Expression and Processing of Uncertain Information

Guoyin Wang[1,2,3], Changlin Xu[1,2], and Hong Yu[2]

[1] School of Information Science & Technology, Southwest Jiaotong University, Chengdu 610031, China
[2] Chongqing Key Laboratory of Computational Intelligence, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[3] Institute of Electronic Information Technology, Chongqing Institute of Green & Intelligent Technology, CAS, Chongqing 401122, China
wanggy@ieee.org

Abstract. Uncertainty is one basic feature in the information processing, and the expressing and processing of uncertain information have attracted more attentions. There are many theories introduced to process the uncertain information, such as probability theory, random set, evidence theory, fuzzy set theory, rough set theory, cloud model theory and so on. They depict the uncertain information from different aspects. This paper mainly discusses their differences and relations in expressing and processing for uncertain information. The future development trend is also discussed.

Keywords: uncertain information, probability theory, evidence theory, random set, fuzzy set, rough set, cloud model.

1 Introduction

In the era of increasing popularity of computer and network, the manifestations of information are more diversified with the development of Internet and multimedia technology, such as text, image, video, audio, etc. Human-computer intersection is more frequent and closer. The expression and reasoning of uncertainty as a fundamental feature of information have always been the important issues of knowledge representation and reasoning [13].

There are many kinds of uncertainties, such as randomness, fuzziness, imprecision, incompleteness, inconsistency, etc.. Correspondingly, there are many theoretical models to study uncertain information. For example, the probability theory and the random set theory mainly study the random uncertainty [17][30]; the evidence theory mainly expresses and processes the uncertainties of unascertained information [7][24]; the fuzzy set theory [39] and their derivations, such as the type-2 fuzzy set, the intuitionistic fuzzy set and the interval-valued fuzzy set, study the fuzzy uncertainty of cognition; the rough set theory [19] and its

P. Lingras et al. (Eds.): RSKT 2013, LNAI 8171, pp. 53–65, 2013.

corresponding expansion models discuss the ambiguity indiscernibility and imprecision of information; the cloud model studies the randomness and fuzziness and their relationships [13][14].

Generally speaking, when talking about uncertainty of information, the uncertainty doesn't mean only one kind of uncertainty, but is the coexistence of multi kinds of uncertainty. In this paper, we will discuss the relations among the probability theory, the evidence theory, the random set theory, the fuzzy set theory and its derivations, the rough set theory and its extended models and the cloud model theory.

2 Uncertainty Expression in Probability Theory

Probability, as a measurement of random event, has been already applied widely. Probability and random variable are two important tools during the research of random phenomena. The axiomatic definition of probability is as follows.

Definition 1. [30] *Given a sample space Ω and an associated sigma algebra Σ, For $\forall A \in \Sigma$, the real-valued set function $P(A)$ defined on Σ is called a probability of the event A, when it satisfies:* (1) $0 \leq P(A) \leq 1$; (2) $P(\Omega)=1$; (3) *If the countable infinite events $A_1, A_2, \cdots \in \Sigma$, $A_i \cap A_j = \emptyset$, $i \neq j$, then $P\left(\bigcup\limits_{i=1}^{\infty} A_i\right) = \sum\limits_{i=1}^{\infty} P(A_i)$.*

For a given probability space (Ω, Σ, P), random variable X is a real-valued function on sample space Ω. Random variables and their probability distributions are two important concepts of studying stochastic system.

From Definition 1, we know that if the countable infinite events $A_1, A_2, \cdots \in \Sigma$, $A_i \cap A_j = \emptyset$, $i \neq j$, and $\bigcup\limits_{i=1}^{\infty} A_i = \Omega$, then $P\left(\bigcup\limits_{i=1}^{\infty} A_i\right) = \sum\limits_{i=1}^{\infty} P(A_i) = 1$. However, in actual applications, the random events A_i and A_j $(i \neq j)$ may not satisfy strictly $A_i \cap A_j = \emptyset$ due to the uncertainty of random events. So, the countable additivity of probability could not be satisfied. In 1967, Dempster gave a probability which does not satisfy countable additivity, and he tried to use a range of probabilities (upper and lower probabilities) rather than a single probability value to depict the uncertainty so as to establish evidence theory, which is further expansion of probability theory. Random set theory is also another expansion of probability theory, in which the value of a random variable is a closed set rather than a real number. Specific contents will be introduced in section 2.1 and section 2.2 respectively.

2.1 Evidence Theory

In evidence theory, belief function and plausibility function are two most fundamental and important notions. Let Ω be the frame of discernment representing all possible states of a system under consideration. Evidence theory assigns a belief mass to each element of the power set. Formally, the definition of a belief mass function is as follows.

