A Distributed Resource Allocation Algorithm in Multiservice Heterogeneous Wireless Networks

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Abstract. In this paper, radio resource allocation and users to access networks assignment in heterogeneous wireless networks is studied. Mobile terminals are assumed to have the capability of using multiple radio access technologies simultaneously. A joint optimization problem is formulated, which guarantees services for terminals and maximizes the sum utility of all base stations/access points. Our model applies to arbitrary heterogeneous scenarios where the air interfaces belong to the class of interference limited systems like CDMA-based UMTS or to a class with orthogonal resource assignment such as TDMA-based GSM, WLAN or OFDMA-based LTE. Dual decomposition is employed to solve this optimization problem and a distributed iterative algorithm is developed. Simulation results demonstrate the validity of the proposed algorithm.

Keywords: heterogeneous wireless networks, resource allocation, distributed algorithm, dual decomposition.

1 Introduction

Currently there exist different wireless access networks with different capabilities in terms of bandwidth, latency, coverage area, load or cost. These networks include 2G/3G cellular, LTE, WLAN, and so on. The integration of such networks can help to support user roaming and provide various class of services with different network resource demands. However, to satisfy the required rates by the mobile terminals via different networks and make efficient utilization of the available resources from these networks, new mechanisms for resource allocation and call admission control are required.

In literature, there exist various works that study the problem of resource allocation in heterogeneous wireless networks(HWNs)[1]-[6]. The existing solutions can be classified in two categories based on whether needing a central resource manager. Most of existing solutions (such as [1]-[4]) need a central resource manager to find the optimum bandwidth allocation. While a distributed mechanism is developed in [5], only a single network is considered in obtaining the required bandwidth. In [6], although a distributed algorithm is developed to find the optimum bandwidth allocation, it neglects the heterogeneity of resource.

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Mobile terminals(MTs) are assumed to have the capability of using multiple radio access technologies(RATs) simultaneously. This paper formulates the user assignment as a utility maximization problem which is constrained by the resource (such as power or bandwidth) of the individual base stations/access points(BSs/APs) as well as users' data rate requirements. Based on the convex formulation and by using structural properties, a decentralized algorithm is presented, which allows each network BS/AP to solve its own utility maximization problem and performs its own resource allocation to satisfy the MTs' rate requirements. The MTs play active role in the resource allocation operation by performing coordination among different BSs/APs.

The rest of this paper is organized as follows: Section 2 describes the system model. In Section 3, after the introduction of utility concept, the optimization problem formulation is developed. Algorithm that solve the problem in a decentralized way is presented in Section 4. Section 5 presents numerical simulation results and discussions. Finally, conclusions are drawn in Section 6.

2 System Model

This paper considers a geographical region where wireless access networks with different RATs is available. Any of the BSs/APs which belongs to network n, access point s can be denoted by (n, s), which $n \in \{1, 2, ..., N\}$, $s \in \{1, 2, ..., S_n\}$. The BSs/APs of each network have different coverage from those of other networks. Different networks have overlapped coverage in some areas. There are M MTs randomly distributed in the region, and MTs can be differentiated by range of rate of service request $[R_m^{min}, R_m^{max}], m \in \{1, 2, ..., M\}$. An exemplary scenario with three RATs is depicted in Fig.1.

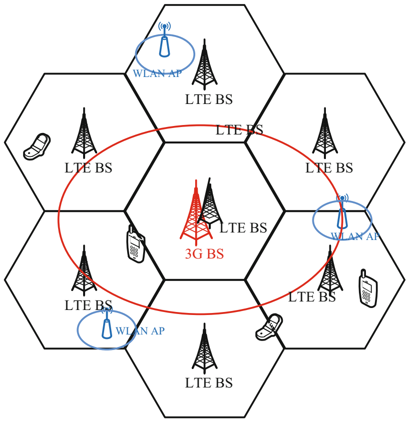


Fig. 1. An exemplary scenario with N=3

Considering the resource at the BSs/APs, the set of RATs can be divided into three subsets. $RATs = RAT_{orth,slot} \cup RAT_{inf,limit} \cup RAT_{orth,subcarriers}$.

