The Robustness of Assortativity
(Short Paper)

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Abstract. Complex networks are ubiquitous in real word and represent a key model for both human made and natural systems. An important characteristics that distinguishes technological networks from biological networks is the assortativity, i.e. the correlation among the degrees of connected nodes. We apply spectral analysis to investigate how assortativity influences the robustness of a network with respect to failure propagations or epidemic spreading. We find a no free lunch situation: while disassortative networks are more robust since they have a higher failure threshold, in assortative networks there is more time for intervention before total breakdown.

1 Introduction

Complex Networks have been applied to a wide range of sectors, from technological fields like the Internet or power grids to biological fields like genomics or ecosystems [1,2]. A network is anything that can be represented by a set of elements called nodes connected by links representing some relationship among nodes: as an example, in social networks the nodes are people and the links between them can be relationships like friendship, political alliance or collaboration. The structure of the networks is linked to topological metrics like the degree distribution (the degree of a node is the number of its neighbours) and it plays a key role in determining the robustness, the resilience ad the response of a network [3]. Real networks in most cases show non-trivial topological correlations; in particular, many networks show "assortative mixing" on their degrees, i.e. high-degree vertices tend to be attached to high-degree ones, while other networks show disassortative mixing, i.e. high-degree vertices tend to be attached to low-degree ones. The network's degree–degree correlation can be quantified by a single scalar α called the assortativity coefficient [4] which assumes values $\alpha = 0$ for degree-uncorrelated networks, $\alpha > 0$ for assortative networks and $\alpha < 0$ for disassortative networks. Assortative correlations are typically observed in social networks [4]; on the other hand, disassortative connections are mainly found in technological and biological networks [5]. We want to investigate the consequences of the assortativity on the characteristics of a network.

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2 Monte Carlo

To randomize a networks one possible procedure consists into reshuffling links while keeping the degree of each node constant [6]; it has already been noticed that link-swap moves can be assortative, disassortative or neutral [7]. We introduced a means to sample the space of networks of different assortativity sharing the initial degree distribution. While our procedure is general, in this paper we will concentrate on initial network configurations obtained by the Barabasi-Albert preferential attachment procedure [8].

We define a fictive energy $H(G) = -\sum_{ij} k_i A_{ij} k_j$ that has the property that on average H decreases if the assortativity increases and vice-versa We can therefore use the fictive energy H to sample the space of assortative networks via a Monte Carlo procedure in which we assign the weight $\propto \exp[-\beta H(G)]$ to the configuration G and we accept a link reshuffling move with probability $\exp\{-\beta [H(G') - H(G)]\}$. The parameter β looks like the analogous of an inverse temperature in the canonical ensemble, but in order to be able to sample both assortative and disassortative configurations we have to allow β to be both positive and negative. The resulting sampling of the assortativity α respect to the parameter β is monotonously increasing.

3 Spectral Analysis

A powerful tool in assessing the general characteristic of a network is the spectral analysis of its associated matrices [9].

Formally, a network (or a graph) is defined as a couple $G = (V, E)$ where V is the set of N_V nodes and E is the set of N_E links; each link joins two nodes. To each graph G we associate its adjacency matrix A, defined as $A_{ij} = 1$ if nodes i,j are connected, $A_{ij} = 0$ otherwise. The networks we are considering are simple (no self loops, i.e. $A_{ii} = 0$) and undirected ($A_{ij} = A_{ji}$). The degree of node i is therefore $k_i = \sum_j A_{ij}$; nodes are labelled for increasing degree: $k_1 \leq k_2 \leq \ldots \leq k_N$.

The eigenvalues of A are real as A is Hermitian (we are considering undirected networks); moreover Λ_N is positive as A is a positive matrix.

The propagation of epidemics on networks is clearly linked to the adjacency matrix A that dictates which nodes can be infected by a virulent node; moreover the dynamics of epidemics in certain cases can be related to the dynamics of failure propagation. The maximum eigenvalue Λ_N has a particular status as it is linked to the epidemic threshold. The epidemic threshold τ of a network can be thought as the fraction of nodes to immunize in order to stop an infection with a fixed disease propagation rate; Wang and coauthors have shown that in networks the epidemic threshold scales as $\tau \sim 1/\Lambda_1$ [10,11].

