, and it may well not have been discovered if S-matrix theory had not been vigorously pursued at the time.*
>
> Stanley Mandelstam

The idea that the Veneziano model might have a basis in a theory of strings was recognised independently by several physicists.[1] The puzzle was to find out what 'lay behind' the Veneziano model and thereby attempt to reconstruct the formula and its predictions from a more fundamental physical picture. Indeed, the notion of providing a 'picture' of the Veneziano model (primarily the N-particle generalisation) was a key feature of this work. Key target features included the Regge trajectories (the tower of resonances described by the Veneziano model) and the DHS duality (linking apparently different kinds of particle processes).[2] Susskind, Nambu, and Nielsen all surmised that the regular presentation of the tower meant that it was being generated by some internal oscillatory motions, lying within hadrons. That is to say, the spectrum of the dual model was suggestive of the spectrum of an oscillating system.

Despite the fact that many of the modern concepts of superstrings come from this work—including the notion of a worldsheet and the idea that the above mentioned duality is a manifestation of the conformal symmetry of the worldsheet—the earliest

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[1] As mentioned earlier, this kind of convergence is common in scientific discovery and often points to 'being on the right track'—or at least to an overall consistency in the methods of science. In the present case, this is not in the least surprising since the Veneziano formula (its spectrum of states) implied that, at the very least, it was describing an infinite family of harmonic oscillators and the harmonic oscillator is one of the best-known examples in physics.

[2] While a mystery from a point-particle point of view, DHS duality is directly understood in hadronic string terms of a perfectly natural geometrical outcome of having a physics of extended objects. Gomez and Ruiz-Altaba explain it thus: "[d]uality is... intrinsic to the string picture, because a Feynman diagram where two rubber bands merge into one and then become two rubber bands again allows for arbitrary definition of s, t, or u channels" [24, p. 54]—they suggest that a more appropriate picture, given this malleability, would be "chewing gum"!

D. Rickles, *A Brief History of String Theory*, The Frontiers Collection,
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string interpretations can be seen in each case (at best) to ‘hedge’ on the issue of *realism* about strings. In much the same way as quarks were used in this period, one finds that the string concept is used heuristically to suggest new research directions or tools to apply, or used analogically, again more for convenience than to provide a faithful representation of actual hadronic processes. It would take further work in establishing the *consistency* of the theory, as well as its demonstrable ability to reproduce the physics of dual models, before strings could be taken seriously as a realistic model of our world. Unfortunately, by the time this was achieved, the dual bootstrap picture was being replaced by quantum field theory, Chew’s “old mistress”. Initially, there was certainly not the slightest intimation that the strings would have anything to do with physics beyond hadrons.

4.1 The Multiple Births of Strings

As we have seen, the Veneziano model as it was understood in the immediate aftermath of its construction was not thought to have anything to do with a dynamical theory of extended objects. The Veneziano formula was exactly that: a formula. It was an example of a mathematical object that would deliver probabilities for scattering events. As such it did not offer any kind of mechanism, or any physical picture, for the kinds of systems that would satisfy it and generate its spectrum. However, what was present was the infinite set of oscillators revealed by the formalism of Fubini and Veneziano. The string models were the fruit of the effort to restore some physical intuition to the dual resonance amplitude: the dual model spectrum could be viewed as issuing from the quantum mechanical behaviour of vibrating, rotating strings. The infinite set of oscillators were modes of vibration of the string.[3]

The intuitive picture that we can draw from this is that a particle is ‘composed’ out of an open spinning string with unconstrained end-points. The string has a tension along its length, and given its rotation is also subject to a centrifugal force. The spin is maximal when the string is straightest, corresponding to the leading Regge trajectory. Given that the strings have a finite, fundamental length one can also see how the slope of the Regge trajectories is determined from the string picture. ‘Classical’ string aspects naturally arise in the various ‘harmonics’ that also contribute to the motion of the string and correspond to the daughter trajectories lying under the leading (fundamental) trajectory. This correspondence between string properties and the Regge trajectories suggest that it might have been possible to guess in advance (without the benefit of the Veneziano model and its operator formulation) that the Regge trajectory was pointing to a unified system generated as the infinitely many excitations of a single fundamental string.

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[3] Note that the naming convention for ‘strings’ appears to have only been firmly established in 1974, in Claudio Rebbi’s survey of dual models and relativistic quantum strings for *Physics Reports* [**?** , p. 4].

This chapter focuses on the key elements involved in this crucial transition in the understanding of the Veneziano amplitude, leading to 'dual string models'. Note, however, that when we speak of the Veneziano model, we are referring in each case (Susskind, Nambu, Nielsen) to the N-point generalisations that occurred later.

Susskind: A Rebel Without a Theory

Just before finding out about the Veneziano amplitude, Leonard Susskind[4] had been investigating relativistic quantum theory in the so-called infinite momentum frame.[5] Susskind argued that physics in the infinite momentum frame was Galilean invariant, implying that standard quantum mechanics was applicable. He wanted to apply this to the problematic hadrons to investigate their internal structure with well-worn tools. He heard about the Veneziano formula in 1968—cf. [57]. There then followed an interaction between his then current research programme and aspects of this formula. In his Galilean-invariant framework m^2 is the energy term, and the same term determines the spacing of the poles of the Veneziano amplitude. Hence, given this correspondence, the energy levels were equally spaced, implying that the system generating the Veneziano formula must be some kind of quantum harmonic oscillator. The remaining task was to try to construct an oscillator model that reproduced the Veneziano formula. As Susskind points out [57, p. 263], while the Veneziano formula looked like:

$$A(s,t) = \int_0^1 x^{-s}(1-x)^{-t}dx \tag{4.1}$$

his own (modeling a hadron oscillating in the infinite momentum frame) looked like

$$A(s,t) = \int_0^1 x^{-s}(\exp - x)^{-t}dx \tag{4.2}$$

where the two formulas are clearly related by the interchange $(1-x) \rightarrow (\exp - x)$. When making a comparison with the more general Veneziano formulae (for more than 4 particles), he found agreement too (modulo the same interchange). Susskind viewed the correspondence as highlighting a potentially fruitful *analogy* rather than anything profound. Hence, the paper containing this idea was entitled "Harmonic Oscillator Analogy for the Veneziano Amplitude" [52]. However, soon after publishing this paper Susskind realised that certain of the properties ("higher harmonics") could

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[4] The subtitle 'Rebel Without a Theory' refers to a remark made by Susskind in *The Cosmic Landscape* [56, p. 204], in which he discusses his distaste for S-matrix ideology.

[5] This is now more commonly referred to as the "light-cone frame", an idea introduced by Fubini and Furlan in 1965 [16] (see [10, pp. 50–52]). The idea is to boost the velocities to such a degree that the dilation effect on the system is very large, allowing one to explore its motions and internal structure more easily.

only be generated by a very specific oscillator such as a violin string rather than, say, a spring [54]:

> I was able to produce formulas that looked a lot like Veneziano's from the simple weight and spring model, but they weren't quite right. During that period I spent long hours by myself, working in the attic of my house. I hardly came out, and when I did I was irritable. I barked at my wife and ignored my kids. I couldn't put the formula out of my mind, even long enough to eat dinner. But then for no good reason, one evening in the attic I suddenly had a "eureka moment". I don't know what provoked the thought. One minute I saw spring, and the next I could visualise an elastic string, stretched between two quarks and vibrating in many different patterns of oscillation. I knew in an instant that replacing the mathematical spring with the continuous material of a vibrating string would do the trick. Actually, the word *string* is not what flashed into my mind. A *rubber band* is the way I thought of it: a rubber band cut open so that it became an elastic string with two ends. At each end I pictured a quark or, more precisely, a quark at one end and an antiquark at the other [47, p. 206].