2.1 Orthogonal Slots RATs ($RAT_{orth,slot}$)

For the class of orthogonal slots RATs systems, a fixed transmission power per BS is assumed. Bandwidth, in terms of time or frequency slots respectively, is the resource continuously distributable between MTs. The signal to interference and noise ratio (SINR) of BS (n, s) and MT m is as follows.

$$SINR_{ns,m} = \frac{g_{ns,m}\overline{P}_{ns}}{I_{ns} + N_{ns}}, \quad \forall(n, s) \in RAT_{orth,slot}, \tag{1}$$

thus depends on the channel gain $g_{ns,m}$, the BS transmission power $\overline{P}_{ns}$, the constant intercell interference I_{ns}, the noise N_{ns}. The amount of bandwidth assigned to MT m by BS (n, s) is denoted by $b_{ns,m}$. It is limited by the total, distributable bandwidth per BS $\overline{B}_{ns}$ and the constraint

$$\sum_{m=1}^{M} b_{ns,m} = B_{ns} \leq \overline{B}_{ns}, \quad \forall(n, s) \in RAT_{orth,slot}, \tag{2}$$

Due to the orthogonality of the MTs' signals and since the bandwidth is the distributable resource the relation between a MT's data rate $r_{ns,m}$ and the assigned resource is linear for this class of RATs[5]:

$$r_{ns,m} = \overline{r}_{ns,m} b_{ns,m}, \tag{3}$$

Here, $\overline{r}_{ns,m} := f(SINR_{ns,m})$ denotes the link rate per time or frequency slot between MT m and base station (n, s), where $f(SINR)$ is a positive, nondecreasing SINR-rate mapping curve corresponding to the coding and transmission technology of the BS (n, s). By substituting (3) into (2) the achievable rate region $R_{ns}$of each individual BS (n, s) results in:

$$\left\{ R_{ns} : \sum_{m=1}^{M} \frac{r_{ns,m}}{\overline{r}_{ns,m}} \leq \overline{B}_{ns}, r_{ns,m} \geq 0 \right\}, \tag{4}$$

which $R_{ns} = (r_{ns,1}, r_{ns,2}, ..., r_{ns,M})$ denotes data rate of MTs through BS (n, s).

2.2 Interference Limited RATs ($RAT_{inf,limit}$)

For the class of interference limited RATs systems, all MTs share the same bandwidth and that resources are distributed in terms of assigned power. The power of BS (n, s) to MT m, which denoted by $p_{ns,m}$ is limited by a sum constraint

$$\sum_{m=1}^{M} p_{ns,m} = P_{ns} \leq \overline{P}_{ns}, \quad \forall(n, s) \in RAT_{inf,limit}, \tag{5}$$

MTs are sensitive to intracell and intercell interference and the SINR between BS (n, s) and MT m is given by

$$
\begin{aligned}
&SINR_{ns,m} = \frac{g_{ns,m} p_{ns,m}}{I_{ns} + N_{ns}}, \\
&\text{with} \quad I_{ns} = \rho g_{ns,m} \sum_{m' \neq m} p_{ns,m'} + \sum_{(n',s') \neq (n,s)} g_{n's',m} P_{n's'}, \\
&\qquad \forall (n, s), (n', s') \in RAT_{inf,limit} \quad m, m' \in \mathcal{M},
\end{aligned}
\tag{6}
$$

with ρ the orthogonality factor which accounts for a reduced intercell interference. In this class of systems all links of one BS share a limited power budget and are impaired by the power assigned to other MTs in the air interface. A wellknown model for the link rate of these systems is given in [7]:

$$
r_{ns,m} = C \log_2 (1 + DSINR_{ns,m}) = C \log_2 \left(1 + D \frac{g_{ns,m} p_{ns,m}}{I_{ns} + N_{ns}} \right) \tag{7}
$$

There, the positive constants C,D parameterize the system characteristics such as bandwidth, modulation, and bit-error rates. However, assuming that all BS transmit with fixed transmission power and that the SINR of all links is not too low, data rate can be approximated as in [5]:

$$
\begin{aligned}
&r_{ns,m} = C \log_2 \left(1 + D \frac{p_{ns,m}}{\beta_{ns,m} - \rho p_{ns,m}} \right) \cong \frac{CD}{I_{ns,m}} p_{ns,m} := \overline{r}_{ns,m} p_{ns,m} \\
&\text{with} \quad \beta_{ns,m} = \frac{\rho g_{ns,m} P_{ns} + \sum_{(n',s') \neq (n,s)} g_{n's',m} P_{n's'} + N_{ns}}{g_{ns,m}},
\end{aligned}
\tag{8}
$$