Definition 2. [3] *Let Ω be a frame of discernment, a function* $m(A)$*:* $2^{\Omega} \rightarrow [0,1]$*, is called a function of basic probability assignment, when it satisfies two properties:* $m(\emptyset)=0$ *and* $\sum_{A \subseteq \Omega} m(A)=1$*.*

From Definition 2, we know that the function of basic probability assignment does not satisfy countable additivity due to $\sum_{A \subseteq \Omega} m(A)=1$, so it is different from probability function.

Based on the function of basic probability assignment, the belief function Bel and the plausibility function Pl are defined as:

Definition 3. [3] *Let Ω be a frame of discernment, $\forall A \subseteq \Omega$, a function Bel :* $2^{\Omega} \rightarrow [0,1]$*, is called a function of belief , when it satisfies:* $Bel(X)=\sum_{A \subseteq X} m(A)$*. A function* $Pl : 2^{\Omega} \rightarrow [0,1]$*, is called a function of plausibility, when it satisfies:* $Pl(X)=\sum_{A \cap X \neq \emptyset} m(A)$*.*

From Definition 3, $Bel(A)$ expresses the confident degree of the evidence supporting the event A being true, while $Pl(A)$ expresses the confident degree of the event A being non-false, and $Bel(A) \leq Pl(A)(\forall A \subseteq \Omega)$. $Bel(A)$ and $Pl(A)$ are called the lower limit and the upper limit of confidence degree for A, respectively.

Thus, another difference from the probability theory is that the evidence theory uses a range $[Bel(A), Pl(A)]$ to depict the uncertainty. The interval-span $Pl(A)$-$Bel(A)$ describes the "unknown part" with respect to the event A. Different belief intervals represent different meanings, see Figure 1.

Obviously, the three intervals are relative to the three-way decisions [38]. That is, the support intervals and reject intervals mean the two-way immediate decisions, and the uncertain interval means the third-way decision which also called the deferred decision.

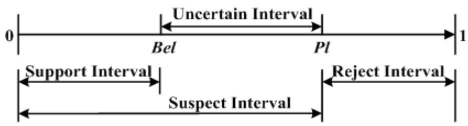

Fig. 1. Uncertainty expression of information

2.2 Random Set Theory

Random set is a set-valued function on sample space Ω, which is a generalization of random variable concept. The strict mathematical definition is as follows.

Definition 4. [17] *Let* (Ω, Σ, P) *be a probability space, and* $(\Psi, \sigma(\beta))$ *be a measurable space, where,* $\beta \subseteq 2^{\Psi}$*, if mapping* F*:* $\Omega \rightarrow 2^{\Psi}$*, is called random set, when it satisfies:* $\forall \Lambda \in \sigma(\beta)$*,* $\{u \in \Omega | F(u) \in \Lambda\} \in \Sigma$*.*

From Definition 4, the difference between random variable and random set is that the former is a random point function, while the latter is a random set-valued function. Thus, random set theory is a generalization from point variable statistics to set variable statistics.

3 Uncertainty Expression in Fuzzy Set Theory

Fuzzy set, which is proposed by Prof. Zadeh as an extension of Cantor set [39], is used to describe the uncertainty of cognition, that is, the extension of concept is not clear and we can not give definitive assessment standard. In Cantor set theory, an element either belongs or does not belong to the set. By contrast, fuzzy set permits the gradual assessment of the membership of elements in a set.

Definition 5. [39] *Let U be a universe of discourse, and A be a fuzzy subset on U, a map μ_A: $U \rightarrow [0,1]$, $x \mapsto \mu_A(x)$, is called membership function of A, and $\mu_A(x)$ is called membership degree respect to A.*

From Definition 5, $\mu_A(x)$ expresses the membership degree of an element x belonging to a fuzzy subset A. Once $\mu_A(x)$ is determined, it will be a fixed value. Thus, the operations between fuzzy sets based on membership degree become certainty calculation. Considering the uncertainty of membership degree, Zadeh proposed interval-valued fuzzy set (IVFS) and type-2 fuzzy set (T2FS) as extension of fuzzy set (FS) [40].

Definition 6. [40] *Let U be a universe of discourse, an interval-valued fuzzy set, denoted A_{IV}, is a map $\mu_{A_{IV}}: U \rightarrow Int[0,1]$, where, $Int[0,1]$ expresses a collection of all closed subintervals on $[0,1]$; A type-2 fuzzy set, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $\forall x \in U$ and $u \in J_x \subseteq [0,1]$, i.e.: $\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u))\}$, or $\tilde{A} = \int_{x \in U} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u)$, where $0 \le \mu_{\tilde{A}}(x,u) \le 1$, $\int\int$ denotes union over all admissible x and u.*

From Figure 2(a), the membership degree of IVFS A_{IV} is $\mu_{A_{IV}}(x_i) = [a_{i-}, a^{i+}]$. For T2FS, each membership degree $\mu_{\tilde{A}}(x,u)$ is a type-1 membership function $u \in J_x$. Therefore, different x may have different membership function u, see Figure 2(b).

