

# Augmented Lagrange Hopfield Network Based Method for Multi-objective Hydrothermal Scheduling

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**Abstract.** The insufficient hydropower resources along with the environmental effects of thermal generations necessitate a proper scheduling scheme to comply with the growing power demand. In fact, the water resource at reservoirs has been affected by fluctuation of whether while exhausting fossil fuels and polluted emissions from thermal plants have become a serious problem. Therefore, the hydrothermal scheduling considering environment constraint becomes a very important problem. This paper proposes an augmented Lagrange Hopfield network (ALHN) for solving the multi-objective short-term hydrothermal scheduling (MOSTHS) problem. In the proposed method, ALHN is used to find a set of non-dominated solutions and the fuzzy decision-making methodology is then exploited to determine the best compromise solution among the obtained ones. The proposed method has been tested on different systems and the obtained results in terms of total fuel cost, emission, and computation time have been compared to those other methods in the literature. The result comparisons have indicated that the proposed method is effective for solving the MOSTHS problem.

**Keywords:** Augmented Lagrange Hopfield network, multi-objective, fixed head, hydrothermal scheduling.

## 1 Introduction

The short term hydro-thermal scheduling (HTS) problem is to determine the power generation among the available thermal and hydro power plants so that the total fuel cost of thermal units is minimized over a schedule time of a single day or a week while satisfying both equality and inequality constraints including power balance, available water, and generation limits of both thermal and hydro plants. In practical systems, thermal power generating stations are the sources of carbon dioxide (CO<sub>2</sub>), sulfur dioxide (SO<sub>2</sub>), and nitrogen oxides (NO<sub>x</sub>) causing atmospheric pollution. Therefore, the optimal scheduling of generation in a hydrothermal system involves the allocation of generation among the hydro and thermal plants to simultaneously

minimize the fuel cost and emission level of thermal plants satisfying the various constraints on the hydraulic and system network becomes a practical requirement.

Several optimization techniques have been proposed to deal with the multiobjective programming problems. Improved quantum-behaved particle swarm optimization (IQPSO) [1] has been applied for short-term combined economic emission hydrothermal scheduling. In this paper, quantum-behaved particle swarm optimization is improved employing heuristic strategies in order to handle the equality constraints especially water dynamic balance constraints and active power balance constraints. As a result, the method has obtained quality solutions. A novel multi-objective optimization procedure based on probability security criteria for optimal generation dispatch has been proposed to obtain a set of non-inferior solutions [2]. Nondominated sorting genetic algorithm-II (NSGA-II) [3] has been used for solving dynamic economic emission dispatch problem. NSGA-II is proposed to handle dynamic economic emission dispatch problem as a true multi-objective optimization problem with competing and noncommensurable objectives. Two novel search methods have been presented in [4] for dealing the problem those are hybrid algorithm and heuristic searches with genetic algorithm (GA). Both techniques can obtain a low maximum generation. However, the computation time of the heuristic searches with GA is very long compared to the hybrid algorithm. An improved bacterial foraging algorithm (BFA) has been applied to solve the short-term HTS problem considering the environmental aspects given in [5]. The research has carried out optimization for each of the objectives individually and optimization of the four objectives. However, the best compromise solution for the combined four objectives is not provided in [5]. Another method, non-dominated sorting genetic algorithm-II (NSGA II) [6] has been applied to economic environmental dispatch of fixed head hydrothermal scheduling problem with both convex and non-convex fuel cost and emission functions. The effectiveness of NSGA-II has been verified on two systems and compared to other methods of real-coded genetic algorithm (RCGA), strength Pareto evolutionary algorithm 2 (SPEA2) and multi-objective differential evolution (MODE). NSGA-II seems to be a powerful method through the comparisons in terms of cost, emission and computation time. However, the computation time is still long and both cost and emission are still high.

In this paper, an augmented Lagrange Hopfield network (ALHN) has been proposed for solving multi-objective short term fixed head hydrothermal scheduling problem. ALHN is a combination of augmented Lagrange relaxation and continuous Hopfield neural network where the augmented Lagrange function is directly used as the energy function of the continuous Hopfield network. ALHN is tested on two systems and the results are compared to those from BFA [5], and NSGA-II, SPEA 2, RCGA, and MODE in [6].

## 2 Problem Formulation

Consider an electric power system network having  $N_1$  thermal plants and  $N_2$  hydro plants;  $N$  is the total number of plants. The basic problem is to find the active power

generation of each plant in the system as a function of time over a finite time period from 0 to  $M$ .

