32. Cosmology with the Cosmic Microwave Background

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The *standard* model of cosmology must not only explain the dynamics of the homogeneous background universe, but also satisfactorily describe the perturbed universe – the generation, evolution and finally, the formation of large-scale structures in the universe. Cosmic microwave background (CMB) has been by far the most influential cosmological observation driving advances in current cosmology. Exquisite measurements from CMB experiments have seen the emergence of a *concordant* cosmological model. Besides precise determination of various parameters of the *standard* cosmological model, observations have also established some important basic tenets that underlie models of cosmology and structure formation in the universe. The article reviews this aspect of recent progress in cosmology for a general science reader.

32.1 Contemporary View of our Cosmos

The universe is the grandest conceivable scale on which the human mind can strive to understand nature. The amazing aspect of cosmology, the branch of science that attempts to understand the origin and evolution of the universe, is that it is largely comprehensible by applying the same basic laws of physics that we use for other branches of physics. Historically, theoretical developments always preceded observations in cosmology up until the past couple of decades. Recent developments in cosmology have been largely driven by huge improvement in quality, quantity, and the scope of cosmological observations.

We will avoid giving a historical perspective. The theoretical model of cosmology, the Hot Big Bang model (HBBM), has broadly remained as it was established and widely accepted by the late 1960s. This is readily available in most standard textbooks, as well as, many semipopular books. The perspective would be to review the theoretical model of cosmology in the light of the available data. The main goal is to convey the excitement in cosmology where amazing observations have now concretely verified that the present edifice of the standard cosmological models is robust. A set of foundation and pillars of cosmology have emerged and are each supported by a number of distinct observations:

- Homogeneous, isotropic cosmology, expanding from a hot initial phase due to gravitational dynamics of the Friedmann equations derived from laws of general relativity.
- The basics constituent of the universe are baryons, photons, neutrinos, dark matter, and dark energy (cosmological constant/vacuum energy).
- The homogeneous spatial sections of spacetime are nearly geometrically flat (Euclidean space).
- Evolution of density perturbations under gravitational instability has produced the large-scale structure in the distribution of matter starting from the primordial perturbations in the early universe.
- The primordial perturbations have correlation on length scales larger than the causal horizon that makes a strong case for an epoch of inflation in the very early universe. The nature of primor-

dial perturbation matches that of the generation of primordial perturbations in the simplest model of inflation.

The cosmic microwave background (CMB), a nearly uniform, thermal black-body distribution of photons throughout space, at a temperature of 2.7 K, accounts for almost the entire radiation energy density in the universe. Tiny variation of temperature and linear polarization of these black-body photons of the cosmic microwave background arriving from different directions in the sky faithfully encodes information about the early universe and have traveled unimpeded across the observable universe, making them an excellent probe of the universe.

There are two distinct aspects to modern day cosmology – the background universe and the perturbed universe. The *standard* model of cosmology must not only explain the dynamics of the homogeneous background universe, but also satisfactorily describe the perturbed universe – the generation, evolution, and, finally, the formation of large-scale structures in the universe. It is fair to say that cosmology over the past few decade has increasingly been more dominated by the interplay between the theory and observations of the perturbed universe – the origin and evolution of large-scale structures in the matter distribution. The past few years have seen the emergence of a *concordant* cosmological model that is consistent both with observational constraints from the background evolution of the universe as well that from the formation of large-scale structures (LSS) in the universe. In particular, the much talked about dawn of *precision* era of cosmology has been ushered in by the study of the perturbed universe. Measurements of CMB anisotropy and polarization have been by far the most influential cosmological observation driving advances in current cosmology in this direction.

32.2 The Smooth Background Universe

In recent years, vast cosmological surveys have provided a three-dimensional map of the distribution of millions of galaxies extending to a billion light-years around us. If theorists were to start building a model of cosmology today, this would be the cosmos they would need to explain. As shown in Fig. 32.1, there is a rich organized structure in the distribution of galaxies in a region of about 100 Mpc. However, this is a typical (statistically speaking) sample of mass distribution. In other words, the mass distribution in the universe averaged over regions of a few hundred mega-parsecs is fairly uniform. A stronger case for the homogeneous cosmology actually comes from the high degree of uniformity in the temperature of the CMB. These provide observational support for the cosmological principle that postulates a homogeneous universe invoked by theorists in the 1920 to 1930s to develop the first physical models of cosmology.

The evolution of the universe is an initial value problem in general relativity that governs Einstein's theory of gravitation – the dynamical evolution in time of the three-dimensional spatial sections in the foliation of spacetime. The, now observationally confirmed, large-scale homogeneity and isotropy of the matter distribution implies that the spatial sections of the universe are homogeneous (i. e., 3-D spaces of constant curvature). This reduces the problem to one of the simplest applications of general relativity formulated as a dynamical system. The dynamics of the spatial sections reduces to the time evolution of the scale factor $a(t)$ of the spatial section. Averaged on large scales, the spatial sections at any time t are simply a scaled version of the present universe at time t_0 – i. e., the physical distance between two points in the universe at time t is given by $a(t)d$, where d is the present distance. It is convenient to define $a(t_0) = 1$ with no loss of generality. Observationally, the expansion of the universe causes a redshift $z = (1-a)/a$, of the spectrum light from a (cosmologically) distant astrophysical object (galaxy or quasar) emitted at a time t, when the universe had a scale factor a. The observation that all galaxies (on the average) appear to have a redshift in the spectra proportional to their distance confirms the expansion of the universe. The cosmic time t, the scale factor $a(t)$, and the redshift z can be used interchangeably to label spatial hyper-surfaces of the evolving universe.