By solving the approximation in (8) and substitution into (5) the achievable rate region of BS $(n, s) \in RAT_{inf,limit}$ can be represented by

$$
\left\{ R_{ns} : \sum_{m=1}^{M} \frac{r_{ns,m}}{\overline{r}_{ns,m}} \leq \overline{P}_{ns}, r_{ns,m} \geq 0 \right\}, \tag{9}
$$

2.3 Orthogonal Subcarriers RATs ($RAT_{orth,subcarriers}$)

For the class of orthogonal subcarriers RATs systems, fixed transmission power per BS is assumed. The overall bandwidth B is divided into K subcarriers for OFDM transmission. Based on the Shannon formula, the average rate between BS (n, k) and MT m on subcarrier k in is given by

$$
r^k_{ns,m} = \frac{B}{K} \log_2 \left(1 + \frac{p^k_{ns,m} l_{ns,m} |h^k_{ns,m}|^2}{\Gamma B N_0 / K} \right), \tag{10}
$$

where $p^k_{ns,m}$ denotes the transmission powers of BS (n, k) to MT m spent on subcarrier k. $h^k_{ns,m}$ represents the small-scale fading coefficients between BS (n, s) and MT m on subcarrier k. The path losses between BS (n, s) and MT

m is $l_{ns,m}$. Γ is the signal to noise ratio gap related to a target bit error rate (BER)[8]. N_0 denotes the power spectral density of the noise.

Assuming that BS (n, s) just allocate one subcarrier to MT m, and the SINR of all links is not too low, data rate of MT m can be approximated by

$$r_{ns,m} = r^k_{ns,m} \cong \frac{l_{ns,m}}{\Gamma N_0} p^k_{ns,m} |h^k_{ns,m}|^2 := \overline{r}_{ns,m} p_{ns,m}, \tag{11}$$

Therefore, the achievable rate region of BS (n, s) can be represented by

$$\left\{ R_{ns} : \sum_{m=1}^{M} \frac{r_{ns,m}}{\overline{r}_{ns,m}} \leq \overline{P}_{ns}, r_{ns,m} \geq 0 \right\}, \tag{12}$$

3 Problem Formulation

Let $u_{ns,m}(r_{ns,m})$ denote utility function of BS/AP (n, s) allocating resource to MT m and data rate of MT m is $r_{ns,m}$, and it is defined as in [2]:

$$u_{ns,m} = \omega \cdot log(\alpha \cdot r_{ns,m}), \tag{13}$$

where ω and α are constants indicating the scale and shape of utility function.

Having the system model and the utility concept introduced, formal problem formulation can be presented. The propose is to find the user assignment that maximizes the sum utility of all networks under the constraint that all MTs are assigned between their rate range $[R^{min}_m, R^{max}_m]$. Based on the earlier presented assumptions, the problem can be formulated as

$$\begin{aligned} &\max_{R_{ns}} \sum_{n=1}^{N} \sum_{s=1}^{S_n} U_{ns}(R_{ns}), \\ s.t. \quad & R^{min}_m \leq \sum_{n=1}^{N} \sum_{s=1}^{S_n} r_{ns,m} \leq R^{max}_m, \forall m \in \{1, 2, ...M\} \\ & \sum_{m=1}^{M} \frac{r_{ns,m}}{\overline{r}_{ns,m}} \leq \Lambda_{ns}, \forall (n, s) \in RAT \end{aligned} \tag{14}$$

with Λ_{ns} denoting available resources,

$$\Lambda_{ns} = \begin{cases} \overline{B}_{ns}, & \forall (n, s) \in RAT_{orth,slot} \\ \overline{P}_{ns}, & \forall (n, s) \in RAT_{inf,limit} \text{ or } RAT_{orth,subcarriers} \end{cases} \tag{15}$$

