Distributed Randomized Broadcasting in Wireless Networks under the SINR Model*

Tomasz Jurdzinski[1], Dariusz R. Kowalski[2], Michal Rozanski[1], and Grzegorz Stachowiak[1]

[1] Institute of Computer Science, University of Wrocław, Poland
[2] Department of Computer Science, University of Liverpool, United Kingdom

Abstract. In the advent of large-scale multi-hop wireless technologies, such as MANET, VANET, iThings, it is of utmost importance to devise efficient distributed protocols to maintain network architecture and provide basic communication tools. One of such fundamental communication tasks is broadcast, also known as a 1-to-all communication. We present a randomized algorithm that accomplishes broadcast in $O(D + \log(1/\delta))$ rounds with probability at least $1 - \delta$ on *any* uniform-power network of n nodes and diameter D, when each station is equipped with its coordinates and local estimate of network density. Next, we develop algorithms for the model where no estimate of local density is available, except of the value n of the size of a given network. First, we provide a simple and almost oblivious algorithm which accomplishes broadcast in $O(D \log n(\log n + \log(1/\delta)))$ rounds with probability at least $1 - \delta$. We further enhance this algorithm with more adaptive leader election routine and show that the resulting protocol achieves better time performance $O((D + \log(1/\delta)) \log n)$ with probability at least $1 - \delta$. Our algorithms are the first provably efficient and well-scalable randomized distributed solutions for the (global) broadcast task in the ad hoc setting with coordinates. This could be also contrasted with the complexity of broadcast by weak devices, for which such scalable algorithms (with respect to D and $\log n$) cannot be obtained [11].

Keywords: Ad hoc wireless networks, Signal-to-Interference-and-Noise-Ratio (SINR) model, Broadcast, Distributed algorithms.

1 Introduction

1.1 The Model

We consider the model of a wireless network consisting of n *stations*, also called *nodes*, deployed into an Euclidean plane and communicating by a wireless medium. *Euclidean metric* on the plane is denoted $\text{dist}(\cdot, \cdot)$. Each station v has its *transmission power* P_v, which is a positive real number.

* This work was supported by the Polish National Science Centre grant DEC-2012/07/B/ST6/01534.

Y. Afek (Ed.): DISC 2013, LNCS 8205, pp. 373–387, 2013.

There are three fixed model parameters: path loss $\alpha > 2$, threshold $\beta \geq 1$, ambient noise $\mathcal{N} > 0$. We also have connectivity graph parameter $\varepsilon \in (0, 1)$. The $SINR(v, u, \mathcal{T})$ ratio, for given stations u, v and a set of (transmitting) stations $\mathcal{T}$, is defined as follows:

$$SINR(v, u, \mathcal{T}) = \frac{P_v \text{dist}(v, u)^{-\alpha}}{\mathcal{N} + \sum_{w \in \mathcal{T} \setminus \{v\}} P_w \text{dist}(w, u)^{-\alpha}} \tag{1}$$

In the *Signal-to-Interference-and-Noise-Ratio (SINR) model* a station u successfully receives a message from a station v in a round if $v \in \mathcal{T}$, $u \notin \mathcal{T}$, and

$$SINR(v, u, \mathcal{T}) \geq \beta \ ,$$

where $\mathcal{T}$ is the set of stations transmitting at that round.

In order to specify the details of broadcasting task and performance analysis, we first introduce the notion of transmission ranges and communication graphs.

Ranges and Uniformity. The *communication range* r_v of a station v is the radius of the ball in which a message transmitted by the station is heard, provided no other station transmits at the same time. A network is *uniform*, when transmission powers P_v and thus ranges of all stations r_v are equal, or *nonuniform* otherwise. In this paper, only uniform networks are considered and without loss of generality we assume that $r_v = r = 1$ for any v, i.e., $(P/(\mathcal{N}\beta))^{1/\alpha} = 1$, where P is the transmission power of a station.

Communication Graph and Graph Notation. The *communication graph* $G(V, E)$ of a given network consists of all network nodes and edges (v, u) such that $\text{dist}(v, u) \leq (1 - \varepsilon)r = 1 - \varepsilon$, where $0 < \varepsilon < 1$ is a fixed model parameter. The meaning of the communication graph is as follows: even though the idealistic communication range is r, it may be reached only in a very unrealistic case of single transmission in the whole network. In practice, however, many nodes located in different parts of the network often transmit simultaneously, and therefore it is reasonable to assume that we may only hope for a slightly smaller range to be achieved. The communication graph envisions the network of such "reasonable reachability". Note that the communication graph is symmetric for uniform power networks. By a *neighborhood* of a node u we mean the set of all neighbors of u in G, i.e., the set $\{w \mid (w, u) \in E(G)\}$. The *graph distance* from v to w is equal to the length of a shortest path from v to w in the communication graph, where the length of a path is equal to the number of its edges. The *eccentricity* of a node is the maximum graph distance from this node to any other node (note that the eccentricity is of order of the diameter D).

Synchronization. It is assumed that algorithms work synchronously in rounds, each station can either act as a sender or as a receiver during a round. We do not assume global clock ticking.

Carrier Sensing. We consider the model *without carrier sensing*, that is, a station u has no other feedback from the wireless channel than receiving or not receiving a message in a round t.