## 2.1 Thermal Model

The objective function to be minimized is the total system operating cost, represented by the fuel cost of thermal generation, over the optimization interval.

$$F_{1sk} = a_{1s} + b_{1s}P_{sk} + c_{1s}P_{sk}^2 \quad (\$/h) \quad (1)$$

where  $a_{1s}$ ,  $b_{1s}$ ,  $c_{1s}$  are cost coefficients for thermal unit  $s$ ,

## 2.2 Emission Model

The atmospheric pollutants such as nitrogen oxides (NOx), sulphur oxides (SO<sub>2</sub>) and carbon oxides (CO<sub>2</sub>) caused by fossil-fueled thermal generator can be modeled separately. The NOx, SO<sub>2</sub>, CO<sub>2</sub> emission objective can be defined as:

$$F_{2sk} = \alpha_{1s} + \beta_{1s}P_{sk} + \gamma_{1s}P_{sk}^2 \quad (\text{Kg/h}) \quad (2)$$

$$F_{3sk} = \alpha_{2s} + \beta_{2s}P_{sk} + \gamma_{2s}P_{sk}^2 \quad (\text{Kg/h}) \quad (3)$$

$$F_{4sk} = \alpha_{3s} + \beta_{3s}P_{sk} + \gamma_{3s}P_{sk}^2 \quad (\text{Kg/h}) \quad (4)$$

where  $\alpha_{1s}$ ,  $\beta_{1s}$ ,  $\gamma_{1s}$  are NO<sub>x</sub> emission coefficients;  $\alpha_{2s}$ ,  $\beta_{2s}$ ,  $\gamma_{2s}$  are SO<sub>2</sub> emission coefficients;  $\alpha_{3s}$ ,  $\beta_{3s}$ ,  $\gamma_{3s}$  are CO<sub>2</sub> emission coefficients.

## 2.3 Hydro Model

Relationship between water discharge and power generated was proposed by Glimm-Kirchmayer [7].

$$q_{hk} = a_h + b_hP_{hk} + c_hP_{hk}^2 \quad (5)$$

where  $q_{hk}$  is rate of water flow ;  $t_k$  is duration of subinterval  $k$ ;  $a_h$ ,  $b_h$ ,  $c_h$  are water discharge coefficients for hydro unit  $h$ .

## 2.4 Equality and Inequality Constraints

1. *Load demand equality constraint:*

$$\sum_{s=1}^{N_1} P_{sk} + \sum_{h=1}^{N_2} P_{hk} - P_{Lk} - P_{Dk} = 0; \quad k = 1, \dots, M \quad (6)$$

$$P_{Lk} = \sum_{i=1}^{N_1+N_2} \sum_{j=1}^{N_1+N_2} P_{ik} B_{ij} P_{jk} + \sum_{i=1}^{N_1+N_2} B_{0i} P_{ik} + B_{00} \quad (7)$$

where  $P_{Dk}$ ,  $P_{Lk}$  are load demand, transmission loss during subinterval  $k$ , in MW;  $P_{hk}$ ,  $P_{sk}$  are generation output of hydro unit  $h$ , thermal unit  $s$  during subinterval  $k$ , in MW;  $B_{ij}$ ,  $B_{0i}$ , and  $B_{00}$  are loss formula coefficients of transmission system.

2. *Water availability constraints:*

$$\sum_{k=1}^M t_k q_{hk} = W_h \quad (8)$$

where  $W_h$  is volume of water available for generation by hydro unit  $h$  during the scheduling period.

3. *Generator operating limits:*

$$P_s^{\min} \leq P_{sk} \leq P_s^{\max}; s = 1, \dots, N_1; k = 1, \dots, M \quad (9)$$

$$P_h^{\min} \leq P_{hk} \leq P_h^{\max}; h = 1, \dots, N_2; k = 1, \dots, M \quad (10)$$

where  $P_h^{\min}$ ,  $P_h^{\max}$  and  $P_s^{\min}$ ,  $P_s^{\max}$  are lower and upper generation limits of hydro unit  $h$  and thermal unit  $s$  respectively.

### 3 ALHN for the Problem

To generate the non-inferior solution to the multi-objective problem, the weighting method is applied. In this method the problem is converted into a scalar optimization as given below [8]:

$$\text{Minimize } \sum_{j=1}^4 w_j F_j \quad (11)$$

subject to (4)-(7) and satisfying

$$\sum_{j=1}^4 w_j = 1; \quad w_j \geq 0 \quad (12)$$

where weighting factors,  $w_j$  is determined based on the relative importance of objective  $j$ , which may vary from place to place and utility to utility.

