# **Super-Resolution from One Single Low-Resolution Image Based on R-KSVD and Example-Based Algorithm**

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**Abstract.** With the development of sparse coding and compressive sensing, image Super-resolution (SR) reconstruction attracts extensive attentions. In this paper, we mainly focus on recovering super-resolution version given only one single low-resolution (LR) image. The proposed method is combined with the example-based algorithm, which also exploits the relationship between the low image patches and the high image patches. Firstly, the proposed method applies guided filter, the first-order and second-order derivatives to extract multiple features from LR images, which superior to using only one feature space. Then, the effective dictionary is constructed by a novel algorithm called Relaxation K-SVD (R-KSVD). R-KSVD relaxes the constraints of Orthogonal Matching Pursuit method (R-OMP) in training dictionary for K-SVD algorithm. Finally, a new approach is presented to estimating better HR residual image in the Back Projection. Experimental results demonstrate the superiority of our algorithm in both visual fidelity and numerical measures.

**Keywords:** Super-resolution, R-KSVD, sparse representation, guided filter.

## **1 Introduction**

Image super-resolution (SR) is currently an increasing active area of research in image processing. The SR problem aims to recover a high-resolution (HR) image from one or more low-resolution (LR) images. It is an inverse problem only under reasonable assumptions and prior knowledge conditions. Existing SR reconstruction algorithms [1-4] mainly include reconstruct-based methods and example-based methods. Reconstruct-ba[sed](#page-7-0) methods degrade rapidly if the magnification factor is too large or if there are not enough LR inputs to constrain the solution [1]. Example-based methods alleviate the difficulty above, which generate a dictionary or other mapping form to identify the relationship between LR and HR images. Then the corresponding HR images can be reconstructed via this relationship.

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However, in terms of SR problem from only one single LR image, those traditional methods demonstrate poor performances. To alleviate it, numerous single-image super-resolution algorithms [6], [13], [14] are proposed, which generate a visually pleasing HR image only from a given LR input image. Comparing with the traditional example-based algorithm, those algorithms learn the dictionary based on the image self-similarity property. They avoid the influence of extra database while maintaining comparable super-resolution image quality.

Motivated by Glasner [6] method, we propose a novel SR approach. At first, we render the dataset by the input LR image according to the self-similarity property. Then, the effective dictionary is constructed by a novel algorithm called Relaxation K-SVD (R-KSVD). Furthermore, in order to obtain better HR residual image, we improve the Back Projection algorithm. Simulational results demonstrate the superiority of the proposed algorithm in both visual fidelity and numerical measures.

This paper is organized as follows: section 2 gives a brief introduction of SR problem. A novel learning dictionary algorithm (R-KSVD) is presented in section 3. Section 4 discusses how to estimate better HR residual image in the Back Projection. The experimental results are shown in Section 5. Finally, the conclusions are provided in Section 6.

### **2 Super-Resolution from One Single Image**

Super-resolution remains extremely ill-posed because of many high-resolution images *X* satisfying the above reconstruction constraint. Only under reasonable assumptions and prior knowledge conditions, the SR problem is an inverse problem. We regularize the problem via the sparse representation prior on small patches *x* of *X* . The patches *x* of the HR image *X* can be represented as a sparse linear combination in a HR sparse dictionary  $D_{h} \in R^{N \times K}$ , namely:

$$
x \approx D_n \alpha \text{ for some } \alpha \in R^k \text{ with } ||\alpha||_0 \ll L \tag{1}
$$

where  $K$  represents the number of atoms in the dictionary,  $L$  is the sparsity constraint parameter.

Next, we will recover the HR image patch from the LR image patch. Here, the coupled dictionaries are learned by R-KSVD. We generate the image patch pairs  $P = \{x_h, y_f\}$  from one single LR image as like Glasner [6], here applies guided filter, the first-order and second-order derivatives to extract features  $\{y_\ell\}$ , and  $\{x_h\}$ . are the corresponding HR image patches. We combine these two objectives together to keep the consistent of two dictionaries, and then learn the coupled dictionaries by forcing the LR and HR sparse representations to operate the same codes, which can be formulated as follows:

$$
\min_{\{D_{\ell}, D_{\mu}, \alpha\}} \frac{1}{N} \|y^{\ell} - D_{\ell} \alpha\|_{2}^{2} + \frac{1}{M} \|x^{\mu} - D_{\mu} \alpha\|_{2}^{2} + \lambda (\frac{1}{N} + \frac{1}{M}) \|\alpha\|_{1}
$$
(2)

where, *M* and *N* are the dimensions of the low-resolution image features and high-resolution image patches in vector form. Equation (2) can be rewritten as

$$
\min_{\{D_{\ell}, D_{h}, \alpha\}} \left\| \hat{Z} - \hat{D}\alpha \right\|_{2}^{2} + \lambda(\frac{1}{N} + \frac{1}{M})\|\alpha\|_{1}
$$
\n(3)

where

$$
\hat{Z} = \begin{bmatrix} \frac{1}{\sqrt{N}} y^{\ell} \\ \frac{1}{\sqrt{M}} x^h \end{bmatrix}, \qquad \hat{D} = \begin{bmatrix} \frac{1}{\sqrt{N}} D^{\ell} \\ \frac{1}{\sqrt{M}} D^h \end{bmatrix}
$$
(4)

We can reconstruct the HR image by  $X \approx D_{\kappa} \alpha$ .

### **3 R-KSVD Algorithm**

Recently, numerous algorithms of dictionary learning mainly focus on training an over-complete dictionary in a single feature space. Many methods follow an iterative scheme that alternates between updates of  $\alpha$  and  $D$ . In the first phase it optimizes for  $\alpha$  while keeping *D* fixed, and in the second phase *D* is updated using the computed coefficient matrix  $\alpha$ . The iteration repeats until some stopping criterion is satisfied. A wide range of dictionary learning algorithms have been developed in the field of SR [11-12]. K-SVD algorithm is very popular for its high efficiency in dictionary learning, which updates the coefficient matrix  $\alpha$  by OMP in the first phase [10].

The main idea of the OMP algorithm is choosing the column of *D* by greedy iterative method, which makes the chosen column and the present redundant vector related to the greatest extent, and then subtracts the related part from measurement vector. The OMP repeats the procedure above until convergence condition meets. In terms of the greatest extent, the OMP algorithm may not obtain the optimal solution.

We propose a novel approach of updating coefficient matrix named R-OMP, which relaxes constrain of choosing the column of  $D$ . At first, we define the relaxation factor *a* . Our method randomly chooses one column of *D* from the candidate columns, which their values of the greatest extent are beyond *a* in training process. That is to say, we find slightly smaller values than the maximum value of the inner product of residual *e* and the columns of sensing matrix  $d_i$ , and then randomly choose one. This paper proposes a novel dictionary learning algorithm, which apply R-OMP to update coefficient matrix  $\alpha$  while keeping D fixed. The algorithm called Relaxation K-SVD (R-KSVD). The complete algorithm is described in Table 1. When we use K-SVD algorithm to obtain the dictionary, there are two schemes to control the precision of the algorithm. One is to constrain the representation error, and the other is to constrain the number of nonzero entries in the sparse representation coefficients. In R-KSVD method, we use the latter scheme because it is required in the R-OMP.

#### **Table 1.** The R-KSVD algorithm

Input: sampling vector  $y$ , the sparsity constraint parameter  $L$ , the relaxation factor *a* ; Output: the sparse matrix  $\alpha^*$ , effective dictionary  $D$ ; Initialization: initial dictionary *D*<sub>0</sub>, the residual  $e_0 = y$ , index set  $\Delta_{0} = \phi$ ,  $t = 1$ ; R-KSVD algorithm is mainly two steps: a. The sparse approximation step: given  $D$ , we estimate  $\alpha$ , using a sparse approximation algorithm by R-OMP; R-OMP: execute steps 1 to 5 until convergence: Step 1: find slightly smaller values (the value beyond *a* magnification of the maximum value)than the maximum value of the inner product of residual *e* and the columns of sensing matrix  $d_i$ , and then randomly choose one column from the candidates, the corresponding foot mark is  $\theta$ ; Step 2: renew the index set  $\Delta_t = \Delta_{t-1} \cup \{ \theta \}$ , the sensing matrix  $D_i = [D_{i-1}, d_i]$ ; Step 3: solve  $\alpha_i^* = \min \|y - D_i \alpha^*\|$ , by least-square method; Step 4: renew the residual  $e_t = y - D_t \alpha^*, t = t + 1;$ Step 5: if  $t > L$ , stop the iteration, else do step 1. b. The dictionary update step: use SVD to jointly re-estimate each atom and its nonzero coefficients to minimize the cost function (3).