Problem (14) is convex, consequently, a variety of ready-to-use algorithms exists to solve it[9]. However, neither give these algorithms insights into the problem structure. We therefore develop a different approach based on duality[9][10]; instead of solving (14) directly we transform it into an alternative problem which is known to have the same solution as (14) but can be solved in a decentralized way by decomposition methods[11]. To obtain an expression for the dual transform the Lagrangian function of (14) is needed, which has the following form:

$$\begin{aligned} L(\boldsymbol{R}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) = \sum_{n=1}^{N} \sum_{s=1}^{S_n} U_{ns}(R_{ns}) + \sum_{n=1}^{N} \sum_{s=1}^{S_n} \lambda_{ns} \left(\Lambda_{ns} - \sum_{m=1}^{M} \frac{r_{ns,m}}{\overline{r}_{ns,m}} \right) \\ + \sum_{m=1}^{M} \nu_m \left(R_m^{max} - \sum_{n=1}^{N} \sum_{s=1}^{S_n} r_{ns,m} \right) + \sum_{m=1}^{M} \mu_m \left(\sum_{n=1}^{N} \sum_{s=1}^{S_n} r_{ns,m} - R_m^{min} \right) \end{aligned} \tag{16}$$

which $\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}$ are nonnegative Lagrangian parameters. The dual function[9] of (14) is defined as

$$g(\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) = \max_{\boldsymbol{R}} L(\boldsymbol{R}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) \tag{17}$$

Due to nonnegativity of the Lagrangian parameters one observes that (17) is always larger than or equal to the solution of (14). Therefore, minimizing the unconstrained dual function over the Lagrangian parameters

$$\min_{\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu} \geq \boldsymbol{0}} g(\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) = \min_{\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu} \geq \boldsymbol{0}} \underbrace{\max_{\boldsymbol{R}} L(\boldsymbol{R}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu})}_{\text{inner problem}} \tag{18}$$

yields an upper bound on the original optimization problem (14) and is called the dual problem of (14). Furthermore, by convexity of (14) and since Slaters conditions[9] hold, the bound is tight and (18) and (14) have the same solution.

3.1 Inner Problem

Rearranging terms in (17) results in the following:

$$\begin{aligned} g(\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) &= \max_{\boldsymbol{R}} L(\boldsymbol{R}, \boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}) \\ &= \sum_{n=1}^{N} \sum_{s=1}^{S_n} \max_{R_{ns}} \left\{ U_{ns}(R_{ns}) - \lambda_{ns} \sum_{m=1}^{M} \frac{r_{ns,m}}{\overline{r}_{ns,m}} - \sum_{m=1}^{M} (\nu_m - \mu_m) r_{ns,m} \right\} \\ &\quad + \sum_{n=1}^{N} \sum_{s=1}^{S_n} \lambda_{ns} \Lambda_{ns} + \sum_{m=1}^{M} \left(\nu_m R_m^{max} - \mu_m R_m^{min} \right) \end{aligned} \tag{19}$$

Consequently, each BS/AP (n, s) can solve its own utility maximization problem, expressed as

$$\max_{R_{ns}} \left\{ U_{ns}(R_{ns}) - \lambda_{ns} \sum_{m=1}^{M} \frac{r_{ns,m}}{\overline{r}_{ns,m}} - \sum_{m=1}^{M} (\nu_m - \mu_m) r_{ns,m} \right\} \tag{20}$$

The optimum allocation R_{ns} for fixed values of $\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}$ can be calculated by each BS/AP by applying the Karush-Kuhn-Tucker(KKT)[10] conditions on (20), and we have

$$\frac{\partial u_{ns,m}(r_{ns,m})}{\partial r_{ns,m}} - \lambda_{ns}/\overline{r}_{ns,m} - (\nu_m - \mu_m) = 0, \tag{21}$$

Using the utility function of (13), (21) results in

$$r_{ns,m} = \frac{\omega}{\lambda_{ns}/\overline{r}_{ns,m} + (\nu_m - \mu_m)}, \tag{22}$$

The optimum values of $\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}$ that give the optimum allocation $r_{ns,m}$ of (22) can be calculated by solving the dual problem of (18).

