Evolutionary Control of Chaotic Lozi Map by Means of Chaos Driven Differential Evolution

Roman Senkerik[1], Michal Pluhacek[1], Ivan Zelinka[2], Donald Davendra[2], Zuzana Kominkova Oplatkova[1], and Roman Jasek[1]

[1] Faculty of Applied Informatics, Tomas Bata University in Zlin
T.G. Masaryka 5555, 760 01 Zlin, Czech Republic
{senkerik,pluhacek,kominkovaoplatkova,jasek}@fai.utb.cz
[2] Technical University of Ostrava, Faculty of Electrical Engineering and Computer Science,
17. listopadu 15,708 33 Ostrava-Poruba, Czech Republic
{ivan.zelinka,donald.davendra}@vsb.cz

Abstract. In this paper, Differential Evolution (DE) is used for the evolutionary optimization of control of chaotic Lozi map system. The novality of the approach is that the identical selected discrete dissipative chaotic system is used as the chaotic pseudo random number generator to drive the mutation and crossover process in the DE. The optimization was performed for two types of case studies and developed cost functions.

Keywords: Differential Evolution, Optimization, Chaos control, Evolutionary algorithms, Lozi map.

1 Introduction

These days the methods based on soft computing such as neural networks, evolutionary algorithms, fuzzy logic, and genetic programming are known as powerful tool for almost any difficult and complex optimization problem.

The interest about the interconnection between evolutionary techniques and control of chaotic systems is spread daily. First steps were done in [1], [2], [3] where the control law was based on Pyragas method: Extended delay feedback control – ETDAS [4], [5], [6]. These papers were concerned to tune several parameters inside the control technique for chaotic system. The big advantage of the Pyragas method for evolutionary computation is the amount of accessible control parameters, which can be easily tuned by means of evolutionary algorithms (EA).

This paper is aimed at investigating the chaos driven Differential Evolution (DE). Although a number of DE variants have been recently developed, the focus of this paper is the embedding of chaotic systems in the form of chaos pseudo random number generator (CPRNG) for DE and its application to optimization of chaos control.

Firstly, the problem design is proposed. The next sections are focused on the description of used cost functions, evolutionary algorithm DE and the concept of chaos driven DE. Results and conclusion follow afterwards.

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2 Motivation

This research is a continuation of the previous successful initial application based experiment with chaos driven DE [7], [8].

This paper extends the research of evolutionary chaos control optimization by means of both SOMA or DE algorithm [3] and initial experiment with chaos driven DE [9].

In this paper the DE/rand/1/bin strategy driven by Lozi chaotic map (system) was utilized to solve the issue of evolutionary optimization of chaos control for the very same chaotic system. Thus the idea was to utilize the hidden chaotic dynamics in pseudo random sequences given by chaotic Lozi map system to help Differential evolution algorithm in searching for the best controller settings for the very same chaotic system.

Recent research in chaos driven heuristics has been fueled with the predisposition that unlike stochastic approaches, a chaotic approach is able to bypass local optima stagnation. This one clause is of deep importance to evolutionary algorithms. A chaotic approach generally uses the chaotic map in the place of a pseudo random number generator [10]. This causes the heuristic to map unique regions, since the chaotic map iterates to new regions. The task is then to select a very good chaotic map as the pseudo random number generator.

Several papers have been recently focused on the connection of DE and chaotic dynamics either in the form of hybridizing of DE with chaotic searching algorithm [11] or in the form of chaotic mutation factor and dynamically changing weighting and crossover factor in self-adaptive chaos differential evolution (SACDE) [12].

The focus of this paper is the embedding of chaotic systems in the form of chaos pseudo random number generator for DE.

The chaotic systems of interest are discrete dissipative chaotic systems. The Lozi map chaotic system was selected as the chaos pseudo random number generator for DE based on the successful results obtained with DE [13] or PSO algorithm [14].

3 Selected Chaotic System

The chosen example of discrete dissipative chaotic system used both as a CPRNG and within the evolutionary optimization of chaos control problem was the two-dimensional Lozi map system.

The Lozi map is a simple discrete two-dimensional chaotic map. The map equations are given in (1). The parameters used in this work are: a = 1.7 and b = 0.5 as suggested in [15].

