Buckling of Non-isotropic Plates with Cut-Outs

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Abstract. To consider the buckling of non-homogeneous elastic thin structures weakened by holes, we analyze the effect of the area of rectangular or circular holes on a critical buckling load under compression of rectangular or circular plates made of isotropic, orthotropic or transversally isotropic materials.

Keywords: Non-isotropic plate buckling, non-homogeneous plate buckling.

1 Introduction

This research is concerned with the buckling analysis of non-homogeneous (weakened by holes or cut-outs) isotropic or non-isotropic (orthotropic or transversely isotropic) thin-walled elastic plates. The purpose of the study is to examine the effect of the area and ratios of rectangular or circular holes on the critical loading of rectangular or circular plates. The effect of the boundary conditions and the plate side ratios are also analyzed. We limit ourselves to the analysis of plates under external compressive planar loadings. The plates are considered to be thin enough to apply the 2D Kirchhoff–Love theory [1]. Mathematically the buckling problems for plates with cutouts are reduced to the solution of boundary value problems for multiply connected domains, which are solved through analytical and/or numerical methods including the Bubnov–Galerkin method [2] and FEM.

2 Buckling of Isotropic Plates

2.1 Rectangular Plates

We start with the analysis of an isotropic thin plate under axial compressive load. The load is directed along the lateral faces of the plate of length a, with side ends of length b where $a \geqslant b$ and the side ratio is $k = a/b$. We consider only simply supported boundary conditions. The free edges of the central hole are parallel to the plate sides and the square hole has length d.

For homogeneous plates the buckling load may be derived analytically [1]

$$N_{cr} = \min_{m,n} \left[((nk)^2 + m^2) \frac{\pi^2 D}{a^2 m^2} \right]; \qquad k = \frac{a}{b}.$$

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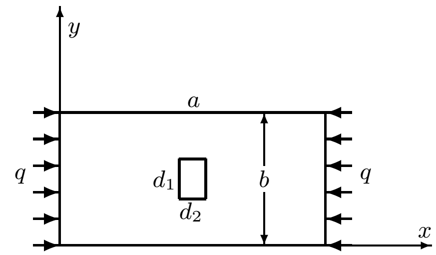

Fig. 1. Buckling of rectangular plate under compression

Here n is the number of waves in the axial direction, m is the number of waves in the transversal direction, and D is the cylindrical stiffness. For plates with holes, the results obtained by means of the Bubnov–Galerkin method are reported in [2]. Here we compare them with the results of the numerical analysis of the problem by means of the FEM package ANSYS. The most important and interesting point is the effect of the hole area on the critical buckling loading and the buckling modes.

In Fig. 2 one can see the effect of the plate sides ratio on the critical loading for a plate with relative thickness $h = 0.01$ and Poisson's ratio $\nu = 0.3$, where N_0 and N_{cr} are the critical buckling loads for a homogeneous plate (see [1]) and for a plate with a square hole respectively, $d = 0.1$.

It appeared that the critical buckling loadings may either increase or decrease. Presumably the effect when "mechanical buckling strengths of the perforated plates, contrary to expectation, increase rather than decrease as the hole sizes grow larger" was firstly reported in [3]. In our research it was found that, for

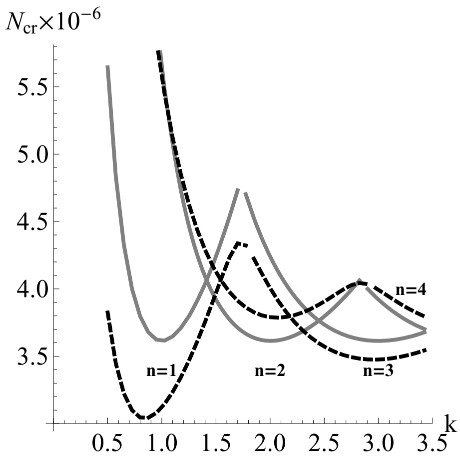

Fig. 2. Buckling of isotropic plates. Effect of the plate side ratio on the critical loading for plates with a hole with $d = 0.1$ (dashed lines) and a homogeneous plate.