The dynamics of the universe is encoded in the simple Friedmann equation

$$\begin{aligned} H^2(t) &\equiv \left(\frac{\dot{a}}{a}\right)^2 \\ &= \frac{8\pi G}{3}\,\rho_c(\Omega_\mathrm{m} + \Omega_\mathrm{r} + \Omega_\Lambda + \Omega_\mathrm{K})\,, \end{aligned} \tag{32.1}$$

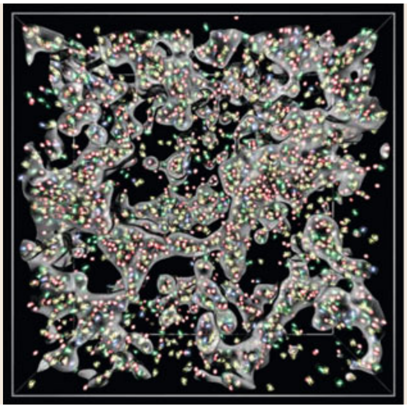

Fig. 32.1 The figure depicts the typical structure in the three-dimensional distribution of galaxies in the universe using a 100 Mpc sized cube carved out from the 2def Galaxy Redshift Survey (2dFGRS). The locations of galaxies are marked by a *toy* image colored according to the galaxy type. The gray shading is a visual aid that highlights the density contrast in the distribution by marking a region of approximately constant density (courtesy Paul Bourke/Swinburne Centre for Astrophysics and Supercomputing and the 2dFGRS Team)

deduced from the Einstein equations. It relates the Hubble parameter $H(t)$, that measures the expansion rate of the universe, to the matter density in the universe. Here we use the conventional dimensionless density parameter $\Omega_i = \rho_i/\rho_c$ in terms of the critical density $\rho_c = 3H^2/8\pi G$ at that time. The key components of the universe are radiation Ω_r, pressure-less gravitating matter Ω_m, and cosmological vacuum (dark) energy Ω_Λ. The departure of the total matter density parameter from unity contributes to the curvature of the space and can, hence, be represented by an effective curvature energy density Ω_K that determines the effect of curvature on the expansion of the universe. (Note that Ω_K is only a convenient notation and not a physical energy density, in particular, the *curvature density* is negative when the spatial section of uniform positive curvature.) Dividing out (32.1) by H^2 on both sides leads to a simple sum rule that summarizes the evolution of the universe

$$\Omega_\mathrm{m} + \Omega_\mathrm{r} + \Omega_\Lambda + \Omega_\mathrm{K} = 1 \,. \tag{32.2}$$

Since the expansion rate $H(t)$ evolves with time, Ω_i are time dependent. Further, the components (species) of matter are assumed to be noninteracting (on cosmological scales), ideal, hydrodynamic fluids, specified by their energy/mass density ρ_i and the pressure p_i (equivalently, by the equation of state w_i, where $p_i = w_i\rho_i$). For given species, the evolution of the density ρ_i is governed by the conservation equation of the energy–momentum tensor. In a volume V_0 in the current universe, the conservation equation implies

$$\begin{gathered} d(\rho_i a^3 V_0) + p_i(3a^2)V_0 = 0\,, \quad \text{or} \\ \mathrm{d}E + p\,\mathrm{d}V = 0 \end{gathered} \tag{32.3}$$

where in arriving at the second equation we use the fact that the physical volume $V = V_0 a^3$. The second equation resembles the first law of thermodynamic for an isentropic system with energy E and work done under pressure p (recall, $\mathrm{d}E + p\,\mathrm{d}V = T\,\mathrm{d}S$). It is straightforward to derive the scaling of the energy density ρ_i with the evolution of the universe as relative to its present value ρ_{0i} as

$$\frac{\rho_i}{\rho_{0i}} = a^{-3(1+w_i)} \,. \tag{32.4}$$

The equation of the state characterizes the ideal cosmological fluid, e.g., $w = 1/3$ for radiation (relativistic matter); $w = 0$ for pressure-less (nonrelativistic matter), curvature density can be expressed as an ideal fluid with $w = -1/3$, and $w = -1$ for a cosmological constant (in general, for the dark energy component $w < -1/3$).