### **4 Back Projection Algorithm**

In this section, the traditional Iterative Back Projection (IBP) approach is presented. This method is the backbone of the proposed algorithm. Though simple IBP method can minimize the restoration error significantly in iterative manner and give reasonable performance, it projected the error back without any edge guidance. In this paper, we apply guided filter to estimating better HR residual image in the Back Projection.

Our IBP procedure starts with the input LR image *Y* .The initial HR image *X* (0) can be recovered from the input LR image by the proposed SR algorithm. The simulated LR image  $Y^{(n)}$  is evaluated by the SR result, as shown in equation (5).

The final HR image is estimated by guided filter for edge preserving and back projecting the error between simulated LR image and the input LR image. The iteration repeats until some stopping criterion is achieved.

The estimated HR image after  $n$  iterations is given by:

a) The recovered *X* should be consistent with the input image*Y* . The simulated LR image  $Y^{(n)}$  can be viewed as a blurred and down sampled version of the HR  $X^{(n)}$ :

$$
Y^{(n)} = DHX^{(n)}\tag{5}
$$

where *H* represents a blurring filter, and *D* the down sampling operator. b) Computing the error from LR images as

$$
E^{(n)} = (Y - Y^{(n)}) U^S
$$
 (6)

where,  $U^s$  is up-sampling by factor *S*, *Y* is initial input LR image,  $Y^{(n)}$  is simulated LR image of the nth iteration,  $E^{(n)}$  is error estimation.

c) Updating the HR image

$$
X^{(n)} = X^{(n-1)} + E^{(n)}
$$
 (1)

 $(7)$ 

$$
X^{(n)} = guidedfilter(X^{(n)}, X^{(n)}, r, \gamma)
$$
\n(8)

where, *r* is a radius of a square window, *y* is the regularization parameter,  $X^{(n)}$  is HR image of the nth iteration. The iteration repeats until some stopping criterion is achieved. The illustration of the complete method is described in Fig 1.



**Fig. 1.** The process of back projection: firstly, we up-sample the input image by the proposed SR algorithm as the original HR image; then we calculate the error between the input image and the simulated LR image, and add it to the HR image; finally, we obtain the further HR image by guided filter. It repeats the procedure above until iterations meets.

# **5 Experimental Results**

In simulations, we magnify the low-resolution test image by a factor 4 in image super-resolution experiments. By experimental results, it is fit to define the relaxation factor as  $a = 0.9$ . We define the square window of a radius  $r = 2$  and regularization parameter  $\gamma = 0.01$  as commonly used in the literature [8], and set sparse parameter  $\lambda = 0.1$  as Yang. [15]. Firstly, we extract the features of low-resolution image by guided filter, the first-order and second-order derivatives. Then the image database from different scales and directions is generated [6]. We use  $2 \times 2$  lowresolution image patches with overlap of one pixel between adjacent image patches in the low-resolution images. Totally 1024 dictionary atoms are learned with each atom of size  $2 \times 2$ .

We apply our method to the luminance channel for color images, because humans are more sensitive to luminance space in visual sense. We interpolate the color layers (Cb, Cr) using bicubic (Bic) interpolation. Numerous natural low-resolution images are simulated, but we only show five images in Fig 2.We apply different algorithms including nearest neighbor (NN) interpolation, bicubic interpolation, Glasner algorithm [6] and our proposed algorithm to reconstruct the HR image. The experimental results are compared from two aspects, RMSE and visual effect. From the Table 2 and Fig 3, we can conclude that our algorithm is superior to the other algorithms in both visual fidelity and numerical measures. What's more the proposed SR method reduces the run time to some extent.