We find that Λ_1^{-1} decreases with assortativiy: in the range of correlation we explore, disassortative networks show an epidemic threshold up to 20% higher than assortative ones (Fig.1). Our findings confirm the idea that avoiding direct connections between hubs (highly connected nodes) may provide protection against epidemics [12].

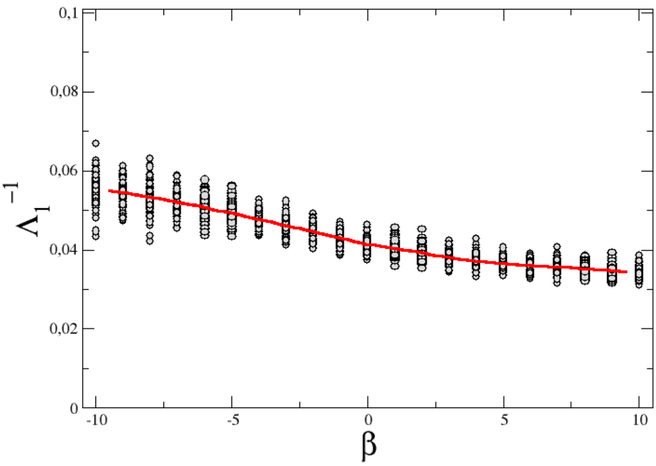

Fig. 1. The epidemic threshold Λ_1^{-1} decreases with assortativity (networks of 10000 nodes)

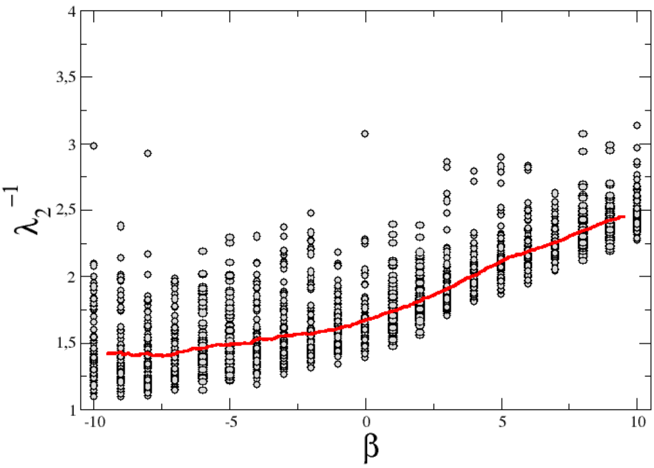

Fig. 2. For diffusion-like processes, the longest time to explore the network is proportional to λ_2^{-1}. For our networks of 10000 nodes, it increases with the assortativity.

The Laplacian matrix of a network is defined as $L \equiv D - A$, where D is the diagonal matrix of degrees $D_{ij} = k_i \delta_{ij}$. It is the analogous of the Laplacian operator and describes the diffusion of random walkers on the network. The eigenvalues of L are $\lambda_1 = 0 \leq \lambda_2 \leq \ldots \leq \lambda_N$; the eigenvector (mode) associated to the zero-th eigenvalue λ_1 is the equilibrium distribution for a diffusive process on the network. The first non-zero eigenvalue λ_2 is the inverse diffusional timescale of slowest mode, i.e. it is a measure of the longest time for a random walker to explore the whole network. Therefore, a lower value λ_2^{-1} means that there is less time for intervention before a network is totally compromised by randomly propagating failures or epidemics; in such respect assortative networks show times up to 60% higher than disassortative ones (Fig. 2).

4 Conclusions

We have investigated via spectral methods some effects of the assortativity on the robustness of a network with respect to randomly propagating failures and epidemics. We have found a "no free lunch" situation: while disassortative networks have a higher failure threshold, assortative networks give more time for intervention before total breakdown.

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