example, for axially compressed rectangular plate for buckling nodes with odd wave numbers the critical loading decreases when the hole area decreases and for even wave numbers the buckling load increases when the hole area decreases [4]. The explanation of this phenomenon is in the initial compressive stresses developing in the narrow strips between the hole and the plate edges. One should remember that a hole not only affects the plate stiffness but also influences initial stress-strain state. These initial stresses are higher for the stronger supports of the lateral edges of the plate and they increase with Poisson's ratio (see [4] and [5]). That leads to the increase of the critical load.

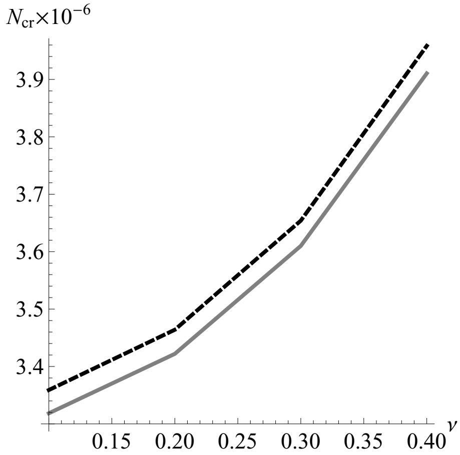

Fig. 3. Effect of Poisson's ratio on critical buckling loading for rectangular plate with a hole

The ratio of the hole sides plays an important role, which is revealed when we analyze plates with holes of equal areas. For buckling under axial loading for all cases, the extension of the hole in the axial direction leads to a decreasie of the critical loading. For a hole elongated in the transversal direction, the width of the strip is smaller and the intensity of the initial stresses is higher and the critical loading increases. The change of the hole side ratio may also cause a switch of the buckling modes. Clearly, this is valid only within some limits for the hole sides. When the width of the side strip becomes too small, the local buckling occurs and one of the strips buckles as a beam under compressive load.

2.2 Circular Plates

For the circular plate (radius R) with a central circular hole (radius r) under radial compressive load q the dependence of the critical load on the hole area is more predictable.

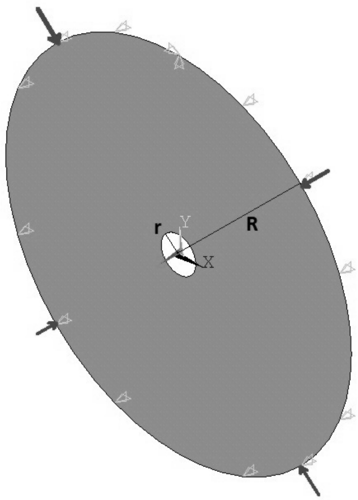

Fig. 4. Buckling of a circular plate under compresion

The main effect is the decrease of the plate stiffness with the hole area and the critical load goes down monotonically with the hole area. In Fig. 5, we compare numerical results for the critical loadings (dashed lines) and those obtained in [6] by the method of initial parameters (solid lines).

3 Buckling of Orthotropic Plates

The buckling behavior of non-isotropic plates has some specific features. As an example, we consider the buckling of a plate made of orthotropic material with Young's moduli E_x, and E_y, Poisson's ratios ν_{xy} and ν_{yx}, and shear modulus G. Since we wish to study the effect of non-isotropy on the buckling load we assume that

$$\begin{gathered} E_x = E_0(1+|\epsilon|)^{\mathrm{sgn}\ \epsilon}, E_y = E_0(1+|\epsilon|)^{-\mathrm{sgn}\ \epsilon}, \\ \nu_{xy} = \nu_0(1+|\epsilon|)^{\mathrm{sgn}\ \epsilon}, \\ E_x\nu_{yx} = E_y\nu_{xy} = E_0\nu_0, G = E_0/(2(1+\nu_0)). \end{gathered} \tag{1}$$

So, for small ϵ, this material is almost isotropic. For positive ϵ, the material is stiffer in the x-direction, for negative ϵ, it is stiffer in the y-direction. Note that for small $\epsilon > 0$, $E_x \approx E_0(1+\epsilon)$ and $E_y \approx E_0(1-\epsilon)$.