On the other hand, in FS, the membership degree $\mu_A(x)$ is a degree of an element x belonging to a fuzzy subset A, which implies that the non-membership degree of x belonging to A is equal to 1-$\mu_A(x)$. Considering the hesitation degree of an element x belonging to A, Atanassov proposed intuitionistic fuzzy set (IFS) [1], and Gau and Buehrer proposed vague set [9] through membership degree and non-membership degree respectively. Afterward, Bustince and Burillo proved that intuitionistic fuzzy set and vague set are equivalent [4]. The definition of IFS is as follows.

Definition 7. [1] *Let U be a universe of discourse, an intuitionistic fuzzy set A is an object of the form: $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in U\}$, where μ_A: $U \rightarrow [0,1]$ and ν_A: $U \rightarrow [0,1]$ are such that $0 \le \mu_A + \nu_A \le 1$, and $\mu_A, \nu_A \in [0,1]$ denote degrees of membership and non-membership of x belonging to A, respectively.*

Comparing the FS and IFS, we find that $\mu_A(x)+\nu_A(x)=1$ in FS, while in IFS, $\mu_A(x)+\nu_A(x)\leq 1$. The IFS is shown in Figure 2(c).

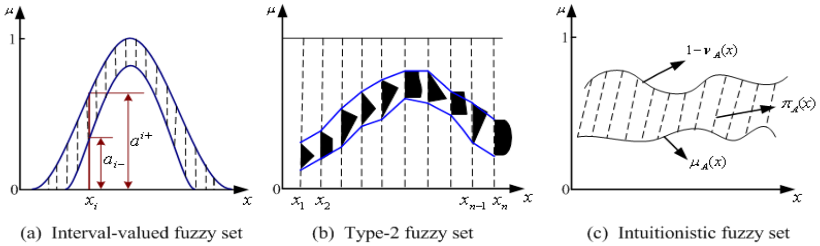

Fig. 2. Uncertainty expression of information

4 Uncertainty Expression in Rough Set Theory

Rough set (RS), proposed by Prof. Pawlak, uses the certain knowledge to depict the uncertain or imprecise knowledge from the perspective of knowledge classification [19], that is, it uses two certain sets (lower approximation set and upper approximation set) to define an uncertain set based on an equivalence relation. The definition of rough set is as follows.

Definition 8. [29] *Let $K=(U,\mathbf{R})$ be a knowledge base, the subset $X \subseteq U$ and the equivalence relation $R \in \mathbf{R}$ ($\mathbf{R}$ is a family of equivalence relation on U), then $\underline{R}X=\{x\in U|[x]_R \subseteq X\}$, $\overline{R}X=\{x\in U|[x]_R\cap X \neq \emptyset\}$, are called the $R-$lower approximation set and $R-$upper approximation set of X respectively.* $\mathrm{BN}_R(X)=\overline{R}X-\underline{R}X$ *is called the $R-$boundary region of X;* $\mathrm{Pos}_R(X)=\underline{R}X$ *is called the positive region of X, and* $\mathrm{Neg}_R(X)=U-\overline{R}X$ *is called the negative region of X. If* $\mathrm{BN}_R(X)=\emptyset$*, then X is definable, otherwise X is a rough set.*

A limitation of Pawlak rough set model is that the classification which it deals with must be totally correct or definite. Because the classification is based on the equivalence classes, its results are accurate, that is, "include" or "not include" certainly. To combat the question, some probabilistic rough set (PRS) models are introduced such as the 0.5 probabilistic rough set (0.5-PRS) model [20], the decision-theoretic rough set (DTRS) model [35], the variable precision rough set (VPRS) model [45], the Bayesian rough set (BRS) model [26], the Game-theoretic rough set (GTRS) model [10], and so on.

RS model is based on equivalence relations, and for each object, there is one and only one equivalence class containing it, then this equivalence class can be regarded as the neighborhood of this object, which constitutes the neighborhood system of this object. In general neighborhood system, the object may have two or more neighborhoods. Lin constructed rough set model based on neighborhood system by means of interior point and closure in topology [32]. It is a more generalized approximation set manifestation and also an extension of RS.

In many cases, the information systems are not complete, such as default attribute values. Thus, the rough set theory and method based on incomplete information systems has been extensively studied and developed [12][33].

In short, we can describe the relations among the above models in Figure 3.