The entire dynamics of the universe is then completely determined by the present matter composition. Explicitly, (32.1) and (32.4) lead to the more commonly seen version of the Friedmann equation

$$\begin{aligned} H^2(t) &\equiv \left(\frac{\dot{a}}{a}\right)^2 \\ &= \frac{8\pi G}{3}\rho_{0c}\left[\Omega_{0\mathrm{m}}a^{-3} + \Omega_{0\mathrm{r}}a^{-4}\right. \\ &\qquad \left. + \Omega_\Lambda + \Omega_{0\mathrm{K}}a^{-2}\right] . \end{aligned} \tag{32.5}$$

Equation (32.5) shows that the energy in an expanding universe is dominated successively by matter with a smaller value of w – i. e., first a radiation dominated phase Ω_r, followed by *matter-dominated* Ω_m, then curvature-dominated Ω_K and finally a cosmological vacuum (dark) energy Ω_Λ.

The relativistic mater density is almost entirely dominated by the CMB and the relic background of three species of light neutrinos (expected to have a density 68% of that of the CMB). The isentropic expansion dictated by the Friedmann equations implies that although, at present (given by the temperature of the CMB), $\Omega_{0\mathrm{r}}$ is negligible, at an early epoch the universe was dominated by relativistic matter density. The

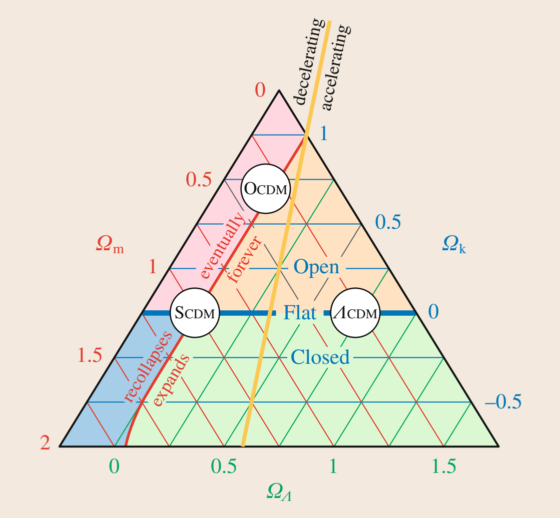

Fig. 32.2 The *cosmic triangle* represents the three key cosmological parameters – Ω_m, Ω_Λ, and Ω_k – where each point in the triangle satisfies the sum rule $\Omega_m + \Omega_\Lambda + \Omega_k = 1$. The *blue horizontal line* (*marked Flat*) corresponds to a flat universe ($\Omega_m + \Omega_\Lambda = 1$), separating an open universe from a closed one. The red line, nearly along the $\Lambda = 0$ line, separates a universe that will expand forever (approximately $\Omega_\Lambda > 0$) from one that will eventually recollapse (approximately $\Omega_\Lambda < 0$). And the *yellow, nearly vertical line* separates a universe with an expansion rate that is currently decelerating from one that is accelerating. The locations of three key models are highlighted: (Flat) standard cold dark matter (SCDM); flat (ΛCDM – Lambda-cold dark matter); and open cold dark matter (OCDM) (after [32.1, 2])

pressure-less matter density $\Omega_m = \Omega_B + \Omega_{cdm} + \Omega_\nu$ minimally consists of three distinct components, the baryonic matter, cold dark matter, and a possibly minor contribution from massive neutrino species. The constraint on the Baryon density $\Omega_B h^2 = 0.022 \pm 0.002$ from the predicted abundances of light elements from Big-Bang nucleosynthesis (BBN) is consistent with that recently obtained from considerations of structure formation.

The present state of the universe in terms the three dominant components can be neatly summarized on the *cosmic triangle* shown in Fig. 32.2 [32.2]. The three axes address fundamental issues regarding background cosmology – Does space have positive, negative or zero curvature (Ω_{0K})? Is the expansion accelerating, or decelerating (determined by Ω_Λ)?, and, what is the fraction of the nonrelativistic matter, (Ω_{0m})?

Historically, the focus has shifted between different sectors of the cosmic triangle depending on which of the three is the dominant player Ω_{0m}, Ω_{0K}, or, Ω_Λ. The canonical standard cold dark matter (SCDM) is a model where the present universe is a flat universe ($\Omega_{0K} = 0$) dominated by nonrelativistic matter density $\Omega_{0m} = 1$ ($\Rightarrow \Omega_\Lambda = 0$). This is also theoretically the simplest since it avoids the fine tuning problem of having a curved universe by invoking inflation and was the favorite in the 1980s. The nonrelativistic matter had to be mostly nonbaryonic dark matter (i. e., matter than does not interact with light), since Big-Bang nucleosynthesis and the absence of CMB temperature fluctuations at the power level of $\approx 10^{-4}$ limit the baryonic fraction to a much smaller value than that inferred for Ω_{0m}. (Nonbaryonic dark matter component has to be nonrelativistic to satisfy power spectrum measurements of the LSS.)

At the end of the 1980s and early 1990s, observations of LSS made it clear that Ω_{0m} was much smaller than unity. The sum rule, (32.2), then implies that either Ω_{0K}, or Ω_Λ, or both had to be non zero. The theoretical discomfort with a nonzero Ω_Λ (that still persists today) led to the era of open cold dark matter (OCDM) models, where $\Omega_{0K} > 0$. The conflict $\Omega_{0K} \neq 0$ with a robust prediction of inflation promptly development of *open* models of inflationary scenarios that could avoid this problem.