Knowledge of Stations. Each station has its unique ID, which is only needed for distinguishing various stations. Each station also knows its location and the number of stations in the network, n. Our algorithms also work when stations share, instead of n, an estimate $\nu \geq n$ of this value which is $O(n)$. We assume that each sender can enclose its ID and location to each transmitted message.[1]

Broadcast Problem and Complexity Parameters. In the broadcast problem, there is one distinguished node, called the *source*, which initially holds a piece of information (also called a *source message* or a *broadcast message*). The goal is to disseminate this message to all other nodes. We are interested in minimizing the *time complexity* of this task being the minimum number of rounds after which, for all communication networks defined by some set of parameters, the broadcast occurs with the probability at least $1 - \delta$ for a given $0 < \delta < 1$. This time is counted since the source is activated. For the sake of complexity formulas, we consider the following parameters: n, D and δ.

Messages and Initialization of Stations Other Than Source. We assume that a single message sent in an execution of any algorithm can carry the broadcast message and at most logarithmic, in the size of the network, number of control bits. A station other than the source starts executing the broadcast protocol after the first successful receipt of the source message; it is often called a *non-spontaneous wake-up model.* We say that a station which receives the source message for the first time is *waken up* at this moment and it is awake afterwards. Our algorithms are described from a "global" perspective, i.e., we count rounds starting from the moment when the source sends its first message. In order to synchronize stations, we assume that each message contains the number of rounds elapsed from the beginning of the execution of the algorithm.

1.2 Our Results

We present randomized distributed algorithms for broadcasting in wireless connected networks deployed in two dimensional Euclidean space under the SINR model, with uniform power assignment and any $\varepsilon \in (0, 1)$. We distinguish two settings: one with local knowledge of density, in which each station knows the upper bound on the number of other stations in its close proximity (dependent on parameter ε) and the other when no extra knowledge is assumed.

In the former model, we develop a randomized broadcasting algorithm with time complexity $O(D + \log(1/\delta))$, where D is the eccentricity of the communication graph, and δ is the maximum error probability. In the latter model, we first provide a simple and almost oblivious algorithm that accomplishes broadcast in $O(D(\log n + \log(1/\delta)) \log n)$ rounds with probability at least $1 - \delta$. Finally, we give a solution with time complexity $O((D + \log(1/\delta)) \log n)$, with probability at least $1 - \delta$, which is only slightly worse than the complexity of the algorithm relying on the density estimates. All these results hold for model parameter $\alpha > 2$ (for $\alpha = 2$ all the solutions are slower by a factor $\log n$).

Our algorithms are the first provably efficient and well-scalable randomized distributed solutions for the (global) broadcast task, which work in the model

[1] For the purpose of algorithms presented in this paper, it is sufficient that each station knows only some good approximation of its coordinates.

with coordinates, without spontaneous wake-up (i.e., no preprocessing is allowed) and for arbitrary value of the parameter ε defining the communication graph. This could be also contrasted with the complexity of broadcast by weak devices, for which such scalable algorithms (with respect to D and $\log n$) cannot be obtained [11]. Due to the space limit, some proofs are deferred to the full version.

1.3 Previous and Related Results

We discuss most relevant results in the SINR-based models and in the older Radio Network model.

SINR Models. One of the first communication problems studied from algorithmic point in distributed ad hoc setting under the SINR model was *local* broadcasting, in which each node has to transmit a message only to its neighbors in the corresponding communication graph. This problem was addressed in [8,10,19] for $\varepsilon > 1/2$. Randomized solutions for contention resolution [14] and packet scheduling (with power control) [13] were also obtained. Usually, the considered setting allowed power control in which, in order to avoid collisions, stations could transmit with any power smaller than the maximal one. Recently, a distributed *randomized* algorithm for multi-broadcast has been presented [18] for uniform networks. Although the problem solved in that paper is a generalization of a broadcast, the presented solution needs the power control mechanism and it is restricted to networks having the communication graph connected for $\varepsilon = \frac{2}{3}r$, where r is the largest possible range. Moreover, spontaneous wake-up of stations is necessary in their algorithm. In contrast, our solutions are efficient and scalable for *any* networks with communication graph connected for *any* value of $\varepsilon < \frac{1}{2}$.[2] Moreover, we do not use the power control mechanism. On the other hand, unlike ours, the algorithm from [18] works even if stations do not know their coordinates (or their estimates).

As shown recently [12], there exists an efficient *deterministic* broadcasting algorithm in the model considered in this paper. More precisely, it is worse than the best algorithm in this work by only a logarithmic factor. Independently, Daum et al. [4] proposed another randomized broadcasting algorithm. Their solution works for a broader family of metrics (not only the Euclidean) and does not rely on the knowledge of coordinates by stations. However, the time complexity of this solution is poly-logarithmic with respect to the ratio R between longest and shortest distance between stations, and R might be even exponential with respect to the size n of a given network.

There is a vast amount of work on centralized algorithms under the classical SINR models. The most studied problems include connectivity, capacity maximization, and link scheduling types of problems; for recent results and references we refer the reader to the survey [9].

Radio Network Model. There are several papers analyzing broadcasting in the radio model of wireless networks, under which a message is successfully heard if

[2] In case of $\varepsilon \in [1/2, 1)$, one could take our algorithm for $\varepsilon' = 1/3$, which guarantees at least as good asymptotic performance.

there are no other simultaneous transmissions from the *neighbors* of the receiver in the communication graph. This model does not take into account the real strength of the received signals, and also the signals from outside of some close proximity.