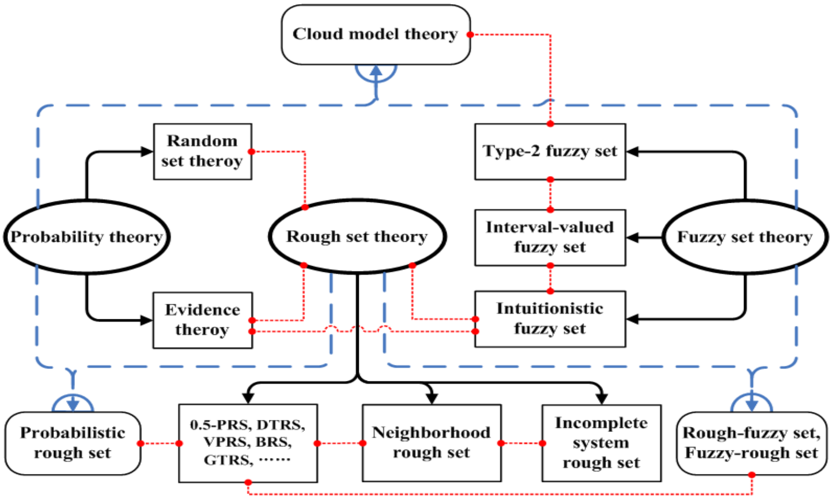

Fig. 3. The relationships between several uncertainty theories

In the foregoing discussion, probability theory, rough set theory and fuzzy set theory are three main uncertainty theories represented with elliptical shape in Figure 3. The random set theory and the evidence theory, IVFS, T2FS, IFS, 0.5-PRS, DTRS, VPRS, BRS, GTRS, the neighborhood rough set and incomplete system rough set are the extended models of probability theory, fuzzy set theory and rough set theory respectively, and they are expressed by rectangle shape. The probabilistic rough set, the rough-fuzzy set, the fuzzy-rough set and the cloud model are obtained by the combination of different theories, and they are expressed by rounded rectangle. The red dotted line expresses the connections among different extended models. The associations and differences between these models will be introduced and analyzed in detail in section 5.

5 Combination, Association and Difference between Different Extended Models

5.1 Probabilistic Rough Set Model

Pawlak RS is based on completion of available information, but the incompletion and statistical information of available information are ignored, so Pawlak RS is often powerless when processing the rule acquisition of inconsistent decision table. Some probabilistic rough set models were introduced to solve problems.

The DTRS was proposed by Yao et al. [35][36], which provides a novel rough set model for studying uncertain information system.

Definition 9. [35] *Let U be a universe of discourse, and R be an equivalence relation on U. A triple $A_p{=}(U,R,P)$ is a probabilistic approximation space, where a probability measure P defined on sigma algebra of subsets of U. In terms of conditional probability, $\forall X{\subseteq}U$, the lower and upper probabilistic approximations of X on parameters $\alpha,\beta(0{\le}\beta{<}\alpha{\le}1)$ are:* $\underline{\mathrm{P}}_{\alpha}(X){=}\{x{\in}U|P(X|[x]_R)\ \ {\ge}\alpha\}$, $\overline{\mathrm{P}}_{\beta}(X){=}\{x{\in}U|P(X|[x]_R){>}\beta\}$. *The corresponding positive region, boundary region and negative region are respectively:* $\mathrm{Pos}(X,\alpha,\beta){=}\underline{\mathrm{P}}_{\alpha}(X)$; $\mathrm{BN}(\mathrm{X},\alpha,\ \beta){=}\{x{\in}\ \ U|\beta{<}P(X|[x]_R){<}\alpha\}$; $\mathrm{Neg}(X,\alpha,\beta){=}\{x{\in}U|P(X|[x]_R){\le}\beta\}$. *If* $\mathrm{BN}(\mathrm{X},\alpha,\beta){\neq}\emptyset$, *then X is called probabilistic rough set on parameters α,β.*

In this context, each subset of U representing a random event is called a "concept". The conditional probability $P(X|[x]_R)$ can be interpreted as the probability that a randomly selected object with the description of concept $[x]_R$ belongs to X.

5.2 Fuzzy-Rough Set and Rough-Fuzzy Set Models

In the above mentioned various rough set models, the concepts and knowledge are all clear, that is, all sets are classical. However, it is mostly fuzzy concept and fuzzy knowledge that involve in people's actual life. There are two types reflected in rough set model, one is that knowledge of knowledge base is clear while the approximated concept is fuzzy, another is that knowledge of knowledge base and the approximated concept are all fuzzy. Based on this point, Dubois and Prade proposed rough fuzzy sets (RFS) model and fuzzy rough sets (FRS) model based on fuzzy set and rough set [8].