3.2 Outer Problem

For a fixed allocation R_{ns}, the dual problem can be expressed as

$$\sum_{n=1}^{N}\sum_{s=1}^{S_n}\min_{\lambda\geq 0}\left\{\lambda_{ns}\left(\Lambda_{ns} - \sum_{m=1}^{M}\frac{r_{ns,m}}{\overline{r}_{ns,m}}\right)\right\} + \sum_{m=1}^{M}\min_{\nu\geq 0}\left\{\nu_m\left(R_m^{max} - \sum_{n=1}^{N}\sum_{s=1}^{S_n} r_{ns,m}\right)\right\}$$
$$+ \sum_{m=1}^{M}\min_{\mu\geq 0}\left\{\mu_m\left(\sum_{n=1}^{N}\sum_{s=1}^{S_n} r_{ns,m} - R_m^{min}\right)\right\} + \sum_{n=1}^{N}\sum_{s=1}^{S_n} U_{ns}(R_{ns}) \tag{23}$$

For a differentiable dual function, a gradient descent method[10] can be applied to calculate the optimum values for $\boldsymbol{\lambda}, \boldsymbol{\nu}, \boldsymbol{\mu}$, given by

$$\lambda_{ns}(i+1) = \left[\lambda_{ns}(i) - \delta_\lambda\left(\Lambda_{ns} - \sum_{m=1}^{M}\frac{r_{ns,m}}{\overline{r}_{ns,m}}\right)\right]^+ \tag{24}$$

$$\nu_m(i+1) = \left[\nu_m(i) - \delta_\nu\left(R_m^{max} - \sum_{n=1}^{N}\sum_{s=1}^{S_n} r_{ns,m}\right)\right]^+ \tag{25}$$

$$\mu_m(i+1) = \left[\mu_m(i) - \delta_\mu\left(\sum_{n=1}^{N}\sum_{s=1}^{S_n} r_{ns,m} - R_m^{min}\right)\right]^+ \tag{26}$$

where i is the iteration index and $\delta_\lambda, \delta_\nu$ and δ_μ are sufficiently small fixed step size. Convergence towards the optimum solution is guaranteed since the gradient of (23) satisfies the Lipchitz continuity condition[10]. As a result, the resource allocation $r_{nm,s}$ of (22) converges to the optimum solution.

4 A Distributed Resource Allocation Algorithm

Based on the optimality conditions of the inner problem and the subgradient of the outer loop in Section 3, we are able to formulate the Algorithm 1. Following the classical interpretation of λ_{ns} as the price of resources, thus, λ_{ns} serves as an indication of the capacity limitation experienced by BS/AP (n, s). ν_m and μ_m are coordination parameters used by MTs with service, and they are used to ensure that allocated resources for an MT with service lie within the specified required rate range.

Algorithm 1. Decentralized resource allocation algorithm

Initialization: Each BS/AP initializes λ_{ns} and broadcasts λ_{ns} to all MTs. Each MT initializes ν_m, μ_m and calculates $\overline{r}_{ns,m}$ for each BS/AP, then broadcasts the parameters to all BSs/APs;
while $r_{ns,m}$ *not converge* **do**
    Each BS/AP calculates $r_{ns,m}$ with (22), updates λ_{ns} with (24) and broadcasts $r_{ns,m}$ and λ_{ns} to all MTs;
    Each MT updates ν_m with (25) and μ_m with (26) , and broadcasts the parameters to all BSs/APs.
end
return $r_{ns,m}$;

5 Simulation Results and Analysis

In simulation, a geographical region showed in Fig.1 is considered. As a result, $N = 3$ with the LTE, 3G cellular network and WLAN indexed as $1, 2$ and 3 respectively. The MTs are randomly distributed. The simulation parameters are listed in Table 2. Numerical results are averaged over 1000 scenarios.