Toward the end of the 1990s, the observation of a high-redshift supernova indicated an acceleration in the present expansion universe. Very soon after, CMB anisotropy observations revealed a flat universe ($\Omega_{0K} = 0$). This leads to the currently favored ΛCDM model in cosmology. The energy density of the cosmological constant (or, more broadly quintessence) can be inferred from the measurement of luminosity distance as a function of redshift using the high-redshift supernova SN Ia as standard candles. In this chapter, we limit our attention to the simplest case of a cosmological constant that has a constant equation of state $w = -1$, which is also completely consistent with all observations to date. Alternative propositions for the nature of the dark energy are discussed in the chapter by Tsujikawa in this volume.

The key program of the Hubble space telescope (HST) mission measured the expansion rate of the universe $H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ in 2001. Recently, new Spitzer calibration, has allowed the systematic uncertainty in H_0 from the HST key project to be decreased by over a factor of 3. Also optical and infrared observations of over 600 Cepheid variables in the host galaxies of eight recent Type Ia supernovae (SNe Ia) determines $H_0 = 73.8 \pm 2.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. This is broadly consistent with the constraints from the CMB anisotropy and large-scale structure observations and combined constraints are remarkably tight. Cosmological observations have definitively determined the present universe to be located in the Λ-CDM sector. (The above improvement in H_0, combined with Wilkinson microwave anisotropy probe (WMAP)-7yr data, results in a strong constraint on the nature of dark energy $w = -1.08 \pm 0.10$, close to a cosmological constant.) The expansion rate and age estimates of the present universe measured from CMB data are again consistent with, and considerably improved in precision by including structure formation consideration.

One the most crucial observational pillars that support the HBBM of the background universe is the CMB discussed in the next section.

32.3 The Cosmic Microwave Background

The CMB, a nearly uniform, thermal black-body (Planck) distribution of photons throughout space, at a temperature of 2.7 K, accounts for almost the entire radiation energy density in the universe. The HBBM ascribes cosmic significance to this microwave radiation background, and hence CMB measurements play a role of great importance. In this widely accepted view, the CMB comprises the oldest photons that last interacted when the universe was only 380 000 yr old (compared to the present age of about 14 billion years). The photons have freely traveled right from the edge of the observable universe a distance of about 43 billion light years (14 Gpc) as explained in Fig. 32.3.

The prediction of the Planck distribution of the CMB in the HBBM dates from the early nucleosynthesis calculations of Gamow and collaborators in 1948. Thermal equilibrium in the early universe establishes a Planck energy distribution for the photons. In the HBBM the universe expands adiabatically conserving the photon entropy per comoving volume. (The observed CMB accounts for almost all the entropy.) The adiabatic Hubble expansion conserves the Planck distribution. However, the energy density of photons $\rho_{\mathrm{r}} \propto a^{-4}$ in an expanding universe (see (32.4) for radiation $w = 1/3$). Recalling, that the energy density of a black body is proportional to the fourth power of the temperature, it is clear that the temperature of the CMB photons $T_{\mathrm{cmb}} = T_{0\mathrm{cmb}}/a = T_{0\mathrm{cmb}}(1+z)$ scales as the inverse of the expansion of universe. At a redshift of $z_{\mathrm{rec}} \approx 1100$, the temperature of CMB falls below the threshold required to keep the hydrogen atoms in the universe ionized. At this epoch of *recombination* at around $t = 380\,000$ yr, the protons and electrons form a neutral hydrogen atom and lose their coupling to the CMB. (This happens a bit earlier for the helium fraction). The baryonic matter in the universe transits from an ionized plasma state to neutral one where CMB photons can freely travel over cosmic distances.

The serendipitous discovery of this extra galactic microwave background Penzias and Wilson in 1965 provided a big boost to the HBBM. This was fol-

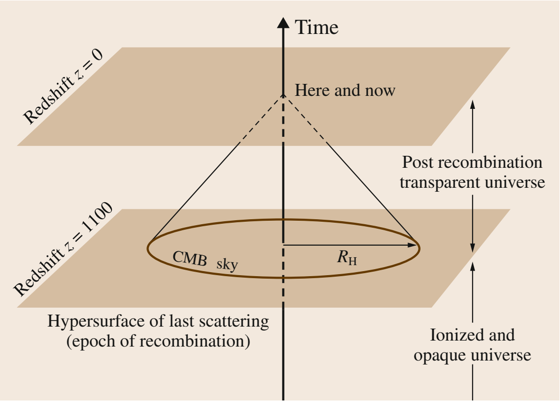

Fig. 32.3 A cartoon explaining the CMB using a space-(conformal) time diagram. The universe became transparent at the epoch of recombination, and CMB photons were able to travel to us freely over cosmic distances along our past light cone. In an expanding universe, the temperature of the Planck black-body CMB is inversely proportional to the expansion factor. When the universe is about 1100 times smaller, the CMB photons are hot enough to keep the baryonic matter in the universe (about three quarters hydrogen, one quarter helium as determined by Big-Bang nucleosynthesis) ionized, accompanied by a sharp transition to an opaque universe. The CMB photons come to us unimpeded directly from this spherical opaque surface of last scattering at a distance of $R_{\mathrm{H}} = 14\,\mathrm{Gpc}$ that surrounds us – a super IMAX cosmic screen. The *brown circle* depicts the sphere of last scattering in the reduced $2+1$-dimensional representation of the universe