It quickly became clear to Susskind that the Veneziano amplitude could be given an interpretation in terms of scattering elastic bands, coming in from infinity, then merging to form a single band, and then splitting before moving out to infinity again. Of course, this achieves a high degree of theoretical (and, correspondingly, ontological) simplification and economy. What was considered to be a series of excited hadronic states (on a trajectory) corresponding to *distinct* particles or resonances, are, in a string picture, states realised in one and the same object (much like Bernoulli's diagram of the superpositions of vibrational modes of a violin string that we saw in Chap. 1).

Susskind notes that the string model for hadrons was "not an immediate success" [56, p. 217]. He traces this to a widespread negative stance against theories that tried to 'picture' what was happening in the world; cracking open the black box of S-matrix theory and looking within the collision region. However, while I think there was an S-matrix motivated reaction against the string model, I don't think that visualisability was the problem. Rather it was a reaction against what the string model *revealed* within the black box, and that was something in direct conflict with nuclear democracy and bootstrap ideals. If there is just a single system underlying all of the different resonances, than that implies a fundamentality that was anathema to most physicists working on the dual resonance models, steeped, as they were, in Chew's philosophical ideas and the notion that duality implied nuclear democracy. Hence, while it is often said that the S-matrix theory was abandoned because of the rise of QCD, it might be said that the S-matrix programme, when carried to its completion with the implementation of duality and the Veneziano model, contained the seeds of its own destruction.[6]

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[6] Susskind claims to pinpoint the time at which Murray Gell-Mann became interested in string theory, and as a result put the 'stamp of approval' on it (ibid). He first refers to a discussion he had with Gell-Mann at a conference in Coral Gables, Florida, in 1970, where Gell-Mann had simply laughed when he mentioned his idea of a string structure of hadrons. Then two years later, at the 'Rochester conference,' at Fermilab, Gell-Mann apparently apologised for his earlier behaviour and pointed out that he was interested in such work, and indeed spoke a little on the subject at the conference. Gell-Mann both outlined his theory of quarks and, though it was not yet known by the name, quantum chromodynamics (with Fritzsch) he gave a summary talk, which is presumably

Nambu's Tale

Yoichiro Nambu too initially spoke in rather more non-committal terms than is often supposed in the more recent literature discussing the origins of string theory. Here too it was by *analogy* that the string picture was initially suggested. There was plenty of historical precedent for the idea that the fundamental objects might not be point particles, but some non-local entities.[7] However, all of these had failed to provide a consistent picture. Hence, there was good reason to be somewhat tentative at this stage. Thus, he writes[8]:

> [T]he internal energy of a meson is analogous to that of a quantized string of finite length (or a cavity resonator for that matter) whose displacements are described by the field $\phi_\alpha(\xi)$ [a Bosonic field—DR] [40, p. 275].

This connection Nambu made on the basis of his derivation of the following expression for the quantum number representing the resonance energy N:

$$N = \frac{-1}{\pi} \int_0^{2\pi} : (\partial_\xi \phi(\xi) \cdot \partial_\xi \phi(\xi) + \pi(\xi) \cdot \pi(\xi)) : d\xi \tag{4.3}$$

where the Bose field and its conjugate are decomposed as follows (with a and a^+ the creation and annihilation operators):

(Footnote 6 continued)
what Susskind is referring to. Gell-Mann had already taken to strings by then. He had been a visitor at CERN in 1971 while Lars Brink and David Olive were also there. Moreover, John Schwarz heard that he would not receive tenure at Princeton in early 1972 and received a position as a research associate at Caltech shortly afterwards (within in a matter of months), to work on string theory (http://resolver.caltech.edu/CaltechOH:OH_Schwarz_J, p. 22). The field of string theory had advanced a great deal within the 2 years from 1970 to 1972. Statistical analysis of citations before and after Gell-Mann's remarks do not reveal any significant differences, though it certainly cannot failed to have put string theory more on the map relative to researchers from the general particle physics community.

[7] Indeed, as Nambu mentions in his reminiscences [43, p. 278], he had been working on non-local field theories as a way of reproducing the seemingly infinite number of Regge trajectories of ever higher spins. His approach involved representing them via a master wave equation in terms of infinite-dimensional representations of groups containing the Lorentz group (in what he called an *infinite-component wave equation*). The work was abandoned since it violated too many desirable properties of quantum field theory, such as CPT, microcausality, tachyon-freedom, crossing, and so on. However, the ability to manipulate creation and annihilation operators in this work would be recycled in his work on the string model.

[8] Nambu's initial suggestion that a string picture might lay behind the Veneziano model was delivered at a somewhat low key conference on Symmetries and Quark Models, held at Wayne State University, June 18–20, 1969. However, this certainly appears to have been the first public mention of the idea that the excitation spectrum of dual models was reducible to a vibrating string. Note also that Nambu's primary concern was not with elucidating the physical content of dual models, but with a factorization method for the Veneziano amplitude.

$$\phi_\alpha(\xi) = \sum_{r=1}^{\infty} \frac{1}{\sqrt{2r}} (a_\alpha^{(r)} + a_\alpha^{\dagger(r)}) \cos r\xi \tag{4.4}$$

$$\pi_\alpha(\xi) = \sum_{r=1}^{\infty} i \sqrt{\frac{r}{2}} (a_\alpha^{(r)} - a_\alpha^{\dagger(r)}) \cos r\xi \tag{4.5}$$

The displacements of the resonator are given by $\phi_\alpha(\xi)$, though as one can see, Nambu had some uncertainty as to the precise nature of the resonator at this stage: it could refer to the oscillations of a string, or the oscillations within a hollow body. Note also that Nambu made crucial usage of Veneziano and Fubini's work on the level structure of the dual amplitude (at the time only available as an MIT preprint). Following his abandonment of his earlier infinite-dimensional representation approach, Nambu describes the initial path to this string idea as follows:

> The Veneziano model realized the linear Regge trajectories and the duality of scattering amplitudes in a simple formula. So I got fascinated by ... First of all, what physics lies behind it? It is a mysterious formula nobody really understands. You can write down the formula in various ways. I wanted to decompose the formula as an infinite sum of Breit-Wigner resonances to see how many of them are at each resonant energy, what their spins are, and if the residues are positive or that they are real physical states. When I started doing this, there was a postdoc, Paul Frampton, arriving from Oxford. So I got his assistance right away and we were more or less convinced that all Breit-Wigner poles were positive [hence, physical—DR]. We also found that the degeneracy of states goes up exponentially with energy in the asymptotic limit.
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> After that I worked further on analyzing the structure of the Veneziano amplitudes. I started from the Koba-Nielsen representation of the beta function, and in the course of this analysis, I discovered that the resonances can be interpreted as the excitations of a string.[9]

Here we see that for Nambu, the Koba-Nielsen Beta function expression was crucial, as it was for Nielsen himself, though inspiring a different approach. His route to the spectrum so suggestive of a harmonic oscillator system proceeded through manipulations of this expression, starting with its factorization. Nambu's factorization yielded:

$$(1-x)^{-\alpha' t - \alpha_t - 1} = e^{\alpha'(p_1 \cdot p_2 - C)(x + x^2/2 + x^3/3 + \cdots)} \tag{4.6}$$

with C describing mass dependence and dependence on the intercept value: $C = m_1^2 + m_2^2 + \alpha_t + 1$. He was then able to link this up to an expression for vector fields decomposed in terms of creation and annihilation operators (as presented above, in Eq. 4.4), giving:

$$\langle \phi(x)\phi(y) \rangle = \sum_k \frac{1}{2E_k} \exp[ik \cdot (x-y)] \tag{4.7}$$

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[9] Interview of Yoichiro Nambu by Babak Ashrafi on July 16, 2004, Niels Bohr Library and Archives, American Institute of Physics, College Park, MD USA http://www.aip.org/history/ohilist/30538.html. Note that, in the portions relevant to string theory at least, Nambu appears to consistently have his dates a year out in this interview. Hence, for years between 1967 and 1970, simply advance them by 1 to get the correct figures.