Definition 10. [8] *Let U be a universe of discourse, and R be an equivalence relation on U. If A is a fuzzy set on U, then $\forall x \in U$,* $\mu_{\underline{A}_R}(x){=}\inf\{\mu_A(y)|y \in [x]_R\}$ *and* $\mu_{\overline{A}_R}(x){=}\sup\{\mu_A(y)|y \in [x]_R\}$ *are called the membership functions of lower approximation fuzzy set $\underline{A}_R$ and upper approximation fuzzy set $\overline{A}_R$ respectively. If $\underline{A}_R{=}\overline{A}_R$, then A is definable, otherwise A is a rough fuzzy set.*

Definition 11. [8] *Let U be a universe of discourse, and $\mathscr{R}$ be a fuzzy equivalence relation on U. If A is a fuzzy set on U, then $\forall y \in U$,* $\mu_{\underline{A}_{\mathscr{R}}}(x){=}\inf\max\{1\text{-}\mu_{[x]_{\mathscr{R}}}(y),\mu_A(y)\}$, $\mu_{\overline{A}_{\mathscr{R}}}(x){=}\sup\min\{\mu_{[x]_{\mathscr{R}}}(y),\mu_A(y)\}$ *are called the membership functions of lower approximation fuzzy set $\underline{A}_{\mathscr{R}}$ and upper approximation fuzzy set $\overline{A}_{\mathscr{R}}$ respectively. If $\underline{A}_{\mathscr{R}}{=}\overline{A}_{\mathscr{R}}$, then A is definable, else A is a fuzzy rough set.*

According to Definition 10, if A is a classical set, then $\underline{A}_R$ and $\overline{A}_R$ are two classical sets. The difference between rough set and rough fuzzy set is whether the approximated concept is a classical set or a fuzzy set. Thereupon, the rough fuzzy set is natural generalization of rough set. From Definition 11, we can see that fuzzy rough set is a further expansion of rough fuzzy set due to the equivalence relation R transformed into fuzzy equivalence relation $\mathscr{R}$. In addition, the reference [23] also studied the fuzzy rough set.

5.3 Cloud Model

Cloud model, proposed by Prof. Li, studies the randomness of sample data and membership degree based on probability theory and fuzzy set theory [13], see Figure 3. A formalized definition is as follows.

Definition 12. [13] *Let U be a universal set described by precise numbers, and C be a qualitative concept related to U. If there is a number $x \in U$, which randomly realizes the concept C, and the membership degree μ of x for C is a random number with a stabilization tendency, i.e., $\mu : U \to [0,1], \forall x \in U, x \to \mu(x)$, then the distribution of x on U is defined as a cloud, and each x is a cloud drop.*

From Definition 12, the membership degree $\mu(x)$ of each cloud drop x is a random number, and all the cloud drops satisfy a certain distribution. The density of cloud drops expresses uncertainty degree of a concept C. Generally, a qualitative concept C is expressed by numerical characteristics (Ex, En, He), wherein, Ex is the most expected value of concept; En is used to figure its granularity scale; He is used to depict the uncertainty of concept's granularity. If the distribution of cloud drops is a normal distribution, then the corresponding cloud model is called a normal cloud.

Definition 13. [13] *Let U be a universal set described by precise numbers, and C be a qualitative concept containing three numerical characters (Ex, En, He) related to U. If there is a number $x \in U$, which is a random realization of the concept C and satisfies $x = R_N(Ex, y)$, where $y = R_N(En, He)$, and the certainty degree of x on U is $\mu(x) = \exp\{-\frac{(x-Ex)^2}{2y^2}\}$, then the distribution of x on U is a normal cloud. Where $y = R_N(En, He)$ denoted a normally distributed random number with expectation En and variance He^2.*

From Definition 13, we can depict an uncertain concept concretely. For example, let (Ex=25, En=3, He=0.3) express "Young", where, Ex=25 represents the expected age of "Young", and the corresponding normal cloud map is shown in Figure 4. The generated cloud drops have randomness (horizontal axis), at the same time, for each cloud drop x, the membership degree $\mu(x)$ also has randomness (vertical axis). That is, different people give different ages for "Young", such as 18, 18.5, 19, 20, 22, 28, 30, $\cdots$, namely these ages have stochastic to a certain extent, and each age may have different membership degree of belonging to "Young", take for 22 years example, $\mu(22)$ may equal to 0.3, 0.35, 0.4, 0.47, 0.51, $\cdots$. Thus, cloud model not only considers the randomness of concept, but also involves the randomness of membership degree of object or sample belonging to the concept.

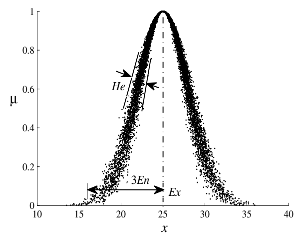

Fig. 4. Normal cloud

5.4 Association and Difference between Different Extended Models

From the above discussion, we know that probability theory, FS theory and RS theory have some extended models respectively, see Figure 3. The associations and differences of these extended models will be discussed.