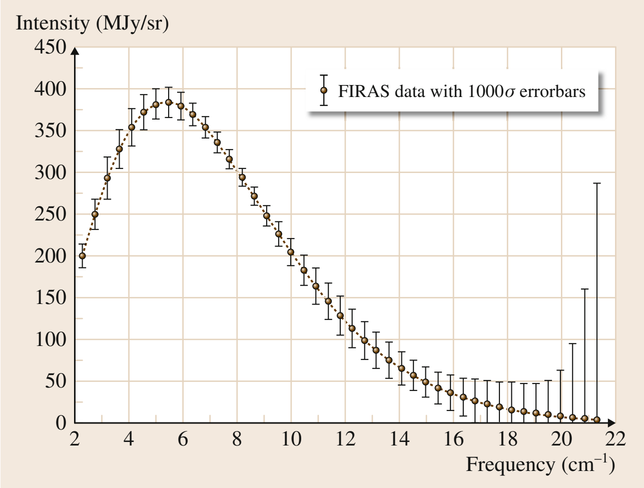

Fig. 32.4 Measurements of the energy spectrum of the CMB photons as a function of frequency (from 60 to 600 GHz). The measurements are from the FIRAS instrument on board the COBE satellite that won the Nobel prize in Physics in 2006. The accuracy of the measurements is apparent from the fact that the error bars have been multiplied by a factor of thousand in the plot. The distribution is extremely well fit by a black-body spectrum at a temperature of $T_0 = 2.726$ (± 0.0013) making the CMB the most perfect black body known in nature (courtesy of Tuhin Ghosh (IUCAA))

lowed up by numerous measurements of the CMB flux at other wavelengths that were broadly consistent with a Planck distribution of CMB photons. The Nobel prize in Physics in 2006 was awarded to John Mather (NASA Goddard Flight Center, USA) and George Smoot (University of Berkeley, USA), who led experimental teams of the pioneering Cosmic Background Explorer (COBE) mission – a US space Administration, NASA, satellite launched in 1989 to measure the cosmic microwave background radiation with unprecedented accuracy over the full sky. The satellite operated for 4 yr in a circumpolar orbit at an altitude of 900 km. COBE carried three different instruments: far-infrared absolute spectrophotometer (FIRAS), differential microwave radiometer (DMR), and diffuse infrared background experiment (DIRBE). John Mather was the principle investigator (PI of the FIRAS experiment that measured the energy distribution of CMB photons to unprecedented accuracy. The FIRAS instrument measurements of the radiation flux in the 60–2880 GHz frequency band shown in Fig. 32.4 confirmed the Planck distribution of CMB photons beyond reasonable doubt. The flux measurement at a given wavelength can be converted into an equivalent thermodynamic temperature T_0 for the CMB. Recent results derived from the FIRAS data combined with WMAP in 2009 find that the energy spectrum of CMB photons is accurately described by a Planck distribution at the precise temperature

$$T_0 = 2.726 \pm 0.0013\,\mathrm{K}\,. \tag{32.6}$$

Over the frequency band 60–630 GHz used to deduce the above FIRAS result, the maximum 1-σ deviation of the CMB spectrum from a Planck is constrained to be $\lesssim 0.01\%$ of the peak brightness. The observationally established Planck distribution of the energy spectrum of the CMB is naturally explained as arising from the thermal equilibrium the baryons and photons set up at very high temperatures and densities, that is expected to exist in the early universe. The present temperature T_0, of the CMB sets the total entropy of the universe (given the number of relativistic neutrino species). The origin of this entropy is not explained within the classical Big Bang model (inflation scenarios do provide an explanation but not a prediction). Working backward in time, adiabatic expansion implies a smaller and hotter universe expected in the HBBM.

32.4 Perturbed Universe: Structure Formation

The *standard* model of cosmology must not only explain the dynamics of the homogeneous background universe, but also describe the perturbed universe – the generation, evolution, and the formation of large-scale structures in the universe. There is a well understood (if not rigorously defined) notion of a *standard* model of cosmology that includes the formation of a large-scale structure. It is fair to say that much of the recent progress in cosmology has come from the interplay between refinements of the theories of structure formation and the improvements in observations.