What is being derived here, from the quantized string picture, is an expression for an N-particle amplitude, along with the Hilbert space and operator formalism (in terms of vertex operators, initially only for open strings). Once one has a picture of one dimensional oscillators, one can imagine their evolution, which will generate a diagram of a kind that exactly satisfies the DHS duality.

The underlying idea is quite simple. Given the Regge trajectories, $J = \alpha(0) + \alpha' m^2$, one can make sense of the spin-mass relationship in terms of a rotating string, with quarks as end-points. As the string rotates it generates a centrifugal force, pushing the quarks outwards, thus stretching the string apart. The longer the string, the more energy per unit length, and therefore the more mass. This physical picture corresponds to the mathematical relationship for mesons. Nambu claims to have realised the confining implications of a string interpretation right away, but was ambivalent about it on account of his own alternative field theoretic approach:

> I hit upon this string interpretation of the Veneziano model, and immediately I knew that that it could confine the quarks, because quarks attached to the ends of the string and can not separate. On the other hand, if you work out the mathematics, it did not quite work out well. Sooner or later people found out that you needed 26 dimensions, something like that. And in the meantime, there emerged a new gauge theory of color which was very nice in explaining the possibility of quark confinement. That is the usual quantum field theory, and my string theory is not quantum field theory but a more general one which also has various problems. Some of them are theoretical so far, they were found already when I worked out this infinite component wave equation. So I was in a sort of quandary, in the following sense. I knew that strings can confine quarks. On the other hand, I had also my pet theory of integral charged colored quarks, so the quarks were free to come out. But anyway, it explained the stability of color-neutral particles, hadrons.
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> So I was in a quandary of which theory I should really side with. And I knew that string theory had mathematical problems. So probably it would not quite work out. Many decided to abandon string theory. I think it was around 1973. There was a summer institute at Aspen, and string theorists of the day got together. And more or less around the time people realized that we had to give up the hadronic string theory.[10]

It is important to note that physicists were not forced down the string theoretic path initially. One could also, as was suggested, substantiate the Veneziano formula in terms of an infinite-component field theory. The latter had very many serious problems, but the dual string model could hardly be said to be problem-free.

Nielsen's 'Almost Physical' Model

Holger Nielsen seems not to have publicly presented his paper, "An Almost Physical Interpretation of the Integrand of the n-point Veneziano Model," though it is often

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[10] Interview of Prof. Yoichiro Nambu by Babak Ashrafi on July 16, 2004, Niels Bohr Library and Archives, American Institute of Physics, College Park, MD USA, http://www.aip.org/history/ohilist/30538.html.

claimed that he presented it at the 1970 'Rochester' conference in Kiev.[11] However, in a note that he appended to the recent release of his original preprint containing the string idea,[12] Nielsen states that he was discussing the idea with people in the spring of 1969—especially at the Lund International Conference on Elementary Particles, held from June 25-July 1 (though his 'official' talk was on the Koba-Nielsen formalism)—and circulated an initial paper identical to the later preprint save for the absence of the word 'almost' (this was a Nordita preprint). It seems that he never published any reference to the work himself, nor did he publicly speak about the work. However, Bunji Sakita did give a review of Nielsen's work at the Kiev conference, though this also was not published in the proceedings. Veneziano did, however, publish a review paper in the proceedings, and mentioned Nielsen's approach, amounting to the first published version of Nielsen's 'fishnet' approach (cf. [45, p. 272]).[13]

Nielsen's approach to a model of hadrons as strings[14] was significantly different to Nambu's and Susskind's (which followed a more or less similar path to one another). Nielsen sought a link to the standard Feynman diagram methodology for representing amplitudes and scattering. He argued that the duality diagram (of the Harari-Rosner sort) could be seen as the limiting case of a class of infinitely complicated Feynman diagrams that formed a mesh, or "fishnet" as it was later labelled.[15] In other words, something like a string world sheet is generated as an approximation to the underlying complex tangle of propagators. However, though Nielsen clearly has in mind a surface generated by the evolution of his threads, he characterises it in terms of two-dimensional conducting disc, with the Harari-Rosner quark flow lines forming its boundary. External lines are characterised as current-carrying 'electrodes' on the conductor's boundary (the analogues of the momenta of the external particles of the Veneziano model). Hence, Nielsen's model employs an electrostatic analogy which allows the integrand of the N-particle Veneziano model to be computed, as the exponential of heat produced by a steady current on the surface of the disc—conformal invariance implies that the result one gets is independent of local

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[11] Veneziano, Zumino, Gervais, Sakita, Volkov, Gross, Migdal, and Polyakov were all present at this conference.

[12] The preprint is [46], and the brief note describing the origins of this preprint can be found at http://theory.fi.infn.it/colomo/string-book/nielsen_note.txt (on the website for the book *The Birth of String Theory*).

[13] Soon after, Sakita, together with Virasroro, published a proof (based on functional integration methods) of Nielsen's fishnet-based claim that the N-particle Veneziano provides an approximate description of planar fishnet-Feynman diagrams in the large-N limit [50] (see below). They also extend the principle to non-planar diagrams: non-simply connected and non-orientable.

[14] Nielsen speaks variously of "one dimensional structures", "thread like structures", "chain molecules", and even "sticks"!

[15] Note that the point particle description was an integral part of the early string proposals, usually entering in the form of an infinite limit of point particles to construct the string, and, as we will see later, an infinite limit of parallel point particle worldlines to construct the worldsheet. Nielsen's fishnet diagrams were precisely of this form, namely a chain of particles linked together (with nearest neighbour interactions).

stretching and rotation of the surface provided one can map it conformally to a disc (cf. [11, p. 71] and [15, p. 286]).[16]

The basic idea underlying Nielsen's path to the string (and the surfaces traced out by such string), from the Veneziano model is quite intuitive: if one is dealing with strong interactions (featuring a large coupling constant) from a Feynman diagram point of view, then one expects higher-order diagrams to dominate (cf. [45, p. 270]). The question he posed to himself was, therefore, whether one could build up the Veneziano amplitude from such high-order Feynman diagrams. The answer was yes, given an $n \rightarrow \infty$ limit (where n is the order of the diagrams) and so long as the diagrams are planar (with no crossed lines). The surfaces of evolution of threads are, as he puts it, "very rough pictures of very complicated Feynman diagrams" such that "only Feynman diagrams having the large scale topological structure of a two-dimensional network are of importance" [46, p. 18]. This provides Nielsen with his picture of the generalised Veneziano model. Nielsen also provided a qualitative account of the splitting and joining of his threads, as follows:

> Hadronic interactions are conceived of then as processes in which threads are connected at the end points into (at first) longer threads which are then again split up into (at first) shorter threads. In fact the mapping $V^{\mu} : \phi \rightarrow$ "Minkowski space" described by the potential of equation (25)[17] could be conceived of as describing the time track of a thread moving around in physical three space [46, p. 13].

Though it is out of chronological sequence (coming in 1973), we should mention Nielsen's later work, with Poul Oleson, which utilised a different analogy, this time involving a (type II) superconductor.[18] The work in question sparked off a field known as *Dual Superconductor Models of Colour Confinement*. The idea here was to provide a physical *grounding* for the still then rather abstract strings, by deriving string-like structures from a local field theory, and from the standpoint of such 'non-fundamental' strings, one could reproduce the behaviour captured by the Veneziano

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[16] It seems that this component of Nielsen's work was devised in close collaboration with David Fairlie, then at Durham University—indeed, the electrostatic analogy was due to Fairlie. Amusingly, Fairlie claims to have come up with the idea of shape independence from a Philips advert in *Scientific American*, showing Ohm's Law (see [15, pp. 286–287]). A general solution (for 'discs' or surfaces of arbitrary genus) of Nielsen and Fairlie's analogue model was later provided by Alessandrini [2], where he understands the problem to consist in solving a harmonic problem on a Riemann surface. He employs Burnside's 1891 analysis of automorphic functions (which had been introduced to study harmonic problems on surfaces with circular holes). This was understood independently by Lovelace, as mentioned in footnote 6. Given this ancestry, David Fairlie makes the following amusing counterpoint to a famous quote from Witten: "Edward Witten was fond of quoting that 'String theory is a piece of mathematics which has fallen out of the twenty first century into the twentieth'. It has seemed to me more like something dragged out of the nineteenth!" [15, p. 285].