(1) In FS theory, T2FS, IVFS and IFS are all the generalization of FS based on membership function. The integration of T2FS, IVFS and IFS can obtain some new models, such as interval-valued intuitionistic fuzzy set [2], interval-valued type-2 fuzzy sets [18], type-2 intuitionistic fuzzy set [44], etc. In rough set theory, VPRS and PRS loose the strict definition of approximate boundary. Compared with RS, the positive region and negative region will become larger, while the boundary region will be smaller in VPRS and PRS due to allowing error classification rate to some extent. In this sense, VPRS and PRS have some similar aspects [27]. In addition, the references [15][16][28] studied the variable precision fuzzy rough set and variable precision rough fuzzy set on the basic of VPRS, FRS and RFS, respectively. For the faults of FRS and VPRS, the reference [43] set up a model named fuzzy VPRS by combing FRS and VPRS with the goal of making FRS a special case. The reference [5] studied the vaguely quantified rough set model which is closely related to VPRS. The references [6] and [11] studied the ordered weighted average based FRS and robust FRS model respectively because the classical model of FRS is sensitive to noisy information. The reference [33] studied the rough set model and attribute reduction based on neighborhood system in incomplete system, and the reference [34] proved the VPRS and multi-granulation rough set model [21][22] are the special cases of neighborhood system rough set model and the neighborhood system rough set is a more generalized rough approach. According to the meanings of belief function and plausibility function, they are similarities with the lower and upper approximation of rough set. The references [25][37] discussed the relationship between them. In incomplete information systems, considering all possible values of the object attributes with incomplete information, then the values of some attribute are no longer a single point value but a set value. Based on this, the references [41][42] made the random set introduce into rough set theory and studied the rough set models based on random sets. The reference [41] discussed the relationships between random set, rough set and belief function. The above relations are shown in Figure 3.

(2) The similarities between evident theory, RS and IFS on the representation of uncertain information: The evidence theory depicts the uncertainty of information based on the belief function Bel and plausibility function Pl. IFS uses the membership degree and non-membership degree to study the fuzziness of information which is caused by the extension unclear. RS gives a characterization of uncertain information through lower approximation set $\underline{R}X$ and upper approximation set $\overline{R}X$ based on a equivalence relation R, and uses the roughness $\rho_R(X)=1-|\underline{R}X|/|\overline{R}X|$ to measure the uncertainty. From the aspects of decision-making, people usually perform three kinds of decision-making in our daily life according to the given information [31][38]: determine decision-making including acceptance decision and refusal decision, and delay decision (we can not make the

acceptance or refusal decision based on the current information, and additional information is required to make a decision). The evidence theory, RS and IFS are all able to describe the three decisions. In evidence theory, $Bel(A)$ expresses the degree of acceptance decision, $1-Pl(A)$ expresses the degree of refusal decision, and $Pl(A)-Bel(A)$ describes the degree of delay decision. In RS, the positive region $\mathrm{Pos}_R(X)$ and the negative region $\mathrm{Neg}_R(X)$ can be used to depict the acceptance decision and the refusal decision respectively, and the boundary region $\mathrm{BN}_R(X)$ depicts the delay decision. In IFS, membership function $\mu_A(x)$ and non-membership function $\nu_A(x)$ describe the degrees of acceptance decision and refusal decision respectively, and the hesitation degree $\pi_A(x)=1-(\mu_A(x)+\nu_A(x))$ describes the degree of delay decision. From this point of view, the three theories have common place in expressing of uncertainty information.

(3) The difference between cloud model and T2FS: T2FS discusses the fuzziness of membership degree using the type-1 fuzzy set. Once the membership function is determined, then it will be fixed. While the membership degree of a object belonging to uncertain concept is not a fixed value, but a random number with a stabilization tendency in cloud model. Thus, they have difference. In addition, cloud model considers the randomness of research object. In this sense, cloud model can well integrate the randomness and fuzziness of information.

6 Conclusions and Prospects

The paper summarizes the research on some uncertainty theory models and the corresponding extended models and discusses the associations and differences between them. But there are still some deficiencies, such as the countable additivity of probability may not be satisfied perfectly in practical applications due to uncertainty; how to determine the values of mass function and membership function objectively; the independence of evidence restricts the application range of evidence theory; RS theory does not take into account the randomness of sample data, which makes the generalization ability of acquired knowledge and rules be relatively low and so on. Thus, these problems will be further studied. In addition, because cloud model can deal with randomness and fuzziness, it will be a good issue, worth to study the combination of rough set and cloud model, and the reasoning mechanism, the combination rule of many uncertain concept, the automatically transformed method among multiple granularities based on cloud model are also urgent problems in the future research.