The problem of broadcasting is well-studied in the setting of *graph radio model*, in which stations are not necessarily deployed in a metric space. The first efficient randomized solution was developed by Bar-Yehuda et al. [1], while the close lower bound was proved in [17]. The algorithms closing the gap between the upper and the lower bound appeared in [3,16]. Since the solutions for a graph model are quite efficient, there are only few studies of the problem restricted to the geometric setting. However, solutions for some other communication problems can be significantly faster in geometric (uniform) radio networks than in general ones [7]. There is also a vast literature on deterministic algorithms for broadcasting in graph and geometric radio models, c.f., [2,15,16,5,6].

1.4 Technical Preliminaries

In this section we formulate some properties and notation that simplify the specification and analysis of algorithms.

Message Content and Global Clock. In the broadcast problem, a round counter could be easily maintained by already informed nodes by passing it along the network with the source message, thus in all algorithms we may in fact assume having a global clock. For simplicity of analysis, we also assume that every message sent during the execution of our broadcast protocols contains the broadcast message; in practice, further optimization of a message content could be done in order to reduce the total number of transmitted bits in real executions.

Successful Transmissions. We say that a station v transmits *c-successfully* in a round t if v transmits a message in round t and this message is heard by each station u in the Euclidean distance at most c from v. A station v transmits *successfully* to u in round t if v transmits a message and u receives this message in round t. We say that a station that received the broadcast message is *informed*.

Grids. Given a parameter $c > 0$, we define a partition of the 2-dimensional space into square boxes of size $c \times c$ by the grid G_c, in such a way that: all boxes are aligned with the coordinate axes, point $(0, 0)$ is a grid point, each box includes its left side without the top endpoint and its bottom side without the right endpoint and does not include its right and top sides. We say that (i, j) are the coordinates of the box with its bottom left corner located at $(c \cdot i, c \cdot j)$, for $i, j \in \mathbb{Z}$. A box with coordinates $(i, j) \in \mathbb{Z}^2$ is denoted $C_c(i, j)$ or $C(i, j)$ when the side of a grid is clear from the context. In the following sections we will always refer to boxes of the grid G_γ, where γ is a parameter specific for a considered algorithm. For a station v, $\text{box}_c(v)$ (or simply $\text{box}(v)$) denotes the box of G_c containing v.

Dilution. For the tuples (i_1, i_2), (j_1, j_2) the relation $(i_1, i_2) \equiv (j_1, j_2) \mod d$ for $d \in \mathbb{N}$ denotes that $(|i_1 - i_2| \mod d) = 0$ and $(|j_1 - j_2| \mod d) = 0$. A set of stations A on the plane is *d-diluted* wrt G_c, for $d \in \mathbb{N} \setminus \{0\}$, if for any two stations $v_1, v_2 \in A$ with grid coordinates $G_c(v_1) = (i_1, j_1)$ and $G_c(v_2) = (i_2, j_2)$, respectively, the relationship $(i_1, i_2) \equiv (j_1, j_2) \mod d$ holds.

2 An Algorithm for Known Local Density

In this section we describe our broadcasting algorithm for networks of known local density, which makes use of some properties exploited e.g., in local broadcasting [8,10]. That is, every station v knows the total number of stations $\Delta = \Delta(v)$ in its box of the grid G_γ. In this section we assume $\gamma = \frac{\varepsilon}{2\sqrt{2}}$. Without loss of generality we can assume, that for some $k \in \mathbb{N}$ the equality $(2k+1)\gamma = 2$ holds. This means that each box B from the grid G_γ lies in the center of some square 2×2 consisting of $(2k+1)^2 = (2/\gamma)^2$ boxes of G_γ. We call this square the *superbox* $S(B)$ of B. Note that all stations in the distance at most $1 - \varepsilon/2$ from B are in $S(B)$.

Algorithm 1. RandBroadcast(Δ, d, T) ▷ code for node v

the source s transmits
for *counter* $= 1, 2, 3, \ldots, T$ **do**
  for each $a, b : 0 \le a, b < d$ **do**
    if $v \in C(i,j) : (i,j) \equiv (a,b) \mod d$ **then**
      v transmits with probability $1/\Delta$

Analysis of Time Performance of RandBroadcast. We define *interference* at a station u with respect to the set of transmitters $\mathcal{T}$ as $\sum_{w \in \mathcal{T} \setminus \{v\}} P \mathrm{dist}(w,u)^{-\alpha}$, see Eq. (1). The boxes $C(i_1, j_1)$ and $C(i_2, j_2)$ are *connected* if there exist stations $v_1 \in C(i_1, j_1)$ and $v_2 \in C(i_2, j_2)$ such that (v_1, v_2) is an edge of the communication graph. We start with stating three general properties regarding interference in the SINR model.

Fact 1. *If the interference at the receiver is at most $\mathcal{N}\alpha x$, then it can hear the transmitter from the distance $1 - x$.*

Proof. By the Bernoulli inequality we get $(1+x)^\alpha \ge 1 + \alpha x$. Thus

$$SINR \ge \frac{P}{(\mathcal{N} + \mathcal{N}\alpha x)(1-x)^\alpha} \ge \frac{P}{\mathcal{N}(1+x)^\alpha (1-x)^\alpha}$$

$$= \frac{P}{\mathcal{N}(1-x^2)^\alpha} \ge \frac{P}{\mathcal{N}} = \beta \ .$$

where the last equality follows from the assumption that the range of stations is equal to 1 which implies $\left(\frac{P}{\mathcal{N}\beta}\right)^{1/\alpha} = 1$. □

We say that a function $d_{\alpha,Q} : \mathbb{N} \to \mathbb{N}$ is *flat* for $\alpha \geq 2$ and a (possibly empty) sequence of constant parameters Q if

$$d_{\alpha,Q}(n) = \begin{cases} O(1) & \text{for } \alpha > 2 \\ O((\log n)^{1/2}) & \text{for } \alpha = 2 \end{cases} \tag{2}$$

Let $C(a,b)$ be a box of G_γ. Assume that, in a given round of a randomized algorithm, only stations in superboxes $S(C(i,j))$ such that $(i,j) \equiv (a,b) \mod d$ transmit. In each box the expected number of transmitting stations is at most 1. We denote by I_d the average maximum of the interference over superbox $S(C(a,b))$ from transmitting stations located outside $S(C(a,b))$:

$$I_d = E\left(\max_{u \in S(C(a,b))} \sum_{v \in \mathcal{T}, v \notin S(C(a,b))} P\text{dist}(u,v)^{-\alpha}\right),$$

provided the algorithm uses the dilution parameter d.