[17] The key featue of the potentials is that they satisfy Ohm's law.

[18] The type II refers to the fact that such superconductors live in a mixture of non-superconducting and superconducting regions, which results in vortices of superconducting current surrounding the non-superconducting regions. The magnetic field enters the interior of the superconductor through such vortices, or 'Abrikosov flux tubes' as they are sometimes called. Such flux tubes have energy per unit length just like strings, growing linearly, with each tube containing one unit of quantized flux (with the number of such tubes determined by controlling the external field)—cf. [39].

model. The motivation was to make sense of the fact that on the one hand the string picture copes remarkably well with the nice features of the Veneziano model (capturing experimental results), yet many of the principles leading to the Veneziano model were drawn from field theory (e.g. crossing symmetry):

> We have good reasons to believe that both field theory (of a kind which is so far not known) and dual strings (with some yet unknown degrees of freedom) are in fact realzed in nature. It is likely that nature has decided to merge some field theory with some dual string structure [47, p. 45]

The Nielsen-Oleson model was thus supposed to offer a compromise, pointing the way from S-matrix theory and bootstrap philosophy, back to more orthodox quantum field theory. Indeed, Nielsen and Oleson hoped that by merging the two in this way, the latter might serve to tame some of the troubling issues facing the former. For example, choosing a positive definite Hamiltonian might eliminate the presence of tachyons in the dual model's spectrum. Moreover, it might serve to throw light on curious features of dual models, such as the condition for a critical dimension of $d = 10$ or $d = 26$. The strategy was to "translate" such notions from dual resonance model language to field theory language (ibid., p. 46) and see if they could be reconceptualised (e.g. as internal symmetries). It was also suggested that such a field theoretic translation might point to potential generalizations of dual models. Hence, in many ways, initially at least, this field-theoretic correspondence was used as a kind of exploratory tool.

Nielsen-Oleson vortex strings were devoid of endpoints, and hence were either closed or infinitely long. A little later, Nambu[42] extended the Nielsen-Oleson idea to the case of open strings (with endpoints), using a formalism developed by Michael Kalb and Pierre Ramond [32]. This paper introduced Dirac monopoles, leading to the conclusion that quarks are sources of magnetic charge, permanently confined by their string bonds—in other words, in order to be of finite length the Nielsen-Oleson had to end on a monopole, to 'capture' the flux.[19] Such 'string/gauge' analogies have continued to play an important role in the development of string theory—a point we shall return to in the final chapter of this book. Further, though we will return to it in the next chapter, we should pause to mention that this Nielsen-Olesen interpretation of dual strings (as Abrikosov flux lines) was highly influential in the subsequent understanding of colour confinement in QCD. Hence, the important role of the hadronic dual string in the construction of QCD should not be underestimated.

4.2 Geometrical Interpretation

The generalised Veneziano model amounts to a formula for computing N-point functions in a field theory. String theory emerged from the recognition that these N-point functions are in exact correspondence with the (expectation values of) N

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[19] This would go on to inspire Gerard 't Hooft [59] in his work on magnetic monopoles in unified gauge theories (cf. [44, p. 381]). We discuss this further in Sect. 6.3.

vertices of a theory of strings (i.e. the two-dimensional surfaces swept out by strings). Hence, one often sees it said that the Veneziano amplitude was ‘really’ a theory of strings. There are, of course, two stories at work here. On the one hand, there is the more abstract algebraic description based on the operatorial formalism, and on the other there is a geometrical approach based on the worldsheet picture. Although the geometric string picture was in some sense derived from the abstract operator approach, the two led curiously separate lives afterwards.

The first published discussion of the string worldsheet (and the coining of the term) appears to have been Susskind [54].[20] Armed with the worldsheet concept, Susskind was also able to show how the duality that kick-started the new work on string theory could be explained as an implication of conformal invariance (though this seems to have been suggested by others too). This must have been roughly contemporaneous with Nambu’s researches (see below) which were then still unpublished. In this section we also see how the worldsheet concept was involved in the construction of the (quadratic) string action.

I mentioned that the worldsheet concept appears to have originated with Susskind. Not long after this, as part of a collaboration with Aage Kraemmer and Holger Nielsen, he wrote down an action principle for strings[21]:

> One of the most exciting things that happened was a correspondence that started when Holger Bech Nielsen sent me a handwritten letter explaining his ‘conducting-disc’ analogy ... Holger understood that the conducting disc was just the world-sheet and that the relation between his work and mine was simply momentum-position duality. Nielsen came to visit me in New York and we excitedly explored the possibility that the world-sheet is a dense planar Feynman diagram which we connected with Feynman’s parton ideas. I believe the paper that we wrote with Aage Kraemmer was the first to contain the quadratic world-sheet action [57, p. 264].

Susskind describes the idea of the worldsheet as follows:

> Let us suppose that a meson is composed of a quark-antiquark pair at the ends of an elastic string as described in previous Sections. As the string moves in space-time a two-dimensional strip bounded by the trajectories of the quarks is generated. In analogy with Minkowski’s world-line we call such a configuration a world-sheet [54, pp. 483–484].

Susskind provides variables, θ and τ, specifying coordinates, labeling the points of the worldsheet, with the dynamical variables given by $X_\mu(\theta, \tau)$ satisfying:

$$\frac{\partial^2}{\partial \tau^2} X_\mu(\theta, \tau) - \frac{\partial^2}{\partial \theta^2} X_\mu(\theta, \tau) = 0 \tag{4.8}$$

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[20] Susskind notes that an earlier version of the paper was rejected by *Physical Review Letters* “on the basis of not having any new experimental prediction” [57, p. 264].

[21] Susskind [55, p. 234] credits Ed Tryon [61] with the discovery that the energy of a string (in the sense of string theory) is proportional to its length. This seems to be true, so far as I have been able to ascertain. This is clearly a forerunner of the idea that the action for a string’s worldsheet is proportional to its area. In the same place (p. 235) Susskind also mentions that one of his students, Henri Noskowitz (sic.), came up with the idea of starting out with an area action from which one can derive the string equations of motion. At the time Susskind pooh poohed the suggestion.

Which he understands simply as a generalization of the equation of motion for the worldline of a point, where $X_\mu \rightarrow$ field, $\theta \rightarrow$ space and $\tau \rightarrow$ time.

