18 Basil Rathbone: Writing a Major Result

Basil Rathbone is not a theorist, but was one of the best—in my opinion—actors ever to portray Sherlock Holmes. Like many, I love his "version" of Holmes—just perfect in my opinion.

Our subject is how to claim a major result: how to write it up, and how to release it to the world.

Holmes will play a role in this discussion, of course. Besides being a wonderful character, he is a great problem solver. In the movies Holmes often plays his violin while thinking, much to the annoyance of Dr. John Watson. The good doctor, as he is often called, hates the "noise" of the playing. Sometimes it is at these moments when the good doctor makes a passing remark that unlocks the problem. Perhaps we should all play the violin or something like it when thinking. Or perhaps the message is that working alone is hard—we all need our Watson.

18.1 My Suggestions

You have just proved the greatest result of all time. Okay, say you have really proved a good result. Here are a few suggestions that you may wish to consider in thinking about your result, in writing it up, and in announcing it to the world. They are just my suggestions—feel free to disagree or to add your own ideas to my list.

- **Be Humble:** The Fundamental Theorem of Algebra states that every non-trivial polynomial over the complex numbers has at least one root. The famous Carl Gauss' doctoral thesis was one of the first "almost correct" proofs of this theorem. It is noteworthy that even the great Gauss entitled his thesis:

> *A New Proof of the Theorem that Every Integral Rational Algebraic Function of One Variable can be Decomposed into Real Factors of the First or Second Degree*

Note the word "new" in his title. He later gave correct proofs; today there are perhaps hundreds of papers published, each with a slightly different proof. Some use algebra, some use analysis, some topology, some complex function theory, and on and on.

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- **Be Humble:** Okay it's a repeat, but it is a key point.

If a problem is a well-known open problem it is likely that some new ideas are needed to solve it. You may think you have them. That is fine. Perhaps you do. But you may wish to step back and ask:

> What is the *trick* that I have seen that eluded everyone else?

If your answer is, "there is no new idea; it just all works," then you should be extremely skeptical.

I once heard a story that I cannot confirm, but it is so cool that I have to repeat it. Today we have just about every type of liquid sold to us in plastic containers. There was a time when milk was sold in plastic containers, like today, but soda was still sold in glass bottles. My understanding is that a clever engineer at a large company that made plastic had the "brilliant" insight: why not sell soda in plastic containers? It would be cheaper—plus it would make money for his company, thus he would get a raise.

The engineer told his co-workers his brilliant idea. He could not understand why no one had done it before. They told him: soda in plastic was impossible. No explanation, just that it was impossible.

He went home and decided he would prove them wrong. So he took a plastic milk container, removed all the milk, cleaned it out, and poured in his favorite soda. Then he placed it in his refrigerator and went to bed.

In the morning he went over to his refrigerator and opened the door. He was shocked. They were right. There was his soda container, only it had increased in size many-fold. Clearly, the gas pressure in the soda had made the plastic container expand to fill all the voids. It was like some sci-fi movie—the container had forced its way into every spare inch of space in the refrigerator.

I can only imagine the mess it made getting the container out. Yuck. The problem of putting soda into plastic containers was not trivial—there was a reason that the "obvious" idea did not work.

The story has a great ending, however. Unlike his co-workers this engineer started to work seriously on the problem. He asked "Why did the plastic container fail?" Eventually he was able to create a new type of plastic that had the right properties so that soda would not destroy the container.

- **Be Recursive:** One way is to divide the great paper up into pieces. Each piece should stand on its own, and together they make the whole proof. But each piece by itself is easier to explain, easier to write up, and easier to check. This method of dividing the "secret" will also damp down any popular press issues.

In "Sherlock Holmes and the Secret Weapon," a 1943 film starring Rathbone, this very trick is used to prevent the theft of a secret bombsight. A bombsight is a device used by planes to decide when to release their bombs. This is a real issue: the plane is moving rapidly, there usually is wind, the bomb falls a great distance before it hits the ground, and so on.

Holmes saves Dr. Franz Tobel and gets him, with his invention, to England. There Tobel divides the invention into four parts. Each part does not reveal the secret of how his new bombsight works.

Of course Professor Moriarty gets involved. There are codes to be broken, escapes to be made since Holmes is captured by his arch rival, and finally a trapdoor that takes the professor by surprise. This was one of the best of the Holmes movies made during the war.

- **Be Partial:** Okay you can prove a huge result. Perhaps a good idea might be to check that your methods at least extend our current knowledge. For example, if you prove that $\mathsf{P} \neq \mathsf{NP}$, perhaps you could first write a paper that proves that SAT cannot be done in linear time. This is open. It would be a great result, and would show off your new methods. But it would be more credible a claim, and would still be a major result. It would likely cause fewer distractions from the press—that can wait for your full paper.