Fact 2. *If in the above described process, the expected number of transmitting stations in a superbox does not exceed x instead of 1, then for any d we have the maximum expected interference in superbox $S(C(a,b))$ equal to $x \cdot I_d$.*

Let $s_\alpha(n) = \min\left\{\frac{\ln n}{2} + \ln 2, \frac{1}{2^{\alpha-2}(\alpha-2)}\right\} + \frac{1}{2^\alpha(\alpha-1)}$ and $d_{\alpha,I,\gamma}(n) = \left\lceil \frac{1}{\gamma}\left(\frac{8Ps_\alpha(n)}{I}\right)^{1/\alpha}\right\rceil$.

Lemma 1. *For any $I > 0$ there exists a flat function d such that $I_d \leq I$. Moreover, for $I \leq \frac{8Ps_\alpha}{2^\alpha}$ we have $I_d \leq I$ when $d = d_{\alpha,I,\gamma}(n)$.*

We proceed with the analysis of algorithm RandBroadcast.

Fact 3. *Consider a round of algorithm RandBroadcast, different from the first one. The probability that in a box $(i,j) \equiv (a,b)$ exactly one station transmits is bigger than $1/e$.*

Fact 4. *Consider a round of algorithm RandBroadcast(Δ, d, T) for $d = d_{\alpha,\mathcal{N}\alpha\varepsilon/4,\gamma}$, different from the first one. The probability that exactly one station in box $C(i,j)$, where $(i,j) \equiv (a,b)$, transmits and the interference from other stations measured in all boxes connected with box $C(i,j)$ is smaller or equal to $\mathcal{N}\alpha\varepsilon/2$ is bigger than $\frac{1}{2e}$.*

Lemma 2. *Consider a Bernoulli scheme with success probability $p < 1 - \ln 2$. The probability of obtaining at most D successes in $2D/p + 2\ln(1/\delta)/p$ trials is smaller than $(D+1)\delta$.*

We say that a subset of nodes W of a graph G is an *l-net* if any other node in G is in distance at most l from the closest node in W.

Fact 5. *If G is of eccentricity D, then there exists a $(1-\varepsilon)$-net W of cardinality at most $4(D+1)^2$.*

Proof. Let $q = 1 - \varepsilon$. Ranges q of all the stations must be all inside the circle of radius $(D+1)q$. The area of this circle is $\pi(D+1)^2q^2$. Let us greedily pick a maximal set of nodes such that any two nodes are in distance at least q. This set is a q-net W. Let us estimate the cardinality of W. All the circles of radius $q/2$ and center belonging to W are disjoint and have areas πq^2. They have total area at most $\pi(D+1)^2q^2$, so $|W| \leq 4(D+1)^2$. □

Using the above results we conclude the analysis.

Theorem 1. *Algorithm RandBroadcast*(Δ, d, T) *completes broadcast in any network in time* $O(d^2(D + \log(1/\delta)))$ *with probability* $1 - \delta$, *for* $d = d_{\alpha,\mathcal{N}\alpha\varepsilon/4,\gamma}(n)$ *and some* $T = O(D + \log(1/\delta))$.

Proof. To complete broadcasting it is enough that all the boxes containing stations of the $(1-\varepsilon)$-net W transmit the message $(1-\varepsilon/2)$-successfully at least once. This is done for box containing $v \in W$ if the message is $(1-\varepsilon/2)$-successfully transmitted at most D times on the shortest path from the source s to v in G, and finally is successfully transmitted by the box(v). The sufficient condition for this to happen is that a chain of altogether at most $D+1$ $(1-\varepsilon/2)$-successful transmissions heard by all potential receivers occurs. In each round the probability of a successful transmission within this chain is bigger than $p = \frac{1}{2e}$, by Fact 4 (recall that Fact 4 uses our assumption $d = d_{\alpha,\mathcal{N}\alpha\varepsilon/4,\gamma}$).

Now we estimate the probability that algorithm RandBroadcast completes the broadcast. Let the number of trials be $T = 2D/p + 2\ln(1/\delta')/p$, for some $\delta' \in \mathbb{R}$. By Lemma 2, Fact 1 and Fact 5, the probability that box(v) transmits $(1-\varepsilon/2)$-successfully for each $v \in W$

$$P \geq 1 - \sum_{v \in W} \Pr(\text{box}(v) \text{ doesn't transmit successfully}) \geq 1 - 4(D+1)^3\delta' .$$

This is bigger than $1-\delta$ for our choice of T. Note also that $T = O(D+\log(1/\delta))$. Because we have a trial every d^2 rounds, we need altogether $O(d^2(D+\log(1/\delta)))$ rounds, for $d = d_{\alpha,\mathcal{N}\alpha\varepsilon/4,\gamma}$. □

We would like also to point out that the knowledge of the density with respect to the grid G_γ (and not just with respect to some small neighborhood of a station) is essential for efficiency of Algorithm 1.