Although the simple homogeneous and isotropic cosmological model does fit the dynamics of the background universe averaged on large scales, the rich structure in the distribution of galaxies shown in Fig. 32.1 suggests that there is more information to be gleaned about the universe from the large-scale structure of mass distribution (LSS). It has been a well-accepted

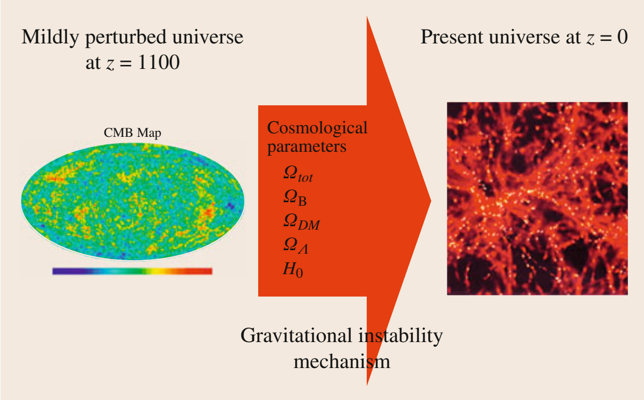

Fig. 32.5 A schematic figure to illustrate how understanding of the perturbed universe *determines* the cosmological parameters. The exquisitely measured CMB anisotropy maps characterize the mildly perturbed universe at early times. Large galaxy surveys and other LSS probe give the final state of the LSS. The cosmological parameters that affect the known structure formation mechanism through gravitational instability have to be dialed to precise values to consistently produce the LSS in the present universe from the mildly perturbed universe observed in the CMB anisotropy

notion that the large-scale structure in the distribution of matter in the present universe arose gradually due to gravitational instability from tiny primordial perturbation in the early universe. Although explosive mechanisms for structure formation in a relatively recent epoch had been proposed, the limits of input into the radiation budget in the recent past due to the tight adherence of the CMB to the Planck form seen in the COBE-FIRAS data make them nonviable. Also, the tiny level of fluctuations in the temperature of the CMB implies that the level of inhomogeneity in the universe at a redshift of $z_{\mathrm{rec}} = 1100$ is at most few 10 ppm. A recent exciting success of observational cosmology has been in detecting the baryon acoustic oscillations that establish the gravitational instability mechanism beyond reasonable doubt.

As schematically summarized in Fig. 32.5, cosmological observations have placed the theory of structure formation in an enviable position for any branch of physics where the initial and final states as well as the dynamical mechanism are known:

- The exquisite maps of CMB anisotropy provide a snap shot of perturbation in the universe at a redshift of $z_{\mathrm{rec}} = 1100$ when the universe is only about 380 000 yr old.
- In the past decade an extensive survey of galaxies has mapped out the distribution of matter in the present 14 Gyr-old universe.
- As mentioned above, the well-understood gravitationally instability is the underlying mechanism for amplifying the tiny perturbations at a redshift of $z_{\mathrm{rec}} = 1100$ to give rise to the observed LSS now.

The recent era of *precision* cosmology arises from the sensitivity of a consistent picture on the cosmological parameters. The parameters have to be dialed to precise values to make a consistent description of the perturbed universe starting with the mildly perturbed universe at $z_{\mathrm{rec}} = 1100$ seen in the CMB to the present universe with a well-developed LSS.

32.5 CMB Anisotropy and Polarization

The CMB photons arriving from different directions in the sky show tiny variations in temperature, at a level of ten parts per million, i. e., tens of micro-Kelvin, referred to as the CMB anisotropy, and a net linear polarization pattern at micro-Kelvin to tens of nano-Kelvin level. The tiny variation of temperature and linear polarization of these black-body photons of the cosmic microwave background arriving from different directions in the sky faithfully encodes information about the early universe and have traveled unimpeded across the observable universe making them an excellent probe of the universe. As illustrated in the cartoon in Fig. 32.3, the cosmic microwave background radiation sky is essentially a *giant, cosmic super IMAX theater screen* surrounding us at a distance of 43 billion light-years displaying a snapshot of the universe at a time very close to its origin.

The CMB anisotropy is imprints of the perturbed universe in the radiation when the universe was only 380 000 yr old. On the large angular scales, the CMB