It was Nambu who, in 1970, extrapolated the notion of a (quasi-geometric, surface) action for a zero-dimensional point particle to that of a one-dimensional string. This generated a dynamics analogous to the minimisation principle for particle worldlines, only in this case the minimisation principle applied to the surface area that an evolving string would trace out in spacetime (roughly, $\int d(\text{worldsheet } area)$). His path to the worldsheet idea was based on a conception of the string as a "limit of a chain of N mass points as $N \rightarrow \infty$" [41, p. 285]. By thinking of each point within the string evolving in the same way as in the classical theory of the motion of a free mass point particle, so that each traces out its own worldline, one can easily see that in the limit of infinitely many such particles, evolving in parallel, one will generate a two-dimensional sheet with the principle of minimisation of worldline length being modified to the area of the sheet. In Nambu's own notation, the sheet is parameterised by two (intrinsic) coordinates: ξ (such that $0 \leq \xi \leq \pi$) and τ (such that $-\infty \leq \xi \leq +\infty$). The action integral is then:

$$I = \frac{1}{4\pi} \int \int \left(\frac{\partial X_\mu}{d\tau} \frac{\partial X^\mu}{\partial \tau} - \frac{\partial X_\mu}{d\xi} \frac{\partial X^\mu}{\partial \xi} \right) d\xi d\tau \tag{4.9}$$

From which he derives:

$$(\partial^2/\partial\tau^2 - \partial^2/\partial\xi^2) X^\mu = 0 \quad (\partial X^\mu/\partial\xi = 0\,, \text{when } \xi = 0, \pi) \tag{4.10}$$

In more modern terms, we would write the action as:

$$S_{Nambu} = -T_0 \int_{\tau_1}^{\tau_2} d\tau \int_{\sigma_1(\tau)}^{\sigma_2(\tau)} d\sigma \sqrt{\left(\frac{\partial X_\mu}{d\tau} \frac{\partial X^\mu}{\partial \sigma} \right)^2 - \left(\frac{\partial X_\mu}{d\tau} \frac{\partial X^\mu}{\partial \tau} \right) \left(\frac{\partial X_\mu}{d\sigma} \frac{\partial X^\mu}{\partial \sigma} \right)} = \int d\tau d\sigma \mathscr{L} \tag{4.11}$$

The idea here involves parameterising the string worldsheet, via parameters σ and τ, which will provide coordinates for the worldsheet in spacetime. T is the tension of the string, and is related to the Regge slope term via $\frac{1}{T} = 2\pi\alpha'$. One then considers the relationship between a point on the worldsheet, labeled by (σ, τ), and the spacetime in which the string is embedded, giving $X^\mu(\sigma, \tau)$ ($\mu = 1, ..., d$, with d the dimension of spacetime)—the action then depends on this rather than a specific parameterisation. In other words, the action is invariant under arbitrary changes of the parameters, $\delta X^\mu(\sigma, \tau) = \xi^\alpha \partial_\alpha X^\mu(\sigma, \tau)$. The symmetry group of such reparameterisations (that leave invariant the action) is infinite dimensional.[22] The clear analogies between the particle theory action and the string theory action enables the carrying over of powerful techniques and ideas from the former to the latter case.[23] The reparameterisation invariance (with respect to σ, τ, or ξ, τ in Nambu's notation),

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[22] The group also reduces to the symmetry groups of general relativity and Yang-Mills theory as the Regge-Mandelstam parameter is sent to zero, though that was not known at the time.

[23] Of course, there are crucial differences too. The worldlines of interacting particles are given by graphs (with nodes) and therefore are not manifolds, whereas the worldsheets of splitting and joining strings *are* smooth manifolds (cf. [31, pp. 50–51].

suggests that the observable (physical) oscillations must be those *transverse* to the worldsheet, since any motions within the sheet can be gauged away by a suitable transformation of the worldsheet variables, σ and τ. In this way, the choice of action gives a representation involving the intrinsic, physical structure (with no ghosts).[24]

Nambu's original presentation of the action that now bears his name was included in notes (on "Duality and Hadrodynamics") that he had prepared in advance for a high energy physics symposium in Copenhagen in August 1970, though had in fact missed as a result of car trouble. Nambu had been invited to speak at the 1970 Copenhagen conference by Koba and Nielsen, and was also due to speak at the Rochester conference. Nambu describes the ensuing events:

> [I]n 1970, I remember there was a Rochester conference in Kiev to which I was invited, so I wanted to attend it. Now, I'd gotten my citizenship in 1970. I got a relief from my problem —I had some sort of immigration problem, but it was solved by then so I was able to go to Kiev. At the same time I got an invitation from the Copenhagen people for a summer institute or something, so I wanted to attend that too, to give a talk on my theory. It was in the summer of 1970 before going to Europe, I wanted to deposit my family in California with my friends. So we drove out from Chicago, and unfortunately on the way we had an accident on the road in the Salt Lake Desert. Actually the whole cooling system ruptured and the engine overheated and was destroyed. So we had to stay three days in the desert to fix it, and managed to get to California. But by then I had missed a plane connection, so I gave up going there and came back to Chicago. But in the meantime I had sent my manuscript to Copenhagen, hoping that it would come out eventually at the proceedings, which it did not.[25]

In fact, the notes were not available in published form until 1995, with the release of his collected papers [41]. Goddard claims that Nambu was known to have considered the geometric action in the advance copy of his paper for the Kiev conference, and news of his idea quickly spread by word of mouth [23, p. 238].[26]

Tetsuo Gotō covered much the same ground as Nambu independently in 1971 [25], with a more detailed (and published!) account of the same action (now called the Nambu-Gotō action).[27] Gotō referred to the string systems in terms of a one-dimensional mechanical continuum ("a finite one-dimensional continuous medium"), following an earlier paper by Takehiko Takabayasi [58].[28] He was primarily concerned with providing a string model explication of the Ward-like ghost

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[24] Lay Chang and Freydoon Mansouri [12] replicate Nambu's basic result, with a solid focus on analogies with gauge theories, constructing a diffeomorphism-invariant action from which gauge symmetries written in terms of string variables are constructed.

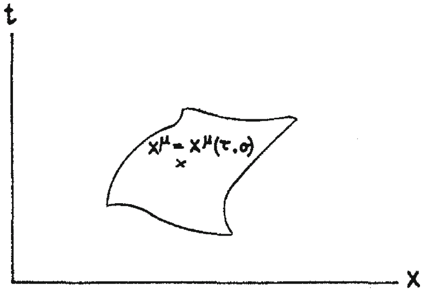
[25] Interview of Prof. Yoichiro Nambu by Babak Ashrafi on July 16, 2004, Niels Bohr Library and Archives, American Institute of Physics, College Park, MD USA, http://www.aip.org/history/ohilist/30538.html.

[26] This is a rare instance in (recent) history of science in which a name has been attached to a concept with neither a public presentation nor a published article.

[27] Gotō mentions Nambu's construction of the same concepts in a footnote, crediting Prof. Iisuka with informing him of Nambu's work after he had arrived at similar results independently [25, p. 1562].

[28] In fact, Gotō is well aware of the "elastic string" terminology, but he does not find elasticity in his model, only straight lines. Therefore, he suggests calling it a "linear *rod*" instead [25, p. 1568].

Fig. 4.1 Motion of Gotō's one-dimensional medium as represented by a two-dimensional world sheet (embedded in four-dimensional Minkowski spacetime). *Image source* [25, p. 1561]



cancellation mechanism of Fubini, Veneziano, and Virasoro: "relativistic quantum mechanics of a one-dimensional object with uniform mass density is equivalent to the so-called 'string' model of hadrons with Virasoro's subsidiary conditions" (p. 1560).

The terminology of 'world sheet' was used in his discussion, and the notation was much the same as Nambu's, though with the σ (rather than Nambu's ξ), for the spatial worldsheet coordinate. Thus, his positional coordinates on the worldsheet are $X^\mu = X^\mu(\tau, \sigma)$, which he labeled "Lagrange coordinates" (see Fig. 4.1).
His action takes the (generally invariant) form (where κ_0 is a mass density):

$$L = \int\int d\tau d\sigma \kappa_0 \sqrt{-\det g} \tag{4.12}$$

Note that neither Gotō nor Nambu (nor Susskind) considered the propagation of the strings in anything other than four-dimensional Minkowski spacetime—though Nambu did suggest including an additional fifth dimension for the oscillators, in order to avoid the problem of being forced into choosing a unit intercept, though this simply trades one unphysical feature for another. The full geometric spacetime interpretation of the quantised string, in terms of worldsheets being swept out in spacetime, was given in Goddard, Goldstone, Rebbi, and Thorn [GGRT: [22]] in which the theory is canonically quantized. When quantized, the action generates the parallel, linear Regge trajectories associated with the particles of the dual model.