3.1 Rectangular Plates

The effect of orthotropy on buckling load of the rectangular plate with a central square hole with different hole area $S^* = d^2$ under axial compression is shown in Fig. 6, where N_0 and N_{cr} are the critical buckling loads for plates without and

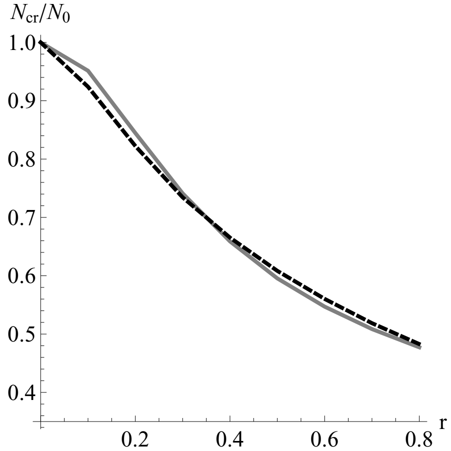

Fig. 5. Buckling of isotropic plates. Effect of the hole area on critical loadings for a circular plate with a circular hole of radius r.

with a hole, respectively. Again for a homogeneous plate we use the analytical formula [1]

$$N_{cr} = \pi^2 \left[D_{11} \left(\frac{n}{a}\right)^2 + 2H \left(\frac{m}{b}\right)^2 \left(\frac{n}{a}\right)^2 + D_{22} \left(\frac{m}{b}\right)^4 \right], \tag{2}$$

where

$$D_{11} = \frac{E_x h^3}{12(1 - \nu_{xy}\nu_{yx})}, \; D_{22} = \frac{E_y h^3}{12(1 - \nu_{xy}\nu_{yx})}, \; H = \nu_{yx} D_{11} + \frac{G h^3}{6}.$$

Even for a relatively small hole, the effect of non-isotropy is very significant: if the plate becomes stiffer in the axial direction and softer in the transversal direction the critical buckling loading decreases very rapidly with $\epsilon > 0$ and for a plate stiffer in the transverse direction the critical buckling loading increases. Again, this underlines the crucial effect for buckling of the initial stresses "carried by the narrow side strips of material along the plate boundaries" [3]. It is interesting that for orthotropic plates even for a very small hole area the buckling mode does not change with the plate side ratio. It looks as the hole fixes the buckling mode wavenumbers that leads to rather monotone dependence of the critical load on the plate material stiffness ratio.

3.2 Circular Lates

Similarly, the critical loading of an orthotropic circular plate with a central circular hole under radial compression depends on the material stiffness ratio. Here the important effect comes from the initial stresses in the circumferential direction. For materials (1) stiffer in the circumferential direction ($\epsilon < 0$) the critical load increases (see Fig. 7).

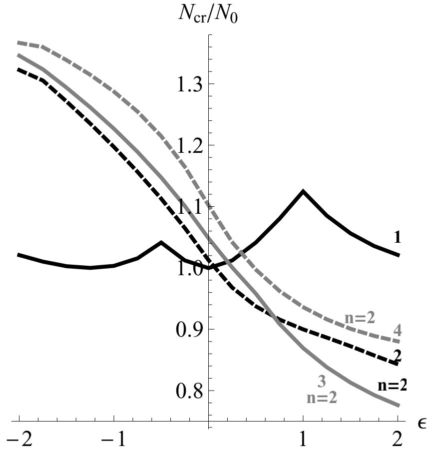

Fig. 6. Buckling of orthotropic plates. Rectangular plate with $k = 2$: 1 – a homogenous plate, and 2,3,4 – a plate with a central square hole of area $S^* = 0.01; 0.04; 0.09$, respectively.

