Robust Continuous Terminal Sliding Mode Control Design for a Near-Space Hypersonic Vehicle

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Abstract. This paper presents a robust continuous terminal sliding mode attitude tracking control approach for a near-space hypersonic vehicle(NSHV) in the presence of parameter uncertainties and external disturbances. Firstly, a novel nonsingular terminal sliding surface is developed. Then, a continuous terminal sliding mode control law is proposed, which is chattering-free. Afterward, considering the presence of parameter uncertainties and external disturbances, a high order sliding mode disturbance observer is introduced to estimate the lumped disturbance to improve the control performance. Finally, numerical simulations applied to the NSHV illustrate the effectiveness of the proposed approach.

Keywords: Terminal sliding mode control, Finite-time convergence, Near-space hypersonic vehicle, Tracking control.

1 Introduction

NSHVs offer a great potential for feasible access to space and high speed civil transportation. The control design for NSHVs faces significant challenging because of NSHVs' strong nonlinearity, highly time-varying dynamics, large parameter uncertainties. Up to now, various nonlinear control approaches have been presented to tackle this problem, such as dynamic inversion [1], backstepping control[2,3], sliding mode control (SMC) [4,5].

Among these control approaches, sliding mode control is a well-known powerful control scheme which has many attractive characteristics such as good transient, fast response and insensitivity to parameter uncertainties and external disturbances. Therefore, SMC has been widely applied for flight control.Howerver, SMC has two disadvantages. The first is that the linear surface usually used in SMC design only can guarantee the asymptotic stability. The second is chattering phenomena, which is the main drawback of SMC.

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To address these issues. in this study, we develop a new robust continuous terminal sliding mode attitude tracking control scheme for the NSHV under the parameter uncertainties and external disturbances. Firstly, a novel nonsingular terminal sliding surface is proposed. Then, based on the Lyapunov method, a continuous terminal sliding mode control law is derived to guarantee the existence of the sliding mode within finite time. Further, to cope with the parameter uncertainties and external disturbances, a high order sliding mode disturbance observer is used to estimate the lumped disturbance. Finally, the proposed approach is applied to design the attitude control system for the NSHV and simulation results is presented to demonstrate the effectiveness of the current method.

2 Robust Continuous Terminal Sliding Mode Control Design

A nonlinear system under model uncertainties and external disturbances is described as

$$\dot{x} = f(x) + \varDelta f(x) + (g(x) + \varDelta g(x))u + d(t) \tag{1}$$

where $x \in \Re^n$ is the state vector of the system. $u \in \Re^n$ is the control input vector. $f(x) \in \Re^n, g(x) \in \Re^{n\times n}$ are known nonlinear system functions of the state variables and time.Moreover, $g(x)$ is invertible. $\varDelta f(x), \varDelta g(x)$ represent model uncertainties. $d(t)$ denotes the external disturbance.The lumped disturbance $\psi = \varDelta f + \varDelta gu + d$ represents a lumped disturbance, which is differentiable and has a known Lipshitz constant $L_i > 0$.

To design nonsingular terminal sliding mode control, a novel nonsingular terminal sliding surface is designed as:

$$s = e + \int_0^t (k_1 sig^{\gamma_1}(e) + k_2 sig^{\gamma_2}(e))d\tau \tag{2}$$

where $e = x - x_d$, $k_1 = diag(k_{11}, \ldots k_{1n})$, $k_2 = diag(k_{21}, \ldots k_{2n})$. $k_{ij} > 0\,(i = 1, 2, j = 1, \ldots n)$, $\gamma_1 \geq 1$ and $0 < \gamma_2 < 1$ are constants.

Once the tracking error reaches the sliding surface, it satisfies the equation $\dot{s} = 0$. Then the sliding mode dynamics is derived as follows:

$$\dot{e} = -k_1 sig^{\gamma_1}(e) - k_2 sig^{\gamma_2}(e) \tag{3}$$

By solving the differential equation (3), it can be obtained that $e_{si} = 0$ will be reached in a finite time determined by

$$\begin{aligned} t_{si} = \int_0^{|e_{si}(0)|} \frac{1}{k_{s1} e_{si}^{\gamma_{s1}} + k_{s2} e_{si}^{\gamma_{s2}}} de_{si} = \frac{|e_{si}(0)|^{1-\gamma_{s1}}}{1-\gamma_{s1}} k_{s1}^{(1-\gamma_{s1})/\gamma_{s1}} \\ \times F(1, \tfrac{\gamma_{s1}-1}{\gamma_{s1}-\gamma_{s2}}; \tfrac{2\gamma_{s1}-\gamma_{s2}-1}{\gamma_{s1}-\gamma_{s2}}; -k_{s2}k_{s1}^{-1} \left|\left|e_{si}(0)\right|\right|^{\gamma_{s2}-\gamma_{s1}}), i = 1, 2, 3 \end{aligned} \tag{4}$$

where $F(\cdot)$ denotes Gauss' Hypergeomeric function.