In recent years, computer and network technology advance rapidly. Along with the development of computer network and the widespread application of the Web technology, the data in database is becoming increasingly complicated. Incomplete information, inconsistent information, etc. are also getting more and more general. However, computer can only perform logic and four arithmetic operations essentially. If there are no good models and algorithms, it is still difficult to get the desired results even if there exists highly efficient large-scale computer. Thus, for the problem solving of large-scale complex systems, it needs more methodological innovations. Granular computing, deep learning, quantum coding and so on may be used to reduce system complexity.

Acknowledgments. This work is supported by National Natural Science Foundation of China under grant 61272060, Key Natural Science Foundation of Chongqing under grant CSTC2013jjB40003, and Chongqing Key Laboratory of Computational Intelligence(CQ-LCI-2013-08).

References

1. Atanassov, K.T.: Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems 20, 87–96 (1986)
2. Atanassove, K.T.: Interval Valued Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems 31(3), 343–349 (1989)
3. Baraldi, P., Compare, M., Zio, E.: Maintenance Policy Performance Assessment in Presence of Imprecision Based on Dempster-Shafer Theory of Evidence. Information Sciences 245, 112–131 (2013)
4. Bustince, H., Burillo, P.: Vague Sets are Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems 79, 403–405 (1996)
5. Cornelis, C., De Cock, M., Radzikowska, A.M.: Vaguely Quantified Rough Sets. In: An, A., Stefanowski, J., Ramanna, S., Butz, C.J., Pedrycz, W., Wang, G. (eds.) RSFDGrC 2007. LNCS (LNAI), vol. 4482, pp. 87–94. Springer, Heidelberg (2007)
6. Cornelis, C., Verbiest, N., Jensen, R.: Ordered Weighted Average Based Fuzzy Rough Sets. In: Yu, J., Greco, S., Lingras, P., Wang, G., Skowron, A. (eds.) RSKT 2010. LNCS, vol. 6401, pp. 78–85. Springer, Heidelberg (2010)
7. Dempster, A.P.: Upper and Lower Probabilities Induced by a Multivalued Mapping. The Annals of Mathematical Statistics 38(2), 325–339 (1967)
8. Dubois, D., Prade, H.: Rough Fuzzy Sets and Fuzzy Rough Sets. International Journal of General Systems 17, 191–209 (1990)
9. Gau, W.L., Buehrer, D.J.: Vague Sets. IEEE Transaction on Systems Man Cybernetics 23(2), 610–614 (1993)
10. Herbert, J.P., Yao, J.T.: Game-theoretic Risk Analysis in Decision-theoretic Rough Sets. In: Wang, G., Li, T., Grzymala-Busse, J.W., Miao, D., Skowron, A., Yao, Y. (eds.) RSKT 2008. LNCS (LNAI), vol. 5009, pp. 132–139. Springer, Heidelberg (2008)
11. Hu, Q., Zhang, L., An, S., Zhang, D., Yu, D.: On Robust Fuzzy Rough Set Models. IEEE Transactions on Fuzzy Systems 20(4), 636–651 (2012)
12. Leung, Y., Wu, W.Z., Zhang, W.X.: Knowledge Acquisition in Incomplete Information Systems: A Rough Set Approach. European Journal of Operational Research 168(1), 164–180 (2006)
13. Li, D.Y., Du, Y.: Artificial Intelligence with Uncertainty. Chapman and Hall/CRC, London (2007)
14. Li, D.Y., Liu, C.Y., Gan, W.Y.: A New Cognitive Model: Cloud Model. International Journal of Intelligent Systems 24, 357–375 (2009)
15. Mieszkowicz-Rolka, A., Rolka, L.: Variable Precision Fuzzy Rough Sets. In: Peters, J.F., Skowron, A., Grzymała-Busse, J.W., Kostek, B., Swiniarski, R.W., Szczuka, M.S. (eds.) Transactions on Rough Sets I. LNCS, vol. 3100, pp. 144–160. Springer, Heidelberg (2004)
16. Mieszkowicz-Rolka, A., Rolka, L.: Fuzzy Rough Approximations of Process Data. Internation Journal of Approximate Reasoning 49, 301–315 (2008)
17. Molchanov, I.S.: Theory of Random Sets. Springer, Berlin (2005)