The augmented Lagrange function  $L$  of the problem is formulated as follows:

$$\begin{aligned} L = & \sum_{k=1}^M \sum_{s=1}^{N_1} t_k (a_s + b_s P_{sk} + c_s P_{sk}^2) + \sum_{k=1}^M \lambda_k \left( P_{Lk} + P_{Dk} - \sum_{s=1}^{N_1} P_{sk} - \sum_{h=1}^{N_2} P_{hk} \right) \\ & + \sum_{h=1}^{N_2} \gamma_h \left[ \sum_{k=1}^M t_k (q_{hk} - r_{hk}) - W_h \right] + \frac{1}{2} \sum_{k=1}^M \beta_k \left( P_{Lk} + P_{Dk} - \sum_{s=1}^{N_1} P_{sk} - \sum_{h=1}^{N_2} P_{hk} \right)^2 \\ & + \frac{1}{2} \sum_{h=1}^{N_2} \beta_h \left[ \sum_{k=1}^M t_k (q_{hk} - r_{hk}) - W_h \right]^2 \end{aligned} \quad (13)$$

where:  $\lambda_k, \gamma_h$  are Lagrangian multipliers associated with power balance and water constraint, respectively.  $\beta_k, \beta_h$  are penalty factors associated with power balance and water constraint, respectively, and

$$a_s = w_1 a_{1s} + w_2 \alpha_{1s} + w_3 \alpha_{2s} + w_4 \alpha_{3s} \quad (14)$$

$$b_s = w_1 b_{1s} + w_2 \beta_{1s} + w_3 \beta_{2s} + w_4 \beta_{3s} \quad (15)$$

$$c_s = w_1 c_{1s} + w_2 \gamma_{1s} + w_3 \gamma_{2s} + w_4 \gamma_{3s} \quad (16)$$

The energy function  $E$  of the problem is described in terms of neurons is determined as:

$$\begin{aligned} E = & \sum_{k=1}^M \sum_{s=1}^{N_1} t_k (a_s + b_s V_{sk} + c_s V_{sk}^2) + \sum_{k=1}^M V_{\lambda k} \left( P_{Lk} + P_{Dk} - \sum_{s=1}^{N_1} V_{sk} - \sum_{h=1}^{N_2} V_{hk} \right) \\ & + \sum_{h=1}^{N_2} V_{\gamma h} \left[ \sum_{k=1}^M t_k (q_{hk} - r_{hk}) - W_h \right] + \frac{1}{2} \sum_{k=1}^M \beta_k \left( P_{Lk} + P_{Dk} - \sum_{s=1}^{N_1} V_{sk} - \sum_{h=1}^{N_2} V_{hk} \right)^2 \\ & + \frac{1}{2} \sum_{h=1}^{N_2} \beta_h \left[ \sum_{k=1}^M t_k (q_{hk} - r_{hk}) - W_h \right]^2 + \sum_{k=1}^M \left( \sum_{s=1}^{N_1} \int_0^{V_{sk}} g^{-1}(V) dV + \sum_{h=1}^{N_2} \int_0^{V_{hk}} g^{-1}(V) dV \right) \end{aligned} \quad (17)$$

where:  $V_{\lambda k}, V_{\gamma h}$  are outputs of the multiplier neurons associated with power balance and water constraint, respectively;  $V_{hk}, V_{sk}$  are output of continuous neuron  $hk, sk$  representing  $P_{hk}, P_{sk}$ , respectively.

The dynamics of the model for updating neuron inputs are defined as follows:

$$\frac{dU_{sk}}{dt} = - \frac{\partial E}{\partial V_{sk}} \quad (18)$$

$$\frac{dU_{hk}}{dt} = - \frac{\partial E}{\partial V_{hk}} \quad (19)$$

$$\frac{dU_{\lambda k}}{dt} = \frac{\partial E}{\partial V_{\lambda k}} \quad (20)$$

$$\frac{dU_{\gamma h}}{dt} = \frac{\partial E}{\partial V_{\gamma h}} \quad (21)$$