For these value, the system exhibits chaotic behavior. The example of this behavior is depicted in the numerical simulation of direct system output (x or y) in the uncontrolled state (Fig. 1).

$$\begin{aligned} X_{n+1} &= 1 - a\left|X_n\right| + bY_n \\ Y_{n+1} &= X_n \end{aligned} \tag{1}$$

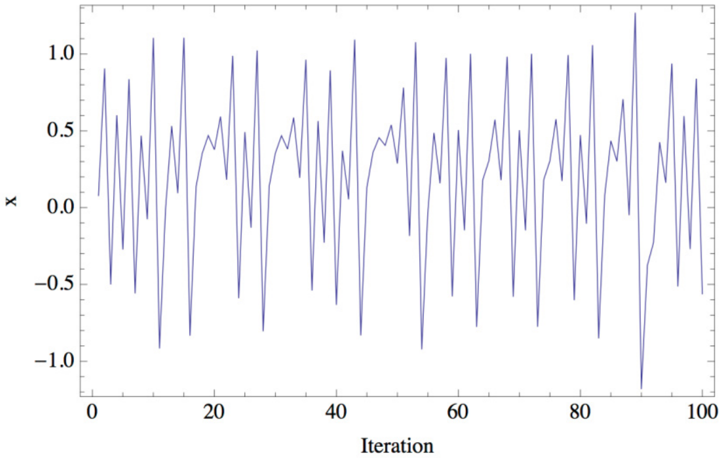

Fig. 1. Iterations of the uncontrolled Lozi map (variable x)

4 Original ETDAS Chaos Control Method

This work is focused on the utilization of the chaos driven DE for tuning of parameters for ETDAS control method to stabilize desired Unstable Periodic Orbits (UPO). In the described research, desired UPO was p-1 (stable state). The original control method – ETDAS in the discrete form suitable for Lozi map has the form (2).

$$
\begin{aligned}
x_{n+1} &= aX_n - Y_n^2 + F_n \\
F_n &= K\left[(1-R)S_{n-m} - x_n\right] \\
S_n &= x_n + RS_{n-m}
\end{aligned}
\tag{2}
$$

Where: K and R are adjustable constants, which have to be evolutionary tuned. F is the perturbation; S is given by a delay equation utilizing previous states of the system, m is the period of m-periodic orbit to be stabilized. The perturbation F_n in equations (2) may have arbitrarily large value, which can cause diverging of the system outside the output interval of Lozi map system {-1.4, 1.4}. Therefore, F_n should have a value between $<-F_{\max}$, $F_{\max}>$. The suitable $F_{\max}$ value was also obtained from evolutionary optimization process.

5 Cost Functions

This research utilizes and compares two cost function design.

The proposal of the first basic cost function (CF) is in general based on the simplest CF, which could be used problem-free only for the stabilization of p-1 orbit. The idea was to minimize the area created by the difference between the required state and the real system output on the whole simulation interval – τ_i. The simple CF is given in (3).

$$CF_{SIMPLE} = \sum_{t=0}^{\tau_i} \left| TS_t - AS_t \right|, \tag{3}$$

Nevertheless this simple approach has one big disadvantage, which is the including of initial chaotic transient behavior of not stabilized system into the cost function value. As a result of this, the very tiny change of control method setting for extremely sensitive chaotic system causing very small change of CF value, can be suppressed by the above-mentioned including of initial chaotic transient behavior.

Another universal cost function had to be used for securing the stabilization of either p-1 orbit (stable state) or higher periodic orbit and having the possibility of adding penalization rules. It was synthesized from the simple CF and other terms were added.

This CF is in general based on searching for desired stabilized periodic orbit and thereafter calculation of the difference between desired and found actual periodic orbit on the short time interval - τ_s (approx. 20 - 50 iterations) from the point, where the first min. value of difference between desired and actual system output is found (i.e. floating window for minimization). The CF_{UNI} has the form (4).

$$CF_{UNI} = pen_1 + \sum_{t=\tau 1}^{\tau 2} \left| TS_t - AS_t \right| \tag{4}$$

Where:
TS - target state, AS - actual state
τ_1 - the first minimal value of difference between TS and AS
τ_2 – the end of optimization interval ($\tau_1 + \tau_s$)
$pen_1 = 0$ if $\tau_i - \tau_2 \geq \tau_s$; $pen_1 = 10*(\tau_i - \tau_2)$ if $\tau_i - \tau_2 < \tau_s$ (i.e. late stabilization)

6 Differential Evolution

DE is a population-based optimization method that works on real-number-coded individuals [16]. DE is quite robust, fast, and effective, with global optimization ability. It does not require the objective function to be differentiable, and it works well even with noisy and time-dependent objective functions. Description of the utilized DERand1Bin strategy is presented in [16], [17], [18] and [19] together the description of all other strategies.

7 Chaos Driven DE

The main principle of this concept is the embedding of chaotic systems in the form of chaos pseudo random number generator (CPRNG) for DE. In this research, direct output iterations of the chaotic map were used for the generation of real numbers in the process of crossover based on the user defined CR value and for the generation of the integer values used for selection of individuals. The initial concept of embedding chaotic dynamics into the evolutionary algorithms is given in [20].