This happened to some extent in the recent resolution to the Poincaré Conjecture. Grigori Perelman proved much more than "just" the Poincaré, but held out the details on all his results. Experts quickly realized that he might have proved more, but even just proving the conjecture was an immense achievement.

- **Be Computational:** I have proved theorems about n-dimensional space, even though my geometric intuition is close to zero. Once I read a claimed proof resolving an open problem about n-dimensional space. Unfortunately, some key lemma failed even for the case when $n = 1$: the simple line.

The suggestion is to try examples of your results. Not all proofs can be checked in this manner, but many can. Many of the great mathematicians have been tremendous calculators. They tried cases, they computed examples, they made tables of values. Some of these calculations were used to discover patterns, but they can also be used to see if your ideas make sense.

One of the neat examples of this is the story of how Percy MacMahon discovered the rough growth of the partition function. Recall $p(n)$ is the number of ways of writing n as a sum, if we do not count order. Thus $p(4)$ is 5:

$$\begin{aligned} 4 &= 1+1+1+1, \\ 4 &= 1+1+2, \\ 4 &= 1+3, \\ 4 &= 2+2, \\ 4 &= 4. \end{aligned}$$

Apparently he kept lists of the number of partitions of numbers and eventually noticed that the number of *decimal digits* in the numbers formed a parabola. This suggested that

$$p(n) \approx e^{c\sqrt{n}}.$$

He was right: although the correct approximate formula is a bit more complicated.

$$\frac{\exp(\pi\sqrt{2n/3})}{4n\sqrt{3}}.$$

- **Be Clear:** Writing mathematics is not easy. But you must do a reasonable job if you hope to be able to see if it works yourself. And of course also if you do this it will help your readers.

  Gary Miller, the "Miller" in the Miller–Rabin primality test, has told me his theory of writing mathematics. He says it is definitions, definitions, and definitions—just like "location, location, location" in real estate. What he means is that one should concentrate on getting the definitions right. They should be clear and crisp. The more precise they are, the better the chances that your proof can be understood. The other advantage of this insight is that even if one of your lemmas has a bug, you might be able to get help fixing it. If the statement of what you tried to prove is clear, then this is possible. If your definitions are murky, or even worse not stated at all, there is no hope.
- **Be Uneven:** What I mean is a more concrete notion of how to be clear in writing down your ideas. A common issue with many wrong proofs is that they spend too much precious time working on the easy part. This is what I mean by being "uneven." Spend little time on the standard facts, and spend lots of time on the new ideas, on the parts of the proof that are tricky, on the places where you do something new.

  I have seen many papers that fell apart after having spent huge space and time working out details to either things that are known, or on things that follow routinely from known theorems. It is a shame the writers do this—sometimes they spend huge amounts of energy on detailed LaTeX tables or figures of facts that I would not contest. Then when the big step occurs there is little or no detail.
- **Be Google-Smart:** Know the literature. If you work on any problem you should try to find out as much as you can on what is known. Almost no major work is done in a vacuum. Use the search engines, talk to experts, send friends and colleagues email, and try to see what is known about your problem.

  I must admit that this is harder and harder to do these days. There is so much work going on across the world that certainly you may miss something. But you can find out a huge amount of information from the Web and related sources.

  When I was a graduate student things were really quite different. There was a time when I flipped through every paper in all of Computer Science. Every one. I sat in the CMU library for months and just looked at every bound journal volume that had anything to do with computing. I will not say I read every paper, but I did scan them all. I loved having the time to do this. Also I loved that I could do it. I suspect today this is hopeless, even trying to do it for a subfield is hard. But the more you know the better your chances will be to succeed.
- **Be** . . .: Add your own suggestion here.

18.2 Open Problems

Good luck and I hope you can indeed prove some new wonderful theorems. I hope these suggestions have helped. What do you think? Also check out Lance Fortnow's view on the same subject.

18.3 Notes and Links

Original post:
http://rjlipton.wordpress.com/2010/09/12/how-to-present-a-big-result/
Gauss' thesis:
http://www.robertnowlan.com/pdfs/Gauss,%20Carl%20Friedrich.pdf
"Sherlock Holmes and the Secret Weapon":
http://en.wikipedia.org/wiki/Sherlock_Holmes_and_the_Secret_Weapon
Partition function:
http://en.wikipedia.org/wiki/Partition_function_(number_theory)
Lance Fortnow on presenting results:
http://blog.computationalcomplexity.org/2010/09/how-to-write-up-major-results.html