3 Algorithms for Unknown Local Density

In this section we describe our broadcasting algorithms for networks of unknown local density. First, we describe a simple almost oblivious algorithm, where the probability of transmitting a message by a station depends merely on the time when it receives the broadcast message for the first time, the current time slot and the fact whether it received a message from a station in its own box. Then, a more involved algorithm is presented which is slower than the (asymptotically optimal) solution for known density only by the multiplicative factor $O(\log n)$.

3.1 A Simple and Almost Oblivious Algorithm

In this section we present an almost oblivious algorithm based only on the size of a network and nodes positions up to a box in the grid G_γ, where here γ is set to $\varepsilon/(2\sqrt{2})$ and $\alpha > 2$. A computation of the algorithm is split into *phases*. A phase consists of $T \log n + R$ rounds, where T and R are some parameters which will be determined later. A station awakes when it receives a message for the first time and after that it is waiting by the end of the current phase. It becomes *active* in the following phase, when it executes Algorithm 2. However, if the station from a box B receives a message from another station in B in *any* round, it switches off and does not transmit any message in the remaining part of the algorithm. We call our algorithm Antibackoff, as each stations starts transmitting using small probabilities and then increases them gradually. This contrasts to classical backoff protocols, where stations are trying to transmit with large probabilities first and then decrease them gradually.

Algorithm 2. Antibackoff-Phase(n, T, R, d) ▷ code for node $v \in C(i, j) = B$

if at any time v receives a message from a station in B **then** switch off
for $i = 1, 2, 3, \ldots, \lceil \log n \rceil - 1$ **do**
  for $k = 1, 2, 3, \ldots, T$ **do**
    transmit with probability $\frac{2^i}{n}$
for $j = 1, 2, 3, \ldots, R$ **do**
  transmit with probability $\frac{1}{8(d+1)^2}$

We refer to iterations of the first loop in Algorithm 2 as to *stages*. The idea behind the algorithm Antibackoff is that the i-th stage deals with boxes containing around $n/2^{i-1}$ active stations by reducing the number of active stations in such boxes to no more than $n/2^i$. Thus, after the last stage, we expect that there is (exactly) one active station in each box containing an active station (at least one) at the beginning of a phase. Then, such a station is supposed to transmit $(1-\varepsilon/2)$-successfully in the "for j" loop, thus transmitting the broadcast message *on behalf* of all stations from its box. Indeed, if a station v transmits $(1-\varepsilon/2)$-successfully, then the message is received by all neighbors in the communication graph of all stations from the box containing v.

Now, we formulate some properties of Algorithm 2 which will conclude in Theorem 2 establishing its time complexity.

The following lemma limits the expected interference at a station caused by stations from distant boxes, provided there is an upper bound on the expected number of transmitters in the same box of G_γ. We define the max-distance between the boxes $C(i_1, i_2)$ and $C(j_1, j_2)$ as $\max\{|i_1 - j_1|, |i_2 - j_2|\}$.

Lemma 3. *Let $I_{B,k}$ be the maximal interference in a box $B = C(i, j)$ caused by boxes in max-distance at least $k+1$ from B under condition that expected number of transmitting station in every box is at most t and let $\kappa(t, x) = \lceil (8tP(\alpha - 1)/x(\alpha-2)\gamma^\alpha)^{1/(\alpha-2)} \rceil + 1$ for $x > 0$. Then $E[I_{B,k}] \le x$ for $k \ge \kappa(t, x)$.*

Now, we evaluate the probability that the number of active stations in each box is at most $n/2^i$ after the ith stage of a phase.

Fact 6. *The probability that at any phase, the number of active stations in any box after the i-th iteration of the first loop is at most* $n/2^i$ *is at least* $1 - n\log n/\exp(T/2^{16c(c+1)+5})$*, where* $c = \kappa(2, \mathcal{N}\alpha(1-\sqrt{2}\gamma)/2)$*.*

The proof of the above fact is obtained by bounding the probability that, for a given box B, the following events appear simultaneously in a round of the ith stage:

- exactly one station from B is transmitting a message;
- no station from boxes within max-distance at most c from B is transmitting;
- maximal interference caused by stations from boxes at max-distance greater than c from B is at most $\mathcal{N}\alpha(1-\sqrt{2}\gamma))$;

provided at least $n/2^i$ stations are active in B and at most $n/2^{i-1}$ stations are active in any other box.

While the previous fact deals with the progress in the process of eliminating stations from dense boxes, now we concentrate on the chance that a (station from a) box containing active stations transmits $(1-\varepsilon/2)$-successfully in the "for j" loop, provided there are no boxes with more than two active stations.

Fact 7. *Consider any phase* K*. Assume that, after the first loop of phase* K*, every box has at most two active stations. Let* r *be the probability that, for every box* B *with active stations in phase* K*, every station connected by an edge with a station* $v \in B$ *in* G *will receive a message from some station in* B*. Then,* r *is at least* $1 - n/\exp(R/64(d+1)^2)$ *with* $d = \kappa(2, \mathcal{N}\alpha\varepsilon/4)$*.*

By combining the above facts, we obtain a time bound of the algorithm.