18. Niewiadomski, A.: Interval-valued and Interval Type-2 Fuzzy Sets: A Subjective Comparision. In: 2007 IEEE Int. Conf. on Fuzzy Systems, London, pp. 1–6 (2007)
19. Pawlak, Z.: Rough Sets. International Journal of Computer and Information Sciences 11(5), 341–356 (1982)
20. Pawlak, Z., Wong, S.K.M., Ziarko, W.: Rough Sets: Probabilistic Versus Deterministic Approach. Information Sciences 29, 81–95 (1988)
21. Qian, Y.H., Liang, J.Y., Dang, C.Y.: Incomplete Multi-granulations Rough Set. IEEE Transactions on Systems, Man and Cybernetics-Part A 40(2), 420–431 (2010)
22. Qian, Y.H., Liang, J.Y., Yao, Y.Y., et al.: MGRS: A Multi-granulation Rough Set. Information Sciences 180(6), 949–970 (2010)
23. Radzikowska, A.M., Kerre, E.E.: A Comparative Study of Fuzzy Rough Sets. Fuzzy Sets and Systems 126(2), 13–156 (2002)
24. Shafer, G.: A Mathematical Theory of Evidence. Princeton University Press, Princeton (1976)
25. Skowron, A.: The Relationship Betweem the Rough Set Theory and Evidence Theory. Bulletiin of the Polish Academy of Sciences Mathematics 37, 87–90 (1989)
26. Ślęzak, D.: Rough Sets and Bayes Factor. In: Peters, J.F., Skowron, A. (eds.) Transactions on Rough Sets III. LNCS, vol. 3400, pp. 202–229. Springer, Heidelberg (2005)
27. Sun, B.Z., Gong, Z.T.: Variable Precision Probabilistic Rough Set Model. Journal of Northwest Normal University (Natural Science) 41(4), 23–26 (2005)
28. Tsang, E.C.C., Ma, W.M., Sun, B.Z.: Variable Precision Rough Fuzzy Set Model Based on General Relations. In: 2012 IEEE Int. Conf. on Machine Learning and Cybernetics, Xi'an, pp. 195–199 (2012)
29. Wang, G.Y.: Rough Set Theory and Knowledge Acquisition. Xi'an Jiao Tong University Press, Xi'an (2001)
30. Wei, L.L., Ma, J.H., Yan, R.F.: An Introduction to Probability and Statistics. Science Press, Beijing (2012)
31. Wu, W.Z., Leung, Y., Zhang, W.X.: Connections Between Rough Set Theory and Dempster-Shafer Theory of Evidence. International Journal of General Systems 31(4), 405–430 (2002)
32. Yang, X.B., Li, X.Z., Lin, T.Y.: First GrC Model–Neighborhood Systems the Most General Rough Set Models. In: 2009 IEEE Int. Conf. on Granular Computer, Beijing, pp. 691–695 (2009)
33. Yang, X.B., Yang, J.Y.: Incomplete Information System and Rough Set Theory and Attraibute Reduction. Science Press, Beijing (2011)
34. Yang, X.B., Yang, J.Y.: Rough Set Model Based on Neighborhood System. Journal of Nanjing University of Science and Technology 36(2), 291–295 (2012)
35. Yao, Y.Y., Wong, S.K.M., Lingras, P.: A Decision-theoretic Rough Set Model. In: Proceedings of the 5th International Symposium on Methodologies for Intelligent Systems, pp. 17–25. North–Holland, New York (1990)
36. Yao, Y.Y., Wong, S.K.M.: A Decision Theoretic Framework for Approximating Concepts. International Journal of Man-machine Studies 37(6), 793–809 (1992)
37. Yao, Y.Y., Lingras, P.J.: Inerpretations of Belief Functions in the Theory of Rough Sets. Information Sciences 104(1-2), 81–106 (1998)
38. Yao, Y.Y.: The Superiority of Three-way Decisions in Probabilistic Rough Set Models. Information Sciences 181(6), 1080–1096 (2011)
39. Zadeh, L.A.: Fuzzy Sets. Information and Control 8(3), 338–353 (1965)
40. Zadeh, L.A.: The Concept of a Linguistic Variable and Its Application to Approximate Reasoning-I. Information Sciences 8, 199–249 (1975)

41. Zhang, W.X., Wu, W.Z.: Rough Set Models Based on Random Sets(I). Journal of Xi'an Jiaotong University 34(2), 75–79 (2000)
42. Zhang, W.X., Wu, W.Z.: Rough Set Models Based on Random Sets(II). Journal of Xi'an Jiaotong University 35(4), 425–429 (2001)
43. Zhao, S., Tsang, E.C.C., Chen, D.: The Model of Fuzzy Variable Precision Rough Sets. IEEE Transactions on Fuzzy Systems 17(2), 451–467 (2009)
44. Zhao, T., Xiao, J.: Type-2 Intuitionistic Fuzzy Set. Control Theory & Applications 29(9), 1215–1222 (2012)
45. Ziarko, W.: Variable Precision Rough Set Model. Journal of Computer and System Sciences 46(1), 39–59 (1993)