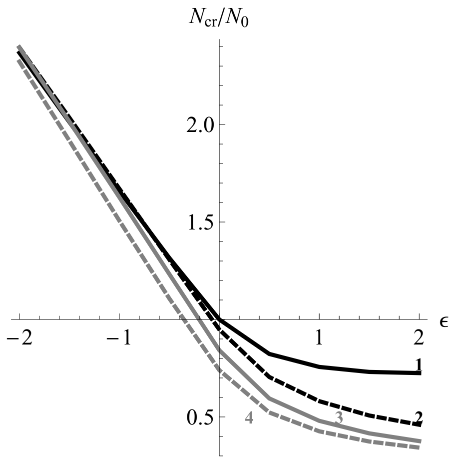

Fig. 7. Buckling of orthotropic plates. Circular plate with $R = 1$: 1 –a homogenous plate, and 2,3,4 – a plate with a central circular hole with $r = 0.1; 0.2; 0.3$, respectively.

4 Buckling of Transversally Isotropic Plates

Finally, we consider the buckling behavior of transversally isotropic plates with the following elastic moduli:

$$\begin{gathered} E_x = E_y = E_0(1+|\epsilon|)^{\mathrm{sgn}\ \epsilon}, E_z = E_0(1+|\epsilon|)^{-\mathrm{sgn}\ \epsilon}, \\ \nu_{xy} = \nu_{yx} = \nu_0(1+|\epsilon|)^{\mathrm{sgn}\ \epsilon}, \\ E_x\nu_{yx} = E_y\nu_{xy} = E_z\nu_{xz} = E_0\nu_0, G = E_0/(2(1+\nu_0)). \end{gathered} \tag{3}$$

For positive ϵ the material is stiffer in the x, y-directions (in the plane), and for negative ϵ, it is stiffer in the z-direction (along the thickness).

4.1 Rectangular Plates

For rectangular transversally isotropic plates, the effect of the material properties is shown in Fig. 8. For stiffer planar materials ($\epsilon > 0$), the buckling load is higher and the buckling mode essentially depends on the stiffness parameter. For small ϵ, the critical load increases with the hole area. Here a buckling mode switching exists. One can see the significant difference with the case of the orthotropic material. For large enough ϵ in absolute value, the critical buckling value becomes smaller than for the isotropic plate.

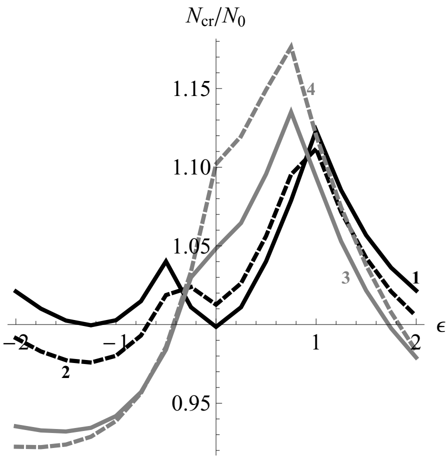

Fig. 8. Buckling of transversally isotropic plates. Rectangular plate with $k = 2$: 1 – homogenous plate, and 2,3,4 – plate with central square hole of area $S^* = 0.01; 0.04; 0.09$, respectively.

4.2 Circular Plates

For transversally isotropic circular plates, the change of planar ratio and in thickness, Young's modulus leads to an increase of the critical buckling load which decreases monotonically with the hole area.

5 Conclusions

The presence of a hole or cut-outs may lead to either increasing or decreasing critical buckling load for compressed plates depending on the boundary conditions, geometric parameters of the plate and the hole and the material properties. For rectangular plates, the main effect comes from stresses in the lateral strips. For buckling of non-isotropic circular plates, the material stifness ratio plays the key role.

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