Theorem 2. *Algorithm Antibackoff*(n, d, T, R) *completes broadcast in any n-node network in time* $O(D\log n\ (\log n + \log(1/\delta)))$ *with probability at least* $1-\delta$*, for some* $T, R \in O(\log n + \log(1/\delta))$ *and* $d = \kappa(2, \mathcal{N}\alpha\varepsilon/4)$*.*

Proof. If events from Fact 6 and Fact 7 occur during an execution of the algorithm, then the maximal number of phases needed for the message to be heard by every station is at most D, since after the K-th phase every node within distance K from the source (in the communication graph) receives the message. Let $c = \kappa(2, \mathcal{N}\alpha(1-\sqrt{2}\gamma)/2)$. One can easily verify that choosing $R \geq 64(d+1)^2(\ln n + \ln(1/\delta_1))$ and $T \geq 2^{16c(c+1)+5}(\ln n + \ln(1/\delta_2))$, the probability that one of the events did not occur is smaller than $\delta_1 + \delta_2$. With $\delta_1 = \delta_2 = \delta/2$, the probability of successful transmission in time $O(D\log n(\log n + \log(1/\delta)))$ is at least $1-\delta$. □

3.2 A Fast Algorithm with Local Leader Election

In this section we describe our broadcasting algorithm for networks of unknown local density. To construct this algorithm we consider the grid G_γ, where $\gamma =$

$\frac{\varepsilon}{6\sqrt{2}}$. For further references observe that this choice of γ satisfies the following property. Let B and $U = C(i,j)$ be boxes of G_γ and let $v \in B, u \in U$, $(u,v) \in E(G)$ for some nodes u and v. In such a case, if any node $v' \in B$ transmits $(1-\varepsilon/2)$-successfully, then its message is received by all stations in all boxes $C(i+a, j+b)$, where $a,b \in [-2,2]$.

We say that two boxes B and U are *adjacent* if the Euclidean distance between any two of their points is at most $1-\varepsilon/2$. But with one exception – boxes that are very close to each other are not adjacent. More precisely the box $C(i,j)$ is not adjacent to any box $C(i+a, j+b)$, where $a,b \in [-2,2]$. Note that if $(u,v) \in E(G)$ and $v \in B, u \in U = C(i,j)$, then any two points $x \in B$ and $y \in C(i+a, j+b), a, b \in [-2,2]$ are in the Euclidean distance at most $1-\varepsilon/2$; that is, B is adjacent to all boxes $C(i+a, j+b)$, unless these boxes are also very close to B. The *neighborhood* of a box B is the set of all boxes U adjacent to B. This definition guarantees that each station $v \in B$ is connected by an edge with each station $u \in U$ if U is in the neighborhood of B. However, the Euclidean distance $\text{dist}(v,u)$ for such u, v is larger than the distance between v and any other station from B. This property is essential for our method of electing leaders in boxes of G_γ.

To formulate the algorithm we define an *octant* of the neighborhood of the box $B = C(i,j)$. Let us place on the plane a Cartesian coordinate system with the origin in the center of the box B. This coordinate system is naturally subdivided into four *quadrants* i.e. the plane areas bounded by two reference axes forming the 90^o angle. The quadrant can be divided by the bisector of this angle into two *octants* corresponding to the angle of 45^o. We attribute one of the rays forming the boundaries of the octants to each octant, so that they are disjoint (and connected) as the subsets of the plane. An *octant* of the neighborhood of B is the set of all boxes U in the neighborhood of B that have centers in a given octant of the coordinate system.

Fact 8. *Each two stations in an octant of the neighborhood of a box B are in the distance at most* $(1-\varepsilon/2)$.

Now we give an intuition how Algorithm 3 works. A station v joins the execution of the algorithm after obtaining the broadcast message (waking up); it can learn the number of executed rounds of the algorithm from the value of the clock attached to each message. The algorithm consists of T iterations of the most external loop. Each of these iterations consists of two parts. The first part is a deterministic broadcast from the leaders of the boxes to all nodes in the distance at most $1-\varepsilon/2$ from these leaders. It is assumed that new nodes are woken up only in the very beginning and in this first part. The second part is a probabilistic algorithm attempting to elect the leaders in all the boxes in which the message was heard in the first part and which currently do not have leaders.

Now, let us fix the values of parameters for which the algorithm will by analyzed. Let $d = d_{\alpha, \mathcal{N}\alpha\varepsilon/2, \gamma}$ which assures that, in the first "**for** each a, b", loop each leader is heard in the distance $1-\varepsilon/2$. Moreover, we take $\bar{d} = d_{\alpha, \mathcal{N}\alpha\varepsilon/28, \gamma}$. This choice guarantees that, if there are on average less than 7 transmitting stations

Algorithm 3. RandUnknownBroadcast(d, T)

```
the source s transmits and becomes the leader of its box of G_γ
for counter ← 1, 2, ..., T do
    for each a, b : 0 ≤ a, b < d do
        if v is the leader of C(i, j) such that (i, j) ≡ (a, b) mod d then v transmits
    for each a, b : 0 ≤ a, b < d̄ do
        for each octant of neighborhood of each B = C(i, j) fulfilling
            (i, j) ≡ (a, b) mod d̄ do
            u ← the leader of the box with lexicographically smallest
                 coordinates in the octant
            for each v ∈ B: conflict(v) ← false
            for k = 0, 1, 2, 3, ..., log n do
                if B has no leader, u exists and not conflict(v) then
                    K1: Each vertex v ∈ B transmits with the probability (1/n)2^k
                    K2: if u hears v in K1 then
                            u transmits "v" and v becomes the leader
                        if v transmitted in K1 and hears nothing in K2 then
                            conflict(v) ← true
                    K3: nodes v transmitting in K1 and u transmit
                        if v not transmitting in K1 does not hear u then
                            conflict(v) ← true
```