One of the major consistency issues with the dual models was the fact that the Fock space of vertex operators includes ghost states (of negative norm). Negative probabilities do not make sense in quantum theory, so some method is needed for eliminating such states, giving the physical space of states.[29] This was achieved

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[29] We might note that Landau became convinced of the inconsistency of quantum electrodynamics as a result of similar ghost states (now called 'Landau ghosts') that appear in the computation of the self-energy of an electron, though the mass of the state was so minuscule as to render the inconsistency empirically inconsequential. In that case one employs the electromagnetic gauge invariance to eliminate the ghost states (the process involves the imposition of certain subsidiary (gauge) conditions that we will see are analogous to those used in the dual model case). (Note that this is not 'ghosts' in the sense of the so-called Faddeev-Popov ghosts, in which additional *unphysical* fields are integrated over to preserve the unitarity of gauge theories.)

through no-ghost theorems, which show how the physical amplitudes for processes do not depend on the ghosts. Such a theorem would give a zero value for amplitudes involving ghosts on external lines, and would ban ghosts from appearing as intermediate states (internal lines) connecting physical states. The same situation can be found in QED, in which the time component of the photon is unphysical, with the physical states being those satisfying the Gauss constraint. Lars Brink and David Olive [5] constructed a projection operator $\tau(k)$ onto physical states (where k is a vector used to build DDF states), allowing them to calculate planar loops with only physical states propagating internally.

The recognition of dual model ghosts came almost immediately with the construction of the operator approach to the dual resonance model devised by Fubini, Gordon, and Veneziano [17]. Recall that this had been initially constructed in order to understand the dual theory's spectrum by establishing factorization of the N-particle generalization of Veneziano's original 4-point amplitude. This involved a representation of the states built up from the vacuum state $|0\rangle$ via the action of an infinite collection of harmonic oscillators, a_n^μ (where the superscript $\mu = 0, 1, 2, 3$ is the Lorentz index, for a flat $\langle -, +, +, + \rangle$ spacetime, implying that the oscillator states must transform as representations of the Lorentz group; the subscript n refers to the integral mode number characterising a bosonic model). The problem with the Landau ghost states (that is the negative-norm states) stemmed from the existence (mathematically speaking) of the timelike modes a_n^0 which automatically point to negative-norm states. A sector of the ghost states was removed by the imposition of $SO(2, 1)$ symmetry, reducing out some of the surplus states. This was discovered independently by both Fubini and Veneziano [18] (while at MIT) and Bardakçi and Mandelstam [3] (at Berkeley). Within the Koba-Nielsen (complex) formalism Möbius invariance can likewise be imposed to reduce out the problematic states. In both cases, the complete elimination would require an infinite family of conditions, one for each possible timelike component in a_n^0—cf. Goddard [23, p. 237]. Interestingly, as mentioned in the previous chapter, the infinite set of such subsidiary conditions was isolated by Virasoro [62], with the infinite set of operators (underlying the conditions) generating what is now known as the Virasoro algebra.[30] But this solution, though it indeed eliminates all time components, had an unwelcome side-effect comparable to the ghost states it was devised to cure. In order to work, the Regge slope of the leading trajectory[31] $\alpha(0)$ had to be *fixed* at a value corresponding to an intercept of 1, which in turn implied that the ground state had $M^2 < 1$ (i.e. a tachyon). Negative probability was thus replaced with negative mass, again trading in one unphysical feature for another.

The *no-ghost theorem* begins with the Nambu-Gōto action and its great virtue of focusing the attention on the physical observables by means of the reparametrization

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[30] These operators satisfy the conditions for the algebra of 2D conformal mappings with a central charge. A fact that led to significant overlap with areas of pure mathematics.

[31] That is, lightest exchanged particle for a given spin.

invariance, forcing physical status on the transverse oscillations only.[32] Their approach was to canonically quantize the transverse degrees of freedom. The consistency conditions, of fixed intercept and spacetime dimension of 26,[33] were shown to arise once again in this context, here demanded by Lorentz invariance. Part of the machinery used was supplied by Brower, in the form of a spectrum-generating algebra, which provides a means of building up a space of physical states from a given physical state (e.g. the vacuum state) by the action of an appropriate operator (the Virasoro generators) [7].

Brower [7] and also Goddard and Thorn [21] were able to prove, independently, the "no-ghost theorem", according to which the dual theory in 26 dimensions doesn't possess negative norm vectors. Again, this number $d = 26$ was clearly playing a central role and, therefore, could not be dismissed so lightly as previously thought. The construction was based on vertex operators and propagators and the association of operator expressions to these. One can then build the S-matrix as a sum of contributions of such terms, in the standard way.

With the clarity provided by the GGRT paper, the string 'picture' was put onto a firmer footing. As Ferdinando Gliozzi puts it:

> only with the GGRT paper were all the consequences of this Nambu-Goto action correctly derived and it became completely clear, even at the quantum level, that the relativistic string was not simply an analogue model used to help intuition, but that it described the underlying microscopic structure of the DRM [20, p. 448].

That is, the string picture could be seen as a genuine physical interpretation of dual-resonance models such that there exists a correspondence between dual amplitudes and the amplitudes for strings. Despite coming after the initial 'golden age' of strings, the no-ghost theorem was pivotal in the theory's development since it firmly established its full mathematical consistency. However, problems still remained: there were massless particles that were not found at the predicted energy scales, and there were spatial dimensions that were demanded by the theory but not observed. Hence, the theory was still inconsistent with physical reality. We return to these problems below.

This marked a stage of development whereby the string theory was somewhat freed from its origins in the dual resonance model on which it had, up to this point,

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[32] Searching for a way to produce string theories in $D = 4$, Bardeen, Bars, Hanson, and Peccei [4] attempted to reintroduce the longitudinal modes into the theory, treating them as analogous to kink solutions of a nonlinear field theory.

[33] One of the additional consistency consistency checks converging on the meaningfulness of the $d = 26$ result (discussed more fully in the next section) was the realisation of the ghost-generating nature of variations of d above 26—for $d < 26$ Brower recalls running a recursive algebraic computer program to enumerate physical states to 30th level finding no ghosts for $d \leq 26$ [9, p. 317]. 26 will reduce to 10 in the case where fermions are included, as we will see in the discussion of supersymmetric strings in the next chapter. These implications, and the demonstration of the reduction to $D = 10$, are laid out in [21]. They had initially believed that this reduction might open up the possibility that "it will be four in some more realistic model" [21, p. 235]. The reasoning is clear: if adding additional structure, such as fermionic coordinates, can reduce the critical dimension so radically, then perhaps there are other structures, not yet understood, that could reduce this all the way down to $D = 4$.

been parasitic: strings were the *reason* for the dual resonance model and so could be pursued in their own right. We might further speculate that this detachment from the dual resonance model contributed to the transition from a model of strong interactions to other interactions. By 1973, then, thanks to the paper by GGRT, there was a fairly complete picture of the quantum mechanics of a relativistic string, albeit with the still peculiar restriction on the space-time dimensionality: physical states of the theory were defined as transverse modes of oscillation of a massless, relativistic string propagating in 26 dimensions.[34]

GGRT only studied the case of *free* strings, in which they can pass through one another. An important task that remained to be solved was that of incorporating interactions, and in such a way so as to not fall foul of the no-ghost theorem (enforcing the restriction to transverse states). Though the idea had been proposed in a qualitative fashion by several people, Mandelstam was responsible for making precise the idea that the scattering represented by the Veneziano amplitude could be understood in terms of the successive splitting and joining of strings, invoking Susskind's worldsheet idea: the dynamics of a string theory is fixed once the vertex for splitting and joining of strings is found.[35] This involves the overlap integral between the two input strings and the output string (or two input strings and a final string: in between interactions, the strings move freely).

