Exchange Markets: Strategy Meets Supply-Awareness [*]

(Abstract)

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The exchange market model, proposed by Leon Walras (1874), has been studied extensively since more than a century due to its immense practical relevance [8,14]. The two implicit assumptions in this model are that agents behave truthfully, and are unaware of the total supply of goods in the market. In this paper we study exchange markets, with each of these assumptions dropped separately, and establish a surprising connection between their solutions.

The strategic behavior of agents is well known; many different types of market games have been formulated and analyzed for its Nash equilibria [1,2,5,6,7,13]. Generalizing the Fisher market[1] game of [1], we define the *exchange market game*, as where agents are the players and strategies are the utility functions that they may pose. We derive a complete characterization of the symmetric Nash equilibria (SNE) of this game, for the case when utility functions are *linear*.

Using the characterization of SNE we obtain: (i) the payoffs at SNE are always Pareto-optimal, and (ii) every competitive equilibrium allocation can be achieved at a SNE. Apart from these, we also obtain structural properties for the SNE set, like (iii) connectedness, and (iv) the necessary and sufficient conditions for uniqueness. These properties are important in equilibrium theory, both competitive and Nash, and a lot of work has been done to characterize such instances [2,7,9,10,11,12,13].

The other assumption that agents are unaware of the total supply of goods in the market, may not hold in many rural and informal markets where supplies are visible. Given that agents know the supply of all the goods, it is rational for them to take the supplies into consideration while calculating their demand bundles. This will change the demand dynamics, and as a consequence the set of competitive equilibria. Such a setting has been analyzed for auction markets [3,4], however to the best of our knowledge no such work for exchange markets is known.

We make significant progress towards understanding the effects of supply-aware agents in exchange markets. We show that the set of competitive equilibria

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[1] Fisher market is a special case of exchange market model.

(CE) of such a market is equivalent to the set of SNE of the corresponding exchange market game. Through this equivalence, we obtain both the welfare theorems, and connectedness and uniqueness conditions of CE for the supply-aware markets with *linear* utilities.

Finally, for markets with arbitrary concave utilities, we derive sufficiency conditions for a strategy to be a symmetric Nash equilibrium, while restricting strategies of the agents to linear functions in the game. Using these conditions we obtain the first two properties, namely, Pareto-optimality and achieving CE allocations at SNEs, for this general setting. Further, we extend the connection between CE and SNE to markets with concave utility functions, and as a consequence obtain both the welfare theorems for the *supply-aware* markets in general.

We note that even though supply-awareness may be thought of as exchange markets with concave utility functions, where no more utility is obtained after the available supply of goods, the welfare theorems do not follow directly as they require non-satiated utility functions [15].

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