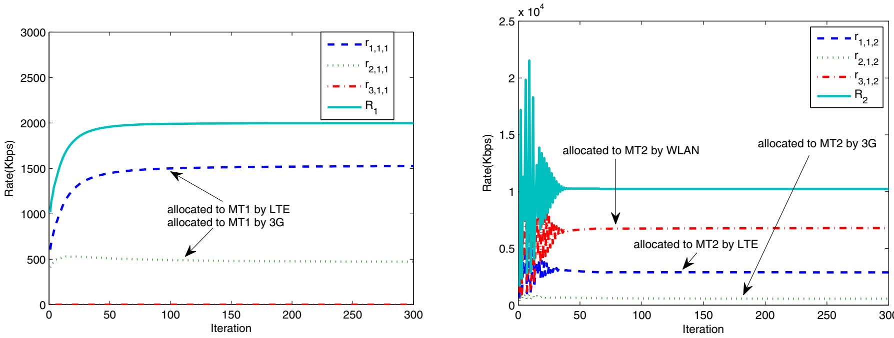
Table 1. Simulation parameters

Parameter	Value	Unit	Parameter	Value	Unit
video $[R^{min}, R^{max}]$	$[256, 2000]$	$Kbps$	C	1.4×10^9	-
data $[R^{min}, R^{max}]$	$[1, 10]$	$Mbps$	D	1×10^{-3}	-
LTE $\overline{P}_{BS}$	40	W	ρ	0.4	-
3G $\overline{P}_{BS}$	20	W	ω	1	-
M	$14, 16, 18, 20, 22$	-	α	0.7	-

Table 2. SINR requirements for different data rates for 802.11a[12]

Rate/Mbps	54	48	36	24	18	12	9	6
SINR/dB	24.6	24	18.8	17	10.8	9	7.8	6

An example for finding an optimal solution of the proposed algorithm is provided in Fig.2. It can be seen that MT1 which applies video service is allocated resource by LTE and 3G, and MT2 which applies data service is allocated by LTE, 3G and WLAN. Therefore, the proposed algorithm is feasible and can efficiently converge to the global optimal solution. Even though the proposed algorithm might be rather complex to implement, it could be utilized as an upper bound on the achievable gains in HWNs.



(a) video (b) data

Fig. 2. The convergent data rate of the proposed algorithm

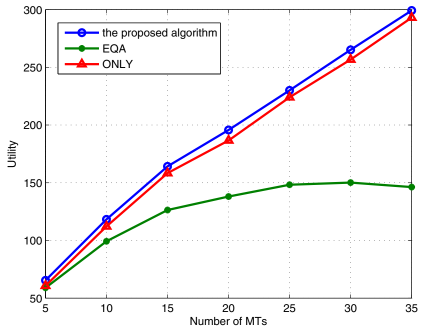


Fig. 3. Utility comparison

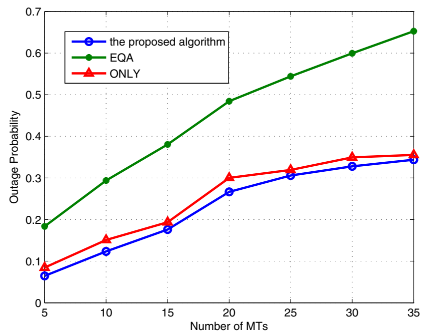


Fig. 4. Outage probability comparison

For performance comparison, we compare the proposed algorithm with EQA algorithm, which is equal resource allocation scheme, and ONLY algorithm, which is changed from the proposed algorithm and MT accesses only one RAT. Fig.3 shows the utility comparison over different number of MTs. The utility of the proposed algorithm achieves more utility than other two algorithms. In Fig.4, the outage probabilities for three algorithms are plotted. The proposed algorithm offers smaller outage probability and increases slowly over the number of MTs. The reason that our proposed algorithm outperforms EQA algorithm, is that its solution determined jointly by resource constraint and service demands. On the other hand, the proposed algorithm outperforms ONLY algorithm because of making use of multi-RAT, which is called RAT-diversity gain.

6 Conclusion

This paper develops an optimization framework for HWNs. Our model applies to arbitrary heterogeneous scenarios where the air interfaces belong to the class of interference limited systems or to a class with orthogonal resource assignment systems. A convex utility maximization problem formulation is introduced, then a distributed resource allocation algorithm is proposed. The algorithm has the following features: 1) it supports different resource (power or bandwidth); 2) Each MT can obtain its required rate from all available RAT simultaneously; 3) It is a distributed algorithm in a sense that each BS/AP solves its own utility maximization problem and performs its own resource allocation. This is very essential to be implemented in a practical environment. The performed simulations observe how the proposed algorithm would work and confirm that the proposed algorithm achieves better performance.

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