One has a many-string formulation once one has the capacity to talks of splitting and joining. The operator formalism (Hilbert space) encodes this. For a non-interacting theory one simply has a term corresponding to the standard Nambu-Gōto Lagrangian per number of strings. Interactions are represented by an interaction vertex term which adds (splits) or subtracts (joins) strings Fig. 4.2.

Mandelstam's interacting strings model was able to recover the dual resonance model in $D = 26$. His method involves an extension of the results of GGRT to interacting particles.[36] String theories come in two varieties: bosonic and supersymmetric

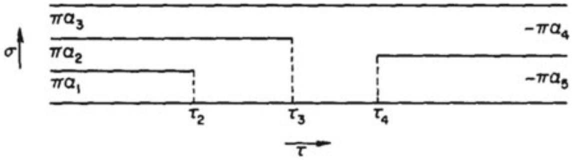

Fig. 4.2 Mandelstam's picture of string interaction. Here, three strings ($\pi\alpha_1$, $\pi\alpha_2$, and $\pi\alpha_3$) come in from $\tau = -\infty$, two of them join at τ_2, after which the resulting string joins a third at τ_3. The single string then splits into two at τ_4, which go out to $\tau = +\infty$. *Image source* [37, p. 208]

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[34] Note that the 'light-cone frame' was introduced in GGRT to provide a formalism in which the dynamics of (bosonic) strings was given in terms of $D-2$ oscillations propagating transverse to the string's length. This was used directly to quantise the string action, and has since been used many times to make calculations easier.

[35] He later extended this to the Ramond-Neveu-Schwarz model [38].

[36] Roscoe Giles and Charles Thorne [19] developed a lattice version of Mandelstam's argument to get around certain divergences associated with using a continuum (see also [60]).

(i.e. including fermions). The bosonic theory came first and is much simpler than the supersymmetric version. Mandelstam included fermions by adding a spinor field $(S_1(x_\mu), S_2(x_\mu))$ (describing a spin-wave respectively going from right to left across the string, and from left to right across the string).[37]

Mandelstam considered the first quantized theory in which the variables are the string coordinates, giving a kind of two-dimensional field theory on the worldsheet. The second quantized version of the theory (a purely bosonic string field theory based on "multilocal relativistic strings") was developed by Kaku and Kikkawa [29, 30]. As they note:

> Notice that, though the string picture presented so far resembles a second-quantized theory because of the presence of an infinite number of harmonic oscillators, it is actually only a first-quantized theory, because we are only quantizing the coordinate $X - \mu(\sigma)$. There are an infinite number of oscillators only because they represent the normal modes of the string, i.e., because the first quantization is performed over an extended object [29, p. 1113].

This was only carried out within the non-supersymmetric case. It wasn't until Green and Schwarz's work in the early 1980s that the superstring field case (in which quantum string-fields create and annihilate complete strings) was considered: [26].

4.3 Bootstrapping Spacetime

I think it is fair to say (and many others have said it) that just after Veneziano's paper was published, the centre of the dual model universe was CERN. Many of the key pieces of the theory of hadronic strings were put into place either by permanent staff members of CERN, visitors, or those just passing through. One of those was Claud Lovelace, who discovered the famous dual model consistency condition that demanded 26 spacetime dimensions—a kind of bootstrapping of spacetime.[38] Lovelace discovered, in 1970, that only if there were 26 dimensions of spacetime would certain problematic branch cuts become simple poles (thereby avoiding a violation of unitarity), with a Regge trajectory possessing an intercept of 2 and a slope of $\frac{1}{2}\alpha'$ (thus allowing for a particle interpretation, then given in term of the Pomeron). That is, unitarity (and so consistency) of the dual model seemed to *demand* $D = 26$. The set of properties (of the pole) corresponded to the Pomeron trajectory, as mentioned. It would not long after be reinterpreted as describing closed strings, eventually associated with the graviton.[39]

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[37] Influenced by Mandelstam's work, the (tree-level) treatment of interactions of closed with open strings was completed soon after, [1] (see also [20, p. 451].

[38] As Gomez and Ruiz-Altaba put it, "[t]he magic of the string approach to quantum gravity is that spacetime is not an ingredient put into the analysis from the start. It is philosophically astounding that spacetime is actually an output of string theory" [24, p. 83].

[39] David Olive recalls 'implanting' in Lovelace, at CERN in late June 1970, the idea that if the branch point singularity (then already identified by Lovelace) could be a simple pole instead of a branch cut then it could be interpreted as corresponding to the propagation of a new kind of particle [48, p. 348]. The branch point in question was discovered in a 1970 paper by Gross, Neveu, Scherk, and Schwarz [28]

In a paper written in the final period of Chew's original bootstrapping approach to physics,[40] he points out that, even if one could produce a unique S-matrix from his scheme, the S-matrix would depend on an underlying *a priori* space-time:

> We must not forget that, in the final analysis, the S-matrix depends on the arbitrary concept of space-time. From an ultimate bootstrap point of view, *all* concepts should be justified by self-consistency, none should be accepted on an *a priori* basis [13, p. 24].

Though it wasn't taken seriously at the time of discovery[41] (and for some time afterwards), the requirement on the space-time dimension, discovered by Lovelace, removes the arbitrariness in the way desired by Chew.[42] It might appear unphysical, but d *is* fixed by consistency conditions in dual models.

Lovelace was English by birth, born to a very wealthy family.[43] They moved to Switzerland when he was young, and he then did his undergraduate studies in Cape Town. In fact, he switched to architecture after completing his Bsc, but later switched back to physics, studying at Imperial with Abdus Salam. He describes his trajectory to the $D = 26$ result as follows:

> Gross et al. ... at Princeton, and Frye and Susskind... at Yeshiva had both found a very strange singularity in the one-loop amplitude. Like everyone else I thought that open strings were Reggeons, so this [singularity] must be the Pomeron, which would be very interesting to a phenomenologist. Unfortunately, it was tachyonic with a continuous spectrum. My notebooks show that I started redoing their calculations on 1 October 1970 at CERN. I needed a realistic model for phenomenology, so the Pomeron intercept had to be 1. By next day I had concluded that the intercept was $D/2$ in spacelike dimensions (i.e. those with oscillators). However, the ensuing calculations turned the cut into a pole by arbitrarily deleting log R factors. They go on for 88 pages until a note written in Princeton in early January says 'I think we need 24 spacelike and 2 *timelike* dimensions to get complete cut cancellation.' I suspect the correct solution came to me suddenly at night, since this note is in different ink. Thus in 26 dimensions, and assuming that two sets of oscillators decoupled, the Pomeron spectrum became discrete... There was still one tachyon, but the next particle had zero mass and spin two.[44] This matched the Shapiro-Virasoro formula [36, p. 199].

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[40] However, the approach did morph, taking in some of the features of duality, into "Dual Topological Unitarization" [DTU], again with the motto of 'no arbitrary parameters' centre stage—see, e.g., [14].

[41] With a few exceptions, by the late 1960s and early 1970s the notion of Kaluza-Klein compactification and theories invoking extra dimensions to perform various functions (though once popular) had dropped out of fashion.

[42] At the end of the same article, Chew writes: "it is plausible that to understand zero-mass phenomena through self-consistency may require bootstrapping space-time itself" (ibid., p. 28).

[43] Lovelace died in 2012, leaving to Rutgers $1.5 million for a chair in *experimental* physics. Clavelli notes that when he arrived in Rutgers, Lovelace was "still living in a motel and driving a rental car" (http://bama.ua.edu/~lclavell/papers/Tension1.pdf).