attributed to each box $C(i', j')$ in the second loop "**for** each a, b", then we have the probability at least $1/2$ that the only station transmitting for $C(i, j)$ does it $(1 - \varepsilon/2)$-successfully. We prove that, during the second part, the probability of electing a leader is bigger than some constant. This is done for each octant in the "**for** k" loop and the result is either selecting the leader of B or silencing all stations in B till the end of this loop (in order to decrease interference in other boxes). To make such an attempt some help from the leader u of a box U adjacent to B is needed. Within an octant the leaders hear each other in the first part, so they all can determine without any additional communication which of them has lexicographically smallest coordinates. Also any node in B knows whether any leader in the octant exists. Let us emphasize here that the second loop lasts $8 \cdot 3 \cdot \bar{d}^2(1 + \log n)$ rounds, since we try to elect a leader in each $B = C(i, j)$ with help of leaders from various octants of its neighborhood separately.

In the loop "**for** k" the transmission probability in K1 grows twice per iteration starting from $1/n$. In rounds K2 and K3 stations from B are "switched off" till the end of the loop "**for** k". It is done in three cases. The first case is when the external noise causes this "switching off" (v cannot hear u in K3). We show that the probability that any stations in B is switched off this way in the whole "**for** k" loop is smaller than $1/2$. In the second case the leader is chosen, because u hears some station transmitting in K1. The station u then notifies deterministically all the stations in B who the leader is. In the third case many stations of B transmit in K1 which causes "switching off" all stations in B.

We now show, that if in some step K1 at least one station of B transmits, then after K3 all stations in B are "switched off". We already considered the case when u hears some of them and the leader is elected. So now assume, that u does not hear anything in K1. Note, that in K2 all stations v transmitting in K1 get the value conflict equal true. In K3 any station v not transmitting in K1 is closer to any of the transmitting stations in B than to u (this fact follows from the properties of neighborhood). So v does not hear u and gets the value conflict equal true.

The above discussion gives the following conclusion.

Fact 9. *Let l be the first round K1 of loop "**for** k" in which some station from a box B transmits a message. Then in the next rounds K2 and K3 either the leader is elected or all stations $v \in B$ set conflict(v) =true.*

Now we formulate an analog of Fact 5 for our algorithm.

Lemma 4. *Let G be of eccentricity D. There exists a set of boxes W of the grid G_γ of cardinality at most $4(D+1)^2$ having the two following properties*

(i) if we choose one station from each box of W then these stations form a $(1-\varepsilon/2)$-net in the set of all the stations,
(ii) for each box B of W there exists a sequence of at most $D+1$ nonempty (i.e., containing stations) boxes, starting from box(s) and ending in B, in which each two consecutive boxes are adjacent.

In what follows, we estimate what is the average maximal number of stations transmitting in the box $C(i,j)$, then we bound the probability of successful leader election in a single call of the loop "**for** k", and finally we conclude the analysis of algorithm RandUnknownBroadcast.

Fact 10. *The expected value of the maximum number of stations transmitting in the box $C(i,j)$ in round K1 during one call of the loop "**for** k" is at most 6.*

Fact 11. *Assume that at least one station from a box $C(i,j)$ is awaken in the first "**for** each (a,b)" loop. Then, the probability, that in one call of the loop "**for** k" the leader of the box $C(i,j)$ is elected is at least $1/18$.*

Theorem 3. *Algorithm RandUnknownBroadcast(d,T) accomplishes broadcast in $O(\bar{d}^2(D+\log(1/\delta))\log n)$ rounds, with probability $1-\delta$, when run for $d = d_{\alpha,\mathcal{N}\alpha\varepsilon/2,\gamma}, \bar{d} = d_{\alpha,\mathcal{N}\alpha\varepsilon/28,\gamma}$ and for some $T = O(D+\log(1/\delta))$.*

Proof. Let W be a set of boxes satisfying the properties (i) and (ii) from Lemma 4. A sufficient condition for the broadcast is that each box of $B \in W$ obtains the message and broadcasts it at least once to all stations in the range $1-\varepsilon/2$. (We say that a box *obtains* a message when at least one station in that box receives it, and a box *broadcasts* a message in a particular range r_0 when at least one of its stations transmits the message r_0-successfully.) This happens, when the message is successfully transmitted at most D times on the shortest sequence of boxes from the source to B and finally is successfully transmitted by the box B.

The sufficient condition for this is that a chain of altogether at most D successful leader elections happen. The probability of such a successful leader election is, by Fact 11, at least $p = 1/18$.

Now we estimate the probability that our algorithm completes the broadcast. Let the number of repetitions of the most external loop be $t = 2D/p + 2\ln(1/\delta')/p$, for some $\delta' \in \mathbb{R}$. By Lemma 2,

$$\Pr(\text{some } B \in W \text{ does not transmit successfully}) \leq$$
$$\leq \sum_{B \in W} \Pr(\text{box } B \text{ does not transmit successfully}) \leq 4(D+1)^2\delta' .$$

To get this probability smaller than δ we need the number of repetitions of the most external loop

$$T = \frac{2D}{p} + \frac{2\ln(1/\delta)}{p} + \frac{2\ln(4(D+1))}{p} = O(D + \log(1/\delta)) .$$

Each run of the most external loop takes $O(\bar{d}^2 \log n)$ rounds, which yields $O(\bar{d}^2(D + \log(1/\delta)) \log n)$ rounds in total. □

4 Conclusions and Future Work

In this work we showed provably well-scalable randomized distributed solutions for the broadcast problem in any wireless networks under the SINR physical model without spontaneous wake-up and without strong assumptions about the connectivity of a given network. Our algorithms rely on the knowledge of its own coordinates by each station; some results without such knowledge were obtained in [4]. We develop a new technique for fast election of local leaders in any network, which may be adopted for the purpose of other communication problems.