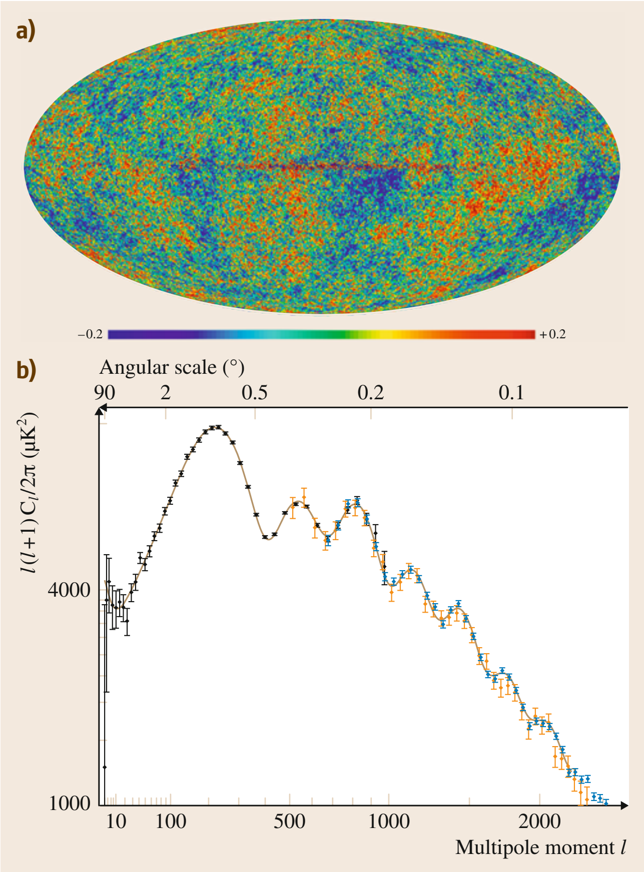

Fig.32.6a,b The exquisite *temperature anisotropy* data that are currently available are shown. **(a)** Color-coded full sky map (in Mollewide projection) of the CMB temperature variations seen in WMAP data. The temperature variations range between $\pm 200\,\mu\mathrm{K}$ with a r.m.s. of about $70\,\mu\mathrm{K}$. The angular resolution of features of the map is about a quarter of a degree. (The map was obtained using a model free approach to foreground removal on WMAP developed by the author's group.) **(b)** Most recent angular power spectrum of CMB obtained from the entire WMAP 9 yr (*black*), the ground-based South Pole Telescope (*blue*), and Atacama Cosmology Telescope (*orange*) data. The solid gray curve shows that the best-fit power law, flat, ΛCDM model obtained from WMAP-9 threads all the data points closely (WMAP-9 publication publicly available at the NASA-GSFC LAMBDA site http://www.lambda.gsfc.nasa.gov)

anisotropy directly probes the primordial power spectrum on scales enormously larger than the *causal horizon*. On smaller angular scales, the CMB temperature fluctuations probe the physics of the coupled baryon–photon fluid through the imprint of the acoustic oscillations in the ionized plasma sourced by the same primordial fluctuations. The physics of CMB anisotropy is well understood, and the predictions of the linear primary anisotropy and their connection to observables are, by and large, unambiguous [32.3–5].

It is convenient to express the sky map of the CMB temperature anisotropy in the direction $\hat{\boldsymbol{n}}$ as a spherical harmonic expansion

$$\Delta T(\hat{\boldsymbol{n}}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{\boldsymbol{n}}) \,.$$

Theory predicts that the primary CMB anisotropy is a Gaussian field (of zero mean), and current observations remain fully consistent with this expectation. The anisotropy can then be characterized solely in terms an angular spectrum

$$C_\ell = \frac{1}{(2\ell+1)} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 \,.$$

The C_ℓ spectra for a wide variety of models share a generic set of features clearly related to basics physics of primary CMB anisotropy.

The acoustic peaks occur because the cosmological perturbations excite acoustic waves in the relativistic plasma of the early universe. The recombination of baryons at redshift $z \approx 1100$ effectively decouples the baryon and photons in the plasma abruptly switching off the wave propagation. In the time between the excitation of the perturbations and the epoch of recombination, a sound wave could have traveled a fixed distance. Modes of different wavelengths can complete different numbers of oscillation periods. This translates the characteristic time into a characteristic length scale and produces a harmonic series of maxima and minima in the CMB anisotropy power spectrum. The acoustic oscillations have a characteristic scale known as the sound horizon, which is the comoving distance that a sound wave could have traveled up to the epoch of recombination. This is a well-determined physical scale imprinted on the CMB fluctuations on the surface of last scattering – the *cosmic super-IMAX* screen.

The angle subtended by this physical scale in the CMB anisotropy sky at the distance of 14 Gpc allows a sensitive determination of the geometry ($\Omega_{0\mathrm{K}}$) of the background universe. This is determined by the location of the harmonic peaks series of C_ℓ seen in Fig. 32.6. The amplitude of baryon–photon oscillations consequently, the height of the peaks in the C_ℓ sensitively determine the baryon density Ω_B. The C_ℓ are sensitive to other important cosmological parameters, such as, the relative density of matter Ω_m; cosmological constant Ω_Λ; Hubble constant H_0, and deviation from flatness (curvature) Ω_K. Implicit in C_ℓ is the hypothesized nature of random primordial/initial metric perturbations – (Gaussian) statistics , (nearly scale invariant) power spectrum, (largely) adiabatic versus isocurvature, and (largely) scalar versus tensor component. The *default* settings in bracket are motivated by inflation.

The transition to precision cosmology has been spearheaded by the measurements of CMB anisotropy and, more recently, polarization. The COBE-DMR detection of CMB anisotropy provided observational evidence for the origin and mechanism of structure formation in the universe. The following decade has been dominated by high-resolution, full sky, CMB anisotropy measurements from the WMAP of NASA that has provided observational support for the basic acoustic physics of the baryon–photon plasma.