The inputs of neurons at step  $n$  are updated:

$$U_{sk}^{(n)} = U_{sk}^{(n-1)} - \alpha_{sk} \frac{\partial E}{\partial V_{sk}} \quad (22)$$

$$U_{hk}^{(n)} = U_{hk}^{(n-1)} - \alpha_{hk} \frac{\partial E}{\partial V_{hk}} \quad (23)$$

$$U_{\lambda k}^{(n)} = U_{\lambda k}^{(n-1)} + \alpha_{\lambda k} \frac{\partial E}{\partial V_{\lambda k}} \quad (24)$$

$$U_{\gamma h}^{(n)} = U_{\gamma h}^{(n-1)} + \alpha_{\gamma h} \frac{\partial E}{\partial V_{\gamma h}} \quad (25)$$

where  $U_{\lambda k}$ ,  $U_{\gamma h}$  are inputs of the multiplier neurons;  $U_{sk}$ ,  $U_{hk}$  are inputs of the neurons  $sk$  and  $hk$  respectively;  $\alpha_{\lambda k}$ ,  $\alpha_{\gamma h}$  are step sizes for updating of multiplier neurons;  $\alpha_{sk}$ ,  $\alpha_{hk}$  are step sizes for updating of continuous neurons.

The outputs of continuous neurons and multiplier neurons:

$$V_{sk} = g(U_{sk}) = (P_s^{\max} - P_s^{\min}) \left( \frac{1 + \tanh(\sigma U_{sk})}{2} \right) + P_s^{\min} \quad (26)$$

$$V_{hk} = g(U_{hk}) = (P_h^{\max} - P_h^{\min}) \left( \frac{1 + \tanh(\sigma U_{hk})}{2} \right) + P_h^{\min} \quad (27)$$

$$V_{\lambda k} = U_{\lambda k} \quad (28)$$

$$V_{\gamma h} = U_{\gamma h} \quad (29)$$

where  $\sigma$  is slope of the sigmoid function.

### 3.1 Initialization

The initial outputs of continuous neurons are set at their middle limits and the multiplier neurons are set as follows:

$$V_{\lambda k}^{(0)} = \frac{1}{N_1} \sum_{s=1}^{N_1} t_k (b_s + 2c_s V_{sk}^{(0)}) / \left( 1 - \frac{\partial P_{Lk}}{\partial V_{sk}} \right) \quad (30)$$

$$V_{\gamma h}^{(0)} = \frac{1}{M} \sum_{k=1}^M V_{\lambda k}^{(0)} \left( 1 - \frac{\partial P_{Lk}}{\partial V_{hk}} \right) / \left( t_k \frac{\partial q_{hk}}{\partial V_{hk}} \right) \quad (31)$$

### 3.2 Stopping Criteria

The algorithm will be terminated when either the maximum error  $Err_{max}$  is lower than a predefined threshold  $\varepsilon$  or maximum number of iterations  $N_{max}$  is reached.

## 4 Best Compromise Solution by Fuzzy-Based Mechanism

In this paper, the best compromise solution for the problem is determined using fuzzy satisfying method [9]. The fuzzy goal is represented in linear membership function as follows [9]:

$$\mu(F_j) = \begin{cases} 1 & \text{if } F_j \leq F_j^{\min} \\ \frac{F_j^{\max} - F_j}{F_j^{\max} - F_j^{\min}} & \text{if } F_j^{\min} \leq F_j \leq F_j^{\max} \\ 0 & \text{if } F_j \geq F_j^{\max} \end{cases} \quad (32)$$

where  $F_j$  is the value of objective  $j$ ;  $F_{jmax}$  and  $F_{jmin}$  are maximum and minimum values of objective  $j$ , respectively. For each  $k$  non-dominated solution, the membership function is normalized as follows [9]:

$$\mu_D^k = \frac{\sum_{i=1}^{N_{obj}} \mu(F_i^k)}{\sum_{k=1}^{N_p} \sum_{i=1}^{N_{obj}} \mu(F_i^k)} \quad (33)$$

where  $\mu_D^k$  is the cardinal priority of  $k$ th non-dominated solution,  $\mu(F_j)$  is membership function of objective  $j$ ,  $N_{obj}$  is number of objective functions, and  $N_p$  is number of Pareto-optimal solutions. The solution that attains the maximum membership  $\mu_D^k$  in the fuzzy set is chosen as the 'best' solution based on cardinal priority ranking [8]:

$$\text{Max } \{\mu_D^k: k = 1, 2, \dots, N_p\} \quad (34)$$

## 5 Numerical Results

The proposed method has been tested on two test systems. The first test system consists of two thermal and two hydropower plants from [5]. The second system comprises of two hydro plants and two thermal plants in [6]. The proposed algorithm has been coded in Matlab 7.2 programming language and executed on an Intel 2.0 GHz PC. For termination criteria, the maximum tolerance  $Err_{max}$  is set to  $10^{-4}$  and  $10^{-3}$  for the first and second system respectively.