Our solutions could be extended to more generalized model settings. In particular, nodes do not have to know their exact coordinates, but only with some $O(\epsilon)$ accuracy. Parameters $\alpha \geq 2$ and $\beta \geq 1$ can be set up individually for every link, which would only change constants hidden in the big-Oh formulas (these constants would depend on the upper and lower bounds on the range of individual parameters α, β). The knowledge of exact number of stations n is also not necessary — an upper bound $O(n)$ is enough to obtain asymptotically the same results.

There are several interesting directions arising from or related with our work. The main one is to extend the proposed approach to other communication problems, such as multi-broadcast, gathering, group communication and routing. The second interesting direction is to study the impact of model setting, such as knowledge of coordinates (or other parameters), or the quality parameter $1-\epsilon$ of the communication graph on the complexity of a communication task. Finally, analyzing algorithms in more advanced models, e.g., with failures, mobility, or other forms of uncertainty, is another perspective research direction.

References

1. Bar-Yehuda, R., Goldreich, O., Itai, A.: On the Time-Complexity of Broadcast in Multi-hop Radio Networks: An Exponential Gap Between Determinism and Randomization. J. Comput. Syst. Sci 45(1), 104–126 (1992)
2. Chrobak, M., Gasieniec, L., Rytter, W.: Fast broadcasting and gossiping in radio networks. J. Algorithms 43(2), 177–189 (2002)
3. Czumaj, A., Rytter, W.: Broadcasting algorithms in radio networks with unknown topology. In: FOCS, pp. 492–501 (2003)
4. Daum, S., Gilbert, S., Kuhn, F., Newport, C.: Broadcast in the ad hoc SINR model. In: Afek, Y. (ed.) DISC 2013. LNCS, vol. 8205, pp. 358–372. Springer, Heidelberg (2013)
5. Dessmark, A., Pelc, A.: Broadcasting in geometric radio networks. J. Discrete Algorithms 5(1), 187–201 (2007)
6. Emek, Y., Kantor, E., Peleg, D.: On the effect of the deployment setting on broadcasting in euclidean radio networks. In: PODC, pp. 223–232 (2008)
7. Farach-Colton, M., Fernandez Anta, A., Mosteiro, M.A.: Optimal memory-aware Sensor Network Gossiping (or how to break the Broadcast lower bound). Theor. Comput. Sci. 472, 60–80 (2013)
8. Goussevskaia, O., Moscibroda, T., Wattenhofer, R.: Local broadcasting in the physical interference model. In: Segal, M., Kesselman, A. (eds.) DIALM-POMC, pp. 35–44.ACM (2008)
9. Goussevskaia, O., Pignolet, Y.A., Wattenhofer, R.: Efficiency of wireless networks: Approximation algorithms for the physical interference model. Foundations and Trends in Networking 4(3), 313–420 (2010)
10. Halldorsson, M.M., Mitra, P.: Towards tight bounds for local broadcasting. In: FOMC 2012, p. 2 (2012)
11. Jurdzinski, T., Kowalski, D.R., Stachowiak, G.: Distributed Deterministic Broadcasting in Wireless Networks of Weak Devices. In: Fomin, F.V., Freivalds, R., Kwiatkowska, M., Peleg, D. (eds.) ICALP 2013, Part II. LNCS, vol. 7966, pp. 632–644. Springer, Heidelberg (2013)
12. Jurdzinski, T., Kowalski, D.R., Stachowiak, G.: Distributed Deterministic Broadcasting in Uniform-Power Ad Hoc Wireless Networks. In: Gąsieniec, L., Wolter, F. (eds.) FCT 2013. LNCS, vol. 8070, pp. 195–209. Springer, Heidelberg (2013)
13. Kesselheim, T.: Dynamic packet scheduling in wireless networks. In: PODC, pp. 281–290 (2012)
14. Kesselheim, T., Vöcking, B.: Distributed contention resolution in wireless networks. In: Lynch, N.A., Shvartsman, A.A. (eds.) DISC 2010. LNCS, vol. 6343, pp. 163–178. Springer, Heidelberg (2010)
15. Kowalski, D.R.: On selection problem in radio networks. In: Aguilera, M.K., Aspnes, J. (eds.) PODC, pp. 158–166. ACM (2005)
16. Kowalski, D.R., Pelc, A.: Broadcasting in undirected ad hoc radio networks. Distributed Computing 18(1), 43–57 (2005)
17. Kushilevitz, E., Mansour, Y.: An omega($d \log (n/d)$) lower bound for broadcast in radio networks. SIAM J. Comput. 27(3), 702–712 (1998)
18. Yu, D., Hua, Q.-S., Wang, Y., Tan, H., Lau, F.C.M.: Distributed multiple-message broadcast in wireless ad-hoc networks under the SINR model. In: Even, G., Halldórsson, M.M. (eds.) SIROCCO 2012. LNCS, vol. 7355, pp. 111–122. Springer, Heidelberg (2012)
19. Yu, D., Wang, Y., Hua, Q.-S., Lau, F.C.M.: Distributed local broadcasting algorithms in the physical interference model. In: DCOSS, pp. 1–8. IEEE (2011)