The measured angular power spectrum of the CMB temperature fluctuations C_ℓ, shown in Fig. 32.6 has become invaluable observables for constraining cosmological models. The position and amplitude of the peaks and dips of the C_ℓ are sensitive to important cosmological parameters. The most robust constraint obtained is that on the spatial curvature of the universe and baryon density. Combining most recent CMB observations from WMAP9, South Pole Telescope (SPT) and Atacama Cosmology Telescope (ACT) can establish that the space on cosmic scales is geometrically flat ($\Omega_\mathrm{K} = 0.001 \pm 0.012$) to nearly within 1% precision. From WMAP9 alone, the dominant energy content in the present universe is a mysterious matter with negative pressure dubbed, dark energy, or a cosmological constant of about 72% ($\Omega_\Lambda = 0.721 \pm 0.025$), followed by cold nonbaryonic dark matter about 23% ($\Omega_\mathrm{m} = 0.233 \pm 0.023$) and ordinary matter (baryons) account for only about 5% ($\Omega_\mathrm{B} = 0.0463 \pm 0.00234$) of the matter budget. Observations of the large-scale structure in the distribution of galaxies, high-redshift supernova, and more recently, CMB polarization, have provided valuable complementary information.

In addition to the temperature anisotropy, there is also linear polarization information imprinted on the CMB at the last scattering surface. Thomson scattering generates CMB polarization anisotropy at decoupling. The coordinate-free description distinguishes two kinds of polarization patterns on the sky by their different parities. In the spinor approach, the even parity pattern is called the E-mode and the odd parity pattern the B-mode. While the CMB temperature anisotropy can also be generated during the propagation of the radiation from the last scattering surface, the CMB polarization signal can be generated only at the last scattering surface, where the optical depth transits from large to small values. The polarization information complements the CMB temperature anisotropy by isolating the effect at the last scattering surface from effects along the line of sight. Since the CMB polarization is sourced by the anisotropy of the CMB at the surface of last scattering, the angular power spectra of temperature and polarization are strongly linked to each other. For adiabatic initial perturbations, the acoustic peaks in the polarization spectra are out of phase with that of the temperature.

The Degree Angular Scale Interferometer (DASI) first measured the CMB polarization spectrum over a limited band of angular scales (multipole band $l \approx 200-440$) in late 2002. Since then, the polarization power spectrum measurements have been further refined by a number of CMB experiments, notably, MAXIMA CBI, QUaD, BICEP (background imaging of cosmic extragalactic polarization), etc. The main results indicated by the E-mode polarization measurements is that the acoustic peaks in the polarization spectra are indeed out of phase with that of the temperature. The strong limit on the nonadiabatic contribution to the primordial perturbations constrains the physics of the early universe.

The CMB polarization is a very clean and direct probe of the energy scale of early universe physics that generate the primordial metric perturbations. In the standard model, inflation generates both (scalar) density perturbations and (tensor) gravity wave perturbations. The relative amplitude of inflationary GW to scalar density perturbations sets the energy scale for inflation. A measurement of B-mode polarization on large-angular scales would give this amplitude, and hence a direct determination of the energy scale of inflation. Besides being a generic prediction of inflation, the cosmological gravity wave background from inflation would be a fundamental test of GR on cosmic scales and the semiclassical behavior of gravity.

32.6 Conclusion

The remarkable transition to precision cosmology has been spearheaded by the nearly two decade long experimental successes of CMB measurements. The first results from the COBE team (awarded the Nobel prize in Physics in 2006) provided only a coarse image of infant universe. The data from the Wilkinson Microwave Anisotropy Probe (WMAP) refined the image of the infant universe considerably in the following decade. It is the precision of these measurements of the CMB fluctuations cosmology that has translated to present day precision cosmology.

The past decade has seen the emergence of a *concordant* cosmological model that is consistent, both, with observational constraints from the background evolution of the universe, and that from the formation of a large-scale structure in the distribution of matter in the universe. Besides precise determination of various parameters of the *standard* cosmological model, CMB and related observations have also established some important basic tenets of cosmology and structure formation in the universe – *acausally* correlated initial perturbations, adiabatic nature primordial density perturbations, gravitational instability as the mechanism for structure formation. We have inferred a spatially flat universe where structures form by the gravitational evolution of nearly scale invariant, adiabatic perturbations, as expected from inflation.

The signature of primordial perturbations observed as the CMB anisotropy and polarization is the most compelling evidence for new, possibly fundamental, physics in the early universe that underlie the scenario of inflation (or related alternatives). Some fundamental *assumptions* rooted in the paradigm of inflation are still to be observationally established beyond doubt. Besides, there are deeper issues and exotic possibilities that no longer remain theoretical speculations, but have now come well within the grasp of cosmological observations (Chap. 39). These include cosmic topology, extra-dimensions, and violations of basic symmetries such as Lorentz transformations. In order to detect the subtle signatures it is also important to identify and weed out systematic effects such as the noncircularity of the beam in the acquisition and analysis of the CMB data.

The progress in the field continues unabated, refining the cosmological parameters into increasingly more precise numbers. Numerous ongoing and near future ground and balloon born CMB experiments at high sensitivity and resolution have sustained a steady pace of progress. The Planck Surveyor mission of ESA (European Space Agency) launched in May 2009 has acquired considerably more refined CMB measurements compared to WMAP. In the near future, exquisite results from the Planck satellite are expected. Planck is arguably the most ambitious cosmological space mission till date. It aims to measure CMB fluctuations at higher sensitivity and angular resolution to eke out almost all the information expected to be available in the CMB sky. Further in the future, dedicated CMB polarization space missions are being studied by both NASA and ESA [32.6].

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