### 5.1 The First System

For this system, the emissions consist of  $\text{NO}_x$ ,  $\text{SO}_2$  and  $\text{CO}_2$ . The optimization for each of the objectives is individually carried out. The results of the fuel cost and each emission with computation time are given in Table 1. The result comparison between ALHN and BFA is given in Table 2. For all cases, the proposed ALHN method can obtain better solution than BFA except for the case of  $\text{CO}_2$  emission individual optimization.

### 5.2 The Second System

In this system, the objective includes one total cost and one emission. The obtained economic distich and emission dispatch from the proposed method are compared to those from RCGA [6] as in Table 3. From the table, the proposed method obtains better results than RCGA method in all cases. Moreover, the proposed method is also much faster than RCGA method for both cases.

**Table 1.** Total cost and emission minimization of individual objective for the first system

	Min $F_1$ (\$)	Min $F_2$ (kg)	Min $F_3$ (kg)	Min $F_4$ (kg)
$F_1$ (\$)	<b>51891.4144</b>	54294.5255	53,104.1251	54,221.8203
$F_2$ (kg)	27443.0377	<b>18,958.608</b>	20,822.2018	18,963.2433
$F_3$ (kg)	73381.1457	72,416.895	<b>71,641.9112</b>	72,358.5684
$F_4$ (kg)	442113.2112	335,810.13	357,415.3897	<b>335,764.1868</b>
CPU time (s)	1.29	1.53	1.79	1.11

**Table 2.** Comparison of the individual minimization of each objective for the first system

	BFA [5]	ALHN
Min $F_1$ (\$)	52,753.291	51,891.414
Min $F_2$ (Kg)	19,932.248	18,958.608
Min $F_3$ (Kg)	71,988.754	71,641.911
Min $F_4$ (Kg)	334,231.219	335,764.187

**Table 3.** Result comparison for economic and emission dispatch of the second system

Economic dispatch				Emission dispatch			
Method	Cost ( $10^4$ \$)	Emis. (lb)	CPU (s)	Method	Cost ( $10^4$ \$)	Emis. (lb)	CPU (s)
RCGA [6]	6.6031	681.1655	21.64	RCGA [6]	6.6892	586.1481	20.28
ALHN	6.4576	668.9824	2.84	ALHN	6.5797	585.6592	3.2

**Table 4.** Comparison of economic emission dispatch for the second system

Method	Cost ( $10^4$ \$)	Emission (lb)	CPU (s)
MODE [6]	6.6354	619.4280	30.72
SPEA2 [6]	6.6332	618.4580	34.88
NSGA-II [6]	6.6331	618.0660	27.86
ALHN	6.4778	615.1773	2.01

For the case of economic emission dispatch, we have determined 11 non-dominated solutions to form Pareto optimal front with the change of weights associated with objectives from 0 to 1 satisfying (12). The best compromise solution from the obtained 11 non-dominated solution is determined by fuzzy based mechanism in Section 4. The obtained the best compromise solution from the proposed method is compared to that from MODE, SPEA2, and NSGA II in [6]. Obviously, the proposed method can obtain better solution than the other methods for both total cost and emission. Moreover, the proposed method is also much faster than the others. Note the computational time in the table is for obtaining one solution and the CPU times of the methods in [6] were from a Pentium-IV, 80 GB, 3.0 GHz. Therefore, the proposed method is very effective and efficient for obtaining the best compromise solution for the problem.



## 6 Conclusion

The paper has been implemented an augmented Lagrange neural network based method for solving the multi-objective short-term hydrothermal scheduling problem. In the proposed method, the ALHN method is implemented for obtaining non-dominated solution and a fuzzy based mechanism is applied for determining the best compromise solution. The ALHN method is an improvement of continuous Hopfield neural network with its energy function based on augmented Lagrange function. Moreover, the ALHN method is recurrent network so it can obtain an optimal solution for an optimization problem in a very fast manner. The proposed method has been tested on two systems with different number of objectives and the obtained results have been compared to those from other methods in the literature. The result comparison has indicated that the proposed method can obtain better solution than other methods with faster computational time. Therefore, the proposed method can be very favorable for solving the multi-objective short-term hydrothermal scheduling problems.

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