After the nonsingular terminal sliding surface is established, then a nonsingular terminal sliding mode controller is proposed without considering the presence of parameter uncertainties and external disturbances, as follows:

$$u = -g^{-1}[f - \dot{x}_d + k_1 sig^{\gamma_1}(e) + k_2 sig^{\gamma_2}(e) - l_1 s + l_2 sig^{\eta}(s)] \tag{5}$$

where $l_1 = diag(l_{11}, \ldots l_{1n}), l_2 = diag(l_{21}, \ldots l_{2n}), 0 < \eta < 1$. η and $l_{ij}(i = 1, 2, j = 1, \ldots n)$ are positive constants.

Theorem 1. Considering the nonlinear system(1) in the absence of parameter uncertainties and external disturbances, if the sliding surface is designed as (2) and the controller is constructed as (5), then the tracking error e will reach the sliding surface in a finite time T, given by

$$T \leq \frac{\ln(\frac{\underline{l}_2 - 2^{\frac{1-\eta}{2}} \bar{l}_1 V^{\frac{1-\eta}{2}}}{\underline{l}_2})}{\bar{l}_1(1-\eta)} \tag{6}$$

Proof: Differentiating sliding variable (2) and using (1) and (5), we can get the closed-loop sliding dynamic equation as

$$\dot{s} = l_1 s - l_2 sig^{\eta}(s) \tag{7}$$

Consider the following Lyapunov function candidate $V = \frac{1}{2} s^T s$

Taking the time derivative of Lyapunov function and using (7), one can get

$$\dot{V} = s^T \dot{s} = s^T [l_1 s - l_2 sig^{\eta}(s)] = s^T l_1 s - s^T l_2 sig^{\eta}(s) \tag{8}$$

$$\leq \bar{l}_1 \|s\|^2 - \underline{l}_2 \|s\|^{\eta+1} = 2\bar{l}_1 V - 2^{\frac{\eta+1}{2}} \underline{l}_2 V^{\frac{\eta+1}{2}} \tag{9}$$

where $\bar{l}_1 = \max_{i=1,\ldots n} \{l_{1i}\} > 0$ and $\underline{l}_2 = \min_{i=1,\ldots n} \{l_{2i}\} > 0$.

Therefore, according to Lemma 2 in [7] , the tracking error $e(t)$ will reach the sliding surface in a finite time T .

Once the sliding mode $s = 0$ is reached, tracking error $e(t)$ will converge to zero in the sliding mode within a finite time .

Remark 1. The terminal sliding mode control given in (5) is a novel continuous second order sliding mode control since the condition $s = \dot{s} = 0$ is satisfied on the sliding surface[6].

When considering the presence of parameter uncertainties and external disturbances of nonlinear system (1), the upper bound of the lumped disturbance ψ is usually unknown. Therefore, a high order sliding mode disturbance observer is introduced to estimate the lumped disturbance. ψ_i can be smoothly estimated by the following high order siding mode disturbance[6].

$$\begin{aligned}
\dot{z_{0i}} &= v_{0i} + u_i \\
v_{0i} &= -2L_i^{1/3} |z_{0i} - s_i|^{2/3} sign(z_{0i} - s_i) + z_{1i} \\
\dot{z_{1i}} &= v_{1i} \\
v_{1i} &= -1.5L_i^{1/2} |z_{1i} - v_{0i}|^{1/2} sign(z_{1i} - v_{0i}) + z_{2i} \\
\dot{z_{2i}} &= -1.1Lsign(z_{2i} - v_{1i}) \\
i &= 1, ..., n
\end{aligned} \tag{10}$$

Then $z_1 = [z_{11}, z_{12}, ..., z_{1n}]^T = \hat{\psi}$ converges to ψ in a finite time, if the sliding variable s and control input u are measured without noise.

Finally, based on the combination of (5) and (10), a robust continuous terminal sliding mode control (RCTSMC) for the nonlinear system(1) is proposed under parameter uncertainties and external disturbances.

$$u = -g^{-1}[f - \dot{x}_d + k_1 sig^{\gamma_1}(e) + k_2 sig^{\gamma_2}(e) - l_1 s + l_2 sig^{\eta}(s) + z_1] \tag{11}$$

where $l_1 = diag(l_{11}, \dots l_{1n}), l_2 = diag(l_{21}, \dots l_{2n}), 0 < \eta < 1$. η and $l_{ij}(i = 1, 2, j = 1, \dots n)$ are positive constants.

Differentiating sliding variable (2) and using (1) and (11), we can obtain the following closed-loop dynamics

$$\dot{s} = l_1 s - l_2 sig^{\eta}(s) - z_1 + \phi \tag{12}$$

After z_1 converges to ψ in finite time.The system is reduced to the system

$$\dot{s} = l_1 s - l_2 sig^{\eta}(s) \tag{13}$$

The system (13) repeats the system (7) which is finite-time stable.