**Fig. 2.** Five low-resolution test images. From left to right: building; raccoon; girl; visual chart; flower

<b>RMSE</b>	building	raccoon	girl	visual chart	flower
<b>NN</b>	23.1935	10.3028	6.4315	38.5656	4.2923
Bic	19.106	8.9977	5.2137	29.4436	3.1928
Glasner [6]	18.2757	8.8952	4.9755	19.4362	2.9632
Our algorithm	18.1713	8.3894	4.8242	19.3914	2.9015
Time $cost(s)$					
Glasner [6]	776.73	2634.59	1875.23	779.43	1472.96
Our algorithm	763.66	2134.69	1400.26	755.05	1302.54

**Table 2.** Comparison results of the RMSE, and Run time cost of the different algorithms



**Fig. 3.** SR results of building, raccoon, girl, visual chart, flower. (a) a LR image; (b) NN result; (c)Bic result;(d)Glasner result;(e)our result;(f)original image.

# **6 Conclusions**

In this paper, we propose a new method for image super-resolution from only one LR input image. Our algorithm reduces the reconstruction error significantly and avoids the influence of extra database while maintaining comparable image quality. Experimental results demonstrate the superiority of the presented method. In the future, we will consider how to extract more effective features and how to reduce the computation time in further.

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# <span id="page-7-0"></span>**References**

- 1. Yang, J., Wright, J., Huang, T., Ma, Y.: Image super-resolution as sparse representation of raw image patches. In: IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pp. 1–8 (2008)
- 2. Goto, T., Suzuki, S., Hirano, S.: Fast and High Quality learning Based Super-Resolution Utilizing TV Regularization Method. In: IEEE International Conference on Image Processing, pp. 1185–1188 (2011)
- 3. Tang, Y., Pan, X.L., Yuan, Y., Yan, P.K., Li, L.Q., Li, X.L.: Local Semi-Supervised Regression for Single-Image Super-Resolution. In: Multimedia Signal Processing (MMSP), pp. 1–5 (2011)
- 4. Yang, S., Wang, M., Chen, Y., Sun, Y.: Single-Image Super-Resolution Reconstruction via Learned Geometric Dictionaries and Clustered Sparse Coding. IEEE Transactions on Image Processing, 4016–4028 (2012)
- 5. Jiang, J.J., Hu, R., Han, Z., Huang, K., Lu, T.: Efficient Single Image Super-Resolution Via Graph Embedding. In: IEEE International Conference on Multimedia and Expo, pp. 610–615 (2012)
- 6. Glasner, D., Bagon, S., Irani, M.: Super-Resolution from a Single Image. In: IEEE 12th International Conference on Computer Vision, pp. 349–356 (2009)
- 7. Chen, S.F., Gong, H.J., Li, C.H.: Super resolution from a single image based on selfsimilarity. In: Computational and Information Sciences (ICCIS), pp. 91–94 (2011)
- 8. He, K., Sun, J., Tang, X.: Guided Image Filtering. In: Daniilidis, K., Maragos, P., Paragios, N. (eds.) ECCV 2010, Part I. LNCS, vol. 6311, pp. 1–14. Springer, Heidelberg (2010)
- 9. Tibshirani, R.: Regression shrinkge and selection via the Lasso. J. Royal Statist., 267–288 (1996)
- 10. Rubinstein, R., Peleg, T., Elad, M.: Analysis K-SVD. IEEE Transactions on Signal Processing & Analysis, 1–17 (2012)
- 11. Zhou, F., Yang, W.M., Liao, Q.M.: Single image super-resolution using incoherent subdictionaries learning. IEEE Transactions on Consumer Electronics, 891–897 (2012)
- 12. Yang, J., Wang, Z., Cohen, Z.S., Huang, T.: Coupled Dictionary Training for Image Super-Resolution. IEEE Transactions on Image Processing, 3467–3478 (2012)
- 13. Kim, K., Kwon, Y.: Single-image super-resolution using sparse regression and natural image prior. IEEE Transactions on Pattern Analysis and Machine Intelligence, 1127–1133 (2010)
- 14. Tang, Y., Yan, P., Yuan, Y., Li, X.: Single-image super-resolution via local learning. International Journal of Machine Learning and Cybernetics, 15–23 (2011)
- 15. Yang, J., Wright, J., Huang, T., Ma, Y.: Image super-resolution via sparse representation. IEEE Trans. Image Process, 2861–2873 (2010)