8 Experimental Results

Within the research a total number of 50 simulations with chaos driven DE by means of Lozi map system were carried out for each CF design. All simulations were successful and have given new optimal settings for ETDAS control method securing the fast stabilization of the chaotic system at required behaviour (p-1 orbit).

Following Tables 2 and 4 contains the simple statistical overview of optimization/simulation results. Tables 3 and 5 contain the best founded individual solutions of parameters set up for ETDAS control method, corresponding final CF value, also the *Istab. value* representing the number of iterations required for stabilization on desired UPO and further the average error between desired output value and real system output from the last 20 iterations.

Graphical simulation outputs of the best individual solutions for both case studies are depicted in Fig. 2 and Fig. 4, whereas the Fig. 3 and Fig 5 shows the simulation output of all 50 runs of CHAOS DE, thus confirm the robustness of this approach.

For the illustrative purposes, all graphical simulations outputs are depicted only for the variable x of the chaotic Lozi map system.

Settings of EA parameters for both processes were based on performed numerous experiments with chaotic systems (Table 1).

Based on the mathematical analysis, the real p-1 UPO for unperturbed Lozi map system has following value: $x_S = 0.4545$.

The ranges of all estimated parameters were these:
$-2 \leq K \leq 2$, $0 \leq F_{max} \leq 0.9$ and $0 \leq R \leq 0.99$,

Table 1. CHAOS DE settings

DE Parameter	Value
PopSize	25
F	0.8
CR	0.8
Generations	250
Max. CF Evaluations (CFE)	6250

8.1 Case Study 1 – Simple Cost Function

From the results presented in the Tables 2 and 3, it follows that the CF-simple is very convenient for evolutionary process, which means that repeated runs of EA are giving identical optimal results (i.e. very close to the possible global extreme). This is graphically confirmed in the Figure 3 when all 50 simulations are basically merged into the one line. On the other hand the disadvantage of including of initial chaotic transient behavior of not stabilized system into the cost function value and resulting very tiny change of control method setting for extremely sensitive chaotic system is causing suppression of stabilization speed and numerical precision.

Table 2. CF-simple values statistic

Statistical data	CF Value
Min	0.520639
Max	0.520639
Median	0.520639
Std.Dev.	$2.41 \cdot 10^{-15}$
Avg. Full Stab. (Iteration)	32

Table 3. Characteristics of the best solution

Parameter	Value
K	-1.11259
F_{max}	0.9
R	0.289232
CF Value	0.520639
Istab. Value	21
Avg. error per iteration	$7.21 \cdot 10^{-15}$

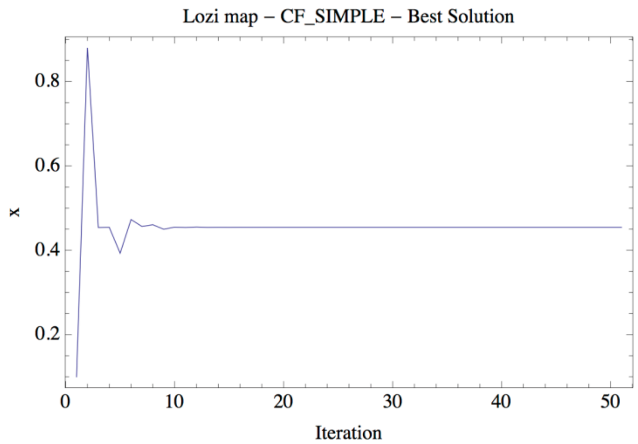

Fig. 2. Simulation of the best individual solution, CHAOS DE - CF Simple

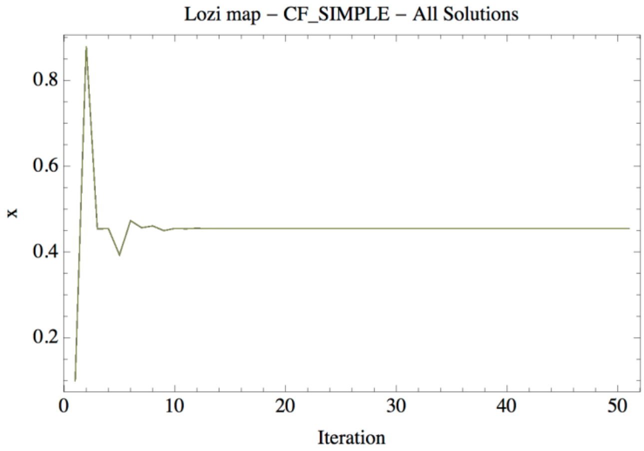

Fig. 3. Simulation of the all 50 solutions, CHAOS DE - CF Simple

8.2 Case Study 2 – Universal Cost Function

Results obtained in this case study lend weight to the argument, that the technique of pure searching for periodic orbits is advantageous for faster and more precise stabilization of chaotic system.