The approach described above will be used to carry out tracking simulations for the NSHV discussed in next Section.

3 Simulation Applied to the NSHV

The considered attitude control model of the NSHV is derived from the six-degree of freedom and twelve-state kinematic equations which can be simplified as the affine nonlinear equation as follows[8]:

$$\begin{cases} \dot{\Omega} = f_s + \Delta f_s + (g_s + \Delta g_s)\omega \\ \dot{\omega} = f_f + \Delta f_f + (g_f + \Delta g_f)M_c + d(t) \end{cases} \tag{14}$$

where $\Omega = [\alpha, \beta, \mu]^T$ is the state vector of the slow loop which is the attitude angle of NSHV including angle of attack, sideslip angle and bank angle. $\omega = [p, q, r]^T$ is the fast-loop state vector. M_c is the control torque vector. $g_s, g_f \in R^{3\times 3}$ are the invertible matrices and $f_s, f_f \in R^3$. The concrete expressions of the above matrixes are specified in [8]. $\Delta f_s, \Delta f_f, \Delta g_s$ and Δg_f are model uncertainties induced by the system parameter uncertainties. $d(t)$ is the external disturbance. Next, according to Theorem 1, we will design the controller for the NSHV(14) to carry out the simulation. For comparative study, the conventional terminal sliding mode control (CTSMC) law based on the constant-rate reaching law is also employed to design the controller for the NSHV. Simulation results are shown in Fig.1.

The simulations are carried out for initial conditions with $V_0 = 2.6km/s$, $H_0 = 30km$, $\alpha_0 = 0.1^{o}$, $\beta_0 = 0^{o}$, $\mu_0 = 0.1^{o}$ and $p_0 = q_0 = r_0 = 0^{o}/s$. The command signals are chosen to be $\alpha_c = 1.5^{o}, \beta_c = 0^{o}, \mu_c = -1^{o}$. Assume that there exist $-30\% \sim +30\%$ random uncertainties in the aerodynamic coefficients. Besides, the external disturbance moment is defined as $d(t) = 10^4 \sin(t)\ [1\ 1\ 1]^T N \cdot M$. The parameters of the controller designed by the proposed approach are set

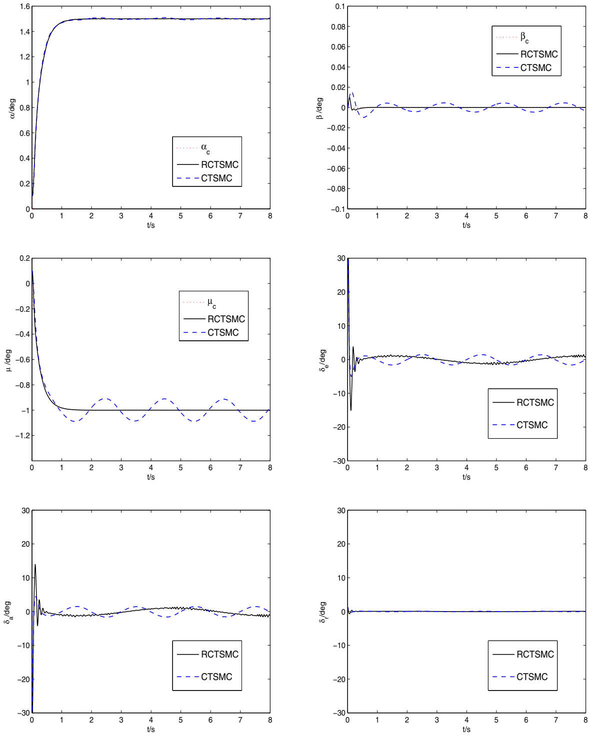

Fig. 1. Comparison of attitude control performance

as $\gamma_{s1} = 1.2, \gamma_{s2} = 0.8, k_1^s = 0.2I, k_2^s = 0.5I\eta_s = 0.7, l_1^s = 3I, l_2^s = 2I, \gamma_{f1} = 1.2, \gamma_{f2} = 0.8, k_1^f = 0.2I, k_2^f = 0.5I, \eta_f = 0.82, l_1^f = 1I, l_2^f = 4I.$

Fig.1 shows that the proposed approach attains a higher precision and better dynamic performance that CTSMC approach. Morover, it can be referred that the proposed approach can eliminate the chattering phenomena and provide a continuous tracking control law.

4 Conclusions

In this paper, we have described the design of a robust continuous terminal sliding mode controller for the NSHV. The proposed approach not only provides a finite-time convergence, but also eliminates the chattering phenomena. Simulation experiments show good performance of the presented approach for the NSHV attitude control.

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