[44] Note that Lovelace writes in terms of Reggeons (with worldsheets described in "ribbon" terms) and Pomerons (with world*tubes*, or the surface of a closed tube). These correspond to what we would now think of open and closed strings. The Pomeron described by Lovelace in this paper was later identified with the graviton (once a scale change had been implemented). In his reminiscences about this paper he writes that, given his knowledge of unified field theory and Kaluza-Klein mechanisms (not least as a result of his studies with Salam), "I was inexcusably stupid not to see in 1971 that my Pomeron was the graviton" [36, p. 199]. This is, of course, overly harsh since there was

This appearance of the spacetime condition comes from the definition of the Pomeron propagator (see footnote 44):

$$(2\pi)^{-1}\int_0^1 dR\int_0^{2\pi} d\sigma\, R^{-1-\alpha_0^P-\frac{1}{2}\alpha' p^2}\mu(R)(Re^{i\sigma})^{n^{\Sigma n a_n^\dagger a_n}}(R)(Re^{i\sigma})^{n^{\Sigma n b_n^\dagger b_n}} \tag{4.13}$$

where

$$\alpha_0^P = \frac{(D-E)}{12} \tag{4.14}$$

$$\mu(R) = (\frac{-\pi}{\log R})^{1-E/2}\omega^{(D-E/24-1)}(1-\omega)^F \tag{4.15}$$

$$\omega = e^{2\pi^2/\log R} \tag{4.16}$$

The Pomeron–Reggeon coupling constant f (with g being the 3-Reggeon) is defined by:

$$f^2 = (2\pi)^{-3}2^{-D/2}g^2 \tag{4.17}$$

One has a self-consistent situation when $D = 26$, $E = 2$, and $F = 0$. Initially, Lovelace did not take the result at all seriously, in the sense of pointing to something deep about the physical world. He notes that in a seminar he gave at the Institute for Advanced Study in Princeton, in February 1971, he made the joke that he had "bootstrapped the dimension of spacetime but the result was slightly too big" (ibid.). In other words, self-consistency had forced the spacetime dimension to be 26, but at this time, of course, there was no connection to spacetime physics or gravitation. Nobody else took it seriously. This was supposed to be a theory of hadrons, pure and simple. It was only after the number $D = 26$ began to reappear, in the context of other consistency conditions such as the no-ghost theorem, that it was taken seriously as something potentially more significant.[45] Despite not thinking much of the result, Lovelace did nonetheless publish, albeit very briefly and with the qualifying remark

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(Footnote 44 continued)
at the time no reason whatsoever to connect up dual models with gravitational physics; that was something that would require the additional investigation of the zero-slope limit of dual models.

[45] Interestingly then, the no-ghost theorem demanded that the maximum number of spacetime dimensions (or a ghost-free theory) be 26, thus providing independent confirmation of the earlier critical dimension result of Lovelace. The decoupling of negative-norm states occurs only for $d \leq 26$ (with the additional Virasoroan condition that the Regge intercept $\alpha(0) = 1$). In this way a kind of mathematical unity was achieved, in which troubles of formalism (tachyon and $d = 26$) were integrated into a single scheme, and shown to be related. Clavelli and Shapiro combined the no-ghost theorem with Lovelace's earlier work on Pomeron factorization to argue forcefully for the existence of a critical dimension in ghost-free dual models such that in this dimension the Pomeron singularity becomes a factorizable Regge pole (which can, therefore, be viewed as a real particle). In the case of the Neveu-Schwarz model (discussed in Sect. 5.2), performing the same kind of procedure Lovelace had applied in the case of Pomerons (reducing cuts to poles, and preserving unitarity), they find $D = 10$, $E = 2$, and $F = 0$. Hence, the restriction on the number of spacetime dimensions was tightly bound to the consistency of the theory. Richard Brower was able to show, using his spectrum-generating algebra that $D = 26$ provides a maximum density of states consistent

that "$D = 26$ is obviously unworldly" [35, p. 502]. He also spread the idea around various colleagues, including many dual theorists at CERN. Since CERN was at the time a hotbed of activity on dual models, the idea was able to infiltrate the research landscape.

The root of the condition is the requirement that the action principle for string theory be conformally invariant. Conformal symmetry allows one to identify any diagrams (or processes) for which all angles are preserved. The laws of string theory are insensitive to conformal transformations of the string worldsheet. An anomaly refers, in this context, to a symmetry that is obeyed at the classical level, but violated quantum mechanically. Hence, given some operation $\mathscr{O}$ for which $\{\mathscr{O}, H\} = 0$, we have $[\hat{\mathscr{O}}, \hat{H}] \neq 0$. If one has such non-conservation for gauge currents (like the conformal symmetry) then the quantum theory is not consistent: it is found to violate unitarity and possibly will be rendered non-renormalizable. String theory was found to have such an anomaly concerning conformal symmetry. That this conformal anomaly cancels in 26 dimensions forms the heart of Lovelace's result.

In 1973, Holger Nielsen and Lars Brink published a paper [6] which analysed the notion of the critical dimension more deeply, providing an explanation (deriving it from a more physical argument)—this analysis covered both the 26 dimensional case, for strings with geometrical degrees of freedom, and the 10-dimensional case, with fermionic degrees of freedom too (i.e. the Neveu-Schwarz model). As they conclude: "we have found a physical interpretation of the ground state mass squared in string models as zero point fluctuations" pointing out that their result makes it "difficult to escape the dependence on the dimension of space-time for such models" [6, p. 336]. The argument was based on the idea that the physical degrees of freedom correspond to transverse degrees of freedom. A radically abbreviated run through goes as follows. The zero point energy of the (ground state) string is given as:

$$E_{zero} = d_{eff} \sum_{n=1}^{\infty} \frac{1}{2} \omega_n = d_{eff} \sum_{n=1}^{\infty} \frac{1}{2} \frac{1}{\alpha'} \frac{n}{2E} \tag{4.18}$$

As they note, for a string with only transverse modes, $d_{eff} = d - 2$. Next, the string is considered in the infinite-momentum frame, and the zero point energy is written as the difference between the quantum mechanical ground state and the classical version: $E_{zero} = E - |p|$, which in the infinite-momentum frame gives $2E = E + |p|$. This lets them rewrite Eq. 4.18 as:

$$E^2 - |p|^2 = \frac{d_{eff}}{4\alpha'} E(E + |p|) \int_0^{\infty} dy y f(y) - \frac{d_{eff}}{24\alpha'} + O\left(\frac{1}{E}\right) \tag{4.19}$$

(Footnote 45 continued)
with a positive-norm space (i.e. an absence of ghosts). He also argues that "at saturation ($D = 26$)" (that is, when thus fixed) the loop theory achieves its "most elegant" form, being non-renormalizable above this value [7, p. 1661]. Note that the $D = 10$ critical dimension was originally discovered by John Schwarz in [51].

They are able to show from this that the theory has a lowest state of mass squared: $m_0^2 = -d_{eff}/24\alpha'$. Since the string has only transverse degrees of freedom, Lorentz invariance forces the spin-1 particle on the leading Regge trajectory to have a mass. This implies:

$$m_0^2 = \frac{1}{\alpha'} = -\frac{d_{eff}}{24\alpha'} = -\frac{d-2}{24\alpha'} \tag{4.20}$$

This latter expression clearly demands $d = 26$.

4.4 Summary

By 1973/4 it was known that the quantization of free open strings and closed strings reproduces the spectra of the generalized Veneziano model and Shapiro-Virasoro models respectively: the oscillators of the dual resonance models corresponded to the normal modes of vibrations of a string. The interacting string theories were established (including open-closed interactions), and the role of the various consistency conditions (involving intercepts and spacetime dimensions) known and understood. Despite the fact that the dual model qua string theory idea was well in place in the early 1970s, it was then still considered tentative: a convenient model in which to think about the mathematical structure. It did not have a robust existence as a picture of string fields living in spacetime, for example. The string model provided a nice way of visualising processes that are rather difficult to handle in the operator approach. Hence, we should not be misled into thinking that string theory in anything like the modern sense (that is, a sense corresponding to the ‘real world’) was in operation in this initial phase.

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