Table 4. CF-universal values statistic

Statistical data	CF Value
Min	$3.5331 \cdot 10^{-15}$
Max	$4.0551 \cdot 10^{-15}$
Average	$3.8063 \cdot 10^{-15}$
Std.Dev.	$1.19 \cdot 10^{-16}$
Avg. Stab. (Iteration)	11

Table 5. Characteristics of the best solution

Parameter	Value
K	-0.859989
F_{max}	0.643099
R	0.065669
CF Value	$3.5331 \cdot 10^{-15}$
Istab. Value	9
Avg. error per iteration	0

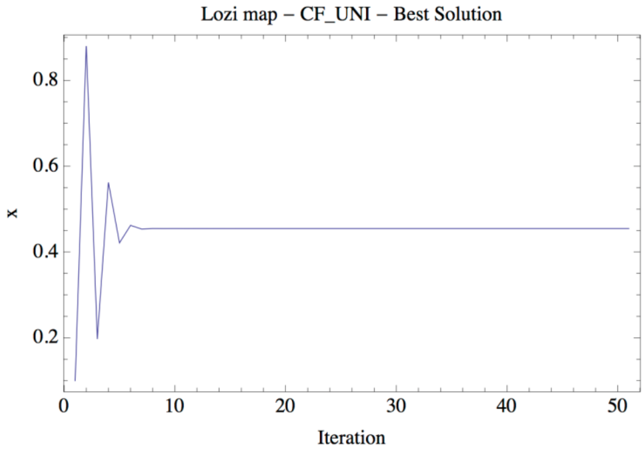

Fig. 4. Simulation of the best individual solution, CHAOS DE - CF Universal

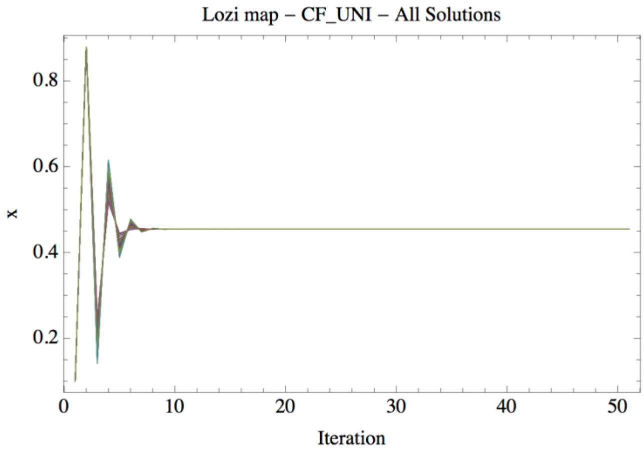

Fig. 5. Simulation of the all 50 solutions, CHAOS DE - CF Universal

9 Conclusions

Based on obtained results, it may be claimed, that the presented Chaos DE driven by selected discrete dissipative chaotic system has given satisfactory results in the chaos control optimization issue.

The results show that embedding of the chaotic dynamics in the form of chaotic pseudo random number generator into the differential evolution algorithm may help to improve the performance and robustness of the DE. Thus to obtain optimal solutions securing the very fast and precise stabilization for both convenient CF surface in case of the CF-simple and very chaotic and nonlinear CF surface in case of the CF-universal.

When comparing the both CF designs, the CF-simple is very convenient for evolutionary process (i.e. repeated runs are giving identical optimal results), but it has many limitations.

The second universal CF design brings the possibility of using it problem free for any desired behavior of arbitrary chaotic systems, but at the cost of the highly chaotic CF surface. Nevertheless the embedding of the chaotic dynamics into the evolutionary algorithms helped to deal with such an issue.

The primary aim of this work was not to develop any new pseudo random number generator, which should normally pass many statistical tests, but to show that through embedding the hidden chaotic dynamics into the evolutionary process in the form of chaotic pseudo random number generators may help to obtain better results and avoid problems connected with evolutionary computation such as premature convergence and stagnation in local extremes.

The issue of possible stagnation in local extremes was tested within the previous initial research with ChaosDE and CEC 2005 benchmark functions. The results lend weight to the argument that no through the distribution of pseudo-random numbers, but the hidden dynamics of chaotic systems representing the sequence of numbers may help to the evolutionary process and drives the population out of the local optimum.

Future plans include testing of different chaotic systems, either manually or evolutionary tuning of chaotic maps parameters, comparisons with different heuristics and obtaining a large number of results to perform statistical tests.

The future research will include the development of better cost functions, testing of different AP data sets, and performing of numerous simulations to obtain more results and produce better statistics, thus to confirm the robustness of this approach.

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